Definitions

Academic

Discipline which involves drawing / displaying graphical objects on graphic devices like video Display unit OR a computer or workstation.

- graphical objects gen. using lines / points / curves / shapes / solids & displayed on VDUs / workstations.

Commercial

- Tools used to make pictures or objects.
- Can be both hardware / software.

H/W tools

↳ video monitor, graphics card, printer, mouse, pen, track ball, joystick.

S/W tools

↳ Prog. langs. have a collection of graphic entities that produce pictures OR dedicated languages for graphics.

- A POINT in CG is the smallest displayable object or a pixel.
- OpenGL - open source graphics libraries.
    - Device independent API.

Libraries in open GL -

① Basic GL ② GLU (Utility lib).
③ GLUT (Utility toolkit) ④ GLUI (User interface).

- Basic GL provides f^ns which are a permanent part of open GL.
Every f^n starts with GL prefix.

- GLU → manages WINDOWS, EVENTS, FULL SCREEN RENDERING.

- GLUT → manages high level processing reqd. for drawing of quadratic surfaces.
→ These involve matrix ops.

→ GLUI → provides sophisticated controls to open GL.

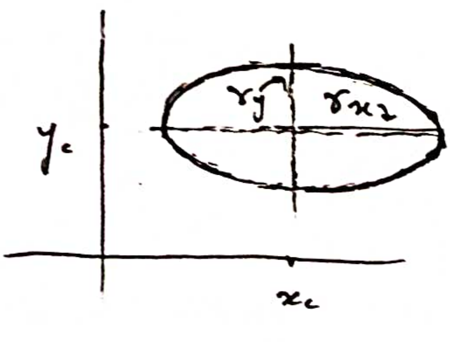
→

General Ellipse Eq^n

$$\frac{(x - x_c)^2}{r_x^2} + \frac{(y - y_c)^2}{r_y^2} = 1$$

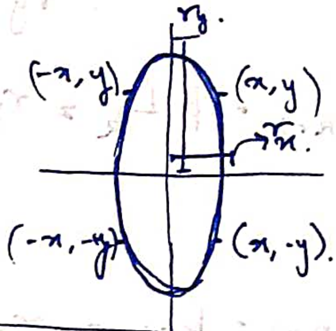
→ when you plot for ellipse centred at x_c, y_c, you shift $x, y \rightarrow (x + x_c, y + y_c)$

→ That's it!



Raster scan Ellipse:

① Mid Point Algo.

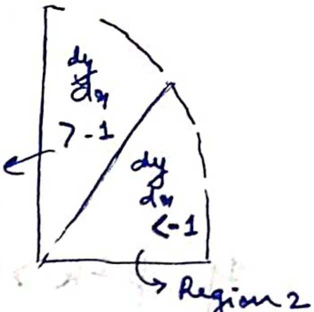


equation $\dfrac{x^2}{r_x^2} + \dfrac{y^2}{r_y^2} = 1$

symmetrical in quadrants.

$$\boxed{f_{ellipse}(x,y) = x^2 r_y^2 + r_x^2 y^2 - r_x^2 r_y^2}$$

$$\boxed{\frac{dy}{dx} = -\frac{r_y^2 x}{r_x^2 y}}$$

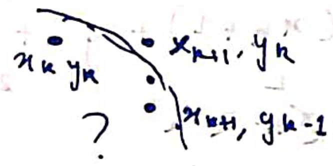


We need to move DOWN.

Region 1.

$(x_k, y_k) \rightarrow x_{k+1}, y_k$ OR x_{k+1}, y_{k-1}

$x_{mid} \rightarrow \dfrac{x_{k+1} + x_{k+1}}{2}$



$y_{mid} \rightarrow \dfrac{y_k + y_{k-1}}{2}$

$\Rightarrow (y_k - \frac{1}{2})$.

Decision parameter in region 1 -

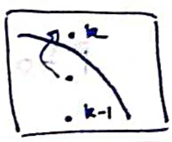
$$P1_k = f_{ellipse}(x_{mid}, y_{mid})$$

$$= f_{ellipse}(x_{k+1}, y_k - \tfrac{1}{2})$$

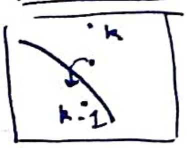
$$= r_y^2 \times (x_{k+1})^2 + r_x^2 (y_k - \tfrac{1}{2})^2 - r_x^2 r_y^2$$

Now, if mid point inside ellipse, plot y_k, else plot y_{k-1}.

$\Rightarrow p1_k < 0$, plot x_{k+1}, y_k



else, plot x_{k+1}, y_{k-1}



Now, $p1_{k+1} = r_y^2(x_{k+1}+1)^2 + r_x^2(y_{k+1}-\frac{1}{2})^2 - r_x^2 r_y^2$

$$= r_y^2(x_k+1+1)^2 + r_x^2(y_{k+1}-\tfrac{1}{2})^2 - r_x^2 r_y^2$$

$$p1_{k+1} - p1_k = r_y^2(x_k+1+1)^2 + r_x^2(y_{k+1}-\tfrac{1}{2})^2 - \cancel{r_x^2 r_y^2}$$
$$- r_y^2(x_k+1)^2 - r_x^2(y_k-\tfrac{1}{2})^2 + \cancel{r_x^2 r_y^2}$$

$$\Rightarrow p1_{k+1} - p1_k = r_y^2[\cancel{x_k}+2-\cancel{x_k}-1][2x_k+3]$$
$$+ r_x^2(y_{k+1}-y_k)(y_{k+1}+y_k-1)$$
$$= 2r_y^2 \cdot [x_k+1] + r_y^2$$
$$\cancel{+ r_x^2[1] - 2y_k} + (y_{k+1}-y_k) \cdot (y_{k+1}+y_k-1)$$

3

If $p1_k > 0$, means mid point outside,

$$\boxed{y_{k+1} = y_k - 1}$$

$$\Rightarrow p1_{k+1} - p1_k = 2r_y^2(x_k+1) + r_y^2 + (-1)r_x^2 \cdot 2(y_k-1)$$
$$\Rightarrow \Delta p = 2r_y^2 x_k + 3r_y^2 - 2y_k r_x^2 + 2r_x^2$$

$$\boxed{\Delta p = 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2(y_k-1)}$$

If $p1_k < 0$, means mid point inside,

$$\boxed{y_{k+1} = y_k}$$

$$\boxed{\Delta p = 2r_y^2 x_{k+1} + r_y^2}$$

AT STARTING POINT →  ? → $(0, r_y)$.

$$P1_0 = f_{ellipse}(x_k+1, y_k-\tfrac{1}{2}) = (1, r_y-\tfrac{1}{2}).$$
$$= r_y^2 + r_x^2(r_y-\tfrac{1}{2})^2 - r_x^2 r_y^2$$
$$= r_y^2 + r_x^2\cancel{r_y^2} + \frac{r_x^2}{4} - r_y r_x^2 - \cancel{r_x^2 r_y^2}$$

$$\boxed{P1_0 = r_y^2 + \frac{r_x^2}{4} - r_y r_x^2}$$

$\boxed{\text{Region 2}}$ → Start point where $\boxed{\frac{dy}{dx} = -1.}$

$\boxed{\text{UNIT STEPS in Y}}$

x_k, y_k

↓ How we are moving ↓. so next cell has $y \to y_k - 1$ → choosing x now!

?

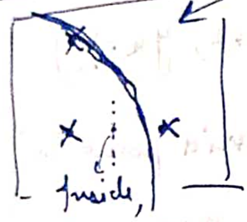
$x_k, y_k - 1$ $x_{k+1}, y_k - 1$

mid point $= \left(x_k + \frac{1}{2}, y_k - 1\right)$

Again same treatment,

$$p2_k = r_y^2 \left(x_k + \tfrac{1}{2}\right)^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2$$

MID is INSIDE

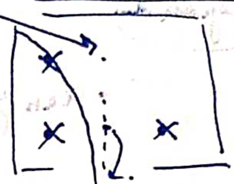


Inside, Draw x_{k+1}.

$p2_k < 0$,

$\boxed{x_{k+1} = x_k + 1}$

MID is OUTSIDE



Outside, Draw x_k.

$p2_k > 0$,

$\boxed{x_{k+1} = x_k}$

$y_{k+1} \to (y_k - 1)$ → next iteration

$$p2_{k+1} = r_y^2 \left(x_{k+1} + \tfrac{1}{2}\right)^2 + r_x^2 \left((y_k - 1) - 1\right)^2 - r_x^2 r_y^2$$

$p2_k < 0$

$\boxed{x_{k+1} = x_k + 1}$

↓

$$p2_{k+1} = r_y^2 \left(x_k + 1 + \tfrac{1}{2}\right)^2 + r_x^2 [y_k - 2]^2 - r_x^2 r_y^2$$

solve Δp in this case

$p2_k > 0$,

$\boxed{x_{k+1} = x_k}$

$$p2_{k+1} = r_y^2 \left(x_k + \tfrac{1}{2}\right)^2 + r_x^2 (y_k - 2)^2 - r_x^2 r_y^2.$$

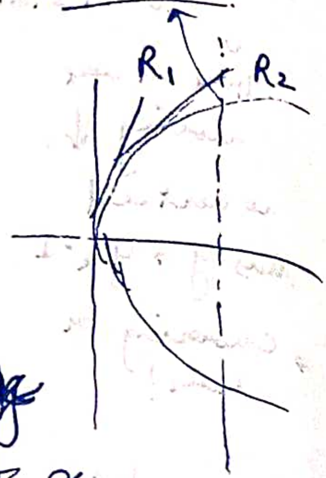
solve Δp in this case.

$$p2(0) = f_{ellipse}\left(x_0 + \tfrac{1}{2}, y_0 - 1\right)$$

RASTER SCAN for Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$\boxed{\frac{dy}{dx} = +1}$



$f_{hyperbola} = b^2x^2 - a^2y^2 - a^2b^2$,

$a > b$

In Region 1, $\frac{dy}{dx} > 1$, unit step in ~~x~~ y

In Region 2, $\frac{dy}{dx} < 1$, unit step in ~~y~~ x

Region 2

$\cdot\, x_{k+1}, y_{k+1}$

$\cdot\, x_k, y_k \qquad \cdot\, x_{k+1}, y_k$

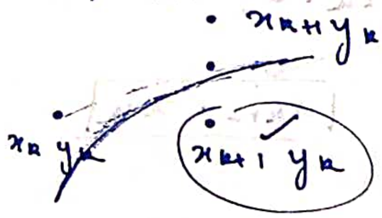
Mid point $= (x_{k+1}, y_k + \frac{1}{2})$

Again, if the mid point is lying inside, then choose y_{k+1}, else choose y_k.

$$p2_k = b^2 \cdot (x_{k+1})^2 - a^2(y_k + \tfrac{1}{2})^2 - a^2b^2$$

$$= b^2(x_k + 1)^2 - a^2(y_k + \tfrac{1}{2})^2 - a^2b^2$$

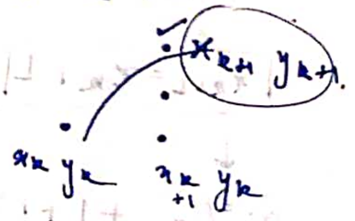
$p2_k > 0$



$$p2_{k+1} = b^2(x_{k+1} + 1)^2 - a^2(y_{k+1} + \tfrac{1}{2})^2 - a^2b^2$$

$$= b^2(x_k + 2)^2 - a^2(y_k + \tfrac{1}{2})^2 - a^2b^2$$

$p2_k < 0$



$$p2_{k+1} = b^2(x_{k+1} + 1)^2 - a^2(y_{k+1} + \tfrac{1}{2})^2 - a^2b^2$$

$$= b^2(x_k + 2)^2 - a^2(y_k + 1 + \tfrac{1}{2})^2 - a^2b^2$$

$\Delta p = p2_{k+1} - p2_k$

when $p2_k > 0$,

$\Delta p = b^2(1)(2(x_k+1)+1) - a^2((y_k+\frac{1}{2})^2 - (y_k+\frac{1}{2})^2)$

$$\boxed{\Delta p = 2b^2(x_k+1) + b^2}$$

$\Delta p = p2_{k+1} - p2_k$

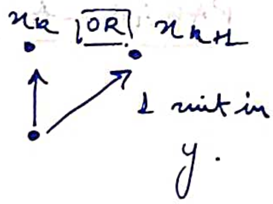
when $p2_k < 0$,

$\Delta p = b^2(1)(2(x_k+1)+1) - a^2((y_k+1+\frac{1}{2})^2 - (y_k+\frac{1}{2})^2)$

$\Delta p = 2b^2(x_k+1) + b^2 - 2a^2[1][y_k+1)$

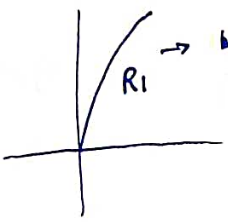
$$\boxed{\Delta p = 2b^2(x_k+1) + b^2 - 2a^2(y_k+1)}$$

Region 1 unit step in y.

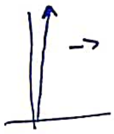


mid $\Rightarrow (x_k + \frac{1}{2}, y_k + 1)$.

same treatment.

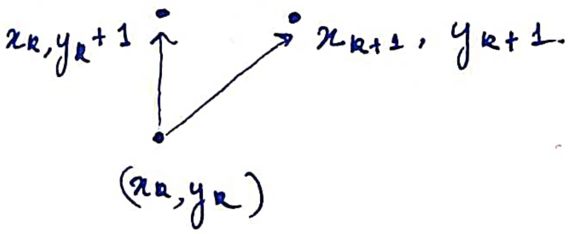


$\rightarrow$ big slope region -



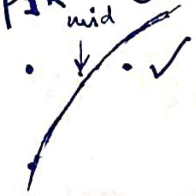
$\rightarrow$ need to take unit steps in y.

Mid Point $\rightarrow (x_k + \frac{1}{2}, y_k + 1)$.



$p1_k = b^2(x_k+\frac{1}{2})^2 - a^2 \times (y_k+1)^2 - a^2b^2$

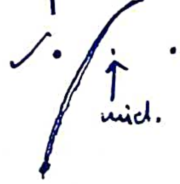
$p1_k > 0$



$\rightarrow x_{k+1}, y_{k+1}$

$p1_{k+1} = b^2(x_k+1+\frac{1}{2})^2 - a^2(y_k+1)^2 - a^2b^2$

$p_1k < 0$.



$\rightarrow x_k, y_{k+1}$.

$p1_{k+1} = b^2(x_k+\frac{1}{2})^2 - a^2(y_k+1)^2 - a^2b^2$

$p1_{k+1} = b^2(x_k + 1 + 1/2)^2$
$- a^2(y_k + 1)^2 - a^2b^2$

Δp, $p1_k > 0$

$\Delta p = b^2(1) \cdot (2)(x_k + 1)$

$\boxed{\Delta p = 2b^2(x_k + 1)}$, then

$x_k + 1$, $y_k + 1$.

$p1_{k+1} = b^2(x_k + 1/2)^2$
$- a^2(y_k + 1)^2 - a^2b^2$

Δp, $p1_k < 0$.

$\boxed{\Delta p = 0}$, then x_k, y_{k+1}

PARABOLA

SCAN CONVERSION of Parabola

using Bresenham Algo,

$$y^2 = 2px \quad , \quad \cancel{2}y\frac{dy}{dx} = \cancel{2}p$$

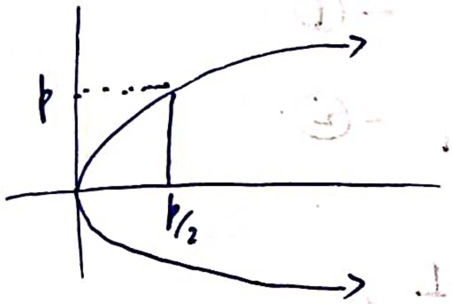
$$\frac{dy}{dx} = \frac{p}{y} \quad \text{if } y = p,$$

$$\boxed{\left(\frac{dy}{dx}\right)_{(p/2,\, p)} = 1}$$

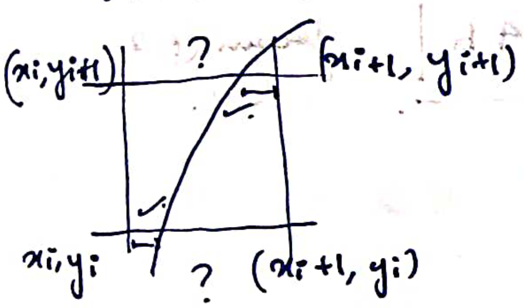
$$\boxed{y = p}$$

$$\frac{p^2}{2p} = x$$

$$\boxed{x = p/2}$$



we are here



if $d_i \geq 0$, then $x_{i+1} = x_i + 1$

else $d_i < 0$, then $x_{i+1} = x_i$

$$d_i = (y_i+1)^2 - px_i - px_{i+1}$$

if $d_i \geq 0$, then $d_{i+1} = (y_{i+1}+1)^2$

Region 2, below the x axis

$$d_1 = (y_i + 1)^2 - y^2$$

$$d_2 = y^2 - y_i^2 \quad , \quad y^2 = 2p x_i$$

$$d2_i = d_1 - d_2$$

$$d2_i = (y_i + 1)^2 + y_i^2 - 2y^2$$

$$d2_i = (y_i + 1)^2 + y_i^2 - 2 \cdot 2p x_i$$

$$d2_i = (y_i + 1)^2 + y_i^2 - 4p x_i \quad - ①$$

$$d2_{i+1} = (y_{i+1} + 1)^2 + y_{i+1}^2 - 4p x_{i+1} \quad - ②$$

If $d2_i < 0$, then $y_{i+1} = y_i + 1$,

$$\boxed{d2_{i+1} = d2_i + 4(y_i + 1) - 4p} \quad \text{from ① \& ②.}$$

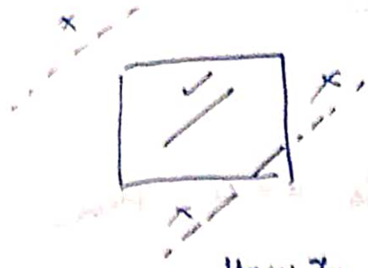
If $d2_i > 0$, then $y_{i+1} = y_i$,

$$\boxed{d2_{i+1} = d2_i - 4p}$$

$$\boxed{\begin{aligned} d1_i\,(0,0) &= 1 - p \\ d1_i\,(p/2, p) &= 1 + p \\ d2_i\,(p/2, p) &= 1 - 2p \end{aligned}}$$

LINE CLIPPING ALGORITHMS.

Chapter - 3, Rogers



How to know if the frame buffer has points which are valid?

How to clip?

WHY don't we have circular display?
→ circular - quad. eqⁿ
→ line → linear,


lin - lin eqⁿ solve


quad - line solve

COHEN - SUTHERLAND Algo.

$b_1 b_2 b_3 b_4$	$b_1 b_2 b_3 b_4$	$b_1 b_2 b_3 b_4$	
1001	1000	1010	
			y_{Top}
0001	0000 screen.	0010	
0101	0100	0110	y_{Bottom}
	x_{Left}	x_{Right}	

4 bit codes are used to represent the regions.

$\Rightarrow x_L, x_R$ & $y_B\ y_T$ are co-ordinates of screen.

$b_1 b_2 b_3 b_4$

If $x < x_L$, then $b_4 = 1$, else $b_4 = 0$.

If $x > x_R$, then $b_3 = 1$, else $b_3 = 0$.

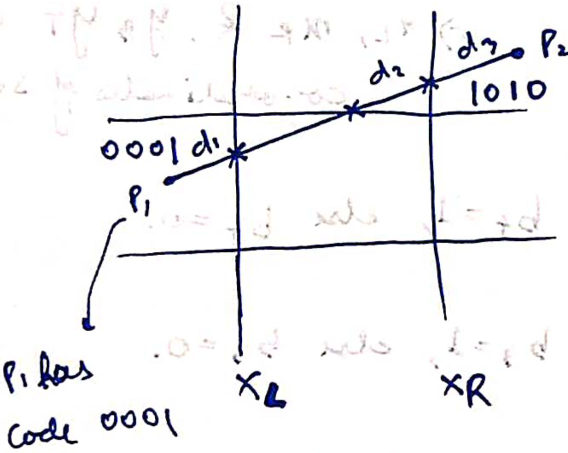
If $y < y_B$ then $b_2 = 1$, else $b_2 = 0$.

If $y > y_T$ then $b_1 = 1$, else $b_1 = 0$.

Nice! each region is identified uniquely by a combo of 4 bits.

Each point has a CODE, if $x_L - x_i > 0$, $b_4 = 1$, else 0,
sign$(x - x_L)$. etc

- Take end points P_1 & P_2 of line
- Find their region codes,
- If logical AND of regions is == 0, then it is visible.
  - If Code P_1 = 0000
    Code P_2 = 0000
    AND = 0 => Fully visible line
  - If code P_1 = 1011
    P_2 = 1001
    AND ≠ 0, Not visible line.



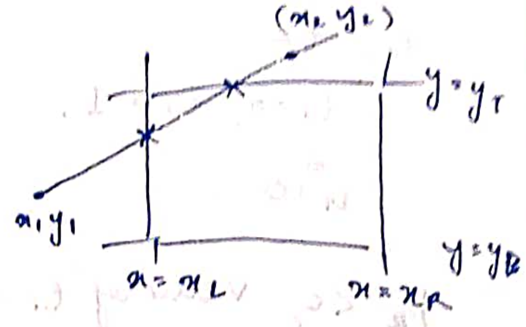
P_2 has code 1010

AND is 0, & P_1 & P_2 non zero, then d_1, d_2 & d_3 need to be discarded.

Liang - Barsky - Line - Clipping

$x = t x_1 + (1-t) x_2$, $y = t y_1 + (1-t) y_2$

$0 \le 1 \le t$



$\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$

$q_1 = x_1 - x_L$

$q_2 = x_R - x_1$

$q_3 = y_1 - y_B$

$q_4 = y_T - y_1$

$p_1 = \Delta x$, $p_2 = -\Delta x$

$p_3 = -\Delta y$, $p_4 = \Delta y$.

$t_k = \frac{q_k}{p_k}$, $k = 1, 2, 3, 4$.

If $p_k = 0$, then line is $\parallel$ to edge.

If $q_k < 0$, the line is completely outside the clipping window.

If $p_k < 0$, the line precedes from outside to the inside of particular clipping window.

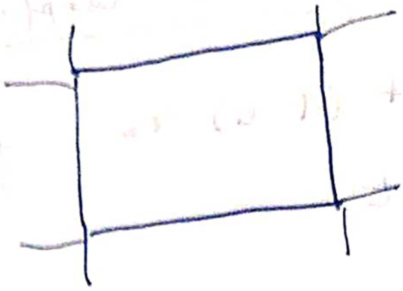
If $q_k \geq 0$ the line is parallel to clipping edges.

If $p_k > 0$ the line precedes from inside to outside.

The value $t = \frac{q_k}{p_k}$, $k = 1, 2, 3, 4$

give us point of intersection.

Consider the values of t as



t_1, t_2

$t_1 = 0, \; t_2 = 1.$

$t_1 = 0$

$p_k < 0$, value of t_1 is found

$p_k > 0$, value of t_2 is found

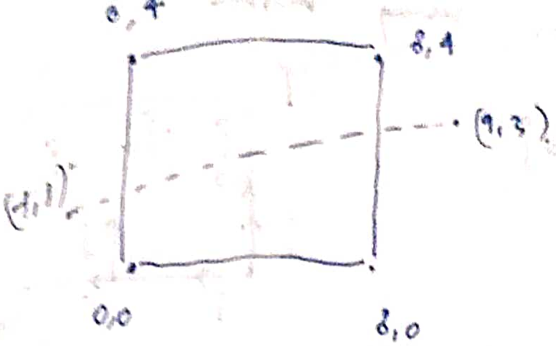
$t_1 = \text{Max}(0, r_k)$ — entering point

$k = 1, 2, 3, 4.$

How?

$t_2 = \text{Min}(1, r_k)$ — leaving point of the line from window

$k = 1, 2, 3, 4.$



$p_1 = -\Delta x = -10 \qquad q_1 = x_1 - x_L = -1$

$p_2 = 10 \qquad q_2 = x_R - x_1 = 9$

$p_3 = -2 \qquad q_3 = y_1 - y_B = 1$

$p_4 = 2 \qquad q_4 = y_T - y_1 = 3$

$r_1 = 1/10$

$r_2 = 9/10$

$r_3 = -1/2$

$r_4 = 3/2$

$p_1 < 0, \; q_1 < 0,$

$t_1 = \text{Max}(0, r_1)$

$= \text{Max}(0, 1/10)$

$\boxed{t_1 = 1/10}$.

$p_2 > 0$, find t_2 now.

$t_2 = \text{Min}(1, \frac{9}{10})$

$\boxed{t_2 = 9/10}$

$p_3 < 0$, find t_1 now,

$t_1 = \text{Max}(t_1, r_3) = \text{Max}(\frac{1}{10}, -\frac{1}{2})$

$\boxed{t_1 = \frac{1}{10}}$

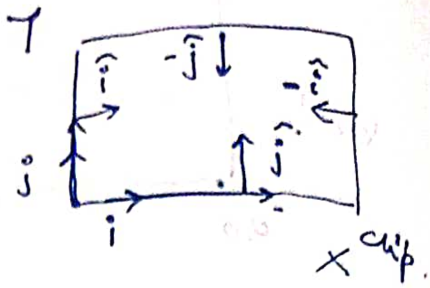
$p_4 > 0$, find t_2 now,

$t_2 = \text{Min}(t_2, r_4)$

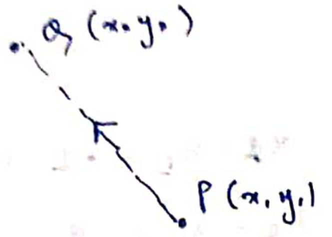
$= \text{Min}(\frac{9}{10}, 3/2) = \boxed{\frac{9}{10} = t_2}$.

find (x, y) for edges by replacing t with t_1 & t_2 & find intersections with edges.

Cyrus – Beck Algo. using vectors.



$$\hat{r} = a\hat{i} + b\hat{j}$$



$$PQ = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j}$$

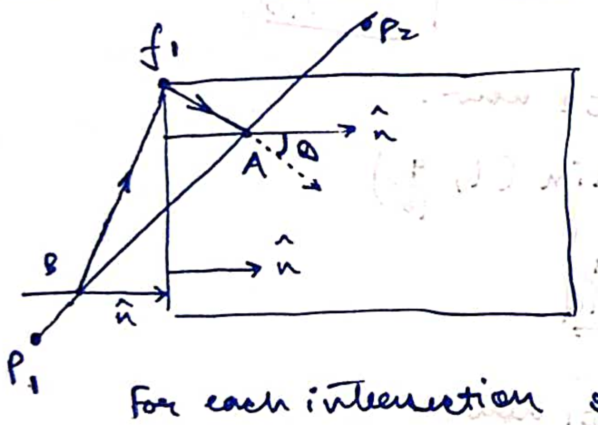
If we have $\hat{r} = a\hat{i} + b\hat{j}$,

normal to it would be

$$r_\perp = b\hat{i} - a\hat{j}.$$

OR

$$r_\perp = -b\hat{i} + a\hat{j}.$$



Goal ?

- Find parameters where point is inside, on & outside clip window.

For each intersection of the edges, we find dot product with edge perpendiculars.

$$P(t) = P_1 + (P_2 - P_1)\,t$$

$$(P(t) - f) \cdot \hat{n} > 0 \text{ at } A$$

$$\& \; (P(t) - f) \cdot n < 0 \text{ at } B$$

Okay, so for all permutations of edges, we find →

$$(P(t) - f_i) \cdot \hat{n}_i$$

$$\downarrow$$

$$\hat{n}_i \cdot (P_1 + (P_2 - P_1) t - f_i]$$

$$\hat{n}_i \cdot (\underbrace{P_1 - f_i}_{w}) + \hat{n}_i \cdot (\underbrace{P_2 - P_1}_{D}) t$$

$$\boxed{\hat{n}_i \cdot \vec{w} + \hat{n}_i \cdot D t}$$

↳ depending on the different values of this product, we will draw the points.

$$\hat{n}_i \cdot \vec{w} + \hat{n}_i \cdot D t = 0$$

if the point is on the edge of clip window.

$$\Rightarrow \boxed{t = -\frac{\hat{n}_i \cdot \vec{w}}{\hat{n}_i \cdot D}}.$$

what if $\bar{n}_i \cdot D = 0$?

1) $D = 0 \Rightarrow P_1 = P_2$

2) $\bar{n}_i \cdot D = 0$,

$D \perp n_i$, means line does not intersect the edges.

If $D = 0$, how to know where the point P lies?

$\vec{w_i} \cdot \vec{n_i} < 0$, point is outside window

$\vec{w_i} \cdot \vec{n_i} > 0$, point is inside the window

$\bar{w}_i \cdot \bar{n}_i = 0$, point is on the i^{th} edge.

$0 \leq t < 1$, If $t > 1$, we reject the line.

$$t_1 = - \frac{\bar{w_i} \cdot \bar{n_i}}{D \cdot \bar{n_i}}, \quad i = [1, 4], \quad D \neq 0.$$

Values of t are grouped in 2 classes.

- One group: correspond to begining of line.
- Other group: " " end of line

- Near the begining of line, largest lower limit which is given by $\boxed{D_i \cdot N_i > 0}$.

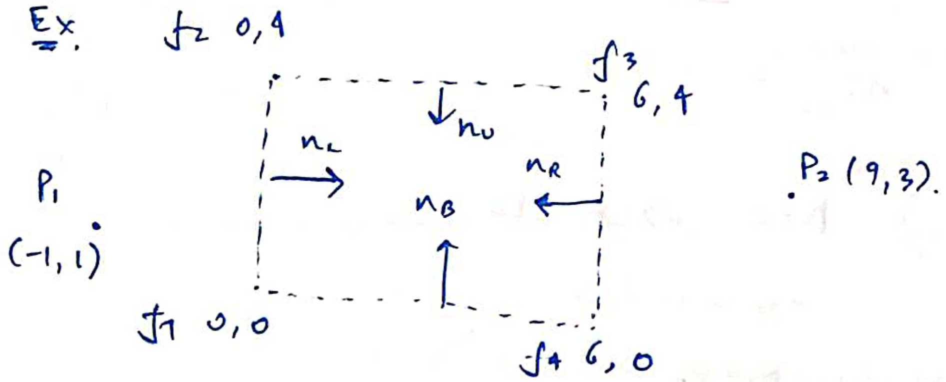
- Near the end of the line, smallest upper limit which is given by $\boxed{D_i \cdot N_i < 0}$.

$t_L = 0$, $t_U = 1$

$D_i \cdot n_i > 0$ ✓ $\quad D_i \cdot n_i < 0$

$t_{Beg} = Max(0, t)$, $\quad t_{end} = Min(1, t_{end})$

Ex.



$D = P_2 - P_1 = (10, 2)$

$w_i = P_t - f_i = (-1, 1) + (P_2 - P_1)t - (0,0)$

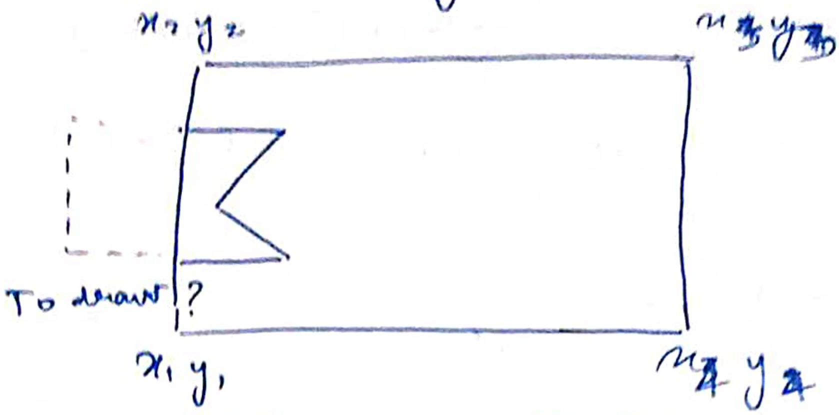
$\boxed{P(t) = (-1, 1) + (10, 2)t}$

Edge	Normal	f_i	$P(t) - f_i$
left	$\hat{i} = (1,0)$	$(0,0)$	$(-1+10t)\hat{i} + (1+2t)\hat{j}$
right	$-\hat{i} = (-1,0)$	$(6,4)$	$(-7+10t)\hat{i} + (-3+2t)\hat{j}$
bottom	$\hat{j} = (0,1)$	$(0,0)$	
up	$-\hat{j} = (0,-1)$	$(6,4)$	

$P_1 - f_i$ w_i	$w_i \cdot n_i$ ($i, -i, j, -j$)	$D \cdot n_i$	$t = -\dfrac{w_i \cdot n_i}{D \cdot n_i}$	t_{Low} t_{up}
$(-1,1)-(0,0)$ $(-1,1)$	-1	$10 > 0$] t_L	$\frac{1}{10}$	$\max(0, \frac{1}{10})$ $\boxed{\frac{1}{10}}$
$(-7,-3)$	7	$-10 < 0$	$7/10$	$\min(1, \frac{7}{10})$ $7/10$
$(-1,1)$	1	$2 > 0$] t_L	$-1/2$	$\max(-\frac{1}{2}, \frac{1}{10})$ $\boxed{\frac{1}{10}}$
$(-7,-3)$	3	$-2 < 0$	$3/2$	$\min(\frac{7}{10}, \frac{3}{2})$ $\Rightarrow 7/10$

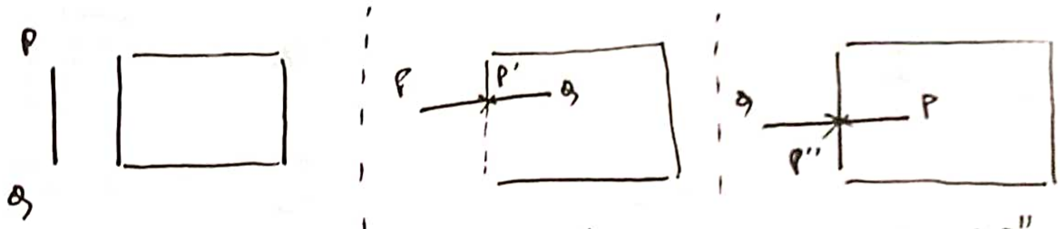
2 groups of values.

Polygon clipping

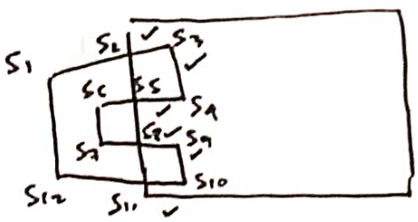


VERTEX	EDGES
$V_1 (x_1, y_1)$	$E_1 : V_1 V_2$
$V_2 (x_2, y_2)$	$E_2 : V_2 V_3$
$V_3 (x_3, y_3)$	$E_3 : V_3 V_4$
$V_4 (x_4, y_4)$	$E_4 : V_4 V_1$

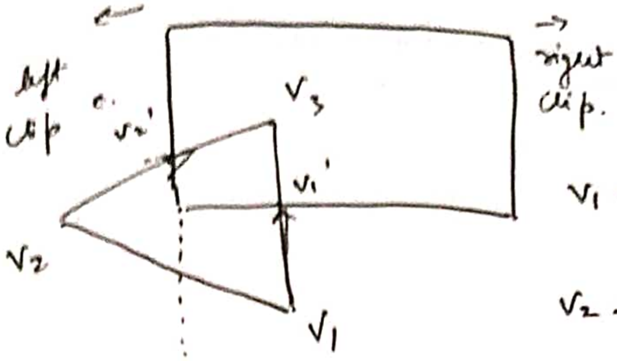
SUTHERLAND - HODGEMAN Algo.



save none of the points | save $P'Q$. | save PP''.



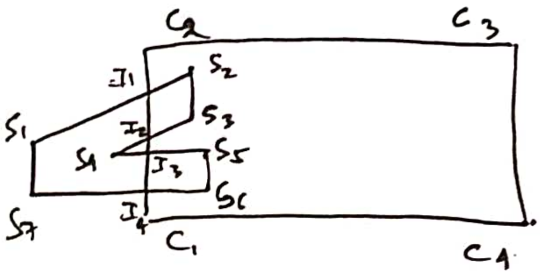
$S_2 S_3$ ✓
$S_3 S_4$ ✓
$S_4 S_5$ ✓
$S_5 S_8$ ✓
$S_8 S_9$ ✓
$S_9 S_{10}$ ✓
$S_{10} S_{11}$ ✓

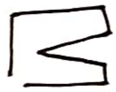


	Left clip	Right clip	Bottom clip	Top clip
$V_1 \rightarrow$	V_1	V_1	V_1'	
$V_2 \rightarrow$	V_2'	V_2	V_2	
$V_3 \rightarrow$	V_3	V_3	V_3	
		no intersection		

Read.

WEILER ATHERTON ALGO.



1. Start from the first vertex of subject polygon  entering the clip polygon $C_1C_2C_3C_4$.

and go along the edge of the subject polygon as we enter the clipping polygon.

2. When we leave the clipping polygon (window).

TURN RIGHT.

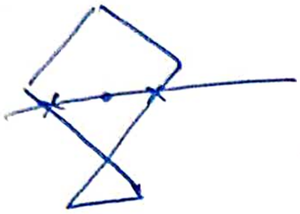
Now subject polygon becomes the clipping polygon and clipped polygon becomes SUBJECT.

Clipping of Lines in Concave Polygons.

Point inside Polygons.

How to know if a point lies inside a polygon?

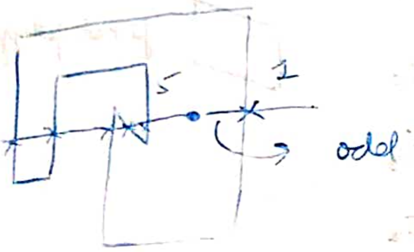
we draw lines, if there are -



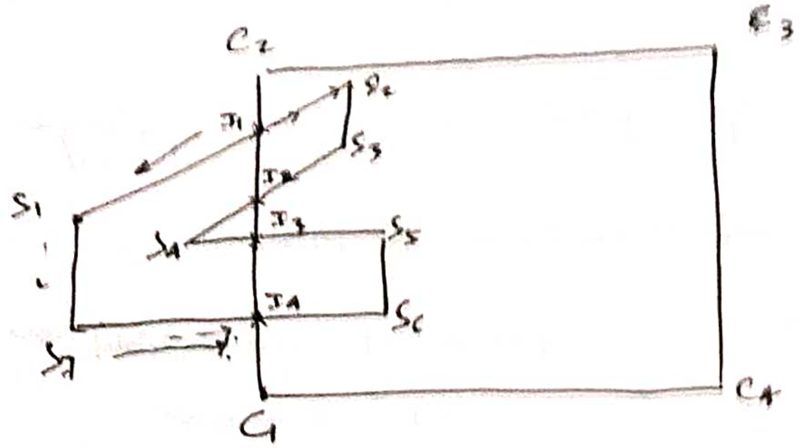
(1) ODD no. of intersections on LEFT & RIGHT

- INSIDE

(2) EVEN no. of intersections on LEFT & RIGHT



weiler-Atherton

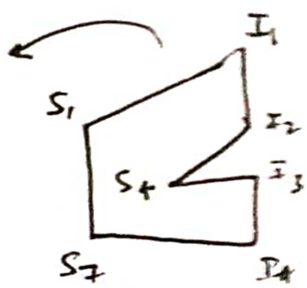


	Subject Polygon	Clip polygon.
Clockwise	I_1	C_1
	S_2	I_4
	S_3	I_3
	I_2	I_2
	S_4	I_1
	I_3	C_2
	S_5	C_3
	S_6	C_4
	I_4	
	S_7	
	S_1	


first po

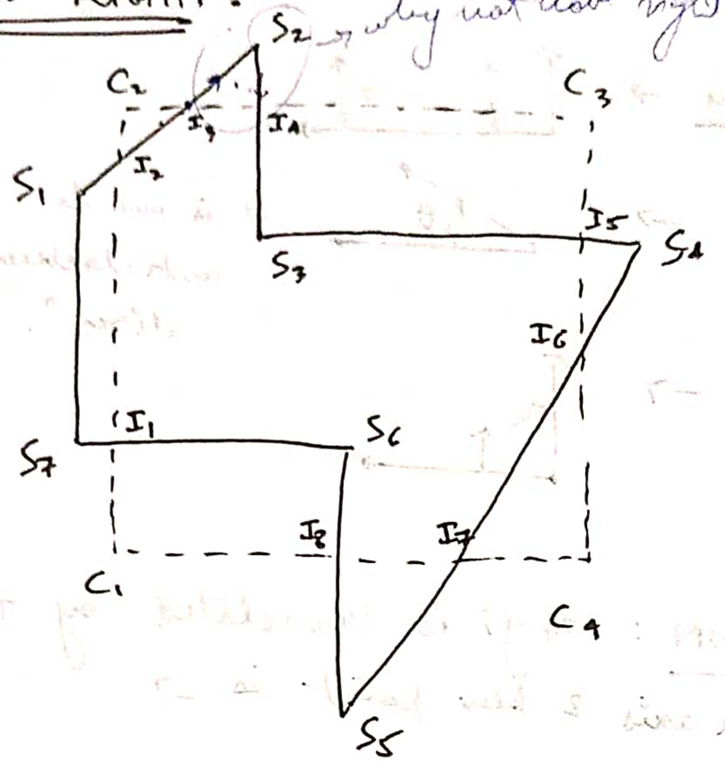

second polygon

	Subject Polygon	Clip polygon
Anticlockwise	I_1	C_1
	S_2	I_4
	S_3	I_3
	I_2	I_2
	S_4	I_1
	I_3	C_2
	S_5	C_3
	S_6	C_4
	I_4	
	S_7	
	S_1	



whenever you are about to leave the clip window, TURN RIGHT.

S2 → why not move right?

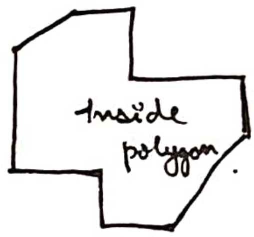


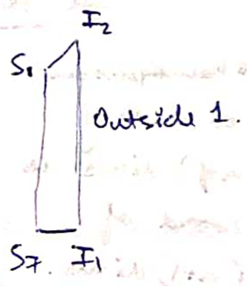
Subject Polygon	Clip Polygon
✓ S1	C1
I2	I1
I3	I2
S2	I3
I4	I4
S3	C3
I5	I5
S4	I6
I6	C4
I7	I7
S5	I8
I8	
S6	
✓ I1 ↳ back to I2 ✓.	
S7	

Inner polygon search.

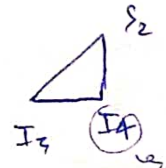
Outer Polygon Search.

1. I1 is the first intersection
2. I2 is used.
3. Use I3.





* From a point, follow the table.



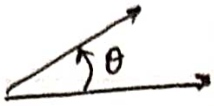
& so on

& so on...

2-D Transformations.

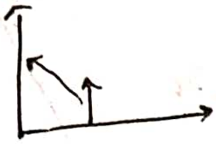
Translation →

Rotation →



θ is +ve in anticlockwise direc^n.

Scaling →



① **TRANSLATION** : (x, y) is translated by t_x along x axis & new point is →

$$x' = x + t_x, \quad y' = y + t_y.$$

② **ROTATION** : Rotation is done by an angle ϕ.

New point → $r\cos(\theta+\phi)$,

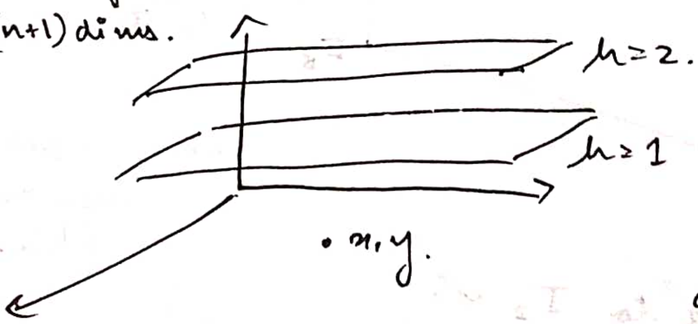
$r\sin(\theta+\phi)$.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Use Homogeneous co-ordinates.

homogeneous co-ords of (n) dims is coord of (n+1) dims.

$$x, y \Rightarrow (x, y, h)$$ — constant



Here, we also see that there is non-unique homogeneous co-ords.

Translation in Homogeneous co-ords.

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad T(x_1, y_1)$$

Rotation in Homogeneous co-ords.

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad R(\theta)$$

Scaling in Homogeneous co-ords.

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} r_x & 0 & 0 \\ 0 & r_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad S(r_x, r_y).$$

LINEARITY.

$$T(t_{x_1}, t_{y_1}) + T(t_{x_2}, t_{y_2}) = T(t_{x_1} + t_{x_2}, t_{y_1} + t_{y_2})$$

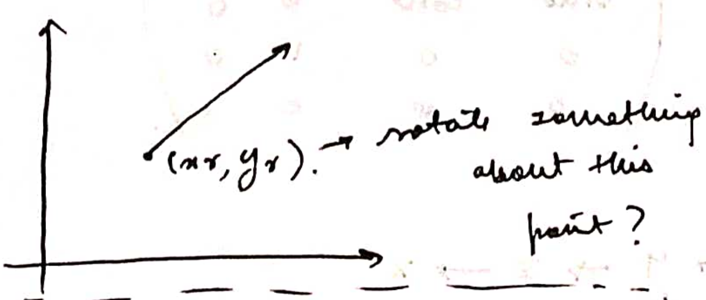
$$R(\theta_1) + R(\theta_2) = R(\theta_1 + \theta_2).$$

$$S(r_{x_1}, r_{x_2}, r_{y_1}, r_{y_2}) = S(r_{x_1}, r_{y_1})\, S(r_{x_2}\, r_{y_2}).$$

We can have combinations of these transformations

TR, ST, ...

Ex.



1. Bring point to origin $T(-x_r, -y_r)$
2. Rotate $R(\theta)$
3. Translate back $T(x_r, y_r)$.

$$T(x_r, y_r) . R(\theta) . T(-x_r, -y_r)$$

$$\begin{bmatrix} r_{11} & r_{12} & t_{x} \\ r_{21} & r_{22} & t_{y} \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

2×2 matrix preserves orthonormality, length & angle.

↳ Rigid Body Transform.

<u>Affine</u> : preserve || lines but not lengths or angle

<u>Shear</u> : ? 1

<u>Reflection</u>: ? 2

Assignment: find T-Matrix for ① & ② about the line $ax + by + c = 0$, $a \neq 0$, $b \neq 0$, $c \neq 0$

<u>3D Transformations.</u>

$$T = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 0 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ Translation

$$S_{caling} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$ Scaling.

$$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$x \to y \to z \to x$

replace x with

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$
$z' = z$

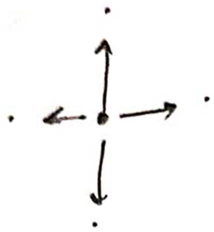
$$R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

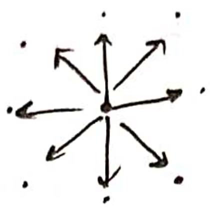
$x' = x$
$y' = y\cos\theta - z\sin\theta$
$z' =$

$$R_y = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

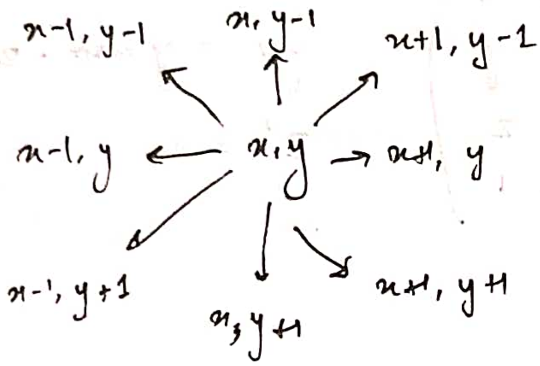
POLYGON FILL ALGOS.

4 connected & 8 connected algos.

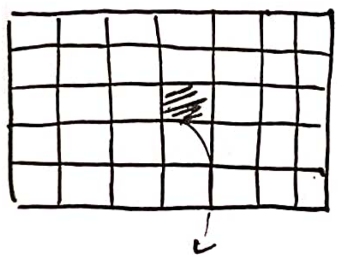




↳ more coherence, so this is chosen.



SEED FILL ALGORITHM.



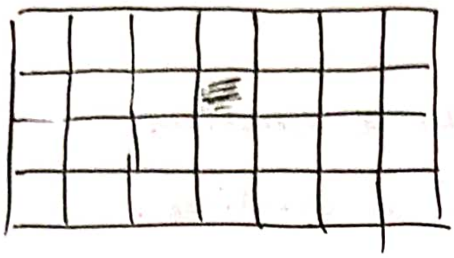
SEED

1. Pick a seed (x, y)
2. Pop seed from the stack
3. Shade the pixel.
4. Find the 4 neighbours of
   - if neighbour unshaded & not a boundary
   - push to stack.
5. Continue till stack empty.

O(N²) space

→ handling pixels in mem, instead of lines.

SCAN-LINE SEED FILL ALGO.



- Seed pixel pushed to the stack

- Now, top pixel is removed from the stack.

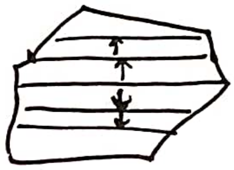
- The scan line containing the seed is filled till left boundary & right boundary.

- Save the left end & right of the currently filled scan line.

- Check if above & below lines are filled in [x_{left}, x_{right}].

- If not filled, push a seed, x_{seed} on stack where $x_L \leq x_{seed} \leq x_{right}$.

- continue for unfilled lines.



& so on

Scan line polygon fill algorithm.

even & odd parity - ?

y = c is the scan line

ALL EDGE Table,
GLOBAL EDGE ",
ACTIVE EDGE ".

Let (x_1, y_1), (x_2, y_2) be the edge of with them 2 vertices.

i) Minimum y value of the 2 vertices.

ii) Maximum y value of the 2 vertices.

iii) x-value associated with min. y-value.

iv) The slope of the edge.

ALL EDGE TABLE (AET).

y_{min}	y_{max}	x val, min	$1/m$