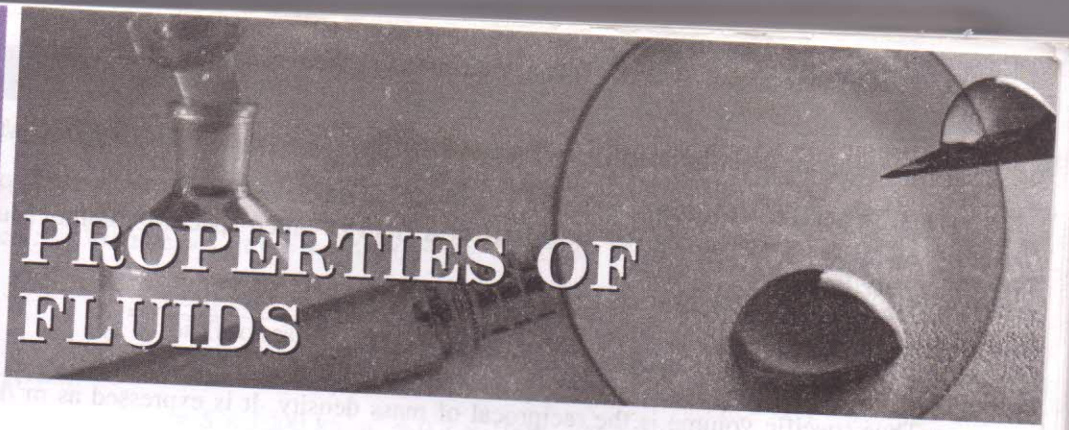
1 CHAPTER



PROPERTIES OF FLUIDS

▶ 1.1 INTRODUCTION

Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion. Thus this branch of science deals with the static, kinematics and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

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▶ 1.2 PROPERTIES OF FLUIDS

1.2.1 Density or Mass Density. Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.*, kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

1.2.2 Specific Weight or Weight Density. Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically,
$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

$$\therefore \quad w = \rho g \qquad \text{...(1.1)}$$

The value of specific weight or weight density (w) for water is 9.81×1000 Newton/m^3 in SI units.

1.2.3 Specific Volume. Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\dfrac{\text{Mass of fluid}}{\text{Volume of fluid}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gases.

1.2.4 Specific Gravity. Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

$$\text{Mathematically, } S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus weight density of a liquid $= S \times$ Weight density of water

$= S \times 1000 \times 9.81$ N/m^3

The density of a liquid $= S \times$ Density of water

$= S \times 1000$ kg/m^3. ...(1.1*A*)

If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water. For example, the specific gravity of mercury is 13.6, hence density of mercury $= 13.6 \times 1000 = 13600$ kg/m^3.

Problem 1.1 *Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.*

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \qquad \left(\because\ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (w) $= \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{7\text{ N}}{\left(\dfrac{1}{1000}\right)\text{m}^3} = \mathbf{7000\ N/m^3.}$ **Ans.**

(*ii*) Density (ρ) $= \dfrac{w}{g} = \dfrac{7000}{9.81}$ kg/m^3 $= \mathbf{713.5\ kg/m^3.}$ **Ans.**

(*iii*) Specific gravity $= \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{713.5}{1000}$ $\{\because$ Density of water $= 1000$ kg/m$^3\}$

$= \mathbf{0.7135.}$ **Ans.**

Problem 1.2 *Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(i) Density (ρ)

Using equation (1.1*A*),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m^3. Ans.**

(ii) Specific weight (w)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m^3. Ans.**

(iii) Weight (W)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

$\therefore$ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

1.3 VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance '*dy*' apart, move one over the other at different velocities, say *u* and *u* + *du* as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to *y*. It is denoted by symbol τ (Tau).

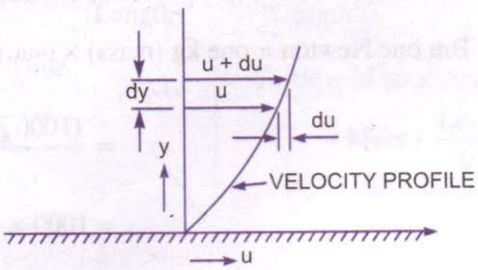


Fig. 1.1 *Velocity variation near a solid boundary.*

Mathematically, $\tau \propto \frac{du}{dy}$

or $$\tau = \mu \frac{du}{dy} \quad \text{...(1.2)}$$

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2), we have $$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} \quad \text{...(1.3)}$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

1.3.1 Units of Viscosity. The units of viscosity is obtained by putting the dimensions of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(Length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

In MKS system, force is represented by kgf and length by metre (m), in CGS system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by metre (m).

$\therefore$ MKS unit of viscosity $= \dfrac{\text{kgf-sec}}{\text{m}^2}$

CGS unit of viscosity $= \dfrac{\text{dyne-sec}}{\text{cm}^2}$

In the above expression N/m^2 is also known as Pascal which is represented by Pa. Hence N/m^2 = Pa = Pascal

$\therefore$ SI unit of viscosity = Ns/m^2 = Pa s.

SI unit of viscosity $= \dfrac{\text{Newton-sec}}{\text{m}^2} = \dfrac{\text{Ns}}{\text{m}^2}$

The unit of viscosity in CGS is also called Poise which is equal to $\dfrac{\text{dyne-sec}}{\text{cm}^2}$.

The numerical conversion of the unit of viscosity from MKS unit to CGS unit is given below :

$$\frac{\text{one kgf-sec}}{\text{m}^2} = \frac{9.81 \text{ N-sec}}{\text{m}^2} \qquad \{\because 1 \text{ kgf} = 9.81 \text{ Newton}\}$$

But one Newton = one kg (mass) × one $\left(\dfrac{\text{m}}{\text{sec}^2}\right)$ (acceleration)

$$= \frac{(1000 \text{ gm}) \times (100 \text{ cm})}{\text{sec}^2} = 1000 \times 100 \frac{\text{gm-cm}}{\text{sec}^2}$$

$$= 1000 \times 100 \text{ dyne} \qquad \left\{\because \text{dyne} = \text{gm} \times \frac{\text{cm}}{\text{sec}^2}\right\}$$

$$\therefore \quad \frac{\text{one kgf-sec}}{\text{m}^2} = 9.81 \times 100000 \frac{\text{dyne-sec}}{\text{m}^2} = 9.81 \times 100000 \frac{\text{dyne-sec}}{100 \times 100 \times \text{cm}^2}$$

$$= 98.1 \frac{\text{dyne-sec}}{\text{cm}^2} = 98.1 \text{ poise} \qquad \left\{\because \frac{\text{dyne-sec}}{\text{cm}^2} = \text{Poise}\right\}$$

Thus for solving numerical problems, if viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical value in MKS.

But $\quad \dfrac{\text{one kgf-sec}}{\text{m}^2} = \dfrac{9.81 \text{ Ns}}{\text{m}^2} = 98.1$ poise

$\therefore \quad \dfrac{\text{one Ns}}{\text{m}^2} = \dfrac{98.1}{9.81}$ poise = 10 poise or One poise $= \dfrac{1}{10} \dfrac{\text{Ns}}{\text{m}^2}$. = 0.1 Ns/m²

Alternate Method. One poise $= \dfrac{\text{dyne} \times \text{s}}{\text{cm}^2} = \left(\dfrac{1\,\text{gm} \times 1\,\text{cm}}{\text{s}^2}\right) \times \dfrac{\text{s}}{\text{cm}^2}$

But dyne $= 1\,\text{gm} \times \dfrac{1\,\text{cm}}{\text{s}^2}$

$\therefore$ One poise $= \dfrac{1\,\text{gm}}{\text{s cm}} = \dfrac{\frac{1}{1000}\,\text{kg}}{\text{s}\,\frac{1}{100}\,\text{m}}$

$= \dfrac{1}{1000} \times 100\,\dfrac{\text{kg}}{\text{s m}} = \dfrac{1}{10}\,\dfrac{\text{kg}}{\text{s m}}$ or $1\,\dfrac{\text{kg}}{\text{s m}} = 10$ poise.

Note. (*i*) In SI units second is represented by 's' and not by 'sec'.

(*ii*) If viscosity is given in poise, it must be divided by 10 to get its equivalent numerical value in SI units.

Sometimes a unit of viscosity as centipoise is used where

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise} \quad \text{or} \quad 1\,\text{cP} = \frac{1}{100}\,\text{P} \qquad [\text{cP} = \text{Centipoise, P} = \text{Poise}]$$

The viscosity of water at 20°C is 0.01 poise or 1.0 centipoise.

1.3.2 Kinematic Viscosity. It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called 'nu'. Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad \text{...(1.4)}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \dfrac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\dfrac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \dfrac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\dfrac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known as stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\dfrac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4}\ \text{m}^2/\text{s}$

Centistoke means $= \dfrac{1}{100}$ stoke.

1.3.3 Newton's Law of Viscosity. It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. Mathematically, it is expressed as given by equation (1.2) or as

$$\tau = \mu \frac{du}{dy}.$$

Fluids which obey the above relation are known as **Newtonian** fluids and the fluids which do not obey the above relation are called **Non-Newtonian** fluids.

1.3.4 Variation of Viscosity with Temperature. Temperature affects the viscosity. The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with the increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. In liquids, the cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity. But in case of gases the cohesive forces are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

(*i*) **For liquids,** $\mu = \mu_0 \left(\dfrac{1}{1 + \alpha t + \beta t^2} \right)$ ...(1.4*A*)

where μ = Viscosity of liquid at t°C, in poise

μ_0 = Viscosity of liquid at 0°C, in poise

α, β = Constants for the liquid

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$.

Equation (1.4*A*) shows that with the increase of temperature, the viscosity decreases.

(*ii*) **For a gas,** $\mu = \mu_0 + \alpha t - \beta t^2$ ...(1.4*B*)

where for air $\mu_0 = 0.000017$, $\alpha = 0.000000056$, $\beta = 0.1189 \times 10^{-9}$.

Equation (1.4*B*) shows that with the increase of temperature, the viscosity increases.

1.3.5 Types of Fluids. The fluids may be classified into the following five types :

1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

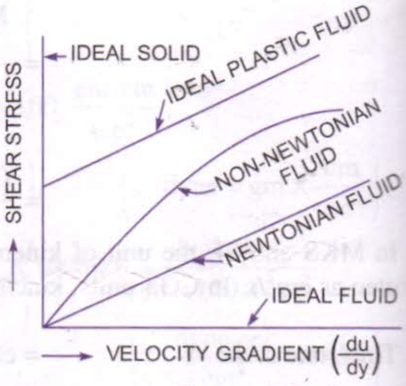


Fig. 1.2 *Types of fluids.*

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

Problem 1.3 *If the velocity distribution over a plate is given by* $u = \frac{2}{3} y - y^2$ *in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at* $y = 0$ *and* $y = 0.15$ *m. Take dynamic viscosity of fluid as 8.63 poises.*

Solution. Given : $u = \frac{2}{3} y - y^2$ $\therefore$ $\frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756 \text{ N/m}^2.\ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167 \text{ N/m}^2.\ Ans.}$$

Problem 1.4 *A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.*

Solution. Given :

Distance between plates, $dy = .025$ mm $= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \frac{\text{N}}{\text{m}^2}$.

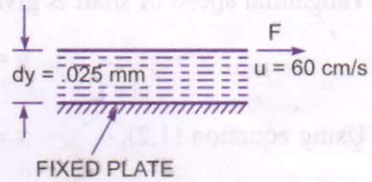


Fig. 1.3

This is the value of shear stress *i.e.,* τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \frac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \frac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise} = \mathbf{8.33 \times 10^{-4} \text{ poise. Ans.}}$$

Problem 1.5 *A flat plate of area 1.5 × 10^6 mm^2 is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.*

Solution. Given :

Area of the plate, $A = 1.5 \times 10^6 \text{ mm}^2 = 1.5 \text{ m}^2$

Speed of plate relative to another plate, $du = 0.4$ m/s

Distance between the plates, $dy = 0.15 \text{ mm} = 0.15 \times 10^{-3}$ m

Viscosity $\mu = 1 \text{ poise} = \dfrac{1}{10} \dfrac{\text{Ns}}{\text{m}^2}$.

Using equation (1.2) we have $\tau = \mu \dfrac{du}{dy} = \dfrac{1}{10} \times \dfrac{0.4}{.15 \times 10^{-3}} = 266.66 \dfrac{\text{N}}{\text{m}^2}$

(*i*) $\therefore$ Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 =$ **400 N. Ans.**

(*ii*) Power* required to move the plate at the speed 0.4 m/sec

$= F \times u = 400 \times 0.4 =$ **160 W. Ans.**

Problem 1.6 *Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.*

Solution. Given : $\mu = 1 \text{ poise} = \dfrac{1}{10} \dfrac{\text{N s}}{\text{m}^2}$

Dia. of shaft, $D = 10 \text{ cm} = 0.1$ m

Distance between shaft and journal bearing,

$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Speed of shaft, $N = 150$ r.p.m.

Tangential speed of shaft is given by

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785 \text{ m/s}$$

Using equation (1.2), $\tau = \mu \dfrac{du}{dy}$,

where du = change of velocity between shaft and bearing $= u - 0 = u$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = \mathbf{52.33 \text{ N/m}^2}. \textbf{ Ans.}$$

Problem 1.7 *Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.*

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300$ N

Velocity of plate, $u = 0.3$ m/s

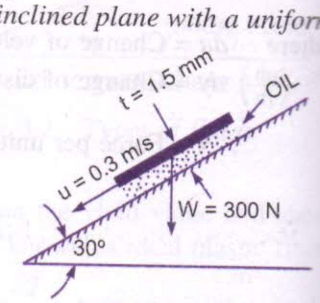


Fig. 1.4

* Power $= F \times u$ N m/s $= F \times u$ W ($\because$ Nm/s = Watt)

Thickness of oil film, $t = dy = 1.5\text{ mm} = 1.5 \times 10^{-3}\text{ m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150$ N

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress, $\tau = \frac{F}{\text{Area}} = \frac{150}{0.64}\text{ N/m}^2$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3}\text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17\text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7\text{ poise.}}\ \textbf{Ans.}$$

Problem 1.8 *Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.*

Solution. Given :

Distance between plates, $dy = 1.25\text{ cm} = 0.0125$ m

Viscosity, $\mu = 14\text{ poise} = \frac{14}{10}\text{ N s/m}^2$

Velocity of upper plate, $u = 2.5$ m/sec.

Shear stress is given by equation (1.2) as, $\tau = \mu \frac{du}{dy}$

where du = Change of velocity between plates $= u - 0 = u = 2.5$ m/sec.

$$dy = 0.0125\text{ m.}$$

$$\therefore \quad \tau = \frac{14}{10} \times \frac{2.5}{.0125} = \mathbf{280\text{ N/m}^2.\ Ans.}$$

Problem 1.9 *The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine :*

(i) the dynamic viscosity of the oil in poise, and

(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

Solution. Given :

Each side of a square plate = 60 cm = 0.60 m

$\therefore$ Area, $A = 0.6 \times 0.6 = 0.36\text{ m}^2$

Thickness of oil film, $dy = 12.5\text{ mm} = 12.5 \times 10^{-3}$ m

Velocity of upper plate, $u = 2.5$ m/sec

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1\text{ N}}{0.36\text{ m}^2}$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $\tau = \mu \frac{du}{dy}$ or $\frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$

$$\therefore \quad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2} \qquad \left(\because \frac{1\text{ Ns}}{\text{m}^2} = 10 \text{ poise}\right)$$

$= 1.3635 \times 10 =$ **13.635 poise. Ans.**

(*ii*) Sp. gr. of oil, S = 0.95

Let ν = kinematic viscosity of oil

Using equation (1.1*A*),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950$ kg/m^3

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435$ m^2/sec $= .001435 \times 10^4$ cm^2/s

$=$ **14.35 stokes. Ans.** ($\because$ cm^2/s = stoke)

Problem 1.10 *Find the kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second.*

Solution. Given :

Mass density, $\rho = 981$ kg/m^3

Shear stress, $\tau = 0.2452$ N/m^2

Velocity gradient, $\frac{du}{dy} = 0.2$ s

Using the equation (1.2), $\tau = \mu \frac{du}{dy}$ or $0.2452 = \mu \times 0.2$

$$\therefore \quad \mu = \frac{0.2452}{0.200} = 1.226 \text{ Ns/m}^2$$

Kinematic viscosity ν is given by

$$\therefore \quad \nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2} \text{ m}^2/\text{sec}$$

$= 0.125 \times 10^{-2} \times 10^4$ cm^2/s $= 0.125 \times 10^2$ cm^2/s

$= 12.5$ cm^2/s $=$ **12.5 stoke. Ans.** ($\because$ cm^2/s = stoke)

Problem 1.11 *Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.*

Solution. Given :

Viscosity, $\mu = 0.05$ poise $= \frac{0.05}{10}$ N s/m^2

Kinematic viscosity, $\nu = 0.035$ stokes

$= 0.035 \text{ cm}^2/\text{s}$ $\{\because \text{Stoke} = \text{cm}^2/\text{s}\}$

$= 0.035 \times 10^{-4} \text{ m}^2/\text{s}$

Using the relation $\nu = \dfrac{\mu}{\rho}$, we get $0.035 \times 10^{-4} = \dfrac{0.05}{10} \times \dfrac{1}{\rho}$

$$\therefore \quad \rho = \frac{0.05}{10} \times \frac{1}{0.035 \times 10^{-4}} = 1428.5 \text{ kg/m}^3$$

$$\therefore \quad \text{Sp. gr. of liquid} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1428.5}{1000} = 1.4285 \simeq \mathbf{1.43.}\ \textbf{Ans.}$$

Problem 1.12 *Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.*

Solution. Given :

Kinematic viscosity $\nu = 6 \text{ stokes} = 6 \text{ cm}^2/\text{s} = 6 \times 10^{-4} \text{ m}^2/\text{s}$

Sp. gr. of liquid $= 1.9$

Let the viscosity of liquid $= \mu$

$$\text{Now sp. gr. of a liquid} = \frac{\text{Density of the liquid}}{\text{Density of water}}$$

or
$$1.9 = \frac{\text{Density of liquid}}{1000}$$

$$\therefore \text{ Density of liquid} = 1000 \times 1.9 = 1900 \frac{\text{kg}}{\text{m}^3}$$

$\therefore$ Using the relation $\nu = \dfrac{\mu}{\rho}$, we get

$$6 \times 10^{-4} = \frac{\mu}{1900}$$

or
$$\mu = 6 \times 10^{-4} \times 1900 = 1.14 \text{ Ns/m}^2$$

$$= 1.14 \times 10 = \mathbf{11.40 \text{ poise.}}\ \textbf{Ans.}$$

Problem 1.13 *The velocity distribution for flow over a flat plate is given by $u = \frac{3}{4} y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate. Determine the shear stress at y = 0.15 m. Take dynamic viscosity of fluid as 8.~~6~~5 poise.*

Solution. Given : $u = \dfrac{3}{4} y - y^2$

$$\therefore \quad \frac{du}{dy} = \frac{3}{4} - 2y$$

At $y = 0.15$,
$$\frac{du}{dy} = \frac{3}{4} - 2 \times 0.15 = 0.75 - 0.30 = 0.45$$

Viscosity,
$$\mu = 8.5 \text{ poise} = \frac{8.5}{10} \frac{\text{N s}}{\text{m}^2} \qquad \left(\because 10 \text{ poise} = 1 \frac{\text{Ns}}{\text{m}^2}\right)$$

Using equation (1.2), $\tau = \mu \dfrac{du}{dy} = \dfrac{8.5}{10} \times 0.45 \dfrac{\text{N}}{\text{m}^2} = 0.3825 \dfrac{\text{N}}{\text{m}^2}$. **Ans.**

Problem 1.14 *The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.*

Solution. Given :

Viscosity $\mu = 6$ poise

$$= \frac{6}{10} \frac{\text{N s}}{\text{m}^2} = 0.6 \frac{\text{N s}}{\text{m}^2}$$

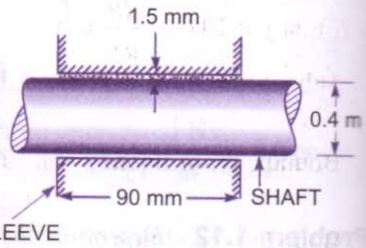


Fig. 1.5

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

0.6

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \dfrac{2\pi NT}{60} = \dfrac{2\pi \times 190 \times 36.01}{60} = $ **716.48 W. Ans.**

Problem 1.15 *If the velocity profile of a fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.*

Solution. Given :

Distance of vertex from plate = 20 cm

Velocity at vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5 \text{ poise} = \dfrac{8.5}{10} \dfrac{\text{N s}}{\text{m}^2} = 0.85.$

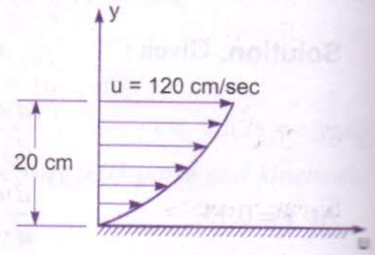


Fig. 1.6

* Power in S.I. unit $= T * \omega = T \times \dfrac{2\pi N}{60}$ Watt $= \dfrac{2\pi NT}{60}$ Watt

The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \quad \text{...}(i)$$

where a, b and c are constants. Their values are determined from boundary conditions as :

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/sec

(c) at $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (a) in equation (i), we get

$$c = 0.$$

Boundary condition (b) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \quad \text{...}(ii)$$

Boundary condition (c) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \quad \text{...}(iii)$$

or $$0 = 2 \times a \times 20 + b = 40a + b$$

Solving equations (ii) and (iii) for a and b

From equation (iii), $b = -40a$

Substituting this value in equation (ii), we get

$$120 = 400a + 20 \times (-40a) = 400a - 800a = -400a$$

$$\therefore \quad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \quad b = -40 \times (-0.3) = 12.0$$

Substituting the values of a, b and c in equation (i),

$$u = -0.3y^2 + 12y.$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = $ **12/s. Ans.**

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = $ **6/s. Ans.**

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = $ **0. Ans.**

Shear Stresses

Shear stress is given by, $\tau = \mu \frac{du}{dy}$

(*i*) Shear stress at $y = 0$, $\tau = \mu \left(\dfrac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2$.

(*ii*) Shear stress at $y = 10$, $\tau = \mu \left(\dfrac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2$.

(*iii*) Shear stress at $y = 20$, $\tau = \mu \left(\dfrac{du}{dy}\right)_{y=20} = 0.85 \times 0 = \mathbf{0}$. **Ans.**

Problem 1.16 *A Newtonian fluid is filled in the clearance between a shaft and a concentric sleeve. The sleeve attains a speed of 50 cm/s, when a force of 40 N is applied to the sleeve parallel to the shaft. Determine the speed if a force of 200 N is applied.*

Solution. Given : Speed of sleeve, $u_1 = 50$ cm/s

when force, $F_1 = 40$ N.

Let speed of sleeve is u_2 when force, $F_2 = 200$ N.

Using relation $\tau = \mu \dfrac{du}{dy}$

where τ = Shear stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A}$

du = Change of velocity $= u - 0 = u$

dy = Clearance $= y$

$$\therefore \quad \frac{F}{A} = \mu \frac{u}{y}$$

$$\therefore \quad F = \frac{A\mu u}{y} \propto u \qquad \{\because \text{ A, } \mu \text{ and } y \text{ are constant}\}$$

$$\therefore \quad \frac{F_1}{u_1} = \frac{F_2}{u_2}$$

Substituting values, we get $\dfrac{40}{50} = \dfrac{200}{u_2}$

$$\therefore \quad u_2 = \frac{50 \times 200}{40} = 50 \times 5 = \mathbf{250 \text{ cm/s.}} \text{ **Ans.**}$$

—x—

Problem 1.17 *A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 r.p.m., determine the viscosity of the fluid.*

Solution. Given :

Diameter of cylinder $= 15$ cm $= 0.15$ m

Dia. of outer cylinder $= 15.10$ cm $= 0.151$ m

Length of cylinders, $L = 25$ cm $= 0.25$ m

Torque, $T = 12.0$ Nm

CHAPTER 2

PRESSURE AND ITS MEASUREMENT

▶ 2.1 FLUID PRESSURE AT A POINT

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by p. Hence mathematically the pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}.$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

$\therefore$ Force or pressure force, $F = p \times A$.

The units of pressure are : (*i*) kgf/m^2 and kgf/cm^2 in MKS units, (*ii*) Newton/m^2 or N/m^2 and N/mm^2 in SI units. N/m^2 is known as Pascal and is represented by Pa. Other commonly used units of pressure are :

$$\text{kPa} = \text{kilo pascal} = 1000 \text{ N/m}^2$$

$$\text{bar} = 100 \text{ kPa} = 10^5 \text{ N/m}^2.$$

▶ 2.2 PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, dx, dy and ds.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and p_x,

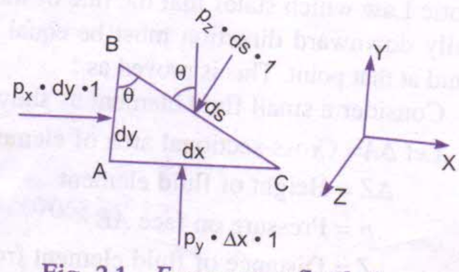


Fig. 2.1 *Forces on a fluid element.*

p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element = (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where ρ = density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1)\sin(90° - \theta) = 0$$

or

$$p_x \times dy \times 1 - p_z\, ds \times 1 \cos\theta = 0.$$

But from Fig. 2.1, $ds \cos\theta = AB = dy$

$\therefore$

$$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or

$$p_x = p_z \qquad ...(2.1)$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or

$$p_y \times dx - p_z\, ds \sin\theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$

$$p_y dx - p_z \times dx = 0$$

or

$$p_y = p_z \qquad ...(2.2)$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \qquad ...(2.3)$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

2.3 PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as :

Consider a small fluid element as shown in Fig. 2.2

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are :

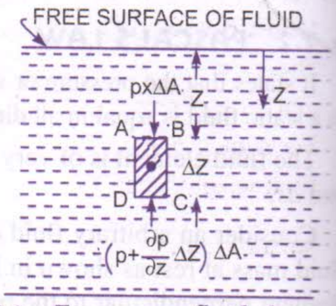


Fig. 2.2 *Forces on a fluid element.*

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.

2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.

3. Weight of fluid element = Density $\times\, g \times$ Volume $= \rho \times g \times (\Delta A \times \Delta Z)$.

4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or
$$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or
$$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or
$$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

$$\therefore \quad \frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \rho \times g = w) \qquad \text{...(2.4)}$$

where w = Weight density of fluid.

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law.**

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g dZ$$

or
$$p = \rho g Z \qquad \text{...(2.5)}$$

where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

From equation (2.5), we have $\quad Z = \frac{p}{\rho \times g} \qquad$ ...(2.6)

Here Z is called **pressure head.**

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Solution. Given :

Dia. of ram, $\quad D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $\quad d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $\quad F = 500 \text{ N}$

Find weight lifted $\quad = W$

Area of ram, $\quad A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $\quad a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = .00159 \text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159}\ \text{N/m}^2.$$

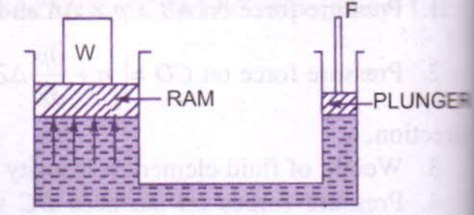


Fig. 2.3

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4\ \text{N/m}^2$$

But pressure intensity at ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068}$ N/m²

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222$ N = **22.222 kN. Ans.**

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20$ cm $= 0.2$ m

$\therefore$ Area of ram, $A = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4}(.2)^2 = 0.0314\ \text{m}^2$

Dia. of plunger $d = 3$ cm $= 0.03$ m

$\therefore$ Area of plunger, $a = \dfrac{\pi}{4}(.03)^2 = 7.068 \times 10^{-4}\ \text{m}^2$

Weight lifted, $W = 30$ kN $= 30 \times 1000$ N $= 30000$ N.

See Fig. 2.3.

Pressure intensity developed due to plunger $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{a}$.

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \dfrac{F}{a}$

$\therefore$ Force acting on ram = Pressure intensity × Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}}\ \text{N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = 675.2\ \text{N.}$$ **Ans.**

Problem 2.3 *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000\ kg/m^3$.*

Solution. Given :

Height of liquid column, $Z = 0.3$ m.

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2}. \textbf{ Ans.}$$

(*b*) For oil of sp. gr. 0.8,

From equation (1.1*A*), we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water}$ (ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}$$

$$= \mathbf{0.2354 \frac{N}{cm^2}}. \textbf{ Ans.}$$

(*c*) For mercury, sp. gr. $= 13.6$

From equation (1.1*A*) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

$\therefore$ Density of mercury, $\rho_s = \text{Specific gravity of mercury} \times \text{Density of water}$

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$$\therefore \quad p = \rho_s \times g \times Z$$

$$= 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}}. \textbf{ Ans.}$$

Problem 2.4 *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Solution. Given :

Pressure intensity, $p = 3.924 \frac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \frac{\text{N}}{\text{m}^2}$.

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \mathbf{4 \text{ m of water}}. \textbf{ Ans.}$$

(*b*) For oil, sp. gr. $= 0.9$

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \mathbf{4.44 \text{ m of oil}}. \textbf{ Ans.}$$

Problem 2.5 *An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.*

Solution. Given :

Sp. gr. of oil, $S_0 = 0.9$

Height of oil, $Z_0 = 40$ m

Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water} = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Intensity of pressure, $p = \rho_0 \times g \times Z_0 = 900 \times 9.81 \times 40 \frac{\text{N}}{\text{m}^2}$

$\therefore$ Corresponding height of water $= \dfrac{p}{\text{Density of water} \times g}$

$$= \frac{900 \times 9.81 \times 40}{1000 \times 9.81} = 0.9 \times 40 = \mathbf{36 \text{ m of water.}} \textbf{ Ans.}$$

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000 \text{ kg/m}^3$

Density of oil, $\rho_2 = \text{Sp. gr. of oil} \times \text{Density of water}$
$= 0.9 \times 1000 = 900 \text{ kg/m}^3$

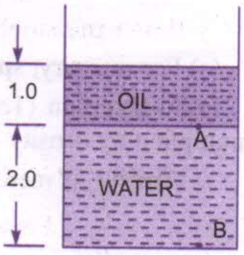


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$
$$= 900 \times 9.81 \times 1.0$$
$$= 8829 \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \mathbf{0.8829 \text{ N/cm}^2.} \textbf{ Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$
$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \mathbf{2.8449 \text{ N/cm}^2.} \textbf{ Ans.}$$

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m³.

Solution. Given :

Dia. of small piston, $d = 3$ cm

$\therefore$ Area of small piston, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{\pi}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(*a*) **When the pistons are at the same level**

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

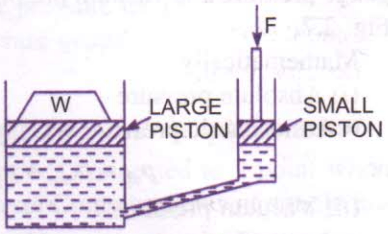


Fig. 2.5

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N.}}$$ **Ans.**

(*b*) **When the small piston is 40 cm above the large piston**

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

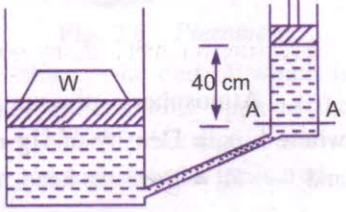


Fig. 2.6

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

$\therefore$ Pressure intensity transmitted to the large piston $= 11.71 \text{ N/cm}^2$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

▶ 2.4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. **Vacuum pressure** is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure – Absolute pressure.

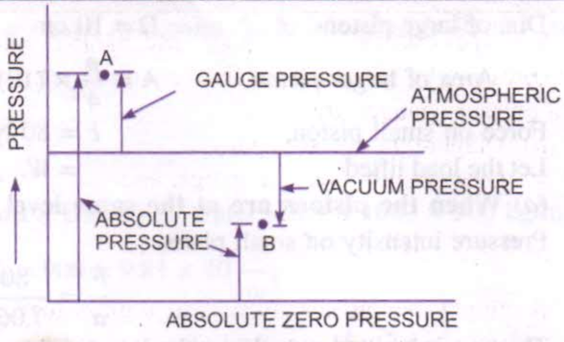


Fig. 2.7 ***Relationship between pressures.***

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem 2.8 *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53 × 10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.*

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

$\therefore$ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \; Z_0 = 0.75)$$
$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$
$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

$\therefore$ Gauge pressure, $p = \mathbf{45028\ N/m^2}$. **Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

2.5 MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers 2. Mechanical Gauges.

2.5.1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

▶ 2.6 SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$

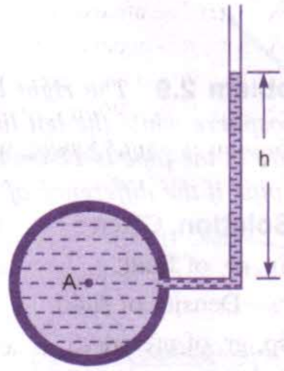


Fig. 2.8 *Piezometer.*

2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

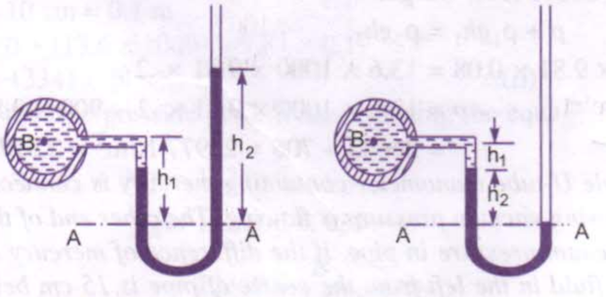


Fig. 2.9 *U-tube Manometer.*

(*a*) **For Gauge Pressure.** Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gr. of light liquid
ρ_1 = Density of light liquid = $1000 \times S_1$
S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1). \quad ...(2.7)$$

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1). \quad ...(2.8)$$

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/m^3

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000$ kg/m^3

Difference of mercury level, $h_2 = 20$ cm $= 0.2$ m

Height of fluid from A-A, $h_1 = 20 - 12 = 8$ cm $= 0.08$ m

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

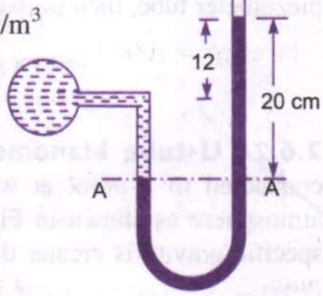


Fig. 2.10

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2. \text{ Ans.}}$$

—x—

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

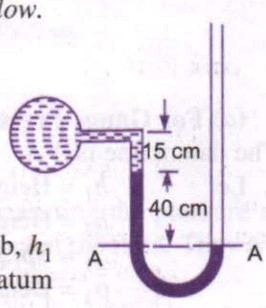
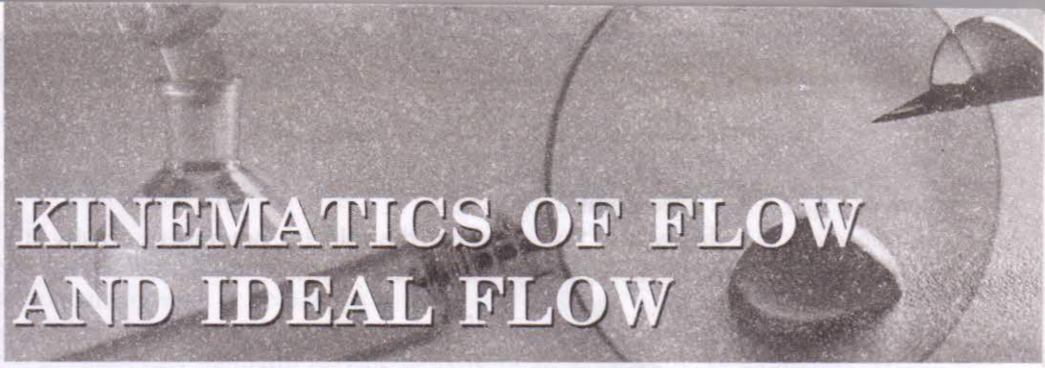


Fig. 2.11

Difference of mercury level, $h_2 = 40$ cm $= 0.4$ m. Height of liquid in left limb, $h_1 = 15$ cm $= 0.15$ m. Let the pressure in pipe $= p$. Equating pressure above datum line A-A, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$



5 CHAPTER KINEMATICS OF FLOW AND IDEAL FLOW

A. KINEMATICS OF FLOW

▶ 5.1 INTRODUCTION

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined. In this chapter, the methods of determining velocity and acceleration are discussed.

▶ 5.2 METHODS OF DESCRIBING FLUID MOTION

The fluid motion is described by two methods. They are —(*i*) Lagrangian Method, and (*ii*) Eulerian Method. In the Lagrangian method, a **single fluid particle** is followed during its motion and its velocity, acceleration, density, etc., are described. In case of Eulerian method, the velocity, acceleration, pressure, density etc., are described **at a point** in flow field. The Eulerian method is commonly used in fluid mechanics.

▶ 5.3 TYPES OF FLUID FLOW

The fluid flow is classified as :

(*i*) Steady and unsteady flows ;
(*ii*) Uniform and non-uniform flows ;
(*iii*) Laminar and turbulent flows ;
(*iv*) Compressible and incompressible flows ;
(*v*) Rotational and irrotational flows ; and
(*vi*) One, two and three-dimensional flows.

5.3.1 Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

5.3.2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity

∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

5.3.3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number,

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

5.3.4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

5.3.5 Rotational and Irrotational Flows. Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then that type of flow is called irrotational flow.

5.3.6 One-, Two- and Three-Dimensional Flows. **One-dimensional flow** is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-dimensional flow

$$u = f_1(x, y),\ v = f_2(x, y) \text{ and } w = 0.$$

Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z),\ v = f_2(x, y, z) \text{ and } w = f_3(x, y, z).$$

▶ 5.4 RATE OF FLOW OR DISCHARGE (Q)

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(*i*) For liquids the units of Q are m^3/s or litres/s

(*ii*) For gases the units of Q is kgf/s or Newton/s

Consider a liquid flowing through a pipe in which

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

Then discharge $\qquad Q = A \times V.$ ...(5.1)

▶ 5.5 CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in Fig. 5.1.

Let V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and V_2, ρ_2, A_2 are corresponding values at section, 2-2.

Then rate of flow at section 1-1 $= \rho_1 A_1 V_1$

Rate of flow at section 2-2 $= \rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(5.2)$$

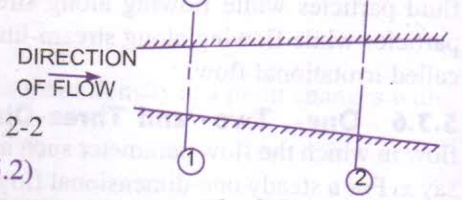


Fig. 5.1 *Fluid flowing through a pipe.*

Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation**. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2 \quad ...(5.3)$$

Problem 5.1 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

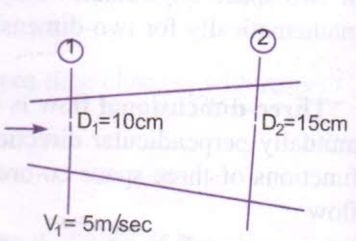


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or $$Q = A_1 \times V_1$$

$$= 0.007854 \times 5 = \mathbf{0.03927 \text{ m}^3/\text{s}.} \text{ **Ans.**}$$

Using equation (5.3), we have $A_1 V_1 = A_2 V_2$

(*ii*) $\therefore$ $$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22 \text{ m/s.}} \text{ **Ans.**}$$

Problem 5.2 *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Solution. Given :

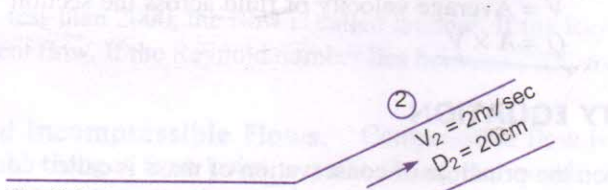


Fig. 5.3

$$D_1 = 30 \text{ cm} = 0.30 \text{ m}$$

$$\therefore \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$\therefore \quad A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$$

$$V_2 = 2 \text{ m/s}$$

$$D_3 = 15 \text{ cm} = 0.15 \text{ m}$$

$$\therefore \quad A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$

Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3 \qquad \text{...(1)}$$

(*i*) **The discharge Q_1** in pipe 1 is given by

$$Q_1 = A_1 V_1 = 0.07068 \times 2.5 \text{ m}^3\text{/s} = \mathbf{0.1767 \text{ m}^3\text{/s}}. \text{ **Ans.**}$$

(*ii*) **Value of V_3**

$$Q_2 = A_2 V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3\text{/s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$$\therefore \quad Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3\text{/s}$$

But $\quad Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$

$$\therefore \quad V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44 \text{ m/s}}. \text{ **Ans.**}$$

Problem 5.3 *Water flows through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter. At C, the pipe branches. Branch CD is 0.8 m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.*

Solution. Given :

Diameter of pipe *AB*, $D_{AB} = 1.2$ m

Velocity of flow through *AB*, $V_{AB} = 3.0$ m/s

Dia. of pipe *BC*, $D_{BC} = 1.5$ m

Dia. of branched pipe *CD*, $D_{CD} = 0.8$ m

Velocity of flow in pipe *CE*, $V_{CE} = 2.5$ m/s

Let the flow rate in pipe $AB = Q$ m^3/s

Velocity of flow in pipe $BC = V_{BC}$ m/s

Velocity of flow in pipe $CD = V_{CD}$ m/s

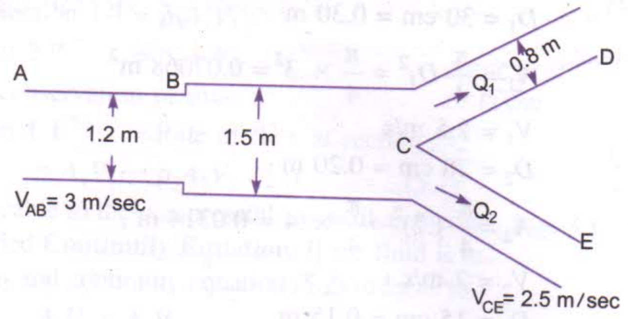


Fig. 5.4

Diameter of pipe $CE = D_{CE}$

Then flow rate through $CD = Q/3$

and flow rate through $CE = Q - Q/3 = \dfrac{2Q}{3}$

(*i*) Now volume flow rate through $AB = Q = V_{AB} \times$ Area of AB

$$= 3.0 \times \frac{\pi}{4}(D_{AB})^2 = 3.0 \times \frac{\pi}{4}(1.2)^2 = \mathbf{3.393\ m^3/s.}\ \textbf{Ans.}$$

(*ii*) Applying continuity equation to pipe AB and pipe BC,

$$V_{AB} \times \text{Area of pipe } AB = V_{BC} \times \text{Area of pipe } BC$$

or $$3.0 \times \frac{\pi}{4}(D_{AB})^2 = V_{BC} \times \frac{\pi}{4}(D_{BC})^2$$

or $$3.0 \times (1.2)^2 = V_{BC} \times (1.5)^2 \qquad \left[\text{Divide by } \frac{\pi}{4}\right]$$

or $$V_{BC} = \frac{3 \times 1.2^2}{1.5^2} = \mathbf{1.92\ m/s.}\ \textbf{Ans.}$$

(*iii*) The flow rate through pipe

$$CD = Q_1 = \frac{Q}{3} = \frac{3.393}{3} = 1.131\ \text{m}^3/\text{s}$$

$\therefore$ $$Q_1 = V_{CD} \times \text{Area of pipe } CD \times \frac{\pi}{4}(D_{CD})^2$$

or $$1.131 = V_{CD} \times \frac{\pi}{4} \times 0.8^2 = 0.5026\ V_{CD}$$

$\therefore$ $$V_{CD} = \frac{1.131}{0.5026} = \mathbf{2.25\ m/s.}\ \textbf{Ans.}$$

(*iv*) Flow rate through CE,

$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262\ \text{m}^3/\text{s}$$

$\therefore$ $$Q_2 = V_{CE} \times \text{Area of pipe } CE = V_{CE}\frac{\pi}{4}(D_{CE})^2$$

or $$2.263 = 2.5 \times \frac{\pi}{4} \times (D_{CE})^2$$

or $$D_{CE} = \sqrt{\frac{2.263 \times 4}{2.5 \times \pi}} = \sqrt{1.152} = 1.0735\ \text{m}$$

$\therefore$ Diameter of pipe CE = **1.0735 m. Ans.**

Problem 5.4 *A 25 cm diameter pipe carries oil of sp. gr. 0.9 at a velocity of 3 m/s. At another section the diameter is 20 cm. Find the velocity at this section and also mass rate of flow of oil.*

Solution. Given :

at section 1, $D_1 = 25 \text{ cm} = 0.25 \text{ m}$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.25^2 = 0.049 \text{ m}^3$$

$$V_1 = 3 \text{ m/s}$$

at section 2, $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_2 = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2$$

$$V_2 = ?$$

Mass rate of flow of oil = ?

Applying continuity equation at sections 1 and 2,

$$A_1 V_1 = A_2 V_2$$

or $$0.049 \times 3.0 = 0.0314 \times V_2$$

$\therefore$ $$V_2 = \frac{0.049 \times 3.0}{0.0314} = \textbf{4.68 m/s. Ans.}$$

Mass rate of flow of oil $= \text{Mass density} \times Q = \rho \times A_1 \times V_1$

Sp. gr. of oil $= \dfrac{\text{Density of oil}}{\text{Density of water}}$

$\therefore$ Density of oil = Sp. gr. of oil × Density of water

$$= 0.9 \times 1000 \text{ kg/m}^3 = \frac{900 \text{ kg}}{\text{m}^3}$$

$\therefore$ Mass rate of flow $= 900 \times 0.049 \times 3.0$ kg/s $=$ **132.23 kg/s. Ans.**

Problem 5.5 *A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, that will be the diameter at a point 4.5 m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s.*

Solution. Given :

Dia. of nozzle, $D_1 = 25 \text{ mm} = 0.025 \text{ m}$

Velocity of jet at nozzle, $V_1 = 12 \text{ m/s}$

Height of point A, $h = 4.5 \text{ m}$

Let the velocity of the jet at a height 4.5 m $= V_2$

Consider the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy).

Initial velocity, $u = V_1 = 12 \text{ m/s}$

Final velocity, $V = V_2$

Value of $g = -9.81 \text{ m/s}^2$ and $h = 4.5 \text{ m}$

Using, $V^2 - u^2 = 2gh$, we get

$$V_2^2 - 12^2 = 2 \times (-9.81) \times 4.5$$

Fig. 5.5

$$\therefore \qquad V_2 = \sqrt{12^2 - 2 \times 9.81 \times 4.5} = \sqrt{144 - 88.29} = 7.46 \text{ m/s}$$

Now applying continuity equation to the outlet of nozzle and at point A, we get

$$A_1 V_1 = A_2 V_2$$

or
$$A_2 = \frac{A_1 V_1}{V_2} = \frac{\frac{\pi}{4} D_1^2 \times V_1}{V_2} = \frac{\pi \times (0.025)^2 \times 12}{4 \times 7.46} = 0.0007896$$

Let $\quad D_2$ = Diameter of jet at point A.

Then $\quad A_2 = \frac{\pi}{2} D_2^2$ or $0.0007896 = \frac{\pi}{4} \times D_2^2$

$$\therefore \qquad D_2 = \sqrt{\frac{0.0007896 \times 4}{\pi}} = 0.0317 \text{ m} = \textbf{31.7 mm. Ans.}$$

▶ 5.6 CONTINUITY EQUATION IN THREE-DIMENSIONS

Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z. Let u, v and w are the inlet velocity components in x, y and z directions respectively. Mass of fluid entering the face $ABCD$ per second

$$= \rho \times \text{Velocity in } x\text{-direction} \times \text{Area of } ABCD$$
$$= \rho \times u \times (dy \times dz)$$

Then mass of fluid leaving the face $EFGH$ per second $= \rho u \, dydz + \frac{\partial}{\partial x}(\rho u \, dydz)\, dx$

$\therefore$ Gain of mass in x-direction

$$= \text{Mass through } ABCD - \text{Mass through } EFGH \text{ per second}$$
$$= \rho u \, dydz - \rho u \, dydz - \frac{\partial}{\partial x}(\rho u \, dydz) dx$$
$$= -\frac{\partial}{\partial x}(\rho u \, dydz)\, dx$$
$$= -\frac{\partial}{\partial x}(\rho u)\, dx\, dydz \qquad \{\because dydz \text{ is constant}\}$$

Similarly, the net gain of mass in y-direction

$$= -\frac{\partial}{\partial y}(\rho v)\, dxdydz$$

and in z-direction
$$= -\frac{\partial}{\partial z}(\rho w)\, dxdydz$$

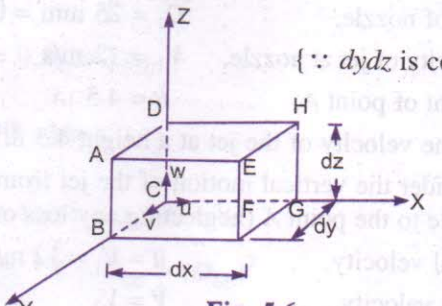


Fig. 5.6

$$\therefore \qquad \text{Net gain of masses} = -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dxdydz$$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass

6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \quad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

Problem 6.1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe = 5 cm = 0.5 m

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0$ m/s

Datum head, $z = 5$ m

Total head = pressure head + kinetic head + datum head

Pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

∴ Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem 6.2 *A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.*

Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

$\therefore$ $$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

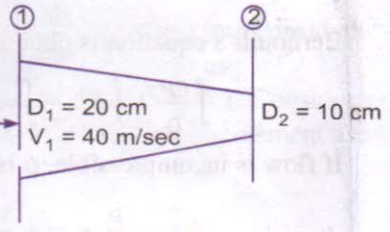


Fig. 6.2

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \textbf{0.815 m. Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$\therefore$ $$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$\therefore$ Velocity head at section 2 $= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \textbf{83.047 m. Ans.}$

(*iii*) Rate of discharge $= A_1V_1$ or A_2V_2

$$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$$

$$= \textbf{125.6 litres/s. Ans.} \qquad \{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$$

Problem 6.3 *State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's equation from first principle and state the assumptions made for such a derivation.*

Solution. Statement of Bernoulli's Theorem. It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are :

$$\text{Pressure energy} = \frac{p}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant.}$$

Derivation of Bernoulli's theorem. For derivation of Bernoulli's theorem, Articles 6.3 and 6.4 should be written.

Assumptions are given in Article 6.5.

Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

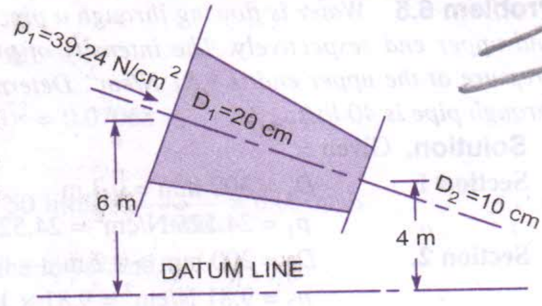


Fig. 6.3

Solution. Given :

At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or

$$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or

$$46.063 = \frac{p_2}{9810} + 5.012$$

$$\therefore \quad \frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$$\therefore \quad p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}. \text{ **Ans.**}$$

Problem 6.5 *Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.*

Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

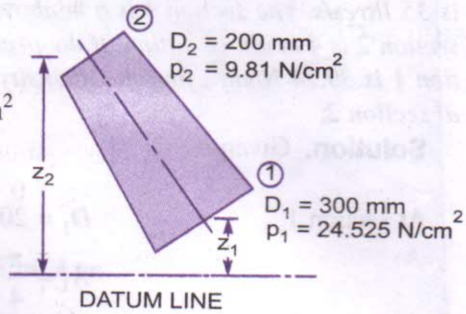


Fig. 6.4

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$\therefore$ $$V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$\therefore$ $$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem 6.6 *The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².*

Solution. Given :

Length of pipe, $L = 100$ m

Dia. at the upper end, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$$

$$= 0.2827 \text{ m}^2$$

p_1 = pressure at upper end

$= 19.62 \text{ N/cm}^2$

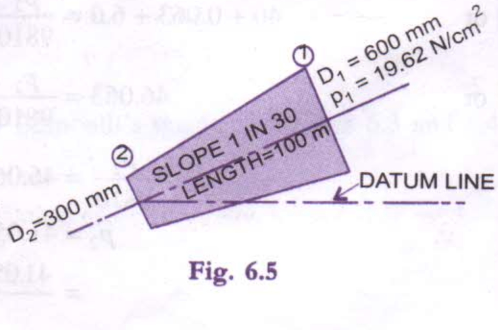


Fig. 6.5

$$= 19.62 \times 10^4 \text{ N/m}^2$$

Dia. at lower end, $D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}$

$$Q = \text{rate of flow} = 50 \text{ litres/s} = \frac{50}{1000} = 0.05 \text{ m}^3\text{/s}$$

Let the datum line passes through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3}$ m

Also we know $Q = A_1 V_1 = A_2 V_2$

$$\therefore \quad V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768 \text{ m/sec} = 0.177 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074 \text{ m/sec} = 0.707 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

or

$$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

or

$$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or

$$p_2 = 23.3 \times 9810 \text{ N/m}^2 = 228573 \text{ N/m}^2 = \mathbf{22.857 \text{ N/cm}^2}. \textbf{ Ans.}$$

▶ 6.6 BERNOULLI'S EQUATION FOR REAL FLUID

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \qquad ...(6.5)$$

where h_L is loss of energy between points 1 and 2.

Problem 6.7 *A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.*

Solution. Given :

Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

Velocity, $V = 25 \text{ m/s}$

At point A, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$

$\therefore$ Total energy at A, $E_A = \dfrac{p_A}{\rho g} + \dfrac{v_A^2}{2g} + z_A$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

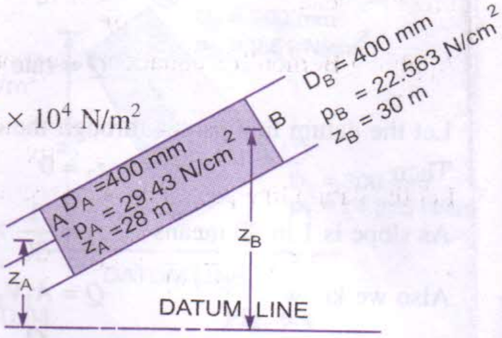


Fig. 6.6

At point B, $p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$

$z_B = 30 \text{ m}$

$v_B = v = v_A = 25 \text{ m/s}$

$\therefore$ Total energy at B, $E_B = \dfrac{p_B}{\rho g} + \dfrac{v_B^2}{2g} + z_B$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

$\therefore$ Loss of energy $= E_A - E_B = 89.85 - 84.85 = \mathbf{5.0 \text{ m.}}$ **Ans.**

Problem 6.8 *A conical tube of length 2.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is* $\dfrac{0.35(v_1 - v_2)^2}{2g}$, *where v_1 is the velocity at the smaller end and v_2 at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.*

Solution. Let the smaller end is represented by (1) and lower end by (2)

Given :

Length of tube, $L = 2.0 \text{ m}$

$v_1 = 5 \text{ m/s}$

$p_1/\rho g = 2.5 \text{ m of liquid}$

$v_2 = 2 \text{ m/s}$

Loss of head $= h_L = \dfrac{0.35(v_1 - v_2)^2}{2g}$

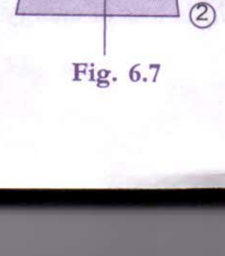


Fig. 6.7

$$= \frac{0.35\,[5-2]^2}{2g} = \frac{0.35 \times 9}{2 \times 9.81} = 0.16 \text{ m}$$

Pressure head, $\dfrac{p_2}{\rho g} = ?$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Let the datum line passes through section (2). Then $z_2 = 0$, $z_1 = 2.0$

$$\therefore \quad 2.5 + \frac{5^2}{2 \times 9.81} + 2.0 = \frac{p_2}{\rho g} + \frac{2^2}{2 \times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{p_2}{\rho g} + 0.203 + .16$$

or $$\frac{p_2}{\rho g} = (2.5 + 1.27 + 2.0) - (.203 + .16)$$

$$= 5.77 - .363 = \mathbf{5.407 \text{ m of fluid. Ans.}}$$

Problem 6.9 *A pipeline carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 metres at a higher level. If the pressures at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 litres/s determine the loss of head and direction of flow.*

Solution. Discharge, $Q = 200 \text{ lit/s} = 0.2 \text{ m}^3/\text{s}$

Sp. gr. of oil $= 0.87$

$\therefore$ ρ for oil $= .87 \times 1000 = 870 \dfrac{\text{kg}}{\text{m}^3}$

Given : **At section *A*,** $D_A = 200 \text{ mm} = 0.2 \text{ m}$

Area, $A_A = \dfrac{\pi}{4}(D_A)^2 = \dfrac{\pi}{4}(.2)^2$

$= 0.0314 \text{ m}^2$

$p_A = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

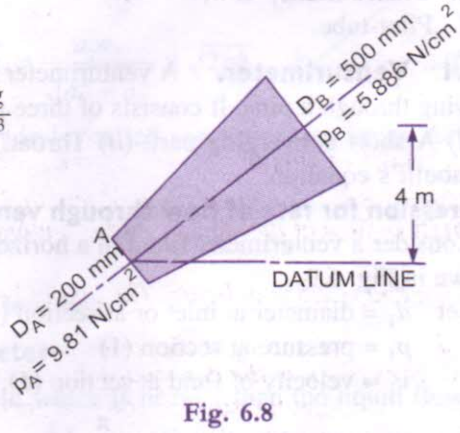


Fig. 6.8

If datum line is passing through *A*, then

$Z_A = 0$

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369 \text{ m/s}$$

At section *B*, $D_B = 500 \text{ mm} = 0.50 \text{ m}$

Area, $A_B = \dfrac{\pi}{4} D_B{}^2 = \dfrac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

$p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$

$$Z_B = 4.0 \text{ m}$$

$$V_B = \frac{Q}{\text{Area}} = \frac{0.2}{.1963} = 1.018 \text{ m/s}$$

Total energy at A $\quad = E_A = \dfrac{p_A}{\rho g} + \dfrac{V_A^2}{2g} + Z_A$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0 = 11.49 + 2.067 = 13.557 \text{ m}$$

Total energy at B $\quad = E_B = \dfrac{p_B}{\rho g} + \dfrac{V_B^2}{2g} + Z_B$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4.0 = 6.896 + 0.052 + 4.0 = 10.948 \text{ m}$$

(*i*) **Direction of flow.** As E_A is more than E_B and hence flow is taking place from A to B. **Ans.**

(*ii*) Loss of head $= h_L = E_A - E_B = 13.557 - 10.948 =$ **2.609 m. Ans.**

▶ 6.7 PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) $= \dfrac{\pi}{4} d_1^{\,2}$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

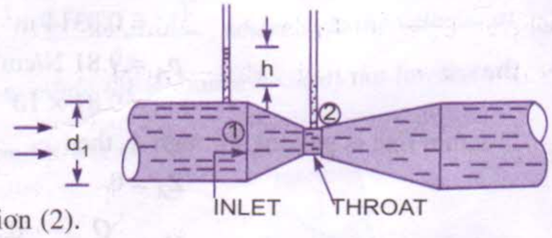


Fig. 6.9 ***Venturimeter.***

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$