8

Mechanics of Solids

8.1. CLASSIFICATION OF LOADS

A load may be defined as the combined effect of external forces acting on a body. The loads may be classified as : (*i*) dead loads, (*ii*) live or fluctuating loads, (*iii*) inertia loads or forces and (*iv*) centrifugal loads or forces.

The other way of classification is (*i*) tensile loads, (*ii*) compressive loads, (*iii*) torsional or twisting loads, (*iv*) bending loads and (*v*) shearing loads.

The load may be a 'point' (or concentrated) or 'distributed'.

Point load. A point load or concentrated load is one which is considered to act at a point. In actual practical, the load has to be distributed over a small area, because, such small knife-edge contacts are generally neither possible, nor desirable.

Distributed load. A distributed load is one which is distributed or spread in some manner over the length of the beam. If the spread is uniform, (*i.e.*, at the uniform rate, say w kN or N/metre run) it is said to be uniformly distributed load and is abbreviated as u.d.l. If the spread is not at uniform rate, it is said to be non-uniformly distributed load. *Triangulary* and *trapezoidally* distributed loads fall under this category.

8.2. STRESS

When a body is acted upon by some load or external force, it undergoes deformation (*i.e.*, change in shape or dimensions) which increases gradually. During deformation, the material of the body resists the tendency of the load to deform the body, and when the load influence is taken over by the internal resistance of the material of the body, it becomes stable. This *internal resistance which the body offers to meet with the load is called* **stress.**

Stress can be considered either as total stress or unit stress. Total stress represents the total resistance to an external effect and is expressed in N, kN or MN. Unit stress represents the resistance developed by a unit area of cross-section, and is expressed in kN/m^2 or MN/m^2 or N/mm^2. For the remainder of this text, the word stress will be used to signify unit stress.

The various types of stresses may be classified as :

1. Simple or direct stress

(*i*) Tension (*ii*) Compression (*iii*) Shear.

2. Indirect stress

(*i*) Bending (*ii*) Torsion.

3. Combined Stress. Any possible combination of types 1 and 2.

This chapter deals with simple stresses only.

8.3. SIMPLE STRESS

Simple stress is often called *direct stress* because it develops under direct loading conditions. That is, simple tension and simple compression occur when the applied force, called load, is in line with the axis of the member (axial loading) (Fig. 8.1 and 8.2), and simple shear occurs, when equal, parallel, and opposite forces tend to cause a surface to slide relative to the adjacent surface (Fig. 8.3).

Fig. 8.1. Tensile stress.

Fig. 8.2. Compressive stress.

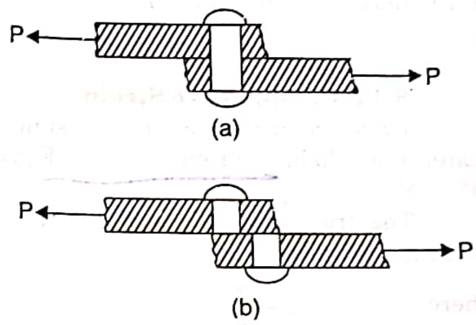

Fig. 8.3. (*a*) Rivet resisting shear (*b*) Rivet failure due to shear.

In certain loading situations, the stresses that develop are not simple stresses. For example, referring to Fig. 8.4, the member is subjected to a load which is perpendicular to the axis of the member (transverse loading) (Fig. 8.5). This will cause the member to bend, resulting in deformation of the material and stresses being developed internally to resist the deformation. All three types of stresses—tension, compression and shear—will develop, but they will not be simple stresses, since they were not caused by direct loading

When any type of simple stress σ (sigma) develops, we can calculate the magnitude of the stress by,

$$\sigma = \frac{P}{A} \qquad \ldots(8.1)$$

where σ = stress, kN/m^2 or N/mm^2

P = load $\left[\begin{array}{l}\text{external force causing} \\ \text{stress to develop}\end{array}\right]$, kN or N

A = area over which stress develops, m^2 or mm^2

It may be noted that in cases of either simple tension or simple compression, the areas which resist the load are perpendicular to the direction of forces. When a member is subjected to simple shear, the resisting area is parallel to the direction of the force. Common situations causing shear stresses are shown in Figs. 8.3 and 8.4.

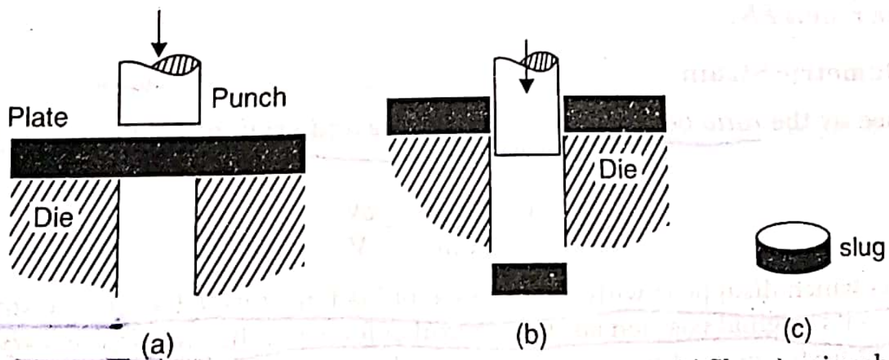

Fig. 8.4. (*a*) Punch approaching plate ; (*b*) Punch shearing plate ; (*c*) Slug showing sheared area.

8.4. STRAIN

Any element in a material subjected to stress is said to be strained. *The strain (e) is the deformation produced by stress.* The various types of strains are explained below :

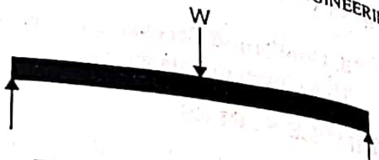

Fig. 8.5. Simply supported beam. (Transverse loading)

8.4.1. Tensile Strain

A piece of material, with uniform cross-section, subjected to a uniform axial tensile stress, will increase its length from l to $(l + \delta l)$ (Fig. 8.6) and the increment of length δl is the actual deformation of the material. The fractional deformation or the tensile strain is given by

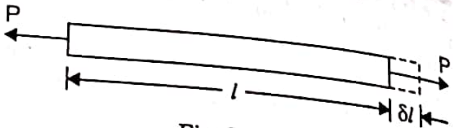

Fig. 8.6

$$e_t = \frac{\delta l}{l} \quad \text{...(8.2)}$$

8.4.2. Compressive Strain

Under compressive forces, a similar piece of material would be reduced in length (Fig. 8.7) from l to $(l - \delta l)$.

The fractional deformation again gives the strain e_c,

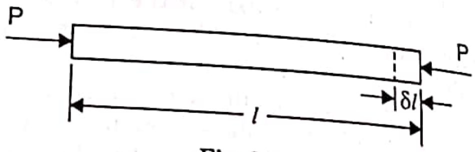

Fig. 8.7

where $$e_c = \frac{\delta l}{l} \quad \text{...(8.2 a)}$$

8.4.3. Shear Strain

In case of a shearing load, a shear strain will be produced which is *measured by the angle through which the body distorts.*

In Fig. 8.8 is shown a rectangular block *LMNP* fixed at one face and subjected to force *F*. After application of force, it distorts through an angle ϕ and occupies new position *LM′ N′P*. The shear strain (e_s) is given by

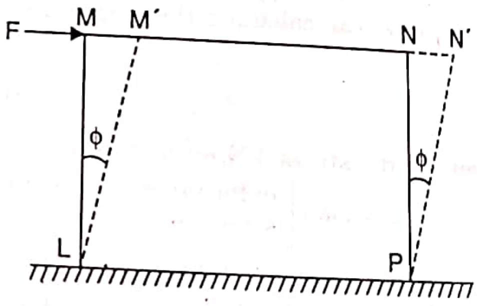

Fig. 8.8

$$e_s = \frac{NN'}{NP} = \tan\phi$$

$$= \phi \text{ (radians) ...... since } \phi \text{ is very small.}$$

The above result has been obtained by assuming *NN′* equal to arc (as *NN′* is small) drawn with centre *P* and radius *PN*.

8.4.4. Volumetric Strain

It is defined as the *ratio between change is volume and original volume of the body*, and is denoted by e_v,

$$\therefore \quad e_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V} \quad \text{...(8.3)}$$

The strains which disappear with the removal of load are termed as *elastic strains* and the body which regains its original position on the removal of force is called an *elastic body*. The body is said to be *plastic* if the *strains exist even after the removal of external force*. There is always a limiting value of load upto which the strain totally disappears on the removal of load—the stress corresponding to this load is called *elastic limit.*

Robert Hooke discovered experimentally that within elastic limit, stress varies *directly* as strain

i.e. stress ∝ strain or $\frac{\text{Stress}}{\text{Strain}}$ = a constant

This constant is termed as *Modulus of Elasticity.*

(*i*) **Young's modulus.** It is the *ratio between tensile stress and tensile strain or compressive stress and compressive strain.* It is denoted by *E*. It is the same as modulus of elasticity.

or

$$E = \frac{\sigma}{e} \left[= \frac{\sigma_t}{e_t} \text{ or } \frac{\sigma_c}{e_c} \right] \quad \text{...(8.4)}$$

(*ii*) **Modulus of rigidity.** It is defined as the *ratio of shear stress τ (tau) to shear strain* and is denoted by *C*, *N* or *G*. It is also called *shear modulus of elasticity.*

or

$$\frac{\tau}{e_s} = C, N \text{ or } G \quad \text{...(8.5)}$$

(*iii*) **Bulk or volume modulus of elasticity.** It may be defined as the *ratio of normal stress* (on each face of a solid cube) *to volumetric strain* and is donated by the letter *K*.

or

$$\frac{\sigma_n}{e_v} = K \quad \text{...(8.6)}$$

Example 8.1. *A square steel rod 20 mm × 20 mm in section is to carry an axial load (compressive) of 100 kN. Calculate the shortening in a length of 50 mm. E = 2.14 × 10⁸ kN/m².*

Sol. Area, $A = 0.02 \times 0.02 = 0.0004 \text{ m}^2$; Length, $l = 50$ mm or 0.05 m

Load, $P = 100$ kN ; $E = 2.14 \times 10^8 \text{ kN/m}^2$

Shortening of the rod, δl :

Stress, $\sigma = \frac{P}{A}$

$\therefore \quad \sigma = \frac{100}{0.0004} = 250000 \text{ kN/m}^2$

Also $E = \frac{\text{stress}}{\text{strain}}$

or $\text{Strain} = \frac{\text{stress}}{E} = \frac{250000}{2.14 \times 10^8}$

or $\frac{\delta l}{l} = \frac{250000}{2.14 \times 10^8}$

$\therefore \quad \delta l = \frac{250000}{2.14 \times 10^8} \times l = \frac{250000}{2.14 \times 10^8} \times 0.05 = 0.0000584 \text{ m or } 0.0584 \text{ mm}$

Hence the shortening of the rod = 0.0584 mm (Ans.)

Example 8.2. *A hollow cast-iron cylinder 4 m long, 300 mm outer diameter, and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m². Assume Young's modulus for cast iron as 1.5 × 10⁸ kN/m² and find (i) magnitude of the load, (ii) longitudinal strain produced and (iii) total decrease in length.*

Sol. Outer diameter, $D = 300$ mm = 0.3 m

Thickness, $t = 50$ mm = 0.05 m

Length, $l = 4$ m

Stress produced, $\sigma = 75000 \text{ kN/m}^2$

$E = 1.5 \times 10^8 \text{ kN/m}^2$

Here diameter of the cylinder, $d = D - 2t = 0.3 - 2 \times 0.05 = 0.2$ m

(*i*) **Magnitude of the load P :**

Using the relation, $\sigma = \dfrac{P}{A}$

or $$P = \sigma \times A = 75000 \times \frac{\pi}{4}(D^2 - d^2) = 75000 \times \frac{\pi}{4}(0.3^2 - 0.2^2)$$

or $$\mathbf{P = 2945.2 \text{ kN} \quad (Ans.)}$$

(*ii*) **Longitudinal strain produced, e :**

Using the relation,

Strain, $$e = \frac{\text{stress}}{E} = \frac{75000}{1.5 \times 10^8} = \mathbf{0.0005 \quad (Ans.)}$$

(*iii*) **Total decrease in length, δl :**

Using the relation,

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\delta l}{l}$$

$$0.0005 = \frac{\delta l}{4}$$

$$\delta l = 0.0005 \times 4 \text{ m} = 0.002 \text{ m} = 2 \text{ mm}$$

Hence decrease in length = 2 mm (Ans.)

Example 8.3. *The following observations were made during a tensile test on a mild steel specimen 40 mm in diameter and 200 mm long.*

Elongation with 40 kN load (within limit of proportionality), $\delta l = 0.0304$ *mm*

Yield load = *161 kN*

Maximum load = *242 kN*

Length of specimen at fracture = *249 mm*

Determine :

(*i*) *Young's modulus of elasticy* (*ii*) *Yield point stress*

(*iii*) *Ultimate stress* (*iv*) *Percentage elongation.*

Sol. (*i*) **Young's modulus of elasticity E :**

Stress, $$\sigma = \frac{P}{A} = \frac{40}{\frac{\pi}{4} \times (0.04)^2} = 3.18 \times 10^4 \text{ kN/m}^2$$

Strain, $$e = \frac{\delta l}{l} = \frac{0.0304}{200} = 0.000152$$

$\therefore$ $$E = \frac{\text{stress}}{\text{strain}} = \frac{3.18 \times 10^4}{0.000152} = \mathbf{2.09 \times 10^8 \text{ kN/m}^2 \quad (Ans.)}$$

(*ii*) **Yield point stress :**

Yield point stress $$= \frac{\text{yield point load}}{\text{area}}$$

$$= \frac{161}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{12.8 \times 10^4 \text{ kN/m}^2 \quad (Ans.)}$$

(*iii*) **Ultimate stress :**

$$\text{Ultimate stress} = \frac{\text{maximum load}}{\text{area}}$$

$$= \frac{242}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{19.2 \times 10^4\ kN/m^2} \quad \textbf{(Ans.)}$$

(*iv*) **Percentage elongation :**

$$\text{Percentage elongation} = \frac{\text{length of specimen at fracture} - \text{original length}}{\text{original length}}$$

$$= \frac{249 - 200}{200} = 0.245 = \mathbf{24.5\%} \quad \textbf{(Ans.)}$$

Example 8.4. *A steel wire 2 m long and 3 mm in diameter is extended by 0.75 mm when a weight W is suspended from the wire. If the same weight is suspended from a brass wire, 2.5 m long and 2 mm in diameter, it is elongated by 4.64 mm. Determine the modulus of elasticity of brass if that of steel be* 2.0×10^5 *N/mm*^{2}.

Sol. *Given :* $l_s = 2$ m, $d_s = 3$ mm, $\delta l_s = 0.75$ mm ; $E_s = 2.0 \times 10^5$ N/mm^2 ; $l_b = 2.5$ m ; $d_b = 2$ mm ; $\delta l_b = 4.64$ m.

Modulus of elasticity of brass, E_b :

From Hooke's law, we know $\quad \delta l = \dfrac{Wl}{AE}$

where, δl = extension, l = length, A = cross-sectional area, and E = modulus of elasticity.

Case I : For steel wire :

$$\delta l_s = \frac{W l_s}{A_s E_s}$$

or

$$0.75 = \frac{W \times (2 \times 1000)}{\left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5}$$

or

$$W = 0.75 \times \left(\frac{\pi}{4} \times 3^3\right) \times 2.0 \times 10^5 \times \frac{1}{2000} \quad \text{...}(i)$$

Case II : For bass wire :

$$\delta l_b = \frac{W l_b}{A_b E_b}$$

$$4.64 = \frac{W \times (2.5 \times 1000)}{\left(\frac{\pi}{4} \times 2^2\right) \times E_b}$$

or

$$W = 4.64 \times \left(\frac{\pi}{4} \times 2^2\right) \times E_b \times \frac{1}{2500} \quad \text{...}(ii)$$

Equating eqns. (*i*) and (*ii*), we get

$$0.75 \times \left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5 \times \frac{1}{2000} = 4.64 \times \left(\frac{\pi}{4} \times 2^2\right) \times E_b \times \frac{1}{2500}$$

or

$$\mathbf{E_b = 0.909 \times 10^5\ N/mm^2} \quad \textbf{(Ans.)}$$

Example 8.5. *A steel bar is 900 mm long ; its two ends are 40 mm and 30 mm in diameter and the length of each rod is 200 mm. The middle portion of the bar is 15 mm in diameter and 500 mm long. If the bar is subjected to an axial tensile load of 15 kN, find its total extension.*

Take $E = 200\ GN/m^2$ (G stands for giga and $1\ G = 10^9$)

Sol. Refer Fig. 8.9.

Load, $P = 15$ kN

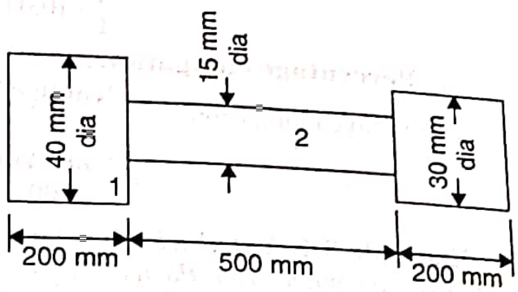

Fig. 8.9

Area, $A_1 = \frac{\pi}{4} \times 40^2$

$= 1256.6\ mm^2 = 0.001256\ m^2$

Area, $A_2 = \frac{\pi}{4} \times 15^2$

$= 176.7\ mm^2 = 0.0001767\ m^2$

Area, $A_3 = \frac{\pi}{4} \times 30^2$

$= 706.8\ mm^2 = 0.0007068\ m^2$

Lengths : $l_1 = 200$ mm $= 0.2$ m, $l_2 = 500$ mm $= 0.5$ m and $l_3 = 200$ mm $= 0.2$ m

Total extension of the bar :

Let δl_1, δl_2 and δl_3 be the extensions in the parts 1, 2 and 3 of the steel bar respectively.

Then, $\delta l_1 = \frac{Pl_1}{A_1E}$, $\delta l_2 = \frac{Pl_2}{A_2E}$, $\delta l_3 = \frac{Pl_3}{A_3E}$ $\left[\because\ E = \frac{\sigma}{e} = \frac{P/A}{\delta l/l} = \frac{P.l}{A.\delta} \text{ or } \delta l = \frac{Pl}{AE}\right]$

Total extension of the bar,

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E} = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

$$= \frac{15 \times 10^3}{200 \times 10^9}\left[\frac{0.20}{0.001256} + \frac{0.50}{0.0001767} + \frac{0.20}{0.0007068}\right]$$

$$= 0.0002454 \text{ m} = 0.2454 \text{ mm}$$

Hence total extension of the steel bar = 0.2454 mm (Ans.)

Example 8.6. *The bar shown in Fig. 8.10 is subjected to a tensile load of 50 kN. Find the diameter of the middle portion if the stress is limited to $130\ MN/m^2$. Find also the length of the middle portion if the total elongation of the bar is 0.15 mm. Take $E = 200\ GN/m^2$.*

Sol. Magnitude of tensile load, $P = 50$ kN

Stress in the middle portion, $\sigma = 130\ MN/m^2$

Total elongation of the bar, $\delta l = 0.15$ mm $= 0.15 \times 10^{-3}$ m

Modulus of elasticity, $E = 200\ GN/m^2$

Diameter of the middle portion, d :

Now, stress in the middle portion, $\sigma = \frac{P}{A} = \frac{50 \times 1000}{(\pi/4)\, d^2} = 130 \times 10^6$

$$\therefore \quad d = \left[\frac{50 \times 1000}{\pi/4 \times 130 \times 10^6}\right]^{1/2} = 0.0221 \text{ m or } 22.1 \text{ mm}$$

Hence diameter of the middle portion = 22.1 mm (Ans.)

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Tensile force in LQ + compressive force in QM = 300000

$$1250 \times 10^{-6} \times \sigma_1 + 1250 \times 10^{-6} \times \sigma_2 = 300000$$

$$\sigma_1 + \sigma_2 = 2.4 \times 10^8 \text{ N/m}^2 \quad ...(ii)$$

Solving (*i*) and (*ii*), we get $\sigma_1 = 1.065 \times 10^8 \text{ N/m}^2 = $ **106.5 MN/m² (tensile) (Ans.)**

$\sigma_2 = 1.335 \times 10^8 \text{ N/m}^2 = $ **133.5 MN/m² (compressive) (Ans.)**

$\sigma_3 = \frac{\sigma_2}{2} = 0.667 \times 10^8 \text{ N/m}^2 = $ **66.7 MN/m² (compressive) (Ans.)**

Example 8.14. *Two parallel steel wires 6 m long, 10 mm diameter are hung vertically 70 mm apart and support a horizontal bar at their lower ends. When a load of 9 kN is attached to one of the wires, it is observed that the bar is 24° to the horizontal. Find 'E' for wire.*

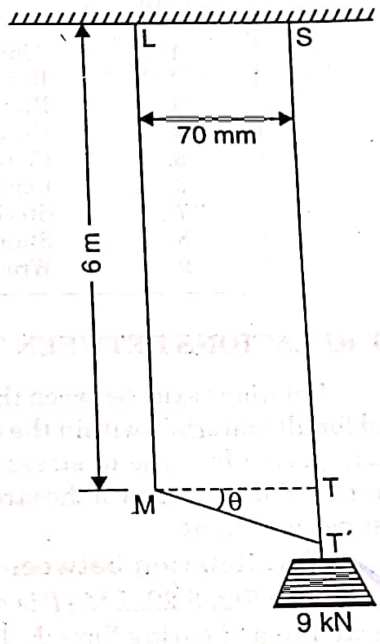

Fig. 8.20

Sol. Refer Fig. 8.20.

Two wires LM and ST made of steel, each 6 m long and 10 mm diameter are fixed at the supports and a load of 9 kN is applied on wire ST. Let the inclination of the bar after the application of the load be θ.

The extension in the length of steel wire ST,

$$\delta l = 70 \tan \theta = 70 \times \tan 2.4° = 70 \times 0.0419 = 2.933 \text{ mm} = 0.00293 \text{ m}$$

∴ Strain in the wire, $e = \frac{\delta l}{l} = \frac{0.00293}{6} = 0.000488$

and stress in the wire

$$\sigma = \frac{P}{A} = \frac{9000}{\frac{\pi}{4} \times \left(\frac{10}{1000}\right)^2} = 11.46 \times 10^7 \text{ N/m}^2$$

Young's modulus $\mathbf{E} = \frac{\sigma}{e} = \frac{11.46 \times 10^7}{0.000488} = 235 \times 10^9 \text{ N/m}^2$

= **235 GN/m² (Ans.)**

8.5. POISSON'S RATIO

If a body is subjected to a load, its length changes ; ratio of this change in length to the original length is known as *linear or primary strain*. Due to this load, the dimensions of the body change ; in all directions at right angles to its line of application the strains thus produced are called *lateral or secondary or transverse strains* and are of *nature opposite to that of primary strains*. For example, if the load is tensile, there will be an increase in length and a corresponding decrease in cross-sectional area of the body (Fig. 8.21). In this case, linear or primary strain will be tensile and secondary or lateral or transverse strain compressive.

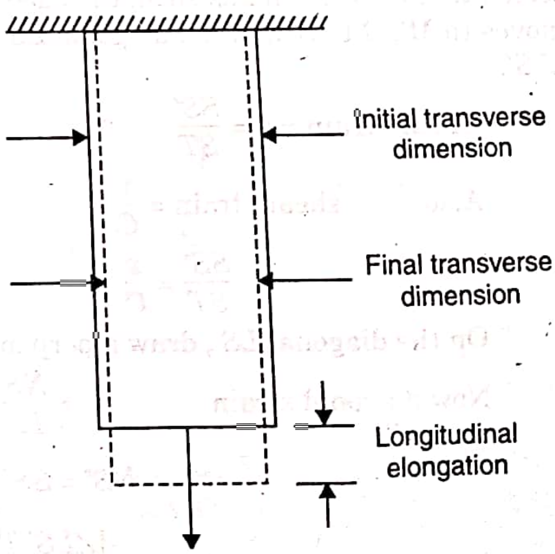

Fig. 8.21

The ratio of lateral strain to linear strain is known as **Poisson's ratio.**

i.e. Poisson's ratio, $\mu = \dfrac{\text{lateral strain or transverse strain}}{\text{linear or primary strain}} = \dfrac{1}{m}$

where m is a constant and its value varies between 3 and 4 for different materials.

Table 8.1 gives the average values of Poisson's ratio for common materials.

Table 8.1. Poisson's ratio for some of the common materials

Sl. No.	*Material*	*Poisson's ratio*
1.	Aluminium	0.330
2.	Brass	0.340
3.	Bronze	0.350
4.	Cast iron	0.270
5.	Concrete	0.200
6.	Copper	0.355
7.	Steel	0.288
8.	Stainless steel	0.305
9.	Wrought iron	0.278

8.6. RELATIONS BETWEEN THE ELASTIC MODULI

Relations exist between the elastic constants for any specific material and these relations hold good for all materials within the elastic range. The relations result from the fact that the application of any particular type of stress necessarily produces other types of stress at other places in the material. Further, each of the stresses produces its corresponding strain and all the strains produced must be consistent.

8.6.1. Relation between E and C

Refer Fig. 8.22. $LMST$ is a solid cube subjected to a shearing force F. Let τ be the shear stress produced in the faces MS and LT due to this shearing force. The complementary shear stress consequently produced in the faces ML and ST is also τ. Due to the shearing load, the cube is distorted to $LM'S'T$, and as such, the edge M moves to M', S to S' and the diagonal LS to $L'S'$.

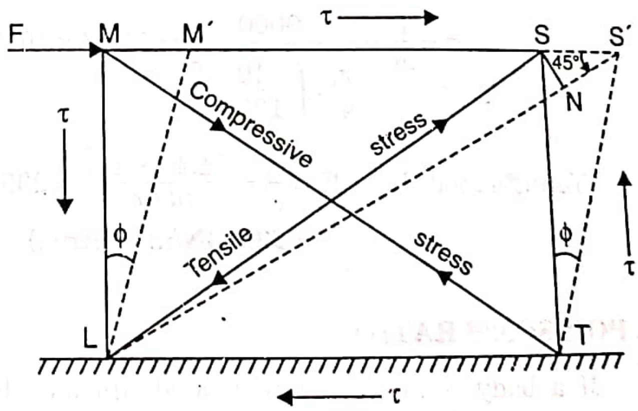

Fig. 8.22

$$\text{Shear strain} = \phi = \frac{SS'}{ST}$$

Also $\qquad \text{shear strain} = \dfrac{\tau}{C}$

$$\therefore \qquad \frac{SS'}{ST} = \frac{\tau}{C} \qquad ...(i)$$

On the diagonal LS', draw a perpendicular SN from S.

$$\text{Now diagonal strain} = \frac{NS'}{LN} = \frac{NS'}{LS} \qquad ...(ii)$$

$$NS' = SS' \cos 45° = \frac{SS'}{\sqrt{2}}$$

[$\angle LS'T$ is assumed to be equal to $\angle LST$ since SS' is very small]

and $\qquad LS = ST \times \sqrt{2}$

Putting the value of LS in (ii), we get

Diagonal strain $= \dfrac{SS'}{\sqrt{2}\,ST \times \sqrt{2}} = \dfrac{SS'}{2\,ST}$

But $\dfrac{SS'}{ST} = \dfrac{\tau}{C}$

$\therefore$ Diagonal strain $= \dfrac{\tau}{2C} = \dfrac{\sigma_n}{2C}$ ...(iii)

where σ_n is the normal stress due to shear stress t. The net strain in the direction of diagonal LS

$$= \frac{\sigma_n}{E} + \frac{\sigma_n}{mE}$$

[Since the diagonals LS and MT have normal tensile and compressive stress σ_n, respectively.]

$$= \frac{\sigma_n}{E}\left[1 + \frac{1}{m}\right] \qquad ...(iv)$$

Comparing (iii) and (iv), we get

$$\frac{\sigma_n}{2C} = \frac{\sigma_n}{E}\left[1 + \frac{1}{m}\right] \quad i.e. \quad E = 2C\left[1 + \frac{1}{m}\right] \qquad ...(8.7)$$

8.6.2. Relation between E and K

If the solid cube in question is subjected to σ_n (normal compressive stress) on all the faces, the direct strain in each axis $= \dfrac{\sigma_n}{E}$ (compressive) and lateral strain in other axis $= \dfrac{\sigma_n}{mE}$ (tensile).

$\therefore$ Net compressive strain in each axis $= \dfrac{\sigma_n}{E} - \dfrac{\sigma_n}{mE} - \dfrac{\sigma_n}{mE} = \dfrac{\sigma_n}{E}\left[1 - \dfrac{2}{m}\right]$

Volumetric strain (e_v) in this case will be,

$$e_v = 3 \times \text{linear strain} = 3 \times \frac{\sigma_n}{E}\left[1 - \frac{2}{m}\right]$$

But $e_v = \dfrac{\sigma_n}{K}$

$$\therefore \quad \frac{\sigma_n}{K} = \frac{3\sigma_n}{E}\left[1 - \frac{2}{m}\right] \quad \text{or} \quad E = 3K\left[1 - \frac{2}{m}\right] \qquad ...(8.8)$$

The relation between E, C and K can be established by eliminating m from the equations (8.7) and (8.8) as follows :

From equation (8.7), $m = \dfrac{2C}{E - 2C}$

$$E = 3K\left[1 - \frac{2}{2C/(E - 2C)}\right] \quad \text{or} \quad E = 3K\left[1 - \frac{E - 2C}{C}\right]$$

or $\dfrac{E}{3K} = \dfrac{C - E + 2C}{C} = \dfrac{3C - E}{C}$ or $\dfrac{E}{3K} + \dfrac{E}{C} = 3$

or $EC + 3\,KE = 9\,KC$

$E\,(3K + C) = 9\,KC$

$$E = \frac{9\,KC}{3K + C} \qquad ...(8.9)$$

Note. When a square or rectangular block subjected to a shear load is in equilibrium, the shear stress in one plane is always associated with a complementary shear stress (of equal value) in the other plane at right angles to it.

Example 8.15. *A concrete cylinder of diameter 150 mm and length 300 mm when subjected to an axial compressive load of 240 kN resulted in an increase of diameter by 0.127 mm and a decrease in length of 0.28 mm. Compute the value of Poisson's ratio* $\mu \left(= \frac{1}{m}\right)$ *and modulus of elasticity E.*

Sol. Diameter of the cylinder, $d = 150$ mm
Length of the cylinder, $l = 300$ mm
Increase in diameter, $\delta d = 0.127$ mm (+)
Decrease in length, $l = 0.28$ mm (–)
Axial compressive load, $P = 240$ kN

Poisson's ratio, μ :

We know that,

Linear strain $= \frac{\delta l}{l} = \frac{0.28}{300} = 0.000933$

and, lateral strain $= \frac{\delta d}{d} = \frac{0.127}{150} = 0.000846$

∴ Poisson's ratio, $\mu = \frac{\text{lateral strain}}{\text{linear strain}} = \frac{0.000846}{0.000933} = 0.907$

Modulus of elasticity, E :

Using the relation, $E = \frac{\text{stress}}{\text{strain (linear)}} = \frac{P/A}{\delta l/l}$

$$E = \frac{240/(\pi/4 \times 0.15^2)}{\left(\frac{0.00028}{0.3}\right)} = \frac{240 \times 4 \times 0.3}{\pi \times 0.15^2 \times 0.00028}$$

$$= 14.55 \times 10^6 \text{ kN/m}^2 = 14.55 \text{ GN/m}^2$$

∴ **Young's modulus, E = 14.44 GN/m² (Ans.)**

Example 8.16. *For a given material, Young's modulus is 110 GN/m² and shear modulus is 42 GN/m². Find the bulk modulus and lateral contraction of a round bar of 37.5 mm diameter and 2.4 m long when stretched 2.5 mm.*

Sol. Young's modulus, $E = 110$ GN/m²
Shear modulus, $C = 42$ GN/m²
Diameter of round bar, $d = 37.5$ mm $= 0.0375$ m
Length of round bar, $l = 2.4$ m
Extension of bar, $\delta l = 2.5$ mm $= 0.0025$ m

Bulk modulus, K :

We know that, $E = 2C\left(1 + \frac{1}{m}\right)$

$$110 \times 10^9 = 2 \times 42 \times 10^9 \left(1 + \frac{1}{m}\right)$$

$$\frac{1}{m} = 1.31 - 1 = 0.31 \quad \text{or} \quad m = \frac{1}{0.31} = 3.22$$

Substituting this value of m in the equation

$$K = \frac{mE}{3(m-2)}, \quad K = \frac{3.22 \times 110 \times 10^9}{3(3.22 - 2)} = \mathbf{96.77 \text{ GN/m}^2} \quad \textbf{(Ans.)}$$

Lateral contraction, δd :

Longitudinal strain, $\frac{\delta l}{l} = \frac{0.0025}{2.4} = 0.00104$

and lateral strain $= 0.00104 \times \frac{1}{m} = 0.00104 \times \frac{1}{3.22} = 0.000323$

$\therefore$ Lateral contraction, $\delta d = 0.000323\, d = 0.000323 \times 37.5 =$ **0.0121 mm (Ans.)**

Example 8.17. *The following data relate to a bar subjected to a tensile test :*

Diameter of the bar, $d = 30$ *mm (= 0.03 m)*

Tensile load, $P = 54$ *kN*

Gauge length, $l = 300$ *mm (= 0.3 m)*

Extension of the bar, $\delta l = 0.112$ *mm*

Change in diameter, $\delta l = 0.00366$ *mm*

Calculate : (i) Poisson's ratio (ii) The values of three moduli.

Sol. (*i*) **Poisson's ratio $\frac{1}{m}$ or (μ) :**

Stress, $\sigma = \frac{P}{A} = \frac{54}{\pi/4d^2} = \frac{54}{\frac{\pi}{4} \times (0.03)^2} = 76394 \text{ kN/m}^2 = 76.4 \text{ MN/m}^2$

Linear strain $= \frac{\delta l}{l} = \frac{0.112}{30} = 3.73 \times 10^{-4}$

Lateral strain $= \frac{\delta d}{d} = \frac{0.00366}{30} = 1.22 \times 10^{-4}$

$\therefore$ Poisson's ratio, $\mu = \frac{1}{m} = \frac{\text{lateral strain}}{\text{linear strain}} = \frac{1.22 \times 10^{-4}}{3.73 \times 10^{-4}} =$ **0.327 (Ans.)**

(*ii*) **The values of three moduli, E, C and K :**

We know that, $\mathbf{E} = \frac{\text{stress}}{\text{strain}} = \frac{76.4}{3.73 \times 10^{-4}} = \mathbf{2.05 \times 10^5 \text{ MN/m}^2}$ **(Ans.)**

Also, $E = 2C\left[1 + \frac{1}{m}\right]$ [Eqn. (8.7)]

or $\mathbf{C} = \frac{E}{2\left[1 + \frac{1}{m}\right]} = \frac{2.05 \times 10^5}{2\,(1 + 0.327)} = \mathbf{0.77 \times 10^5 \text{ MN/m}^2}$ **(Ans.)**

Again $E = 3K\left[1 - \frac{2}{m}\right]$

$\therefore$ $\mathbf{K} = \frac{E}{3\left[1 - \frac{2}{m}\right]} = \frac{2.05 \times 10^5}{3\,(1 - 2 \times 0.327)} = \mathbf{1.97 \times 10^5 \text{ MN/m}^2}$ **(Ans.)**

Example 8.18. *A C.I. Flat, 300 mm long and of 30 mm × 50 mm uniform section, is acted upon by the following forces uniformly distributed over the respective cross-section ; 25 kN in the direction of length (tensile) ; 350 kN in the direction of the width (compressive) ; and 200 kN in the direction of thickness (tensile). Determine the change in volume of the flat.*

Take $E = 140$ *GN/m*^{2} *and* $m = 4$.

Sol. Refer Fig. 8.23.

The stresses in the direction of the axes (X, Y, Z) are :

$$\sigma_x = \frac{350000}{0.03 \times 0.3} = 38.8 \times 10^6 \text{ N/m}^2 \text{ (compressive)}$$

The maximum stress of 110 MN/m^2 in the rod and 80 MN/m^2 in the tube will not occur simultaneously, rather the magnitudes of induced stresses in the two materials will be related to each other by eqn. (*ii*). Thus if it is assumed that the stress in steel is the limiting case, then from eqn. (*ii*) for σ_s = 110 MN/m^2,

$$\sigma_b = 0.667 \times 110 + 33.3 = 106.7 \text{ MN/m}^2$$

which is more than the allowable stress in brass leading to the conclusion that the assumption is wrong. Then again assuming the stress in brass to be the limiting case, from (*ii*) we have, for σ_b = 80 MN/m^2,

$$80 = 0.667\,\sigma_s + 33.3$$

$$\therefore \quad \sigma_s = \frac{80 - 33.3}{0.667} = 70 \text{ MN/m}^2$$

which is safe, being less than the maximum allowable stress in steel. Thus substituting the values of σ_b and σ_s in eqn. (*i*), we get

$$P = 706.86 \times 10^{-6} \times 70 + 373.06 \times 10^{-6} \times 80 = 0.079325 \text{ MN or } 79.325 \text{ kN}$$

Hence maximum permissible load, P = 79.325 kN (Ans.)

(*ii*) **The amount by which the tube will be shortened, δl_b :**

In this case

$$\sigma_s = \sigma_b = 110 \text{ MN/m}^2$$

$$\delta l_b = \frac{\sigma_b l_b}{E_b} = \frac{110 \times 0.3}{100 \times 10^3} = 0.00033 \text{ m or } 0.33 \text{ mm}$$

Hence $\quad$ **δl_b = 0.33 mm (Ans.)**

Example 8.24. *Two vertical rods, one of steel and the other of bronze, are rigidly fastened at upper ends at a horizontal distance of 760 mm apart. Each rod is 3 m long and 25 mm in diameter. A horizontal cross-piece connects the lower ends of the bars. Where should a load of 4.5 kN be placed on the cross piece so that it remains horizontal after being loaded ?*

Determine the stresses in each rod. Given :

$$E_s = 210 \text{ GN/m}^2,\ E_b = 112.5 \text{ GN/m}^2.$$

Sol. Refer Fig. 8.30.

Length of each rod $\quad$ = 3 m

Diameter of each rod $\quad$ = 25 mm = 0.025 m

Load, $\quad P$ = 4.5 kN

Let $\quad \sigma_s$ = stress in steel rod,

σ_b = stress in brass rod,

e_s = strain in steel rod,

e_b = strain in bronze rod.

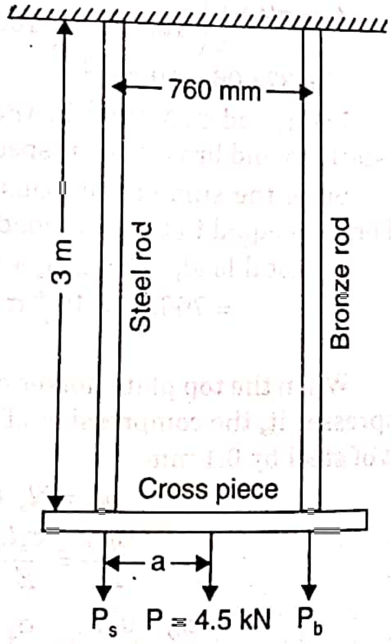

Fig. 8.30

Since the load is placed in such a manner that the cross-piece remains horizontal,

$$\therefore \quad e_s = e_b \text{ or } \frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

i.e.

$$\sigma_s = \sigma_b \cdot \frac{E_s}{E_b} = \sigma_b \cdot \frac{210 \times 10^9}{112.5 \times 10^9}$$

$$= 1.867\,\sigma_b \qquad ...(i)$$

Let P_s and P_b be the loads shared by the steel and bronze rods respectively.

Then $$P = P_s + P_b$$

or $$P = \sigma_s \,.\, A_s + \sigma_b \,.\, A_b \qquad \text{...(ii)}$$

where A_s = cross-sectional area of steel rod, and
A_b = cross-sectional area of bronze rod.

$$\therefore \quad P = 1.867\ \sigma_b \times \frac{\pi}{4} \times (0.025)^2 + \sigma_b \times \frac{\pi}{4} \times (0.025)^2$$

$$4500 = 0.000916\ \sigma_b + 0.000491\sigma_b$$

$$\sigma_b = 3.19 \times 10^6\ \text{N/m}^2 = 3.19\ \text{MN/m}^2$$

and $$\sigma_s = 1.867 \times 3.19 = 5.956\ \text{MN/m}^2$$

$$\therefore \quad P_b = \sigma_b A_b = 3.19 \times 10^6 \times \frac{\pi}{4} \times (0.025)^2 = 1566\ \text{N}$$

Let 'a' be the distance from the steel rod where the load P should be placed so that the cross-piece remains horizontal after being loaded.

Then, $$P_b \times 760 = P \times a$$

$$\therefore \quad a = \frac{P_b \times 760}{P} = \frac{1566 \times 760}{4500} = \mathbf{265\ mm} \quad \textbf{(Ans.)}$$

Example 8.25. *A beam weighing 450 N is held in a horizontal position by three vertical wires, ne attached to each end of the beam, one to the middle of its length. The outer wires are of brass of iameter 1.25 mm and the central wire is of diameter 0.625 mm. If the beam is rigid and wires of the ame length and unstressed before the beam is attached, estimate the stresses induced in the wires. Take Young's modulus for brass as 86 GN/m² and for steel 210 GN/m².*

Sol. Refer Fig. 8.31.

Let P_b = load taken by the brass wire, P_s = load taken by the steel wire.

Then $$2P_b + P_s = P \qquad \text{...(i)}$$

Since the beam is horizontal, all wires will extend by the same amount.

i.e. $e_b = e_s$ ($\because$ length of each wire is same)

where e_b = strain in brass wire, and
e_s = strain in steel wire.

$$\frac{\sigma_b}{E_b} = \frac{\sigma_s}{E_s}$$

$$\frac{P_b}{A_b \,.\, E_b} = \frac{P_s}{A_s \,.\, E_s} \quad \text{or} \quad P_s = \frac{P_b A_s E_s}{A_b E_b}$$

$$= \frac{P_b \times \frac{\pi}{4} \times (0.625 \times 10^{-3})^2 \times 210 \times 10^9}{\frac{\pi}{4}(1.25 \times 10^{-3})^2 \times 86 \times 10^9}$$

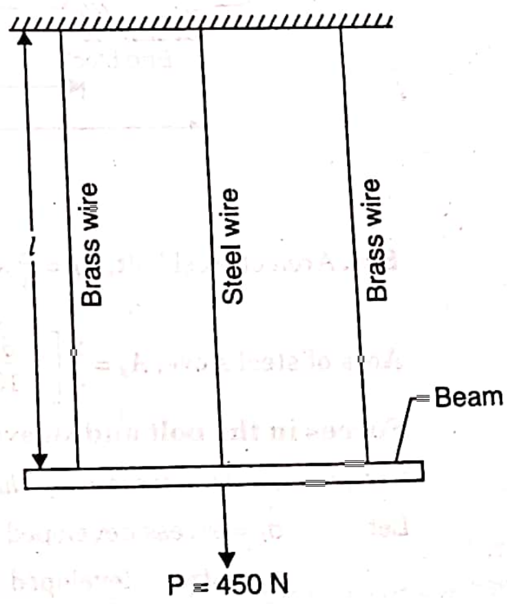

Fig. 8.31

or $$P_s = 0.61\ P_b \qquad \text{...(ii)}$$

Substituting the value of P_s in equation (*i*), we get

$$2P_b + 0.61\ P_b = P$$

$$2.61\ P_b = 450$$

$$P_b = 172.4\ \text{N}$$

and $$P_s = 0.61 \times 172.4 = 105.2\ \text{N}$$

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Now, *stress, induced in the brass wire,*

$$\sigma_b = \frac{P_b}{A_b} = \frac{172.4}{\frac{\pi}{4} \times (1.25 \times 10^{-3})^2} = 1.40 \times 10^8 \text{ N/m}^2$$

$$= \mathbf{140\ MN/m^2} \quad \textbf{(Ans.)}$$

and *stress induced in a steel wire,*

$$\sigma_s = \frac{105.2}{\frac{\pi}{4}(0.625 \times 10^{-3})^2} = 3.429 \times 10^8 \text{ N/m}^2$$

$$= \mathbf{342.9\ MN/m^2} \quad \textbf{(Ans.)}$$

Example 8.26. *A steel bolt and sleeve assembly is shown in Fig. 8.32. The nut is tightened up on the tube through the rigid end blocks until the tensile force in the bolt is 40 kN. If an external load 30 kN is then applied to the end blocks, tending to pull them apart, estimate the resulting force in the bolt and sleeve.*

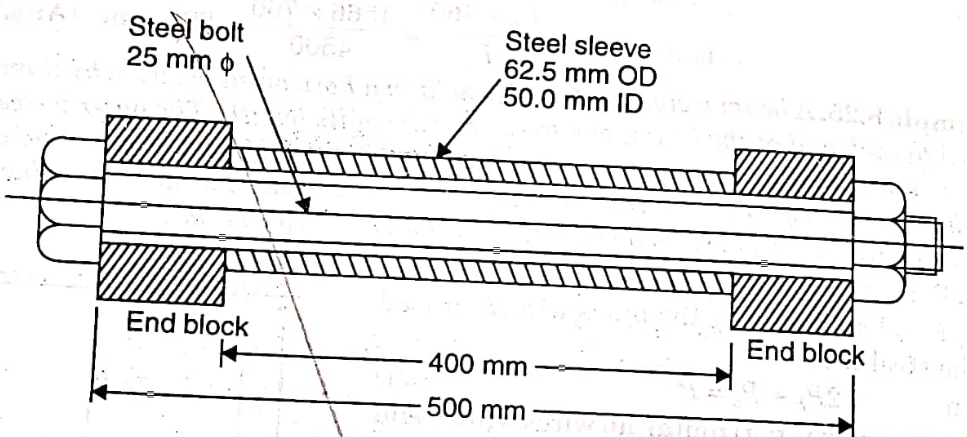

Fig. 8.32

Sol. Area of steel bolt, $A_b = \frac{\pi}{4} \times \left(\frac{25}{1000}\right)^2 = 4.908 \times 10^{-4} \text{ m}^2$

Area of steel slove, $A_s = \frac{\pi}{4}\left[\left(\frac{62.5}{1000}\right)^2 - \left(\frac{50}{1000}\right)^2\right] = 1.104 \times 10^{-3} \text{ m}^2$

Forces in the bolt and sleeve :

(*i*) *Stresses due to tightening the nut :*

Let σ_b = stress developed in steel bolt due to tightening the nut, and

σ_s = stress developed in steel sleeve due to tightening the nut.

Tensile force in the steel bolt = 40 kN = 0.04 MN (given)

Hence, $\sigma_b \times A_b = 0.04$ or $\sigma_b \times 4.908 \times 10^{-4} = 0.04$

$$\therefore \quad \sigma_b = \frac{0.04}{4.908 \times 10^{-4}} = 81.5 \text{ MN/m}^2 \text{ (tensile)}$$

Compressive force in steel sleeve = 0.04 MN

$$\sigma_s \times A_s = 0.04 \quad \text{or} \quad \sigma_s \times 1.104 \times 10^{-3} = 0.04$$

But at the stabilized or common position dd,

$$\text{Push on copper rod} = \text{pull on steel tube}$$

i.e. $$\sigma_c \cdot A_c = \sigma_s \cdot A_s$$

or $$\sigma_c \times 225 = \sigma_s (576 - 324)$$

or $$\sigma_s = \sigma_c \times \frac{225}{(576 - 324)} = 0.89\,\sigma_c \qquad ...(iv)$$

Substituting for σ_s in eqn. (*iii*), we have

$$2.1\,\sigma_c + 0.89\,\sigma_c = 279.3 \quad \text{or} \quad 2.99\,\sigma_c = 279.3$$

$\therefore$ $$\sigma_c = \frac{279.3}{2.99} = \mathbf{93.41\ N/mm^2} \quad \textbf{(Ans.)}$$

From eqn. (*iv*), we get $$\sigma_s = 0.89 \times 93.41 = \mathbf{83.13\ N/mm^2} \quad \textbf{(Ans.)}$$

8.9. STRAIN ENERGY OR RESILIENCE

When an elastic body is loaded it undergoes deformation *i.e.* its dimensions change and when it is relieved of the load it regains its original shape. For the time loaded energy is stored in it, the same is given up or released by the loading when the load is removed. This energy is called **strain energy.** *The strain energy stored by the body 'within' elastic limit, when loaded externally is called* **'resilience'** and the *maximum energy which a body stores 'upto' elastic limit is called* **'Proof resilience'.**

Proof resilience is the mechanical property of materials and it indicates their *capacity to bear shocks.* Proof resilience *per unit volume of* piece is called **'Modulus of resilience'.**

8.10. STRAIN ENERGY IN SIMPLE TENSION AND COMPRESSION

Let us take the case of a bar of cross-sectional area A and length l and subjected to a load W. Suppose this load extends the bar by an amount δl and produces a maximum stress σ.

The work done by W and hence the strain energy (U) stored in the material is equal to the area (shaded in Fig. 8.40) under the force-extension curve.

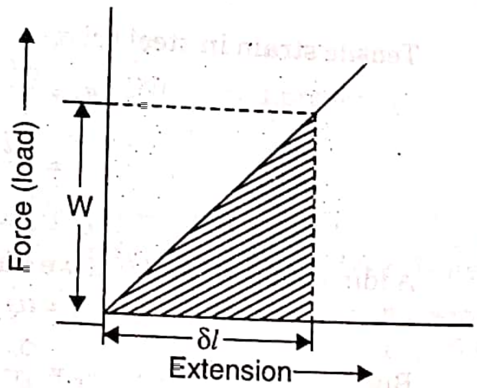

Fig. 8.40

Strain energy stored in the bar

= work done by the load.

$$U = \frac{1}{2} \cdot W \cdot \delta l$$

$$= \frac{W}{2} \cdot \frac{\delta l}{E} \qquad \left[\because \ \delta l = \frac{\sigma l}{E}\right]$$

$$= \frac{\sigma A}{2} \cdot \frac{\sigma l}{E} \qquad \left[\begin{array}{ll} \because & \text{Load} = \text{strees} \times \text{area} \\ \text{or} & W = \sigma \times A \end{array}\right]$$

or $$U = \frac{\sigma^2 A l}{2E} = \frac{\sigma^2 V}{2E} \qquad ...(8.13) \quad \left[\begin{array}{ll} \because & \text{Volume} = \text{area} \times \text{length} \\ \text{or} & V = A \times l \end{array}\right]$$

If σ_p be the proof stress or maximum stress to which the bar is stressed upto the elastic limit, then

Proof resilience, $$U_p = \frac{\sigma_p^{\,2}}{2E} \times V \qquad ...(8.14)$$

and *modulus of resilience* $= \dfrac{\sigma_p^{\,2}}{2E}$. ...(8.15)

8.11. STRESSES DUE TO DIFFERENT TYPES OF LOADS

A body may be subjected to following types of loads :

1. Gradually applied loads
2. Suddenly applied loads
3. Falling or impact loads

1. Gradually applied loads :

A body is said to be acted upon by a gradually applied load if the load increases from zero and reaches its final value stepwise. Let W be the load applied gradually on a body and let δl and σ be the corresponding change in length and maximum stress induced in it.

Energy due to external load $= \dfrac{1}{2}\sigma \,.\, A \,.\, \delta l$

Also, work done on the body $= \dfrac{1}{2} W \,.\, \delta l$

But stain energy stored = work done on the body

$$\frac{1}{2}\sigma A \,.\, \delta l = \frac{1}{2} W \,.\, \delta l \text{ or } \sigma = \frac{W}{A}$$

It may be remembered that *unless specifically mentioned*, the *load is always gradually applied*.

2. Suddenly applied loads :

When the load is applied *all of a sudden* and not stepwise is called *suddenly applied load*. Now let the load W is applied all of a sudden and maximum stress thus produced be σ_{su} ; the extension being the same as δl.

Then, energy stored = external work done

$$W \times \delta l = \frac{1}{2}\sigma_{su} A \times \delta l \text{ or } \sigma_{su} = \frac{2W}{A}$$

This shows that stress (σ_{su}) *due to suddenly applied load is double that of gradually applied load*. Evidently the *instantaneous strain will also be in the same ratio*.

3. Falling or impact loads.

The load which falls from a height or strike the body with certain momentum is called *falling or impact load*.

Refer Fig. 8.41 Consider a weight W falling through a height 'h' on a collar fitted on the rod which is of length 'l' and has a cross-sectional area 'A'. Let extension and maximum stress thus produced be δl_i and σ_i respectively.

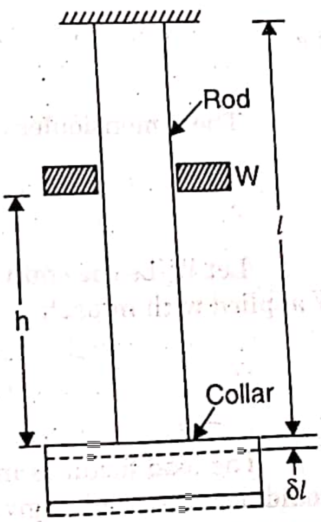

Fig. 8.41

Now, external work done on the bar = energy stored in the bar

i.e. $$W(h + \delta l) = \frac{1}{2}\delta_i A \times \delta l_i$$

or $$W\left[h + \frac{\sigma_i l}{E}\right] = \frac{1}{2}\sigma_i A \times \frac{\sigma_i l}{E} \qquad \left[\because \ \delta l_i = \frac{\sigma_i l}{E}\right]$$

or $$W\left[h + \frac{\sigma_i l}{E}\right] = \frac{\sigma_i^{\,2} A l}{2E}$$

$$\therefore \quad \frac{U_H}{U_S} = \frac{D_H^4 - d_H^4}{D_H^4} = \frac{(n^4 - 1)\, d_H^4}{n^4\, d_H^4} = \frac{n^4 - 1}{n^4}$$

Hence $\dfrac{U_H}{U_S} = \dfrac{n^4 - 1}{n^4}$. **(Ans.)**

From eqns. (1) and (2), we have $\dfrac{T_H}{T_S} = \dfrac{U_H}{U_S}$. ...(2)

8.14. MECHANICAL PROPERTIES OF METALS

1. Strength :

The strength of metal is its ability to withstand various forces to which it is subjected during a test or in service. It is usually defined as tensile strength, compressive strength, proof stress, shear strength, etc. *Strength of materials is a general expression for the measure of capacity of resistance* possessed by solid masses or pieces of various kinds to any cause tending to produce in them a permanent and disabling change of form or positive fracture. Materials of all kinds owe their strength to the action of the forces residing in and about the molecules of the bodies (the molecular forces) but mainly to that ones of these known as *cohesion*; certain modified results of cohesion as toughness or tenacity, hardness, stiffness and elasticity are also important elements, and strength is in relation of the toughness and stiffness combined.

2. Elasticity :

A material is said to be *perfectly elastic* if the whole of the stress produced by a load *disappears completely* on the removal of the load, the modulus of elasticity of Young's modulus (E) is the proportionally constant between stress and strain for eastic materials. *Young's modulus is the indicative of the property called stiffness ; small values of E indicate flexible materials and large value of E reflect stiffness and rigidity.* The property of spring back is a function of modulus of elasticity and refers to the extent to which metal springs back when an elstic deforming load is removed. In metal cutting, modulus of elasticity of the cutting tools and tool holder affect their rigidity. Values of modulus of elasticity for some important metals are given below in Table 8.2.

Table 8.2. Modulus of Elasticity of Some Important Metals

S. No.	*Metals*	*Young's modulus of elasticity, E (GN/m^2)*
1.	Cast iron	100–150
2.	Wrought iron	180–200
3.	Mild steel	210
4.	Aluminium	72
5.	Copper	70–120
6.	Zinc	100
7.	Tungsten	430
8.	Molybdenum	350
9.	Tin	42
10.	Lead	18

3. Plasticity :

- *Plasticity is the property that enables the formation of permanent deformation in a material*. It is reverse of elasticity; a plastic material will retain exactly the shape it takes under load, even after the load is removed. Gold and lead are the highly plastic materials. *Plasticity is used in stamping images on coins and ornamental work.*

- During plastic deformation there is the displacement of atoms within metallic grains and consequently the shapes of the metallic components change. It is because of this property that certain synthetic materials are given and name "plastics". These materials can be changed into required shape easily.

4. Ductility :

It is the ability of a metal to withstand elongation or bending. Due to this property, wires are made by drawing out through a hole. The material shows a considerable amount of plasticity during the ductile extension. This is a valuable property in chains, ropes etc., because they do not snap off, while in service, without giving sufficient warning by elongation.

5. Malleability :

This is the property by virtue of which a material may be hammered or rolled into thin sheets without rupture. This property generally *increases with the increase of temperature*. The metals in order of their ductility and malleability (at room temperature) are given below in Table 8.3.

Table 8.3. Common Metals in order of their Ductility and Malleability

Ductility	*Malleability*
Gold	Gold
Silver	Silver
Platinum	Copper
Iron	Aluminium
Nickle	Tin
Copper	Platium
Aluminium	Lead
Zinc	Zinc
Lead	Nickel

6. Toughness (or Tenacity) :

Toughness (or tenacity) is the strength with which the material opposes rupture. It is due to the attraction which the molecules have for each other; giving them power to resist tearing apart.

The area under the stress-strain curve indicates the toughness (*i.e.*, energy which can be absorbed by the material upto the point of rupture). Although the engineering stress-strain curve is often used for this computation, a more realistic result is obtained from a *true-stress curve*. Toughness is expressed as energy absored (Nm) per unit volume of material participating in absorption (m^3) or Nm/m^3. This result is obtained by multiplying the ordinate by the abscissa (in appropriate units) of stress-strain plot.

7. Brittlenss :

Lack of ductility is brittleness. When a body breaks easily when subjected to shocks it is said to be *brittle*.

8. Hardness :

- *Hardness is usually defined as resistance of material to penetration*. Hard materials resist scratches or being worn out by friction with another body.
- Hardness is primarily a function of the elastic limit (*i.e.*, yield strength) of the material and to a lesser extent a function of the work hardnening co-efficient. The modulus of elasticity also exerts a slight effect on hardness.
- In the most generally accepted test, an indentor is pressed into the surface of the material by slowly applied known load, and the extent of the resulting impression is measured

mechanically or optically. A large impression for a given load and indentor indicates soft material, and the opposite is true for small impression.

- The converse of hardness is known as *softness.*

9. Fatigue :

- When subjected to flucturating or repeating loads (or stresses), materials tend to develop a characteristic behaviour which is different from that (or materials) under *steady loads. Fatigue is the phenomenon that leads to fracture under such conditions.* Fracture takes place under repeated or fluctuating *stresses whose maximum value is less than the tensile strength of the material* (under steady load). Fatigue fracture is progressive, beginning as minute cracks that grow under the action of the fluctuating stress.
- *Fatigue fracture starts at the point highest stress*. This point may be determined by the shape of the part; for instant, by stress concentration in a groove. It can also be caused by surface finish, such as tool marks or scratches, and by *internal voids such as shrinking cracks and cooling in castings and weldments and defects introduced during mechanical working and by defects, stresses introduced by electroplating*. It must be remembered that surface and internal defects are stress raisers, and the point of highest actual stress may occur at these rather than at the minimum cross-section of highest normal stress. Thus processing methods are extremely important as they affect fatigue behaviour.

10. Creep :

- *Creep is the slow plastic deformation of metals under constant stress or under prolonged loading usually at high temperature. It can take place and lead to fracture at static stresses much smaller than those which will break the specimen by loading it quickly*. Creep is specially taken care of while designing I.C. engines, boilers and turbines.
- The creep at a room temperature is known as *low temperature creep* and occurs in load pipes, roofings, glass as well as in white metal bearings. The creep at high temperatures is known *high temperature creep*. It mainly depends upon metal, service temperature to be encountered and the *stress involved*. For studying its effects, the specimens are put under a constant load; the creep is measured during various time intervals and results then plotted to get a *creep curve.*

Importance of mechanical tests :

- Structures, machines and products of various kinds are usually subjected to load and deformation. Therefore, the properties of materials under the action of load and deformation so produced under various environments become an important engineering consideration. *The microscopic properties of materials under applied forces or loads are broadly classed as mechanical properties.* They are a measure of the strength and lasting characteristic of a material in service and are of great importance particular to the design engineer. Unfortunately these properties cannot be desired from the structural or bonding considerations alone since most of them are structure-sensitive, are much more *affected by crystal imperfections* and other factors such as composition, grain size, heat treatment etc. Therefore, mechanical properties do not depend on them in all situations. A great number of mechanical properties, are, therefore, best evaluated by mechanical testing of the materials like metals and alloys.
- The following important mechanical tests give valuable information about metals and alloys as given below :

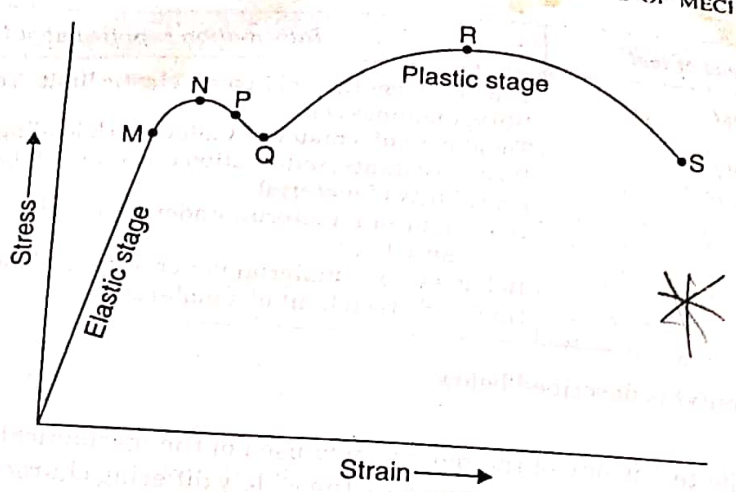

Fig. 8.49. Stress-strain curve.

the mild steel yielding commences immediately and two points P and Q, the *upper and lower yield points* respectively are obtained. On further increasing the load slightly, the *strain increases rapidly* till R when *neck or waist* is formed. When this point (R) is reached the deformation or extension continues even with lesser load and ultimately *feature occurs.*

The various properties connected with this test are given more elaborately in the following paragraphs :

(*i*) **Proportional limit.** *It is the maximum stress at which stress remains directly proportional to strain*. The proportional limit is determined from the stress strain curve by drawing straight line tangent at the origin and noting the first deviation of the plot from the line.

The proportional limit has *limited engineering significance* because of its great dependence upon the precision available for its determination.

(*ii*) **Elastic limit.** *The elastic limit is the maximum stress which the material can withstand without causing permanent deformation which remains after removal of stress.*

For engineering usage the elastic limit has *little significance.*

(*iii*) **Yield strength.** *The yield strength is the stress at which a material exhibits a specified limiting permanent set.*

The yield strength of a metal is a property of *considerable significance*. The tensile yield strength indicates resistance to permanent deformation produced by tensile loads. It is related to resistance to permanent deformation by shearing, bending, compressive, and complex combination of forces. Because of this and the ease of its measurement the tensile yield strength is *used widely as a factor of design*; it is preferable in most instances to the use of tensile strength. The yield strength also is *indicative of the ease of forming or shaping metals by mechanical means.*

(*iv*) **Yield point.** The *yield point is the stress at which there first occurs a marked increase in strain without an increase in stress.* The yield point can be determined by noting the first load at which there is visible increase in the distance between two gauge marks on a tensile specimen. This is conveniently accomplished by checking the length with a pair of dividers. If an extensometer is used, the length can be observed to increase rapidly without an increase in load. Still a third method is to coat the specimen with lacquer which cracks when the yield point is reached. *The yield point most commonly is observed in mild steels, although it has been detected in a few other alloys as well.*

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(*v*) **Tensile strength** (Ultimate or maximum strength). *It is calculated by dividing the maximum load carried by the specimen during a tension test by the original cross-sectional area of the specimen.*

Tensile strength is widely used design factor, although there is more justification for yield strength.

(*vi*) **Rupture strength.** *It is determined by dividing the load at the time of fracture by the original cross-sectional area.* If the rupture load is divided by the actual cross-section at the time of fracture, *the time rupture strength is obtained.*

The rupture strength is of indirect and limited interest to engineers. It provides the terminal point of the stress-strain curve and makes possible a computation of *static toughness.*

(*vii*) **Elongation.** Elongation of a specimen after fracture may be determined by placing the parts of the broken specimen closely together and holding them in place by a vice. The distance between gauge marks may be measured by means of dividers.

$$\text{Percentage elongation} = \frac{\text{final length} - \text{original length}}{\text{original length}} \times 100$$

Elongation has considerable engineering significance because it indicates ductility or the ability to deform appreciably without rupture. Ductility is essential in forming operation for metals, where it is desirable to achieve as much deformation as possible in one operation without danger of causing rupture. Ductility is also essential to avoid local failures leading to overall failures in metal members which are locally highly stressed as a result of design or fabricating techniques.

(*viii*) **Reduction of area :** After the metal has fractured the percentage reduction in area is calculated by measuring the test piece diameter at the point of fracture, calculating the cross-sectional area at this point, and expressing it as a percentage of original area.

$$\text{Percentage reduction of area} = \frac{\text{original area} - \text{area at fracture}}{\text{original area}} \times 100$$

(*ix*) **Modulus of elasticity :** Below the proportional limit stress and strain are related to one another by a constant of proportionality known as modulus of elasticity.

Thus, $$E = \frac{\text{stress}}{\text{strain}}$$

The value of E is determined from the stress strain curve by measuring the slope of initial straight line portion of the plot.

E indicates resistance to elastic deformation. Resistance to elastic deformation is more commonly called *stiffness.*

Note : *Proof stress* is the stress at which the stress strain curve departs from a straight line by not more than 0.1 percent of length of test piece. The material is said to have passed the proof stress test if application of certain load for 15 seconds does not produce more than 0.1 percent elongation.

Stress-strain curve for 'Brittle materials' :

Structural steel is the only material that exhibits a marked yield point. Most of the other materials show a gradual change from linear to the non-linear range. **Brittle materials have a very low proportional point and do not show the yield point.** Fig. 8.50 shows a typical stress-strain curve for cast iron.

Note : The stress-strain curve for *compression* can *similarly* be plotted to determine the characteristic stress such as proportional stress, yield stress, and ultimate stress. In case of *steel these stresses are the same both in tension and in compression.*

Solution. Initial, original or nominal diameter, $d_0 = 12.7$ mm
Instantaneous or final diameter, $d = 7.87$ mm
Load applied = 14 kN

(*i*) **True breaking strength** $= \dfrac{14 \times 1000}{\frac{\pi}{4} d^2} = \dfrac{14 \times 1000}{\frac{\pi}{4} \times 7.87^2} = \mathbf{287.8\ N/mm^2}$. **(Ans.)**

(*ii*) **Nominal breaking strength** $= \dfrac{14 \times 1000}{\frac{\pi}{4} \times 12.7^2} = \mathbf{110.5\ N/mm^2}$. **(Ans.)**

(*iii*) **True fracture strain** $\varepsilon_x = 2 \log_e \left(\dfrac{d_0}{d}\right) = 2 \log_e \left(\dfrac{12.7}{7.87}\right) = \mathbf{0.957}$. **(Ans.)**

Example 8.51. *The following observations were made during a tensile test on a mild steel specimen 40 mm in diameter and 200 mm long.*

Elongation with 40 kN load (within limit of proportionality), $\delta l = 0.0304$ mm.

Yield load = 161 kN ; Maximum load = 242 kN ; Length of specimen at fracture = 249 mm.

Determine :

(*i*) *Young's modulus of elasticity,*
(*ii*) *Yield point stress,*
(*iii*) *Ultimate stress, and*
(*iv*) *Percentage elongation.*

Solution. (*i*) **Young's modulus of elasticity E :**

$$\text{Stress } \sigma = \frac{W}{A} = \frac{40}{\frac{\pi}{4} \times (0.04)^2} = 3.18 \times 10^4 \text{ kN/m}^2$$

$$\text{Strain } e = \frac{\delta l}{l} = \frac{0.0304}{200} = 0.000152$$

$$E = \frac{\text{stress}}{\text{strain}} = \frac{3.18 \times 10^4 \text{ kN/m}^2}{0.000152} = \mathbf{2.09 \times 10^8\ kN/m^2}. \quad \textbf{(Ans.)}$$

(*ii*) **Yield point stress** $= \dfrac{\text{yield point load}}{\text{area}} = \dfrac{161}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{12.8 \times 10^4\ kN/m^2}$. **(Ans.)**

(*iii*) **Ultimate stress** $= \dfrac{\text{maximum load}}{\text{area}} = \dfrac{242}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{19.26 \times 10^4\ kN/m^2}$. **(Ans.)**

(*iv*) **Percentage elongation** $= \dfrac{\text{length of specimen at fracture} - \text{original length}}{\text{original length}}$

$$= \frac{249 - 200}{200} = 0.245 = \mathbf{0.245\%}. \quad \textbf{(Ans.)}$$

Example 8.52. *In order to evaluate various mechanical properties, a steel specimen of 12.5 mm diameter and 62.5 mm gauge was tested in a standard tension test. Following observations were made during the test :*

Yield load = 40.0 kN ; Maximum load = 71.5 kN ; Fracture load = 50.5 kN ; Gauge length at fracture = 79.5 mm ; Strain at load of 20 kN = 7.75×10^{-4}.

Determine :

(*i*) *Yield point stress*
(*ii*) *Ultimate tensile stress*
(*iii*) *Percentage elongation*
(*iv*) *Modulus of elasticity*
(*v*) *Modulus of resilience*
(*vi*) *Fracture stress.*

Solution. (*i*) **Yield point stress σ_y :**

$$\text{Yield stress, } \sigma_y = \frac{\text{load at lower yield point}}{\text{original area}}$$

or
$$\sigma_y = \frac{40 \times 10^3}{\frac{\pi}{4} \times 12.5^2} = \mathbf{325.95\ N/mm^2.}\quad \textbf{(Ans.)}$$

(*ii*) **Ultimate tensile strength, σ_u :**

$$\text{Ultimate tensile strength} = \frac{\text{maximum load}}{\text{original load}} = \frac{71.5 \times 10^3}{\frac{\pi}{4} \times (12.5)^2} = \mathbf{582.6\ N/mm^2.}\quad \textbf{(Ans.)}$$

(*iii*) **Percentage elongation :**

$$\text{Percentage elongation, } \sigma_u = \frac{\text{length at fracture} - \text{original length}}{\text{original length}}$$

$$= \frac{79.5 - 62.5}{62.5} \times 100 = \mathbf{27.2\%.}\quad \textbf{(Ans.)}$$

(*iv*) **Modulus of elasticity, E :**

$$\text{Stress at 20 kN} = \frac{20 \times 10^3}{\frac{\pi}{4} \times (12.5)^2} = 162.97\ \text{N/mm}^2$$

$$\text{Strain at 20 kN} = 7.75 \times 10^{-4}$$

$$\therefore\ \text{Modulus of elasticity, } E = \frac{\text{stress}}{\text{strain}} = \frac{162.97}{7.75 \times 10^{-4}} = 2.1 \times 10^5\ \text{N/mm}^2.\quad \textbf{(Ans.)}$$

(*v*) **Modulus of resilience :**

$$\text{Modulus of resilience} = \frac{\sigma_y^2}{2E} = \frac{(325.95)^2}{2 \times 2.1 \times 10^5} = \mathbf{0.252.}\quad \textbf{(Ans.)}$$

(*vi*) **Fracture stress :**

$$\text{Fracture stress} = \frac{\text{fracture load}}{\text{original area}} = \frac{50.5 \times 10^3}{\frac{\pi}{4} \times (12.5)^2} = \mathbf{411.5\ N/mm^2.}\quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. A load may be defined as the combined effect of external forces acting on a body.
   The loads may be classified as :
   (*i*) Dead loads, (*ii*) Live or fluctuating loads, (*iii*) Inertia loads or forces and (*iv*) Centrifugal loads or forces.
   Or (*i*) Tensile loads, (*ii*) Compressive loads, (*iii*) Torsional or twisting loads, (*iv*) Bending loads and (*v*) shearing loads, or (*i*) Point loads, (*ii*) distributed loads.
2. The internal resistance which the body offers to meet the load is called *stress*. When any type of *simple* stress develops, we can calculate the magnitude of the stress by
   $$\sigma = \frac{P}{A}$$
   where σ = stress, P = load, and A = area over which stress develops.
3. The strain (e) is the deformation produced by the stress.

   Tensile strain, $$e_t = \frac{\delta l}{l} = \frac{\text{increase in length}}{\text{original length}}$$

   Compressive strain, $$e_c = \frac{\delta l}{l} = \frac{\text{decrease in length}}{\text{original length}}$$

Shear strain, $e_s = \tan \phi = \phi$ radian. (Since ϕ is very small)

Volumetric strain, $e_v = \frac{\delta V}{V}$.

4. Hooke's law states that within elastic limit, stress varies directly as strain

i.e., stress α strain or $\frac{\text{Stress}}{\text{Strain}}$ = constant

Modulus of elasticity, $E = \frac{\sigma}{e}$

Modulus of rigidity, C (or N or G) $= \frac{\tau}{e_s}$

Bulk modulus of elasticity, $K = \frac{\sigma_n}{e_v}$.

5. Stress (σ) and elongation (δl) produced in a bar due to its own weight :

$$\sigma = 9.81 \rho y \text{ N/m}^2, \delta l = \frac{9.81 \rho l^2}{2E}$$

(where ρ = density of material, kg/m^3).

6. Elongation in case of a tapered rod, $\delta l = \frac{4Pl}{\pi E d_1 d_2}$.

7. Elongation of *e* conical bar due to its self-weight, $\delta l = \frac{9.81 \rho l^2}{6E}$.

8. In case of a bar of uniform strength, $A_2 = A_1 e^{9.81 \rho l/\sigma}$.

9. Maximum stress (σ_{max}) and the total extension (δl) in a bar due to rotation :

$$\sigma_{max} = \frac{1}{8} \rho \omega^2 . l^2 \quad \text{(where } \omega = \text{angular velocity)}$$

$$\delta l = \frac{\rho \omega^2 l^2}{12E}.$$

10. The ratio of lateral strain to linear strain is known as *Poisson's ratio*

i.e., Poisson's ratio $= \frac{\text{lateral or secondary strain}}{\text{linear or primary strain}} = \frac{1}{m}$

where m is a constant and it value varies between 3 and 4 for different metals.

11. Relations between elastic moduli :

$$E = 2C\left(1 + \frac{1}{m}\right) \quad ...(i)$$

$$E = 3K\left(1 - \frac{2}{m}\right) \quad ...(ii)$$

$$E = \frac{9KC}{3K + C} \quad ...(iii)$$

12. If the temperature of a body is lowered or raised, its dimensions will decrease or increase correspondingly. If these changes, however, are checked, the stresses thus developed in the body are called *temperature stresses* and the corresponding strains are called *temperature strains*.

Temperature strain $= \alpha (t_2 - t_1)$ ...(Compressive) When the temperature

Temperature stress developed $= \alpha (t_2 - t_1) E$ ... (Compressive) of the body is raised

If however, the temperature of the body is lowered, the temperature strain will be *tensile* in nature.

OBJECTIVE TYPE QUESTIONS

A. Choose the Correct Answer :

1. The combined effect of external forces acting on a body is called

(*a*) stress (*b*) strain (*c*) load (*d*) none of the above.