5107

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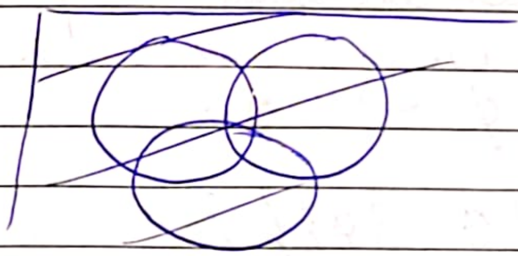
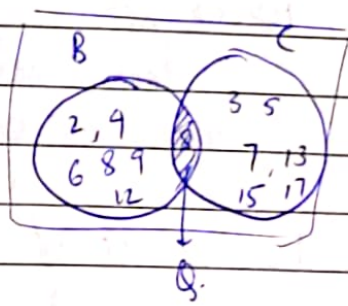
Discrete Mathematics

Q.) Show Venn diagram for the following
$A - ((B \cap C) \cup D)$

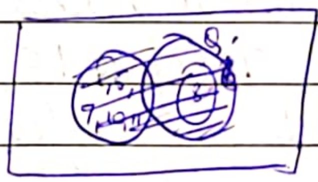
$A = \{1, 3, 5, 7, 8, 9, 10, 11, 13, 15\}$
$B = \{2, 4, 6, 8, 9, 12\}$
$C = \{3, 5, 7, 8, 13, 15, 17\}$
$D = \{1, 5, 7, 10, 11\}$



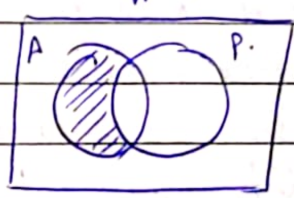


$B \cap C = \{8\}$
$P = (B \cap C) \cup D \Rightarrow \{1, 5, 7, 8, 10, 11\}$
$A - P = \{3, 9, 13, 15\}$

P =

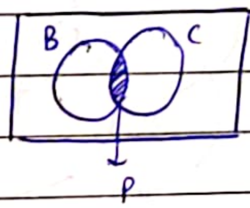


A−P.

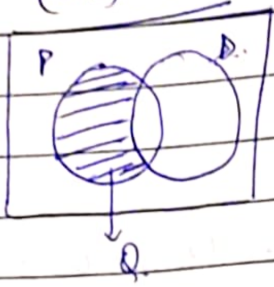


b) $A - ((B \cap C) - D)$ (No data).

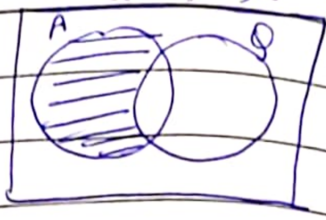
$B \cap C$



$(B \cap C) - D$



$A - ((B \cap C) - D)$.



⇒ Empty set : represented by ϕ or $\{\}$.

NOTE : $\{\phi\}$ is a singleton set.

⇒ Singleton set : set with just 1 element.

⇒ Power set : set containing all possible sub-sets.

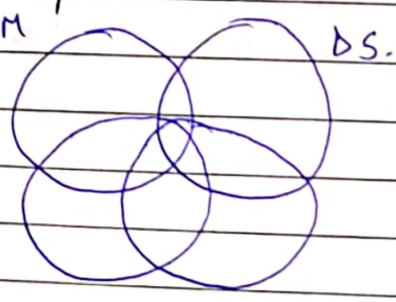
Power set allways contains ϕ & that set.

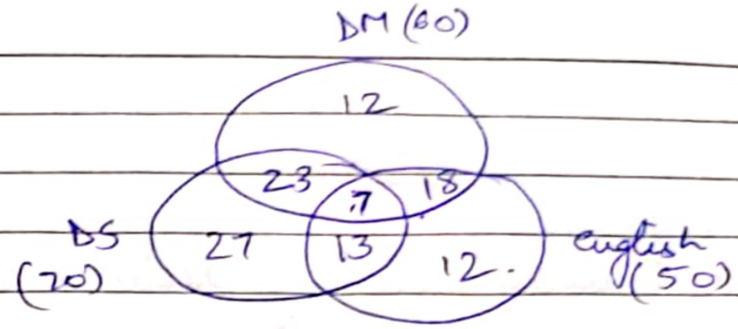
Power set of ϕ = $\{\phi\}$.

Q. Power set of $\{\phi, \{\phi\}\}$?

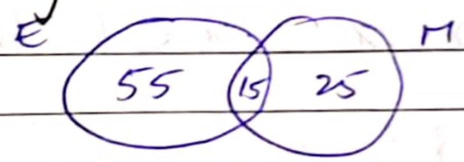
$\{\phi, \{\phi\}, \{\phi, \{\phi\}\},$
$\{\{\phi\}\}\}$.

Q.). Suppose there are 130 students in a class. Out of them 60 opted Discrete mathematics, 70 opted DS, 50 opted english, 30 opted both DM & DS, 20 opted DS & english 25 opted english & DM. Find number of student not opted any thing. (7 opted all 3).



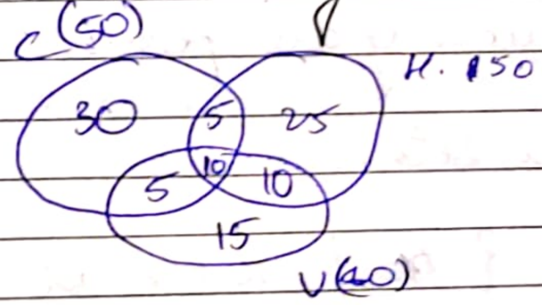


In a class 40% of student enrolled for maths and 70% enrolled for eco. If 15% enrolled for both, what % of students of class did not enroll in anything



5 %.

Among a group of students, 50 played cricket, 15 played hockey, 40 played volleyball, 15 played cricket & hockey. 20 played hockey & volleyball, 15 played cricket & volleyball, 10 played all 3. If every student played atleast 1 game, find number of students?



Unit 1
Sets.

Collection (unordered) of distinct elements is sets.

Subset

$A \subseteq B$ A is a subset of B.

if $\forall x \in A \Rightarrow x \in B$.

Strict subset / Proper Subset:

$A \subset B$ A is proper subset of B

if $\exists$ (for some) x, $x \in B$ but $x \notin A$.

Null set (ϕ or $\{\}$)

Set that contains no element.

Singleton Set

set that contains a single element

Power set

Set of collection of all possible subsets.

* Operations on Sets

Union

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Intersection

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Subtraction

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Compliment

A' or $A^c = \{x \mid x \in \text{Universal set and } x \notin A\}$

Cartesian product

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

$$N(A \times B) = N(A) \times N(B)$$

No. of elements in a set are called cardinality of set

Identities

Identity Law.

$A \cap U = A$.

$A \cup \phi = A$.

B. Domination Law.

$A \cup U = U$

$A \cap \phi = \phi$

Complementation law.

$(A^c)^c = A$.

Commutative Law.

$A \cup B = B \cup A$.

$A \cap B = B \cap A$.

5) Associative Law.

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

6) Distributive Law.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

7) De Morgan's Law

$\cancel{\overline{A \cup B} \Rightarrow \overline{A \cap B}}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\cancel{AA}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$

8) Absorption law

$$A \cup (B \cap A) = A.$$

$$A \cap (B \cup A) = A.$$

Q1) Prove that for all sets S and T, $S = (S \cap T) \cup (S - T)$

using distributive law.

$$S = (S \cup (S-T)) \cap (T \cup (S-T))$$
$$= S \cap (T \cup (S-T)).$$

Let $x \in S$.

Case I: $x \in T$.

$\Rightarrow$ $(S \cap T)$ contains x.

$\Rightarrow$ S will be a subset of RHS.

Case II: $x \notin T$.

$\Rightarrow$ $(S \cap T) \not\ni x$

However

$x \in (S-T)$

on taking union, x will be there

$\Rightarrow$ S will be subset of RHS. —— 1

Now.

Let : $x \in (S \cap T) \cup (S-T)$ — P

Case I: $x \in (S \cap T)$

this means

$x \in S$ & $x \in T$.

$\Rightarrow$ P is subset of S.

Case II: $x \in (S-T)$.

$\Rightarrow$ $x \in S$ but $x \notin T$.

$\Rightarrow$ P is a subset of S. —— 2

from 1 & 2.

$P = S$.

Q2) Prove Absorption law.
$A \cup (A \cap B) = A$.

$A \cup (A \cap B)$
using distributive law.

~~$(A \cup A)$~~
$= (A \cup A) \cap (A \cup B)$

$= A \cap (A \cup B)$

Since $(A \cup B)$ will contain all elements of A.
$\Rightarrow$ on taking intersection with A.
we will get A.
$= \underline{\underline{A}}$.

Q3) Prove Demorgan's law.
$\overline{A \cup B} = \bar{A} \cap \bar{B}$

Let
$x \in \overline{A \cup B}$
$\Rightarrow x \notin (A \cup B)$ i.e. $x \notin A \cup B$
i.e. $x \notin A$ and $x \notin B$

$\Rightarrow x \in \bar{A}$ and $x \in \bar{B}$

$\Rightarrow x \in \bar{A} \cap \bar{B}$

$\Rightarrow \overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$ —(1)

Now. Let $x \in \overline{A} \cap \overline{B}$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow x \notin A$ and $x \notin B$.

$\Rightarrow x \notin A \cup B$.

$\Rightarrow x \in \overline{A \cup B}$

$\Rightarrow \overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$ — ②

from ① & ②

LHS = RHS

* Relations

$R = \{(a,b) \mid R \subseteq A \times B$ where $a \in A$, $b \in B\}$.

Properties

1) Reflexive

$\forall a \in A$, $(a,a) \in R$.

(only for $R \subseteq A \times A$)

2). Symmetric

if a is related to b.

$\Rightarrow$ b must be related to a.

3). ~~Ir~~ Antisymmetric

if a is related to b
and b is related to a.
$\Rightarrow a = b$.

4). Asymmetric
if a is related to b,
b should not be related to a.
i.e. (1, 1) is not asymmetric.

5) Transitive
if a is related to b,
b is related to c,
then, a should be related to c.

A = {1, 2, 3}
R = {(1,2), (2,1), (1,1), (3,1)}
NOT Transitive

6) Equivalence

Equivalence Relation
A relation R is said to be equivalence relation if it follows all 3 properties :- reflexive, symmetric and transitive.

Q1) Let R be drawn over A×A. It hold that
$R = \{ (a,b) \mid (a \leq b) \}$. ~~NEA~~. A = {1, 2, 3, 4}.
Find the relation R & check its equivalence.
A = {1, 2, 3, 4}.
R = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3)
(3,4), (4,4)}.

Q. Classify the following as reflexive, symmetric, transitive, anti symmetric, asymmetric & irreflexive.

i) $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$

$A = \{1, 2, 3\}$.

Reflexive; Antisymmetric, transitive

Q) ii) $R = \{(3,3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,6), (3,9), (3,12)\}$. $A = \{3, 6, 9, 12\}$

reflexive transitive, Anti symmetric

iii) $R = \{(3,12), (3,6), (3,15), (9,6), (6,12), (6,15), (15,12), (12,6), (3,9)\}$. $A = \{3, 6, 9, 12, 15\}$

irreflexive ~~asymmetric~~

Q Let $R \to Z$: by $\{(x, y)\} \in R$ if $x - y = 10$.

$R = \{(10,0), (11,1) \ldots (\infty, \infty - 10)\}$.

asymmetric, irreflexive.

Q.)

1.

2.

i)

ii)

Check for equivalence.

$A = \{a, b, c, d, e, f\}$.

$R = \{(a,a), (b,b), (a,b), (b,a), (c,c), (d,d), (c,d), (d,d), (c,d), (d,c), (e,e), (f,f), (e,f), (f,e)\}$.

Reflexive ✓

symmetric ✓

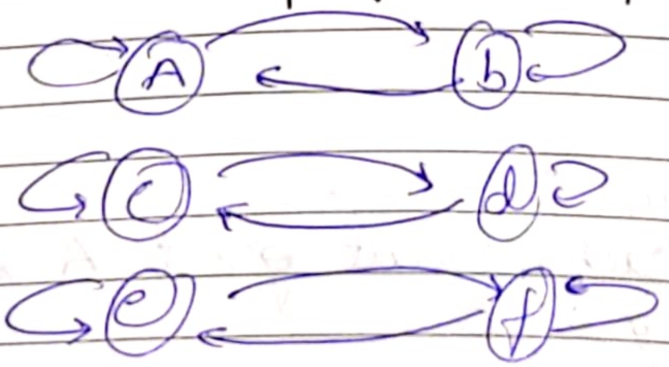
transitive ✓

] equivalence relation.

Graphical Representation

Draw 1 vertex for each element in A.

Draw edges for each pair.



for an equivalence Relation we get a complete graph.

Partition

A partition Π of a non-empty set A is a collection of non empty subsets of A such that $\forall$

$\forall \; s, t \in \Pi$

either, $s = t$ or $s \cap T = \phi$.

Union of all subsets of $\Pi = A$.

eg = A = {1, 3, 5, 7, 9, 11, 13}.

then

π = {{1, 3}, {5, 7, 9}, {11, 13}}.

* Function *

Let A and B, be 2 sets. A Function from A to B denoted by :-

$$f : A \xrightarrow{\text{domain}} B \xrightarrow{\text{codomain}}$$

is a relation from $A \rightarrow B$ such that for every $a \in A$, there exists a unique ~~B~~ $b \in B$.

eg $f : N \rightarrow N$.

$f(x) = 1$ $x \in$ odd.

$f(x) = x/2$ $x \in$ even

g(f(x)).

Type.

1. One-to-one Mapping injective.

Function for which each element of set A is mapped to a unique element of set B. Q.

eg

2. Onto mapping / Surjective.

~~For~~ Every element in ~~do~~ co-domain must have a pre-image in domain. Q.

3. Bijective

A ~~Relation~~ relation is said to be bijective if it is both surjective ~~or~~ ~~to~~ and injective. Q.

Many to One mapping

If some of the elements in co-domain have more than 1 pre-image.

~~Funct~~ Composition of function

Function composition is a operation that takes 2 function f and g and produces a third function h such that

$$h(x) = g(f(x)).$$

eg if ~~f~~ $f(x) = \{(1,3), (2,1), (3,4), (4,6)\}.$
$g(x) = \{(1,5), (2,3), (3,4), (4,1), (5,3), (6,2)\}$

$$gof(x) = \{(1,4), (2,5), (3,1), (4,2)\}.$$

$f : R \to R \Rightarrow f(x) = 2x + 4.$
$g : R \to R \Rightarrow g(x) = x^3$
] Find $gof(x)$ & $fog(x)$

~~g(x~~ $gof(x) = (2x+4)^3$
$fog(x) = 2x^3 + 4$

$f(x) = \sqrt[3]{x-4}$ $\qquad g(x) = x^3 + 4$

$fog(x) = ?$

$fog(x) = x.$

$f(x) = 2x + 4$ $\qquad g(x) = \cancel{\sqrt[3]{x-4}}\ x^3 \cdot h(x) = x^3 + 4$

$fogoh(x) = ?$

$\cancel{goh(x)} = x.$

$$(f\circ g(x))^{-1} = (g^{-1}\circ f^{-1})(x)$$

__/__/____

$$g\circ h(x) = (x^3+3)^3$$
$$f\circ g\circ h(x) = 2(x^3+3)^3 + 4$$

2.

* Proposition Logic

Proposition is a declarative statement which can have either true value or false value but not both.

3.

eg

$2+3=5$	True	✓
$2+3=9$	True	✓
$x+1=5$	maybe	✗.
This is a book.	maybe	✗.
This is a wrong statement	✗	(Lier's para(-dox)).
Answer this question!	✗	not a declarative stmt

4.

Logic Operators

#

1. Conjunction ($\wedge$ or AND)

p	q	$p\wedge q$
0	1	0
1	0	0
1	1	1
0	0	0.

$0 \rightarrow F$

$T \leftarrow 1$

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Disjunction operator (v- OR)

p	q	p ∨ q
F	F	F
T	F	T
F	T	T
T	T	T.

If both proposition p & q are false, answer is false else answer is True.

Negation operator (~ or ¬)

p	~p
T	F
F	T.

exclusively OR (⊕)

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

Conditional operator.

Let p & q be 2 propositions then conditional operator is represented by $p \to q$ and infered as "if p then q." It will have only false if p is true and q is false

p	q	$p \to q$
T	T	T
T	F	F.
F	T	T
F	~~T~~ F	T

⇒ $p \to q$ can be expressed as

if p then q p implies	if p, q p only if q	p is sufficient for q q if p	q unless negation p.

Q. Draw Truth Table for $\sim p \vee q$.

p	q	$\sim p \vee q$
T	T	T
T	F	F
F	T	T
F	F	T.

6. Bi-directional Condition ($p \leftrightarrow q$)

Infered as "p if and only if q". Value is true if both proposition contain same value.

equivalent to $(p \to q) \wedge (q \to p)$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T.

p = male
q = ↓4
x = >16.

Q. For a real x, $f(x) = 5x + 1 + x^3$. Find preimage of 0.

pre image b/w $-1, 0$.

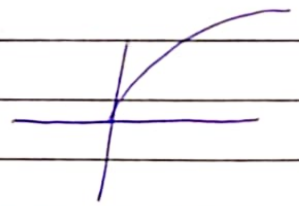
Q. b) $x^2 - 1$

quadratic, many-one
into.

c) $f(x) = \sqrt{x}$.

$y^2 = x$.

one-one; onto.



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Write down proposition for following. -

a) You can access the internet from campus only if you are a CS major or not a freshman

$$(\text{access internet}) \longrightarrow ((\text{CS major}) \vee \sim(\text{freshman}))$$

$$A \longrightarrow (B \vee (\sim C))$$

b) You cannot ride the roller coaster if you are under 4ft tall unless you are older than 16yr old.

p ⇒ ride roller coaster
q ⇒ < 4 ft tall
r ⇒ older than 16.

$$\boxed{\sim(\sim q \vee r) \longrightarrow \sim p}$$

$$A_2 \quad p \longrightarrow (r \vee (\sim q))$$

~~$\sim p \Rightarrow (q \Rightarrow r)$~~ ~~$\sim p \Rightarrow (q \Rightarrow r)$~~

p	q	r	A_1	A_2
T	T	T	T.	T.
T	T	F	F	F
T	F	T	T	T.
T	F	F	T.	T.
F	T	T	T.	T
F	T	F	T.	T
F	F	T	T	T
F	F	F	T.	T.

__/__/__

* NOTE

There are some other ways to express are: $p \rightarrow q$:

→ If p then q. → q when p.
→ p implies q. → q unless not p
→ If p, q. → a sufficient condition for q is p
→ p only if q. → q whenever p.
→ p is sufficient for q. → q is neccessary for p.
→ q if p → q follows from p.

⇒ for bidirectional.

→ p is neccessary and sufficent for q
→ if p then q and converely
→ p if and only if q.

Q. Write proposition using p and q and logical connective for following
p = it is below freezing
q = it is snowing

1) it is below freezing & snowing
2) it is " " but not snowing
3) it is not below " and it is not snowing
4) it is either snowing or below freezing or both
5) if it is below freezing, it is also snowing
6) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing
7) it is below freezing is neccessary and suffcient for it to be snowing

1) $p \wedge q$
2) $p \wedge (\sim q)$
(3) $(\sim p) \wedge (\sim q)$
(4) $p \vee q$
(5) $p \rightarrow q$
(6) $(p \oplus q) \wedge (p \rightarrow (\sim q))$
(7) $p \leftrightarrow q$

*

AND

OR

XOR

*

#

#

Q.

a)

contra

__/__/__

* BITWISE Operation

A = 1 0 1 1 0 1 0
B 0 1 1 0 1 1 0.

0 0 1 0 0 1 0

1 1 1 1 1 1 0.

1 1 0 1 1 0 0.

Converse of a Conditional statement

Let condition statement is $p \rightarrow q$

converse will be $q \rightarrow p$.

Contrapositive

Let conditional statement be $p \rightarrow q$.

Contrapositive statement $\Rightarrow$ $\sim q \rightarrow \sim p$

~~negateinverse~~ + converse.

Inverse

Let conditional statement be $p \rightarrow q$

inverse = $\sim p \rightarrow \sim q$.

What are contrapositive converse & inverse of:-

Home team wins whenever it is raining.

~~if it is not raining, the home team doesn't~~ whenever

if the home team doesn't win, then it is not rainin

__/__/__

Convers — If home team wins, it is raining

Inverse — ~~to~~ If it does not rain home team does not win.

Q. Design True Table.

1). $(p \to q) \to (q \to p)$

2) $(p \oplus q) \to (p \wedge q)$

3) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

4) $(p \oplus q) \to (p \oplus \neg q)$

5) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

6) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg p) \wedge p$

1)

p	q	$p \to q$	$q \to p$	A
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T.

2)

p	q	$p \oplus q$	$p \wedge q$	A
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F.	F	T.

3)

p	q	r	$p \leftrightarrow q$	$\sim p \leftrightarrow \sim r$	A
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	T	F	T
F	F	F	T	T	F

p	q	$p \oplus q$	$\rightarrow$	$p \oplus \sim q$	A
T	T	F		T	T
T	F	T		F	F
F	T	T		F	F
F	F	F		T	T

p	q	$p \leftrightarrow q$	$\oplus$	$\sim p \leftrightarrow q$	A
T	T	T		F	T
T	F	F		T	T
F	T	F		T	T
F	F	T		F	T

p	q	$p \leftrightarrow q$	$\oplus$	$(p \leftrightarrow \sim p)$	$\wedge p$	A
T	T	T		F	F	T
T	F	F		F	F	F
F	T	F		F	F	F
F	F	T		F	F	T

Tautology & fallacy

A compound proposition is said to be tautology if all its truth values are True for any truth value associated with its propositional variables.
If its false for all possible ~~tt~~ truth values of its propositi-onal variable it is Contradiction / fallacy.

If a proposition is neither fallacy nor Taulology is Contigency

Logical Equivalency

2 compound propositions p & q are said to be logically equivalent if $p \leftrightarrow q$ is a taulology.

Prove logicall Equivalence.

Precedence

$\sim, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$

$(p \vee (q \wedge r))$ & $(p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$	A
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

Tautology

Hence logically Equivalent.

Q. b) $(p \leftrightarrow p \wedge q) \equiv (p \rightarrow q)$

p	q	$p \wedge q$	LHS	RHS	$\equiv$ A
T	T	T	T	T	T
T	F	F	F	F	T
F	T	F	T	T	T
F	F	F	T	T	T

Hence logically equivalent.

Logical Equivalence Rules

1) Identity Law.

$A \wedge T \equiv A$

$A \vee F \equiv A$

2) Domination Law.

$A \wedge F \equiv F$

$A \vee T \equiv T.$

Idempotence

$$p \wedge p = p$$
$$p \vee p = p.$$

Double negation Law

$$\sim(\sim p) \equiv p.$$

Commutative Law

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

Associative law.

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Distributive Law.

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

De-Morgan's Law.

$$\sim(p \wedge q) \equiv \sim p \vee \sim q.$$
$$\sim(p \vee q) \equiv \sim p \wedge \sim q.$$

Absorption Law.

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

For conditional operators

1) $p \to q \cong \sim p \vee q$.

2) $p \to q \cong \sim q \to \sim p$

3) $p \vee q \cong \sim p \to q$. | $p \wedge q \cong \sim(p \to \sim q)$

4). $\sim(p \to q) \cong p \wedge \sim q$

5) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

6) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

7) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

8) $(p \to r) \vee (q \to r) = (p \wedge q) \to r$

9) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

10) $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

11) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

12) $\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$.

Q. Show that $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$

$\sim((p \vee (\sim p)) \wedge (p \vee q))$.

$\sim(T \wedge (p \vee q))$

$= \cancel{\sim(p \vee p \quad \sim((p \wedge p) \vee (p \wedge q)))}$

$= \sim p \wedge \sim q$

Q. $p \leftrightarrow p \wedge q \equiv p \rightarrow q$.

$\cancel{p \leftrightarrow (p \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow p)}$.

$\cancel{(p \wedge (p \rightarrow q)) \wedge (p \vee p(q \rightarrow p))}$.

$\cancel{\Rightarrow (p \wedge (p \wedge q)) \vee (\sim p \wedge (\sim p \vee \sim q)))}$.

$\cancel{\Rightarrow (p}$

$= (p \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow p)$.

$= (\sim p \vee (p \wedge q)) \wedge ((\sim p \vee \sim q) \vee p)$

$= (T \wedge (\sim p \wedge q)) \wedge (T)$

$= \underline{(\sim p \wedge q)}$

__/__/__

Q. Let 2 kinds of ~~inhabited~~ inhabitant in an island ~~there are~~ ~~1)~~ knights who always tell truth and their opposites knaves who always lie. You encounter 2 pupil A & B. What are A & B if A says B is a knight & B says the 2 of us are of ~~diff~~ opposite ~~tf~~ type?

A B $p \to$ B is knight
$q \to$ both are opposite type.

A	B	Ans
N	N	F
N	K	F
K	N	F
K	K	T

* Predicates & Quantifiers

Propositional Satisfiability

A compound proposition is satisfiable if there is a assignment of truth values to its variables that makes it true.

When no such arrangement exists, the compound proposition is false for all arrangement of truth values, it is unsatisfyable.

eg $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

p	q	r	A
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

$\rightarrow$ satisfiable

1).
2)
*
Q.
#

DUAL

$\wedge \to \vee$ | $T \to F$ | negation DO NOT change. __/__/____
$\vee \to \wedge$ | $F \to T.$ |

Quantifiers are used to optimise predicates.

Types:-

1. Universal Quantifier (forall $\forall$)
Existential Quantifier ($\exists$)

eg some students in class opted DM.
$x \to$ students ^(in class)^ opted DM.
$\exists x : P(x)$ is proposition.

eg.

Variables bounded by some quantifiers $\Rightarrow$ bounded variable
Variables not bounded by quantifier $\Rightarrow$ free variable.

Logical Equivalence involving Quantifiers.

$$\forall x \, (P(x) \wedge Q(x)) \equiv \forall x \, P(x) \wedge \forall x \, Q(x).$$

Let a be a element in Domain.

$\rightarrow \forall a \; (\underbrace{P(a)}_{\text{TRUE}} \wedge \underbrace{Q(a)}_{\text{TRUE}})$

$\Rightarrow \forall a \; P(a) \to T$
$\forall a \; Q(a) \to T.$] T.

Negation of Quantified statement.

$$\sim (\forall x \, (P(x))) \equiv \exists x \sim P(x)$$

~~Tab~~ That is:-

_/_/_

$\exists \longrightarrow \forall$

$\forall \longrightarrow \exists$

Q. Every student in your class has taken a course in Calculus

$P(x)$ = x has taken a course in calculus.

~~$Q(x)$ → is student of our class~~.

$\cancel{\forall x \; (Q(x) \rightarrow P(x))}$

$\sim (\forall x \; P(x)) \Rightarrow$ This is not the case that every student has taken a course in calculus

$\Rightarrow \; \exists x \; \neg P(x)$

Q. Express the statement using Predicates & Quantifiers

a) Every student in this class has studied calculus.

$P(x)$ = x has studied calculus.

$\forall x : P(x)$

$Q(x)$ x is a student

$\forall x \; (Q(x) \rightarrow P(x))$.

Q. b) Some students in this class have visited Agra

$P(x) \Rightarrow$ x is student in our class.

$Q(x, y) \Rightarrow$ x has visited Agra

~~M = agra~~.

$\exists x : P(x) \rightarrow Q(x)$

This is false as. if P(x) is
Q(x) is
But ans ... conditional might ...

NOTE
In case of ~~there exists~~ $\Rightarrow$ conditional

Ans $\exists x : P(x) \rightarrow Q(x)$.

Inferences from proposition

- Arguments: Collection of 2 things:- (a) Premises (b) Conclusion

eg

If you have a voter id, you can vote] premises
You have voter id
You can vote $\Rightarrow$ Conclusion

$\rightarrow$ Argument

OR

P $\Rightarrow$ You have voter id
Q $\rightarrow$ You can vote.

$$\begin{array}{c} P \rightarrow Q \\ P \\ \hline Q \end{array}$$

] Argument form of proposition

Rules

1) If ($P \rightarrow Q$ and P) is given, you can Q.

This Rule is called MODUS PONENS

{ Tautology: $[(P \rightarrow Q) \wedge Q] \rightarrow Q$ }

__/__/__

2) If ($p \to q$ and $\neg q$) is given, you can conclude $\neg p$.
This rule is called MODUS TOLLENS.

3) If ($p \to q$ and if $q \to r$) is given, you can conclude $p \to r$
This rule is called HYPOTHETICAL SYLLOGISM

4) If (($p \vee q$) and $\neg p$) is given, you can conclude that q
This rule is called DISJUNCTIVE SYLLOGISM

5) If p is given, you can conclude $p \vee q$.
This rule is called ADDITION.

6) If ($p \wedge q$) is given you can conclude p
This rule is called SIMPLIFICATION.

7). If (p and q) is given you can conclude $p \wedge q$
This rule is called CONJUNCTION

8) If (($p \vee q$) and ($\neg p \vee r$)) is given you can conclude ($q \vee r$)
This rule is called RESOLUTION.

Show that the hypothesis
"It is not sunny this afternoon & it is colder than yesterday"
"We will go swimming only if it is sunny"

c) "If we do not go swimming, then we will take a manili trip" ~~and~~

d) "If we take a manili trip, then we will be home by sunset".

Lead to conclusion :- "We will be home by sunset -t."

p ⇒ it is sunny this afternoon.

q ⇒ it is colder than yesterday.

r ⇒ we will go swimming

s ⇒ we will take manilli Trip

t ⇒ we will be home by sunset.

premises :-

① $\neg p \wedge q$.

② $r \rightarrow p$.

③ $\neg r \rightarrow s$.

④ $s \rightarrow t$.

t.

Applying Simplification rule on ①.

⑥ $\neg p$.

using ⑥ and ② & MODES TOLLENS.

⑦ $\neg r$.

using ⑦ and ③, using MODES PONENS.

⑧ s.

using MODES PONENS on ④ & ⑧.

⑨ t.

__/__/__

2) a) "If today is tuesday, then I have a test in Maths or Economics"

b) "If my economics professor is sick, then I have no test of Economics"

c) Today is tuesday & my eco. prof. is sick then conclude that I have a test in Mathematics".

p : ~~If~~ today is Tuesday

q : I have a test in Maths

r : " " " " " Eco.

s ⇒ My Eco professor is sick

1) $p \rightarrow (q \vee r)$

2) $s \rightarrow \neg r$

3). $(p \wedge s) \Rightarrow q$.

Using simplification in 3.

4 p 5 s.

using MODES PONENS in 2 & 5 & 1 & 4

6 $\neg r$.

7 $q \vee r$

using disjunctive Syllogism. 6 & 7.

q.

~~Resto~~ Resolution Principle.

Represent every premise in form of Disjunction. If it is conjunctive, bifurcate it into a new ~~claus~~. clauses and combine them into disjunctive.

i.e. cancel negative & positive & represent rest in disjunctive

eg ① $p \to (q \vee r)$ $\longrightarrow$ $c_1 : \sim p \vee (q \vee \cancel{r})$
② $s \longrightarrow \neg r$ $\longrightarrow$ $c_2 : \sim s \vee \cancel{\neg r}$
③ $p \wedge s$ $\longrightarrow$ $c_3 : p$
④ q. $c_4 = s$.

~~cancel~~ $c_1 + c_2 = \sim p \vee q \vee \sim s$.
$c_1 + c_2 + c_3 \Rightarrow q \vee \neg s$.
$c_1 + c_2 + c_3 + c_4 \Rightarrow \boxed{q}$ conclusion

Rules of Inference for Quantifiers.

Rules of Inference	Name
$\dfrac{\forall x \; P(x)}{\therefore \; P(c)}$	Universal Instantiation
$\dfrac{P(c) \text{ for an arbitary } c}{\therefore \; \forall x \; P(x)}$	Universal generalisation
$\dfrac{\exists x \; P(x)}{\therefore \; P(c)}$	Existential Instantiation
$\dfrac{P(c) \text{ for some arbitary } c}{\therefore \; \exists x \; P(x)}$	Existential Generalisation

Show that the premises
a) "A student in this class has not read the book"

__/__/____

b) "Everyone in this class passed the first exam"
Imply the conclusion
"Someone who passed the first exam has not read the book.

$P(x) \Rightarrow$ x is a student of class ~~who has~~
$Q(x) \Rightarrow$ x has ~~not~~ read the book.
$R(x) \Rightarrow$ x passed the first exam.

$\exists x \; (P(x) \wedge \neg Q(x))$ ①
$\forall x \; (P(x) \rightarrow R(x))$ ②
$\exists x \; R(x) \wedge \neg Q(x)$

$P(c) \wedge R(c)$

In ① , using existential instantiation
$P(c) \wedge \neg Q(c)$
$\Rightarrow$ $P(c)$ is true
$\neg Q(c)$ is true.
in ② using universal instantiation
$P(c) \rightarrow R(c)$.
if $P(c)$ is true $\Rightarrow$ $R(c)$ is true
(using MODUS PONES Rule)

using $R(c)$ true & $\neg Q(c)$ trues,
use conjuctive rule.
$\Rightarrow$ $R(c) \wedge \neg Q(c)$
Now using existential generalisation
$\exists x : (R(c) \wedge \neg Q(c))$.

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__/__/____

Q. Validate the following
" All humming birds are richly coloured."
" No large birds lives on honey "
" Birds that do not live on honey are dull in colour"
concl. " Humming Birds are small".