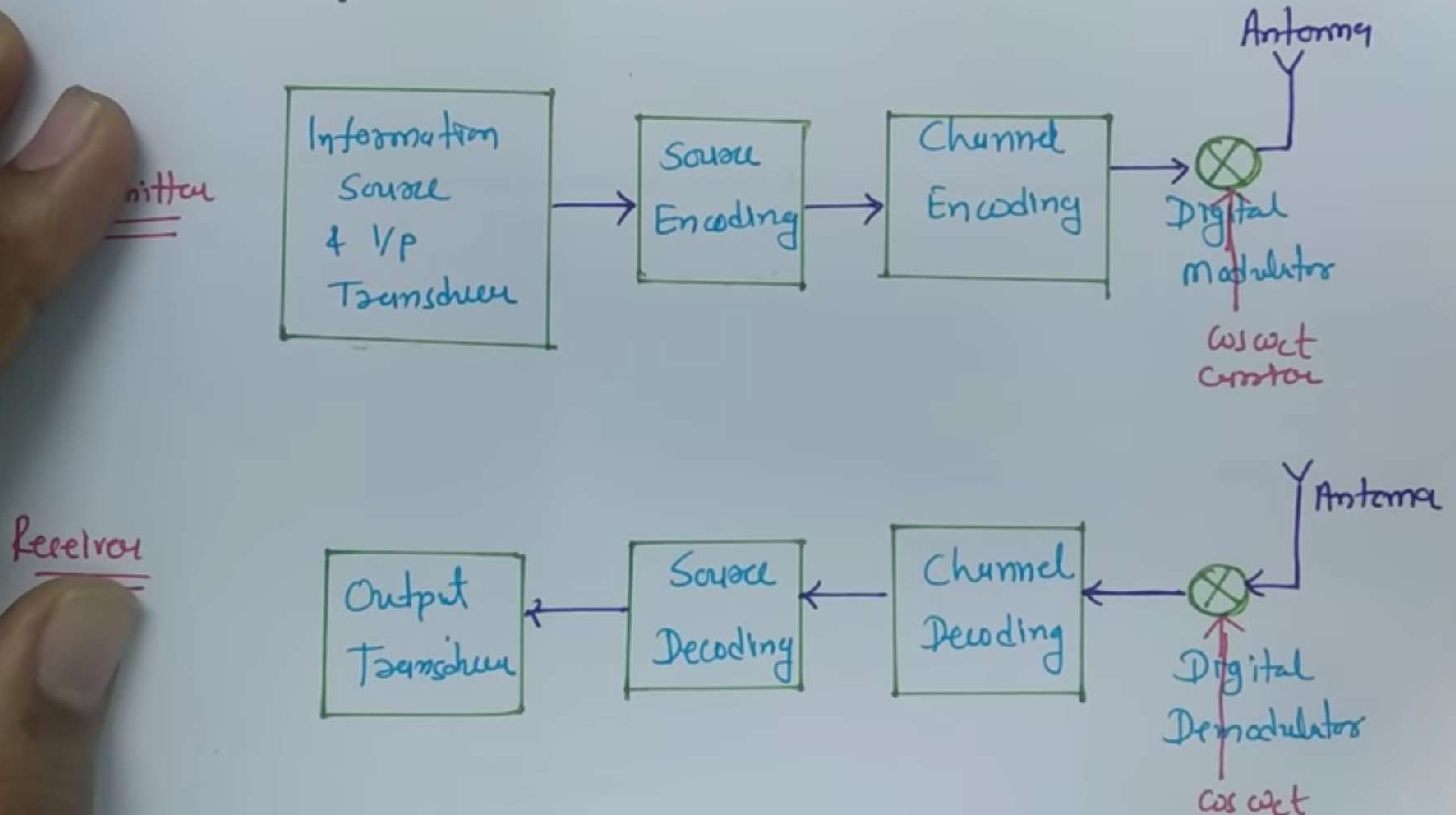
Basic Blockdiagram of digital communication system



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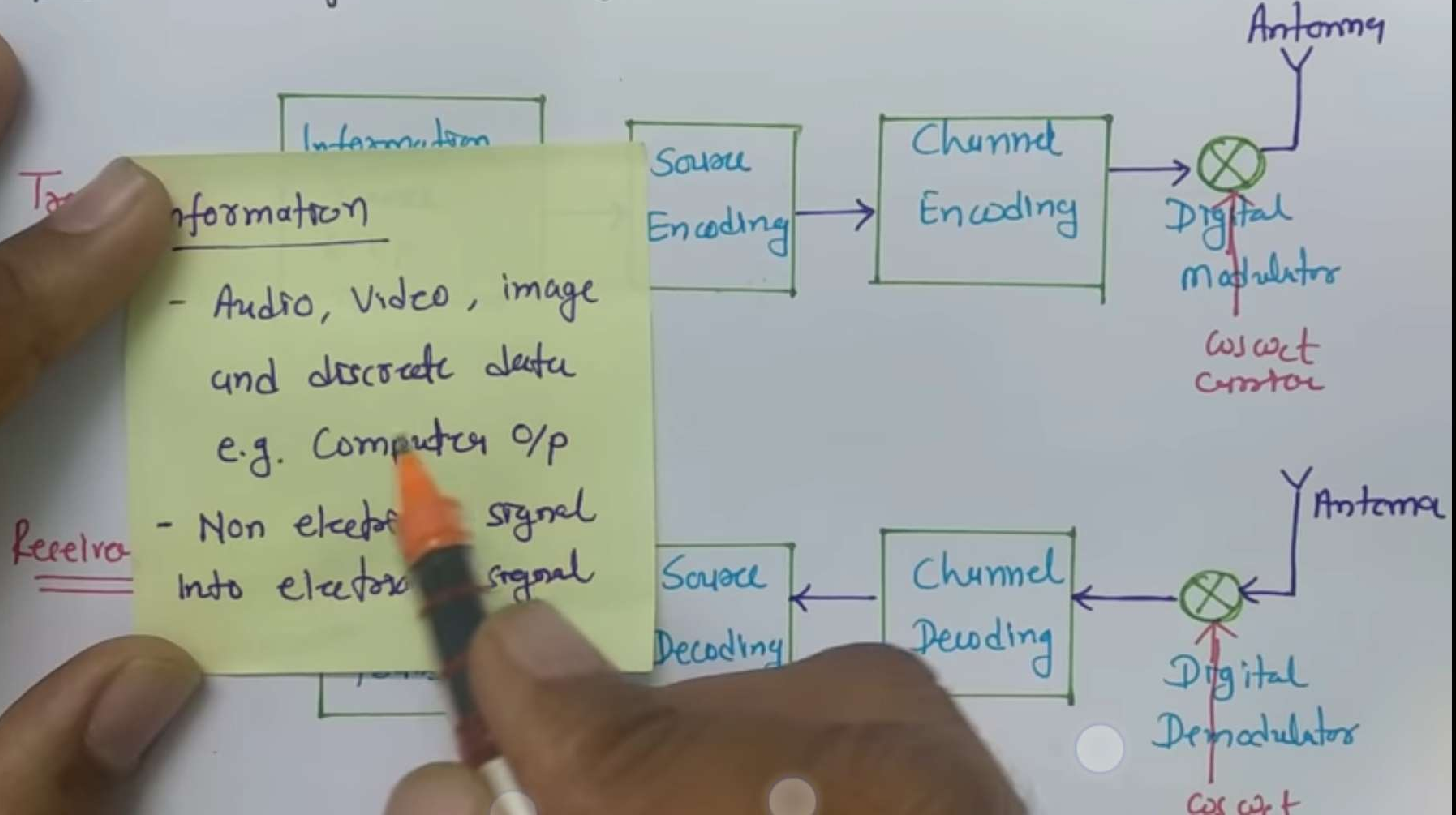


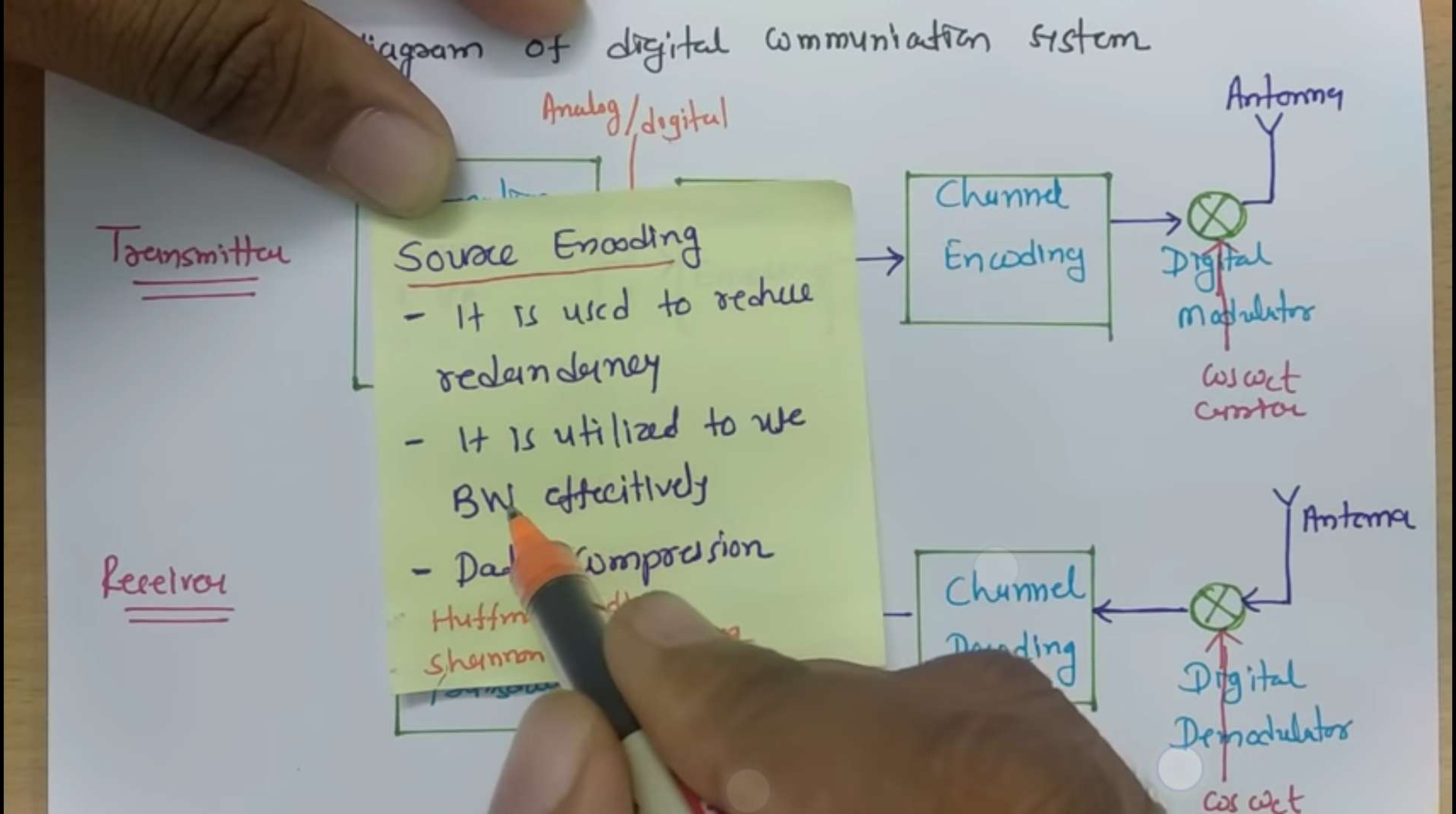


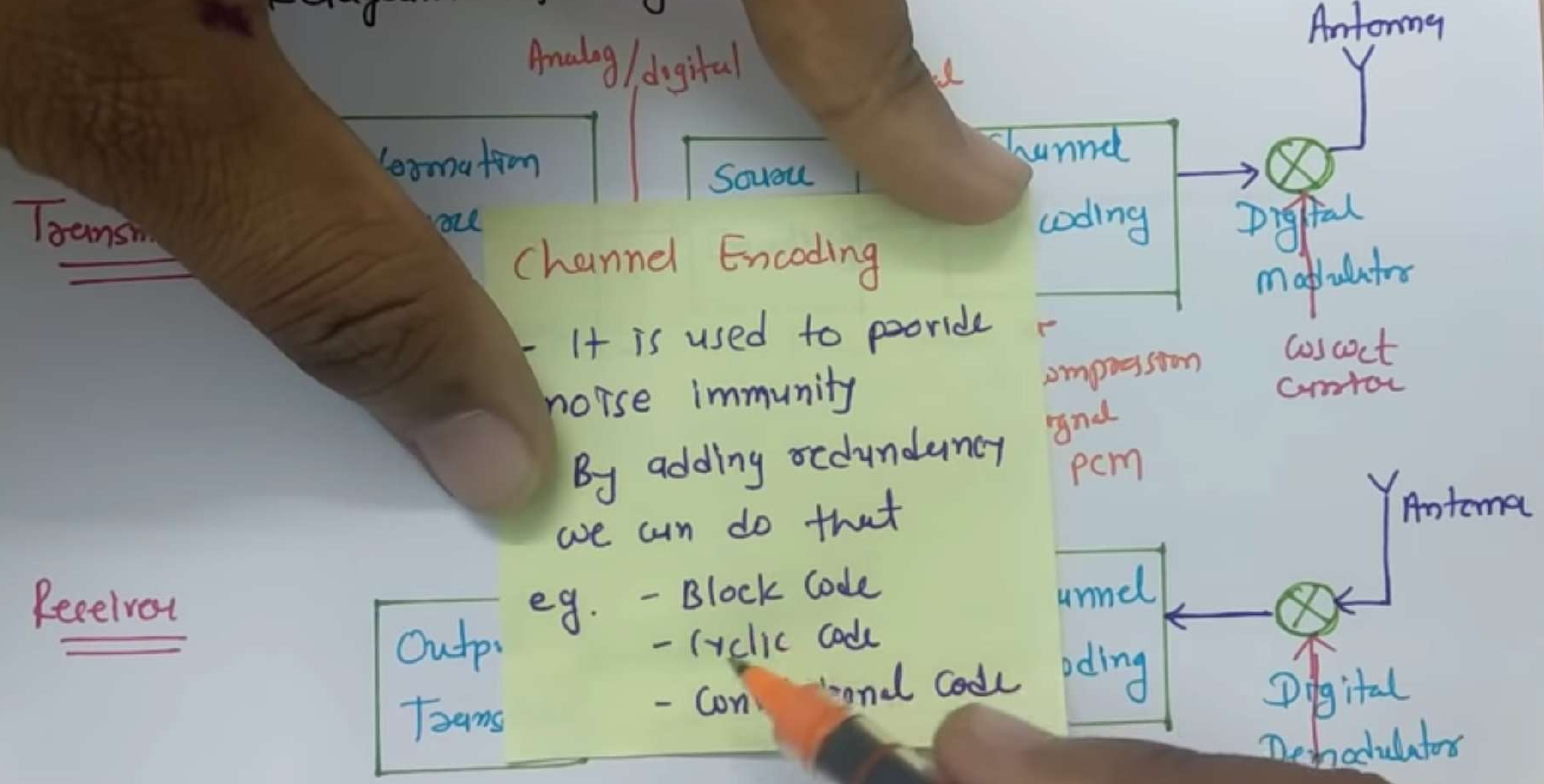
diagram of digital communication system
Analog/digital
Transmitter
Source Encoding
- It is used to reduce redundancy
- It is utilized to use BW effectively
- Data compression
Huffman
Shannon
Channel Encoding
Antenna
Digital Modulator
cos ωct
Carrier
Receiver
Channel Decoding
Antenna
Digital Demodulator
cos ωct

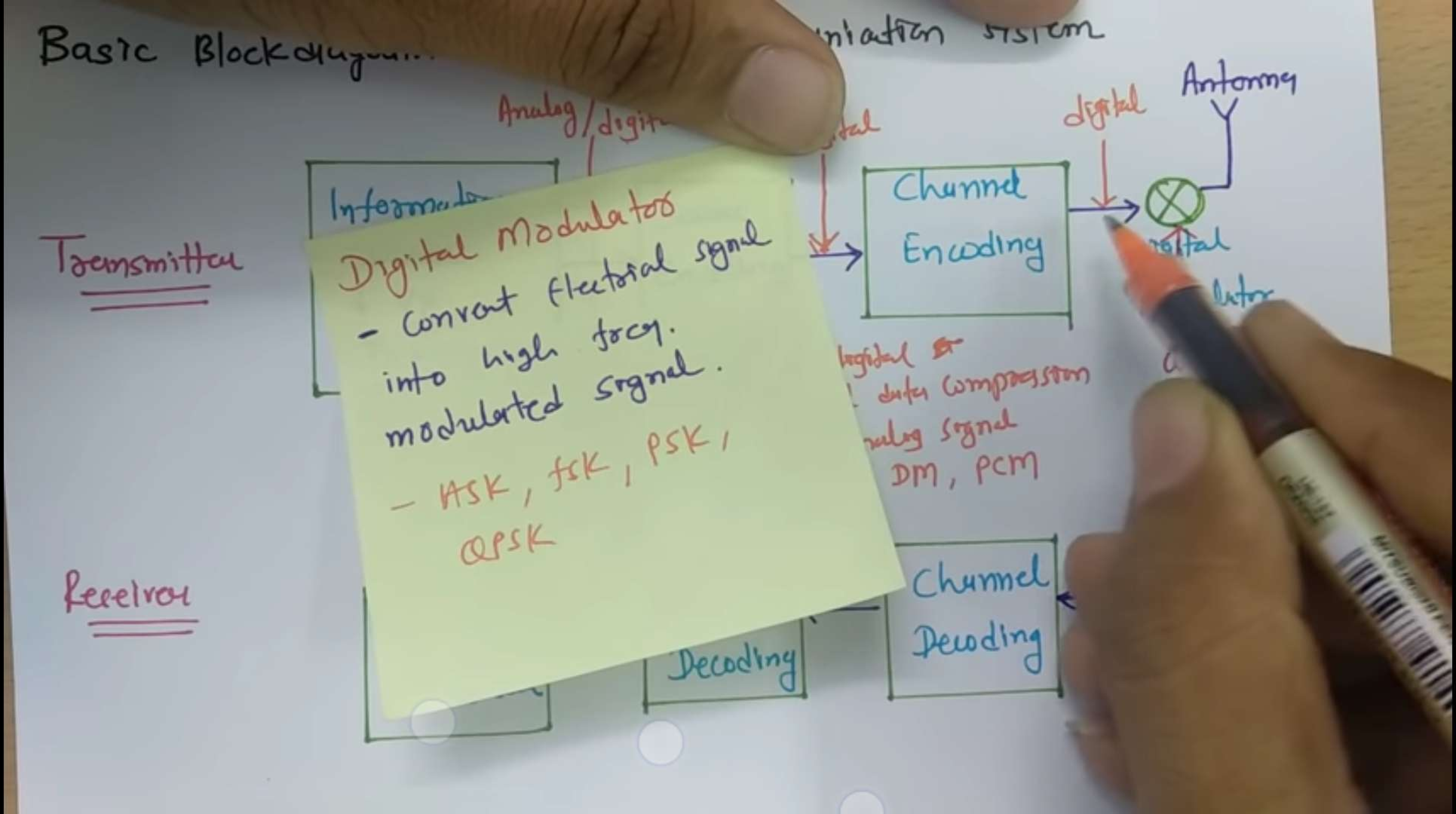
k diagram of digital ... nication system

Channel Encoding

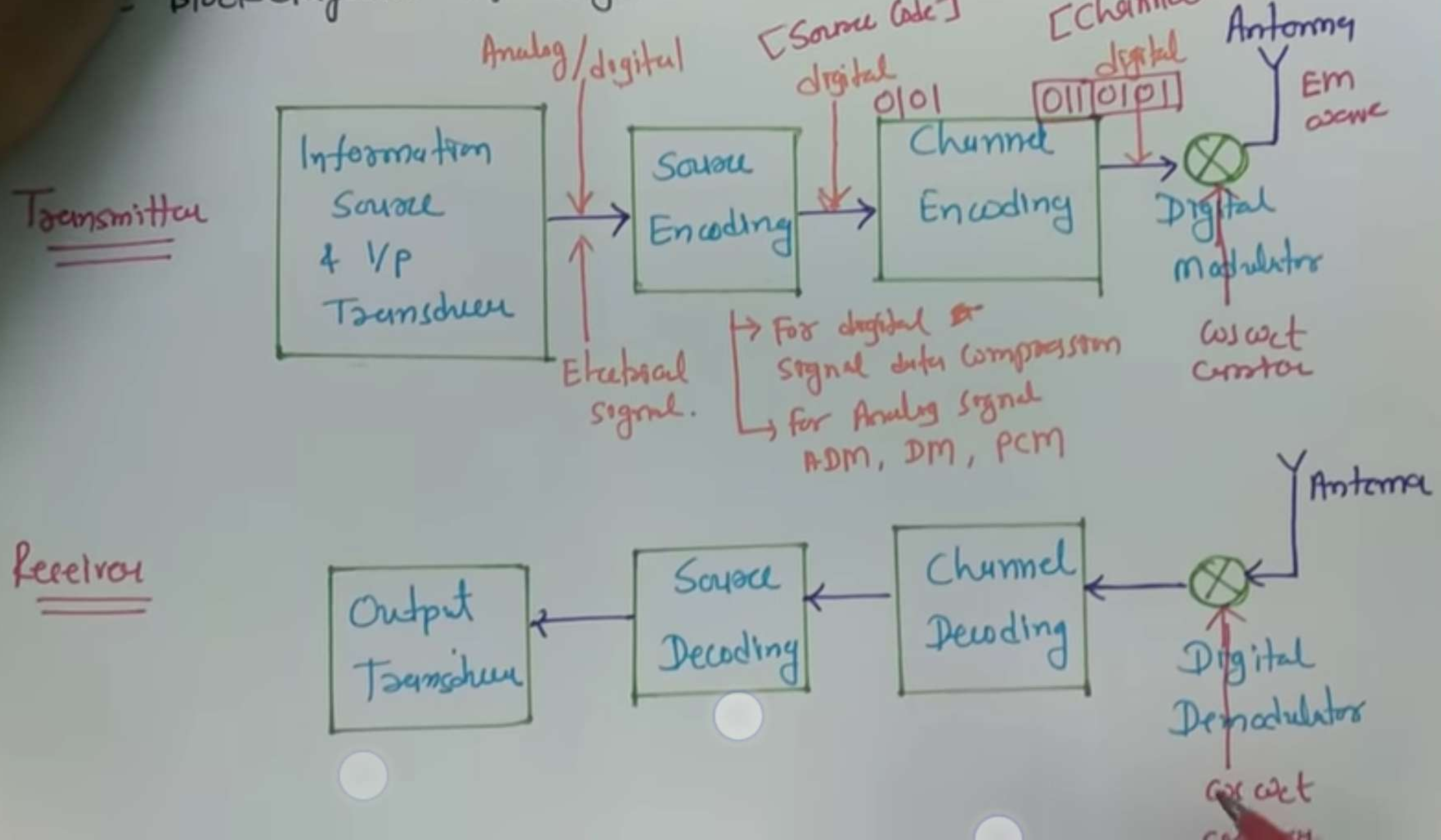
- It is used to provide noise immunity
- By adding redundancy we can do that

e.g.
- Block Code
- Cyclic Code
- Con... ...onal Code

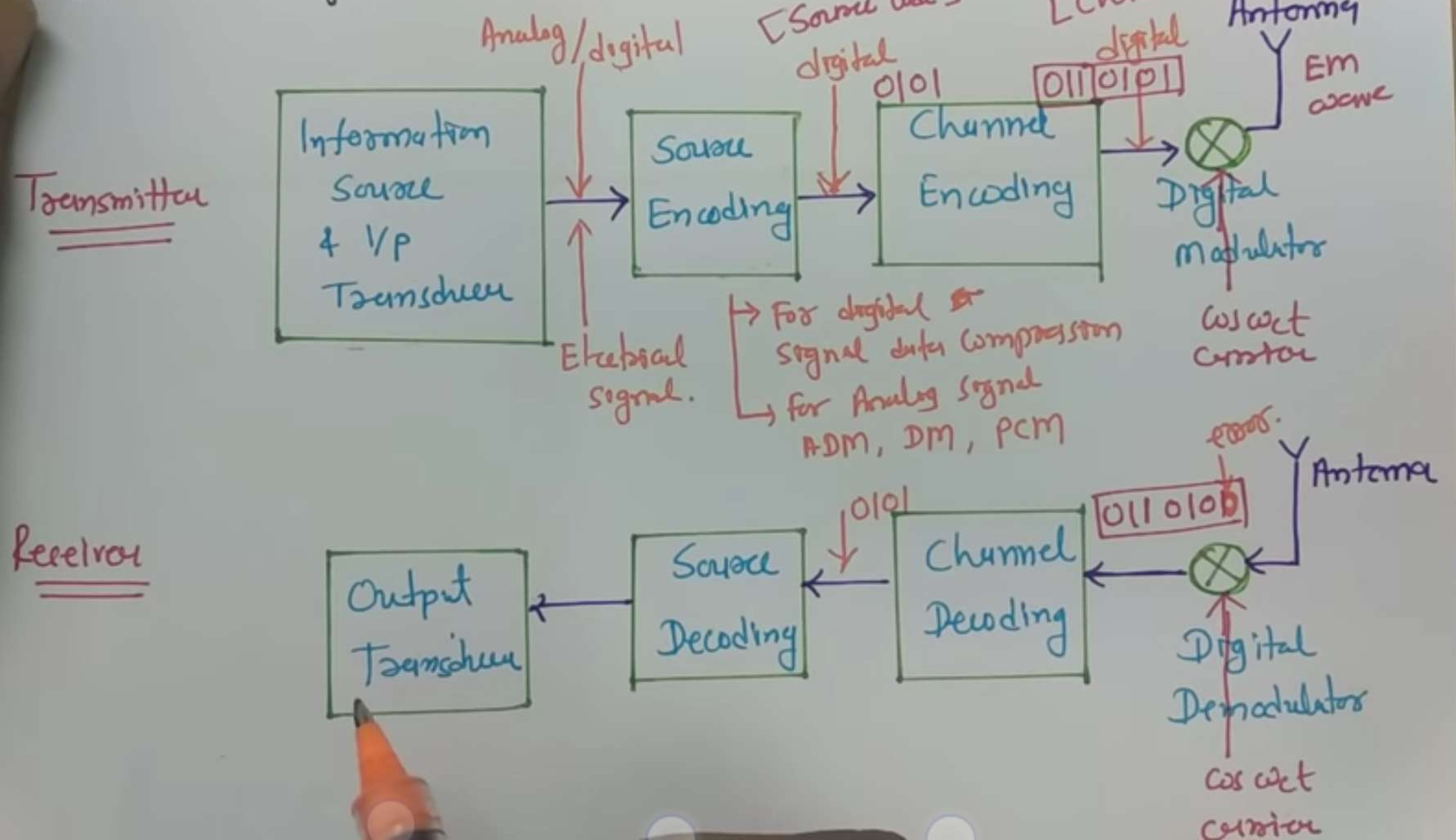



Basic Block
Antenna
digital
Channel Encoding
Transmitter
Digital Modulator
- Convert electrical signal into high freq. modulated signal.
- ASK, FSK, PSK, QPSK
data compression
DM, PCM
Receiver
Decoding
Channel Decoding

Block diagram of digital communication system



Basic Block diagram of digital



Advantages and Disadvantages of Digital Communication System

Advantages

- Storing Capability
- Inexpensive
- Use of repeaters
- Privacy and Security through the use of encryption
- Data compression, error detection & error correction is possible
- Flexible Hardware Implementation
- Easier & efficient multiplexing by TDMA & CDMA.

- Storing capability
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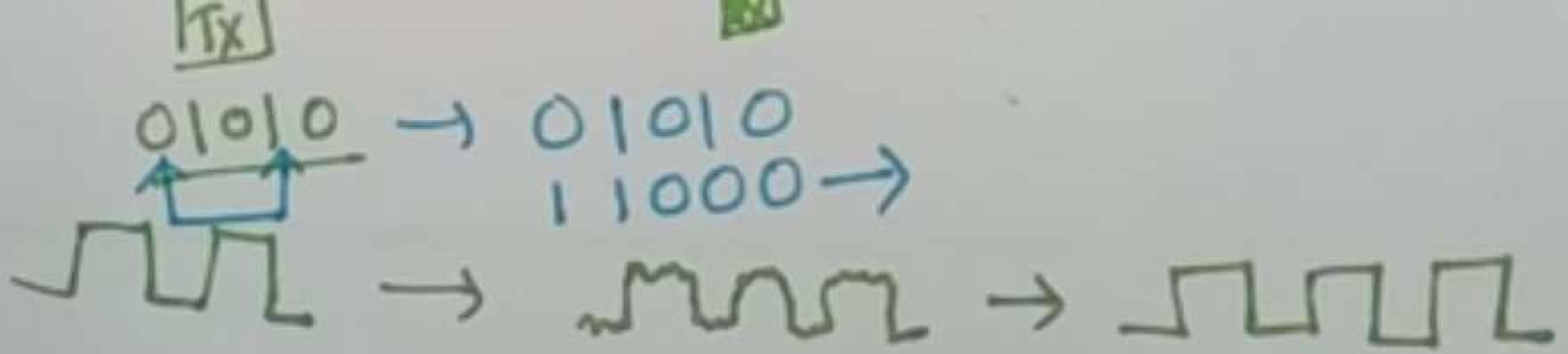


- Privacy and Security through the use of encryption
- Data compression, error detection & error correction is possible
- Flexible Hardware Implementation
- Easier & efficient multiplexing by TDMA & CDMA.

Disadvantages

- Bandwidth is high for channel
- Synchronization complexity.
- High power consumption due to multiple stages.
- Complex Circuit.

- basics of Scrambling
- Significance of Scrambling
- Working of Scrambling

Basics of Scrambling

- It is rearrangement of data sequence.

  01101100 ⟶ 01010110

Significance of Scrambling

- It prevents unauthorized access of data

  11001100 → 1010101 0

- Timing extraction of data

  11111000 → Problem at R_x [timing extraction]

  00000 11 → $\underline{010101010}$

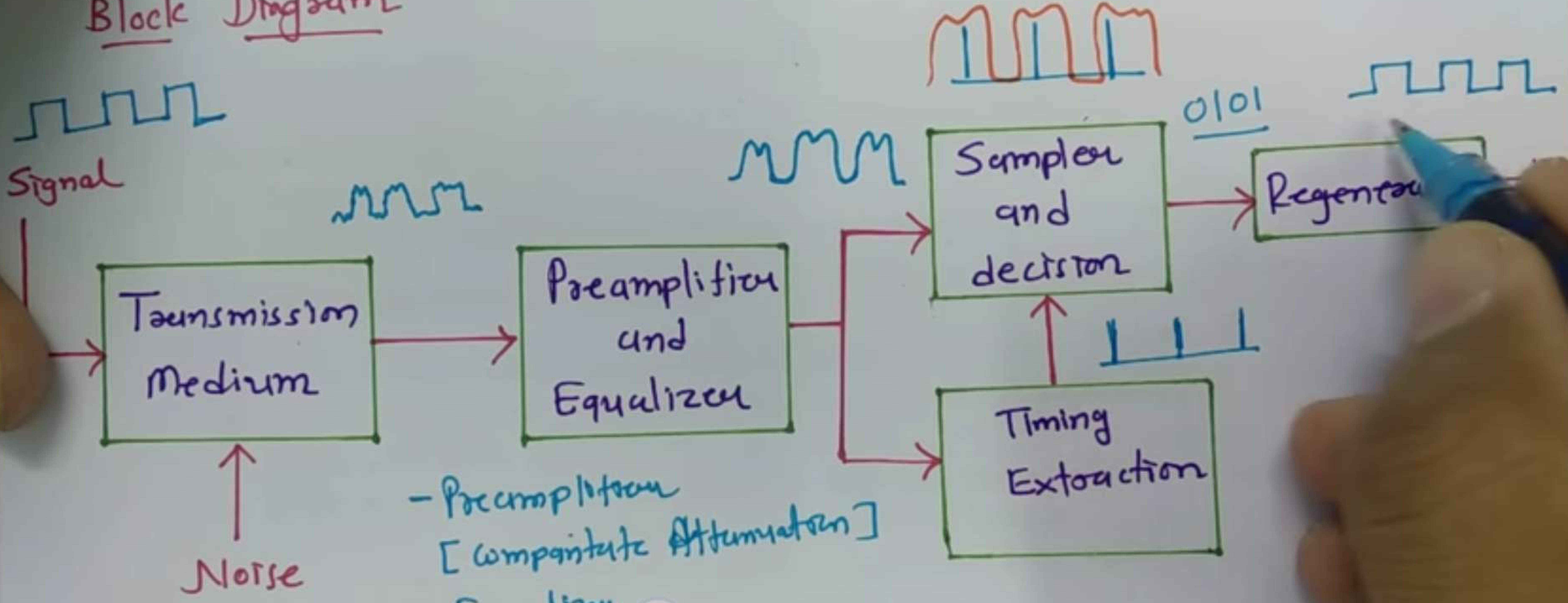
Regenerative Repeater or Digital Receiver

Objectives

- Reshaping Incoming Pulses
- Extracting the timing Information
- Making symbol detection decision

- Making symbol

Block Diagram



- Preamplification
  [compensate Attenuation]
- Equalizer
  [compensate distortion]

Equalizer & Preamplification

Equalizer - Distortion in pulse happen due to ISI, main reason is pulse dispersion.

- Char. of equalizer should be inverse of transmission medium.

Noise Amplification - Equalizer eliminates pulse dispersion but preamplifier enhances the received channel noise by boosting noise signal. This undesirable phenomena is known as noise amplification.

- In digital Rx, It is not compulsory to receive exact shape of signal. By decision making we identify data is 0 or 1.
- At this block, we compromise bet.^{n} distortion reduction and noise amplification.

Timing Extraction

1. Derivation from a Primary or Secondary standard
   - Device working under master timing Information.
   - high cost
   - Used in high Vol.^{m} data or high speed.

2. Transmitting a Seperate Synchronizing Signal
   - Send Saparate pilot carrier with signal
   - Uses more power.
   - Channel capacity should be higher then signal BW.

3. Self Synchronization
   - No pilot carrier
   - low cost

Attenuation of Signal & Significance of decible dB.

- Attenuation

- It is loss of energy
- It happens due to Conduction loss, dielectric loss & Propagation loss
- It is usally measured in terms of dB.
- To Compensate Attenuation we use Amplifier.

Attenuation of Signal & Significance of decible dB.

- **Attenuation**

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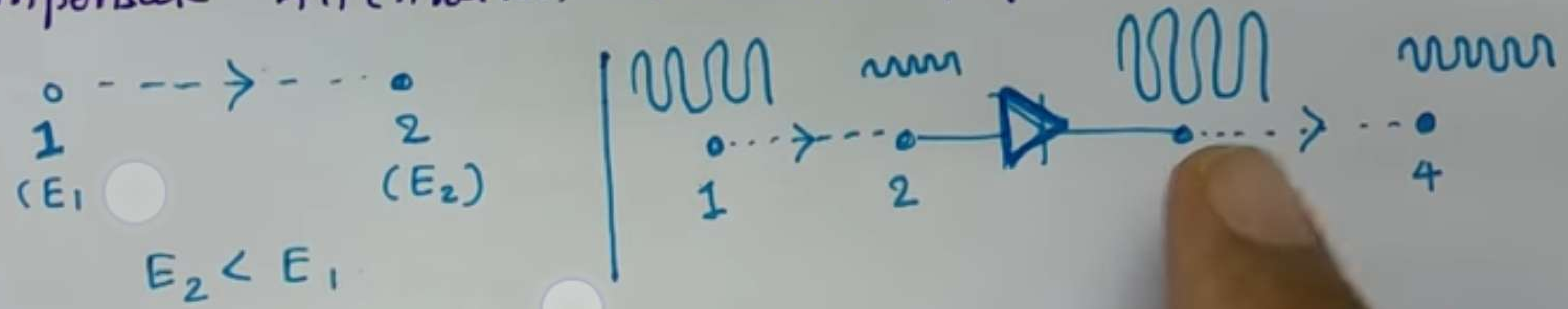

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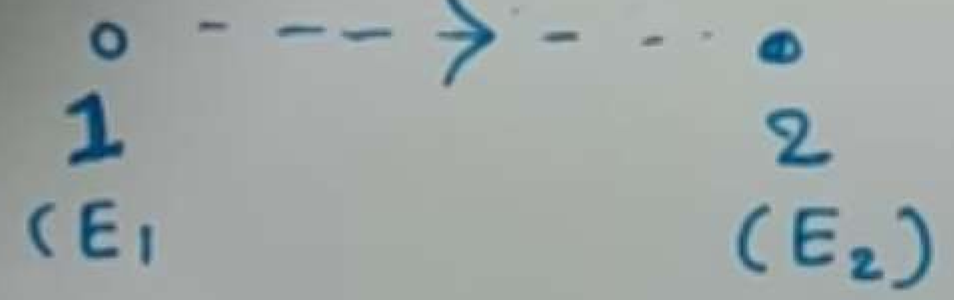
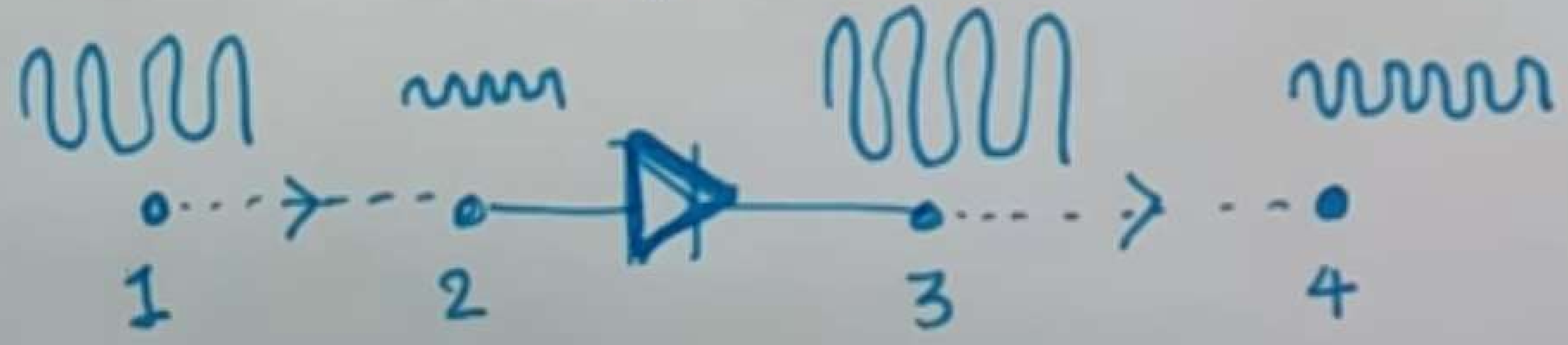


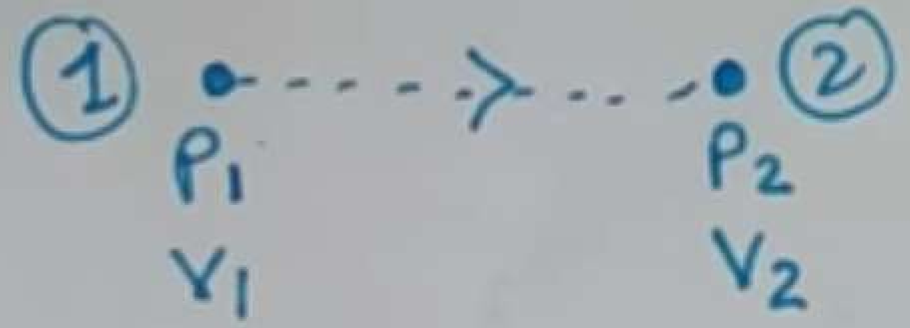
$E_2 < E_1$



- Attenuation decerys amplitude of signal.
-

- dB (decible) & It's Signifiance.

- It is used to measure loss/gain.
- It is used to measure relative power or relative voltage loss/gain



- $\text{gain (dB)} = 10 \log \left(\frac{P_2}{P_1}\right) = 20 \log \left(\frac{V_2}{V_1}\right)$
- $P \propto V^2$

- dB (decible) & It's significance.

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(1) $\bullet \dashrightarrow \bullet$ (2)

P_1 $\quad$ P_2

V_1 $\quad$ V_2

- gain (dB) $= 10 \log \left(\frac{P_2}{P_1}\right) = 20 \log \left(\frac{V_2}{V_1}\right)$
- $P \propto V^2$

- dB is +Ve means, there is gain of signal
- dB is -Ve means, there is attenuation of signal.
- By dB, we can measure gain/atten[illegible] by addition/subtraction.
-

- dB (decible) & it's significance.

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$$① \; \overset{P_1,\ V_1}{\bullet} \dashrightarrow \overset{P_2,\ V_2}{\bullet} \; ②$$

- $\text{gain (dB)} = 10 \log\left(\frac{P_2}{P_1}\right) = 20 \log\left(\frac{V_2}{V_1}\right)$
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- dB is +ve means, there is gain of signal
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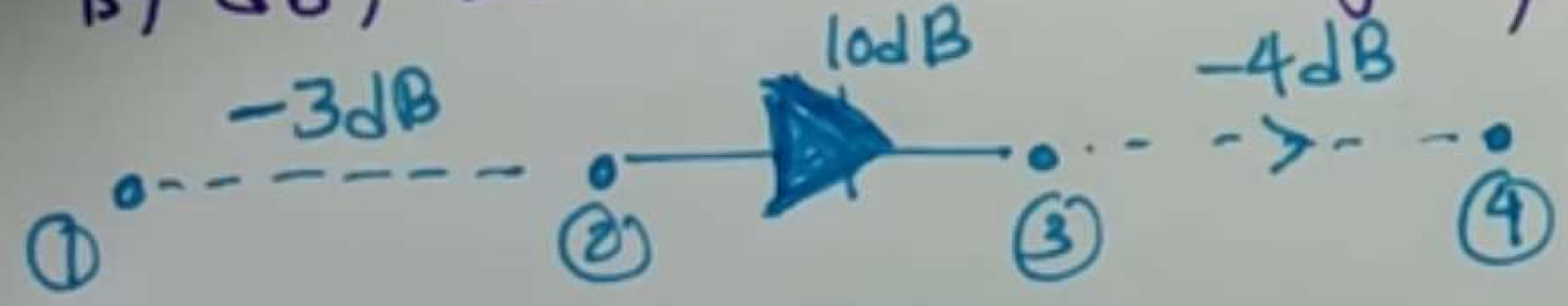
- By dB, we can measure gain/attenuation by addition/subtraction.

- $① \bullet \xrightarrow{-3\,dB} \bullet ② \xrightarrow{10\,dB} \bullet ③ \xrightarrow{-4\,dB} \bullet ④$

$$\text{gain (dB)} = -3 + 10 - 4$$
$$= \underline{+3\ dB}.$$

- By dB, we can measure gain/attenuation by addition/subtraction.

- 

$$\text{gain (dB)} = -3 + 10 - 4 = \underline{+3\text{ dB}}.$$

Example - Suppose a signal travels through a transmission medium and its power is reduced to one fourth. In this case attenuation can be calculated as.

$P_1 \longrightarrow P_2 = \frac{P_1}{4}$

$$\rightarrow (\text{dB}) = 10 \log\left(\frac{P_2}{P_1}\right)$$

$$= 10 \log\left(\frac{P_1}{4P_1}\right)$$

$$= 10 \log(1/4)$$

$$= \boxed{-6\text{ dB}}$$

Bit rate and Baud rate

- Bit rate (R) - It is number of bits/sec.
- Baud rate (r) - It is number of symbols/sec [elements/sec]
- If n = number of bits/symbol [bits/element].
- $\boxed{r = \frac{R}{n}}$
- Total number of symbols [elements] = $L = 2^n$

n

- Total number of Symbols [elements] = $L = 2^n$

Example - An Analog signal carries 4 bits / signal elements. If 1000 signal elements are sent per second. Find the bit rate.

- $n = 4$ bits / elements
- $r = 1000$ baud $\left[\frac{\text{elements}}{\text{sec}}\right]\left[\frac{\text{symbols}}{\text{sec}}\right]$
- $R = n r$

$= 4 \times 1000 = 4000$ bits/sec $= 4$ Kbps

$\rightarrow L = 2^n$

$= 2^4$

$= 16$

Example - An analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

- $R = 8000$ bps
  $r = 1000$ baud
  $n = ?$
  $L = ?$

- $n = \frac{R}{r}$

  $= \frac{8000}{1000}$

  $= 8$ bits/element

- $L = 2^n$

  $= 2^8$

  $= 256$

Amplitude shift keying (ASK)

Outlines

- basics of ASK
- Defination of ASK
- Bandwidth of ASK
- Modulation of ASK
- Demodulation of ASK
- Advantages of ASK
- Disadvantages of ASK
- Applications of ASK

- Advantages of ASK
- Disadvantages of ASK
- Applications of ASK

- Basics of ASK
- It is digital to Analog conversion technique

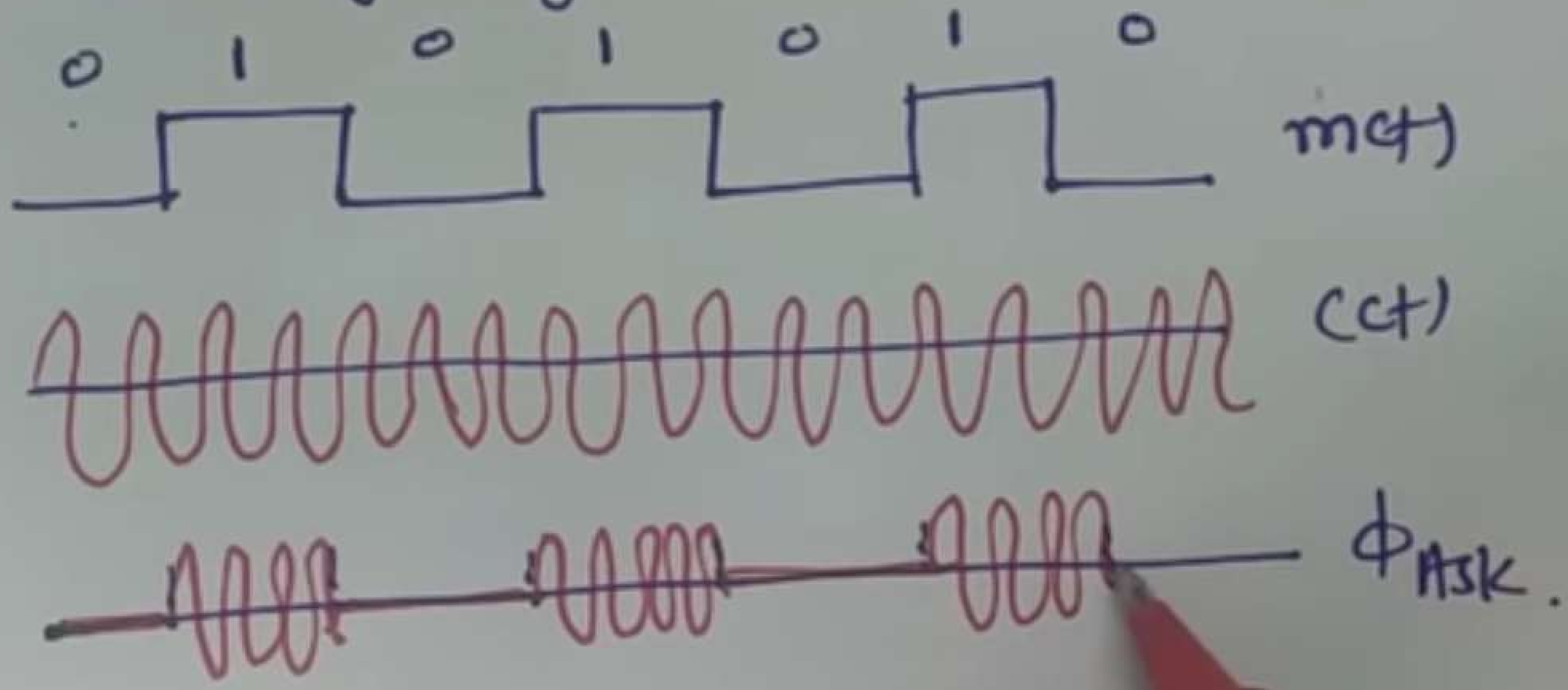
<u>Definition</u> : The amplitude of carrier signal varies w.r.t amplitude of message signal.

010101

$m(t) \longrightarrow \otimes \longrightarrow \phi_{ASK}$

$\uparrow$ carrier

$c(t) = \cos \omega_c t$



- Advantages of ASK
- Disadvantages of ASK
- Applications of ASK

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010101

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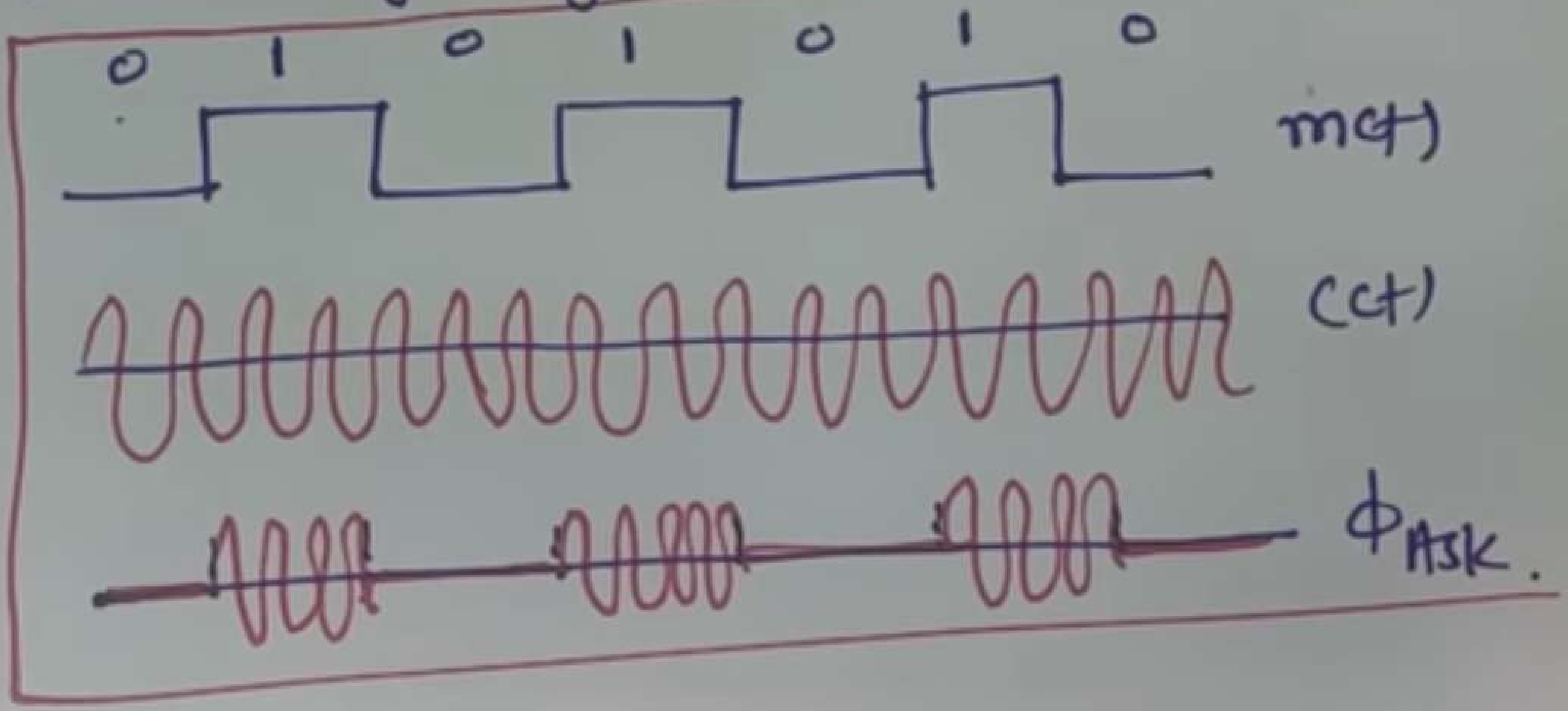


→ for binary $n = 1$

BASK or OOK

BASK or OOK

- Bandwidth of ASK

$$m(t) \longrightarrow \otimes \longrightarrow \phi_{ASK} = m(t)\cos\omega_c t$$

$$\uparrow$$

$$c(t) = \cos\omega_c t$$

$\Rightarrow$ $BW \propto r$

$\Rightarrow$ $BW = (1+d)\,r$

$\Rightarrow$ $\boxed{BW = (1+d)\dfrac{R}{n}}$

where r = baud rate

R = data rate

n = no of bits req^d for sample.

d = Factor for modulation & filtering process.

$d \in (0, 1)$

$\boxed{BW = r}$ ← For ideal (d = 0)

worse modulation (d = 1) $\boxed{BW = 2r}$

Modulation of ASK

$$
\begin{tikzcd}
m(t) \arrow[r, "01010"] & \otimes \arrow[r] & \phi_{ASK} = m(t)\cos\omega_c t \\
& c(t) = \cos\omega_c t \arrow[u] &
\end{tikzcd}
$$

where, $m(t) = \begin{cases} 1 & \text{+ve voltage} \\ 0 & \text{No voltage} \end{cases}$

Demodulation of ASK

Demodulation of ASK

Synchronous (Coherent)	Non Synchronous (Non Coherent).

$$m(t)\cos\omega_c t \longrightarrow \otimes \xrightarrow{m(t)\cos^2\omega_c t} \boxed{\text{LPF}} \longrightarrow m(t).$$

$$\uparrow \cos\omega_c t$$

<u>Adv</u> - ...t is efficient

<u>Disadv</u> - It is Costly

$$m(t)\cos\omega_c t \longrightarrow \boxed{\text{Envelope}} \longrightarrow m(t).$$

<u>Adv</u> - Cost is low.

<u>Disadv</u> - Performance is poor with less SNR received signal.

FSK - Freq. Shift Keying

Outlines

- Basics of FSK
- definition of FSK
- Spectrum and BW of FSK
- modulation of FSK
- Demodulation of FSK
- Applications of FSK

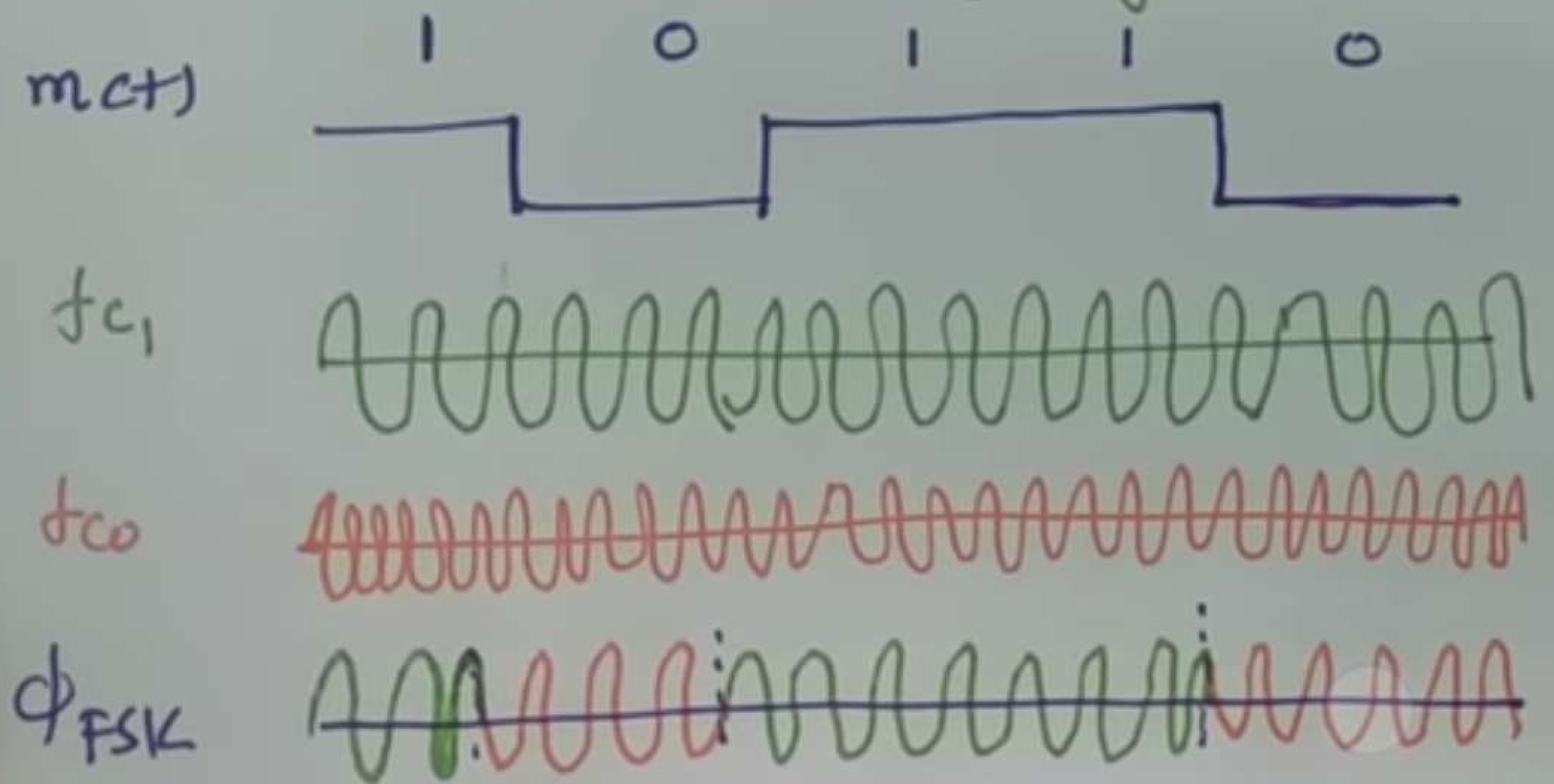
- Demodulation of FSK

- Applications of FSK

- Basics of FSK

- It is used to convert digital data into analog data.

<u>Definition</u> - Freq. of carrier signal varying w.r.t amplitude of message signal.



- Spectrum and BW of FSK

- Demodulation of FSK
- Applications of FSK

- Basics of FSK
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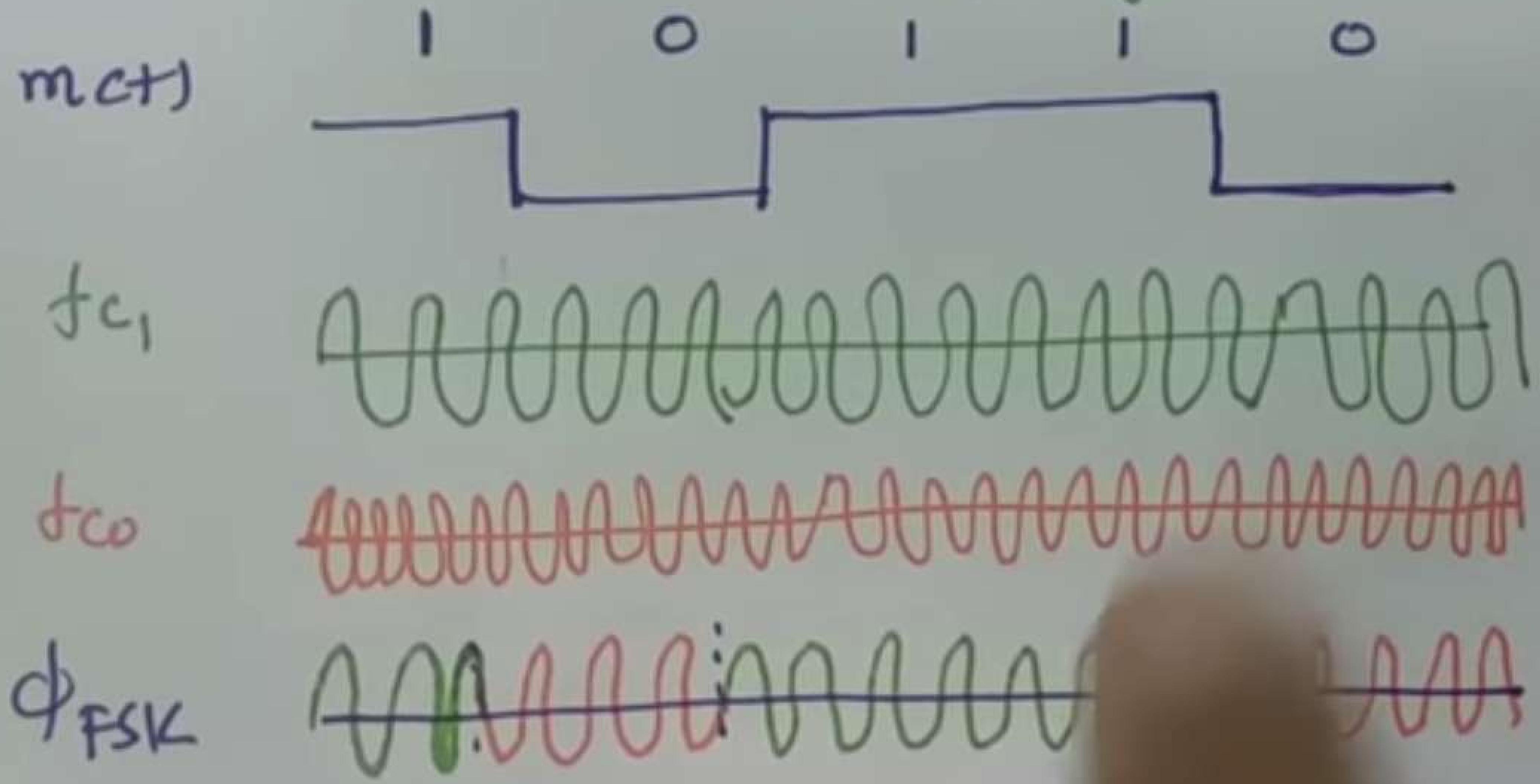
Definition – Freq. of carrier signal varying w.r.t amplitude of message signal.

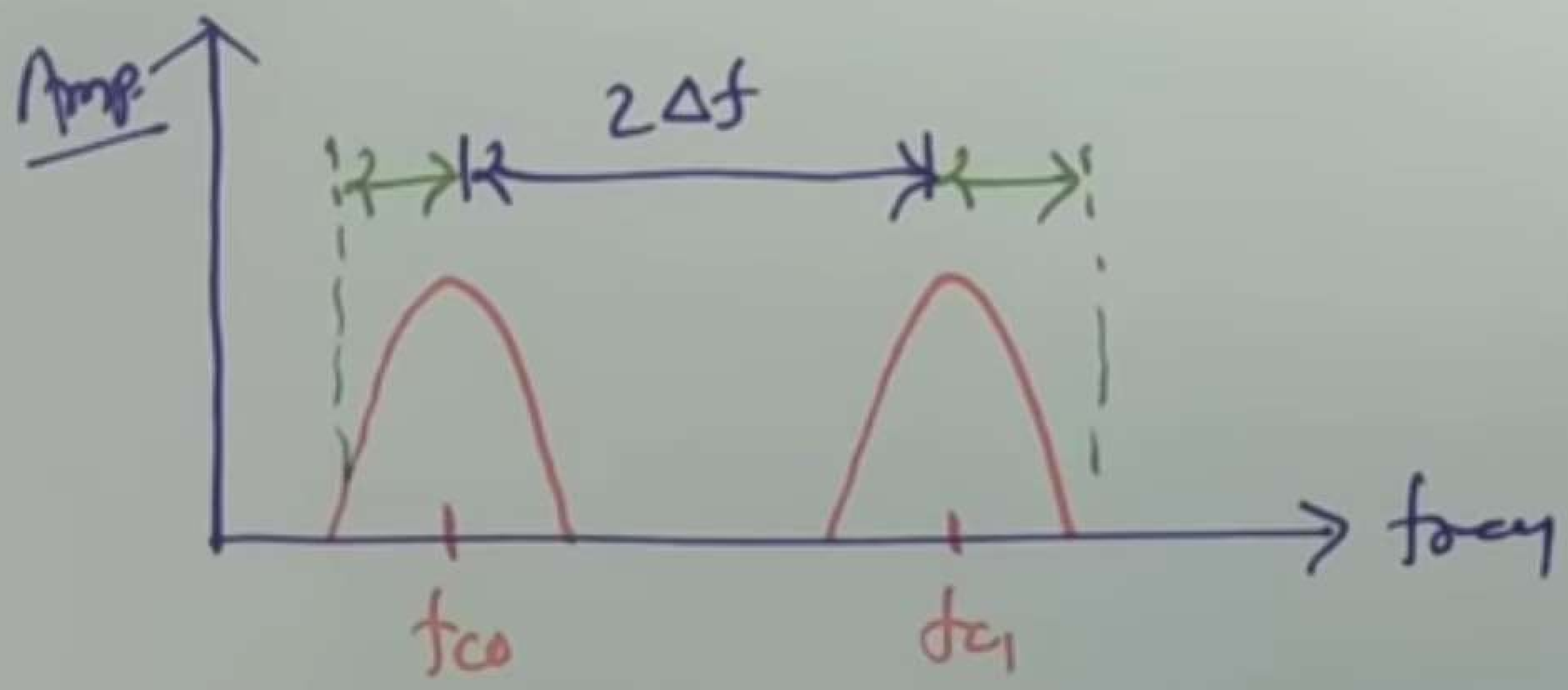


$$\phi_{FSK} = m_0(t)\cos\omega_{c_0}t + m_1(t)\cos\omega_{c_1}t$$

$$m(t) = \begin{cases} 1 \longrightarrow f_{c_1} \\ 0 \longrightarrow f_{c_0} \end{cases}$$

- Spectrum and BW o

- Spectrum and BW of FSK



$BW = (1+d)\gamma$

— total BW of FSK

$$= \frac{(1+d)\gamma}{2} + \frac{(1+d)\gamma}{2} + 2\Delta f$$

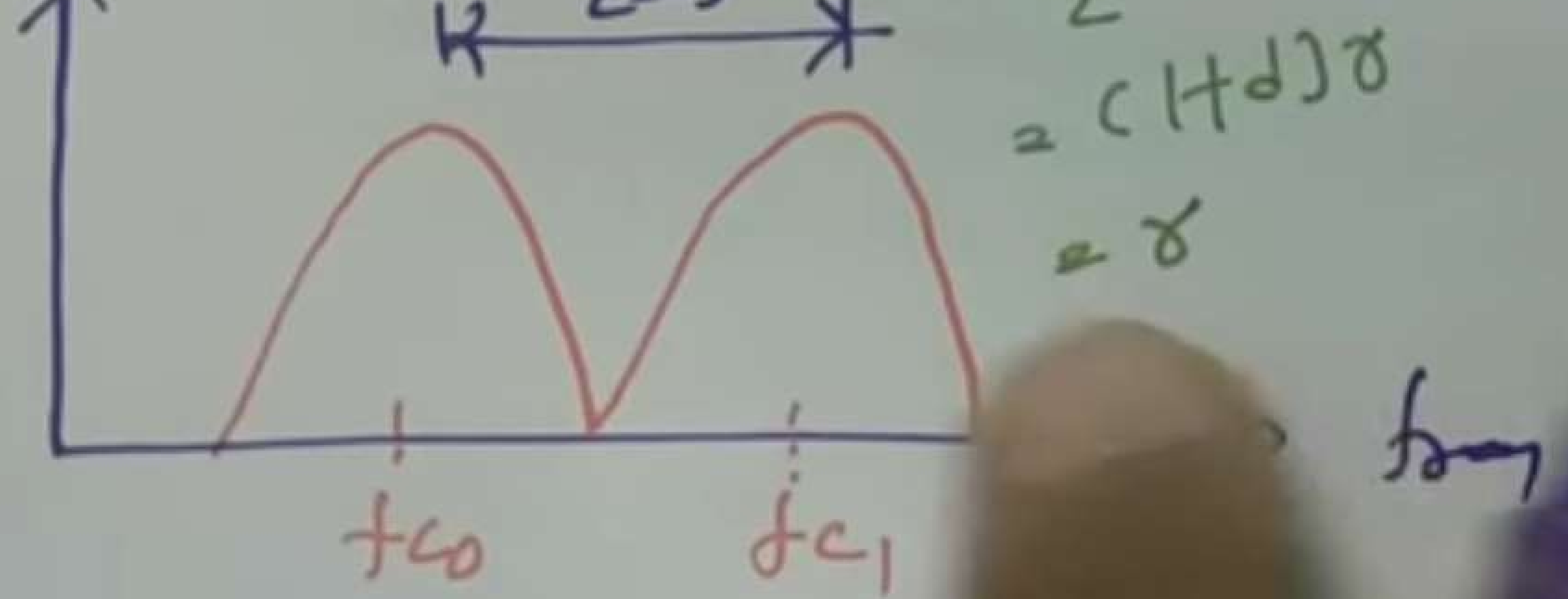
$$= (1+d)\gamma + 2\Delta f$$

→ for min BW. $(d=0)$

$$2\Delta f = \frac{(1+d)\gamma}{2} + \frac{(1+d)\gamma}{2}$$

$$= (1+d)\gamma$$

$$= \gamma$$



$$\rightarrow (BW_{min})_{FSK} = \gamma + \gamma$$

$$= 2\gamma$$

Multi Level FSK

$BW = (1+d)r + 2\Delta f$

→ For BFSK

$L = 2 = 2^n \rightarrow n = 1 \rightarrow f_{c0}, f_{c1}$

→ $L = 4 = 2^n \rightarrow n = 2 \rightarrow f_{c0}, f_{c1}, f_{c2}, f_{c3}$

→ $L = 8 = 2^n \rightarrow n = 3 \rightarrow f_{c0}, f_{c1}, \ldots, f_{c7}$

→ For L Level FSK BW.

$$BW = (1+d)r + (L-1)(2\Delta f)$$

→ min BW. ($d = 0$)

$$BW = (1+0)r + (L-1)r = Lr$$

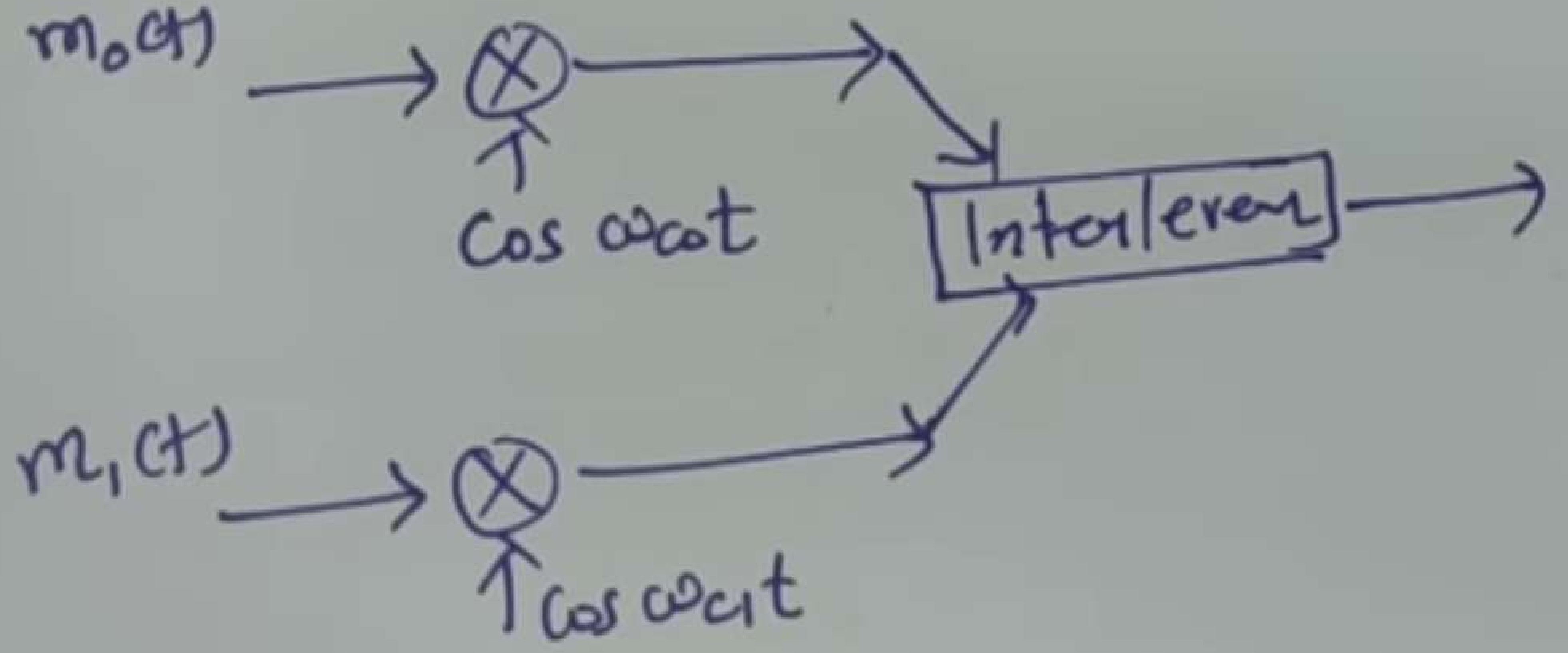
→ For L Level FSK BW.

$$BW = (1+d)r + (L-1)(2\Delta f)$$

→ min BW. (d = 0)

$$BW = (1+0)r + (L-1)r = Lr$$

FSK modulation

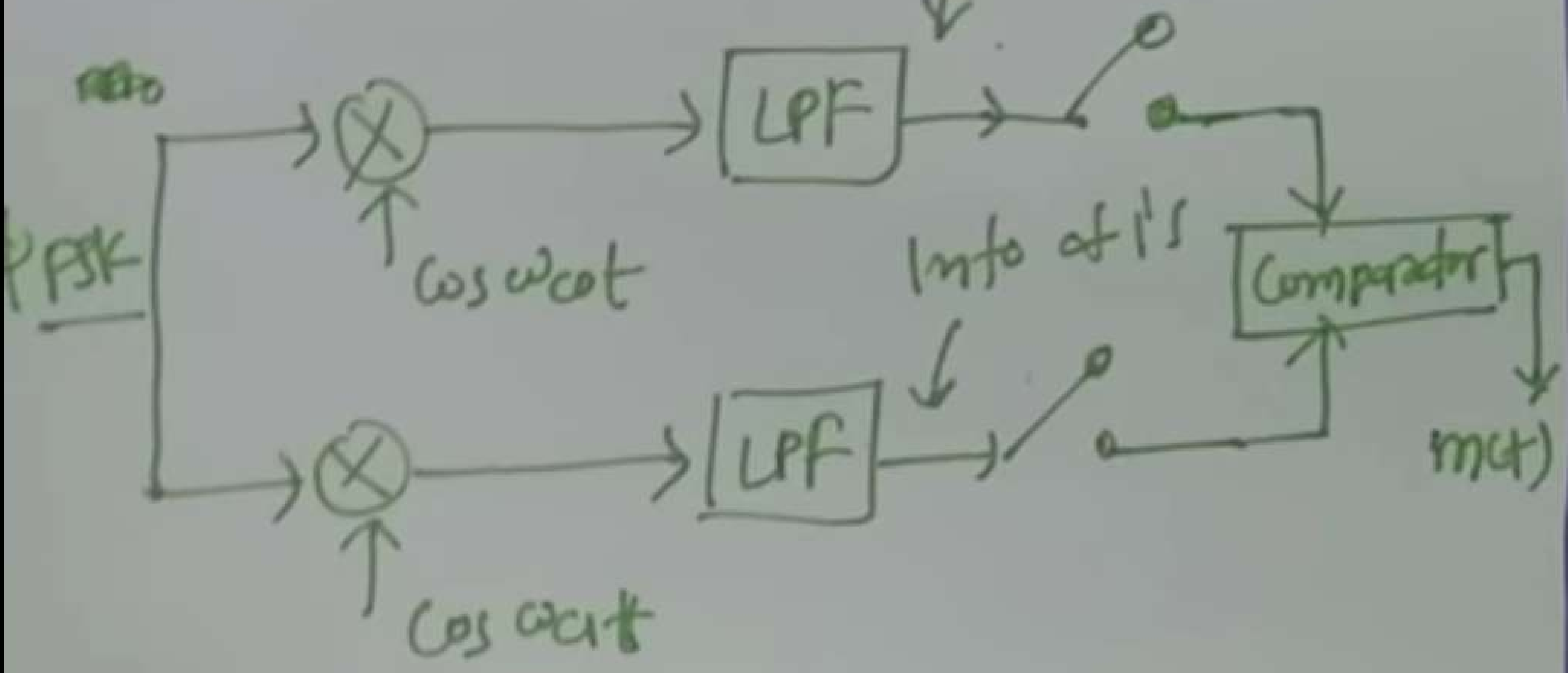


$$\phi_{FSK} = m_0(t)\cos\omega_{c0}t + m_1(t)\cos\omega_{c1}t$$

$1 \cos \omega_{c1} t$

FSK Demodulation

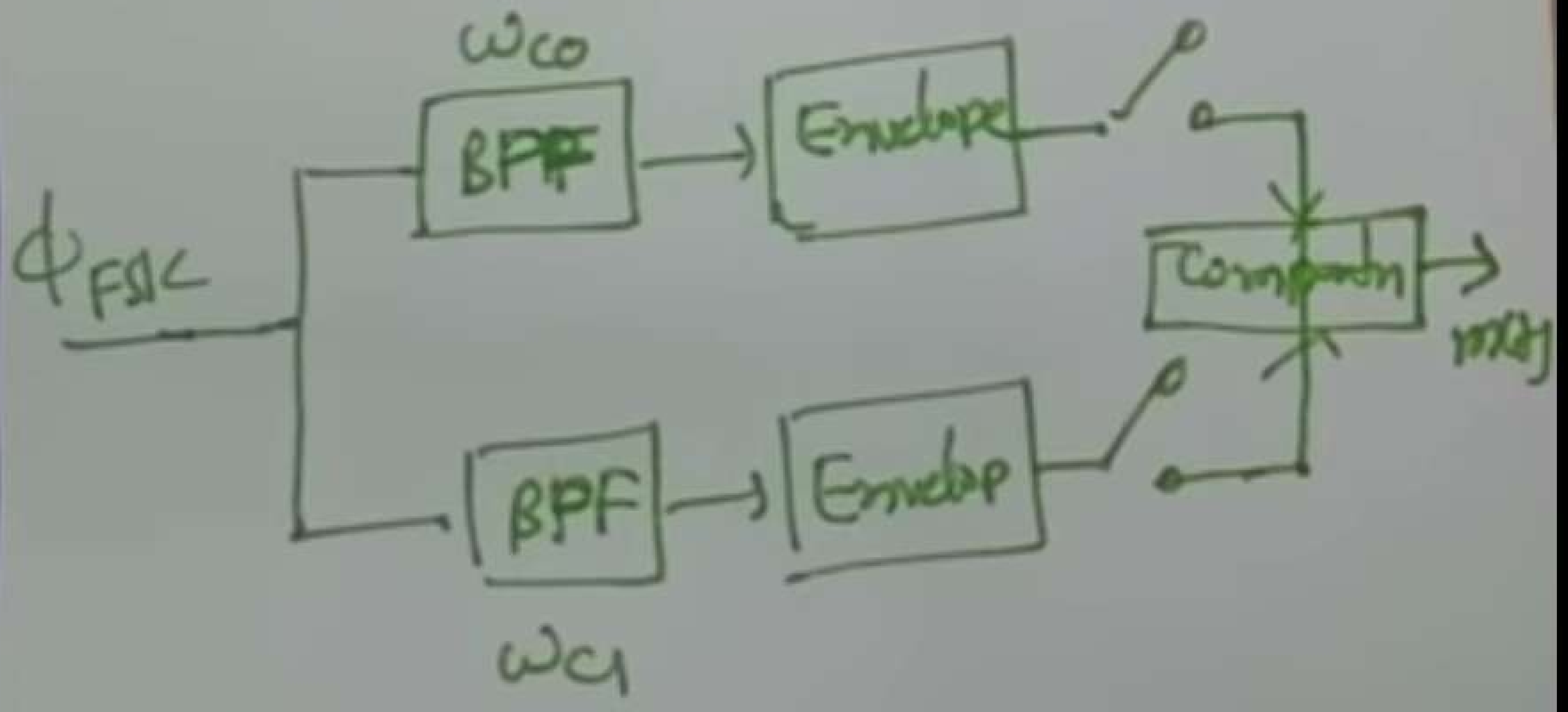
→ Coherent



Adv - simple

Disadv - costly

→ Non coherent



Adv - low cost

We need to send data 3 bits at a time at a bit rate of 3 mbps. The carrier freq is 10 MHz. Calculate the number of Levels, the baud rate and the BW.

→ $n = 3$ bits

$R = 3$ mbps

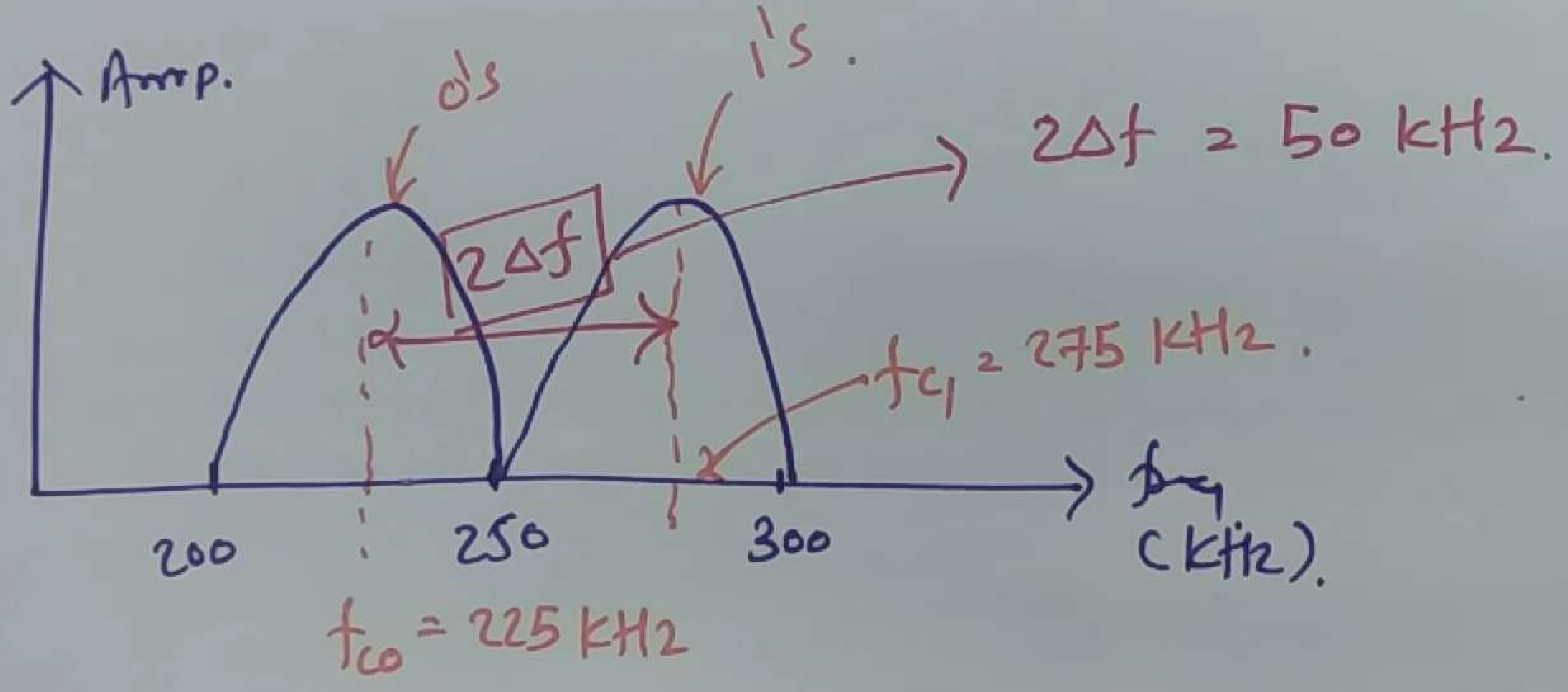
$f_c = 10$ MHz.

→ $L = 2^n = 2^3 = 8$

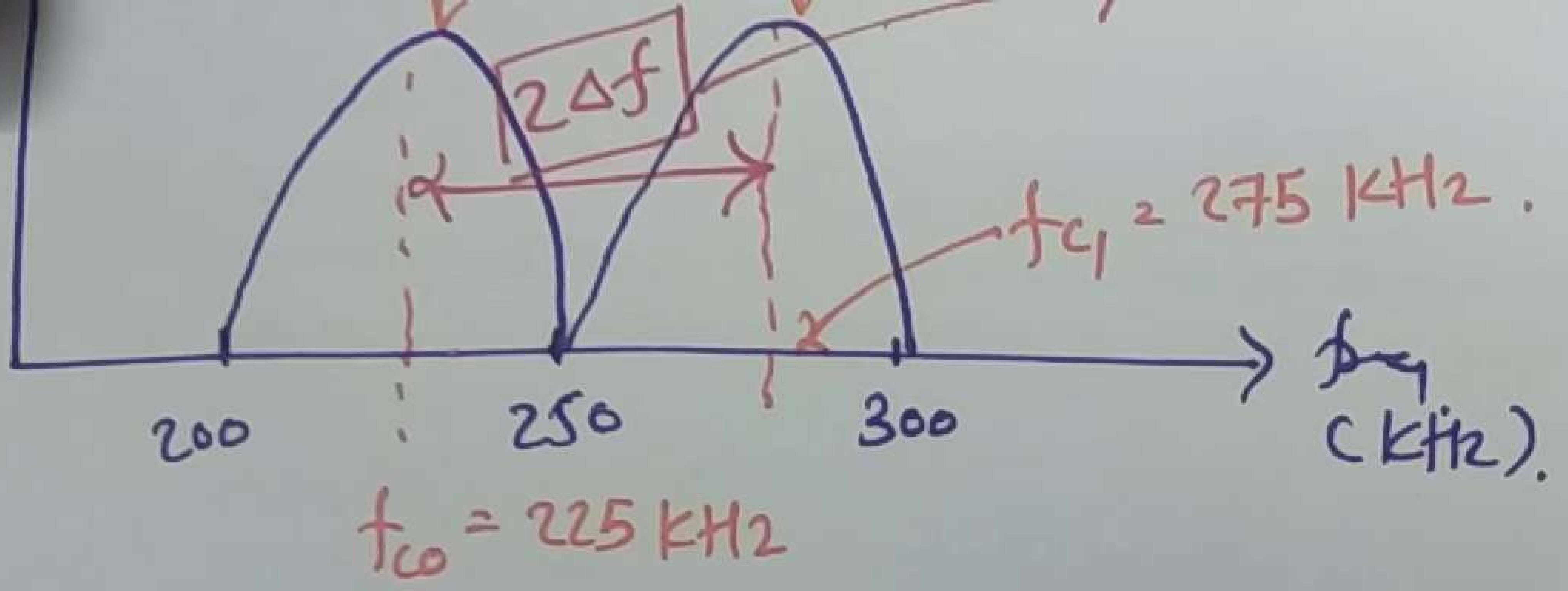
→ $r = \frac{R}{n} = \frac{3}{3} = 1$ mbaud

→ $BW = Lr = 8 \times 1 = 8$ MHz.

We have an available BW of 100 kHz which spans from 200 to 300 kHz. What should be carrier freq. and the bit rate if we modulated our data by using FSK with $d = 1$?



— BW = 1



→ For 2 Level FSK

$L = 2 = 2^n$

→ $\boxed{n = 1\ bit}$

$BW = 100\ kHz$

$2\Delta f = 50\ kHz$

$\Rightarrow BW = (1+d)r + 2\Delta f$

$\Rightarrow 100\ kHz = (1+1)r + 50\ kHz$

$\Rightarrow 50\ kHz = 2r$

$\Rightarrow r = 50/2 = 25\ kbaud$

$\Rightarrow r = \frac{R}{n}$

$\Rightarrow R = nr$

$= 1 \times 25$

$= 25\ kb$

PSK [Phase Shift Keying]

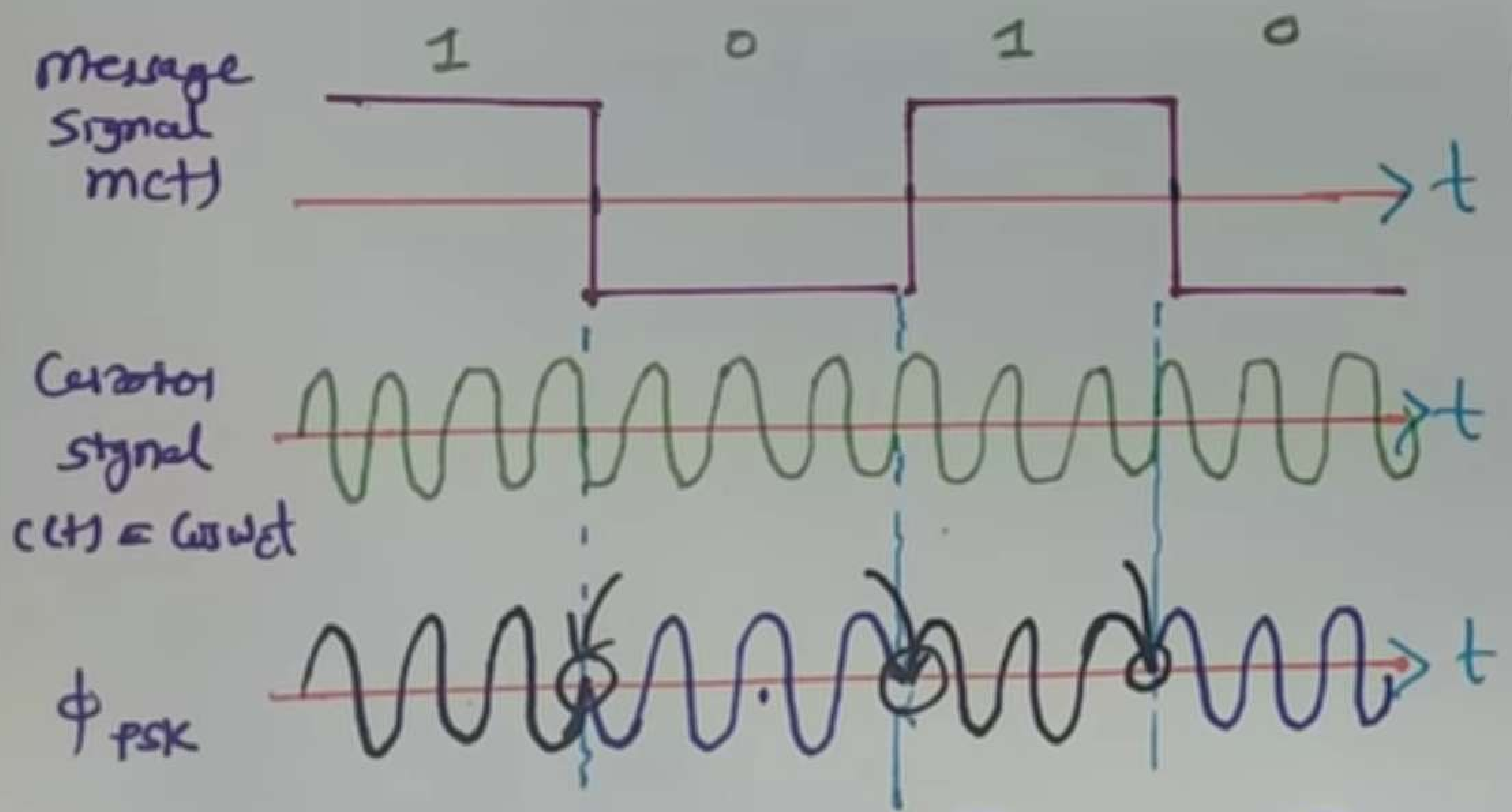
Outlines

- Defination of PSK
- Waveforms of PSK
- Bandwidth of PSK
- Multilevel PSK
- Modulation of PSK
- Demodulation of PSK
- Advantages of PSK
- Disadvantages of PSK
- Application [illegible] of PSK

- Applications of PSK

- Defination of PSK
  - Carrier phase is varying according to the amplitude of message signal.

- waveforms of PSK

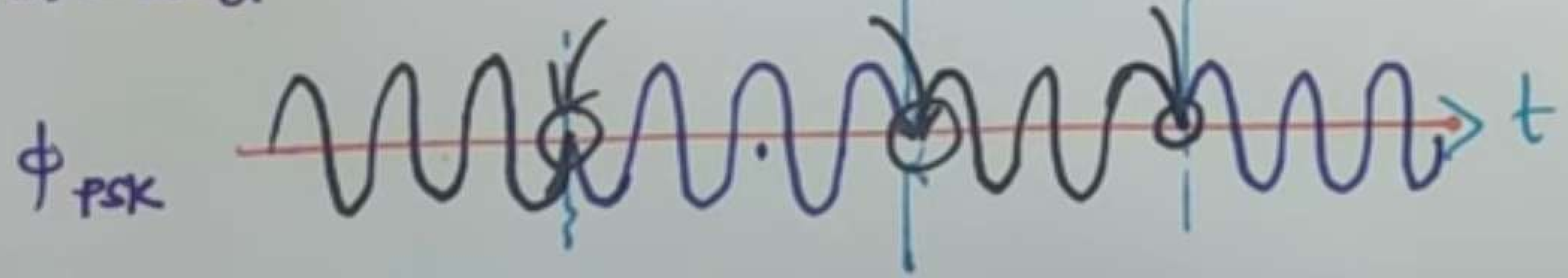


Polar wave NRZ

→ for 1 to 0 phase reversal.

→ 1 → $\phi = 0°$
0 → $\phi = 180°$

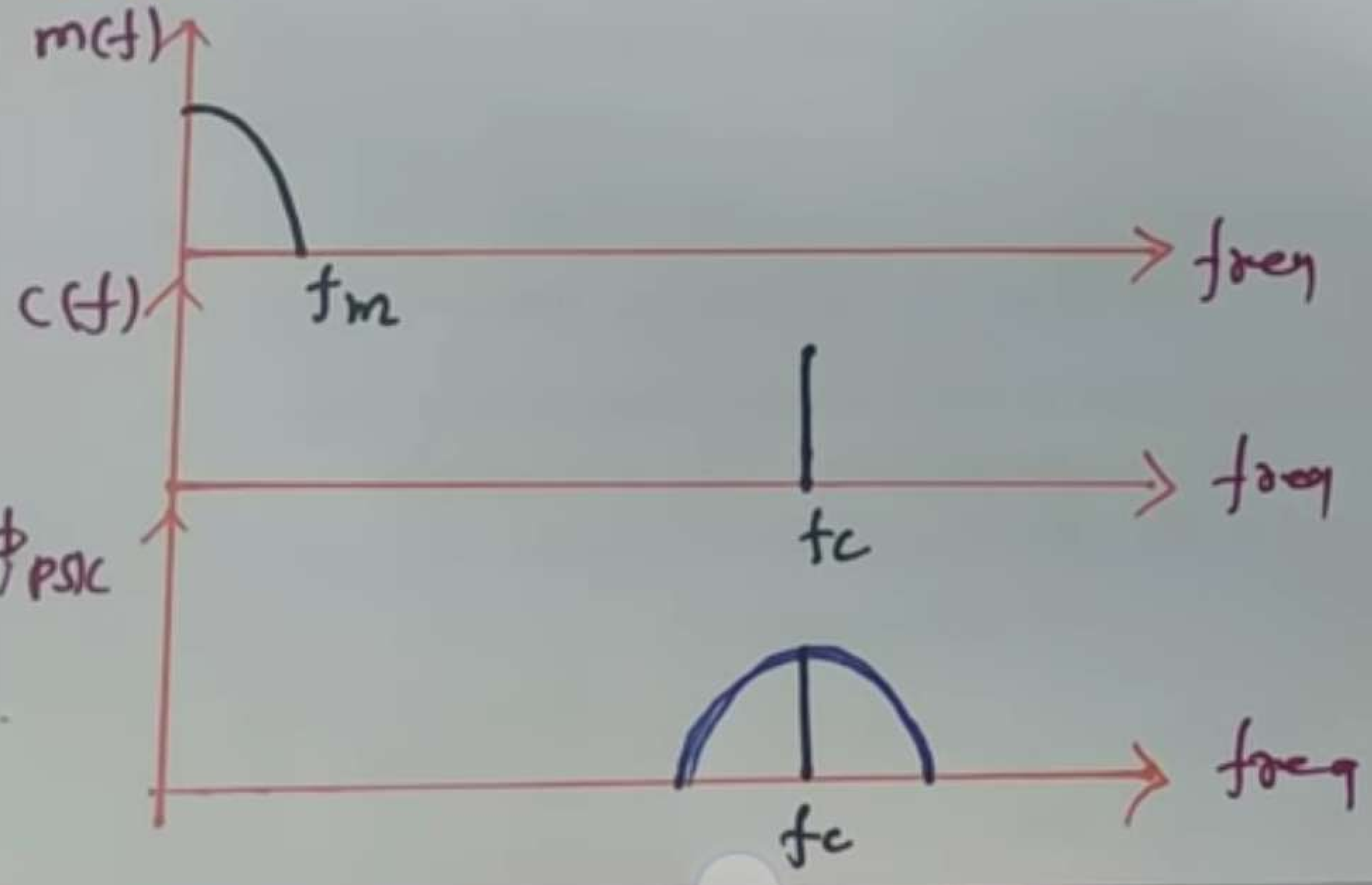
$c(t) = \cos \omega_c t$



$\rightarrow 1 \rightarrow \phi = 0^\circ$

$0 \rightarrow \phi = 180^\circ$

- Bandwidth of PSK



$\Rightarrow BW_{PSK} \propto r$

$\Rightarrow BW_{PSK} = (1+d)\, r$

where $d \in (\quad)$

loaded

Multi level PSK

→ BPSK → $L = 2 = 2^n$, → $n = 1$ → $\phi = 0, 180^\circ$

→ $L = 4 = 2^n$ → $n = 2$ → $\phi = 0, 90, 180, 270$

→ $L = 8 = 2^n$ → $n = 3$ → $\phi = 0, 45, 90, 135, 180, 225, 270, 315$

$$\boxed{BW = (1+d)r}$$

$\rightarrow L = 4 = 2^n \rightarrow n = 2 \rightarrow \phi = 0, 90, 180, 270$

$\rightarrow L = 8 = 2^n \rightarrow n = 3 \rightarrow \phi = 0, 45, 90, 135, 180, 225, 270, 315$

$$\boxed{BW = (1+d)r}$$

for $L = 4$, $n = 2$

$\phi = \underset{00}{0}, \underset{01}{90}, \underset{10}{180}, \underset{11}{270}$

odulation of PSK

1010

$m(t) \longrightarrow \otimes \longrightarrow \phi_{PSK} = \pm m(t) \cos \omega_c t$

$c(t) = \cos \omega_c t$

* For multilevel PSK

$\phi_{PSK} = m(t) \cos(\omega_c t + \phi)$

$m(t) \longrightarrow \otimes \longrightarrow \phi_{PSK}$

$c(t) = \cos\omega_c t$

* For multilevel PSK

$$\phi_{PSK} = m(t)\cos(\omega_c t + \phi)$$

Demodulation of PSK

$$\phi_{PSK} \longrightarrow \underset{\uparrow \atop \cos\omega_c t}{\otimes} \xrightarrow{y(t)} \boxed{\text{LPF}} \xrightarrow{\pm m(t)} \boxed{\text{Comparator}} \longrightarrow \overset{1010}{m(t)}$$

$$- \; y(t) = \phi_{PSK}\cos\omega_c t$$
$$= \pm m(t)\cos^2\omega_c t$$

$$\Rightarrow y(t) = \pm m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right]$$
$$= \underbrace{\boxed{\pm\frac{m(t)}{2}}}_{\text{data}} \pm \frac{m(t)}{2}\cos 2\omega_c t$$

Advantages of PSK

$\cos\omega_c t$

$- \; y(t) \cdot \phi_{PSK} \cos\omega_c t$

$= \pm m(t) \cos^2 \omega_c t$

$\Rightarrow y(t) = \pm m(t) \left[\frac{1 + \cos 2\omega_c t}{2}\right]$

$= \underbrace{\boxed{\pm \frac{m(t)}{2}}}_{\text{data}} \pm \frac{m(t)}{2} \cos 2\omega_c t$

Advantages of PSK

- Better then ASK, FSK
- BW is better then FSK
- Noise Immunity - Datarate better then FSK

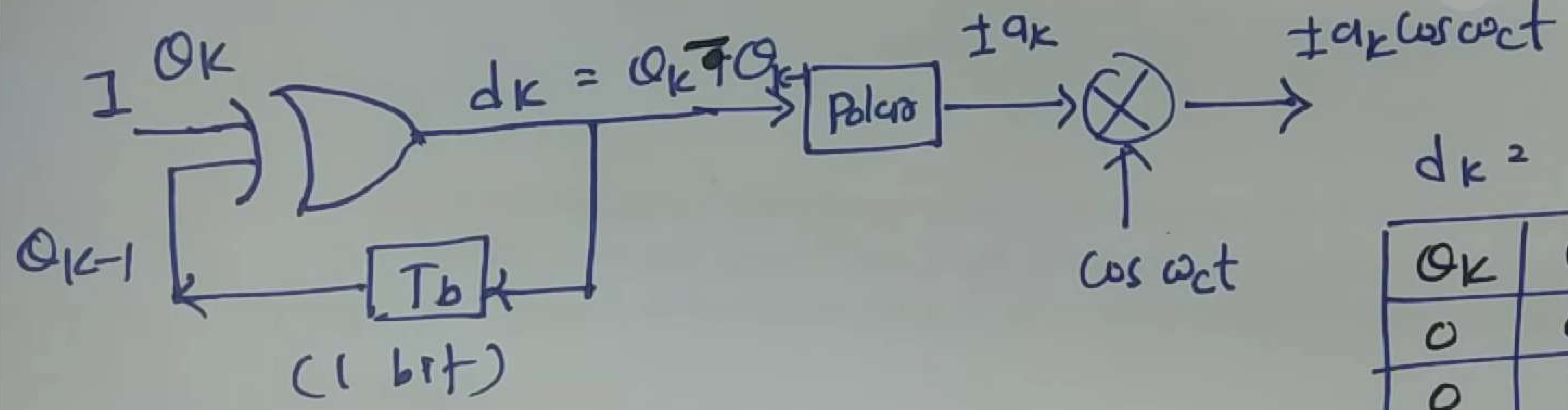
Drawbacks of PSK

- No Non coherent detection
- Costly

Applications of PSK

- In digital Communication
- It was also used in earliest telephone modems with data rate [2400 and 4800 bits/sec]

- Basics of DPSK
  - It is not possible to have non coherent detection of PSK
  - to detect non coherent detection of phase we use DPSK
  - It reduces cost of circuit.
- DPSK transmitter



$d_K = \theta_K - \theta_{K-1}$

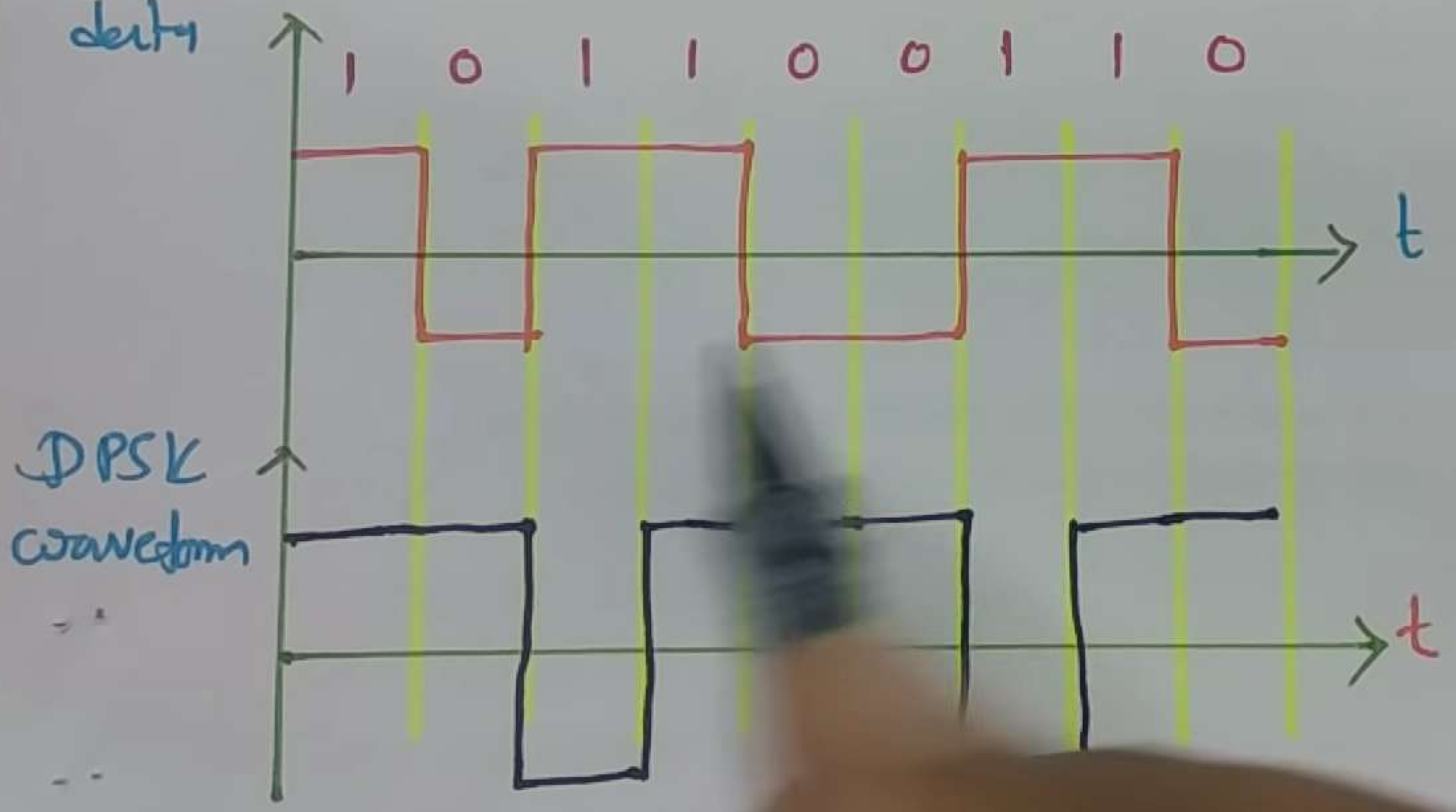
θ_K	θ_{K-1}	d_K
0	0	0
0	1	1
1	0	1
1	1	0

- DPSK ~~Receiver~~ waveforms

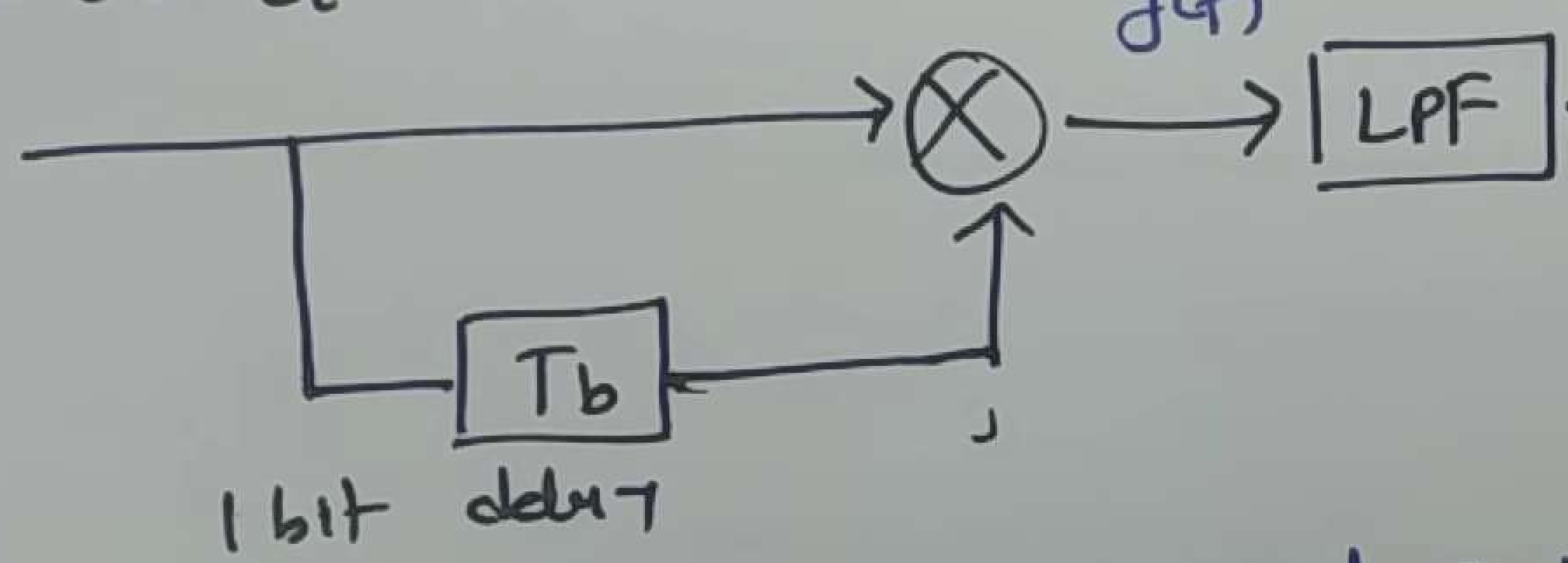
1	1	0

- DPSK ~~[illegible]~~ waveforms

  - If next data is 1, then change polarity of o/p.
  - If next data is 0, then do not change polarity of o/p.



$\pm a_k \cos\omega_c t$



$z(t) = \frac{a_k^2}{2} \rightarrow$ bit 1

$= -\frac{a_k^2}{2} \rightarrow$ bit 0

- Case-I [same polarity of Input & delayed Signal]
- $y(t) = (a_k \cos\omega_c t)(a_k \cos\omega_c t) = a_k^2 \cos^2\omega_c t$

$= \frac{a_k^2}{2}[1 + \cos 2\omega_c t] = \frac{a_k^2}{2} + \frac{a_k^2}{2}\cos 2\omega_c t \xrightarrow{LPF} = \frac{a_k^2}{2}$

- Case-II [opposite polarity of Input & delayed Signal]
- $y(t) = (a_k \cos\omega_c t)(-a_k \cos\omega_c t) = -\frac{a_k^2}{2} - \frac{a_k^2}{2}\cos 2\omega_c t$

$\xrightarrow{LPF} = -\frac{a_k^2}{2}$

Advantages of DPSK

QPSK [Quadrature Phase Shift Keying]

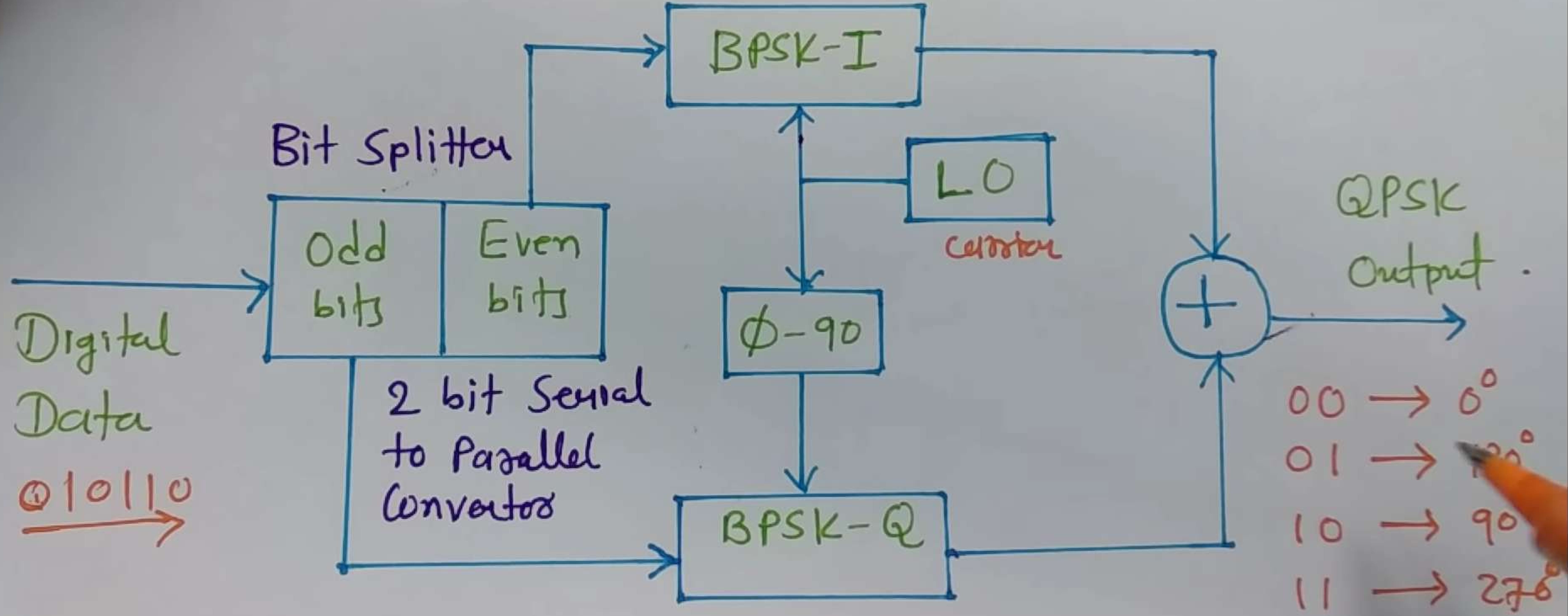
Outlines

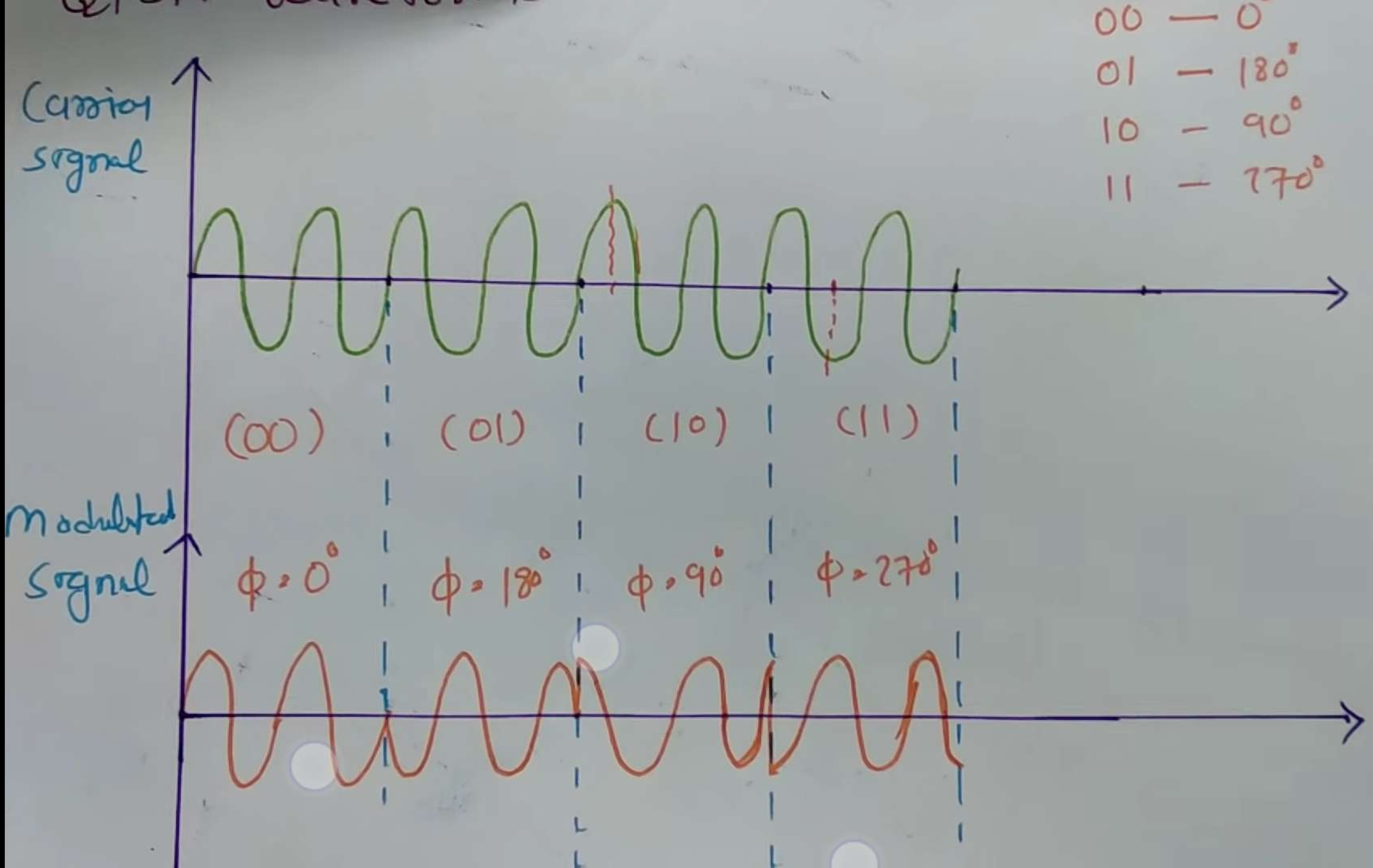
- Basics of QPSK
- QPSK modulator
- waveforms of QPSK
- QPSK demodulator.

- Quadrature phase shift keying (QPSK) is a form of PSK (Phase shift keying), in which two bits are modulated at once.
- It selects one of four possible carrier phase shifts [0°, 90°, 180°, 270°]
- QPSK allows the signal to carry twice as much information as ordinary PSK using the same BW.
- QPSK is used for satellite transmission of MPEG2, cable modem, cellular phone system etc.

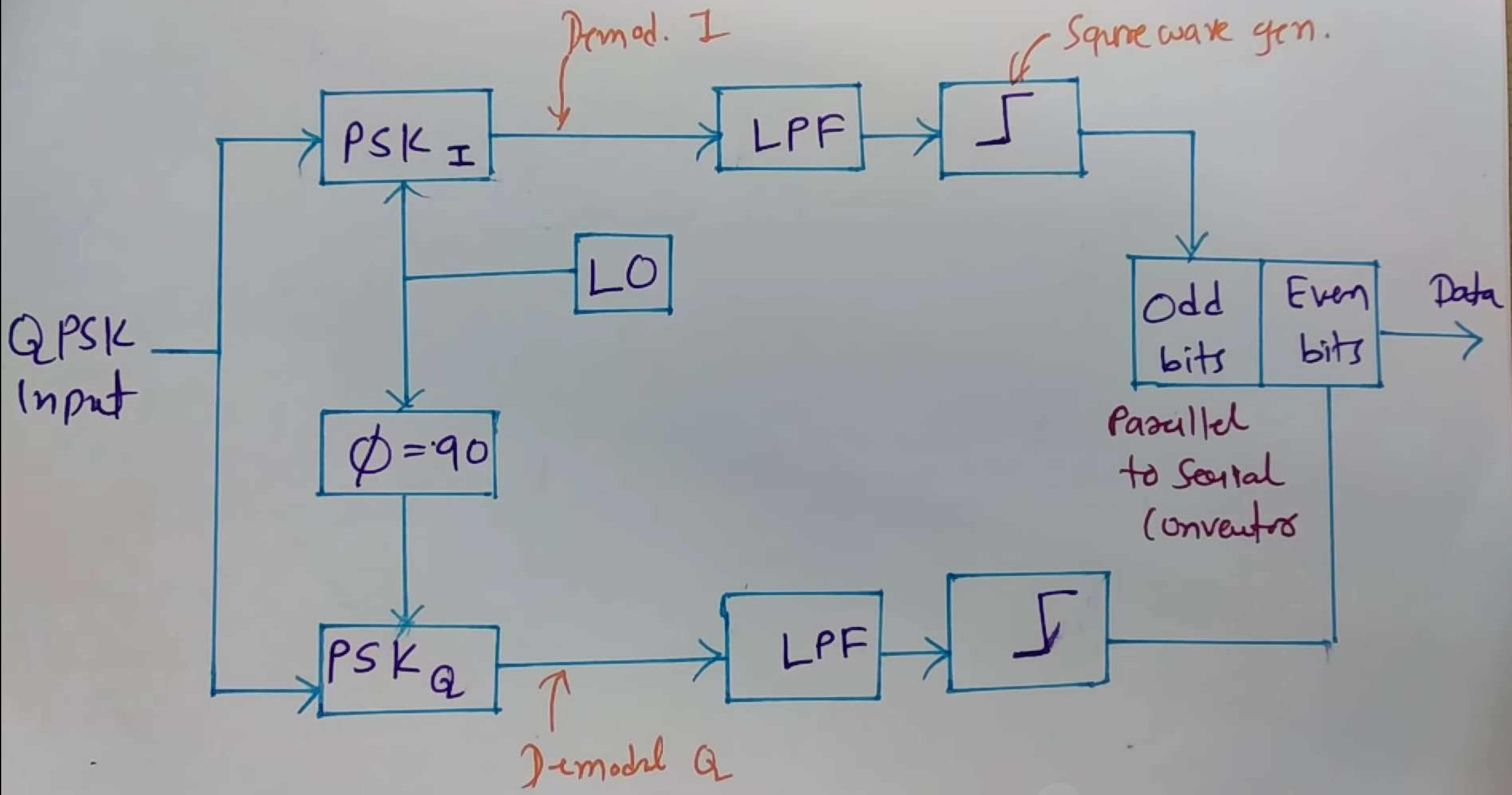
Cable modem, cellular phone system etc.

QPSK Modulator





00 — 0°
01 — 180°
10 — 90°
11 — 270°
Carrier signal
(00)
(01)
(10)
(11)
Modulated signal
φ = 0°
φ = 180°
φ = 90°
φ = 270°



Demod. I
Square wave gen.
PSK I
LPF
LO
QPSK Input
Odd bits
Even bits
Data
ϕ = 90
Parallel to Serial Convertor
PSK Q
LPF
Demodul Q

FSK - Freq. Shift Keying

Outlines

- Basics of FSK
- defination of FSK
- Spectrum and BW of FSK
- modulation of FSK
- Demodulation of FSK
- Applications of FSK

BPSK [Binary Phase Shift Keying]

$\uparrow$

$m = 2$

$\rightarrow$ Here Carrier signal

$x_1(t) \longrightarrow 1$

$x_2(t) \longrightarrow 0$

$c(t) = A\cos(2\pi f_c t + \phi)$

bit 0 $\longrightarrow$ $\phi = 0$

bit 1 $\longrightarrow$ $\phi = 180$

- $x_1(t) = A\cos(2\pi f_c t) \longrightarrow$ bit 1

$x_2(t) = -A\cos(2\pi f_c t) \longrightarrow$ bit 0.

$[\ 0 \le t \le T_b\ ]$

- E_b is Energy per bit

$$E_b = \int_0^{T_b} x_1^2(t)\, dt$$

$$= \int_0^{T_b} A^2 \cos^2(2\pi f_c t)\, dt$$

$$= A^2 \int_0^{T_b} \frac{1 + \cos(4\pi f_c t)}{2}\, dt$$

$$= \frac{A^2}{2} \int_0^{T_b} \underline{1} + \underline{\cos(4\pi f_c t)}\, dt$$

$$\Rightarrow E_b = \frac{A^2}{2}(T_b) \Rightarrow A = \sqrt{\frac{2E_b}{T_b}}$$

- $x_1(t) = A\cos(2\pi f_c t) \longrightarrow$ bit 1

$x_2(t) = -A\cos(2\pi f_c t) \longrightarrow$ bit 0.

$$[\,0 \le t \le T_b\,]$$

- E_b is Energy per bit

$$E_b = \int_0^{T_b} x_1^2(t)\,dt$$

$$= \int_0^{T_b} A^2\cos^2(2\pi f_c t)\,dt$$

$$= A^2\int_0^{T_b} \frac{1+\cos(4\pi f_c t)}{2}\,dt$$

$$= \frac{A^2}{2}\int_0^{T_b} 1 + \cos(4\pi f_c t)\,dt$$

$$\Rightarrow E_b = \frac{A^2}{2}(T_b) \Rightarrow A = \sqrt{\frac{2E_b}{T_b}}$$

- $x_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$

$x_2(t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$.

$m = 2$ → Here Carrier signal

$x_1(t) \longrightarrow 1$

$x_2(t) \longrightarrow 0$

$c(t) = A\cos(2\pi f_c t + \phi)$

bit $0 \longrightarrow \phi = 0$

bit $1 \longrightarrow \phi = 180$

- $x_1(t) = A\cos(2\pi f_c t) \longrightarrow$ bit 1

$x_2(t) = -A\cos(2\pi f_c t) \longrightarrow$ bit 0.

$[\, 0 \le t \le T_b \,]$

- E_b is Energy per bit

$$E_b = \int_0^{T_b} x_1^2(t)\,dt$$

$$= \int_0^{T_b} A^2\cos^2(2\pi f_c t)\,dt$$

$$= A^2 \int_0^{T_b} \frac{1+\cos(4\pi f_c t)}{2}\,dt$$

$$= \frac{A^2}{2}\int_0^{T_b} 1 + \cos(4\pi f_c t)\,dt$$

$$\Rightarrow E_b = \frac{A^2}{2}(T_b) \Rightarrow A = \sqrt{\frac{2E_b}{T_b}}$$

- $x_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \longrightarrow$ bit 1

$x_2(t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \longrightarrow$ bit 0

→ As per Gram Smith Orthogonalization procedure, we will find number of basis function.

$$N \le m = 2$$

$$\rightarrow \phi_1(t) = \frac{x_1(t)}{\sqrt{E_b}} = \frac{\sqrt{2E_b/T_b}}{\sqrt{E_b}}\cos(2\pi f_c t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t).$$

→ Here, $x_1(t) = -x_2(t)$

$N = 1$, BPSK → 1D Modulation

→ As per orthogonalization

$$\int_0^{T_b} \phi_1^2(t)\,dt = 1$$

→ Scatter plot / Constellation point / Space diagram

- $x_{ij} = \int_0^{T_b} x_i(t)\,\phi_j(t)\,dt$ $\quad i = 1, 2, \dots m$; $\quad j = 1, 2, \dots N$

- $x_{11} = \int_0^{T_b} x_1(t)\,\phi_1(t)\,dt$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_1^2(t)\,dt$$

$$= \sqrt{E_b}\int_0^{T_b}\phi_1^2(t)\,dt$$

$$= \sqrt{E_b}$$

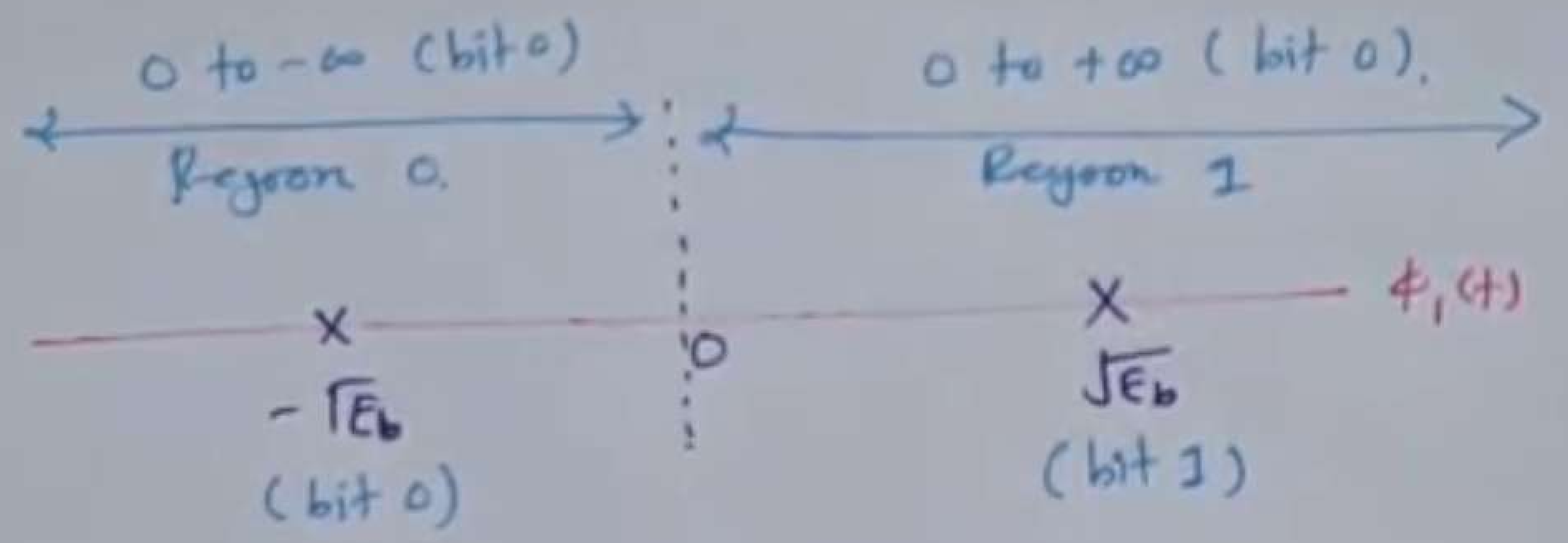
- $x_{21} = \int_0^{T_b} x_2(t)\,\phi_1(t)\,dt$

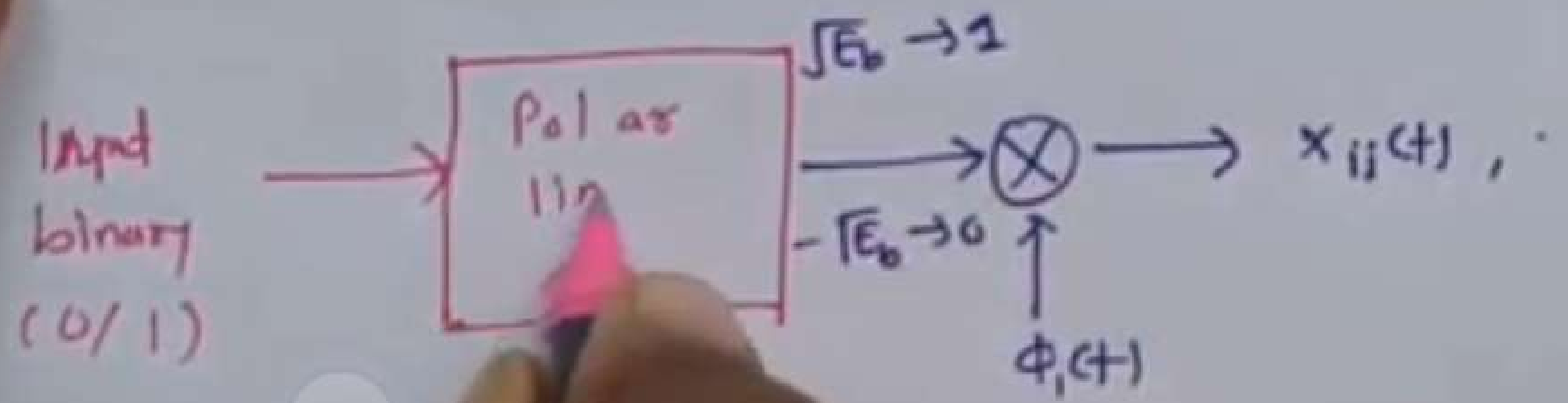
$$= \int_0^{T_b} -\sqrt{E_b}\,\phi_1^2(t)\,dt$$

$$= -\sqrt{E_b}\int_0^{T_b}\phi_1^2(t)\,dt$$

$$= -\sqrt{E_b}.$$

Constellation Diagram / Space diagram / Scatter plot





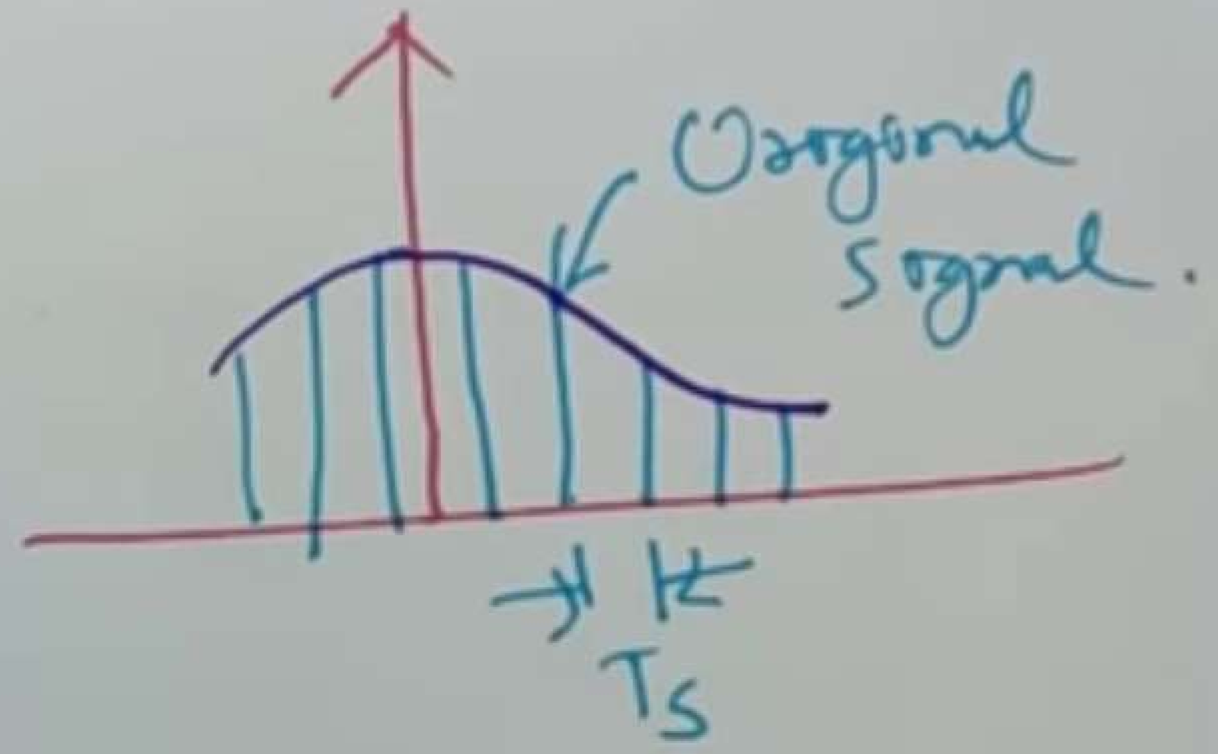
- It is a process to convert continous time signal ...
discreate signal.

Sufficient no of samples must be taken, so that the Orginal signal is reconstructed properly.

- No of samples to be taken depends on maximum signal freq. present in the signal.

- Different types of sampling.
    - Ideal samples.
    - Natural samples.
    - Flat-top samples.

- Different types of Sampling.
  - Ideal samples.
  - Natural samples.
  - Flat-top samples.



- Statement of sampling theorm

i] A band limited signal of finite energy, which has no freq. component higher than f_m (Hz), is completely described by its sample values at uniform intervals less than or equal to $1/2f_m$

$$\boxed{T_s \leq 1/2f_m}$$

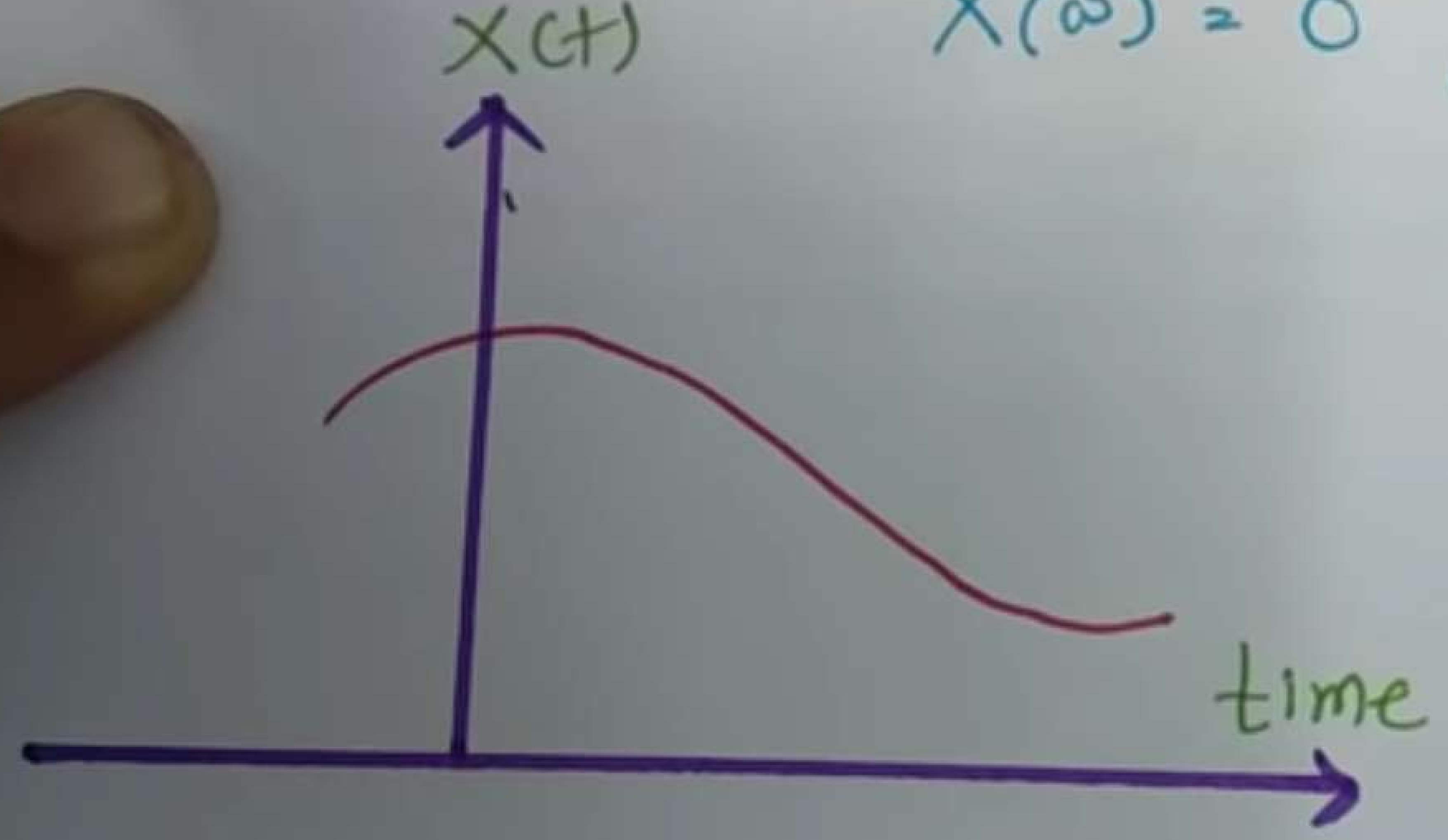
Intervals less than ...

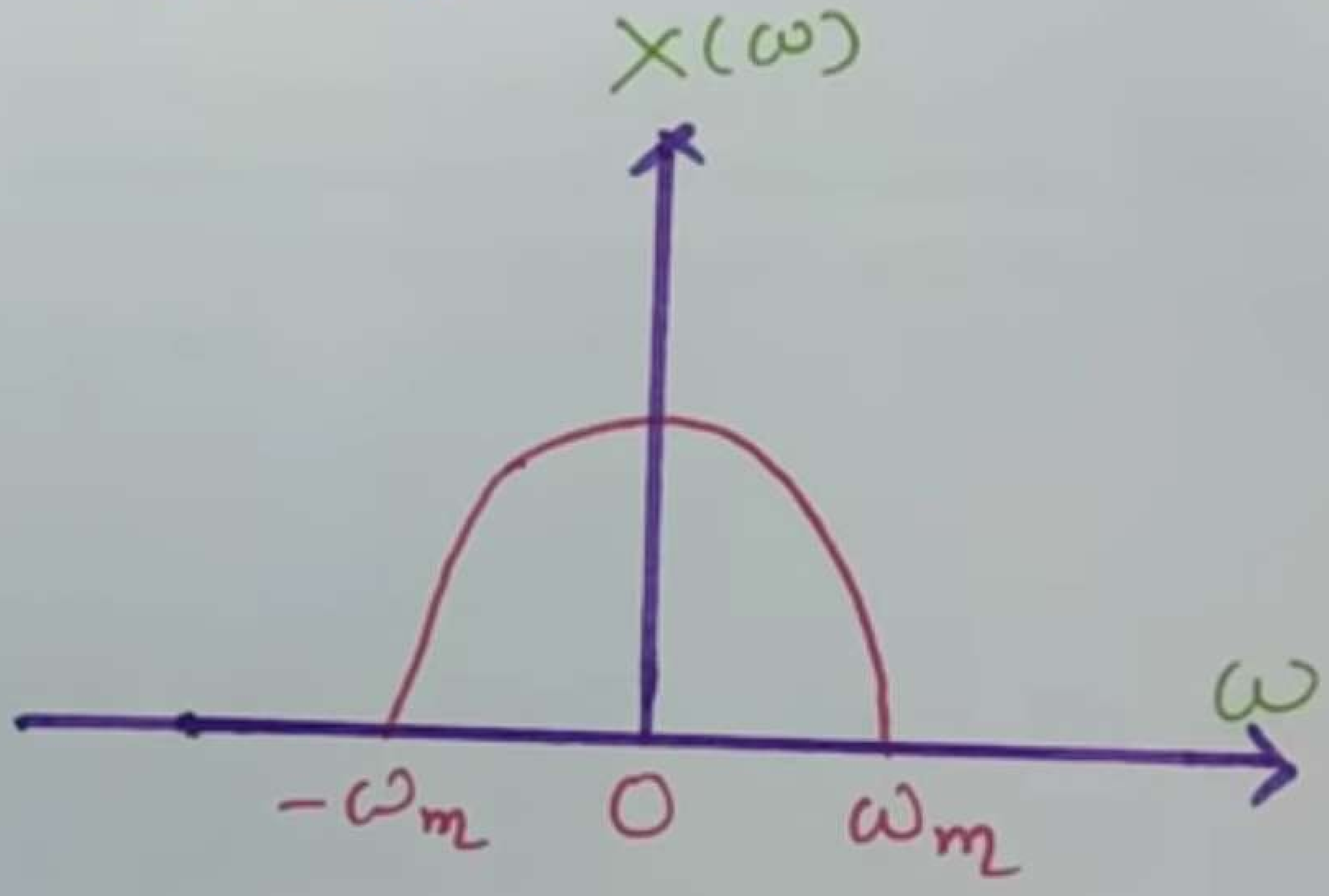
$$\boxed{T_s \le 1/2f_m}$$

ii] A band limited signal of finite energy, which has no freq. components higher than f_m (Hz), may be completely recovered from the knowledge of its samples taken at the rate of $2f_m$ samples per second.

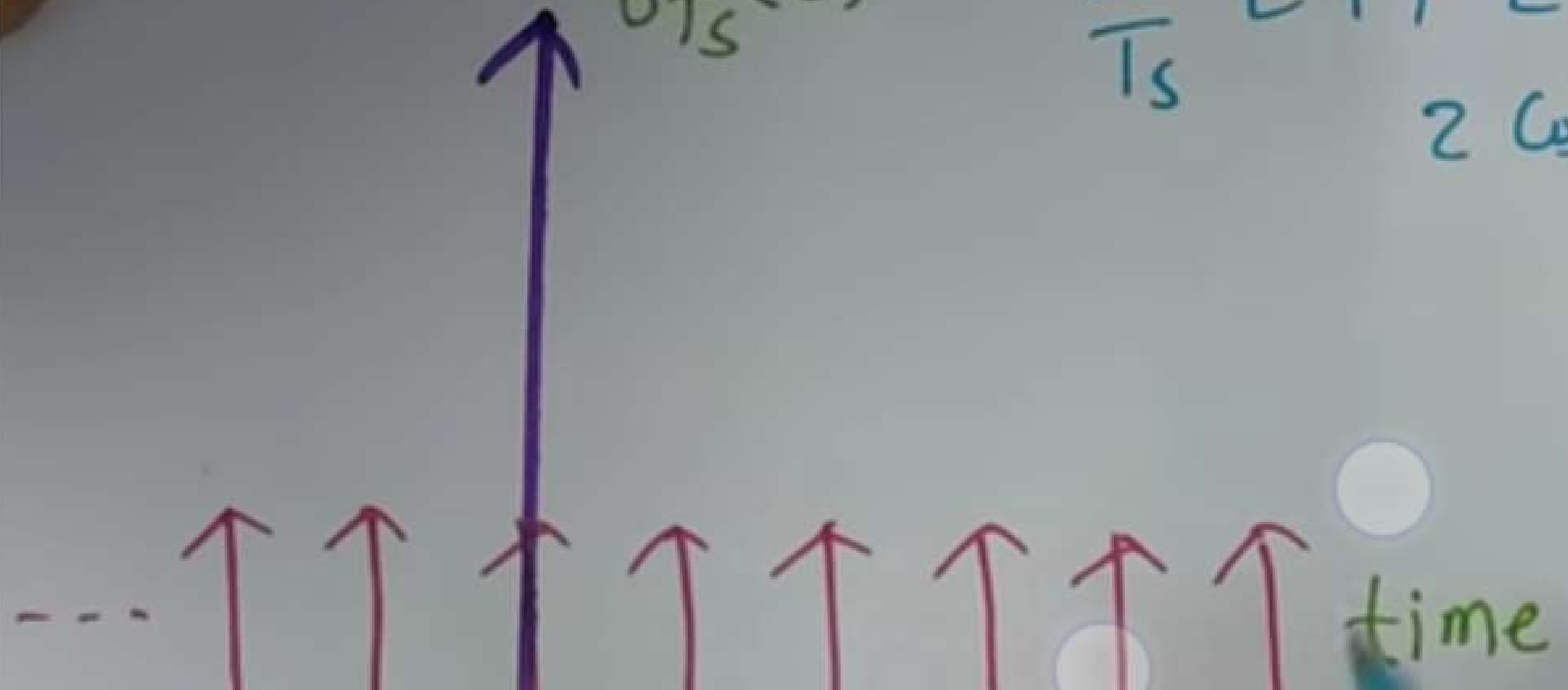
$$\boxed{f_s \geq 2f_m}$$

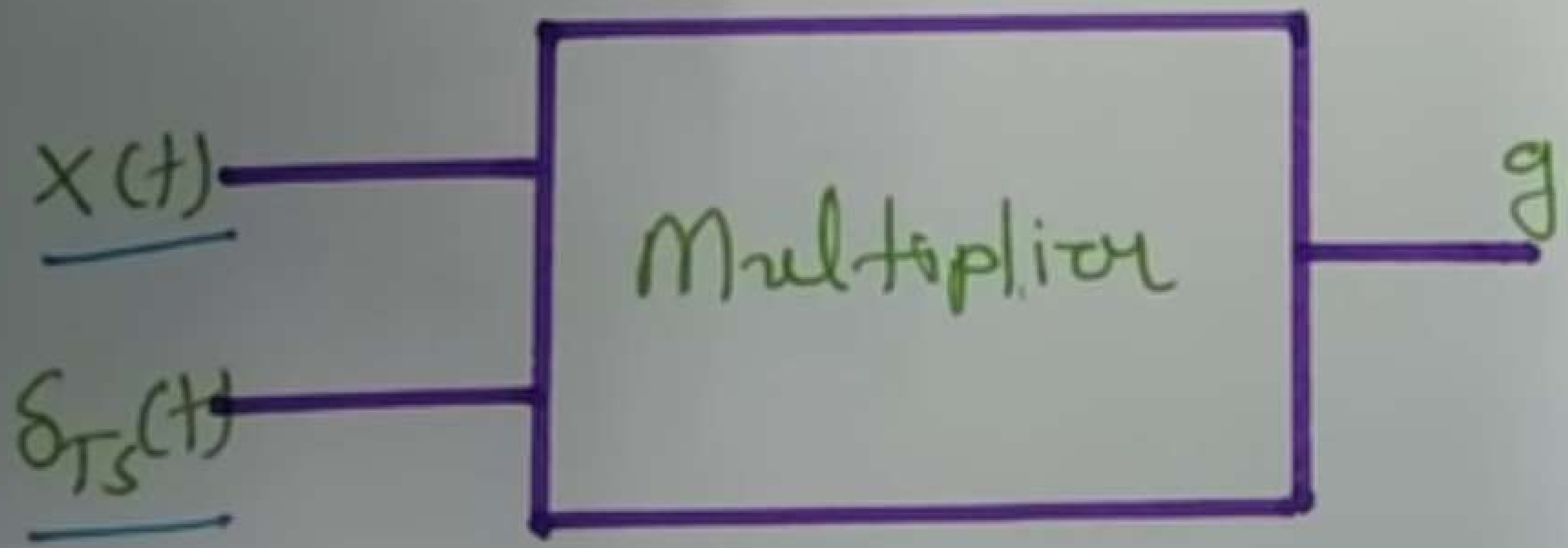
$X(\omega) = 0 \quad , \quad \omega > \omega_m$



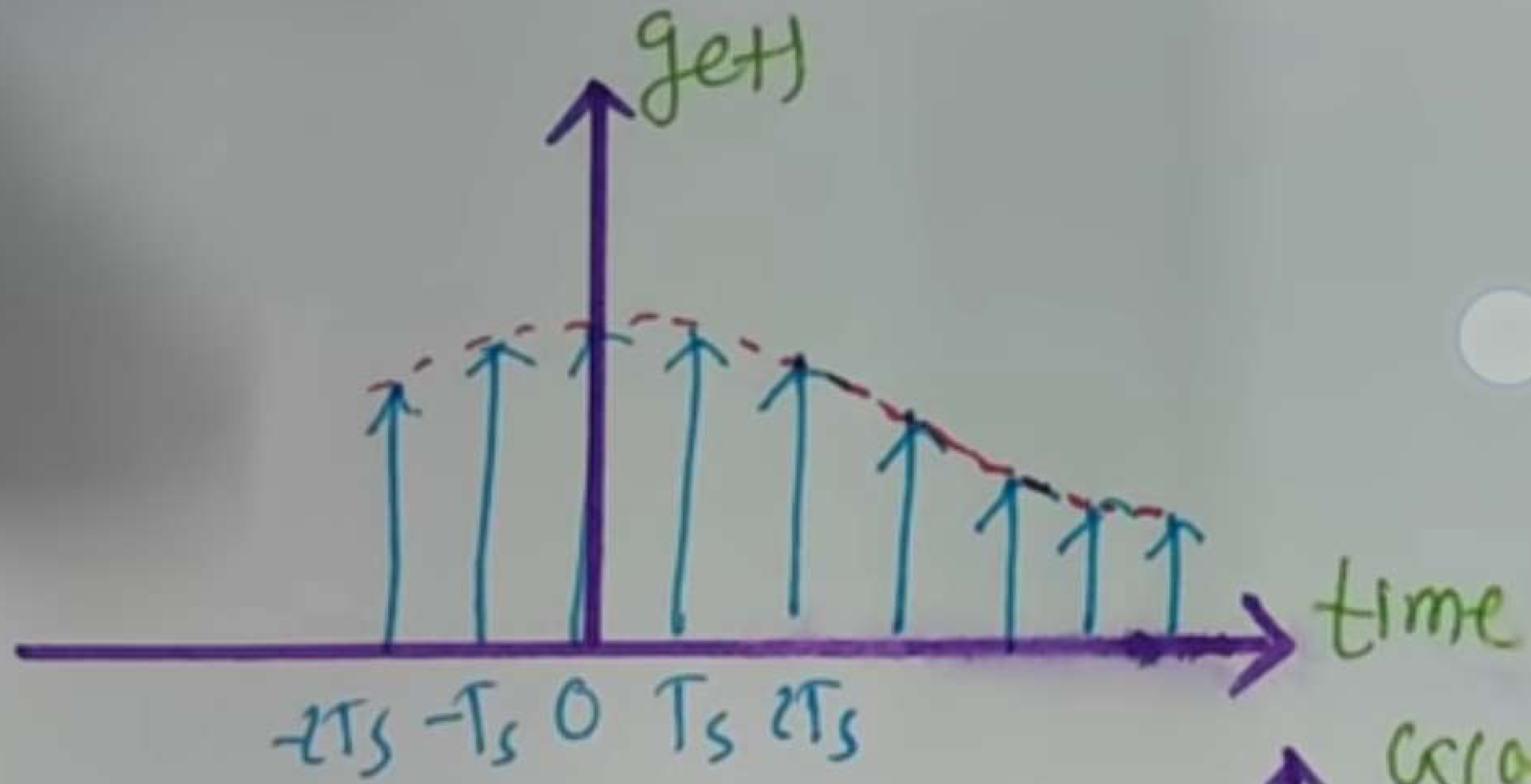


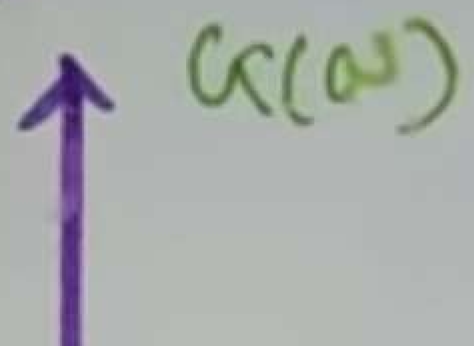
$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \dots\right]$$



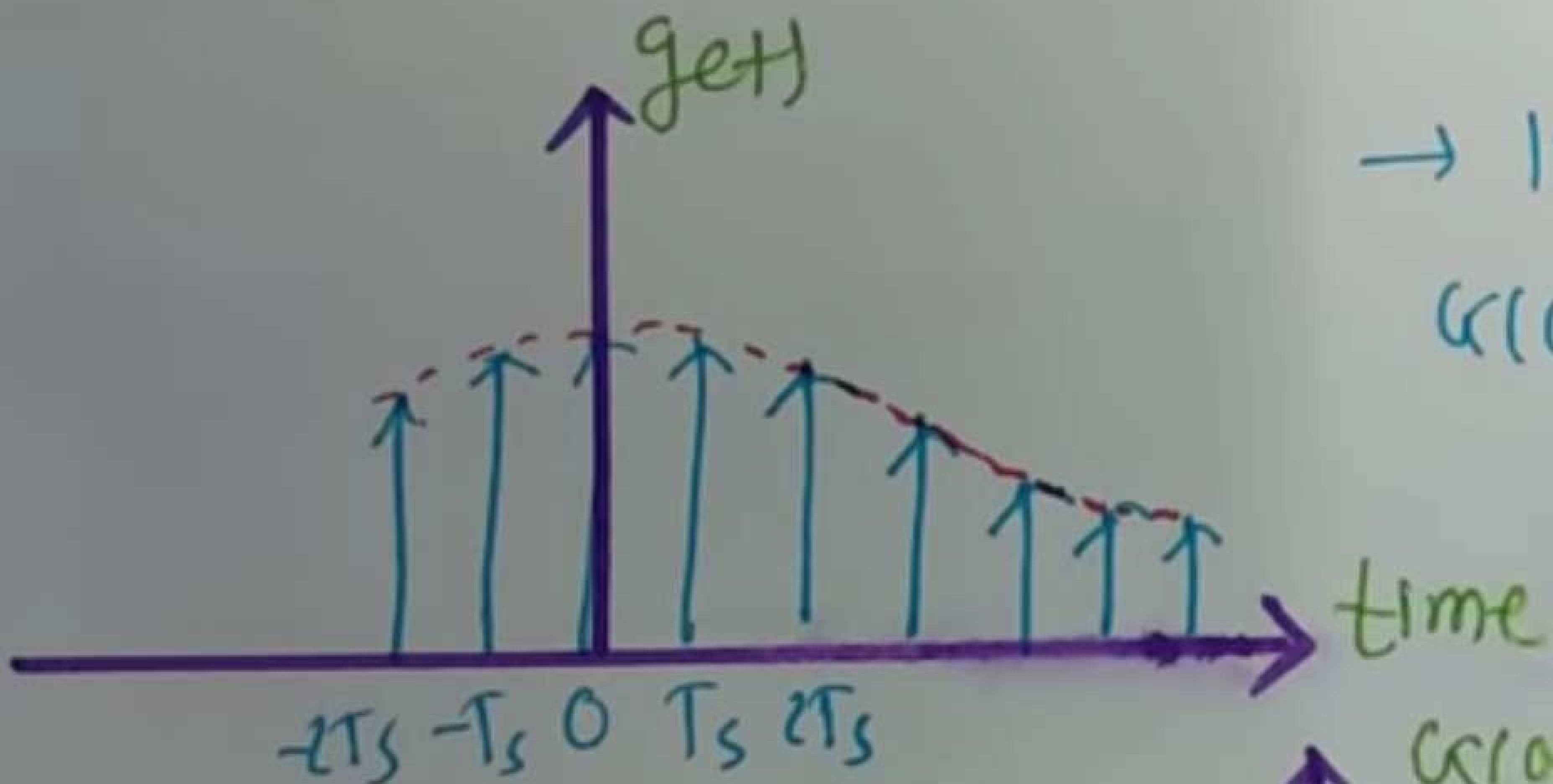


$$g(t) = x(t)\,\delta_{T_s}(t)$$



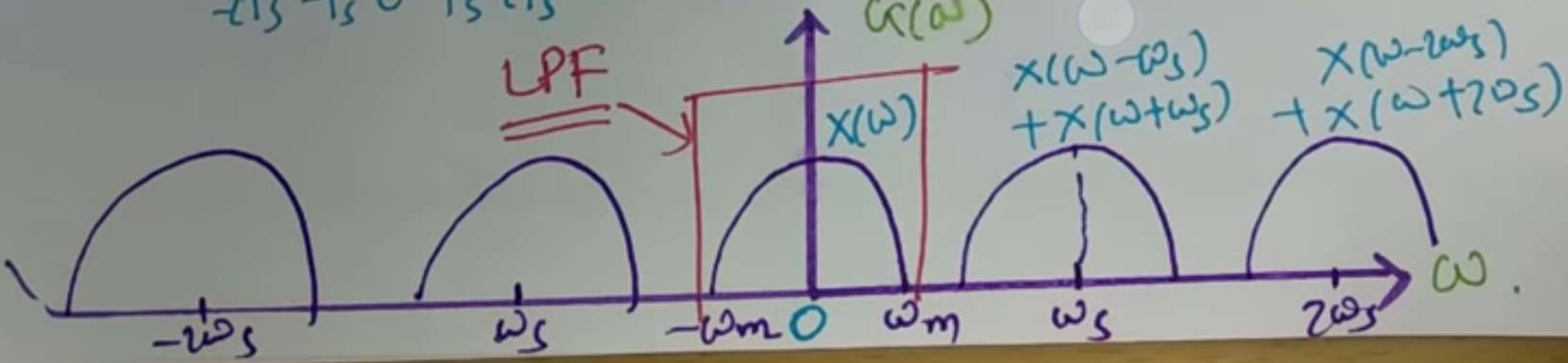


$$2\omega_s\, 3\omega_s t + \ldots \big]\Big)$$



→ In freq domain

$$G(\omega) = \frac{1}{T_s}\big[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + \ldots\big].$$



→ As long as $f_s > 2f_m$, $G(\omega)$ will repeat Periodically without overlapping.

→ Spectrum $G(\omega)$ extends upto ∞ freq. but our Perpuse is to extract original Spectrum $X(\omega)$ out of the Spectrum $G(\omega)$.

→ At receiver we place LPF of freq ω_m. So we can extract original Information.

→ $f_s > 2f_m$, To avoid successive cycles not to overlape

→ Spectrum $K(\omega)$ extends upto ∞ freq. but over

Perpuse is to extract original Spectrum $X(\omega)$ out of the Spectrum $K(\omega)$.

→ At receiver we place LPF of freq ω_m. So we can extract original Information.

→ $f_s > 2f_m$, To avoid successive cycles not to overlape

$f_s = 2f_m$, successive cycles just touch eachother.

$f_s < 2f_m$, Successive cycles overlape eachother

→ Hence, for reconstruction without d[illegible]rtion

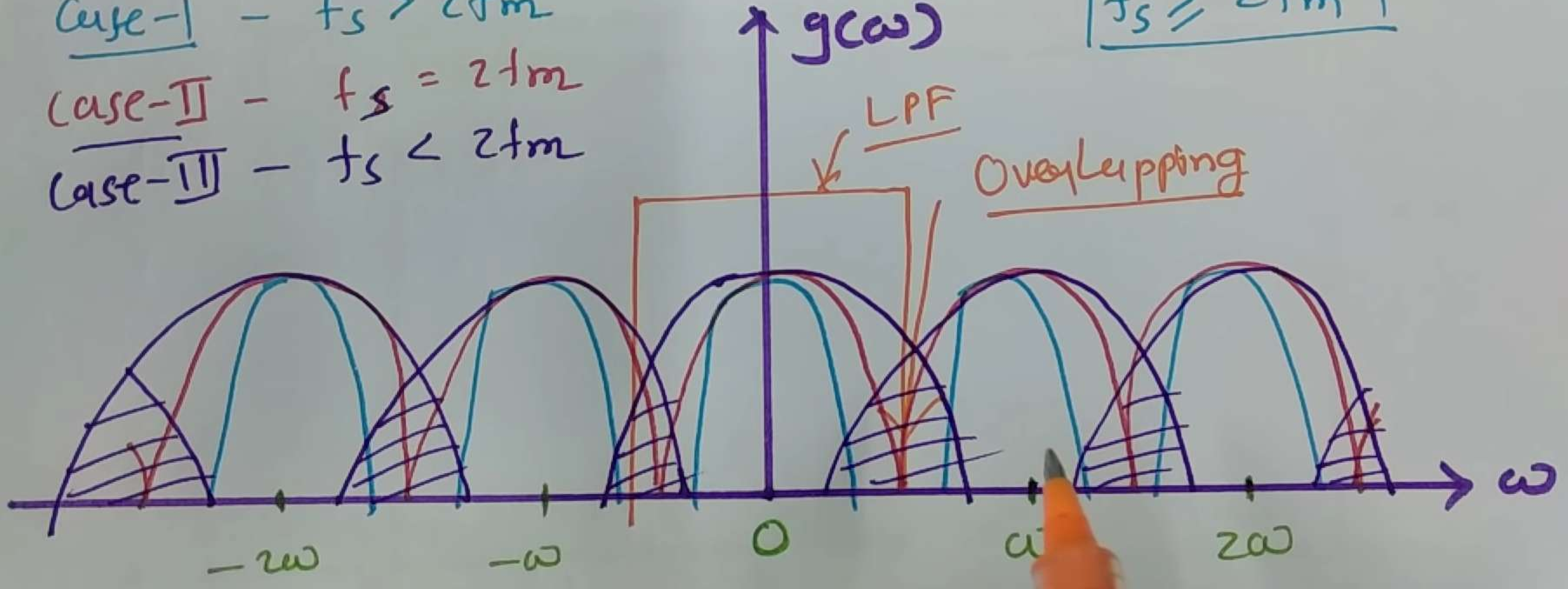
✲ Effect of under sampling - Aliasing

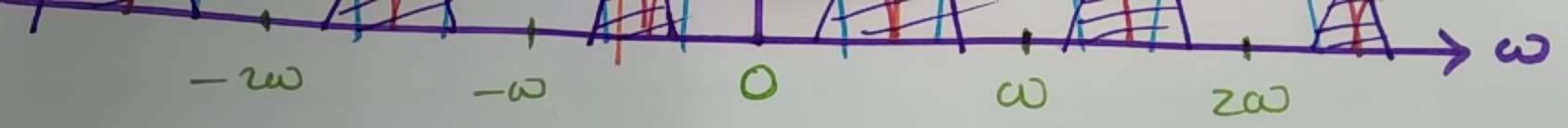
<u>Case-I</u> - $f_s > 2f_m$

<u>Case-II</u> - $f_s = 2f_m$

<u>Case-III</u> - $f_s < 2f_m$

$\boxed{f_s \geq 2f_m}$





- If $f_s < 2f_m$, then successive samples (gets) of $X(\omega)$ will overlap each other.
- Due to Aliasing effect, It is not possible to recover original signal $x(t)$ by LPF.
- Here due to overlape of one region to other region, signal $x(t)$ is distorted.
- So before we go for sampling, we pass original

due to overlape of one region to other region, Signal x(t) is distorted.

- So before we go for sampling, we pass Original Signal through LPF. This is even reffered as pre-alias filter, other name is band limit filter.

- In Short, to avoid aliasing.

  1) Pre alias Filter can be used

  2) $f_s \geq 2f_m$

Examples on Sampling & Nyquist Rate

1) $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$

Calculate the Nyquist rate for this signal.

$f_1 = \frac{\omega_1}{2\pi} = 25\ Hz$

$f_2 = \frac{\omega_2}{2\pi} = 150\ Hz$

$f_3 = \frac{\omega_3}{2\pi} = 50\ Hz$

$\rightarrow$ max freq. $f_m = 150\ Hz$.

$\rightarrow f_s = 2f_m$

$= 2 \times 150$

$= 300\ Hz$

... rate & Nyqu... Interval for th...

2) Find the Nyquist rate & Nyquist Interval for the signal $x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$

$$= \frac{1}{4\pi} [2 \cos(4000\pi t) \cos(1000\pi t)]$$

$$= \frac{1}{4\pi} [\underline{\cos(3000\pi t)} + \underline{\cos(5000\pi t)}]$$

$\rightarrow$ $f_1 = \frac{\omega_1}{2\pi} = 1500$ Hz

$f_2 = \frac{\omega_2}{2\pi} = 2500$ Hz

$\rightarrow$ Max freq.

$f_m = 2500$ Hz.

$\rightarrow$ $f_s = 2 f_m$

$= 2(2500)$

$= 5000$ Hz

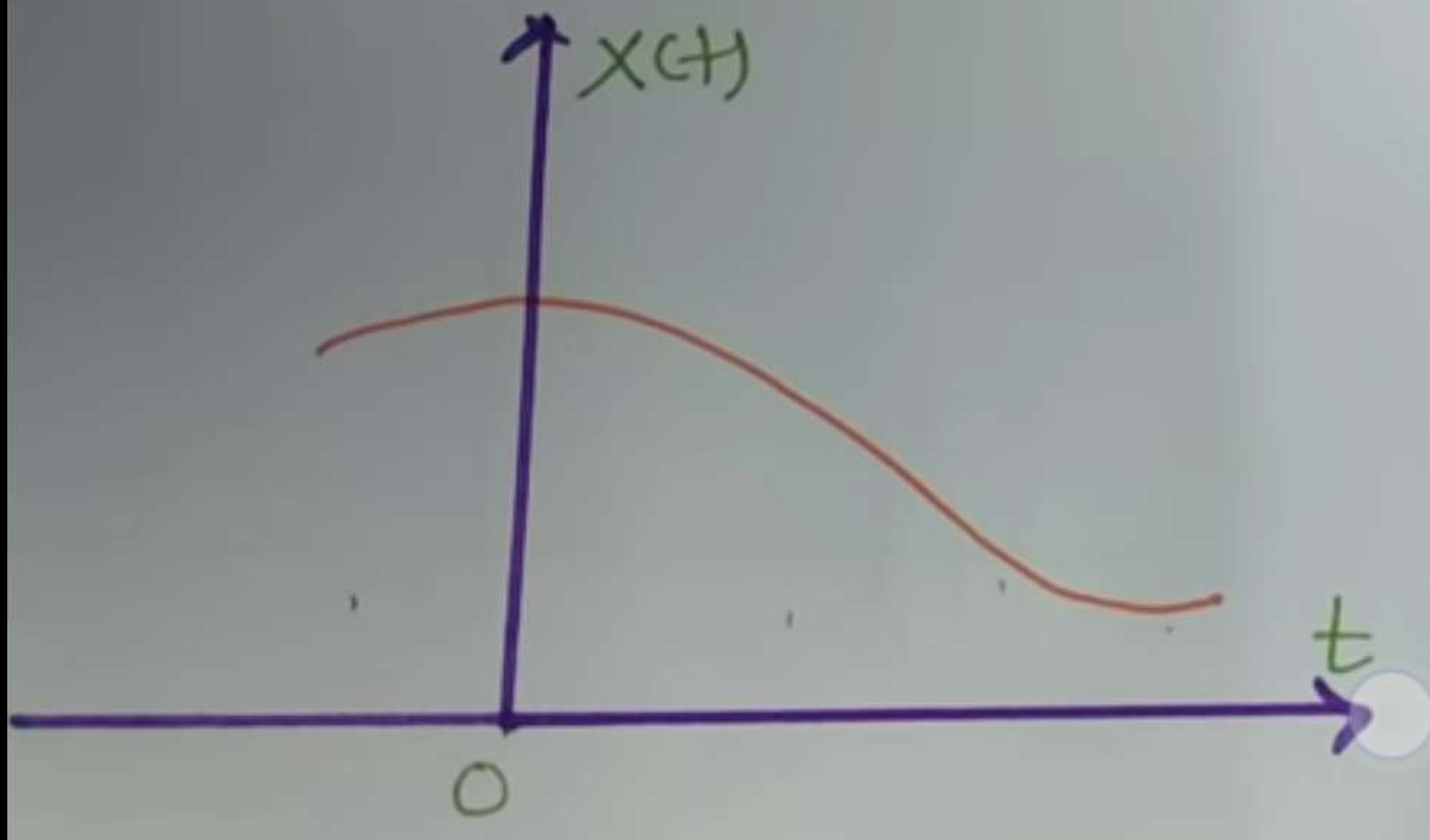
$\rightarrow$ $T_s = 1/f_s$

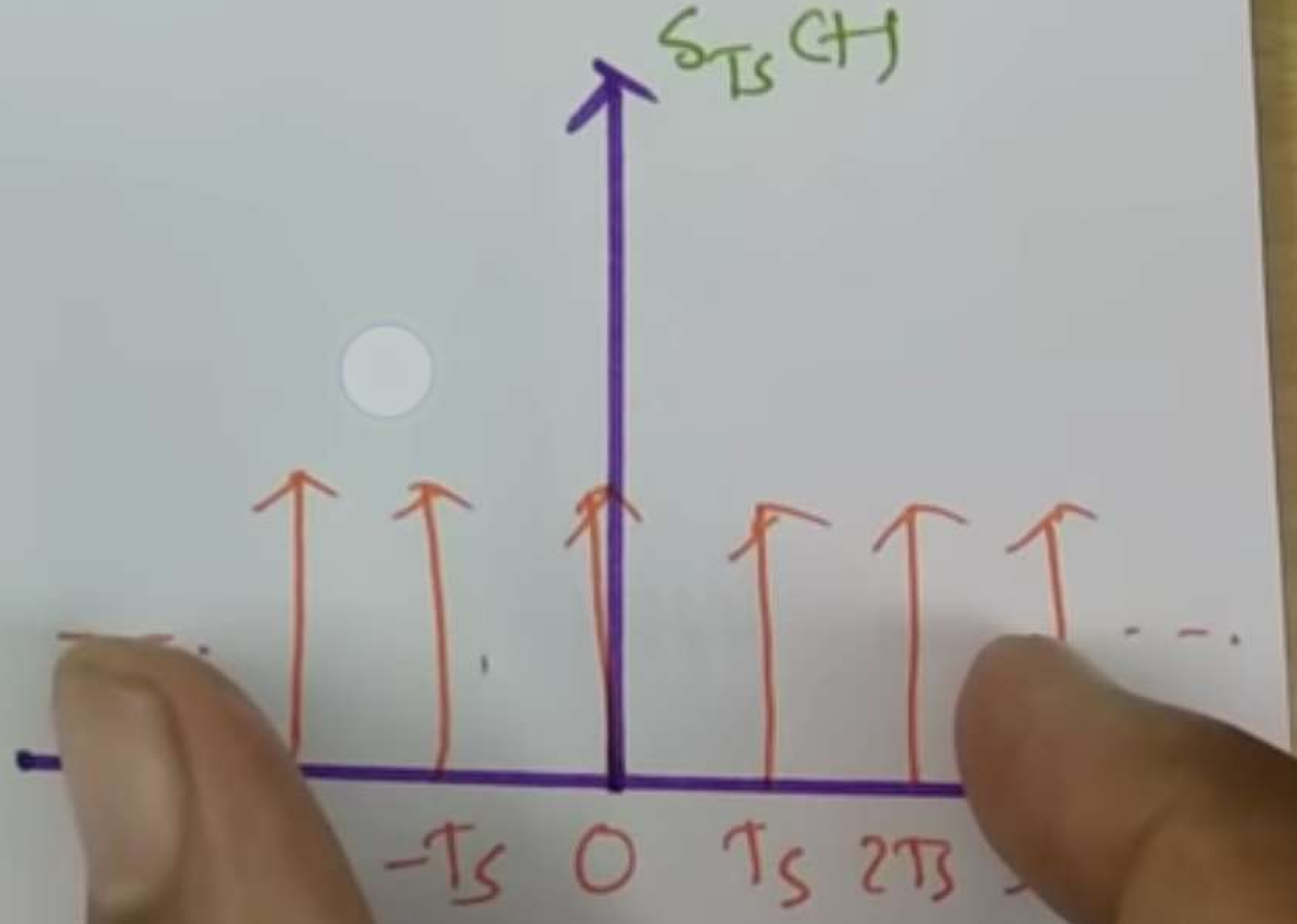
$= 1/5000$

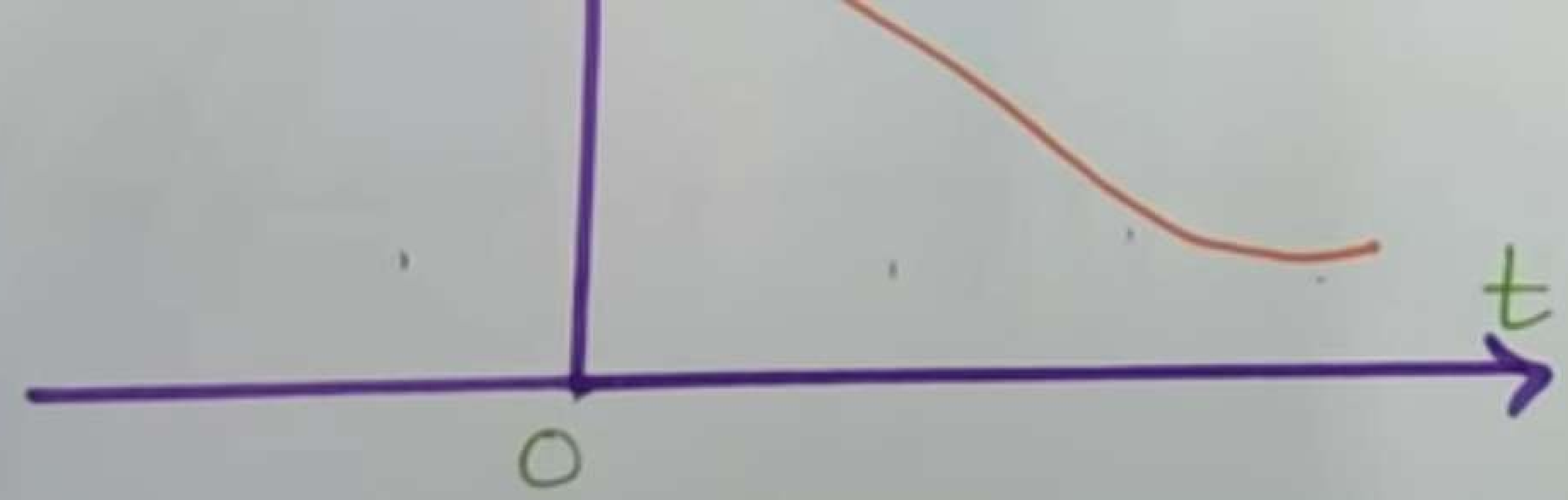
$= 0.2$

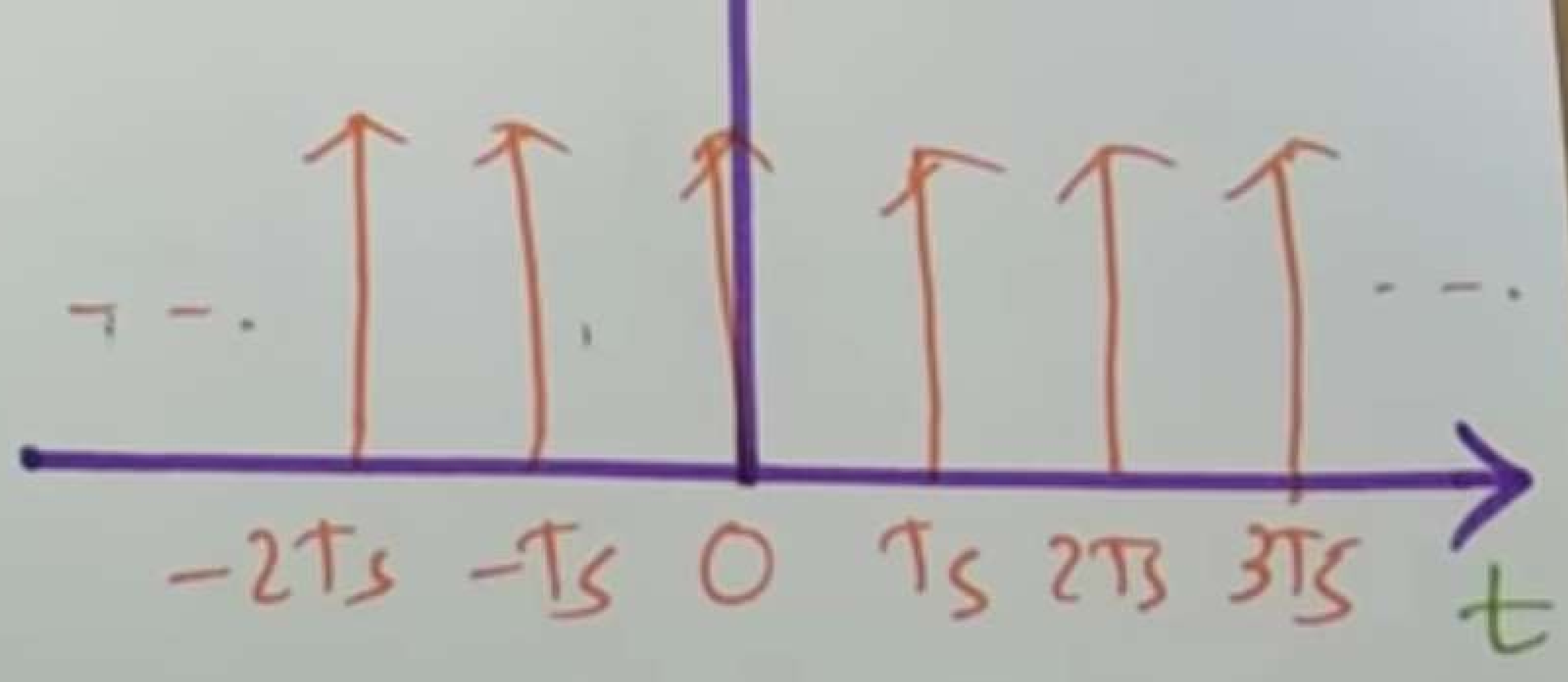
stantaneous Sampling or Impulse Sampling or Ideal Sampling

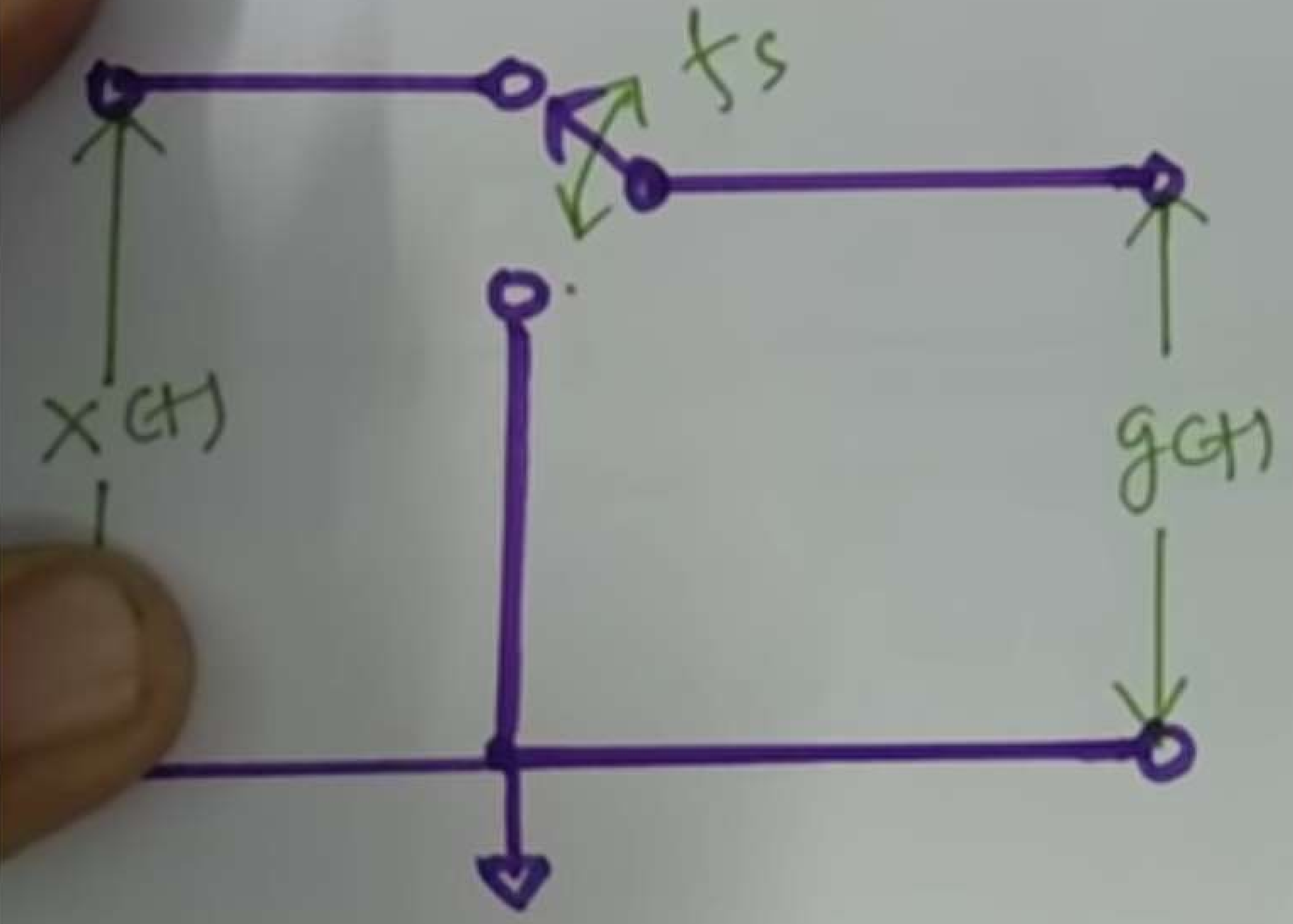
→ It uses principle of multiplication

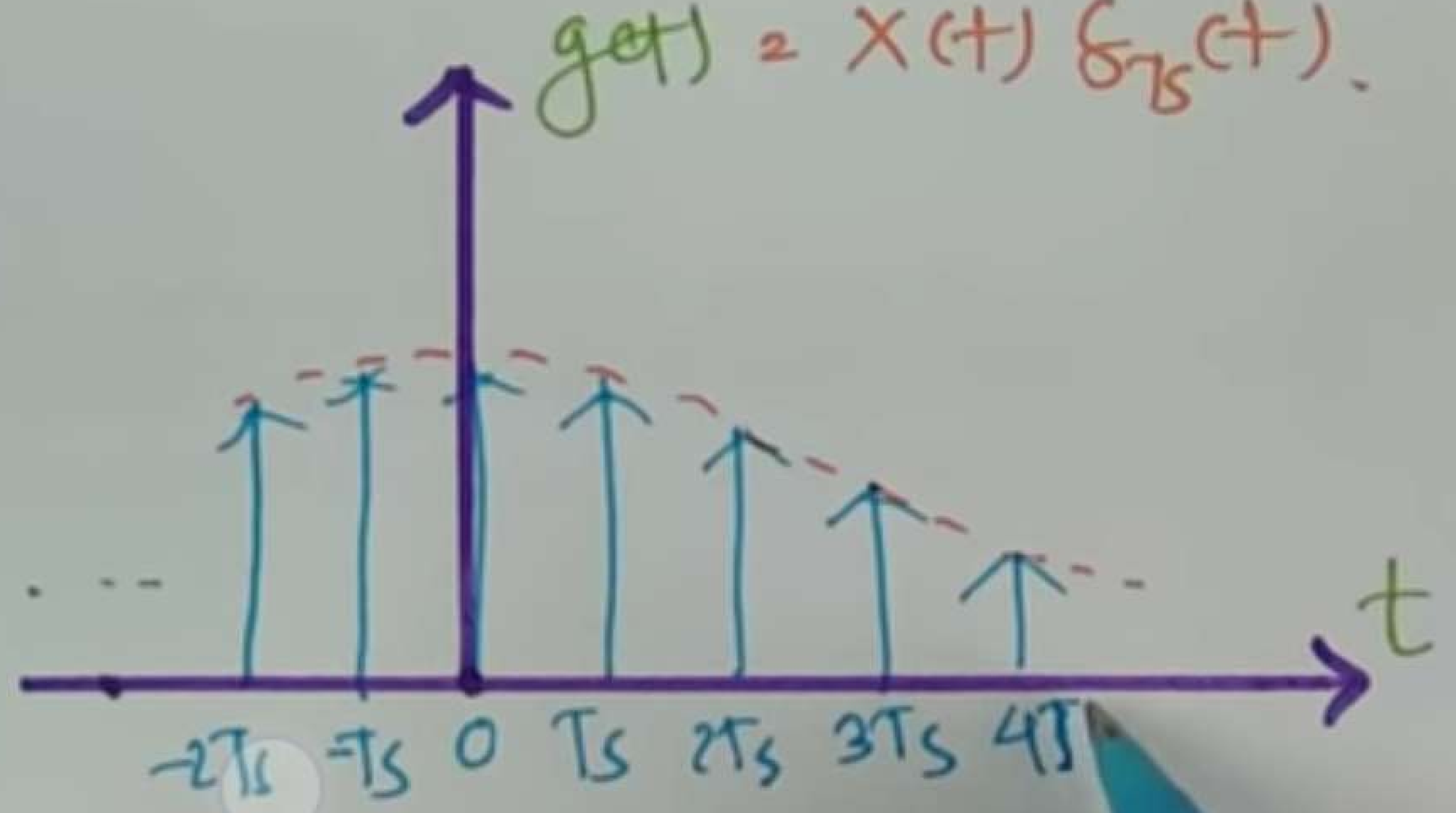












- To generate ideal samples train, we use Sw

- To generate ideal samples train, we use switching sampler.
- If we assume, closing time $t \to 0$, then it has to be consider ideal impulse ~~train~~.
- Impulse train

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

- Output

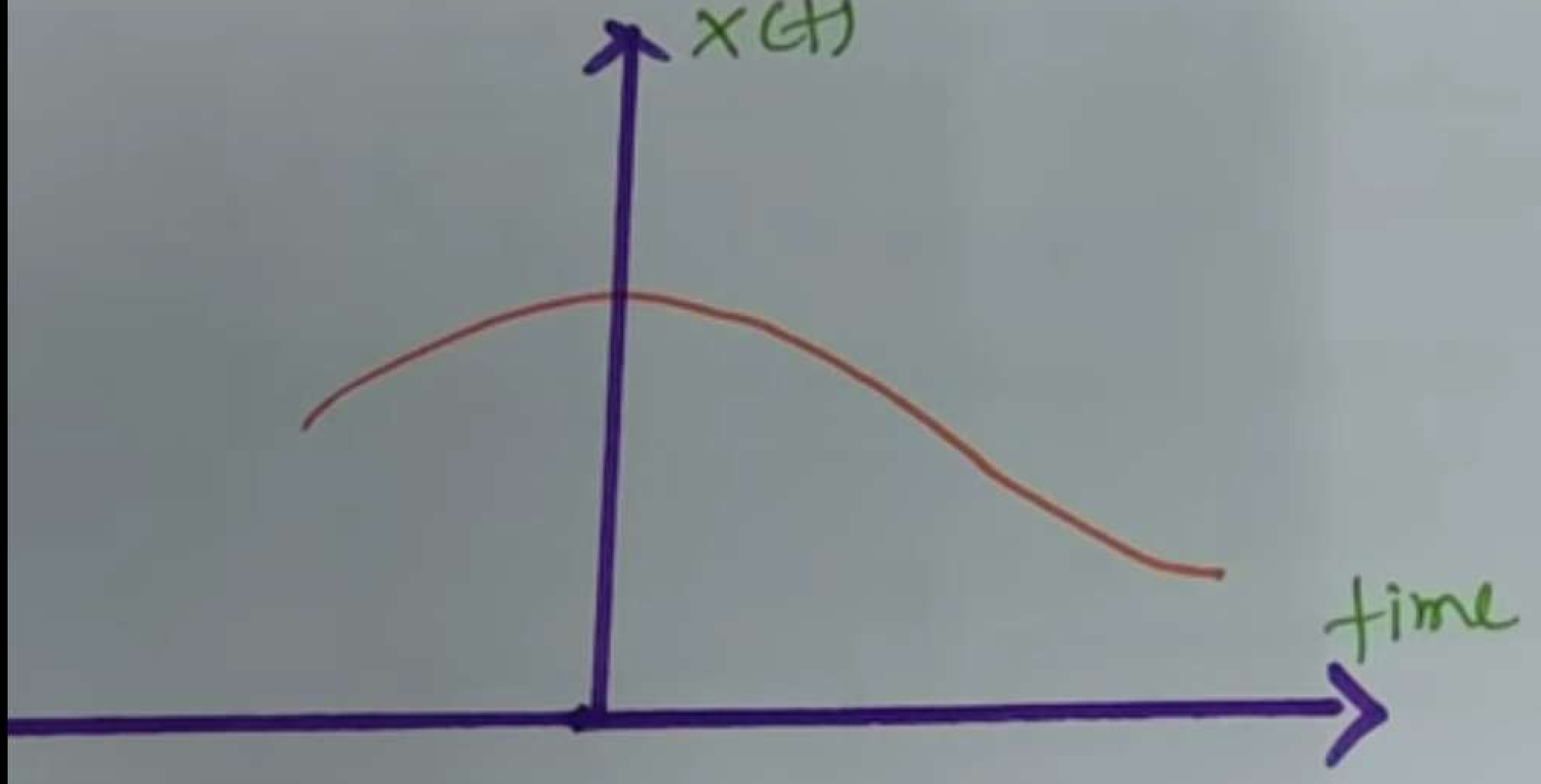
$$g(t) = x(t)\,\delta_{T_s}(t)$$
$$= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

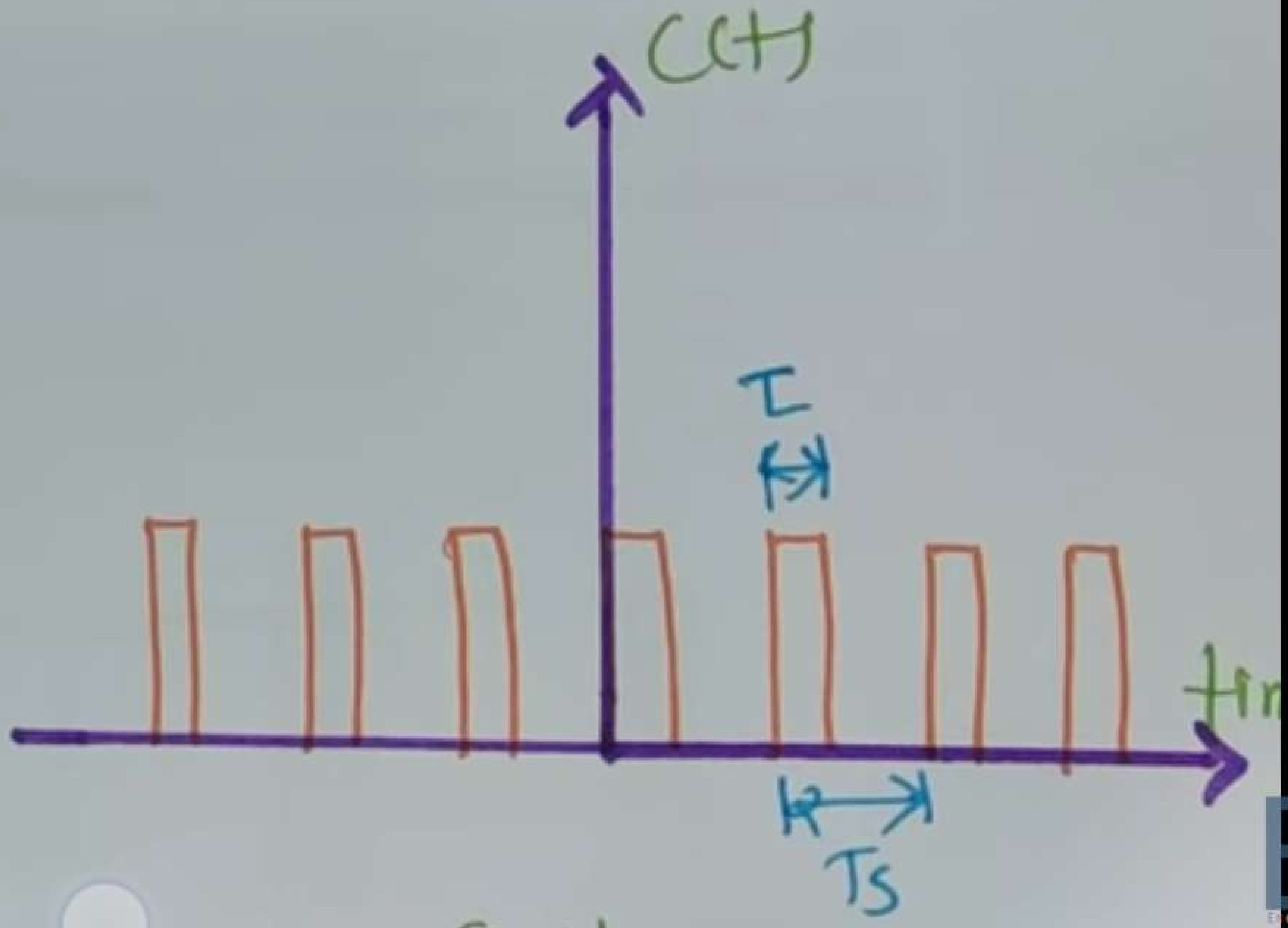
- In Freq Domain,

$$G(\omega) = f_s \sum_{n=-\infty}^{\infty} x(f - nf_s)$$

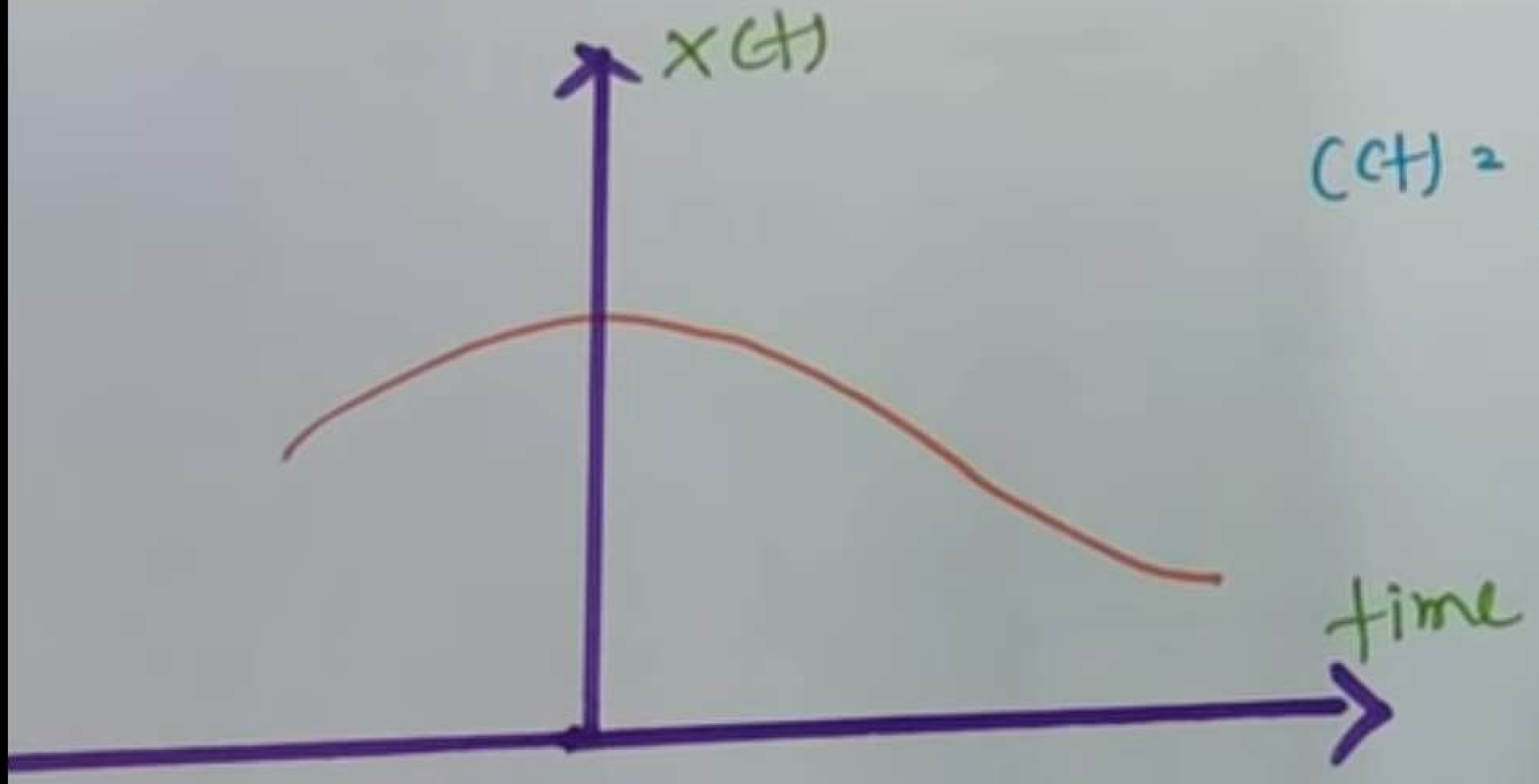
Natural Sampling

- It uses chopping principle

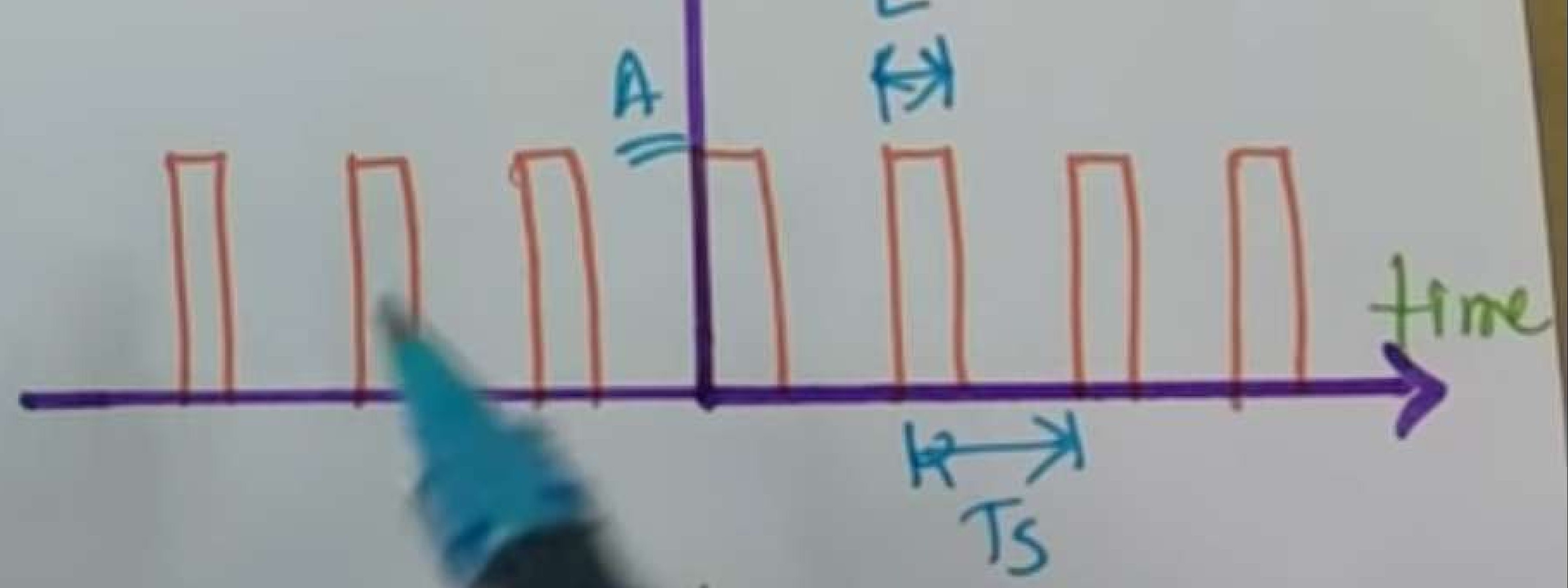


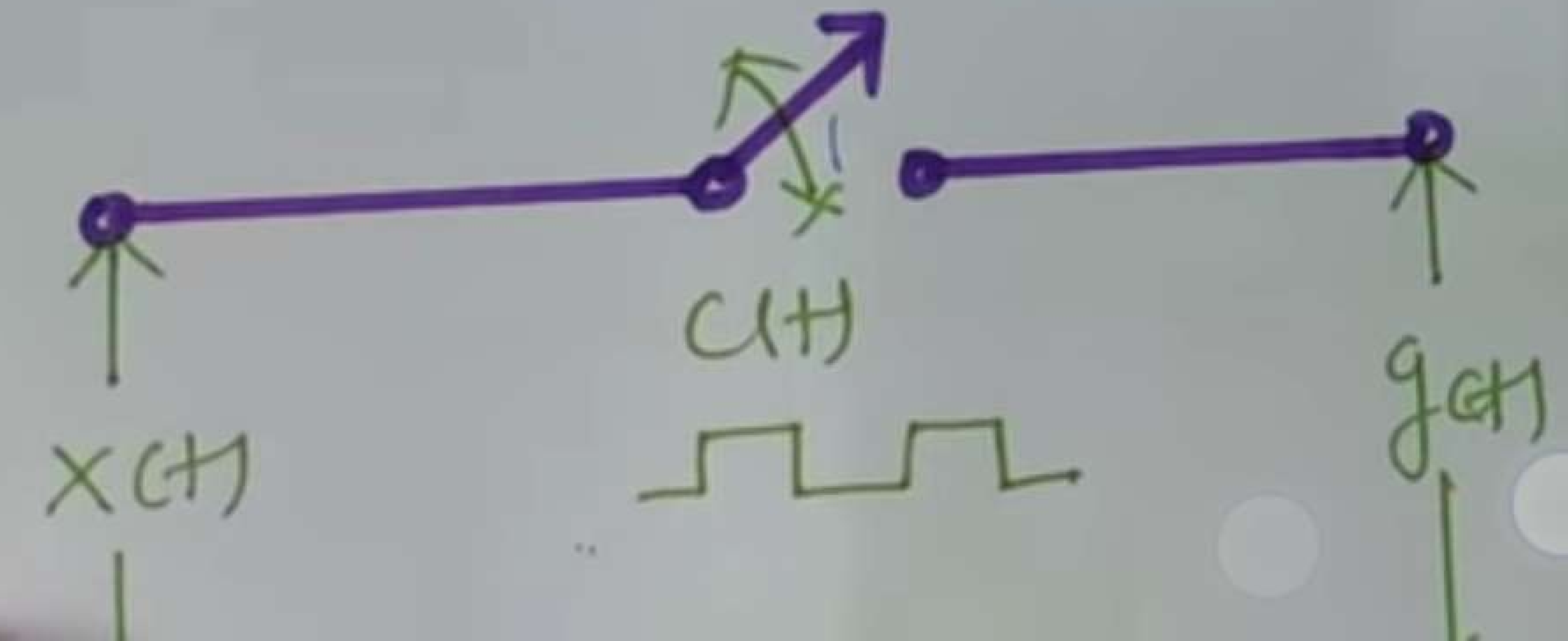


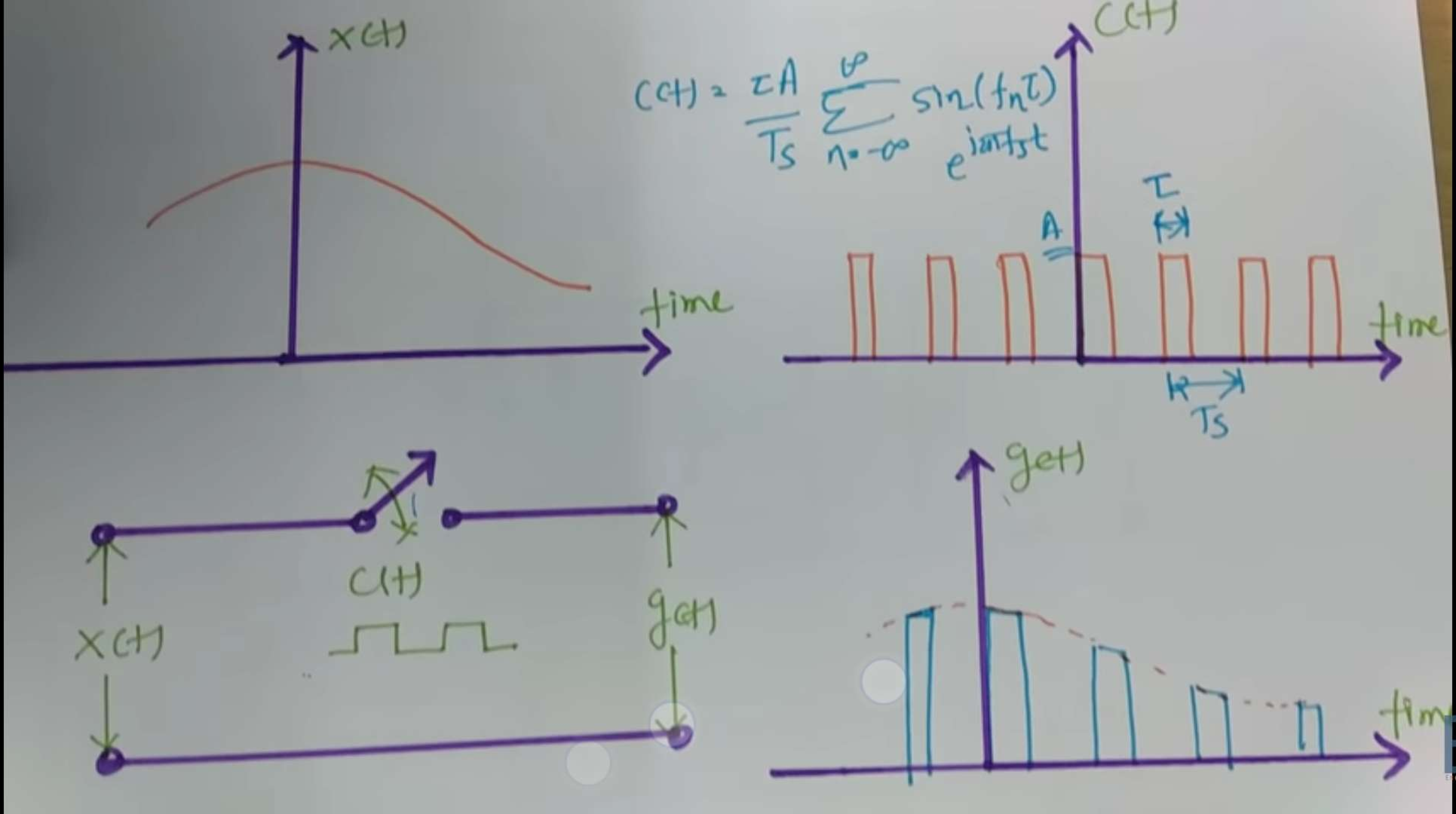
- It uses chopping principle



$$c(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \sin(f_n \tau)\, e^{j2\pi f_s t}$$






x(t)
time
c(t)
$c(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \sin(f_n \tau)\, e^{j2\pi n f_s t}$
A
τ
time
Ts
c(t)
x(t)
g(t)
g(t)
time

- $g(t) = x(t)$, $c(t) = A$

$g(t) = 0$, $c(t) = 0$

- So Mathematically

$g(t) = x(t)\, c(t)$

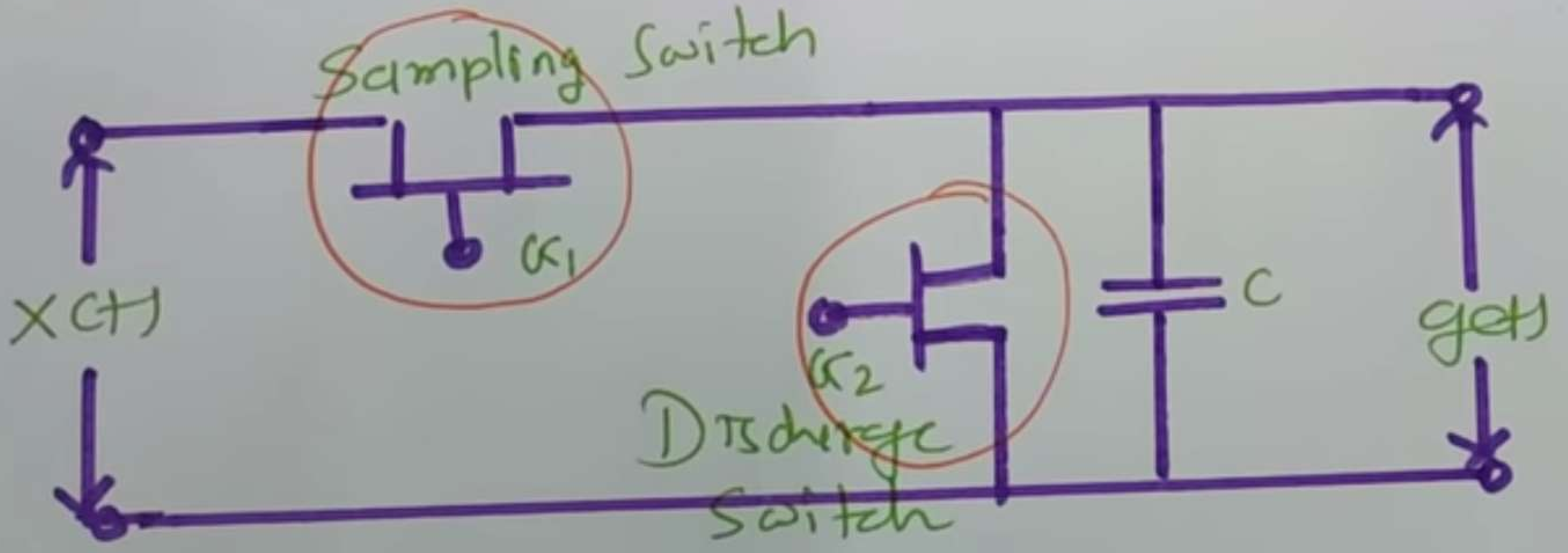
$$= \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t)\ \mathrm{sinc}\,(f_n \tau)\ e^{j2\pi f_s t}$$

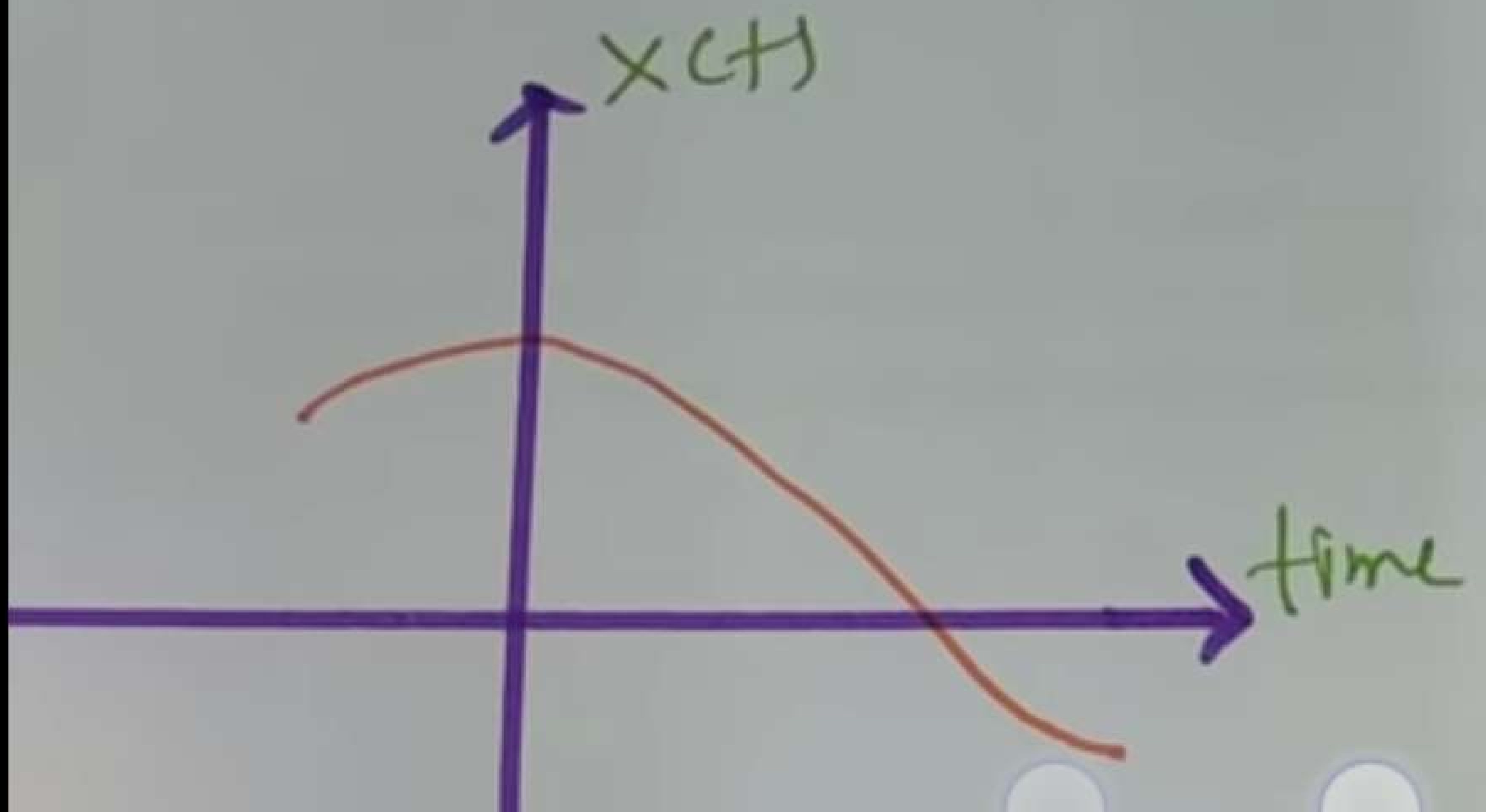
- freq. domain

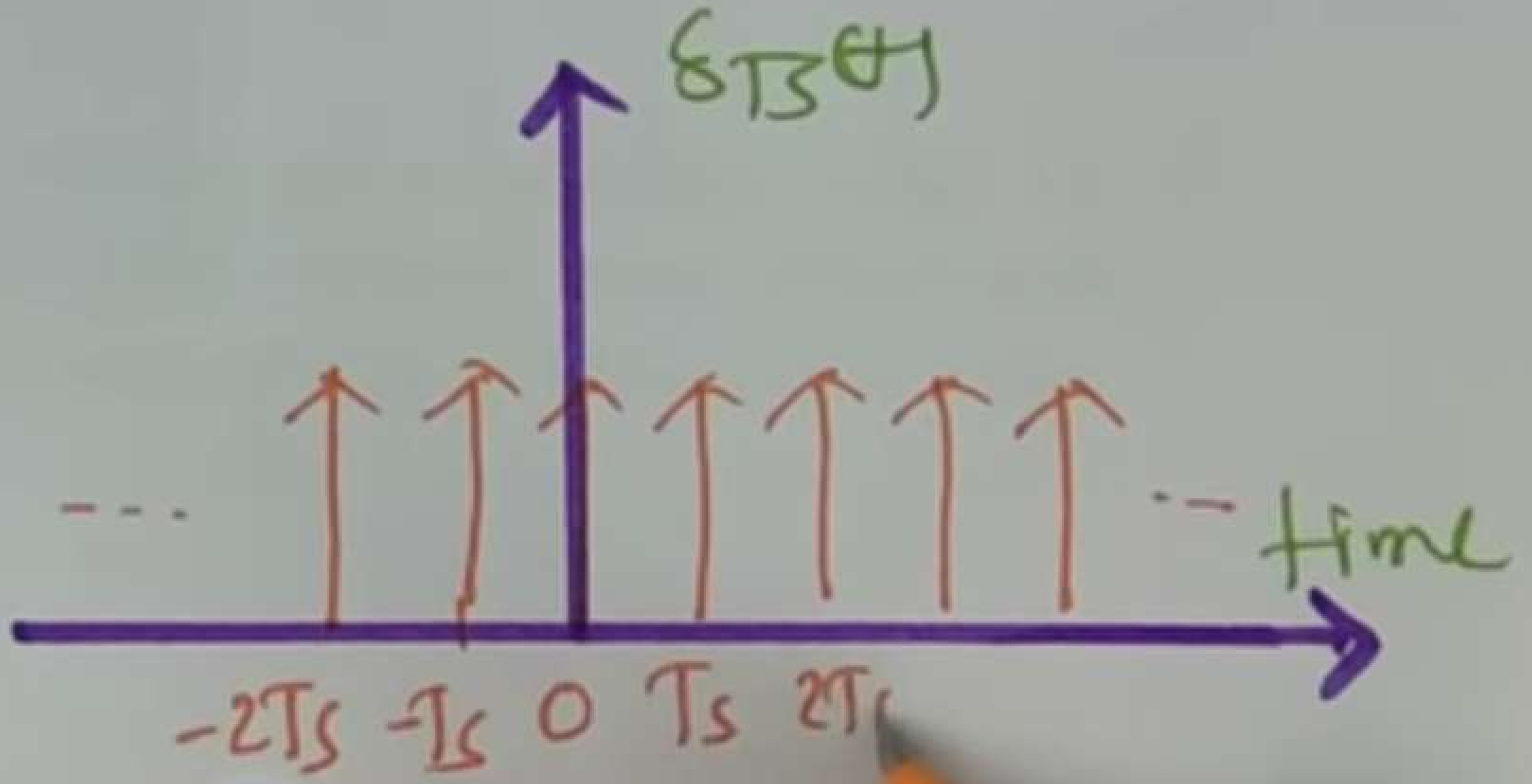
$$G(\omega) = \frac{\tau A}{T_s} \sum_{n}^{\infty} \sin\,(n f_s \tau)\ X\,(f - n f_s)$$

Flat - top Sampling [PAM]

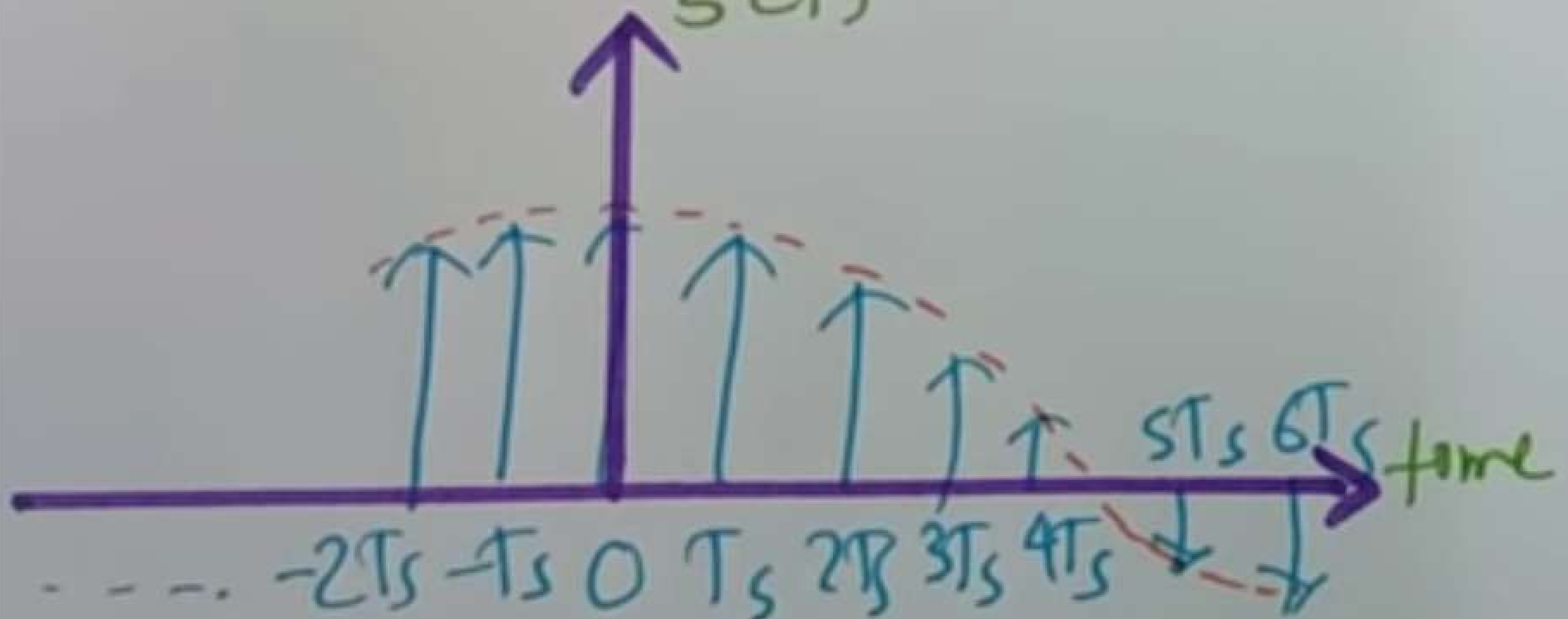
- It uses Sample and hold circuit
- It is practically possible like natural sampling but Flat top sampling is easier compared to natural sampling
- It has very high noise Interference.

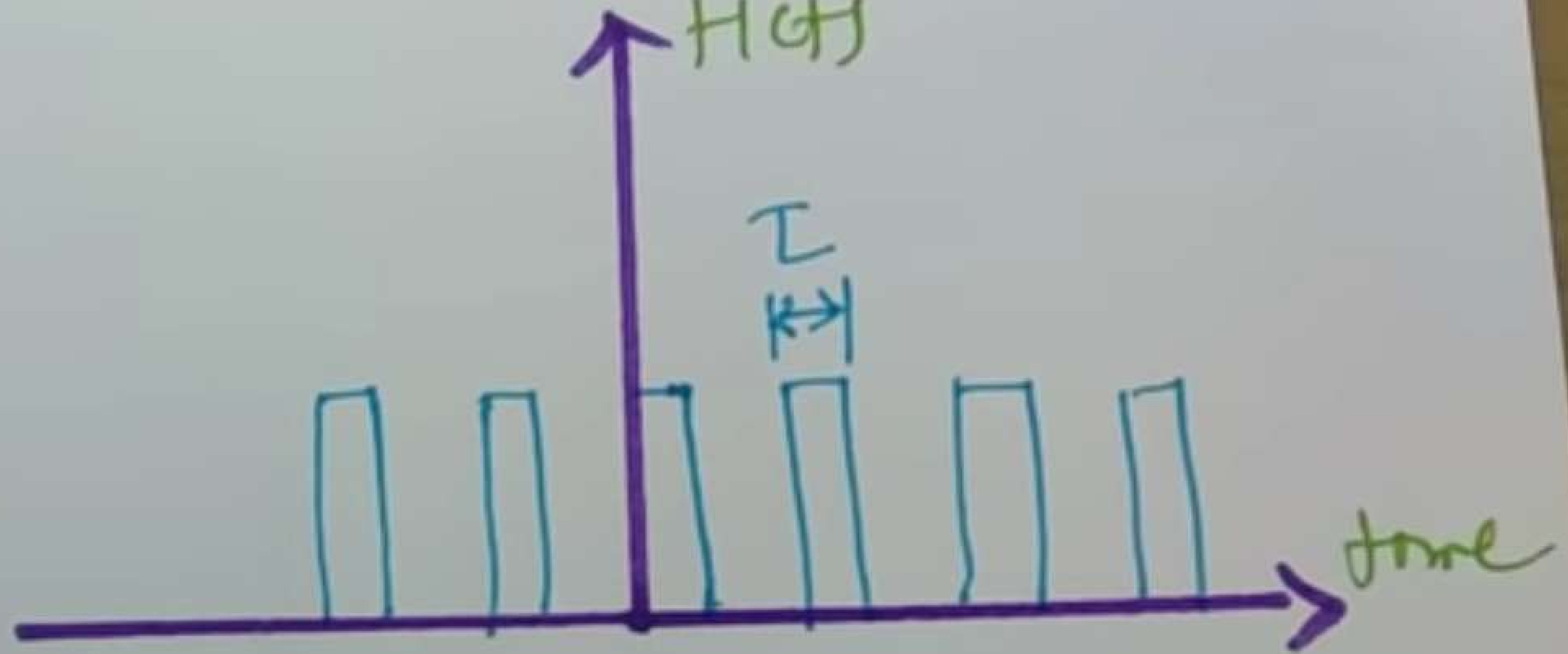

Sampling Switch
G1
x(t)
G2
C
y(t)
Discharge Switch

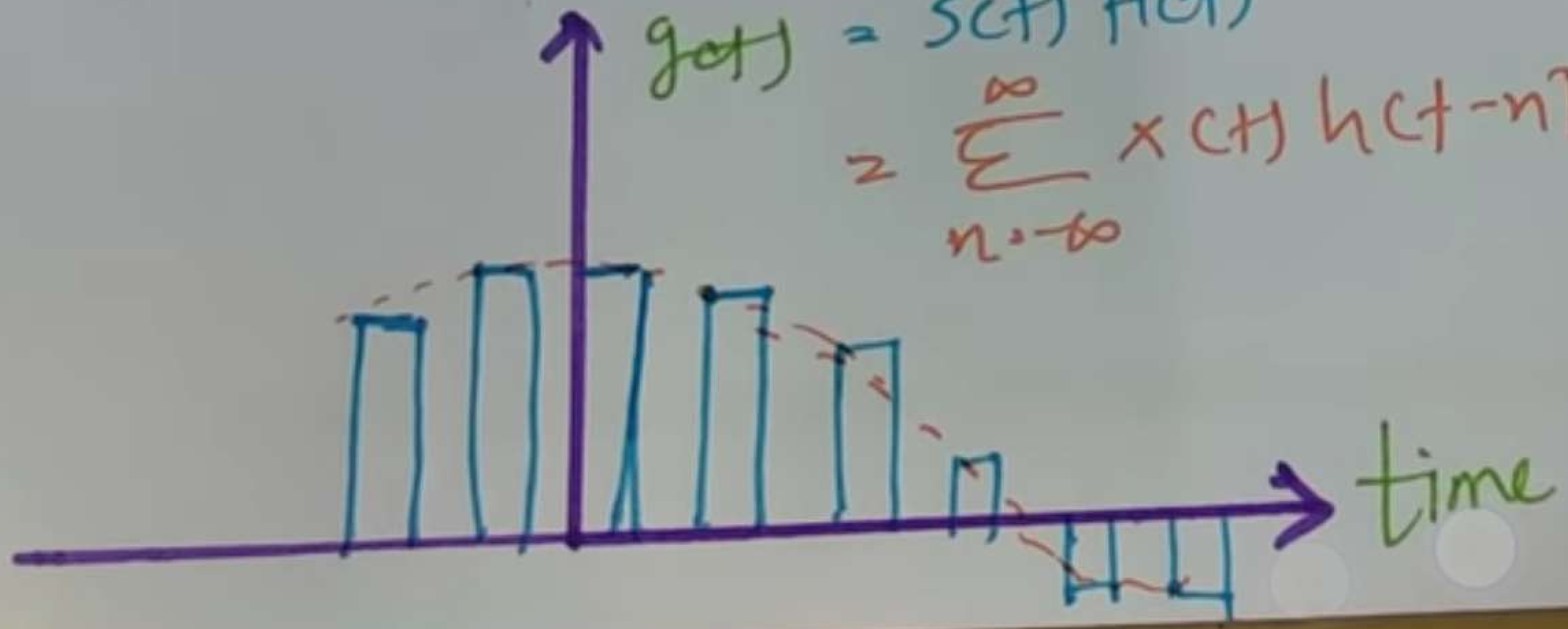

x(t)
time


δTs(t)
time
-2Ts
-Ts
0
Ts
2Ts

$s(t) = x(t)\,\delta_{T_s}(t)$







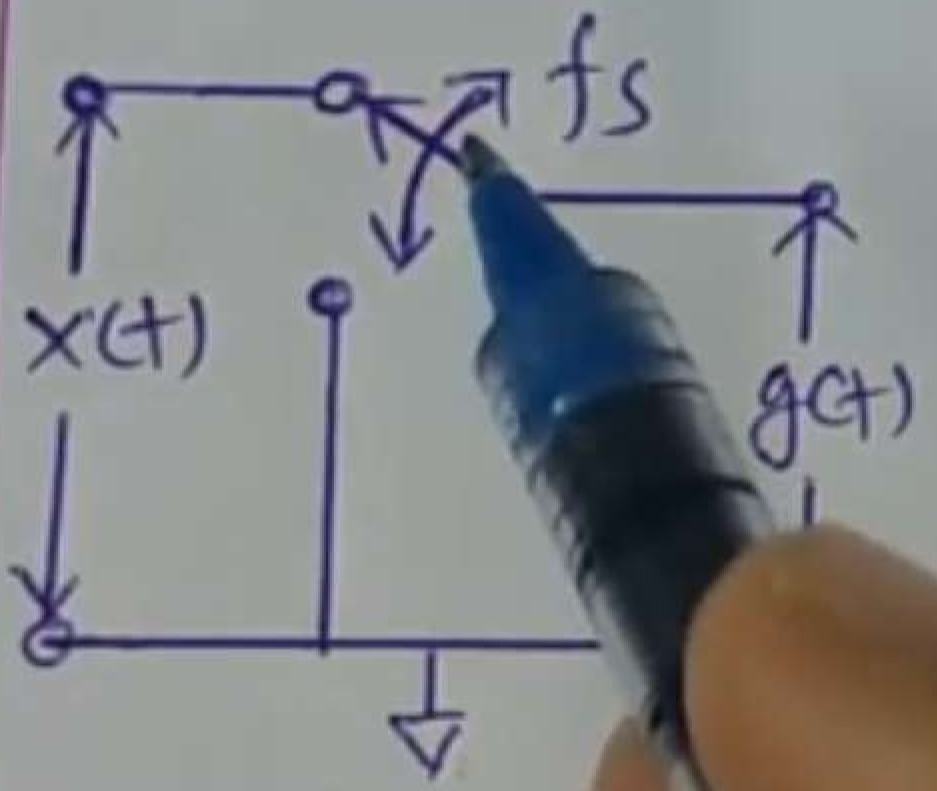
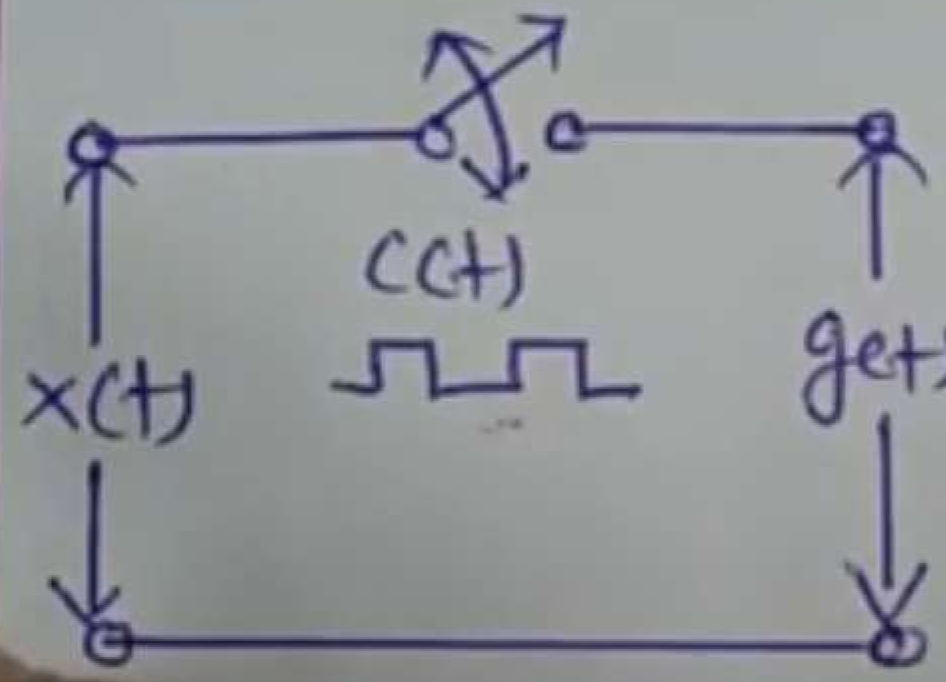
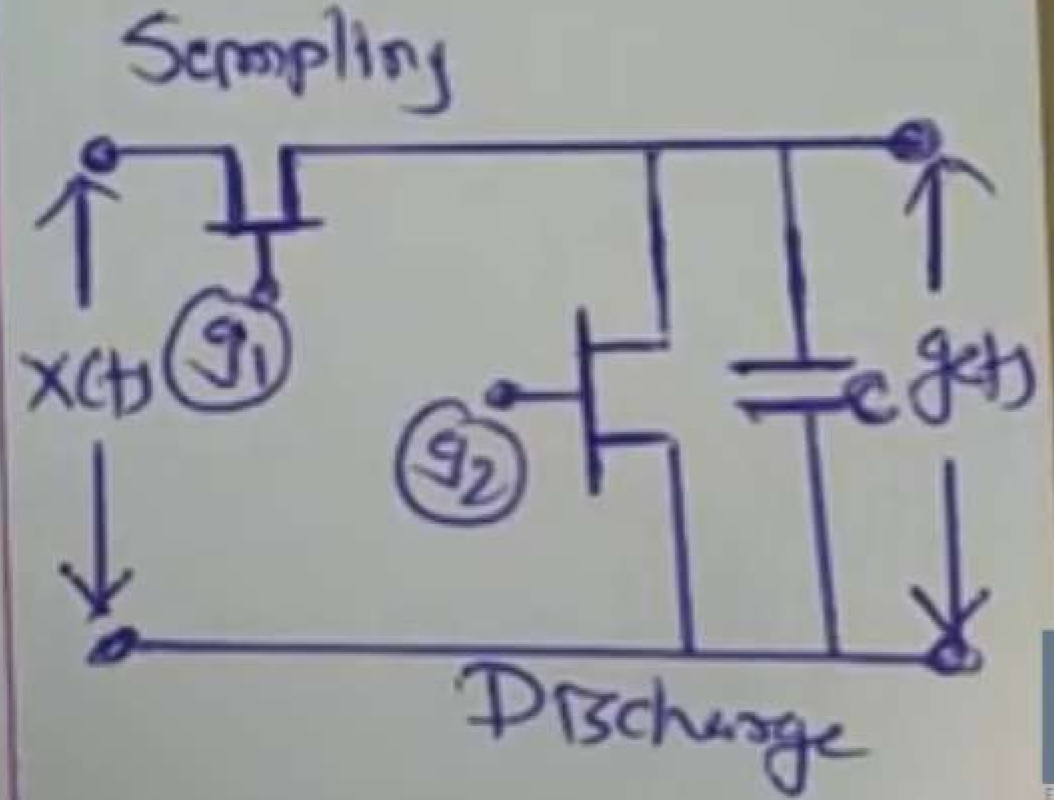
$g(t) = s(t)\,H(t)$

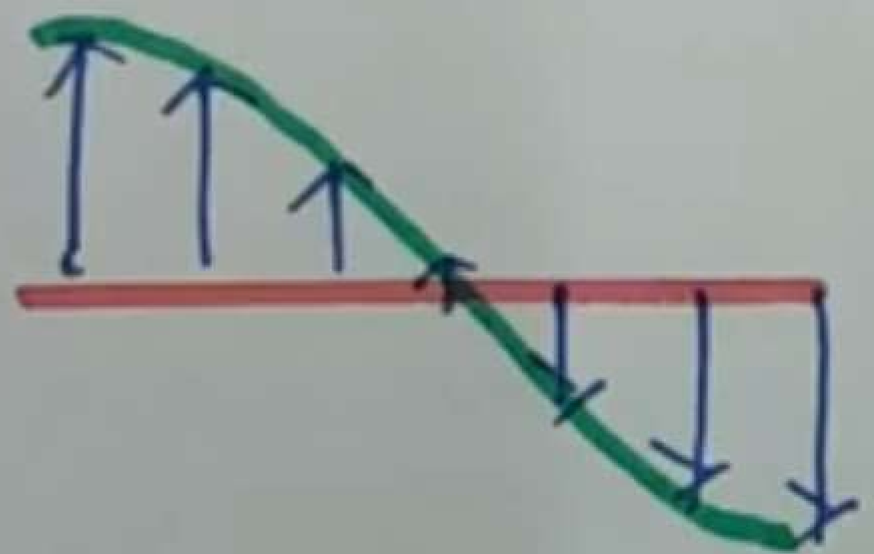
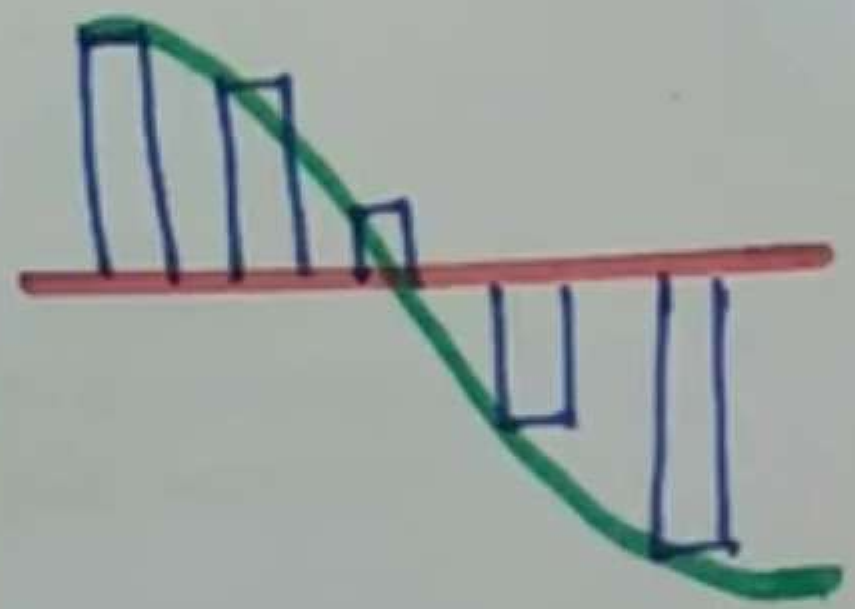
$= \sum_{n=-\infty}^{\infty} x(t)\,h(t-nT_s)$

→ freq. domain

$G(\omega) = f_s \sum_{n=-\infty}^{\infty} x(f-nf_s)\,H(f)$

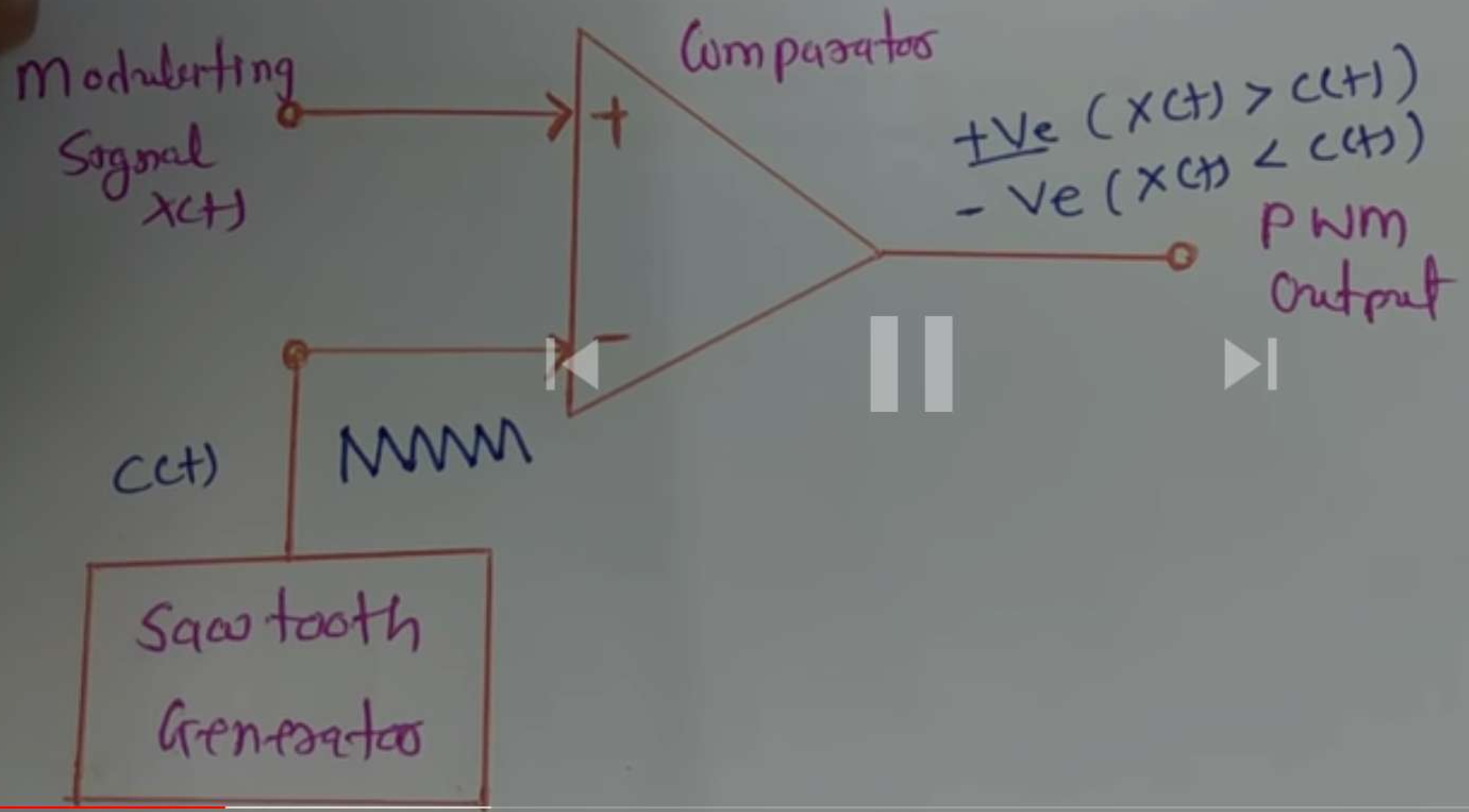
Performance comparison of Sampling techiques.

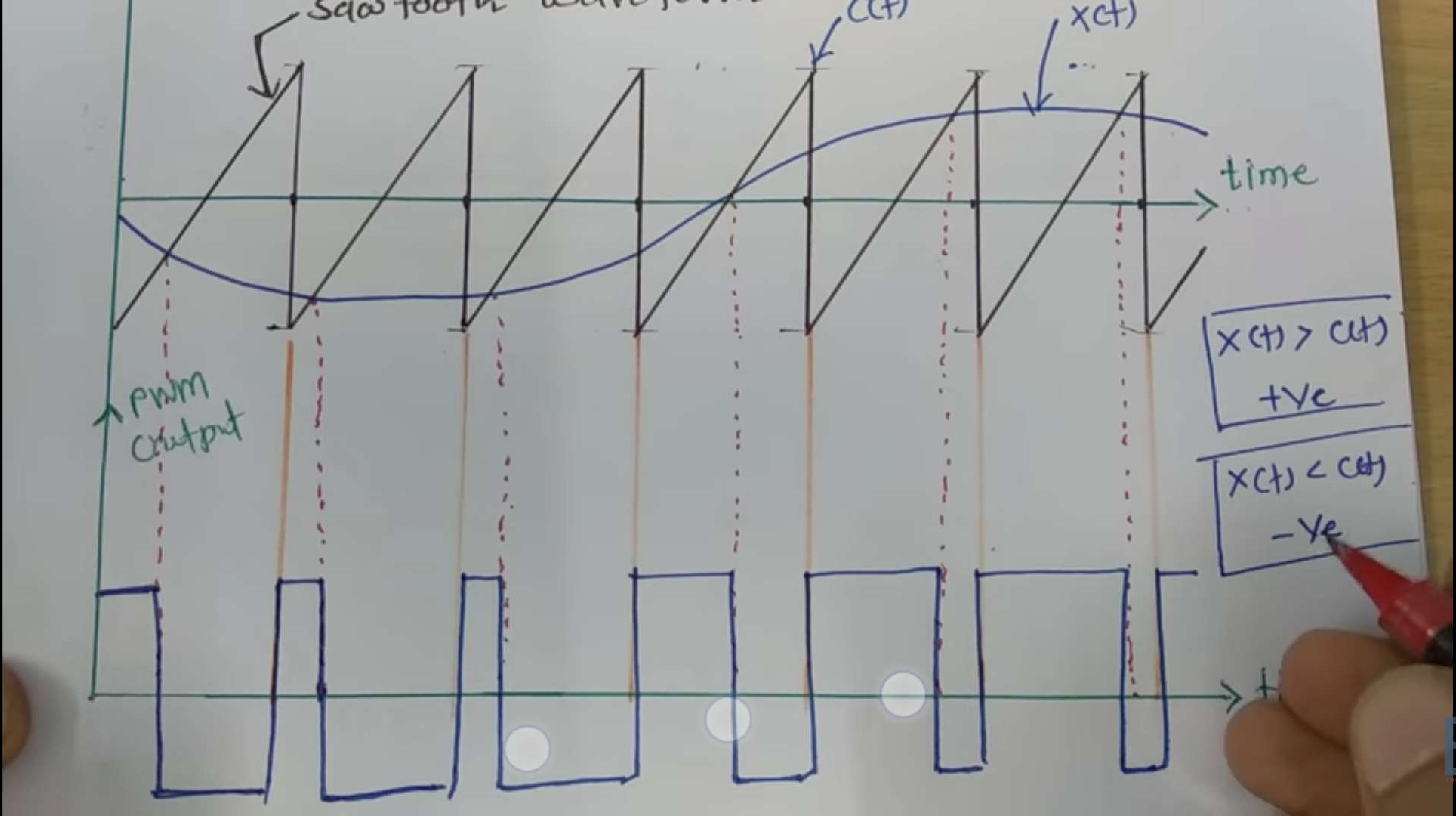
Performance Parameter	Ideal Sampling	Natural Sampling	Flat top Sampling
Sampling Principle	Multiplication	Chopping	Sample & Hold Circuit
Generation Circuit			

Waveforms			
Feasibility	Practically not possible	used Practically	used Practically.
Noise Interference	Very high	Less	high

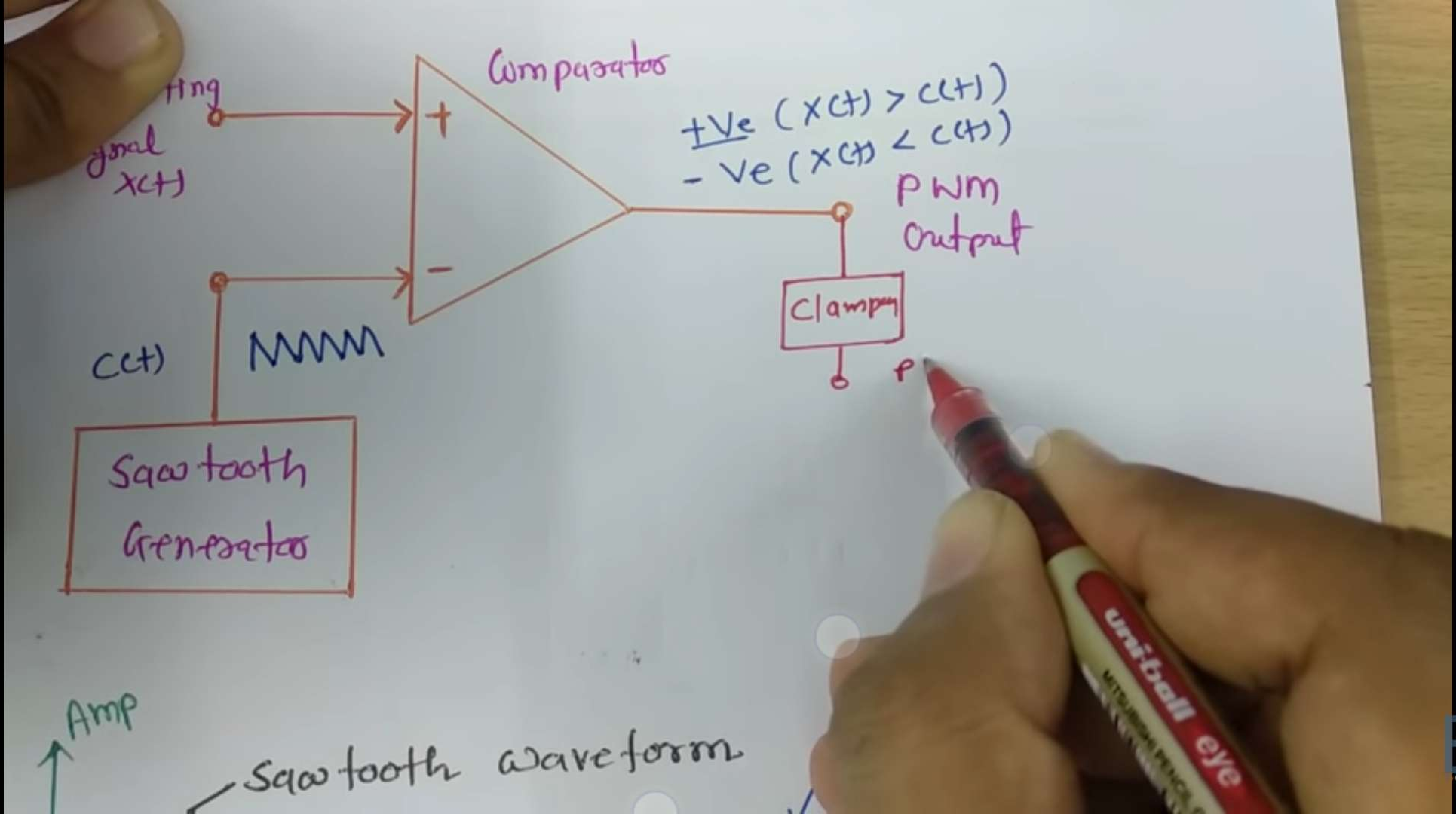
Feasibility	Practically not possible	used Practically	used Practically.
Noise Interference.	Very high	Less	high
Time domain Representation	$g(t) = \sum_{n=-\infty}^{\infty} x(t)\, \delta(t-nT_s)$	$g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t)\, \mathrm{Sinc}(nf_s\tau)\, e^{j2\pi nf_s t}$	$g(t) = \sum_{n=-\infty}^{\infty} x(t)\, h(t-nT_s)$
Freq. domain Representation	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)$	$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \mathrm{Sinc}(nf_s\tau)\, X(f-nf_s)$	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)\, H(f)$

PWM (Pulse Width Modulation)

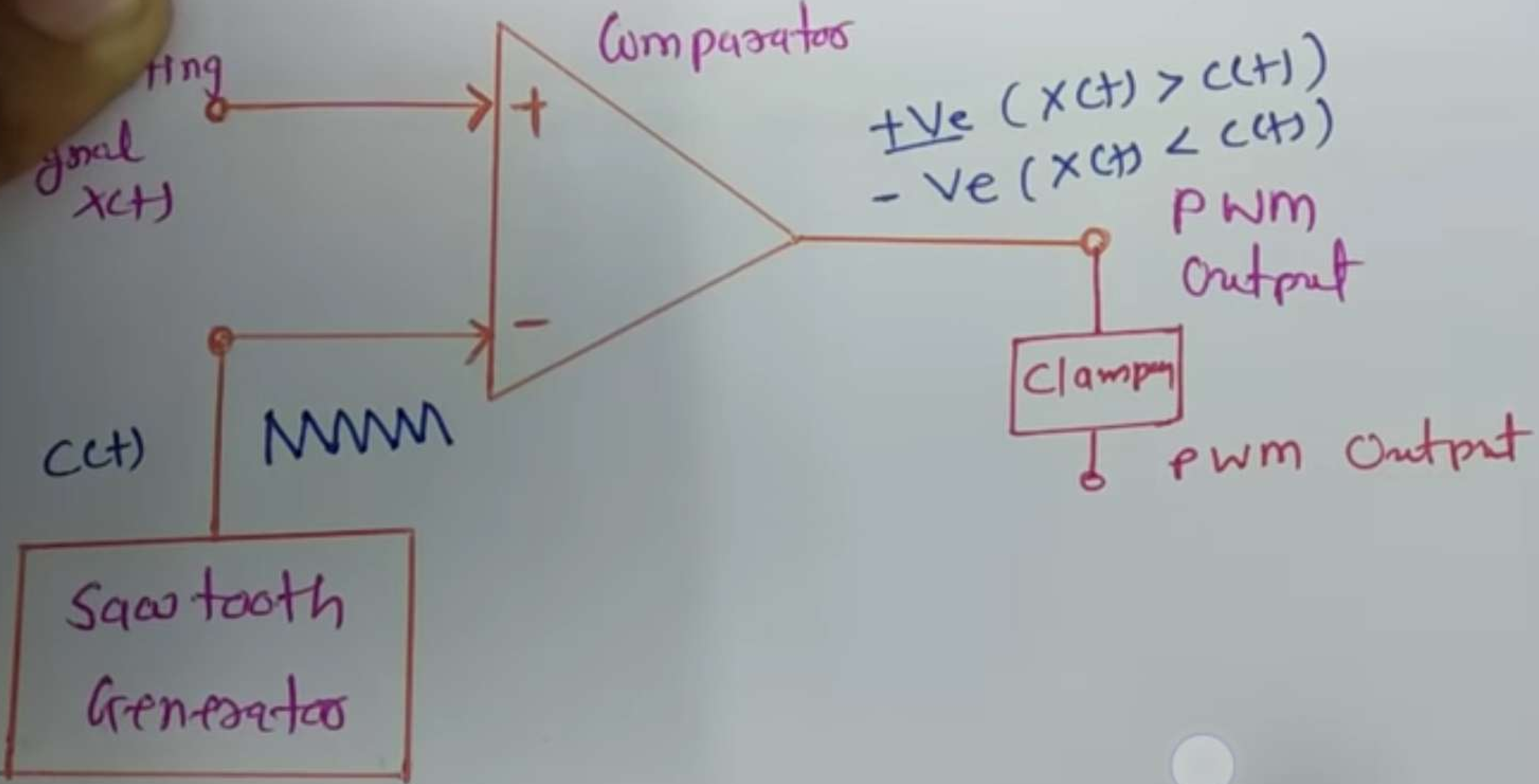


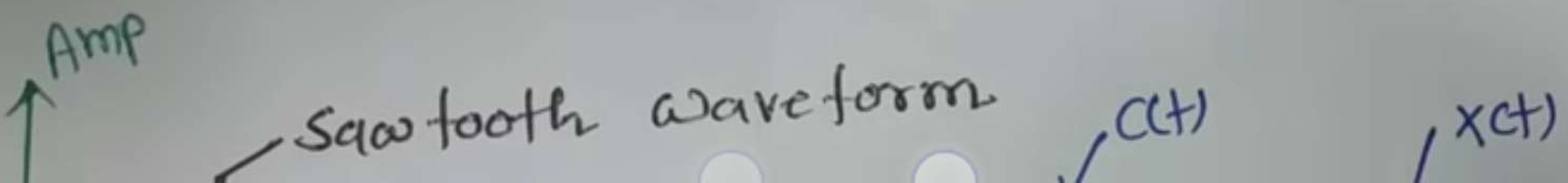


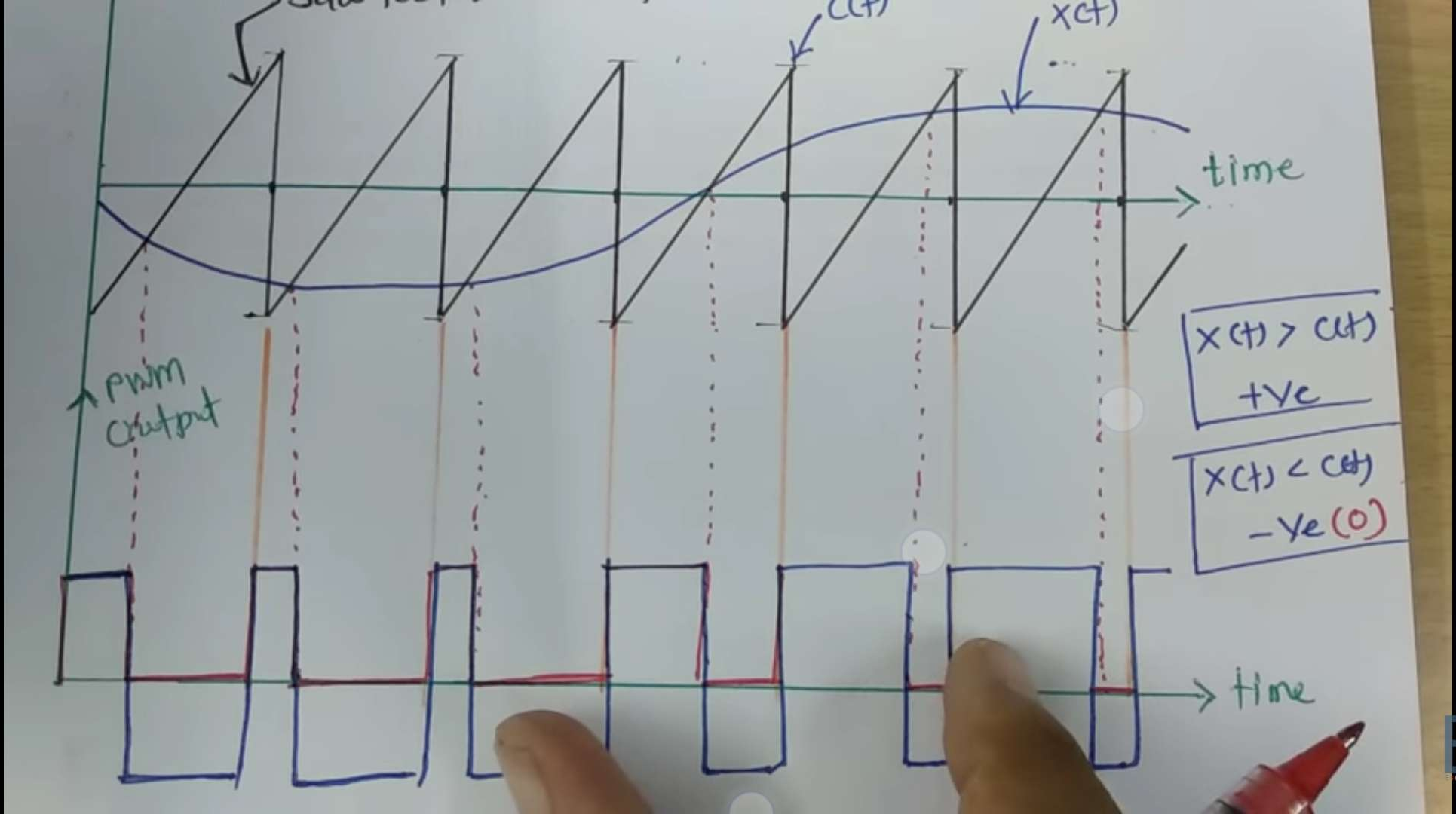
c(t)
x(t)
time
pwm output
x(t) > c(t)
+ve
x(t) < c(t)
-ve



Comparator
+
-
x(t)
+Ve (x(t) > c(t))
-Ve (x(t) < c(t))
PWM Output
Clamper
c(t)
Sawtooth Generator
Amp
Sawtooth waveform

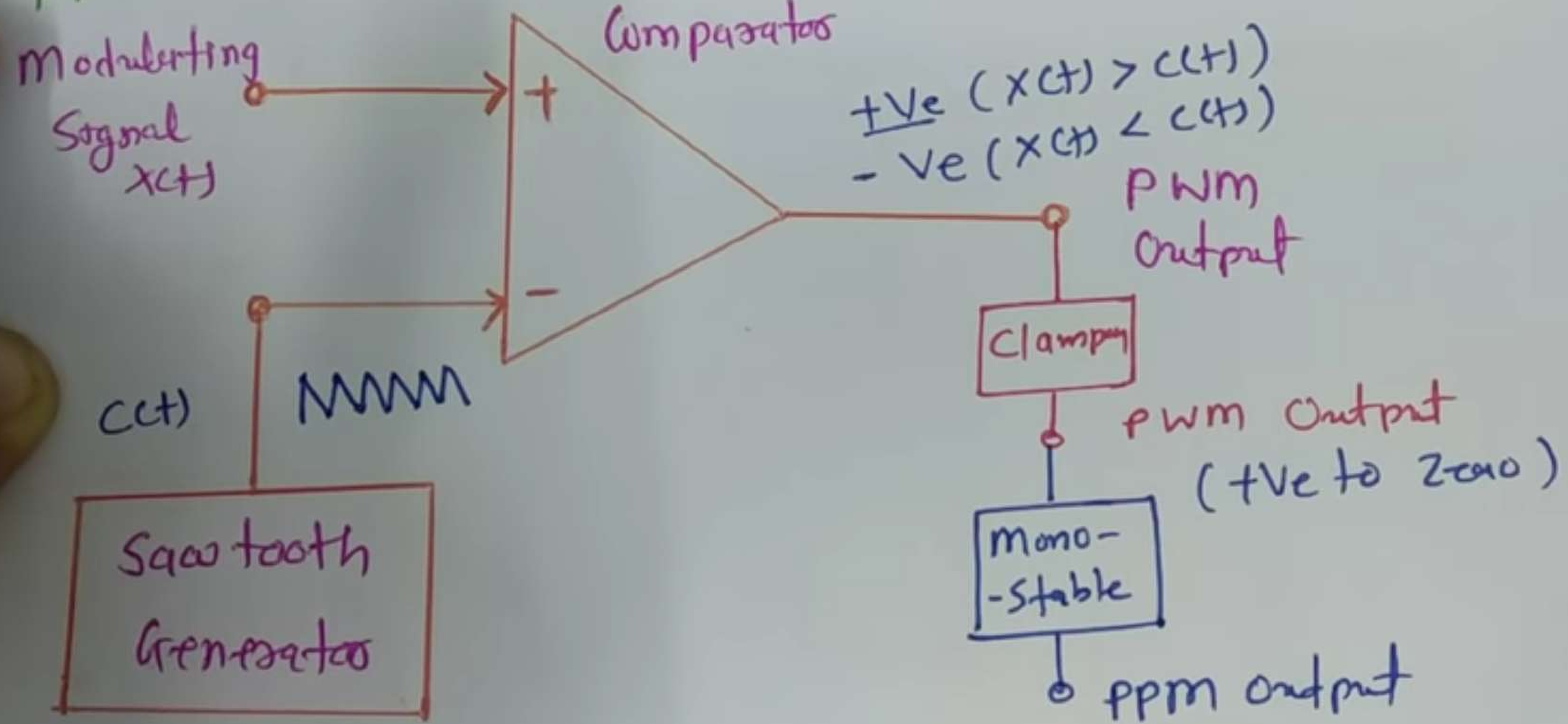

x(t)
Comparator
+
−
+Ve (x(t) > c(t))
− Ve (x(t) < c(t))
PWM Output
Clamper
PWM Output
c(t)
Sawtooth Generator

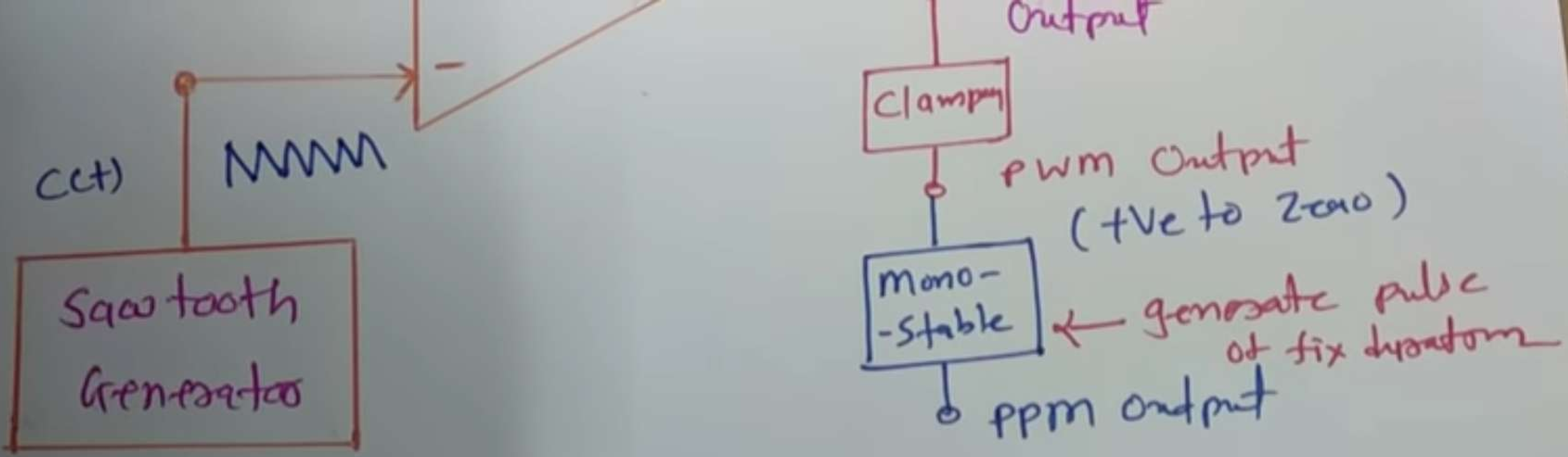

Amp
Sawtooth waveform
c(t)
x(t)

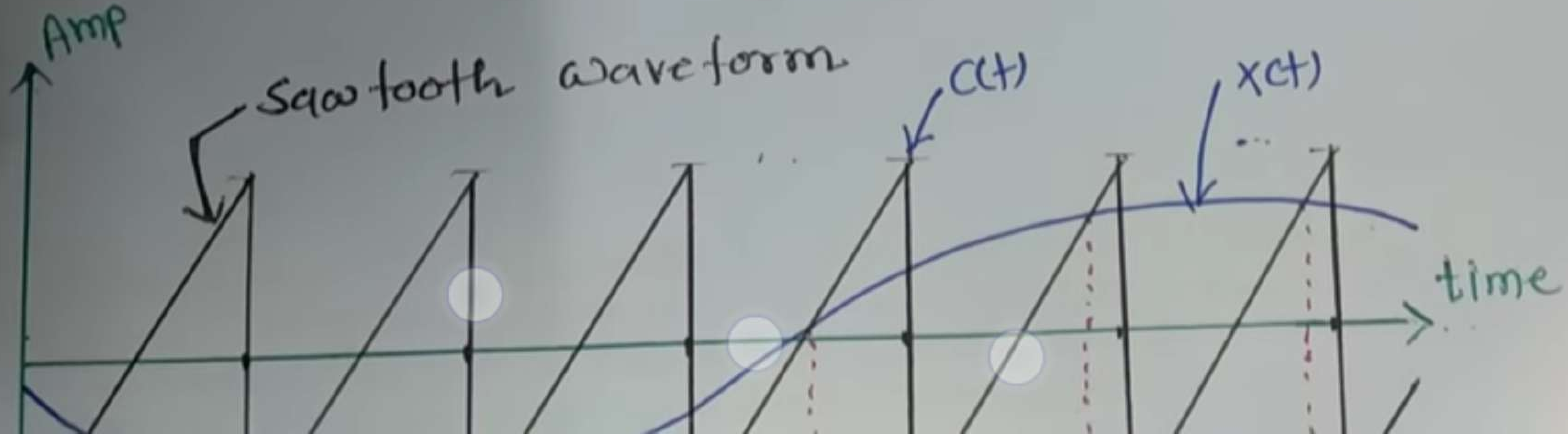

c(t)
x(t)
time
pwm output
x(t) > c(t)
+Ve
x(t) < c(t)
-Ve (0)
time

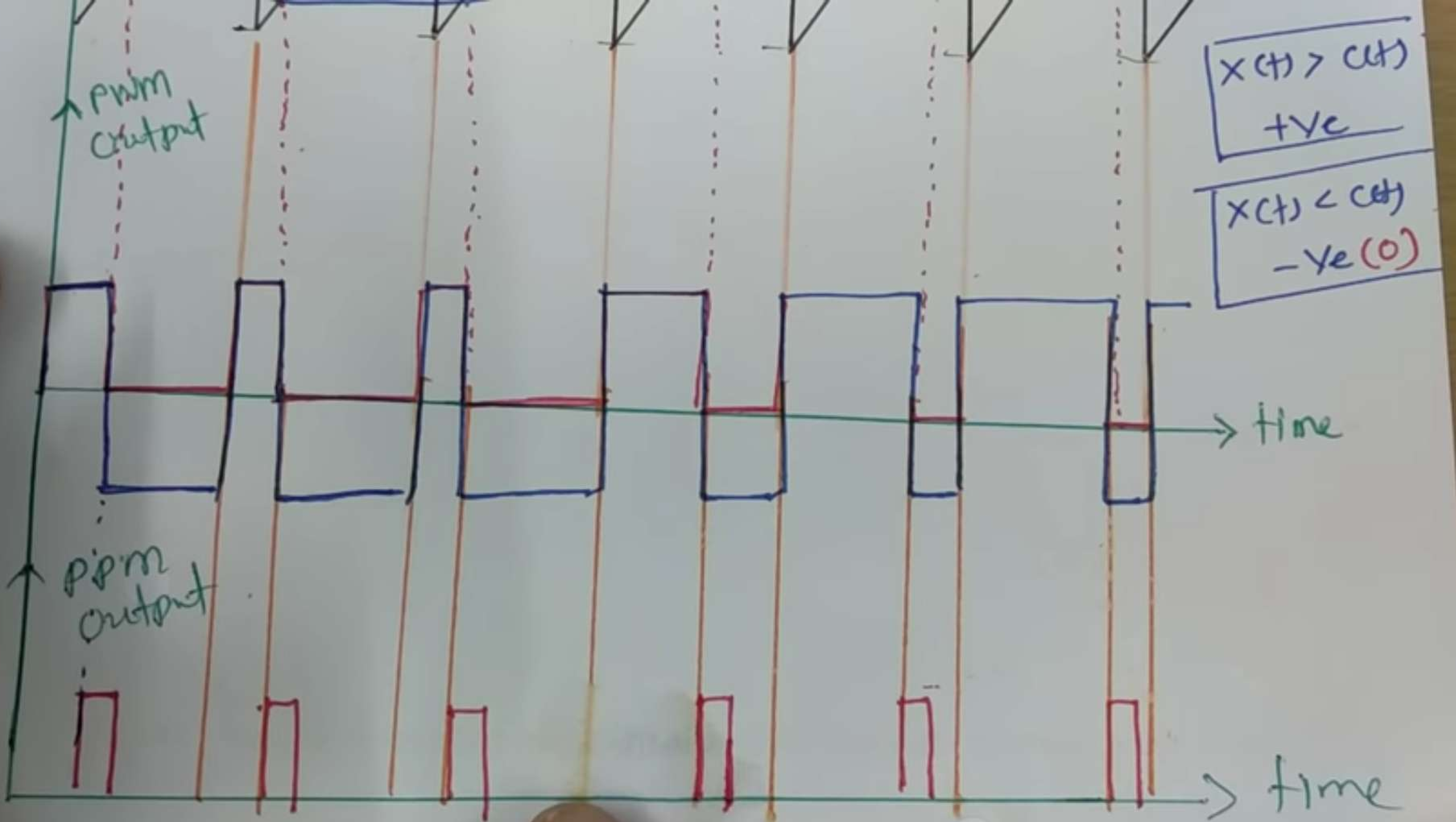
PWM (Pulse Width Modulation)

PPM (Pulse Position Modulation)



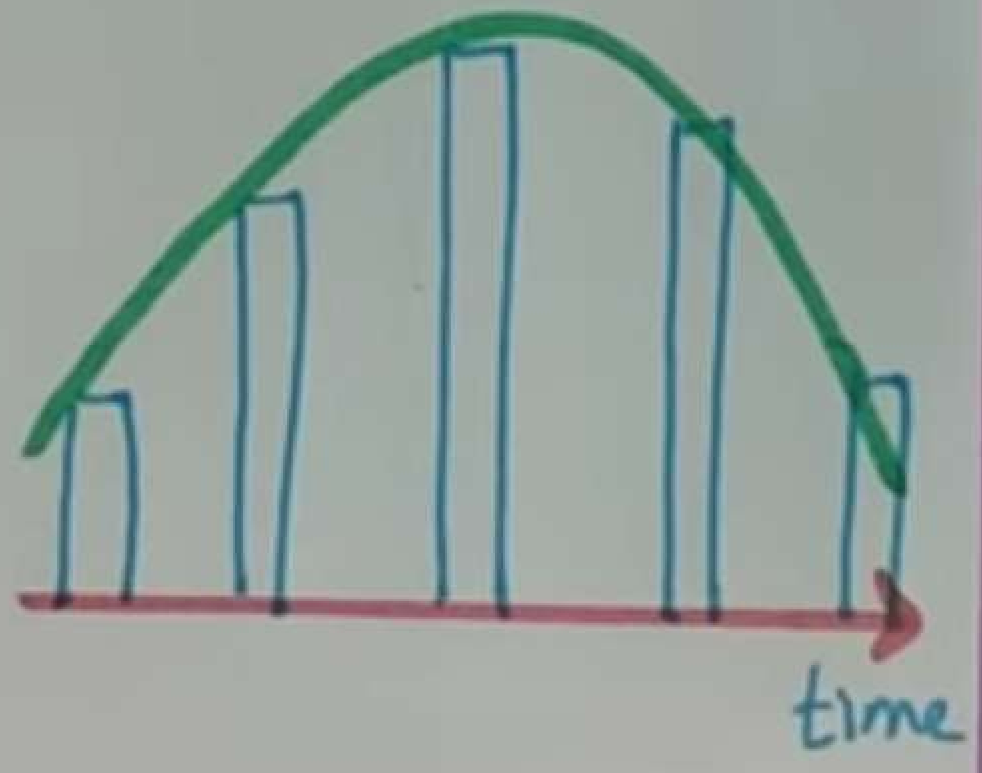
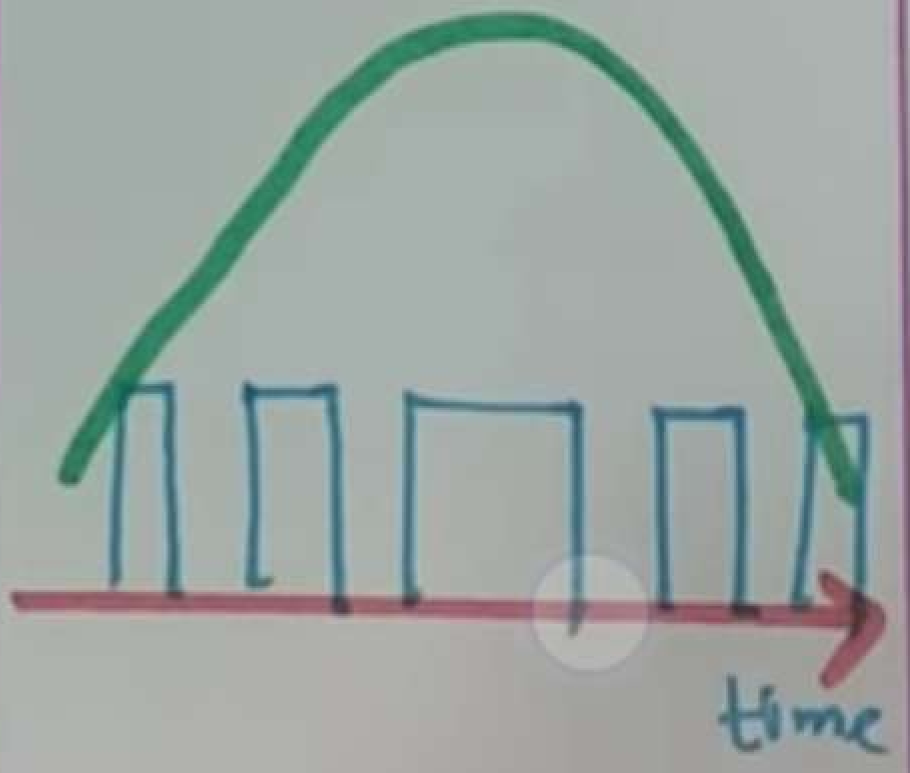
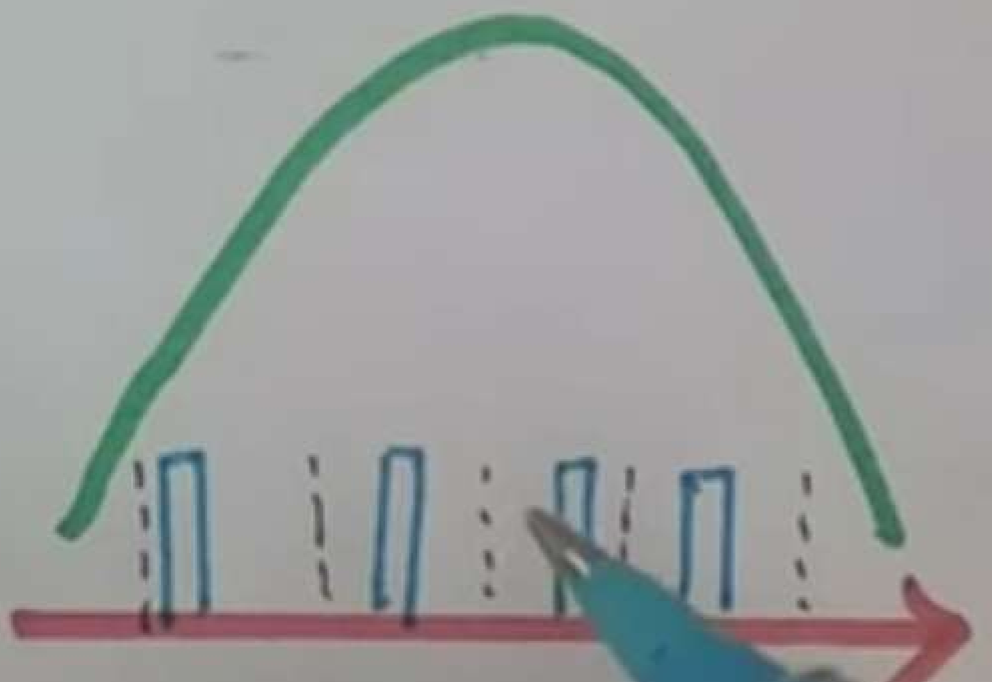

Output
Clamp
PWM Output
(+Ve to Zero)
c(t)
Mono-
-Stable
generate pulse
of fix duration
Sawtooth
Generator
PPM Output


Amp
Sawtooth waveform
c(t)
x(t)
time

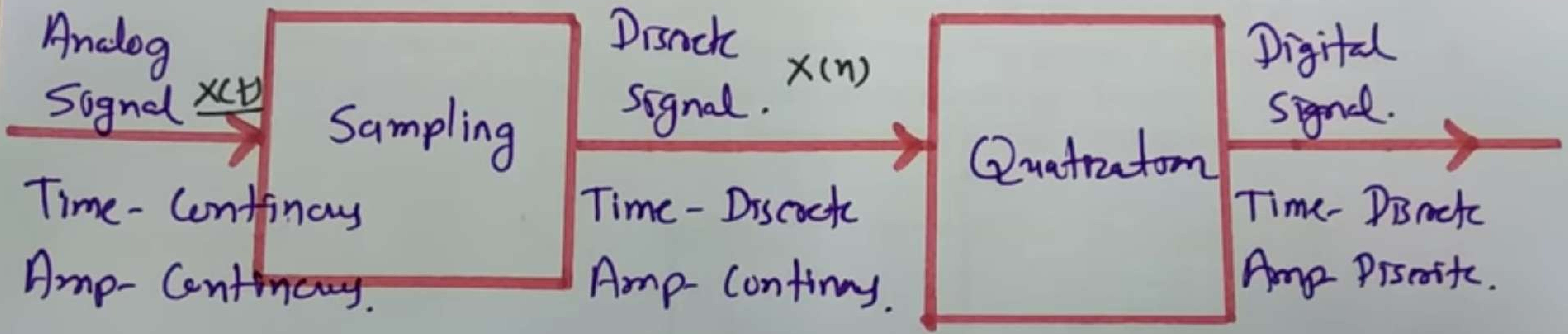


pwm output
ppm output
time
time
x(t) > c(t)
+ve
x(t) < c(t)
-ve (0)

Performance Comparison of Pulse analog Modulation

Performance Parameter	PAM	PWM	PPM
Waveform			

	time	time	time
Working Principle	Amplitude of Pulse Proportional to amp. of modulating Signal.	- Width of Pulse Is proportional to amplitude of modulating Signal	- Relative Position of Pulse is proportional to amp. of modulating signal
Bandwidth	- BW is depending on width of Pulse.	- BW depends on rise time of Pulse.	- BW depends on rise time of pulse.
Transmitted Power	- Varies w.r.t time	- Varies w.r.t time	- Constant.

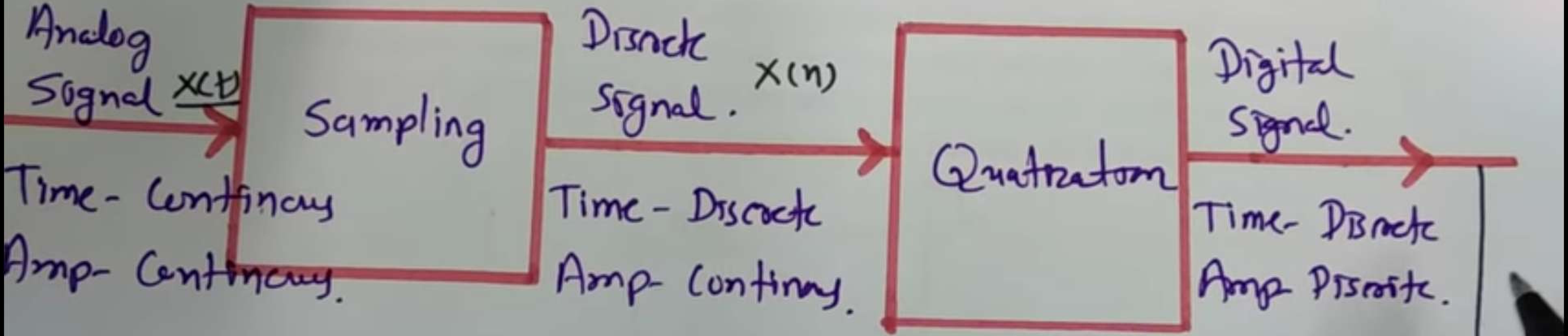


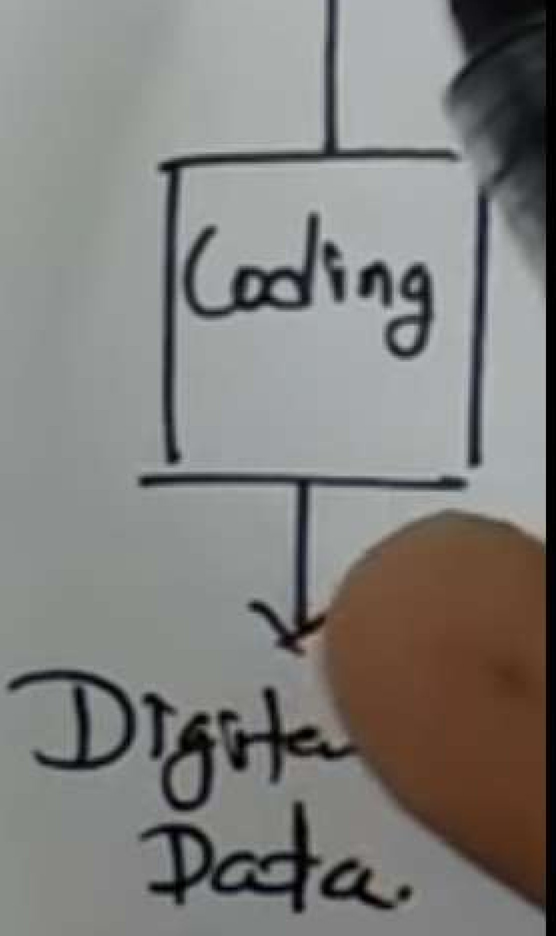
- Analog signal - $x(t) = \sin(2\pi f t)$

$t = 0.1,\ 0.11,\ 0.112,\ \ldots$

- Discrete signal - $x(n) = \sin(2\pi f n T_s)$

$n = 1, 2, 3, \ldots\ldots$





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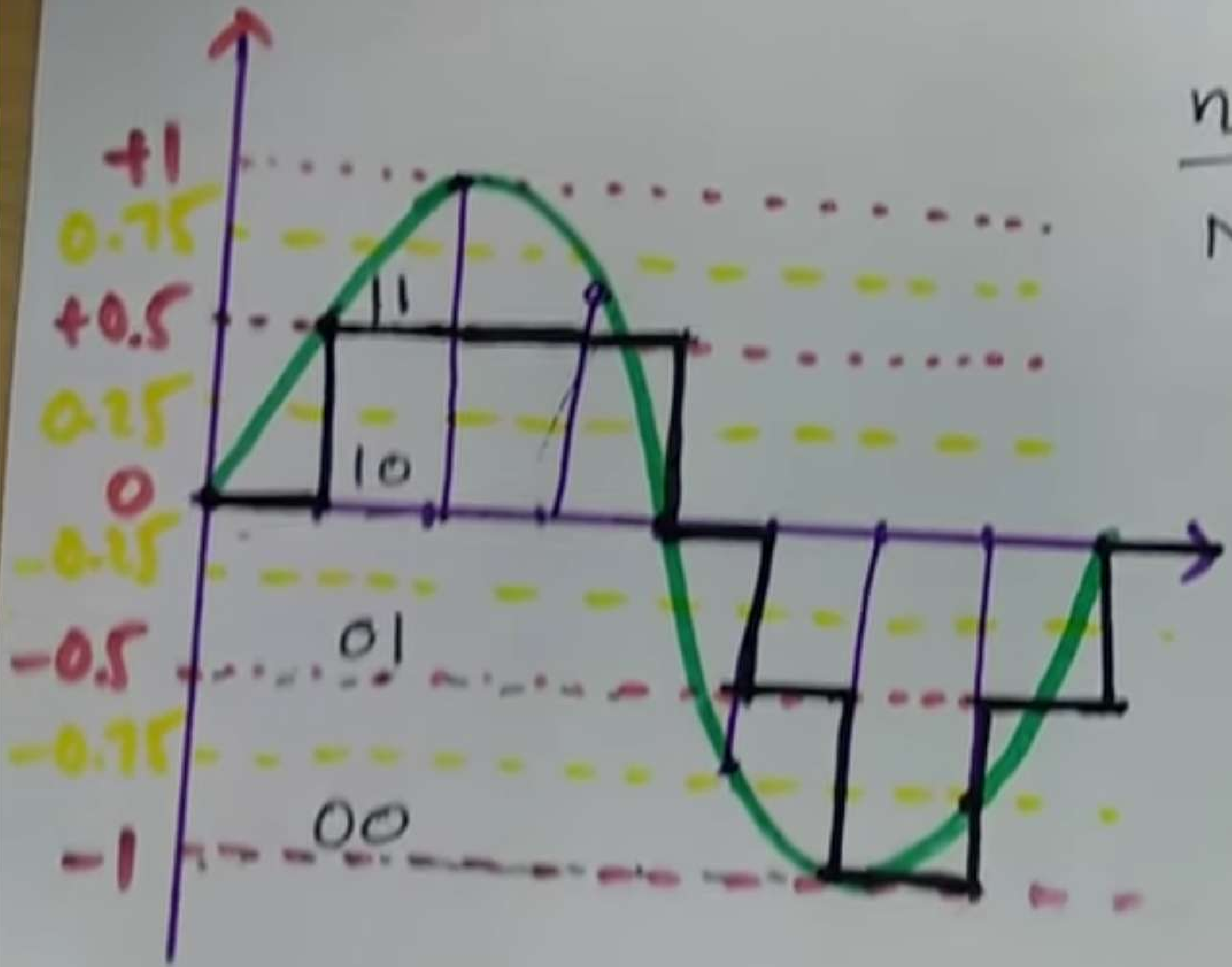
  $t = 0.1,\ 0.11,\ 0.112,\ \ldots$

- Discrete signal - $x(n) = \sin(2\pi f n T_s)$

  $n = 1, 2, 3, \ldots$

n = 1, 2, 3, ...

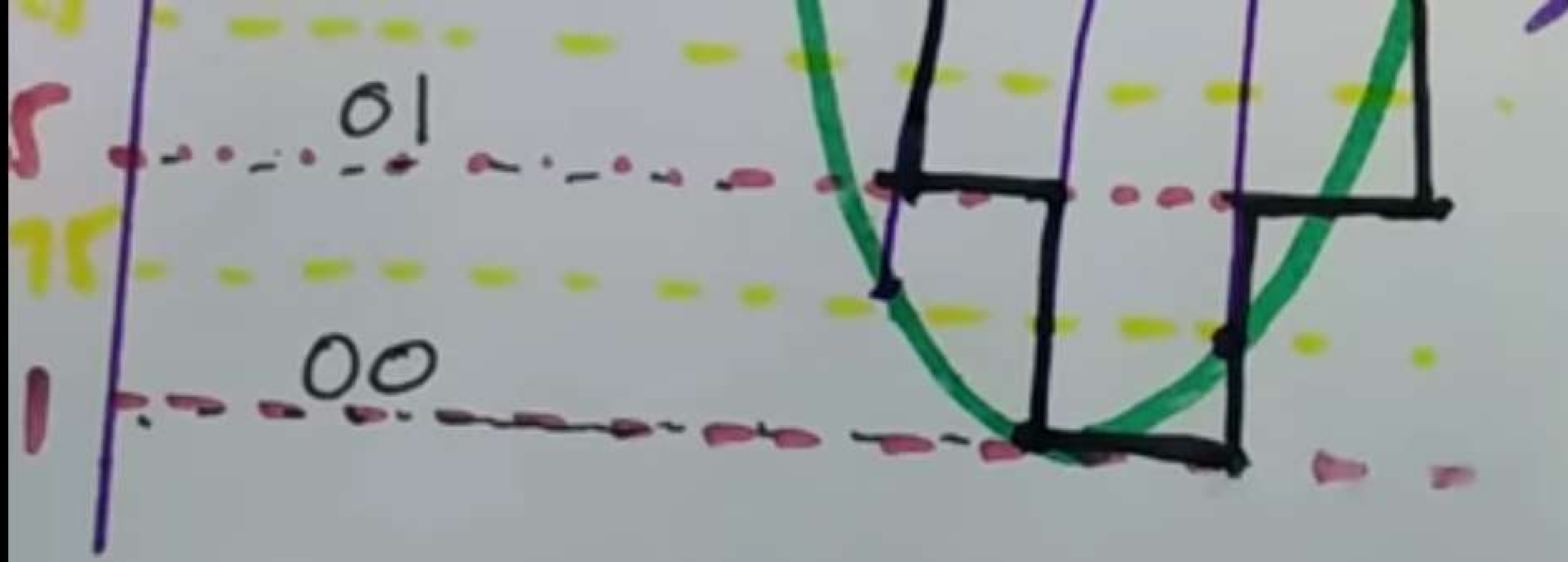
Digital Data



$n = 2$ bit

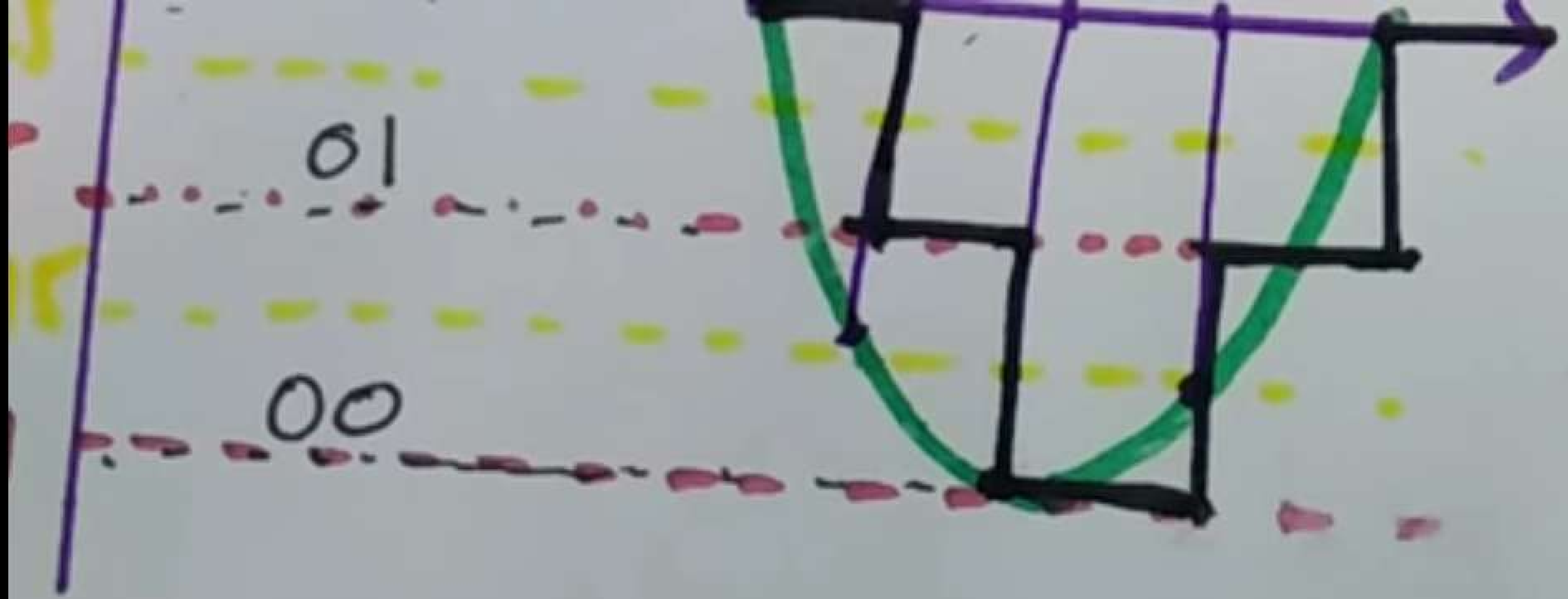
No of Quantize level $= 2^n$
$= 2^2$
$= 4$

$$\Delta = \frac{X_{max} - X_{min}}{L}$$



L

X (Sample Value)	0	0.707	1	0.707	0	-0.701	-1
X_q (Quantized Value)	0	0.5	0.5	0.5	0	-0.5	-1
Digital signal	10	11	11	11	10	01	00
Quantized error	0	0.207	0.5	0.207	0	-0.207	0



$$\Delta = \frac{x_{max} - x_{min}}{L}$$

$$e_q = x_q - x$$

x (Sample Value)	0	0.707	1	0.707	0	-0.701	-1
x_q (Quantized Value)	0	0.5	0.5	0.5	0	-0.5	-1
Digital signal	10	11	11	11	10	01	00
Quantized error	0	0.207	0.5	0.207	0	-0.207	0

Formulas of Quantization

- Mapping the continuous range of values into a finite se[t] of values.
- Changing the infinite precision to the finite precision
- Rounding off the samples to nearest quantization level.

→ Bit depth $= n$ = no of bits

→ No of Quantized Levels

$$L = 2^n$$

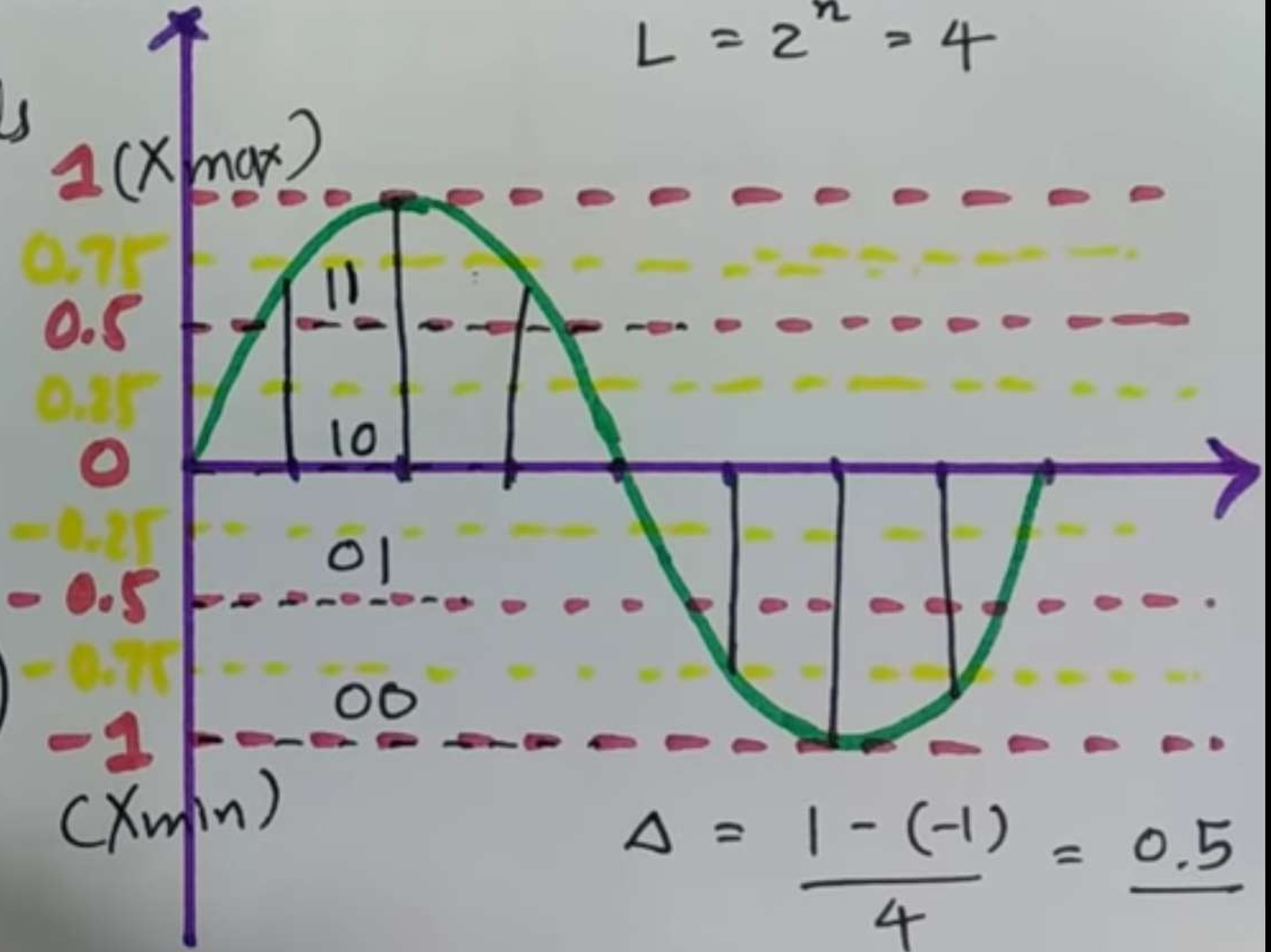
→ Step-size

$$\Delta = \frac{X_{max} - X_{min}}{L}$$

→ $I = \text{Round}\left(\frac{X - X_{min}}{\Delta}\right)$

$n = 2$

$L = 2^n = 4$



$$\Delta = \frac{1-(-1)}{4} = \underline{0.5}$$

→ Bit depth = n = no of bits

$n = 2$

$L = 2^n = 4$

→ No of Quantized Levels

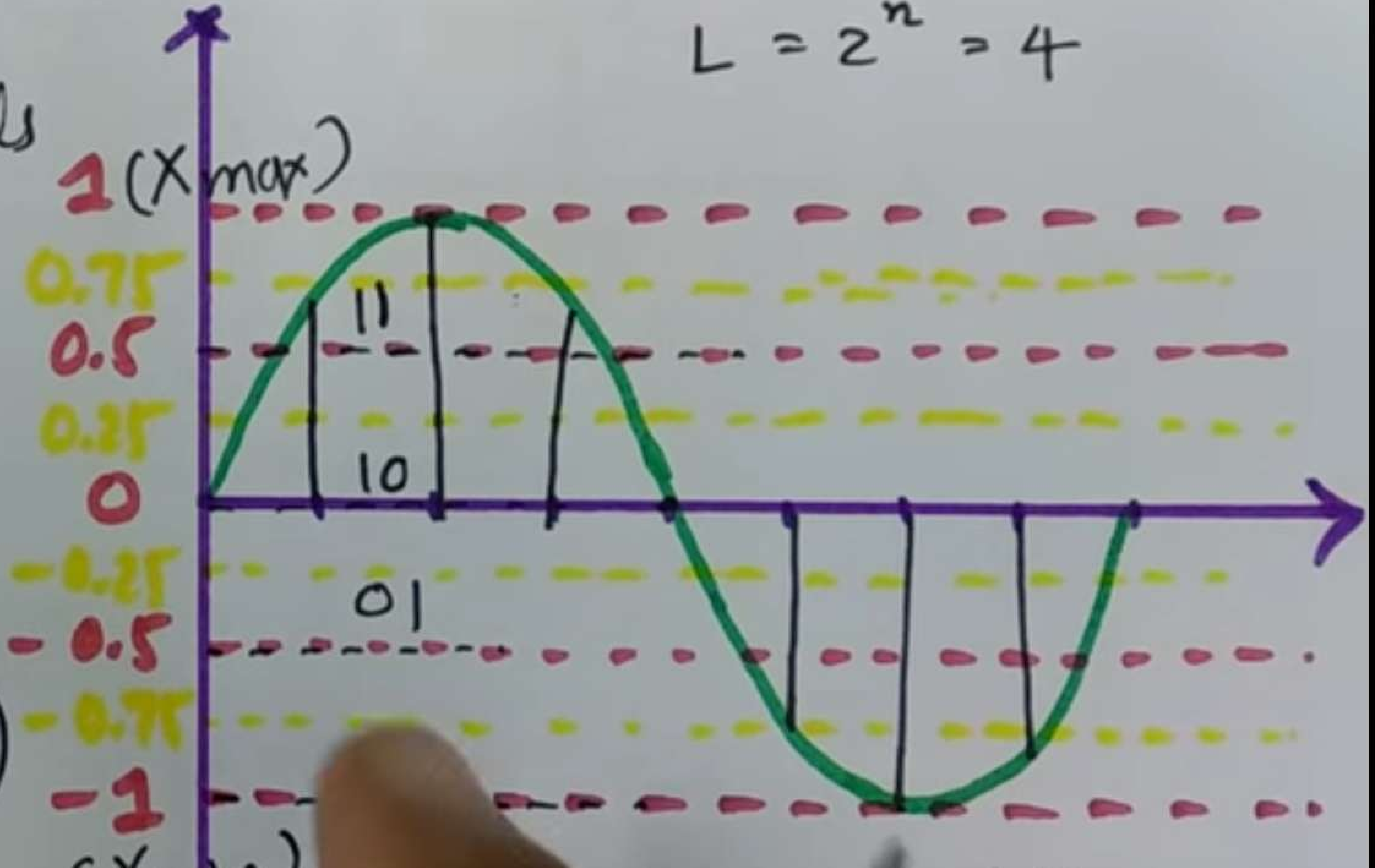
$L = 2^n$

→ Step-size

$$\Delta = \frac{X_{max} - X_{min}}{L}$$

→ $I = \text{Round}\left(\frac{X - X_{min}}{\Delta}\right)$

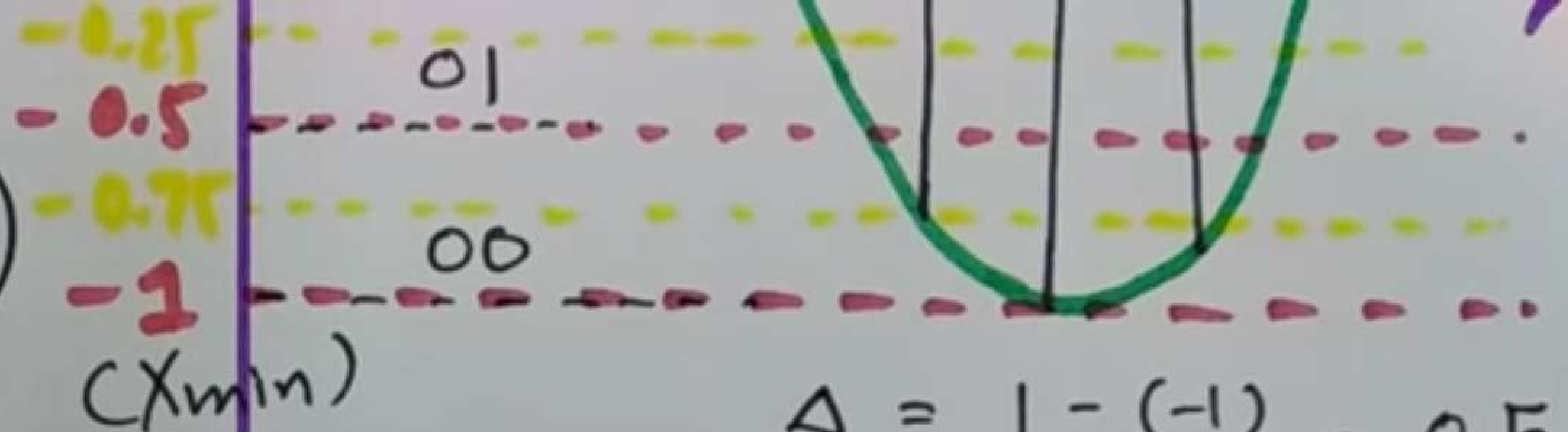
→ $X_q = X_{min} + I\Delta$



$$\Delta = \frac{\;\; - (-1)}{4} = \underline{0.5}$$

L

→ $I = \text{Round}\left(\frac{x - x_{min}}{\Delta}\right)$



→ $x_q = x_{min} + I\Delta$

$\Delta = \frac{1-(-1)}{4} = \underline{0.5}$

→ Example $\underline{x = 0.707}$

$$I = \text{Round}\left(\frac{x - x_{min}}{\Delta}\right) = \text{Round}\left(\frac{0.707-(-1)}{0.5}\right)$$

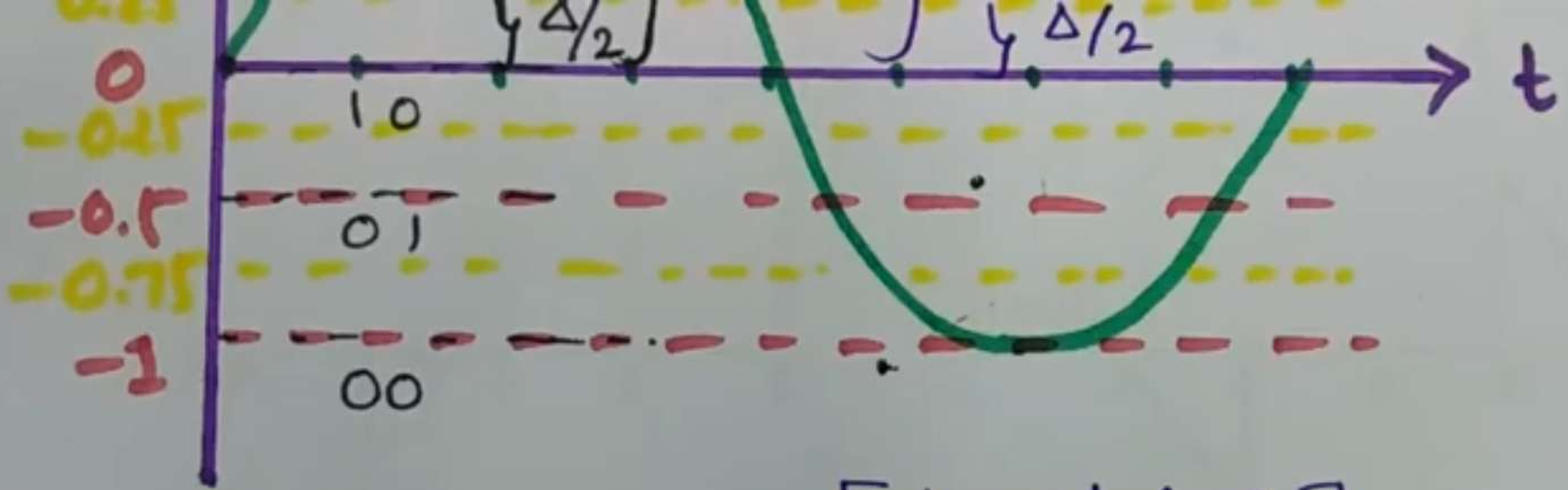
$$= \text{Round}(3.414) = 3$$

→ $x_q = x_{min} + I\Delta = -1 + 3(0.5) = 0.5$

$= \text{Round}(3.414) = 3$

$\rightarrow \quad x_q = X_{min} + I\Delta = -1 + 3(0.5) = 0.5$

x (Sample Value)	0	0.707	1	0.707	0	-0.707	–
I (Index value)	2	3	4	3	2	1	0
x_q (Quantized Value)	0	0.5	1	0.5	0	-0.5	–



→ Dynamic Range $= 20 \log \left[\dfrac{\text{Largest Amp}}{\text{Smallest Amp}} \right]$

$$= 20 \log \left[\frac{2^{n-1}}{2^{-1}} \right]$$

$$= 20 \log 2^n = 20 n \log 2$$

$$\boxed{\text{Dynamic Range} = 6.02\, n}$$

$$\rightarrow \text{SNR} = 20 \log \left[\frac{\text{Signal rms Voltage}}{\text{Noise rms Voltage}} \right]$$

$$\Rightarrow \overline{q_e^2} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 \, dq = \frac{1}{\Delta} \left[\frac{q^3}{3} \right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta 3} \left[\frac{\Delta^3}{8} - \left(-\frac{\Delta^3}{8} \right) \right]$$

$$= \frac{\Delta^2}{12}$$

$$\Rightarrow q_e = \frac{\Delta}{\ }$$

$\rightarrow$ SNR = 20 log $\left[\frac{\text{Signal rms Voltage}}{\text{Noise rms Voltage}} \right]$

$$\Rightarrow \overline{q_e^2} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 \, dq = \frac{1}{\Delta} \left[\frac{q^3}{3} \right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta 3} \left[\frac{\Delta^3}{8} - \left(-\frac{\Delta^3}{8} \right) \right]$$

$$= \frac{\Delta^2}{12}$$

$$\Rightarrow \boxed{q_e = \frac{\Delta}{2\sqrt{3}}}$$

$$\Delta = \frac{V_{max}}{2^n} = \frac{\dots}{2^n} \Rightarrow \sqrt{3}$$

- $\boxed{\text{Signal rms voltage} = \frac{2^n \Delta}{2\sqrt{2}}}$ — ①

- $$SNR = 20 \log \left(\frac{2^n \cancel{\Delta}}{\cancel{2}\sqrt{2}} \Big/ \frac{\cancel{\Delta}}{\cancel{2}\sqrt{3}} \right)$$

$$= 20 \log \left(2^n \sqrt{\frac{3}{2}} \right)$$

$$= 20 \log 2^n + 20 \log \sqrt{3/2}$$

$$\boxed{SNR = 6.02n + 1.76}$$

Basics of Pulse Code Modulation

- Pulse Code Modulation is used to convert Analog signal into Digital data.
- In PCM, 1^{st} we do sampling to convert analog signal into discrete signal.
- After that, we do quantization to convert discrete signal into digital signal.

Basics of Pulse Code Modulation

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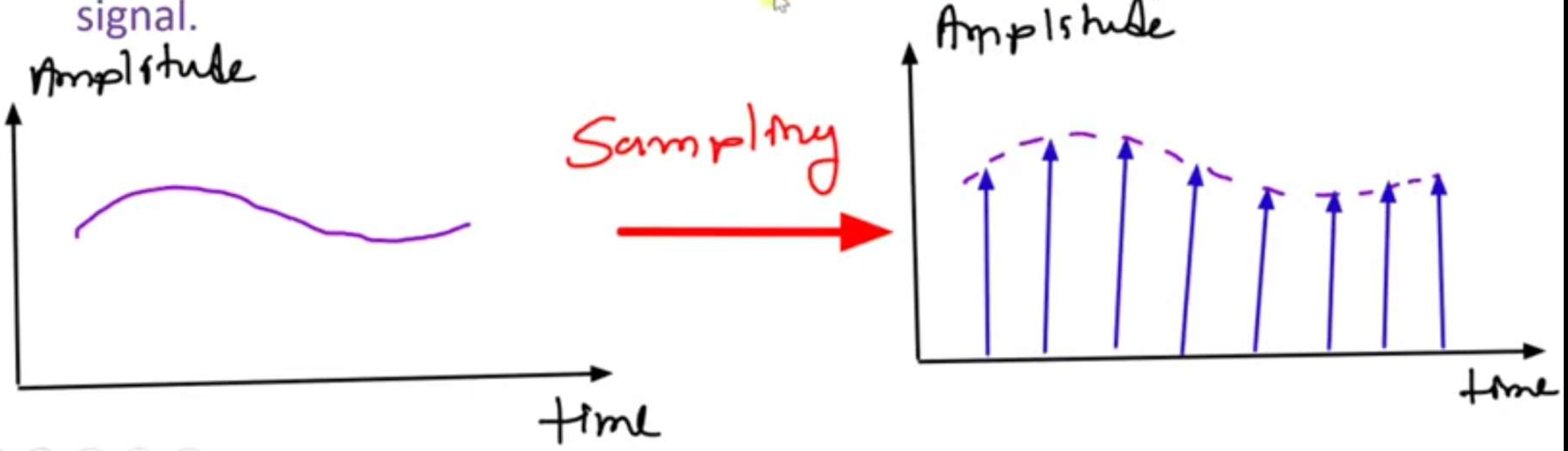
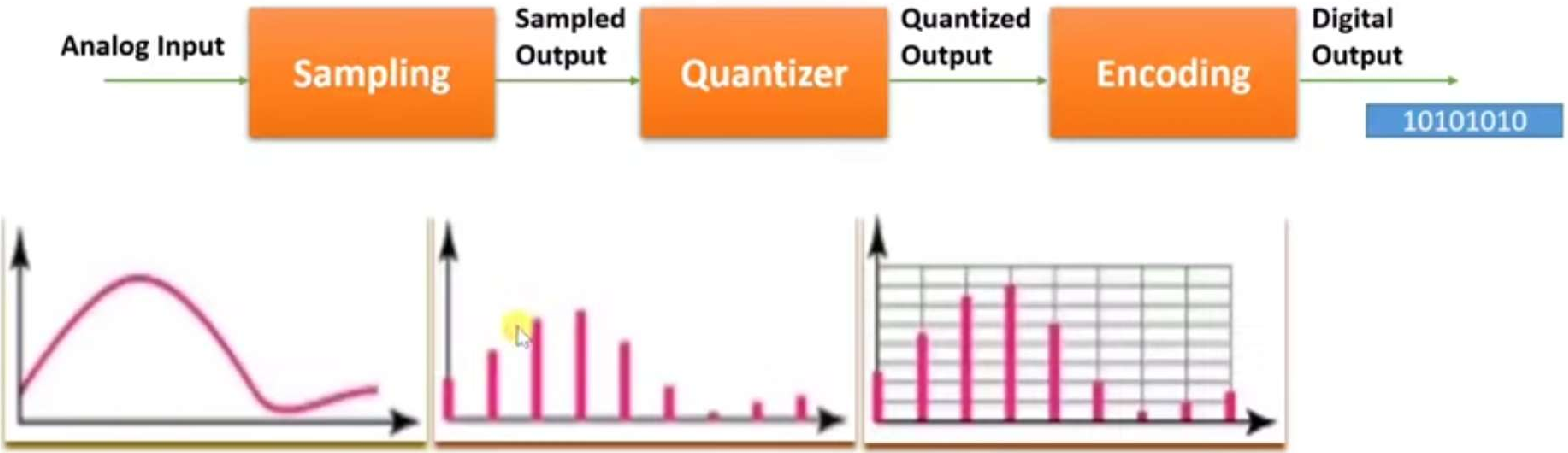


Basics of Pulse Code Modulation

- Pulse Code Modulation is used to convert Analog signal into Digital data.
- In PCM, 1st we do sampling to convert analog signal into discrete signal.
- After that, we do quantization to convert discrete signal into digital signal.
- After that, we do encoding of that digital signal.

Block Diagram of Pulse Code Modulation



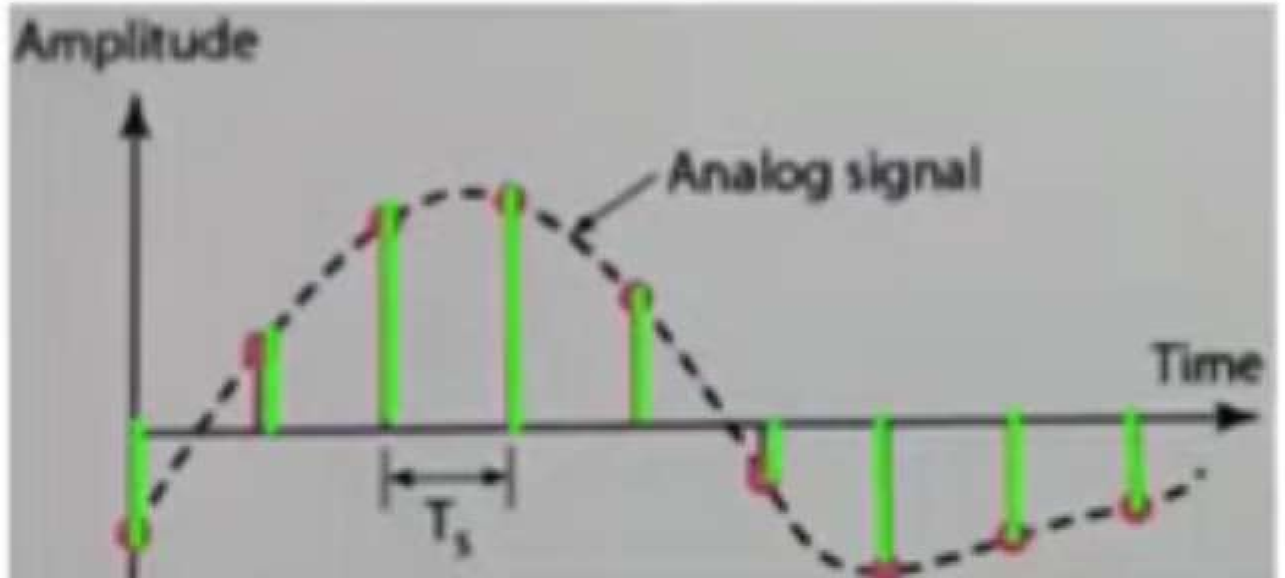
Note : If high frequency components are there at Analog input, then we should use LPF at input before we give signal to sampling

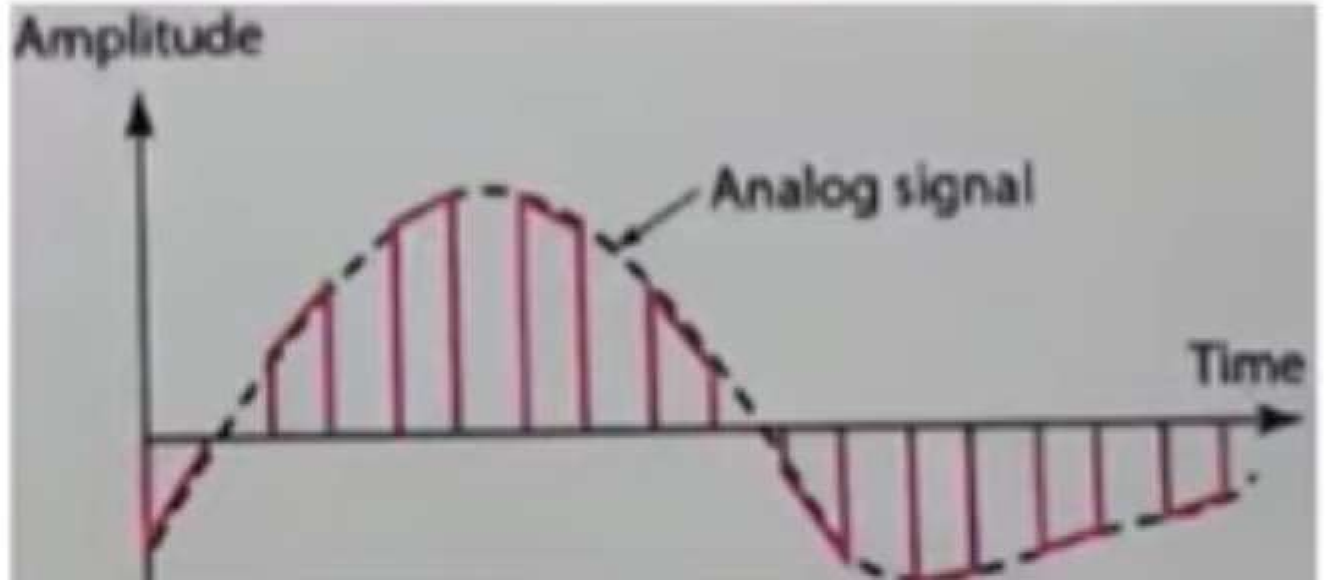
Process of Pulse Code Modulation

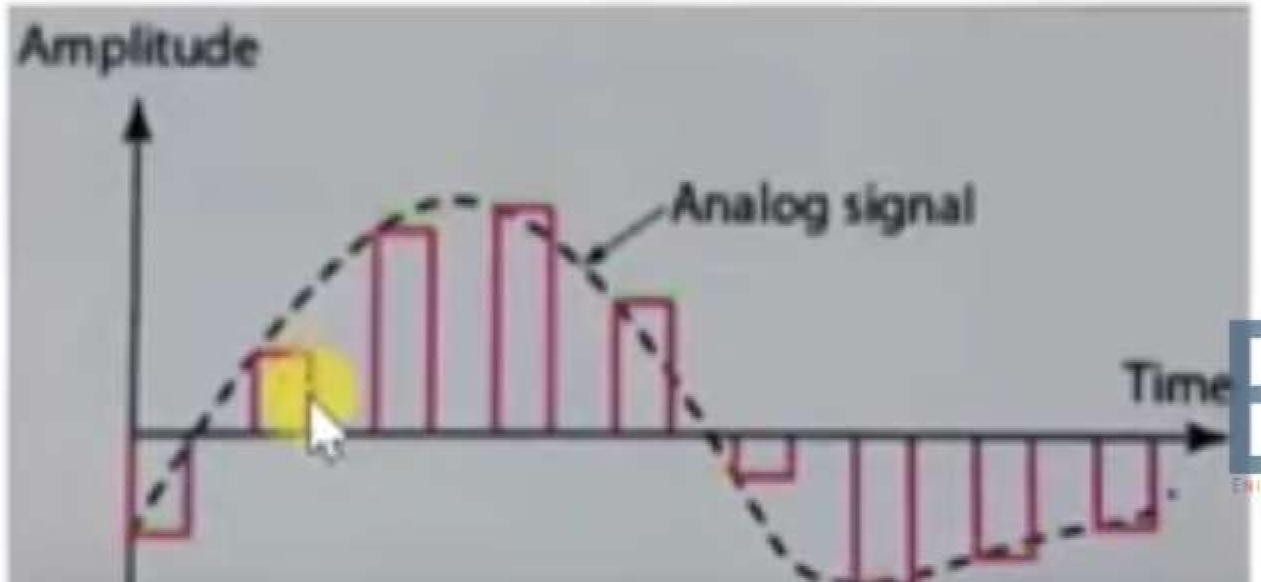
- Filtering
- Sampling
- Quantization
- Encoding

Sampling in Pulse Code Modulation

- Analog signal is sampled after every Ts interval.
- So sampling frequency is fs = 1/Ts.
- There are three sampling method
  - Ideal sampling : An impulse at each instant.
  - Natural sampling : A pulse of short width with varying amplitude.
  - Flat Top sampling : A pulse of short width with fixed amplitude.







Quantization in Pulse Code Modulation

- The process of measuring the numerical values of the samples and giving them a table value in a suitable scale.
- The finite number of amplitude intervals is called 'quantizing interval'.
- There can be two categories
  - Linear quantization
  - Non Linear quantization
- Difference between sampled output and quantized output is quantization distortion.

→ Sampled Output = 0.129 Volt

→ Quntized Output = 0.125 Volt

→ Quantization error

= 0.129 − 0.125

Quantization in Pulse Code Modulation

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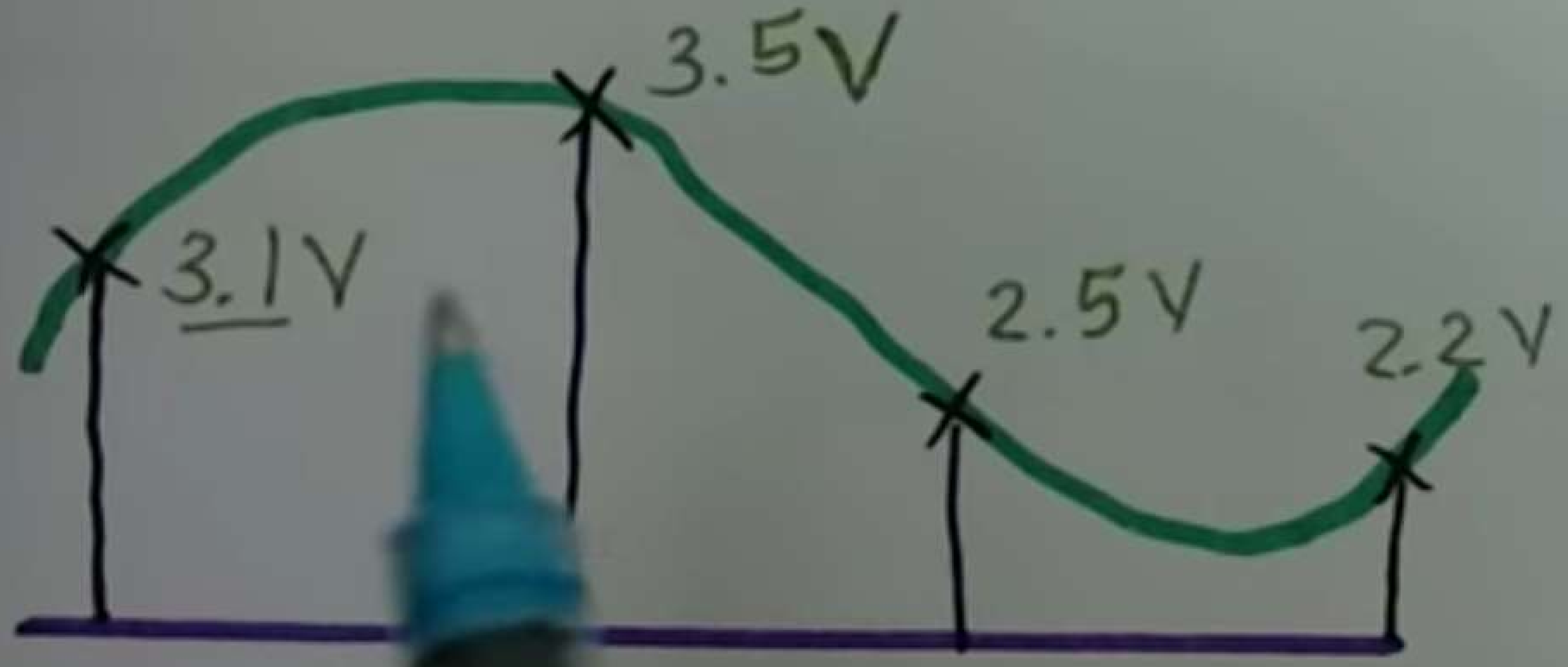
- There can be two categories
  - Linear quantization
  - Non Linear quantization
- Difference between sampled output and quantized output is quantization distortion.
- To decrease this distortion, we can increase number of levels by increasing number of bits.

fferential Pulse Code Modulation

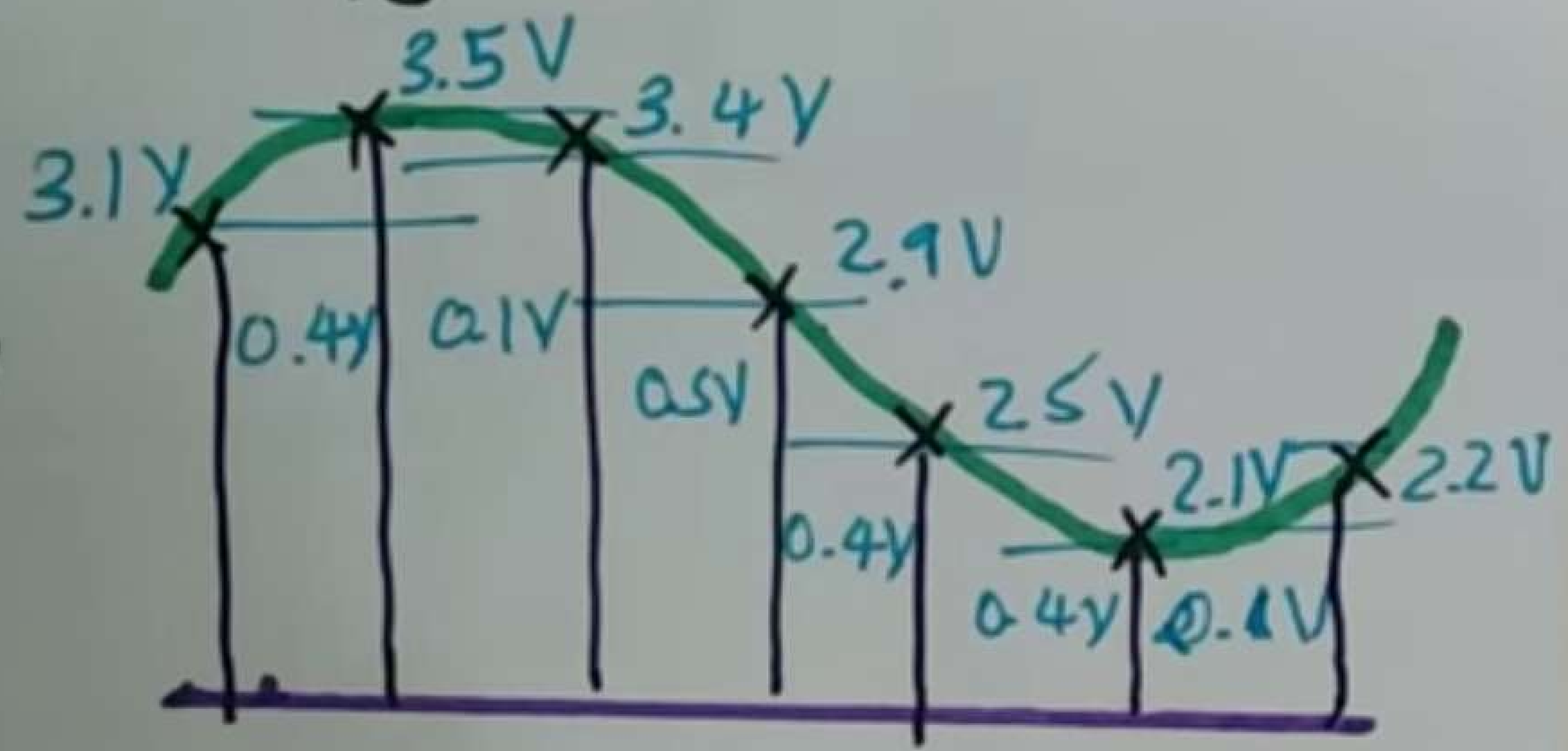
[DPCM]

$f_s = 2W$



cm

$f_s = 5W$

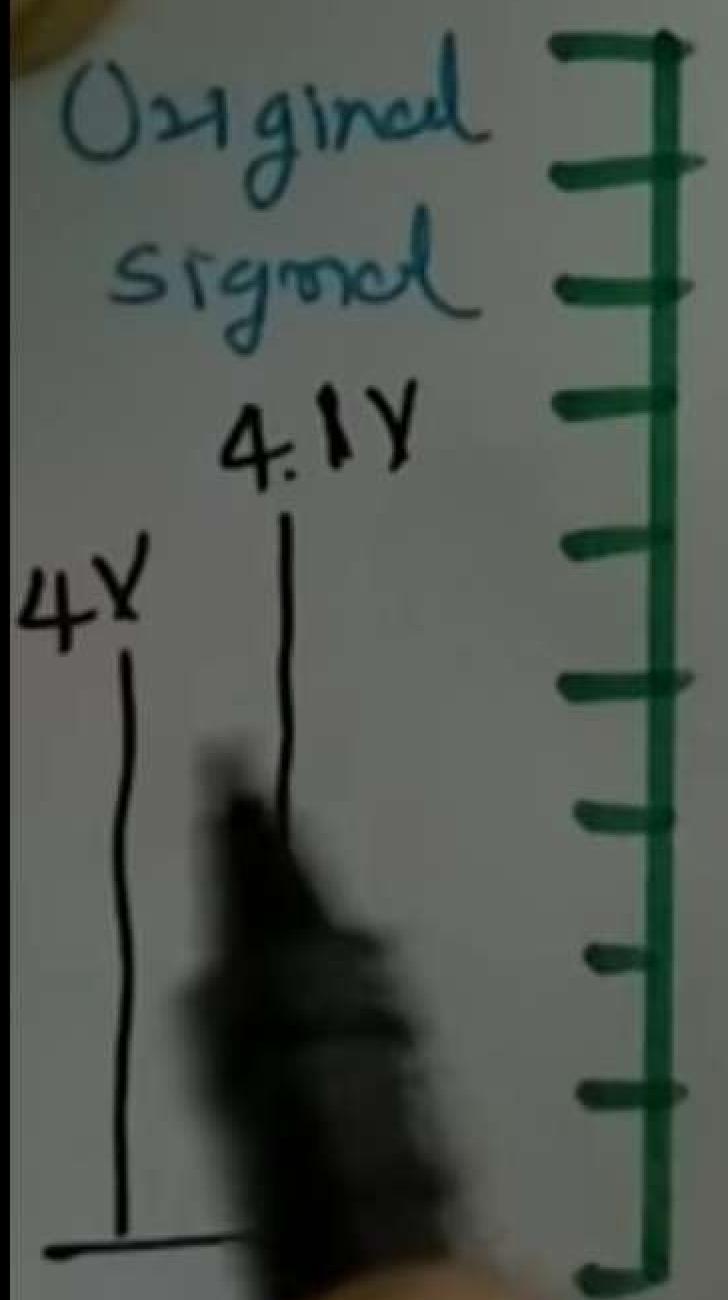


DPCM

at a rate higher than the Nyquist rate, the successive samples become more correlated.

- Their exist a very little difference between the amplitudes of successive samples.
- When all these samples are quantized and encoded there exist more redundant information in the transmitted signal.
- In DPCM, to reduce the redundant information & to achieve more compression, only the difference between the successive samples are transmitted.

- In DPCM, to ...
& to achieve more compression, Only the difference between the successive samples are transmitted.

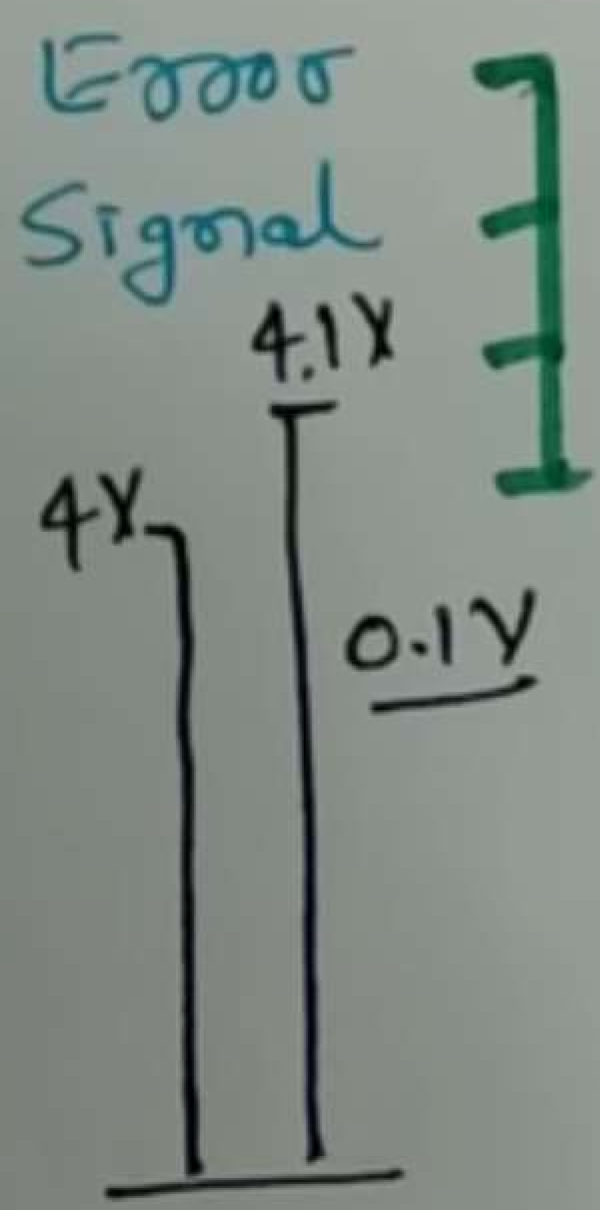


More Levels

- More number of bits per level (samples)

$$L = \frac{2(X_{max})}{\Delta}$$

$L\uparrow \rightarrow n\uparrow$

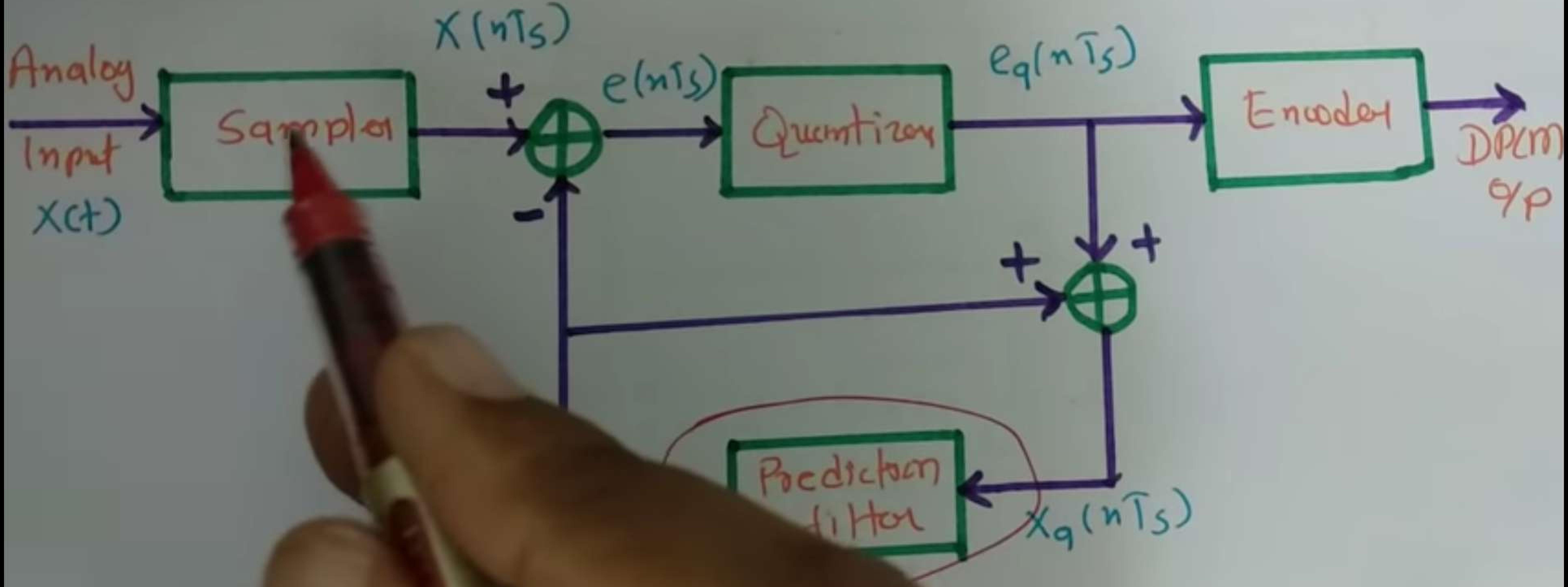


Less Levels

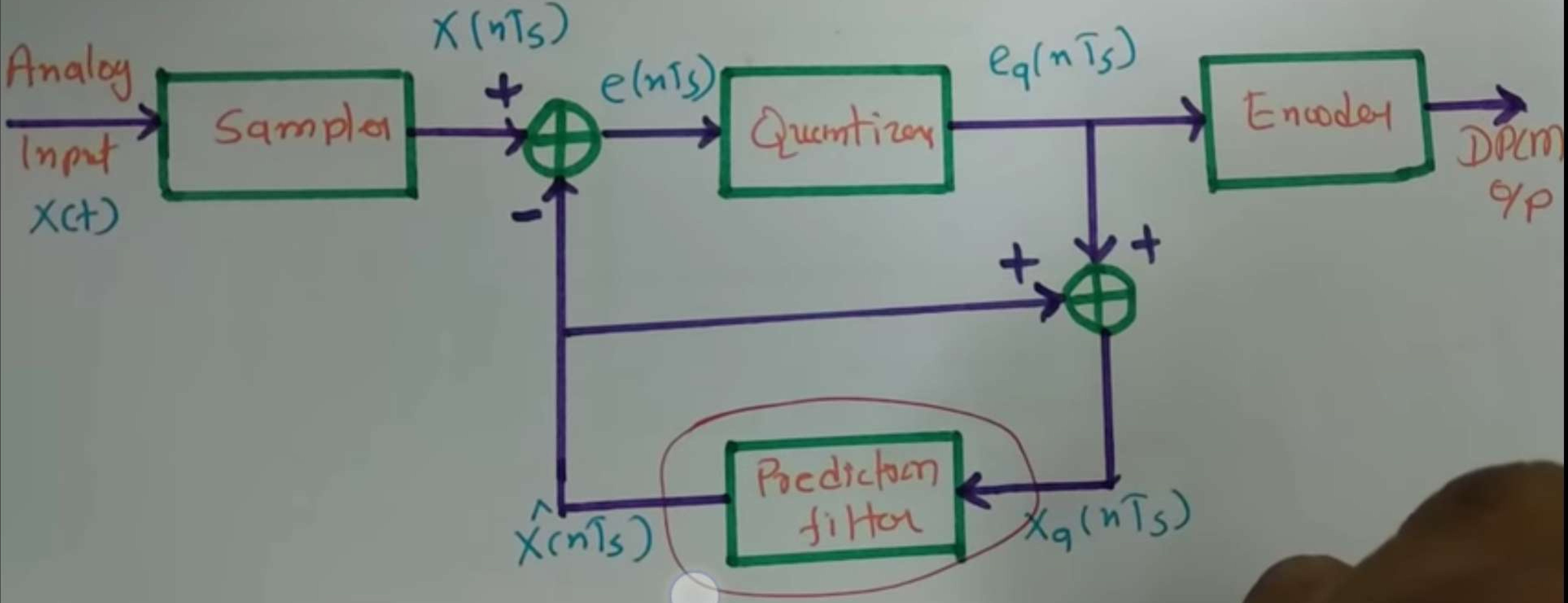
- Less no of bits per levels (Sampl...

$L\downarrow \rightarrow n\downarrow$

DPCM Encoder



DPCM Encoder



$x_q(nT_s)$

- Error signal

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

- Quntized error signal

$$e_q(nT_s) = e(nT_s) + q_e(nT_s)$$

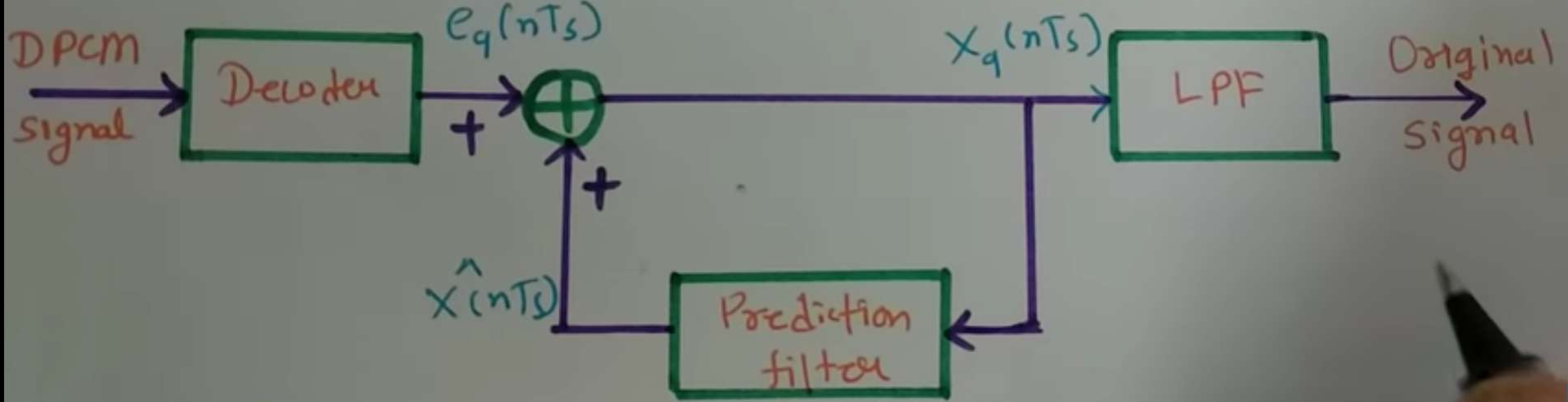
- Input to prediction filter.

$$\begin{aligned} x_q(nT_s) &= e_q(nT_s) + \hat{x}(nT_s) \\ &= e(nT_s) + q_e(nT_s) + \hat{x}(nT_s) \\ &= x(nT_s) - \cancel{\hat{x}(nT_s)} + q_e(nT_s) + \cancel{\hat{x}(nT_s)} \\ &= x(nT_s) + q_e(nT_s) \end{aligned}$$

$$= x(nT_s) - \cancel{\hat{x}(nT_s)} + q_e(nT_s) + \cancel{\hat{x}(nT_s)}$$

$$= x(nT_s) + q_e(nT_s)$$

DPCM Decoder



Example of DPCM

Consider the Input samples $x(n) = \{2.1, 2.2, 2.3, 2.6, 2.7, 2.8\}$

Explain how encoding and decoding is done in DPCM,

Assume first Order prediction filter $\hat{x}(n) = x_q(n-1)$

Encoder

Encoder

$x(n)$	$\hat{x}(n) = x_q(n-1)$	$e(n) = x(n) - \hat{x}(n)$	$e_q(n)$	$x_q(n) = \hat{x}(n) + e_q(n)$
2.1	0	2.1 − 0 = 2.1	2	0 + 2 = 2
2.2	2	2.2 − 2 = 0.2	0	2 + 0 = 2
2.3	2	2.3 − 2 = 0.3	0	2 + 0 = 2
2.6	2	2.6 − 2 = 0.6	1	2 + 1 = 3
2.7	3	2.7 − 3 = −0.3	0	3 + 0 = 3
2.8	3	2.8 − 3 = −0.2	0	3 + 0 = 3

1010 000 000 001 000 0001 Digital Data

Decoder

$e_q(n)$	$\hat{x}(n) = x_q(n-1)$	$x_q(n) = \hat{x}(n) + e_q(n)$
2	0 Initially	$2+0=2$
0	2	$0+2=2$
0	2	$0+2=2$
1	2	$1+2=3$
0	3	$0+3=3$
0	3	$0+3=3$

	0 Initially	2 + 0 = 2
0	2	0 + 2 = 2
0	2	0 + 2 = 2
	2	1 + 2 = 3
	3	0 + 3 = 3
0	3	0 + 3 = 3.

Received Signal = {2, 2, 2, 3, 3, 3}.

Transmitted signal = {2.1, 2.2, 2.3, 2.6, 2.7, 2.8}.

Hamming Code Basics

→ It is given by RW Hamming.

→ It is used to detect and correct error.

→ In Hamming Code, we send data along with parity bits or Redandent bits.

→ It is represented by (n, k) code.

$$(\underbrace{n}_{\text{total bits}},\ \underbrace{k}_{\text{message bits.}})$$

→ It is represented by (n, k) code.

n → total bits

k → message bits.

→ Parity bits $p = n - k$

→ To Identify parity bits, It should satisfy given Cond.^{n}

$$\Rightarrow 2^p \geq p + k + 1$$

→ So for $k = 4$ message bits.

$$\Rightarrow 2^p \geq p + 4 + 1$$

$$\Rightarrow 2^p \geq p + 5$$

For $p = 1$	For $p = 2$	For $p = 3$
$2^1 \geq 6$	$2^2 \geq 7$	$2^3 = 8$
×	×	✓

$\Rightarrow p = 3$ for $k = 4$ bits.

$\Rightarrow n = 3 + 4 = 7$ bits.

→ This is $(7, 4)$

$2^p \geq p+k+1$

→ So for $K = 4$ message bits.

$$\Rightarrow 2^p \geq p+4+1$$

$$\Rightarrow 2^p \geq p+5$$

For $p=1$	For $p=2$	For $p=3$
$2^1 \geq 6$	$2^2 \geq 7$	$2^3 = 8$
×	×	✓

$\Rightarrow p = 3$ for $K = 4$ bits.

$\Rightarrow n = 3+4 = 7$ bits.

→ This is (7,4) code.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

→ $P_1 \rightarrow D_3\ D_5\ D_7$ (XOR)

→ $P_2 \rightarrow D_3\ D_6\ D_7$ (XOR)

→ $P_4 \rightarrow D_5\ D_6\ D_7$ (XOR).

Generation of Hamming Code

- 5 bit data 01101 is given. Represent given data in Hamming code.

→ $K = 5$ bits.

→ Identify parity bits.

$$\Rightarrow 2^P \geq P + K + 1$$

$$\Rightarrow 2^P \geq P + 6$$

For $P=1$	For $P=2$	For $P=3$	For $\boxed{P=4}$
$2^1 \geq 7$	$2^2 \geq 8$	$2^3 \geq 9$	$2^4 \geq 10$
×	×	×	✓

→ So for $p = 4$, $K = 5$, $n = 9$

→ This is $(9, 5)$ hamming Code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ Pattern of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3\ D_5\ D_7\ D_9$

$P_2 \longrightarrow D_3\ D_6\ D_7$

$P_4 \longrightarrow D_5\ D_6\ D_7$

$P_8 \longrightarrow D_9$

9	8	7	6	5	4	3	2	1
0 D_9	P_8	1 D_7	1 D_6	0 D_5	P_4	1 D_3	P_2	P_1

→ ~~Pattern~~ of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3 \; D_5 \; D_7 \; D_9$

$P_2 \longrightarrow D_3 \; D_6 \; D_7$

$P_4 \longrightarrow D_5 \; D_6 \; D_7$

$P_8 \longrightarrow D_9$

→ $P_1 = 1 \oplus 0 \oplus 1 \oplus 0 = 0$

→ $P_2 = 1 \oplus 1 \oplus 1 = 1$

→ $P_4 = 0 \oplus 1 \oplus 1 = 0$

→ $P_8 = D_9 = 0$

$\boxed{00[illegible]100110}$

Hamming Code Error detection & Error Correction

* If Received Hamming Code is 1110101 with even Parity then detect and correct error.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	1	1	0	1	0	1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$$\left.\begin{array}{l} P_1 = D_3 \oplus D_5 \oplus D_7 \\ 1 = 1 \oplus 1 \oplus 1 \end{array}\right] \text{—} \; P_1 = 0 \; \checkmark$$

$$\left.\begin{array}{l} P_2 = D_3 \oplus D_6 \oplus D_7 \\ 0 = 1 \oplus 1 \oplus 1 \end{array}\right] \text{—} \; P_2 = 1 \; \times$$

$$\left.\begin{array}{l} P_4 = D_7 \oplus D_6 \oplus D_5 \\ 0 = 1 \oplus 1 \oplus 1 \end{array}\right] \text{—} \; P_4 = 1 \; \times$$

$D_7 \oplus D_6 \oplus D_5$ — $P_4 = 1$ ×

$0 = 1 \oplus 1 \oplus 1$

$P_4 P_2 P_1 = 110 = \underline{6}^{th}$ bit error

E =

0	1	0	0	0	0	0

R =

1	1	1	0	1	0	1

→ Corrected data = $R \oplus E$

=

1	0	1	0	1	0	1

Linear codes basics & property with example

Defination - A Block code is said to be linear code If its codewords satisfy the condition that the sum of any two codewords gives another codeword.

i.e. $c_p = c_i + c_k$

Property

i) The all-zero words [0, 0, 0, ... 0] is always a codeword.

ii) Given any three codewords c_i, c_j and c_k such that

$c_p = c_i + c_k$, then $d(c_i, c_j) = W(c_p)$

iii) Minimum distance of the code

- (7, 4) Hamming Code

$$\begin{array}{lcl} c_1 & = & 0\,0\,0\,1\,0\,1\,1 \quad \boxed{3} \\ c_{10} & = & 1\,0\,1\,0\,0\,1\,1 \quad \boxed{4} \\ \hline c_{11} & = & 1\,0\,1\,1\,0\,0\,0 \quad \boxed{3} \end{array}$$

$c_{11} = c_1 + c_{10}$

- $c_0 = [0\ 0\ 0\ 0\ 0\ 0\ 0]$

$d(c_1, c_{10}) = 3$

$w(c_{11}) = 3$

$d(c_1, c_{10}) = 3 = w(c_{11})$

- $c_{15} = [1\ 1\ 1\ 1\ 1\ 1\ 1]$, $w = 7$

other than c_{15} codes are having weight 3 & 4.

$d_{min} = w_{min}$

Show that (4, 3) Even-parity code is a linear and (4, 3) odd parity is not linear.

→ (4, 3) even parity code.

C

(4, 3) odd parity is not linear.

→ (4, 3) even parity code.

C	d_1	d_2	d_3	P
C_0	0	0	0	0
C_1	0	0	1	1
C_2	0	1	0	1
C_3	0	1	1	0
C_4	1	0	0	1
C_5	1	0	1	0
C_6	1	1	0	0
C_7	1	1	1	1

$$
\begin{array}{cccccc}
 & & & & & W \\
0 & 0 & 1 & 1 & C_1 & 2 \\
0 & 1 & 0 & 1 & C_2 & 2 \\
\hline
0 & 1 & 1 & 0 & \boxed{C_3} & 2
\end{array}
$$

$d_{min}(C_1, C_2) = 2$

$W(C_3) = 2$

− $\boxed{d_{min} \quad W}$

- Odd parity (4,3) code

C	d_1	d_2	d_3	P
C_0	0	0	0	1
C_1	0	0	1	0
C_2	0	1	0	0
C_3	0	1	1	1
C_4	1	0	0	0
C_5	1	0	1	1
C_6	1	1	0	1
C_7	1	1	1	0

$$\begin{array}{cccc|c} 0 & 0 & 1 & 0 & C_1 \\ 0 & 1 & 0 & 0 & C_2 \\ \hline 0 & 1 & 1 & 0 & \end{array}$$

- $C_1 + C_2$ is not present in odd parity (4,3) code
- It is not linear block

Generator matrix in Linear Code

to generate code words

- using a matrix to generate codewords is a better approach.

$$[c] = [i][G]$$

$[c]$ = Code word
$[i]$ = Information words
$[G]$ = Generator matrix

- The Generator matrix of an (n, K) linear code has 'K' rows and 'n' columns.

- Generator matrix for $(7, 4)$ code is given by

$$[G] = [I : P]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

Identity Matrix I_K Parity Matrix

$\rightarrow$ $[C] = [i][G]$

Example - Generate Codeword for $i = (1110)$ with (7, 4) generator matrix code.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & \vdots & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & \vdots & 0 & 1 & 1 \end{bmatrix}$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$c = [i][G]$$

$$1.1 + 1.0 + 1.0 + 0.0$$

$$= [1\ 1\ 1\ 0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [1\ 1\ 1\ 0\ 1\ 0\ 0]$$

Determine the set ... codewords for the (6, 3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$K = 3$

$\rightarrow$ Message bits $= 3$

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$\rightarrow C = [i][G]$

$$\rightarrow C_0 = [0\ 0\ 0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 0\ 0\ 0\ 0]$$

$$\rightarrow C_1 = [0\ 0\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1\ 0]$$

	m_0	m_1	m_2	p_0	p_1	p_2
c_0	0	0	0	0	0	0
c_1	0	0	1	1	1	0
c_2	0	1	0	1	0	1
c_3	0	1	1	0	1	1
c_4	1	0	0	0	1	1
c_5	1	0	1	1	0	1
c_6	1	1	0	1	1	0
c_7	1	1	1	0	0	0

1 1 1