3(i) Q.17, Q.19.

Q.9, Q.10, Q.11

Dt.
Pg. B+

Partial differentiation | 3(a), 3(b), 3(c), 3(e), 3(f), 3(i)

outer limit point to point vary करेगा
inner limit function to function vary करेगा

1. Evaluate the following integral:-

(i) $\int_0^1 \int_0^{\sqrt{a^2-x^2}} y \, dy \, dx$

Ans. $\int_0^1 \left| \frac{y^2}{2} \right|_0^{\sqrt{a^2-x^2}} dx = \int_0^1 \frac{a^2-x^2}{2} dx = \left| \frac{a^2x - \frac{x^3}{3}}{2} \right|_0^1$

$= \frac{a^2 - \frac{1}{3}}{2}$

(ii) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy \, dx}{1+x^2+y^2}$

Ans. $\int_0^1 \frac{1}{\sqrt{1+x^2}} \left| \tan^{-1}\left(\frac{y}{\sqrt{1+x^2}}\right) \right|_0^{\sqrt{1+x^2}} dx$

$\int_0^1 \frac{1}{\sqrt{1+x^2}} \left(\frac{\pi}{4} - 0\right) dx$

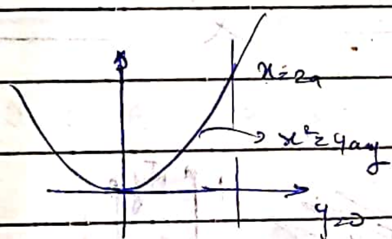
(iii) $\int_0^{2a} \int_{y^2/4a}^{3a-y} (x^2+y^2) \, dx \, dy$

Dt.
Pg.
B+

3(g), 3(i) Ans.
$$\int_0^{2a}\left[\frac{x^3}{3}+y^2x\right]_{\frac{y^2}{4a}}^{3a-y} dy$$

$$\int_0^{2a}(3a-y)\left[\frac{(3a-y)^2}{3}+y^2\right]-\frac{y^2}{4a}\left[\frac{y^4}{16a^2\times 3}+y^2\right]dy$$

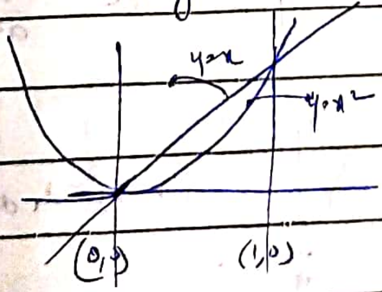
Ques. Given $\iint_A xy\,dx\,dy$ where a is the region bound by x-axis, ordinate $(x=2a)$ and $\boxed{x^2=4ay}$.

Ans.


$$\int_0^{2a}\int_0^{x^2/4a} xy\,dx\,dy$$

$$\boxed{x^2=4ay}$$

Ques. evaluate $\iint x\,y(x+y)\,dx\,dy$ over the area b/w $y=x^2$ and $y=x$.

Ans.


$$\int_0^1\int_{x^2}^{x} xy(x+y)\,dx\,dy$$
x → upper
x² → lower

$$\int_0^1\int_{x^2}^{x} x^2y+xy^2\,dx\,dy = \int_0^1\left|\frac{x^2y^2}{2}+\frac{xy^3}{3}\right|_{x^2}^{x} dx$$

SUCCESS

Dt.
Pg. B+

Ans. $\int_0^1 \frac{x^2}{2}(x^2 - x^4) + \frac{x}{3}(x^3 - x^6)\, dx$

$\int_0^1 \frac{1}{2}(x^4 - x^6) + \frac{1}{3}(x^4 - x^7)\, dx$

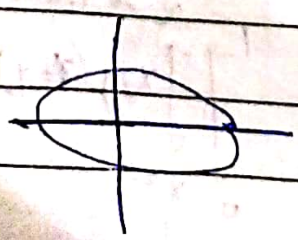
$\frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \frac{1}{3}\left(\frac{1}{5} - \frac{1}{8}\right)$

$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{35} + \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{40} = \frac{1}{35} + \frac{1}{40} = \frac{\cancel{75}^{\,\cancel{15}\,3}}{\cancel{35 \times 40}_{\,7 \ 8}} = \frac{3}{56}$

$\cancel{\int_0^1 \int_y^{\sqrt{y}} xy(x+y)\, dx\, dy}$ $\qquad \int_0^1 \int_y^{\sqrt{y}} xy(x+y)\, dx\, dy$

Ques. Evaluate :- $\iint (x+y)^2\, dx\, dy$
over the area bounded by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Ans.



$\int_{-a}^{a} \int (x+y)^2\, dx\, dy$

Dt.

Pg.

B+

Ques. $\iint_R e^{ax+by}\,dx\,dy$

Where R is the region bounded by the lines $x=0$, $y=0$, $ax+by=1$

Ans. $\int_0^{1/a}\int_0^{\frac{1-ax}{b}} e^{ax+by}\,dx\,dy.$

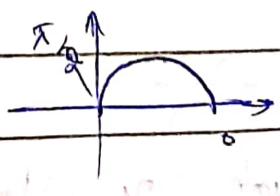
$$\int_{\theta_1=\alpha}^{\theta_2=\beta}\int_{f_1(\theta)}^{f_2(\theta)} f(r,\theta)\,dr\,d\theta$$

Ques. Ques The area b/w the circles $r=a\sin\theta$, $r=b\sin\theta$

Ans. $\int_0^{\pi}\int_{a\sin\theta}^{b\sin\theta} r^3\,dr\,d\theta$ $\boxed{\frac{3(a^4-b^4)\pi}{64}}$

Ques. $\iint_R r^2\sin\theta\,dr\,d\theta$ where R is the region bounded by semicircle. $\boxed{r=2a\cos\theta}$ above the initial line.

Ans. $\int_0^{\pi/2}\int_0^{2a\cos\theta} r^2\sin\theta\,dr\,d\theta$

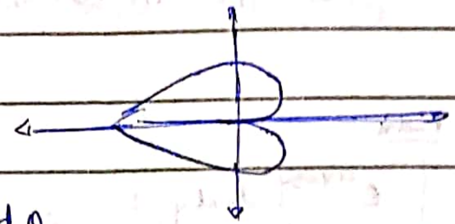


Dt.
Pg.

B+

Ques. Integrate $r\sin\theta$ over the area of quadrant $r = a(1-\cos\theta)$ above the initial line.

Ans.



$$\int_0^{\pi}\int_0^{a(1-\cos\theta)} r\sin\theta \, dr\, d\theta$$

How to convert into polar form :-

$x = r\cos\theta \quad y = r\sin\theta \quad \int dx\,dy = r\,dr\,d\theta$

replace करना है

Ques. $$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dx\,dy$$

Imp. Polar form में बदलो

$$\int_0^{\pi/2}\int_0^{\infty} e^{-r^2}\, r\,dr\,d\theta$$

$$-\frac{1}{2}\int_0^{\pi/2} \left| e^{-r^2} \right|_0^{\infty} = \frac{\pi}{4}$$

Ques. Evaluate the following integral by changing into polar coordinates.

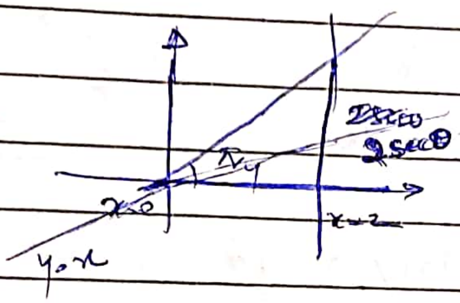
$$\int^{2}\int^{x} y\,dy\,dx$$

Dt.
Pg. B+

Ans.
$$\int_0^{\pi/4}\int_0^{2\sec\theta} r\sin\theta \, r\,d\theta\,dr$$

$y = r\sin\theta$

$x = r\cos\theta$



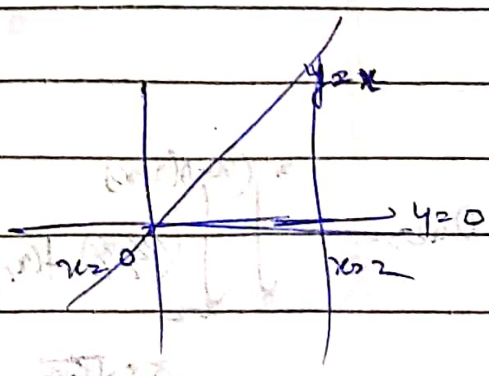
Change of order of integeration :-

$$\int_{x=a}^{x=b}\int_{y=f_1(x)}^{y=f_2(x)} f(x,y)\,dx\,dy$$

$$\int_{y_1}^{y_2}\int_{x=f_1(y)}^{x_2=f_2(y)} f(x,y)\,dy\,dx$$

Change order of integeration in the following integral.

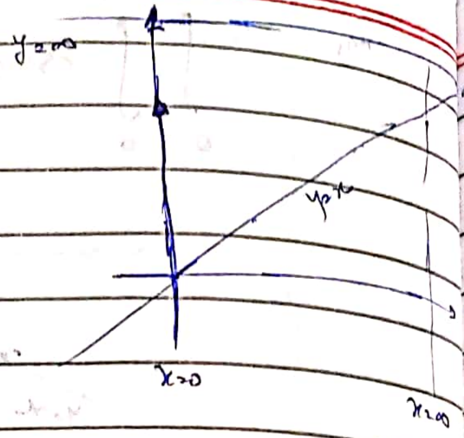
Ques. $\int_0^2\int_0^x f(x,y)\,dx\,dy$



Ans.

$$\int_0^2\int_y^2 f(x,y)\,dy\,dx.$$

Ques $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dx \, dy$



Ans. $\int_0^\infty \int_0^y \frac{e^{-y}}{y} \, dy \, dx$

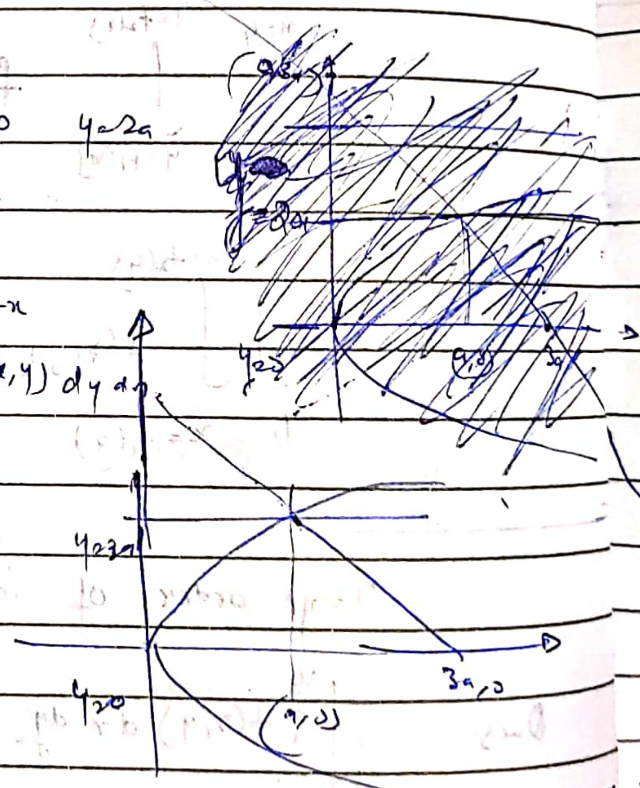
Ques. $\int_0^{2a} \int_{y^2/4a}^{3a-y} f(x,y) \, dx \, dy$

##

Ans $y^2 = 4ax$ $\quad y=0 \quad y=2a$

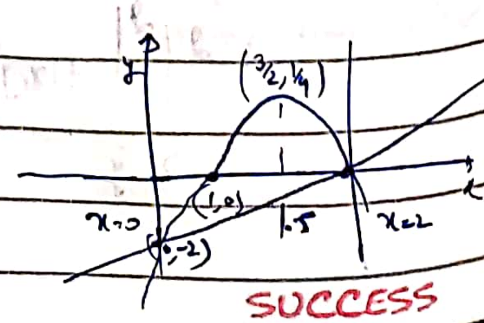
$x+y=3a$

$\int_0^a \int_0^{\sqrt{4ax}} f(x,y) \, dy \, dx + \int_a^{2a} \int_0^{3a-x} f(x,y) \, dy \, dx$



Ques. $\int_0^2 \int_{x-2}^{(x-1)(2-x)} f(x,y) \, dy \, dx.$

Ans $\frac{3}{2} \pm \frac{\sqrt{1-4y}}{2}$



SUCCESS

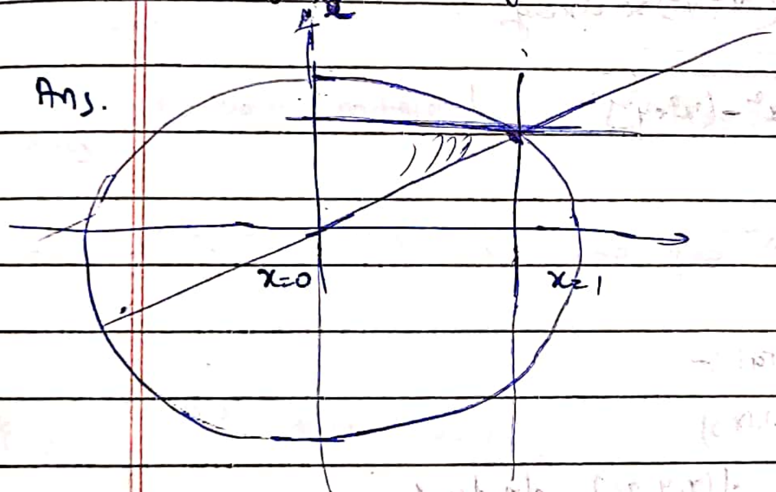
$$\int_{-2}^{0}\int_{\frac{3-\sqrt{1-4y}}{2}}^{y+2} f(x,y)\,dx\,dy + \int_{0}^{1/4}\int_{\frac{3-\sqrt{1-4y}}{2}}^{\frac{3+\sqrt{1-4y}}{2}} f(x,y)\,dx\,dy$$

Ques. Evaluate the following integral

(i)

$$\int_{0}^{1}\int_{x}^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$$

Ans.



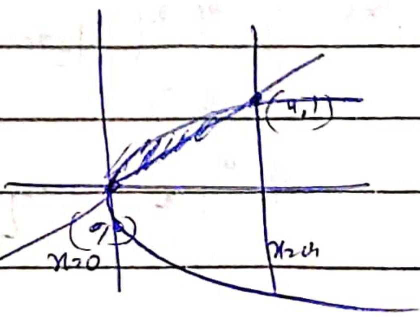
$y^2+x^2=2$

$$\int_{0}^{1}\int_{0}^{y} f(x,y)\,dy\,dx + \int_{1}^{\sqrt{2}}\int_{0}^{\sqrt{2-y^2}} f(x,y)\,dy\,dx.$$

Ques. $\displaystyle\int_{0}^{a}\int_{x/a}^{\sqrt{x/a}} (x^2+y^2)\,dx\,dy$

Ans. $y = x-2$

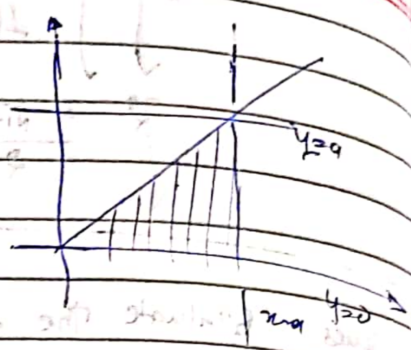
$$\int_{0}^{1}\int_{ay^2}^{ay} (x^2+y^2)\,dy\,dx$$



Dt.
Pg.

Ques- $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$

Ans.



Ques. $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \frac{\phi'(y)(x^2+y^2)x\,dx\,dy}{\sqrt{4a^2x^2-(x^2+y^2)^2}}$

##

#

Multiple Integral :-

$$\int_{x=x_1}^{x=x_2} \int_{y=f_1(x)}^{y_2=f_2(x)} \int_{z=f_1(x,y)}^{z=f_2(x,y)} f(x,y,z)\,dx\,dy\,dz$$

Ques. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$

Ans. $\int_0^a \int_0^x \left|e^{x+y+z}\right|_0^{x+y} dy\,dz$

$$\int_0^a \int_0^x \left(e^{2x+2y} - e^{x+y}\right) dy\,dz = \int_0^a \left| \frac{e^{2x+2y}}{2} - \frac{e^{x+y}}{1} \right|_0^x dy\,dz$$

SUCCESS

Dt. ______
Pg. ______

$$\int_0^a \left| \frac{e^{4x}-1}{2} - (e^{2x}-1) \right| dx\, dy$$

$$\frac{1}{8}\left(e^{3x}\right) - \frac{x}{2}\Big|_0^a + \left| -\frac{e^{2x}}{2} + x \right|_0^a$$

$$\frac{e^{4a}}{8} - \frac{1}{8} - \frac{a}{2} - 0 - \frac{e^{2a}}{2} + \frac{1}{2} + a$$

$$\frac{e^{4a}}{8} + \frac{a}{2} - \frac{e^{2a}}{2} + \frac{3}{8}$$

Ques. Evaluate integral $\iiint (x+y+z)\, dx\, dy\, dz$

over the region $x \geq 0, y \geq 0, z \geq 0$ and $x+y+z \leq 1$,

Ans Put $z=0$ $\quad x+y=1$

$x=0 \quad y=0$

Plane का fig. बनाने की जरूरत नहीं है।

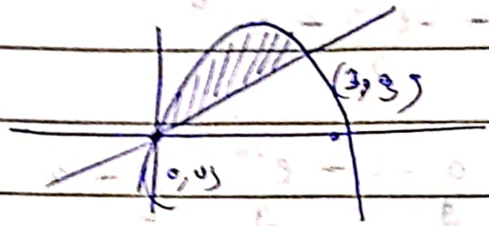
$x \in (0, 1)$

$y = (0, 1-x)$

$z = (0, 1-x-y)$

Ques. Find by double integration the area ~~line~~ of parallel
$y = 4x - x^2$
$y - x = 0$

Ans.



$x = 4x - x^2$
$x^2 = 3x$

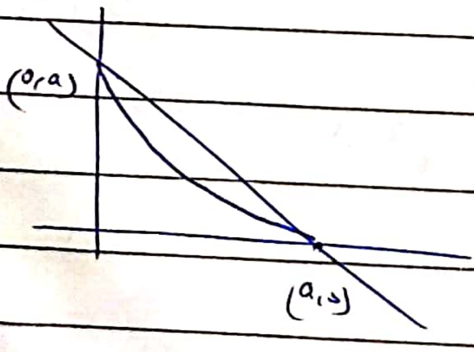
$$\int_0^3 \int_x^{4x-x^2} dx\,dy$$

$$\int_0^3 4x - x^2 - x$$

$$\left| \frac{3x^2}{2} - \frac{x^3}{3} \right|_0^3 = \frac{27}{2} - \frac{27}{3} = \frac{27}{6} = \frac{9}{2}$$

Ques. Find by double integration the area of region closed by curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and $x + y = a$

Ans.



$$\int_0^a \int_{(\sqrt{a}-\sqrt{x})^2}^{a-x} dx\,dy.$$

B+

Dt.
Pg.

B+

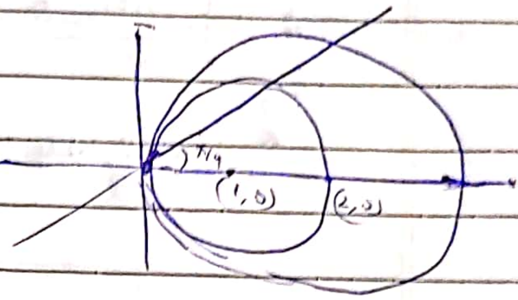
Ques Change into polar coordinates find the area bounded by curve $\boxed{x^2+y^2 = 2x}$ $\boxed{x^2+y^2 = 4x}$ $y = x$ and $y = 0$

Ans. $r^2 = 2r\cos\theta$

$\boxed{r = 2\cos\theta}$

$\boxed{r = 4\cos\theta}$

$\boxed{\theta = \pi/4}$ $\boxed{\theta = 0}$



$$\int_0^{\pi/4}\int_{2\cos\theta}^{4\cos\theta} r\,dr\,d\theta =$$

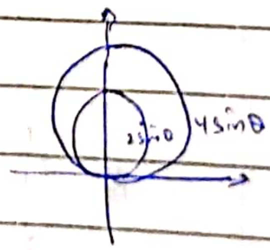
x ——— x

Ques. (i) Find the area bounded by circles $r = 2\sin\theta$ and $r = 4\sin\theta$

(ii) Find by double integeration the area of one loop of the curve $r^2 = a^2\cos 2\theta$

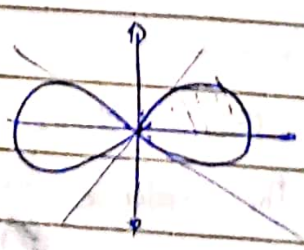
Ans. (i)

$$\int_0^{\pi}\int_{2\sin\theta}^{4\sin\theta} r\,dr\,d\theta$$



(ii)

$$2\int_0^{\pi/4}\int_0^{a\sqrt{\cos 2\theta}} r\,dr\,d\theta$$



Ans (i)

$$\int_0^{\pi}\left|\frac{r^2}{2}\right|_{2\sin\theta}^{4\sin\theta} d\theta = \int_0^{\pi}\frac{16\sin^2\theta - 4\sin^2\theta\,d\theta}{2}$$

$$\int^{\pi}\frac{6}{2}(1-\cos 2\theta)\,d\theta$$

Dt.
Pg.

B+

$$3\left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_0^{\pi} = 3\pi$$

Ans 2

$$\cancel{2}\int_0^{\pi/4} \frac{a^2\cos 2\theta}{\cancel{2}}\, d\theta$$

$$\int_0^{\pi/4} \frac{a^2 \sin 2\theta}{2} = \boxed{\frac{a^2}{2}}$$

Ques. Find by double integration, the volume of sphere

$$x^2+y^2+z^2 = 4$$

Ans. Put $z=0$

$$2\int_0^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2}\, dy\, dx$$

$$\int_0^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dx\, dy\, dz$$

$$8\int_0^{\pi/2}\int_0^{2} \sqrt{4-r^2}\, r\, dr\, d\theta$$

Ques. Find the volume bounded by the coordinate plane and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Ans.

$$\int_0^{a}\int_0^{b(1-x/a)} \left(1 - \frac{x}{a} - \frac{y}{b}\right) c\, dx\, dy$$

SUCCESS

Dt. ______
Pg. ______ B+

B+

Ques. Compute the volume of region bounded by surface $z = 4 - x^2 - y^2$ and the xy plane.

Ans.
$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{+\sqrt{4-x^2}}\int_{0}^{4-x^2-y^2} dx\,dy\,dz \quad \text{or} \quad 4\int_{0}^{\pi/2}\int_{0}^{2}(4-r^2)\,r\,dr\,d\theta$$

$$\cancel{2}\!/\!\cancel{4}\int_{0}^{\pi/2}\left(4t - \frac{t^2}{\cancel{2}}\right)\Bigg|_{0}^{4}$$

$$2\int_{0}^{\pi/2}\left(16 - \frac{16}{\cancel{2}}\right)$$

$$8 \times \cancel{2}\,\pi/\cancel{2} = 8\pi$$

sphere

Ques. Find the volume cut off by sphere $x^2+y^2+z^2=a^2$ by the cylinder $x^2+y^2=ax$.

Ans.
$$2 \times 2\int_{0}^{\pi/2}\int_{0}^{a\cos\theta}(a^2-r^2)\,r\,dr\,d\theta$$

$$4\int_{0}^{\pi/2}\frac{1}{2}\left(a^2 t - \frac{t^2}{2}\right)\Bigg|_{0}^{a^2\cos^2\theta} d\theta$$

$$\cancel{4}^{2}\int_{0}^{\pi/2}\frac{1}{\cancel{2}}\left(a^2 a^2\cos^2\theta - \frac{a^4\cos^4\theta}{2}\right)d\theta$$

$$\int_{0}^{\pi/2}\left(2a^3\cos^2\theta - a^3\cos^4\theta\right)d\theta$$

a^4 (solve by γ function)

plane

SUCCESS

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Dt. ______ B+
Pg. ______

$$\int_0^a \int_{-\sqrt{ax-x^2}}^{\sqrt{ax-x^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dx\, dy\, dz$$

x ——————— X

Ques. ##

$2\pi \iint y\, dx\, dy \rightarrow$ x-axis

$2\pi \iint x\, dx\, dy \rightarrow$ y-axis

Ques- (i) Calculate by double integration the volume generated by revolution of cardiod $r = a(1-\cos\theta)$ about its axis

(ii) Find volume of solid bounded by paraboloid $y^2+z^2 = 4ax$ and the plane $x=5$.

(iii) Find volume by cylinder $x^2+y^2=4$ and plane $y+z=4$ and $z=0$

(iv) Find the volume of cylindrical column standing on area common to the parabolas $y^2=x$ and $x^2=y$ and cut by the surface $z = 12 + (y - x^2)$

$\Gamma n = \int_0^\infty e^{-t}\, t^{n-1}\, dt$

Ques. $\Gamma(n+1) = n\Gamma n$

Proof :- $\int_0^\infty e^{-t}\, t^{n}\, dt$

$\left[-e^{-t} t^n\right]_0^\infty + n\int t^{n-1} e^{-t}\, dt = n\Gamma n$

SUCCESS

formulas.

$\Gamma_{n+1} = n!$

$\Gamma_{1/2} = \sqrt{\pi}$

Proof $\to$ $\int_0^\infty e^{-t} t^{1/2-1} dt = \int_0^\infty e^{-t} t^{-1/2} dt$

$t \Rightarrow x^2$

$\Gamma_{\frac{1}{2}} = 2\int_0^\infty e^{-x^2} dx$

$\Gamma_{\frac{1}{2}} = 2\int_0^\infty e^{-y^2} dy$

$\left(\Gamma_{\frac{1}{2}}\right)^2 = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} dy\, dx$

$x = r\cos\theta \quad y = r\sin\theta$

$$\boxed{\Gamma_{\frac{1}{2}} = \sqrt{\pi}}$$

$$\boxed{4\int_0^{\pi/2}\int_0^\infty e^{-r^2} r\,dr\,d\theta = \pi}$$

$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$

$= \int_0^1 (1-x)^{m-1} x^{n-1} dx = \beta(n,m)$

Proof

$\beta(m,n) = \frac{\Gamma_m \Gamma_n}{\Gamma_{m+n}}$

$\Gamma_m = \int_0^\infty e^{-t} t^{m-1} dt$

$\boxed{t = x^2}$

$\Gamma_m = 2\int_0^\infty e^{-x^2} x^{2m-1} dx$

$\Gamma_n = 2\int_0^\infty e^{-y^2} y^{2n-1} dy$

SUCCESS

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Dt
Pg
B+

$$\Gamma m \, \Gamma n = 4 \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} x^{2m-1} y^{2n-1} \, dx \, dy$$

$\downarrow$

convert into polar.

$$\Gamma m \, \Gamma n = 4 \int_0^{\pi/2} \int_0^\infty e^{-r^2} \, dr \, d\theta \, \cos\theta^{2m-1} \sin\theta^{2n-1} \, r^{2m+2n-1}$$

$$= 2 \int_0^{\pi/2} \int_0^\infty e^{-r^2} r^{2m+2n-2} \, 2r\,dr \, \sin\theta^{2n-1} \cos\theta^{2m-1} \, d\theta$$

$r^2 = t$

$$= 2 \, \Gamma m{+}n \int_0^{\pi/2} \sin\theta^{2n-1} \cos\theta^{2m-1} \, d\theta$$

$\sin^2\theta = t$

$$= \Gamma m{+}n \int_0^{\pi/2} t^{n-1} (1-t)^{m-1} \, dt$$

$$\boxed{\beta(n,m) = \beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m{+}n}}$$

$\sin^2\theta = t$

$2\sin\theta\cos\theta = \frac{dt}{d\theta}$

$$\Gamma m \, \Gamma n = 2 \, \Gamma m{+}n \int_0^{\pi/2} (\sin\theta)^{2n-1} \cos\theta^{2m-1} \, d\theta$$

$2n-1 = p \qquad 2m-1 = q$

$n = \frac{p+1}{2} \qquad m = \frac{q+1}{2}$

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2} \, \Gamma\frac{q+1}{2}}{2 \, \Gamma\frac{p+q+2}{2}} = \frac{1}{2} \left(\frac{\frac{p-1}{2}! \; \frac{q-1}{2}!}{\frac{p+q}{2}!} \right)$$

Ques
Ans.
Ques
Ans
Ques
Ans

Ques. $x = \frac{y}{1+y}$

Ans. $x + xy = y$

$\frac{x}{1-x} = y$

Ques Evaluate $\int_0^\infty x^4 e^{-x^2} dx$

Ans $x^2 = t$ $\qquad 2x\,dx = dt$

$$\int_0^\infty t^2 e^{-t} \frac{dt}{2\sqrt{t}} = \int_0^\infty \frac{t^{3/2}}{2} e^{-t} dt$$

$$\Gamma_{5/2} = \frac{1}{2} \int_0^\infty t^{5/2-1} e^{-t} dt$$

Ques. $\int_0^\infty e^{-k^2 x^2} dx$

Ans $k^2 x^2 = t$ $\qquad 2k^2 x\, dx = dt$ $\qquad dx = \frac{dt}{2k^2 \sqrt{t}} \times$

Dt.
Pg.

B+

$$\int_0^\infty e^{-t}\,\frac{dt}{2k}\,t^{-1/2} = \frac{\Gamma\frac{1}{2}}{2k}$$

x ———— x

Ques. $$\int_0^\infty \frac{x^c}{c^x}\,dx = \frac{\Gamma c+1}{(\log c)^{c+1}}$$

Ans. $$\int_0^\infty x^c c^{-x}\,dx$$

$$\int_0^\infty x^c e^{\log c^{-x}}\,dx = \int_0^\infty x^c e^{-x\log c}\,dx$$

$$\int_0^\infty \left(\frac{t}{\log c}\right)^c e^{-t}\,\frac{dt}{\log c} = \frac{\Gamma c+1}{(\log c)^{c+1}}$$

$x\log c = t$

$dx = \frac{dt}{\log c}$

x ———— x

Ques. $$\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1} dy = \frac{\Gamma p}{q^p}$$ $\boxed{\begin{matrix} p>0 \\ q>0 \end{matrix}}$

Ans.

$y = e^{-t}$ $\quad -\log y = t$

$dy = -e^{-t}\,dt$

$$-\int_\infty^0 e^{-t(q-1)}\,e^{-t}\,dt\;t^{(p-1)} \qquad \int_0^\infty e^{-t(q)}\,dt\;t^{p-1}\,dt$$

$qt = x$

$dt = \frac{dx}{q}$

$$\int \frac{dx}{q}\,e^{-x}\left(\frac{x}{q}\right)^{p-1} = \frac{\Gamma p}{q^p}$$

SUCCESS

Ques. $\beta(n,n) = \dfrac{\sqrt{\pi}\,\Gamma n}{2^{2n-1}\,\Gamma\left(n+\frac{1}{2}\right)}$

Ans. $\beta(n,n) = \dfrac{\Gamma n\,\Gamma n}{\Gamma 2n} = \dfrac{\cancel{(n-1)(n-2)\ldots\ldots 1}\;\Gamma n}{(2n-1)(2n-2)\ldots\ldots \cdot (n-1)(n-2)\ldots 1}$

(even vali term उपर cancel हो जाएगी)

$\dfrac{\Gamma n \quad \Gamma\frac{1}{2}}{2^{2n-1}\left(n-\frac{1}{2}\right)\left(n-\frac{3}{2}\right)\left(n-\frac{5}{2}\right) \ldots \frac{1}{2}\,\Gamma\frac{1}{2}} = \dfrac{\Gamma n\sqrt{\pi}}{2^{2n-1}\,\Gamma\left(n+\frac{1}{2}\right)}$

Ques $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n}\,\frac{\Gamma\frac{1}{n}}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$

Ans $\displaystyle\frac{2}{n}\int_0^{\pi/2} \frac{\sin^{2/n-1}\theta\,\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}$ Put $x^n = \sin^2\theta$

$x = \sin^{2/n}\theta$

Ans. $\boxed{\dfrac{2}{n}\cdot\dfrac{1}{2}\cdot\dfrac{\Gamma\frac{1}{n}}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}}$

4(a), 4(b), 4(c), 4(d), 4(e),

Ex. 12 b Q1–20

Dt.
Pg. B+

1) Solve the eqⁿ using Rows by Partial Pivoting

$x_1 + x_2 + x_3 = 6$

$3x_1 + 3x_2 + 4x_3 = 20$

$2x_1 + x_2 + 3x_3 = 30$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 30 \end{array}\right]$$

पहले first column में जो बड़ा है वो ऊपर आ जायेगा, फिर solve करेगा बाद second column में जो बड़ा होगा वो R_2 से replace करेंगे similarly solve it.

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 3 & 3 & 4 & 20 \\ 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 30 \end{array}\right]$$

$R_1 \to R_1/3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 4/3 & 20/3 \\ 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 30 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 4/3 & 20/3 \\ 0 & 0 & -1/3 & -2/3 \\ 2 & 1 & 3 & 30 \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 4/3 & 20/3 \\ 2 & 1 & 3 & 30 \\ 0 & 0 & -1/3 & -2/3 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

Dt.
Pg.

B+

$$\left[\begin{array}{ccc|c} 1 & 1 & 4/3 & 20/3 \\ 0 & -1 & 1/3 & 50/3 \\ 0 & 0 & -1/3 & 2/3 \end{array}\right]$$

$3 - 8/3^2$
$30 - 40/3^2$

LU decomposition method :-

If $U_{ii} = 1$ — Crout's method,
$L_{ii} = 1$ — Doolittle method.

$A = LU$

$L \rightarrow$ lower triangular
$U \rightarrow$ upper triangular.

Use decomposition method solve :-

Ques

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

Ans

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$LU = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21} u_{11} & l_{21} u_{12} + u_{22} & l_{21} u_{13} + u_{23} \\ l_{31} u_{11} & l_{31} \times u_{12} + l_{32} u_{22} & l_{31} \times u_{13} + l_{32} u_{23} + u_{33} \end{bmatrix}$$

SUCCESS

Dt
Pg.

B+

$u_{11} = 1$

$u_{12} = 1$

$u_{13} = 1$

$Ax = B$

$l_{21} = 4$

$u_{22} = -1$

$u_{23} = -5$

$l_{31} = 3$

$= l_{32} \times -1 + 3 \times 1 = 3$

$l_{32} = 0$

$\cancel{3 \times 1} + l_{32} \times -5 + u_{33} = \cancel{3}$

$\boxed{u_{33} = 0}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$L\boxed{UX} = B$

let $UX = z$

Gauss Seidel :-

Ques. Solve the following eqn by Gauss Seidel method upto 2 eqn

i) $27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

Ans.

Ques.

Ans.

Dt.
Pg.
B+

Ans.

$$x = \frac{1}{27}(85 + z - 6y)$$

$$y = \frac{1}{15}(72 - 2z - 6x)$$

$$z = \frac{1}{54}(110 - x - y)$$

(i)

$$x_1 = \frac{85}{27}$$

$$y_1 = \frac{1}{15}\left(72 - \frac{\cancel{6}^{2} \times 85}{\cancel{27}_{9}}\right)$$

$$y_1 = \frac{1}{15}\left(\frac{288 - 170}{9}\right) = \frac{118}{9 \times 15}$$

$$z_1 = \frac{1}{54}\left(110 - \frac{85}{27} - \frac{118}{9 \times 15}\right)$$

Newton Raphson (Method of tangents)

For coincident root

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad x_{n+1} = x_n - \frac{f'(x_m)}{f''(x_m)}$$

how to find x_0

जहाँ $f(x) f''(x) > 0$

Ques. $x^4 - x - 10 = 0$

Ans. let $x = \underset{-ve}{1}, \underset{+ve}{2}$

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x	1	1.5	1.75	1.85	1.95	2
y	-10	-6.4			2.59	4

$$x_{n+1} = x_n - \frac{(x_n^4 - x_n - 10)}{4x_n^3 - 1}$$

Ques. Use Newton method to solve the eqn

$$3x - \cos x - 1 = 0$$

Ans. for $x = 0$ $\quad 0 - 1 - 1 = -2$

$x = 1$ $\quad 3 - 1 - \sim = +ve$

x	0	0.5	0.75	1
y	-2	-0.127		+ve

$$\frac{0.5 + 0.75}{2} = \frac{1.25}{2} = 0.625$$

$x_1 = 0.60710$

$x_2 = 0.60710$

General Quadrature

$$\int_a^b f(x)\,dx \qquad \text{let } a = x_0, \; x_0 + h, \; \ldots\ldots, \; x_0 + nh = b$$

$$\int_{x_0}^{x_0+nh} f(x)\,dx \qquad u = \frac{x - x_0}{h}$$

$$h\int_0^n f(x_0+hu)\,du$$

$$h\int_0^n \left[f(x_0) + u f'(x)\Delta + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n f(x_0) \right] du$$

$$h\int_0^n \left(y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)\cdots(u-n+1)}{n!}\Delta^n y_0 \right) du$$

$$\boxed{\Delta = \text{Forward difference operator}}$$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta^2 f(x) = f(x+2h) - f(x+h)$$

$$\Delta y_0 = y_1 - y_0$$

$$\Delta^2 y_0 = \Delta(\Delta y_0) = \Delta(y_1 - y_0)$$
$$= \Delta y_1 - \Delta y_0$$

$$\Delta^2 y_0 = y_2 - 2y_1 + y_0$$

$$\int_{x_0}^{x_0+h} f(x)\,dx = h\left(y_0 + \frac{1}{2}(y_1 - y_0)\right)$$

$$= \frac{h}{2}(y_0 + y_1)$$

$$\int_{x+h}^{x+2h} f(x)\,dx = \frac{h}{2}(y_1 + y_2)$$

Pg. B+

$$\int_{x+h}^{x+2h} f(x)\,dx = \frac{h}{2}\left(y_2 + y_3\right)$$

$$\int_{x+(n-1)h}^{x+nh} f(x)\,dx = \frac{h}{2}\left(y_{n-1} + y_n\right)$$

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left(\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + y_3 + \cdots + y_{n-1}\right)\right)$$

$$\int_{x_0}^{x_0+2h} f(x)\,dx = h\left(2y_0 + 2(y_1 - y_0) + \frac{1}{2}\left(\frac{8}{3} - 2\right)(y_2 - 2y_1 + y_0)\right)$$

$$= h\left(2y_1 + \frac{1}{3}\left(y_2 - 2y_1 + y_0\right)\right)$$

$$= \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

$$\int_{x_0+2h}^{x_0+4h} f(x)\,dx = \frac{h}{3}\left(y_2 + 4y_3 + y_4\right)$$

$$\int_{x}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left(\left(\text{First} + \text{Last}\right) + 4\left(\text{odd degree}\right) + 2\left(\text{mid term}_{\text{even}}\right)\right)$$

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$$\int_{x_0}^{x_0+3h} f(x)\,dx = h\left(3y_0 + \frac{9}{2}(y_1 - y_0) + \right.$$

Ques $y = \int_0^1 \frac{dx}{1+x}$

Ans. let $h = 10$

$$u = \frac{1-0}{10} = 0.1$$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
y	1	a	b	c	d	e	f	g	h
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

$$y = \frac{h}{2}\left((1+h) + 2(a+b+c+d+e+f+g\ldots)\right)$$

<u>Euler's Method</u> :-

$$y(x_0) = y_0 \qquad \frac{dy}{dx} = f(x,y)$$

$$\int_{y_0}^{y_1} dy = \int_{x_0}^{x_1} f(x,y)\,dx$$

$$y_1 - y_0 = (x_1 - x_0) f(x_0, y_0)$$

$$y_1 = y_0 + h f(x_0, y_0)$$

$$\boxed{y_{n+1} = y_n + h f(x_n, y_n)}$$

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Ques $\frac{dy}{dx} = \frac{y-x}{y+x}$ $x=0$ $y=1$ $x=c$ $y=?$

Ques. $\frac{dy}{dx} = x+y$ $x=0$ $y=1$ find y when $x=0.20$

Ans. $\boxed{\text{let } h = 0.04}$

$y_{n+1} = y_n + h f(x_0, y_0)$

$y_1 = 1 + (1) 0.04$

$\boxed{y_1 = 1.04}$

$y_2 = 1.04 + 0.04(1.04 + 0.04)$

$y_2 = 1.04 + 0.04(1.08)$

$y_3 = y_2 + 0.04(f(x_0 + y_0))$

Runge Kutta fourth order :

$$y_1 = y_0 + \frac{h}{6}\left(k_1 + 2(k_2 + k_3) + k_4\right)$$

$k_1 = f(x_0, y_0)$

$k_2 = f(x_0 + h/2, y_0 + k_1/2)$

$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_4 = f(x_0 + h, y_0 + k_3)$

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Dt.______ B+

Pg.______

Ques. Apply fourth order method approximate value of y when $x = 0.2$, $y = 1$ at $x = 0$

$$\frac{dy}{dx} = x + y^2 = f(x, y)$$

Ans. let $h = 0.1$

$$k_1 = f(x_0, y_0) = f(0, 1)$$

$$K_1 = 0 + 1 = 1$$

$$k_2 = f(0 + 0.05,\ 1 + 0.5)$$

$$k_2 = f(0.05,\ 1.5)$$

$$= 0.05 + 2.25$$

$$k_2 = 2.30$$

$$k_3 = f(0.05,\ 1 + 1.15)$$

$$k_3 = f(0.05,\ 2.25)$$

$$k_3 = 0.05 + 5.0625 = 5.1125$$

$$k_4 = f(0.05,\ 1 + 2.55625)$$

$$k_4 = f(0.05,\ 6.1125) = 37.4126$$

$$y_1 = 1 + \frac{0.1}{6}\left(1 + 2(2.3 + 5.1125) + 37.4126\right)$$

$$= 1 + \frac{0.1}{6}\left(38.4126 + \dots\right)$$

$$= 1 + \frac{0.1}{6}\left(53.2376\right) = 1.88729$$

When $p \to 0$ and $n \to \infty$ then we apply Poisson's formula

Dt.
Pg.

B+

$$\lim_{n \to \infty} \frac{n!}{r!(n-r)!} \left(\frac{m}{n}\right)^r \left(1-\frac{m}{n}\right)^{n-r}$$

$$\frac{n(n-1)(n-2)\cdots(n-(r-1))}{r!\left(1-\frac{m}{n}\right)^r} \left(\frac{m}{n}\right)^r \left(\left(1-\frac{m}{n}\right)^{-\frac{n}{m}}\right)^{-m}$$

$$\frac{1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{(r-1)}{n}\right) m^r}{r!\left(1-\frac{m}{n}\right)^r} \left(\left(1-\frac{m}{n}\right)^{-\frac{n}{m}}\right)^{-m}$$

$$\frac{1}{r!} m^r e^{-\frac{n}{m}\left(\cancel{1-\frac{m}{m}-1}\right)^{-m}}$$

$$\boxed{P(r) = \frac{m^r e^{-m}}{r!}}$$

Poisson Rule

$$\text{Mean} = e^{-m}\left(\frac{1m}{1} + \frac{2m^2}{2!} + \frac{3m^3}{3!} + \frac{4m^4}{4!} + \cdots\right)$$

$$\text{mean} = e^{-m}m\left(\frac{1}{0!} + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right)$$

$$\boxed{\text{mean} \Rightarrow m}$$

Pg.
B+

$$P(r+1) = \frac{m}{r+1} P(r)$$

Ques. If the variance of the poisson distribution is 2. Find the probability $r = 1, 2, 3, 4$ form the recurrence relation of the poisson distribution. Also find $P(r \geq 4)$

Ans. Poisson distribution की mean and variance same होता है

$$P(1) = \frac{2}{1} P(0) = 2e^{-2}$$

$$P(0) = \frac{e^{-2}\, 2^0}{0!} = e^{-2}$$

$$P(2) = \frac{\cancel{2}}{\cancel{2}} P(1) = P(1) = 2e^{-2}$$

$$P(3) = \frac{2}{3} P(2) = \frac{4e^{-2}}{3}$$

$$P(4) = \frac{\cancel{2}}{\cancel{4}} P(3) = \frac{P(3)}{2} = \frac{2}{3} e^{-2}$$

Ques. Assume that the probability of an individual coal minor be killed in a mine accident in a year $\frac{1}{2400}$ use appropriate distribution to calculate the probability that in a mine include 200 minors there will be atleast one accident in a year

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Pg.

B+

$m = n \times p = \frac{2\cancel{0}}{240\cancel{0}} = \frac{1}{12}$

$P(\text{atleast}) = 1 - P(0) = 1 - \frac{e^{-1/12} (1/12)^0}{0!} = 0.$

x———————x

Ques. Suppose 3% of the ~~boards~~ boults made by a machine are defective. The defects occuring at random during production if boults are packed 50 per box find.

i) exact probability.

ii) poison approx. to it that a given box contain 5 defectives.

Ans. $\boxed{p = \frac{3}{100}}$ $\boxed{n = 50}$

$m = \frac{3 \times \cancel{50}}{\cancel{100}} = \frac{3}{2} = 1.5$

$P(x) = {}^{50}C_5 \left(\frac{3}{100}\right)^5 \left(1 - \frac{3}{100}\right)^{45}$ — exact

$P(5) = \frac{e^{-1.5} (1.5)^5}{5!}$ — approximation.

x———————x

Ques. The no. of arrival of constumers during any day follows poison distribution with the mean of 5. What is the probability that the total no. of customers on two days selected at random is less than 2.

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Ans.

0	0	1
0	1	0

$P(0) \times P(0) + P(0)\,P(1) + P(1)\,P(0)$

Ques. Using poison distribution That the ace of spade drawn from a well suffled card at least once in 104 consecutive trials.

Ans.

$n = 104$

$p = \frac{1}{52}$

$m = \frac{104}{52} = 2$

$P(0) = \frac{e^{-2}\, 2^0}{0!}$

$\boxed{\text{Ans.} = 1 - P(0)} = 1 - e^{-2}$

Ques In a cycle factory there is a small chance of 1 in 500 tyres to be defective. The tyres supplied in lots of 10.

Ans. $p = \frac{1}{500}$ $n = 10$

$m = \frac{1}{50}$

$P(0) = \frac{e^{-m}\,(m)^0}{0!} = e^{-m}$

Pg. B+

$$P(1) = \frac{e^{-1/50} \times (1/50)^1}{1!}$$

$$P(2) = \frac{e^{-1/50} \times (1/50)^2}{2!}$$

$$\text{For 1 lot} = e^{-1/50}\left(\frac{(\frac{1}{50})^0}{0!} + \frac{(\frac{1}{50})^1}{1!} + \frac{(\frac{1}{50})^2}{2!}\right)$$

For 1000 lot = 1000 × (For 1 lot) = 9802

X ——— X

Ques. A company has two cars which has no. of demands for return in each day is distributed as poison distribution with mean (1.5) calculate the no. of days in a year in which is type is not used (ii)

Ans (i) $m = 1.5$

$$P(0) = \frac{e^{-1.5} (1.5)^0}{0!} = e^{-1.5}$$

In a year = $365 \times e^{-1.5} = 81.44$

X ——— X

Ques. If the probability that an individual suffer fods from an certain injection 0.001 determine the probability out of 2000 individuals 1) exactly 3 2) more than 2 (3) none (4) more than 1 individual

SUCCESS

Dt.
Pg. B+

Ques. Suppose that the book of 600 pages contains 40 printing mistakes assume that these errors are randomly distributed throughout the book and x the no. of errors per page has a poison distribution. What is the probability that 10 pages selected at random will be free of errors.

Ans. $m = 600 \times$

A function is said to be probability density function. it
$$f(x) \geq 0 \quad \forall x$$
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_a^b f(x)\,dx = P(a < x < b)$$

Ques. A function is defined as
$$\begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \leq x \leq 4 \\ 0, & x > 4 \end{cases}$$

Show that it is the probability density function.

Ans.
$$\int_2^4 \frac{2x+3}{18} = \left|\frac{\frac{2x^2}{2} + 3x}{18}\right|_2^4 = \frac{(16-4) + 3(2)}{18} = \frac{18}{18} = 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{2} f(x)\,dx + \int_2^4 f(x)\,dx + \int_4^{\infty} f(x)\,dx = 1$$

SUCCESS

Dt.
Pg.
B+

Normal Distribution :-

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The general eqⁿ of the normal distribution is given by this where the variable can from $-\infty$ to ∞ called the parameter of the distribution are, μ and σ repectively the mean and standard deviation of distribution.

$f(x)$ is probability density funcⁿ of distribution. If a variable x is a normal distribution $N(\mu, \sigma^2)$

$$z = \frac{x-\mu}{\sigma}$$

Then $f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$

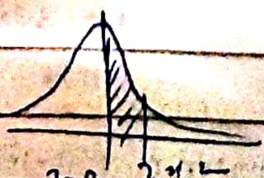
Ques Find the area of the normal curve in each of the cases

i) $z = 0$ $z = 1.2$

ii) $z = 0.68$, $z = 0$

iii) $z = -0.46$, $z = 2.21$

Ans.



$= 0.3849$

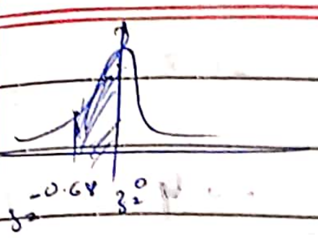
$z=0$ $z=1.2$

SUCCESS

0.1772

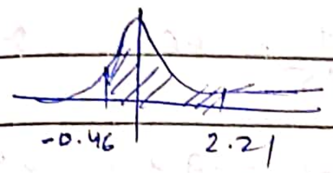
Dt. $\boxed{0.6637}$ B+

Pg.

(ii) 

$\boxed{0.2518}$

$$\begin{array}{r} 0.5 \\ -\ 0.2257 \\ \hline 0.2743 \end{array}$$

(iii) 

$\boxed{0.6637}$

Ques. To the left of $z = -0.6$

Ans. 0.2743 $\qquad 0.5 - 0.2257$

Ques. Well given an test there marks were found to be normally distributed $\mu = 60$ and $\sigma = 5$ question what % of students scored more than 60 marks

Ans. $z = \frac{60 - 60}{5} =$ ~~0~~

$z > 0$ Answer $0.5 \times 100 = 50\%$

Ques. In a sample of 1000 cases the mean of a certain test of 14 and $\sigma = 2.5$. Assuming it normaly distributed. find

i) How many students score 12 and 15.

SUCCESS

Dt.
Pg.
B+

Ans $z_1 = \frac{12-14}{2.5} = \frac{-2}{2.5} = -0.8 = 0.2881$

$z_2 = \frac{15-14}{2.5} = \frac{1}{2.5} = 0.4 = 0.1554 = 0.4435 \times 1000$

$z_1 = \frac{18-14}{2.5} = \frac{4}{2.5} = 1.6 = 0.4452 = 0.5-0.4452 = 0.0548 \times 1000$

$z_1 = \frac{8-14}{2.5} = \frac{-6}{2.5} = -2.4 = 0.4918 = 0.5-0.4918 = 0.0082 \times 1000$

~~# exact case नहीं पूछा है~~

Ques. A manufacture of envelop the weight of envelop is normally distributed with mean = 1.9 gram and variance = 0.01 gram
S.D = 10^{-1} gram.
for 1000 envelope

Ans. $\frac{2-1.9}{0.1} = \frac{0.1}{0.1} = 1 = \boxed{0.3413}$

Ques The life of army shoes is normally distributed with mean 8 months and S.D = 2 months. If 5 thousand pairs are issued. How many pairs for need replacement after 12 months.

$\frac{12-8}{2} = \frac{4}{2} = 2$

$0.5 - 0.4772 = 0.0228 \times 5000 = 114$

$5000 - 114 = 4886$ $\boxed{\text{shoes}}$