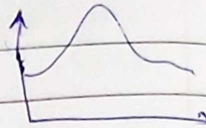
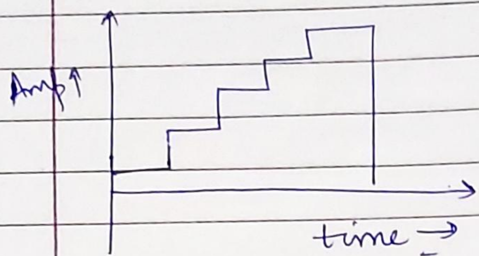
Data Communication

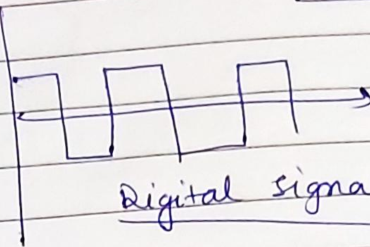


Analog and digital Signal

Analog signal



Digital signal.

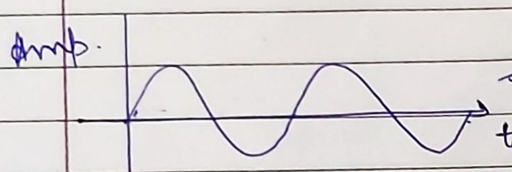


Digital signal.

~~It is discrete.~~

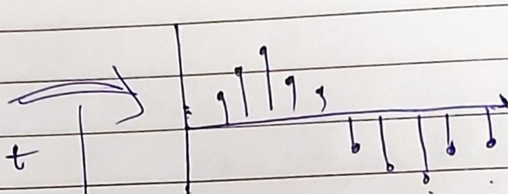
Digital Signal :- where fixed predefined amplitude sa labels.

Discrete :- when signal exit only on specific value of time called discrete time signal.



Continuous.

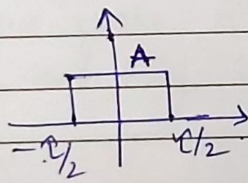
conversion is called sampling.



Discrete

Pulse signal

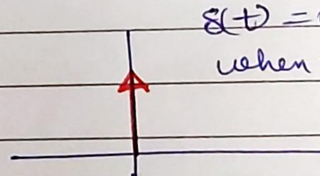
Gate / window



$\delta(t) = \begin{cases} A & \text{when } -\tau/2 \le t < \tau/2 \\ 0 & \text{Otherwise} \end{cases}$

→ with a finite duration.

Impulse



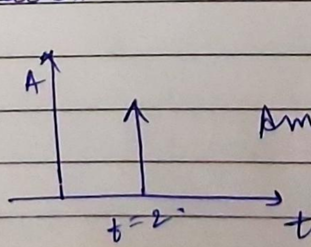
$\delta(t) = 0$ when $t \neq 0$

exist at $t = 0$.

→ Exist only at $t = 0$

Examples

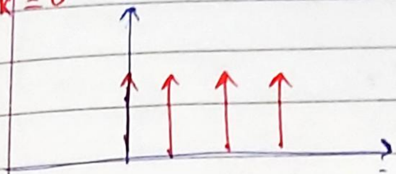
① $\delta(t-2)$ Impulse



Amp = ∞.

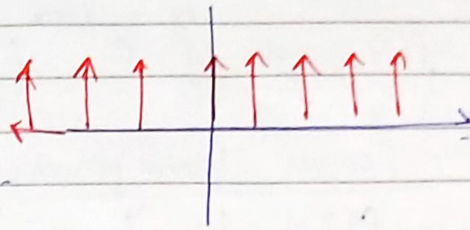
width = 0.

② $\sum_{k=0}^{\infty} \delta(t-k)$



Train of impulses.

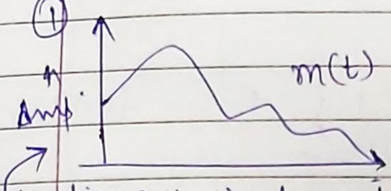
③ $\sum_{k=-\infty}^{\infty} \delta(t-k)$



Again on sampling.

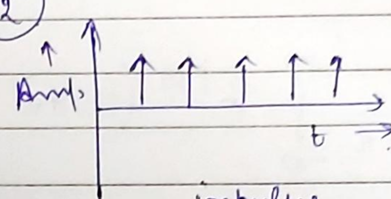
Original source signal can be of any shape and size.

①



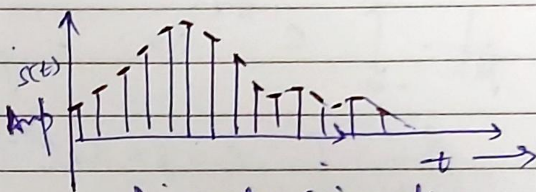
Continuous signals

②



impulses

① × ② =



Discrete signals.

Fourier Transform :- Technique which converts time-domain signal representation into frequency-domain representation.

$$F(t) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi f t}\, dt$$

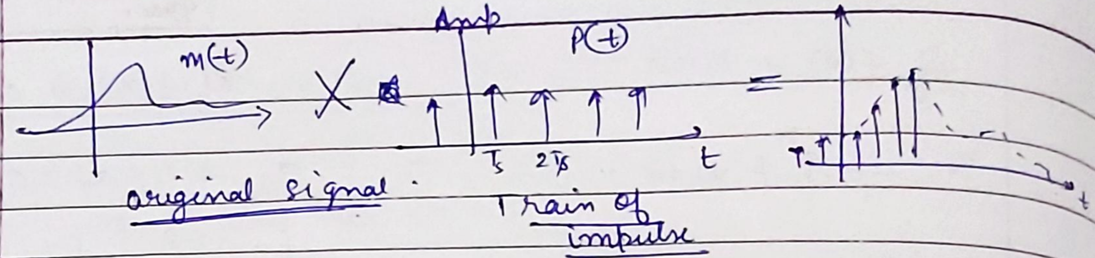
Fourier series :- is a representation of periodic signal in terms of sine and cosine harmonics

Sampling Theorem says that if the sampling frequency is greater than twice the maximum f component exist in our original signal.

$$\boxed{f_s \geq 2 f_m}$$

→ Multiplicative Property of F.T.

Date 13-01-21



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

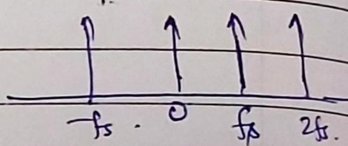
$$s(t) = m(t) \times p(t) = m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)$$

⇒ $m(t) \xleftrightarrow[\text{Tranform}]{\text{fourier}} M(j\omega)$

⇒ $p(t)$: It is a periodic function. F.T. of periodic function is also train of impulse with different frequency

$$p(t) \longleftrightarrow P(f) = f_s \sum_{k=-\infty}^{\infty} \delta(f - kf_s)$$

$\boxed{f_s = 1/T_s}$



H.W. $C_k = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t)\, e^{-jk\omega t}\, dt \quad \forall\ k = \frac{1}{T_s} = f_s$

Extra

$x(t) \longrightarrow$ Network $\longrightarrow y(t)$

$y(t)$ depends on $x(t)$ and Network.

$y(t) = x(t) ? \underbrace{h(t)}$ impulse function of Network.

$= x(t) \circledast h(t)$

↑ convulation.

Acc. to Multiplicative property of F.T. $Y(j\omega) = X(j\omega) \times H(j\omega)$

$$\boxed{H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}}$$

Transfer function.

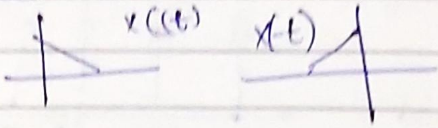
Impulse response of system

F. tranform of impulse function is 1.

→ There is no existence of −ve frequency in the real world

Convolution

Flip $x(t)$ $x(-t)$



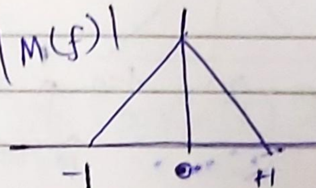
Convolution $\begin{cases} \text{Flip} \\ \text{Shifting} \\ \text{Multiplication} \\ \text{Addition} \end{cases}$

$$x(t) \circledast h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

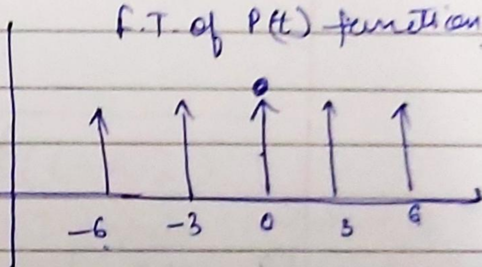
Back to topic

$$S(f) = f_s \sum_{K=-\infty}^{\infty} M(f - Kf_s)$$

$|M(f)|$



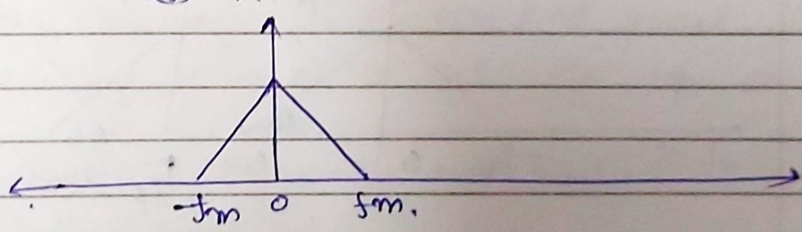
F.T. of P(t) function



Assumption.

$\boxed{f_s = 2 f_m}$

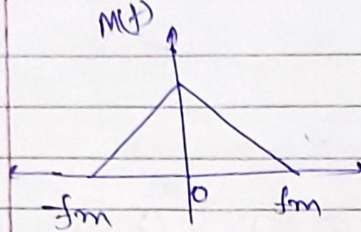
$M(f)$ X

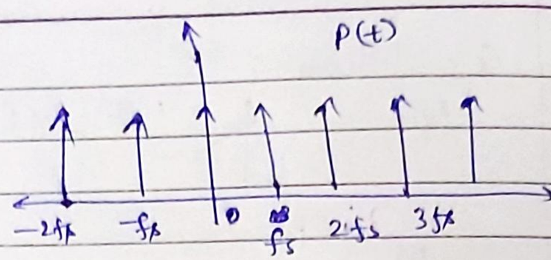


At $k = 0$

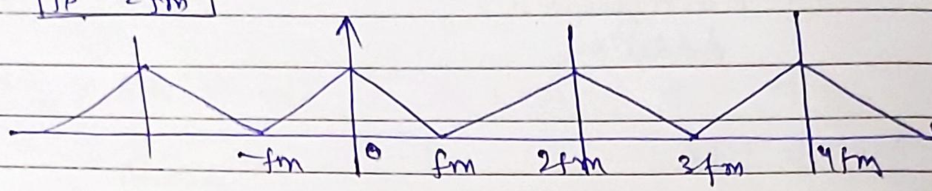
Using Multiplicative property of F.T.

After F.T $\begin{cases} s(t) \longleftrightarrow S(f) \\ m\,t \longleftrightarrow M(j\omega) \\ p(t) \longleftrightarrow P(t) \end{cases}$ (This will train of impulses but with different frequencies)

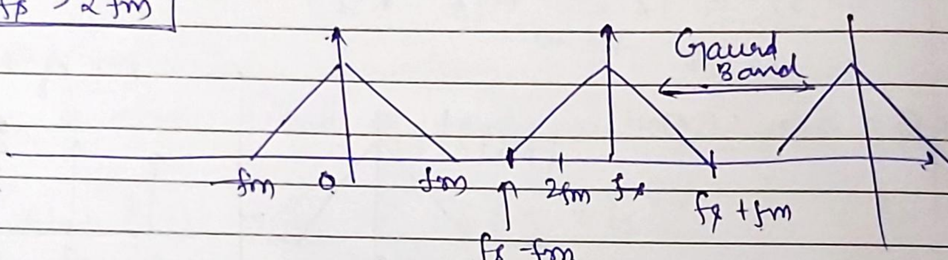




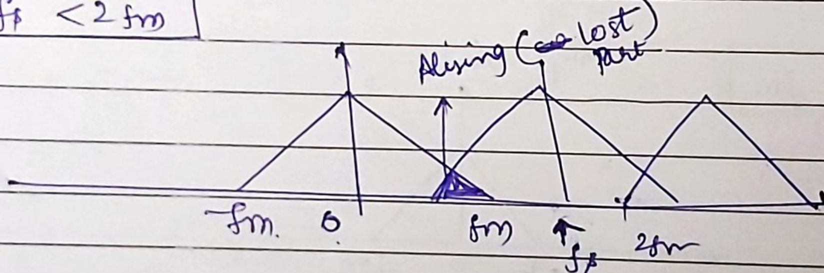
$\boxed{f_s = 2f_m}$



$\boxed{f_s > 2f_m}$



$\boxed{f_s < 2f_m}$

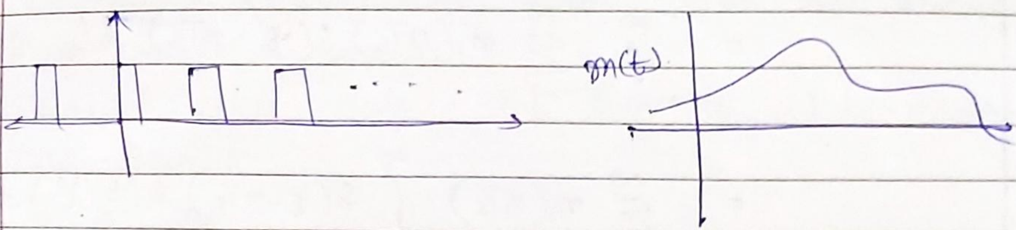


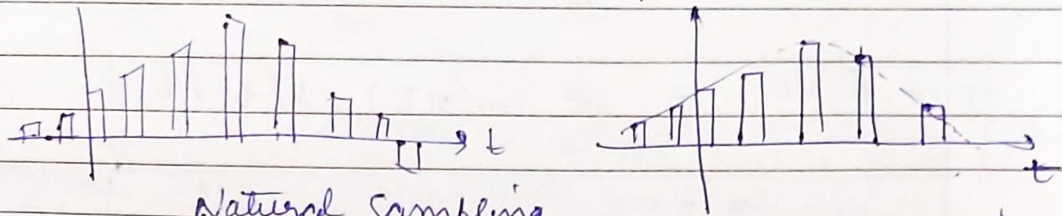
$f_s = 2f_m$ is an ideal case (Not feasible in real world)

So, $f_s > 2f_m$.

14-01-21

Pulse Sampling :- use train of pulses.





Natural Sampling

flat top sampling

Flat-Top Sampling

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t - nT_s)$$

↑ impulse response of pulse broadening function

→ We take the instantaneous sample and passed it through pulse broadening circuit to get flat top sample.

or

Took the instantaneous sample and kept it for more period of time

$$h(t) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{elsewhere} \end{cases}$$

$$m(t) = \sum_{n=\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)$$

Discrete time signal of m(t)

$\delta(t) \cdot x(t) = x(0)$

$\delta(t-2) \cdot x(t) = x(2)$

$$m_1(t) * h(t) = \int_{-\infty}^{\infty} m_1(\tau)\, h(t-\tau)\, d\tau .$$

$$= \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} m(nT_s)\, \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

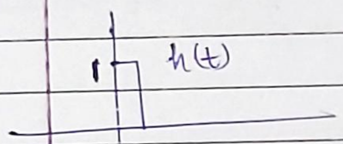
$$= \sum_{-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

$$\boxed{m_1(t) \circledast h(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \cdot h(t - nT_s)}$$

function for flat top sampling

for $n = 0$.

$m(0) \cdot h(t)$



H.W. Natural Sampling

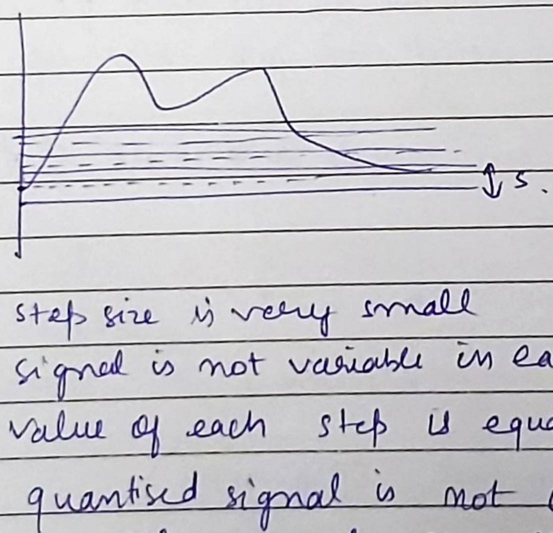
Quantization :- Process in which we restrict our function with fixed ~~magnitude~~ ~~[illegible]~~ levels.

V_H = Max. Voltage

V_L = Min "

$$S = \frac{V_H - V_L}{M}$$ M ⟶ No. of levels.

S ⟶ step size



→ step size is very small

→ signal is not variable in each step.

→ value of each step is equal to middle value.

→ As quantised signal is not original signal, so it will generate some error known as quantization error.

Quantization error $= m(t) - m_q(t)$

Max. Q. error = $S/2$.

Page :
Date :

→ To reduce quantization error, step size should be as small as possible

→ Channel noise says step size should be as large as possible

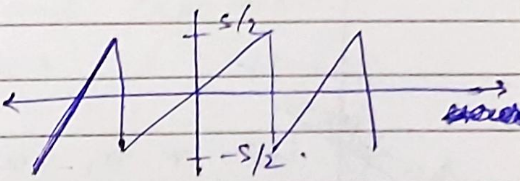
Trade off .

Date 15/01/21

Quantization → linear
→ Non-linear .

Quantization error varies from $S/2$ to $-S/2$.

Linear Quantization
→ Midtread
→ Midrise



Quantization noise :- Diff. b/w original signal and Quantized signal .

. Random variable X, which is uniformly distributed

$$f_x(x) = \begin{cases} 1/S & -S/2 \le x \le S/2 . \\ 0 & \text{otherwise} . \end{cases}$$

Mean square error (Noise variance)

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 f_x(x)\, dx$$

$$= \frac{1}{S} \int_{-S/2}^{S/2} x^2\, dx .$$

$$= \frac{1}{S} \left. \frac{x^3}{3} \right]_{-S/2}^{S/2} = \frac{1}{S} \times \frac{1}{3} \left(\frac{S^3}{4} \right)$$

$$\boxed{\sigma_x^2 = \frac{S^2}{12}}$$

Quantization error : signal to noise ratio

→ 2^{nd} way of calculating error.

Let $f(m)\,dm$ be the probability that $m(t)$ lies in a range $m - \frac{dm}{2}$ to $m + \frac{dm}{2}$

Then the mean square quantization error.

$$\overline{e^2} = \int_{m_1 - S/2}^{m_1 + S/2} f(m)(m - m_1)^2\,dm + \int_{m_2 - S/2}^{m_2 + S/2} f(m)(m - m_2)^2\,dm + \cdots$$

m_1 : mid point of 1st step.

Assumptions: let $m - m_k \equiv x$,
step size small so, probability distribution is constant

$$\overline{e^2} = \left\{ f^{(1)} + f^{(2)} + f^{(3)} + \cdots \right\} \int_{-S/2}^{S/2} x^2\,dx.$$

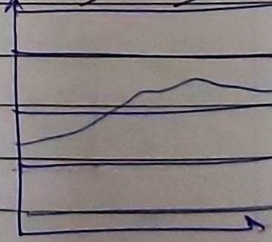
$$\overline{e^2} = \left\{ f^{(1)} + f^{(2)} + \cdots \right\} S^3/12.$$

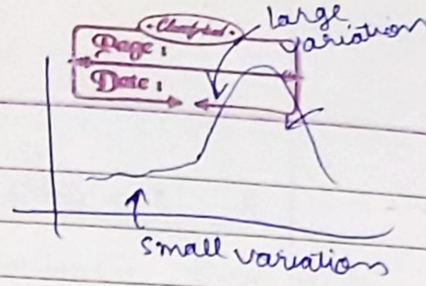
$$\overline{e^2} = \{ \underbrace{S.f^{(1)}}_{\text{Probability that signal lies in 1st step}} + S.f^{(2)} + S.f^{(3)} \} S^2/12.$$

f → probability distribution.

$$\overline{e^2} = 1 \cdot S^2/12 = S^2/12.$$

If step size is large such that whole function lies b/w two or few step. Then we will not get the replica of exact signal



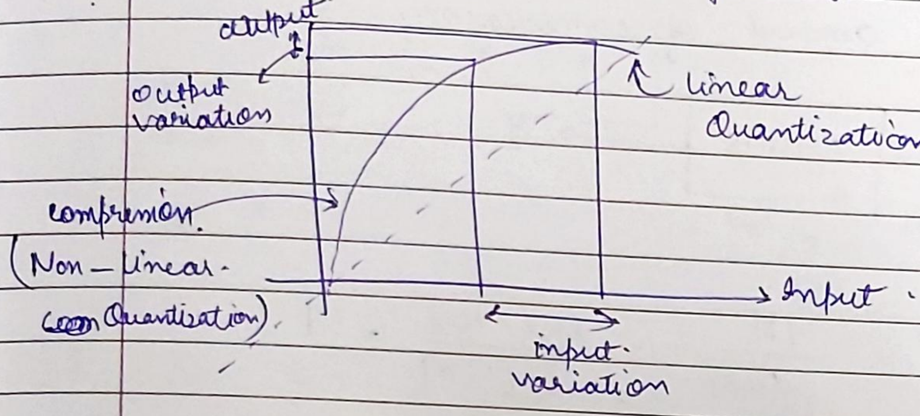


If step is small, channel noise become very large

→ So, large variation we should take large step size and for small variation we should take small step size.
→ So, we can't take fixed step size.
→ So, in Non-linear Quantization. there is variable step size exist.

Non-linear Quantization.

Companding :- compression and expansion.
→ Step size keeps on compressing and expanding



~~Compressor → Uniform Quantizer → Expander~~.

For getting Non-linear Quantization
→ Compressor
→ Uniform Quantizer
→ Expander.

Laws (In non-linear Quantization)
1. A - Law
2. μ - Law.

Application of sampling :- Can send more than one signals simultaneously from youtube

μ - Law Companding → compression + expansion
(at input side) (at output side)

→ I/P → O/P relationship is given by

$$\frac{|y|}{x_{max}} = \frac{\ln\left[1 + \mu\frac{|x|}{x_{max}}\right]}{\ln[1+\mu]}$$

where,

x = Amp. of I/p signal at a particular instant
y = compressed o/p.
μ = unitless parameter used to define the amount of compression.

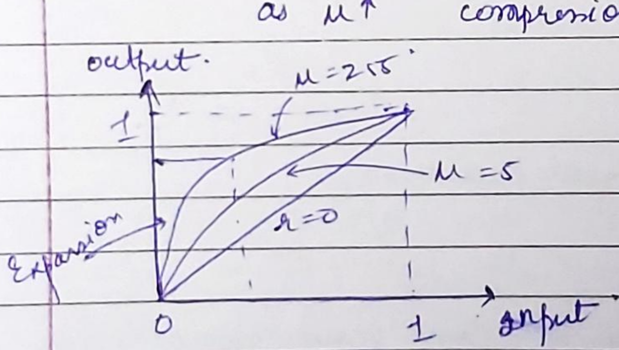
$x \rightarrow$ $\boxed{\mu\text{-law}}$ $\rightarrow y$

$\left(\frac{|x|}{x_{max}}\right) \rightarrow 0$ to 1

for $\mu = 0$, $\frac{|y|}{x_{max}} = \frac{\ln[1+0]}{\ln[1+0]} = 1$

$|y| = x_{max}$.
↳ un-compressed output.

→ There is no compression.
as $\mu\uparrow$ compression will happen.



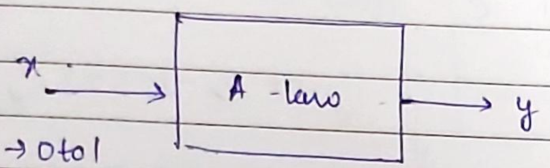
→ for smaller values of input, ~~out vari~~ output variation is large and for larger values of input, output variation is small.

→ Larger value of μ, more is the compression

A-law companding

→ It has slightly flatter o/p characterstic then μ-law
→ I/o → O/p relationship is given by.

$$\frac{|y|}{x_{max}} = \begin{cases} \dfrac{\dfrac{A|x|}{x_{max}}}{1+\ln A} & ; \quad 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln\left[\dfrac{A|x|}{x_{max}}\right]}{1+\ln A} & ; \quad \dfrac{1}{A} < \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

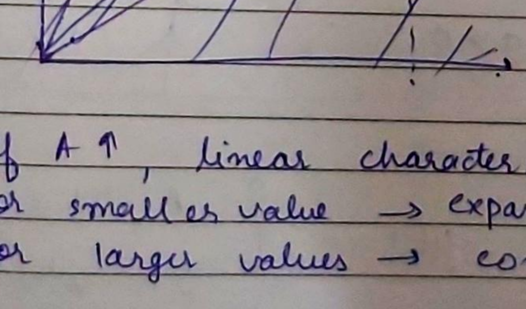


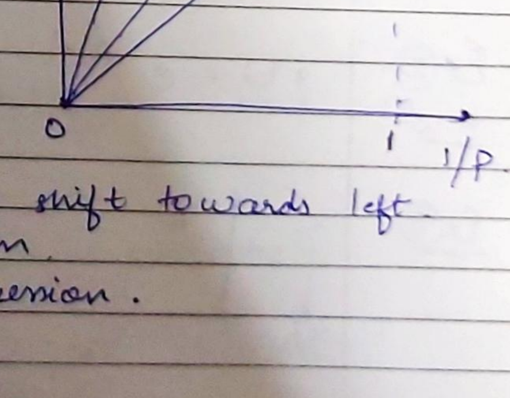
$\frac{|x|}{x_{max}} \rightarrow 0$ to 1

→ If $A = 1$ $\quad \frac{|y|}{x_{max}} = \frac{A|x|/x_{max}}{1+\ln A} = \frac{|x|}{x_{max}}$.

$|y| = |x|$

→ As $A \uparrow$ compression $\uparrow$.





→ If $A \uparrow$, linear characterstic shift towards left.
→ for smaller value → expansion
for larger values → compression.

Date

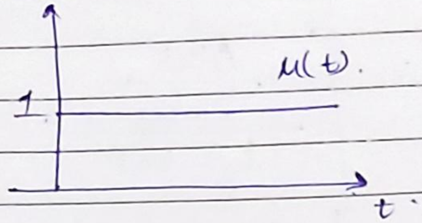
18-01-21

Fourier Transform

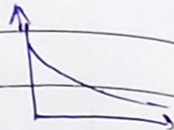
$$F\{g(t)\} = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt.$$

Unit step function.

$$u(t) = \begin{cases} 1 & \text{when } t \geq 0 \\ 0 & t < 0 \end{cases}$$



Ex (1) $g(t) = e^{-at} u(t)$ Fourier Transform ?



$$F\{g(t)\} = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j2\pi f t}\, dt.$$

$$= 0 + \int_{0}^{\infty} e^{-at}\, e^{-j2\pi f t}\, dt.$$

$$= \int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt = \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)}\right]_0^{\infty}$$

$$= \frac{-1}{a+j2\pi f}[0-1]$$

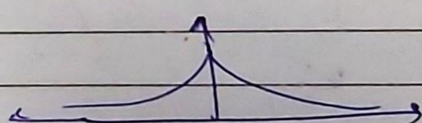
$$G(f) = F\{g(t)\} = \frac{1}{a+j2\pi f}.$$

$$|G(f)| = \frac{1}{\sqrt{a^2+(2\pi f)^2}}. \qquad \angle G(f) = -\tan^{-1}\left(\frac{2\pi f}{a}\right)$$

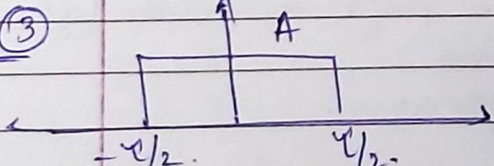
Phase angle

Ex (2) $g(t) = e^{-a|t|}$, $a > 0$

H.W.



Ex (3)



$$g(t) = \begin{cases} A, & -\tau/2 \leq t \leq \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

$$F\{g(t)\} = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} A.\, e^{-j2\pi f t}\, dt = A \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-\tau/2}^{\tau/2}$$

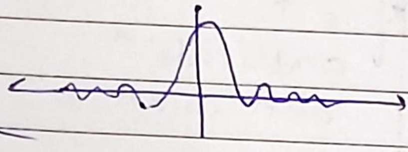
$$= \frac{A}{j2\pi f} \left[e^{j\pi f \tau} - e^{-j\pi f \tau} \right]$$

$$\left[\frac{e^{i\theta} - e^{-i\theta}}{2j} = \sin\theta \right]$$

$$= \frac{A}{\pi f} \sin(\pi f \tau)$$

$$\frac{\sin x}{x} \Rightarrow Sa(x)$$

$$= A\tau\, \text{sinc}(f\tau) \xleftarrow{\text{use}} \left[\frac{\sin \pi x}{\pi x} \rightarrow \text{sinc}(x) \right]$$



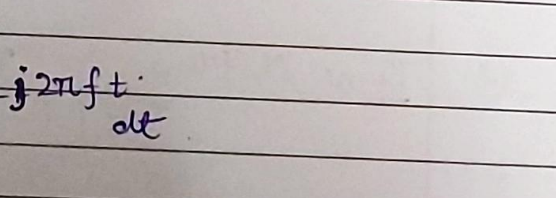
Ex (4)

Fourier Tranform of impulse.

$$f\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi f t}\, dt \qquad \left[\begin{array}{l} \delta(t) = 0 \\ \text{when } t \neq 0 \end{array} \right]$$

At $t = 0$

$$F\{\delta(t)\} = 1$$



Ex (5) $g(t) = \cos(\omega_0 t)$

$$F\{g(t)\} = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-j2\pi f t}\, dt$$

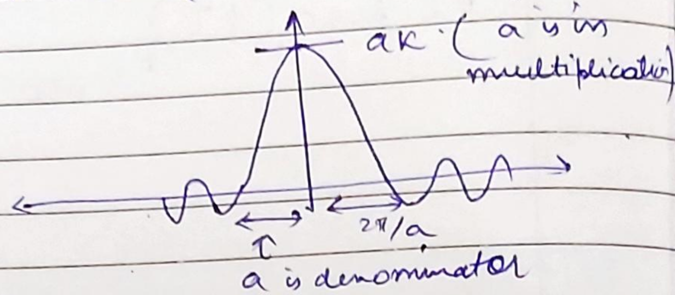
Ex-6. $g(t) = k$.

$$F\{g(t)\} = \int_{-\infty}^{\infty} K \cdot e^{-j2\pi f t} \, dt.$$

$$= \cancel{\left.\frac{k\, e^{-j2\pi f t}}{-j2\pi f}\right]_{-\infty}^{\infty}} \;\; \cancel{\frac{K}{j2\pi f}\left[e^{j2\pi f \infty} - 0\right]}$$

$$F\{g(t)\} = \lim_{a\to\infty} \int_{-a}^{a} K e^{-j2\pi f t} \, dt.$$

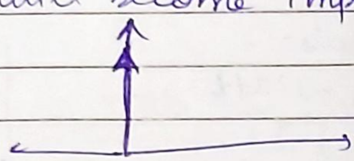
$$= \lim_{a\to\infty} aK \ \text{sinc}(f a)$$



If a ↑
amplitude ↑
and width ↓

As $a \to \infty$

it will become impulse.



<u>Properties of Fourier Transform</u>

① <u>Linearity</u>

$x_1(t) \longleftrightarrow X_1(f)$.

$x_2(t) \longleftrightarrow X_2(f)$

So. $a\, x_1(t) + b\, x_2(t) \longleftrightarrow a X_1(f) + b X_2(f)$.

② <u>Time Shifting</u>

$x(t) \longleftrightarrow X(f)$.

$x(t - t_0) \longleftrightarrow X(f)\, e^{-j2\pi f t_0}$

③ <u>Frequency Shifting</u>

$x(t)\, e^{j2\pi f_0 t} \longleftrightarrow X(f - f_0)$

Page :
Date :

(4). <u>Scaling</u>

$$x(t) \longleftrightarrow X(f)$$
$$x(at) \longleftrightarrow \frac{1}{|a|} X(f/a)$$

(5). <u>Time Reversal</u>

$$x(t) \longleftrightarrow X(f)$$
$$x(-t) \longleftrightarrow X(-f)$$

(6). <u>Differentiation in Time Domain</u>

$$x(t) \longleftrightarrow X(f)$$
$$\frac{d}{dt} x(t) \longleftrightarrow j2\pi f \, X(f)$$
$$\frac{d^2}{dt^2} x(t) \longleftrightarrow (j2\pi f)^2 X(f).$$

<u>Ex (2)</u>.

$$g(t) = e^{-a|t|}$$

$$F\{g(t)\} = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j2\pi f t} dt.$$

$$= \int_{-\infty}^{0} e^{(a-j2\pi f)t} dt + \int_{0}^{\infty} e^{(-a-j2\pi f)t} dt$$

$$= \left. \frac{e^{(a-j2\pi f)t}}{a-j2\pi f} \right]_{-\infty}^{0} + \left. \frac{e^{(-a-2j\pi f)t}}{-a-2j\pi f} \right]_{0}^{\infty}$$

$$= \frac{1}{a-j2\pi f} + \frac{1}{a+2j\pi f}$$

$$= \frac{2a}{a^2+4\pi^2 f^2}$$

21-01-21 Q signal $-A$ to $+A$ (Peak to peak signal)

Signal to Noise Ration → $\frac{\text{Signal Power}}{\text{Noise Power}}$

Signal to Q. Noise Ration → ?

Step size $= \frac{2A}{m}$ Signal Power (From Area) $= A^2/2$.

$$\overline{e^2} = \frac{s^2}{12} = \frac{A^2}{3m^2}.$$

m: No. of steps.

Signal to Q. Noise Ratio (SQNR) $= \frac{A^2/2}{A^2/3m^2} = \frac{3}{2}m^2$.

$$(SQNR)_{dB} = 10\log(SQNR)$$
$$= 10\log 3/2 + 10\log m^2$$
$$= 1.7 + 20\log m.$$

PAM, PWM, PPM

Pulse modulation

└ Process where one parameter of carrier signal ~~matches with~~ varied according to modulating signal.

Modulating signal → Low freq signal which want to transfer.

carrier signal → High freq.
(Amp or freq or Phase)

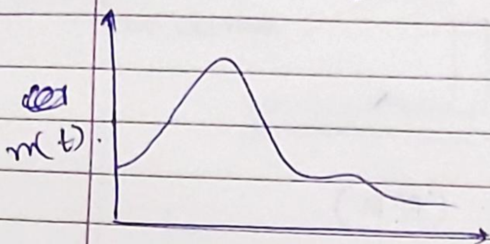
let modulating signal → 3 KHz

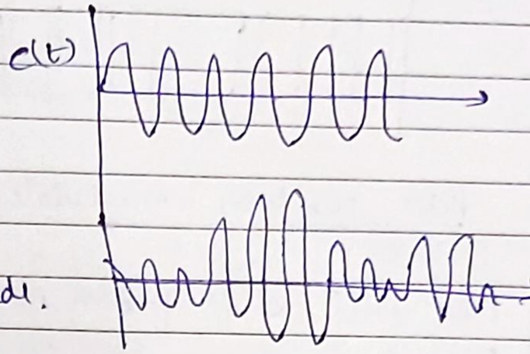
Size of Antenna $= \lambda/4$ $\lambda = c/f = \frac{3\times10^8}{3\times10^3} = 10^5$

$$= \frac{10^5}{4} = 25\times10^3$$
$$= 25\text{ KM}.$$

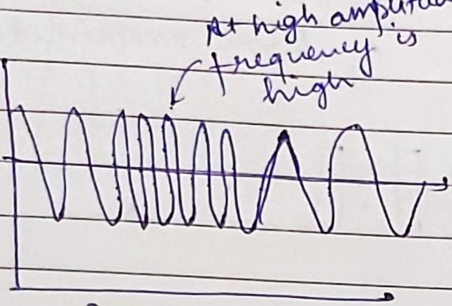
All diagrams are in time-domain

So, we have to ↑ the frequency to decrease the size of antenna.





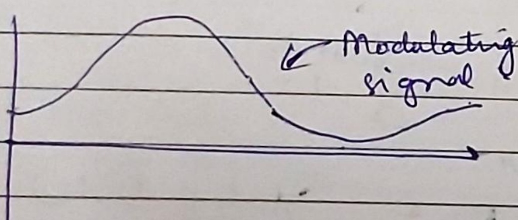
Amplitude modulated signal.

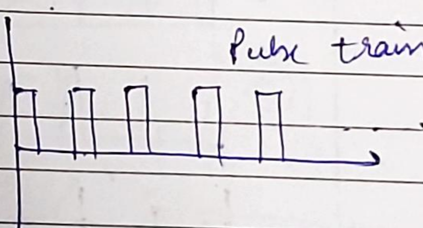


frequency modulated signal.

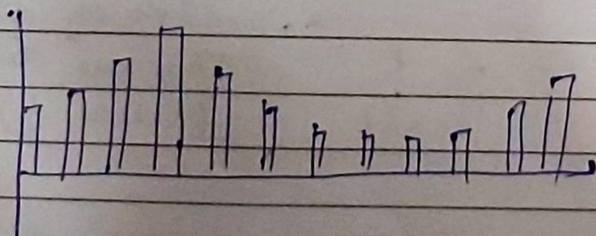
If we modulate signal with pulse train i.e. is called pulse modulation.

3 types.



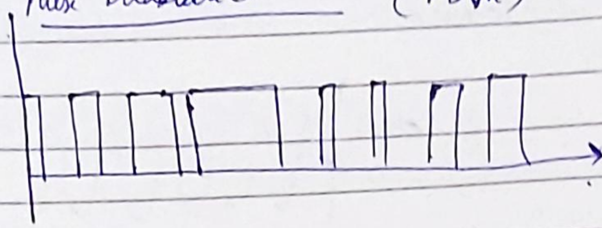


Pulse Amplitude Modulation (PAM) $\cong$ flat top sampling



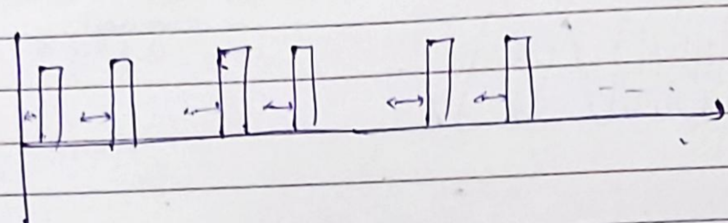
Pulse width modulation (PWM)
Pulse Duration " (PDM)

Amplitude = constant
width varies according to amplitude.

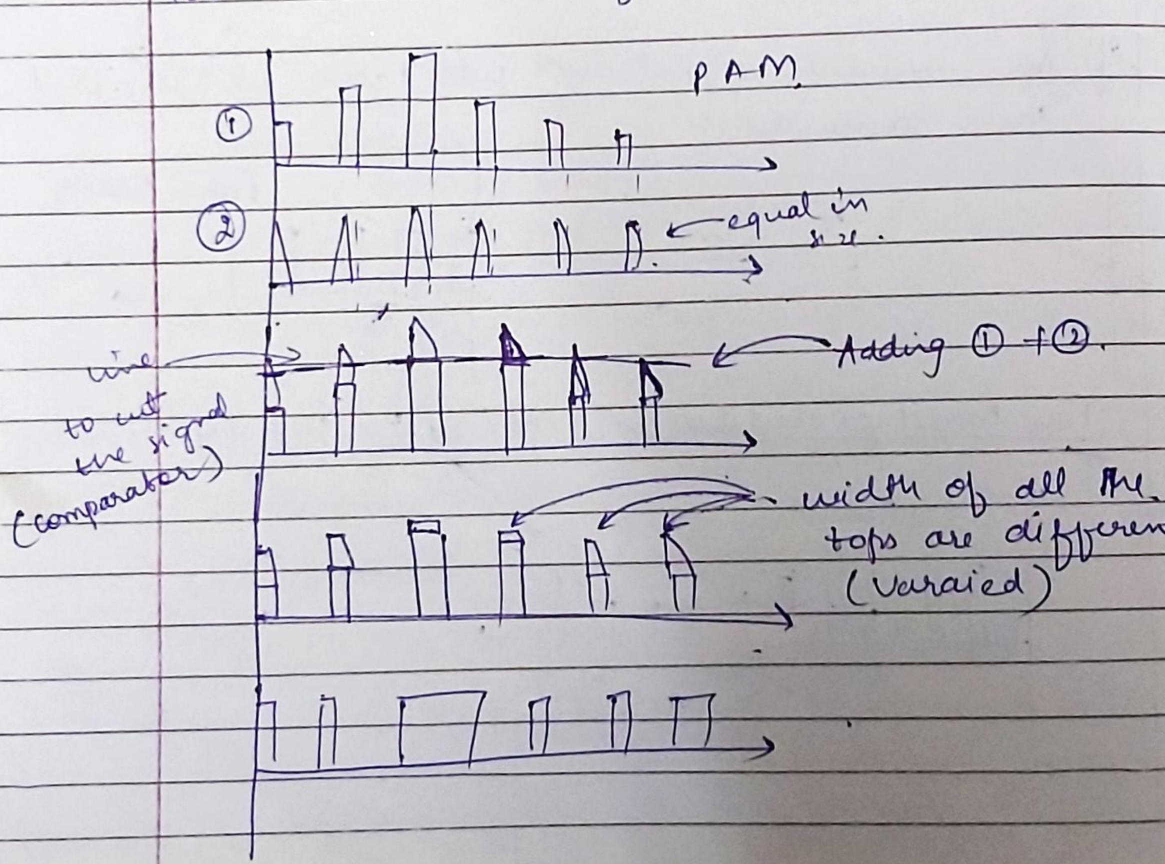


Pulse position modulation (PPM)

Pulse will shift from its position.
Shifting is more in case of high amplitude



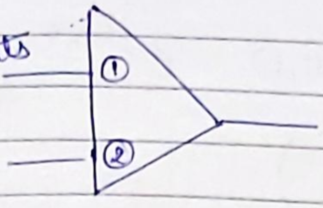
How PWM and PPM generated?

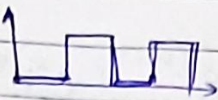


① > ② +ve voltage

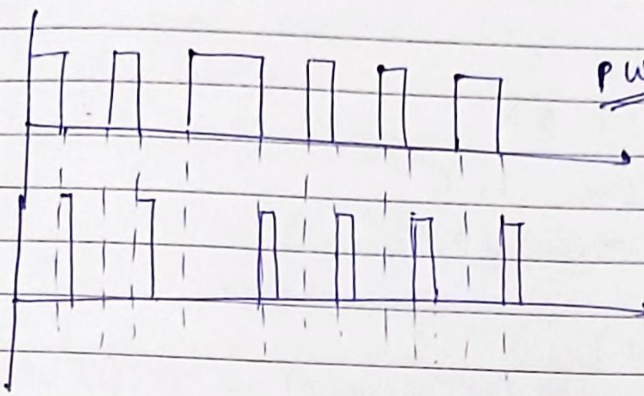
① < ② 0 voltage

2 inputs





comparator (Use for slicing)

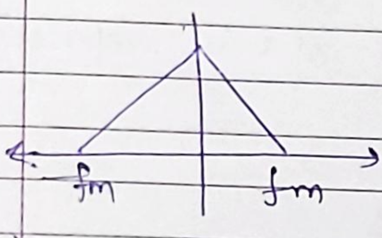


PWM → Rising time is same
→ falling time changed.

PPM

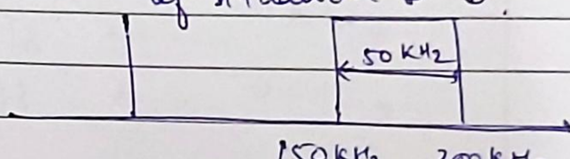
→ Monostable / Bistable vibrator

Sampling



Nyquist rate $\geq 2f_m$

If situation is I.



Exist near zero
↳ we can say low pass signal.

$f_s \geq 2f_m$

we require 400 Kz.
↳ Not a nice choice

→ Large data is generated with low signal variable if we are using high sampling frequency.

Band pass sampling

$f_s = ?$

Encoding

22|01|

Q. $g(t) = \cos(\omega_0 t)$ $\qquad \cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$$F\{g(t)\} = \int_{-\infty}^{\infty} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right)$$

$$F\{g(t)\} = \pi(\delta(\omega-\omega_0) + \delta(\omega+\omega_0))$$

Q. $g_1(t) = \delta(t-3)$

$$F\{g_1(t)\} = 1 \cdot e^{-j\omega \times 3}$$
$$= e^{-j3\omega}$$

Q. $g(t) = \sin(\omega_0 t)$

$$F\{g(t)\} = \pi(\delta(\omega+\omega_0)) - \pi(\delta(\omega-\omega_0))$$

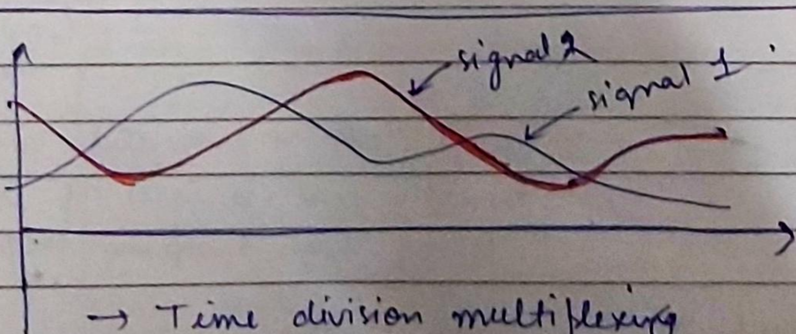
Q. $g(t) = e^{-j\omega_0 t}$

$1 \longleftrightarrow \delta(f)$ (F.T. of k is impulse)

$1 \cdot e^{j\omega_0 t} \longrightarrow \delta(\omega - \omega_0)$

$e^{-j\omega_0 t} \longrightarrow \delta(\omega + \omega_0)$

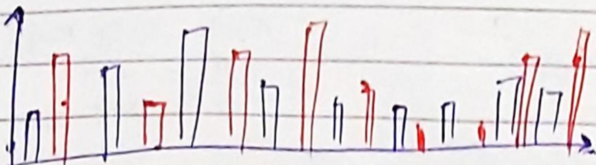
→ signum function



→ Time division multiplexing

Page :
Date :

T_1 system :- 24 users data can be multiplexed.



→ Taking sample at alternate frequency
→ To send multiple data using ~~simple~~ same of resources.

About T_1 system $f_m = 3.4$ KHz.

$f_s = 8$ KHz.

Quantization level = 256

Every sample values are encoded in 8 bits
→ 8 bit × 24 users = 192 bits
→ 1 bit for frame synchronization.

Total bits = 193 bits / frame

Data rate = 193 bits × 8 KHz.

= 1.544 mbps.

25/01/21

Modulation

- Analog Modulation ×
  - Types
    - → AM, ~~P~~
    - → FM,
    - → PM.
- Digital Modulation. ✓
  - Types
    - → ASK. (Amp Shift Key)
      - → Binary
      - → Quadrature
      - → M-ary.
    - → PSK
      - → B
      - → Q
      - → M-ary.
    - → FSK
      - → B
      - → Q.
      - → M-ary.
- Pulse ✓
  - Types
    - → PAM
    - → PPM
    - → PWM

PCM (Pulse code Modulation)
DPCM (Differential " " ")
DM (Delta Modulation)
ADM (Adaptive Delta Modulation)

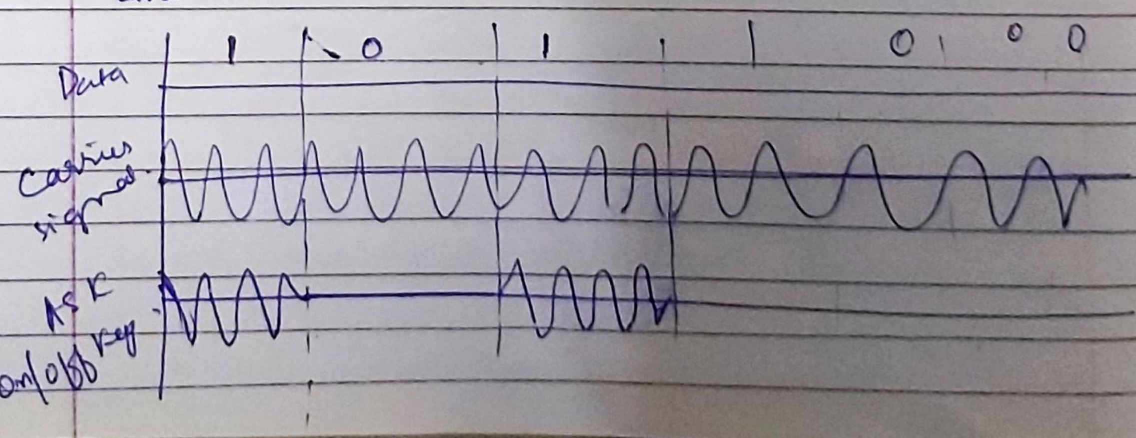
} These are not modulation techniques.

These techniques helps → Analog $\xrightarrow[\text{convert}]{\text{to}}$ Digital Data form.

→ Now we have data that to be modulated not signal.

signal → [Sampled] → [Quantized these sampled value] → [Encoded the data.] → data.

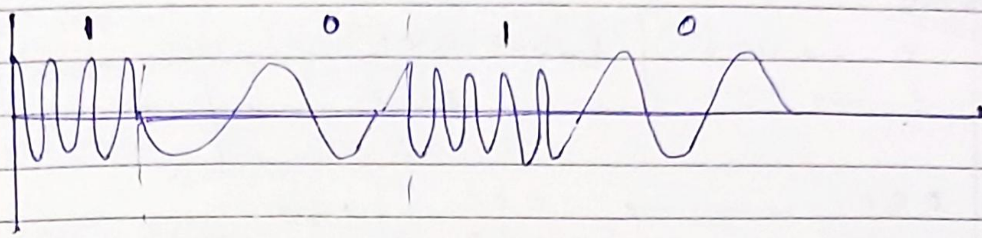
→ This data is need to be transmitted from one to another end



→ bit Duration
→ Symbol Duration. $f = nf_c$

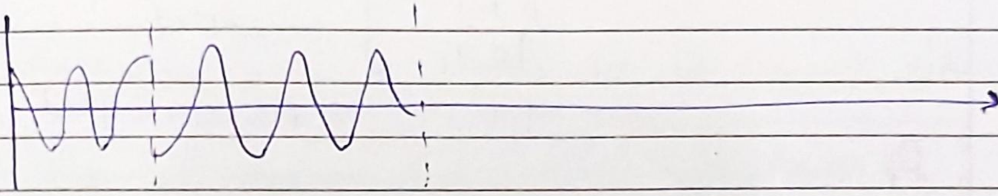
ASK → for '1' data, ~~Signals are~~ carrier wave is transmitted
↘ " '0' " , No signal is transmitted

PSK



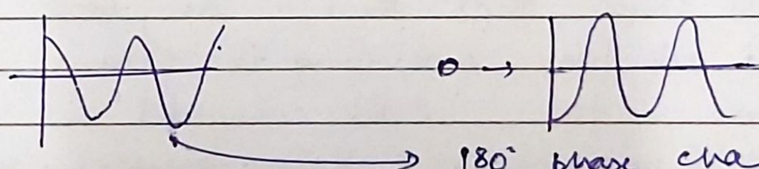
FSK → for '1' data → High frequency.
For '0' " → Low frequency.

PSK.



PSK → For '1' data, with one phase.
For '0' data, with opposite phase.

1 → 0 →

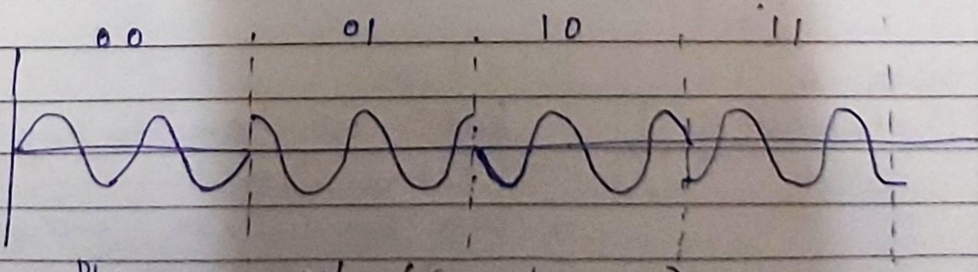


180° phase changed

Type → Binary → Single bit is modulated at a time
→ Quadrature → 2 bits
Mary → General (m = ~~2~~ 8, 16, 32 ... etc.)

For Quadrature (4 combinations are possible)

~~ASK~~
PSK.



Phase Diff = $\pi/2$ (for 4 states)

Page :
Date :

→ No of ~~data~~ levels ↑ Probability of error also ↑.

Example

0 → 000
1 → 111] Probability of error = ? [We are considering 0 → 000, 1 → 111]

000 ⟶ 001, 010, 100] 1 bit corrupted.

110, 011, 101] 2 bits corrupted

111 3 bits corrupted.

~~Probability of error =~~

→ When '1 bit corrupted' data is received, reciever will detect that there is an ~~error~~ error and ask for resending the data.
→ Same for '2 bit corrupted'.
→ But when all 3 bits corrupted now reciever can't identified that it is a corrupted data or not.
So, probability of error $= 1/8$.

If 0 → 0000
1 → 1111] Probability of error $= \frac{1}{16}$

Probability of error ↓, but we have to pay for extra data.
→ Bandwidth, energy, buffer size etc.

→ For improving system we to require lot of extra things.

Given $A \cos(2\pi f_c t)$ Power $= A^2/2$

$E/T = P$. $\frac{E}{T} = \frac{A^2}{2} \Rightarrow A = \sqrt{\frac{2E}{T}}$

→ So, we have some limitations for adding extra data.

Date 27/01/21

Binary Phase Shift Keying (BPSK)

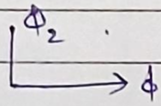
$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

where $E_b \rightarrow$ Bit energy . $T_b \rightarrow$ Bit duration

• Orthogonal Signal :- signals lie in different directions → interference is minimum.

Orthonormal function ⇒ $\boxed{\int_T \phi_1(t)\, \phi_2(t)\, dt = 0}$



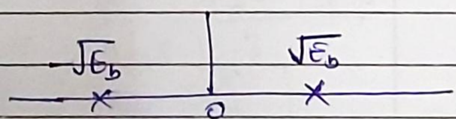
If $\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$

$$S_1(t) = \sqrt{E_b}\, \phi_1(t)$$

$$S_2(t) = -\sqrt{E_b}\, \phi_1(t)$$

S_1 and S_2 are represented using one orthogonal function. which means both signal lies in one dimension.

$$S_{11} = \int_0^{T_b} S_1(t)\, \phi_1(t)\, dt$$



constillation diagram

$$= \int_0^{T_b} \sqrt{E_b}\, \phi_1(t) . \phi_1(t)\, dt$$

$$= \sqrt{E_b} \int_0^{T_b} \phi_1(t) . \phi_1(t)\, dt$$

$$\boxed{S_{11} = \sqrt{E_b}}$$

orthonormal

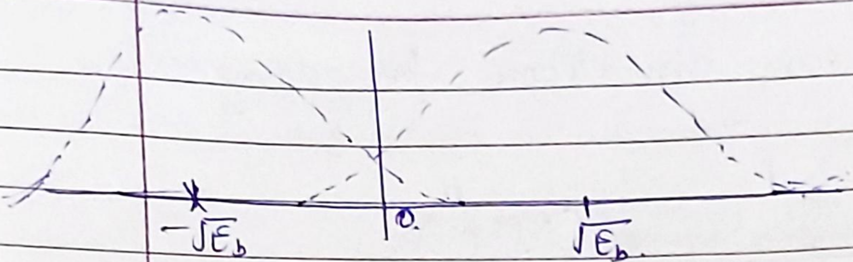
$\int_T \phi_i(t)\, \phi_j(t) = 1$ $i = j$

$\int_T \phi_i(t)\, \phi_j(t) = 0$ $i \neq j$

$$S_{21} = \int_0^T S_2(t)\, \phi_1(t)\, dt$$

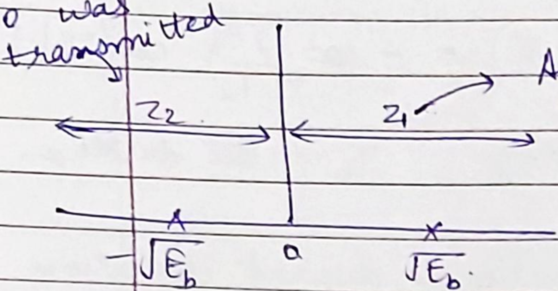
$$= -\sqrt{E_b}$$

Page :
Date :



constellation diagram.

$s_{11}, s_{21} \rightarrow$ coordinates of signal space



$\rightarrow$ After detection of any values in this region, it means 1 was transmitted

$\rightarrow$ Only one signal is transmitted at a time through the channel.

$\rightarrow$ If recieved signal is $x(t)$

Observation scalar $\Rightarrow x_1 = \int_0^{T_b} x(t)\, \phi_1(t)\, dt$

Acc. to x_1 we will decide 0 is transmitted or 1

Error probability (Calculation)

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right]$$

likelyhood function when signal 0 was transmitted.

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$

$$\text{Prob of Error (0)} = \int_0^{\infty} f_{x_1}(x_1|0)\, dx$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$

let $\frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b}) = z$

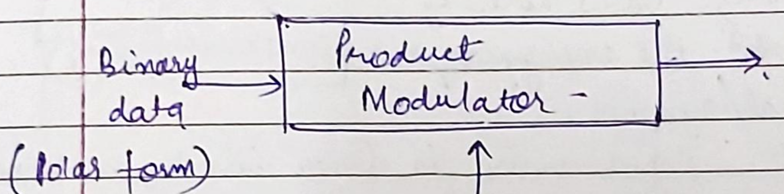
$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz.$$

$$P_e(0) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

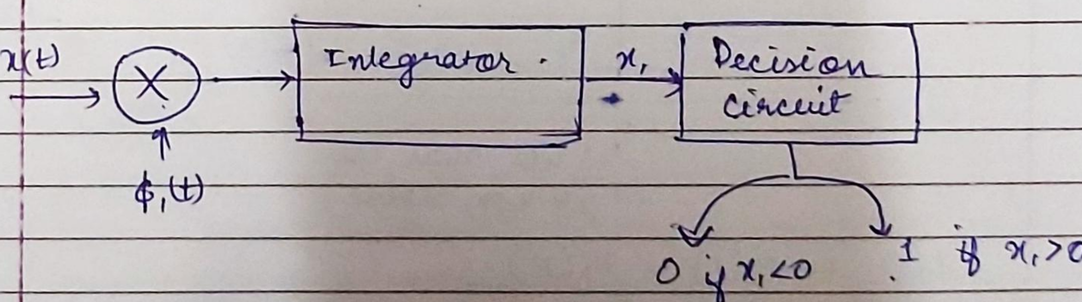
$$\boxed{\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-x^2}\, dx.}$$

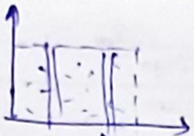
$$P_e(1) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_{e_T} = \frac{1}{2}\left(P_e(0) + P_e(1)\right)$$



$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$





28/01/21

uniform Distribution

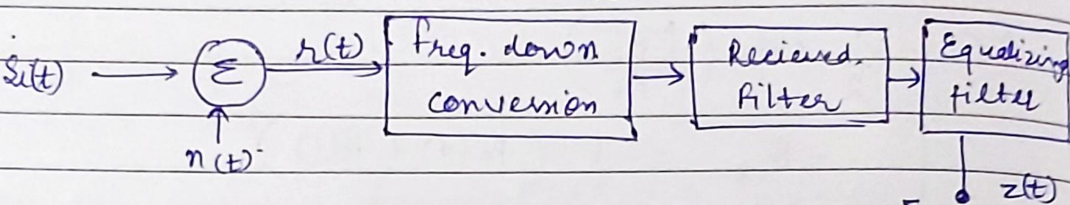
For any binary modulation we require two functions.

$$S_i(t) = \begin{cases} S_1(t) & 0 \le t \le T \Rightarrow 0 \\ S_2(t) & 0 \le t \le T \Rightarrow 1 \end{cases}$$

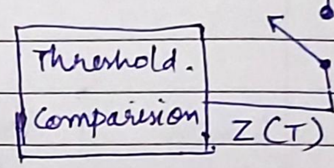
$$\underbrace{r(t)}_{\text{recieved signal}} = s_i(t) \circledast \underbrace{h(t)}_{\text{impulse response of channel}} + \overbrace{n(t)}^{\text{noise}}$$

For ideal distortionless channel

$$r(t) = S_i(t) + \underbrace{n(t)}_{\text{noise}} \qquad i = 1 \text{ or } 2.$$



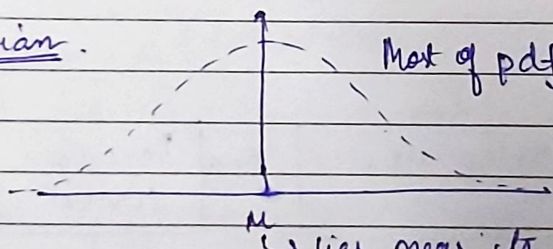
→ Z(T) has a voltage value directly proportional to the energy of the recieved signal/symbol.



→ We consider that input noise is Gaussian by nature.
Input noise is Gaussian random process.

→ Recieved filter is a linear

Gaussian.



↳ lies near to mean value.

Most of pdf. are Gaussian.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

$$Z(T) = a_i(T) + n_0(T)$$

It will also be gaussian

$$Z = a_i + n_0.$$

Z(T) is also a Gaussian random variable with a mean either a_1 or a_2.

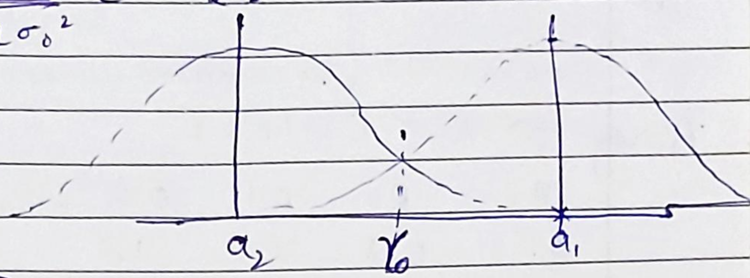
$$P(n_0) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{(n_0 - 0)^2}{2\sigma^2}}$$ let mean of $n_0(t) = 0$

$$= \frac{1}{2\pi\sigma^2} e^{-1/2\left(\frac{n_0}{\sigma}\right)^2}$$

$$P(z|s_1) = \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{1}{2}\frac{(z-a_1)^2}{\sigma_0^2}}$$

$$P(z|s_2) = \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{(z-a_2)^2}{2\sigma_0^2}}$$

Diff $a_1 - a_2 \uparrow$
error $\downarrow$



Where should be γ_0?

$\boxed{P(s_1) = P(s_2)}$ Probability of s_1 = Prob. of s_2

the likelihood ration.

$$\frac{P(z/s_1)}{P(z/s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$ Bay's Theorem.

Due to this

$$z \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1 + a_2}{2}.$$

$$P(e|s_1) = \int_{-\infty}^{\gamma_0} p(z/s_1)\, dz.$$

$$p(e|s_2) = \int_{\gamma_0}^{\infty} p(z|s_2)\, dz.$$

P_b (Probability of bit error) $= P(e/s_1)\,(P(s_1)) + P(e/s_2).\,P(s_2)$

$$P_b = \frac{1}{2}\left[P(e|s_1) + P(e|s_2)\right]$$

Page :
Date :

$P(e/s_1) = P(e/s_2)$

$P(H_2/s_1) = P(H_1/s_2)$

Probability of error $= \int_{\gamma_0}^{\infty} P(z/s_2)\,dz = \int_{\frac{a_1+a_2}{2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{z-a_2}{\sigma}\right)^2} dz$

29/1/21 for Practicals → MATLAB.

Commands (On command window)

Some commands:
- >> clear a
- >> clear all.
- >> help ...
- >> help help.

Array >> x = 0:1/10:1

x = 0. 0.1 0.2 0.3 0.4 0.5 0.6 0.7
0.8 0.9 1.0

space, commas → Both are valid

Array >> A = [1 2 4 7 ,9 , 2]

A = 1 2 4 7 9 2.

>> A = [1 2 3 ; 2 -1 0 ; 0 3 5] ← 2-D array 3X3 Matrix

>> A + B → Both are matrices

>> A - B

>> C = A + 2.

>> A * B

>> A' (Transpose of A)

>> A + 2

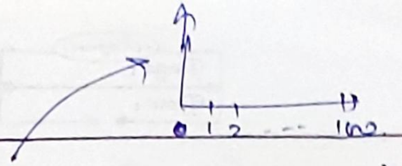
→ for writing program we have to open script window

→ Initiate your program with clc and clear all.

```
- clc; clear all;
- t = 0 : 1/100 : 1
- f = 2;
- x = cos(2* pi * f * t);
- Plot (x) → y variable case I
```

Plot (t, x) (Case II)
↑ x-var ↑ y-var.

Example of program →



case I. x – contains index of array starting with 1

case II x – contain values of t.

for running more than 1 plots simultaneously.

- Subplot (2, 1, 1) position.
  row. columns
- plot (t, x_1);
- f_2 = 10 ;
- x_2 = cos (2* pi*f_2*t);
- Subplot (2, 1, 2)
- plot (t, x_2)

→ for array operations we have use . before operators.

eg :- .*, .^, ./

.^ ↳ Power operator.

Loop

```
for n = 1 : 2 : 30;
    a = 2* n ;
end
```

if-else

```
if
else
end
```

```
for i = 1 : 2 : 30;
    a(i) = 2* i ;
end;
```

Random variable

a = Rand (10) → generate 10×10 matrix

a = Rand (1, 10) . → generate 1×10
↳ uniform Random variable (lies from 0 to 1)

b = Randn (1, 10)
↳ Gaussian random variables

hist (a)

hist (b)

Date
01-12-21

Page :
Date :

Binary frequency Shift Keying (BFSK)

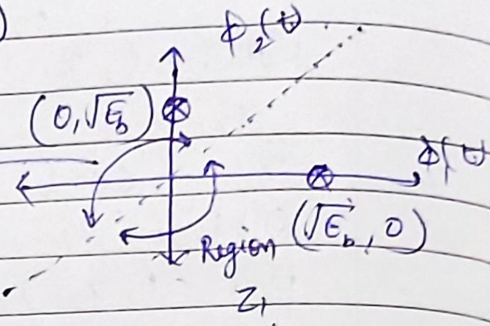
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \qquad s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t)$$

the orthonormal basis function.

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t)$$

$$s_1(t) = \sqrt{E_b}\,\phi_1(t)$$
$$s_2(t) = \sqrt{E_b}\,\phi_2(t)$$



$$s_{11} = \int_0^{T_b} s_1(t)\cdot\phi_1(t)\,dt = \sqrt{E_b}$$

$$s_{12} = \int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\phi_2(t)\,dt = 0.$$

$$s_{21} = \int_0^{T_b} \sqrt{E_b}\,\phi_2(t)\,\phi_1(t)\,dt = 0.$$

$$s_{22} = \int_0^{T_b} \sqrt{E_b}\,\phi_2(t)\,\phi_2(t)\,dt = \sqrt{E_b}$$

Middle point should be boundary.

→ Error in BFSK is more than BPSK because of less distance between points $(0,\sqrt{E_b})$ and $(\sqrt{E_b}, 0)$

If the recieved signal is $x(t)$.

Observation scalar

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt.$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt$$

when $x_1 > x_2$. ⇒ Region Z_1

$x_1 < x_2$ ⇒ Region Z_2

Page:
Date:

⇒ define a new random variable L whose sample value l is

$$l = x_1 - x_2.$$

if $l > 0$ ⇒ Region Z_1 $l < 0$ ⇒ Region Z_2

Expectation value

$$E[L/1] = E[X_1/1] - E[X_2/1]$$
$$E(L/1] = \sqrt{E_b} - 0 = \sqrt{E_b}$$
$$E[L/0] = E(X_1/0] - E[X_2/1]$$
$$E[L/0] = 0 - \sqrt{E_b} = -\sqrt{E_b}$$

$$V[L] = Var[X_1] + Var[X_2] = N_0/2 + N_0/2 = N_0$$

L → mean $= 0$, variance $= N_0$

⇒ The conditional prob. of Random var. L when 0 was transmitted

$$f_L(l/0] = \frac{1}{2\pi N_0} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right]$$

$$P(e/0) = P(l > 0 \mid \text{symbol 0 was transmitted})$$
$$= \int_0^\infty f_L(l/0)\, dl.$$
$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[\frac{-(l+\sqrt{E_b})^2}{2N_0}\right] dl$$

let

$$\frac{l+\sqrt{E_b}}{\sqrt{2N_0}} = z$$

$$P(e/0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/2N_0}}^{\infty} e^{-z^2} dz.$$

$$P(e/0) = \frac{1}{2}\, \text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Page :
Date :

Similarly, $P(e/1) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$P_e = \frac{1}{2}\left[P(e/0) + P(e/1)\right]$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

Error is more in case of BFSK.

Home Assign :- Binary Amplitude shift Keying or Binary ON-OFF Keying

Error Correcting codes → To minimize effect of noise in channel.

→ source coding

→ Channel coding

parity code → even parity

P	data		Tx	1-bit corrupted →	Rx
0	1001	→	01001	→	01101
1	1000	→	11000	→	01000
0	1111	→	01111	→	01101
1	1101	→	11101	→	11101
1	0010	→	10010	→	10010

added bit to maintain even parity

It has odd parity

So, we can say this is wrong data

→ If 2 bit corrupted then we can't identify if it is corrupted or not

Linear Block code (n, k)

n → No. of coded bits

k → No. of data bits

n−k → No. of Parity bits

$\bar{c} = \bar{m}\,G$

m → message array (data array) bits

G − Generator matrix

Hamming code (7, 4)

$\bar{c} = [P : m]$ $m = [1010]$

Generator Matrix $G = [\hat{P} : I_k]$

$n \to 7$

$k \to 4$

$n-k \to 3$

$$\bar{G} = \begin{bmatrix} 1 & 1 & 0 & \vdots & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & \vdots & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & \vdots & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & \vdots & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\bar{C} = [1010]\,\bar{G}$

$\bar{c} = [0 0 1 : 1010]$

$$\boxed{\begin{aligned} c_i + c_j &= m_i G + m_j G \\ &= (m_i + m_j) G \end{aligned}}$$ → Property of code

H — parity check matrix

$\bar{H} = [I_{n-k} : P^T]$

$H G^T = [I_{n-k} : P^T] \begin{bmatrix} P^T \\ I_k \end{bmatrix}$ $- P^T + P^T = 0$

$c = m G.$

$c H^T = m G H^T$

$\boxed{c H^T = 0}$

Recieved code vector ← $r = c + e$
↳ error vector

$c = 1010011$
$e = 0100000$
$r = 11\boxed{1}0011$
↳ error bit

$S = r H^T$

$S = (c + e) H^T$

$= c H^T + e H^T$ → 0

$\boxed{S = e H^T}$

↳ Syndrome :- Gives info about error location.

If $S = 0$ then there is no error

Page :
Date :

5-02-21

Hamming Distance

↳ No. of elements which are differ from one code to another.

Eg :- C_i = 1011001

C_j = 1101001

H. Distance = 2.

d_{min} → minimum distance.

→ An (n, k) linear block code of minimum distance d_{min} can correct upto 't' ~~errors~~ errors if and only if

$$t \le \left\lfloor \frac{1}{2}(d_{min} - 1) \right\rfloor$$

For (7,4) $d_{min} = 3$. ⇒ $\boxed{t \le 1}$

→ We can only correct single bit error in (7,4) code word.

⇒

Syndrome	Error Pattern	
0 0 0	0 0 0 0 0 0 0	(No error)
1 0 0	1 0 0 0 0 0 0	(1st bit corrupted)
0 1 0	0 1 0 0 0 0 0	(2nd " ")
0 0 1	0 0 1 0 0 0 0	
1 1 0	0 0 0 1 0 0 0	
0 1 1	0 0 0 0 1 0 0	
1 1 1	0 0 0 0 0 1 0	
1 0 1	0 0 0 0 0 0 1	

Eg :- recieved code vector r = [1 1 0 0 0 1 0]

Find the correct transmitted code word.

$S = rH^T$

$S = [1100010]\ H^T$

$S = 001$.

$$H^T = \begin{bmatrix} I_{n-k} \\ \cdots \\ P^T \end{bmatrix}$$

$S = 001 \quad e = 0010000$

→ 3rd bit is corrupted

$r = 1100010$

$= 1110010$

<u>Prop. of syndrome</u>

The syndrome depends only on the error pattern and not on the transmitted code word.

$$S = rH^T = (c+e)H^T$$

$$S = eH^T$$

All error patterns that differ at most by a code word have the same syndrome

#. The syndrome is the sum of those columns of the matrix H corresponding to the error location.

$t \leq \left[\frac{1}{2}(d_{min}-1) \right]$ Proof = ?

Hamming weight

channel coding or source coding

$s = 001$ $\quad e = 0010000$

→ 3rd bit is corrupted.

$r = 1100010$

$= 1110010$

<u>Prop. of syndrome.</u>

The syndrome depends only on the error pattern and not on the transmitted code word.

$$s = rH^T = (c+e)H^T$$

$$s = eH^T$$

All error patterns that differ at most by a code word have the same syndrome

#. The syndrome is the sum of those columns of the matrix H corresponding to the error location.

$t \le \left\lfloor \frac{1}{2}(d_{min} - 1) \right\rfloor$ <u>Proof</u> = ?

Hamming weight

<u>06-2-21</u> <u>Pulse Code Modulation (PCM)</u>

Analog Signal → Sampler → Quantizer → Encoder → Digital Data

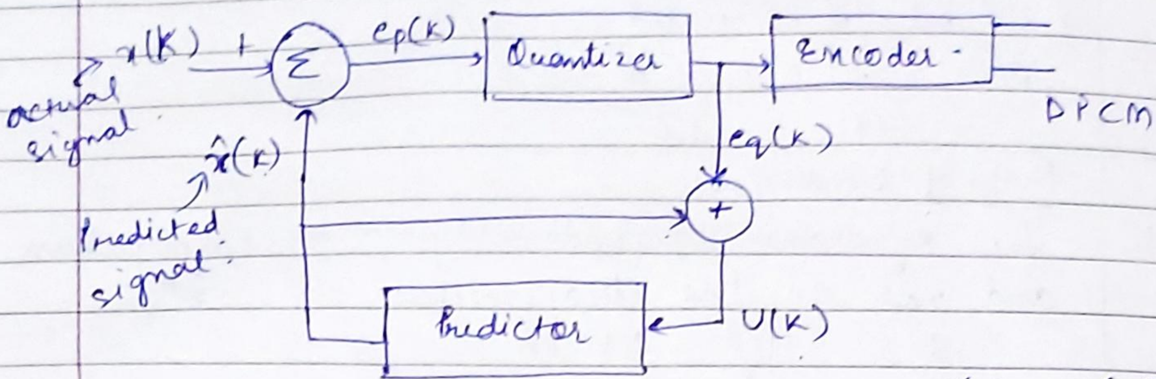
Encoder: using source coding

→ <u>DPCM</u> Problem :- large data size.

For reducing data size inplace exact sampled value we are sending the difference b/w two consecutive sampled value.

Page :
Date :

Differential PCM



$x(K) = x(KT_s)$

$e_p(k) = x(k) - \hat{x}(k)$

→ Prediction error of k^{th} sample.

$e_q(k)$ = Quantized prediction error.

$q(k)$ = Quantization error

$e_q(k) = e_p(k) \pm q(k)$

I/P to the predictor.

$u(k) = \hat{x}(k) + e_q(k)$

$u(k) = \hat{x}(k) + e_p(k) \pm q(k)$

$u(k) = x(k) \pm q(k)$

SQNR (signal to quantization

$$SQNR = \frac{\text{Var of } x(k)}{\text{Var of } q(k)} = \left[\frac{Var[x(k)]}{Var[e_p(k)]}\right] \times \frac{Var[e_p(k)]}{Var[q(k)]}$$

SQNR = prediction Gain × SNR_p
(Gp) ↑ signal to noise ratio.

→ Gp should be as high as possible

<u>One Tap Predictor</u> :- k^{th} value depend on $k-1^{th}$ value

$\hat{x}(k|k-1) = \hat{x}(k)$

Page:
Date:

$\hat{x}(k) = a.u(k-1)$ $\quad$ $u(k+1|k-1) = u(k-1)$

Prediction coefficient

$e_p(k) = x(k) - \hat{x}(k)$

$e_p(k) = x(k) - a.u(k-1)$

Mean square error

$$E[e_p^2(k)] = E[\{x(k) - au(k-1)\}^2]$$

$E[x^2(k)]$ = signal power = $R(0)$

$E[-2a\,x(k)\,u(k-1)] = -2a\,E[x(k).u(k-1)]$

let quantization noise is very small.

$= -2a\,E[x(k)\cdot x(k-1)]$

$= -2a\,R(1)$

auto co-relation

$E[e_p^2(k)] = R_x(0) - 2a\,R_x(1) + ?$

$? = E[a^2 u(k-1)\,u(k-1)]$

$= E[a^2 x(k-1)\,x(k-1)]$

$= a^2 E[x(k-1)\,x(k-1)]$

$= a^2 E[x(k)\,x(k)]$

$= a^2 R_x(0)$

$E[e_p^2(k)] = R_x(0) - 2a\,R_x(1) + a^2 R_x(0)$

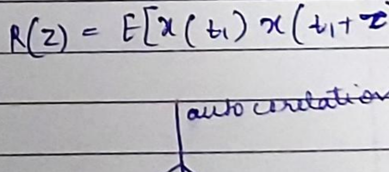
$E[e_p^2(k)] = R_{ep}(0)$

$R_{ep}(0) = R_x(0) - 2a\,R_x(1) + a^2 R_x(0)$

Prediction error power $\quad$ signal power

Auto corelation

$R(\tau) = E[x(t_1)\,x(t_1+\tau)]$



$R(0) = E[x(t_1)\,x(t_1)]$

= signal power

Page :
Date :

$$R_{ep}(0) = R_x(0)\left[1 - 2a\frac{R_x(1)}{R_x(0)} + a^2\right]$$

This relation is used to calculate value of a

$$\boxed{G_p = \frac{R_x(0)}{R_{ep}(0)}}$$

$$\boxed{G_p = \frac{1}{1 - 2a\frac{R_x(1)}{R_x(0)} + a^2}} \rightarrow$$

~~Maxima it~~
Maximize it to get value of a