

Mutually Exclusive:  $A \cap B = \emptyset$ ;  $P(A \cup B) = P(A) + P(B)$

Collectively Exhaustive events:  $A \cup B = \text{sample space}$ .

Independent Events  $\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Random V(X)

Discrete

$$\text{PMF: } f(n) \geq 0$$

$$\sum P(n) = 1$$

$\rightarrow$  Binomial

$\rightarrow$  Poisson

continuous.

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$\rightarrow$  Uniform

$\rightarrow$  Normal

$\rightarrow$  Exponential

PMF / PDF are same, depends on context.

$x$	0	1	2
$P(n)$	a	b	c

$x$	Interval	$I_2$	$I_3$
$P(n)$	a	b	c

Cumulative DF

$x$	0	1	2
$f(n)$	a	a+b	a+b+c

Prefix sum.

Mean =  $\bar{x}$

$$E(x) = \sum x P(n)$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

Sum of sq. - sq. of sum.

$$\text{Gamma function} \quad \int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma(n)}{a^n}$$

Mean of general function:

$$E(\phi(x)) = \sum \phi(n) P(n)$$

$$\text{or } \int \phi(x) f(x) dx$$

Binomial Distribution:  $n \rightarrow \text{finite}$

$$P(n) = {}^n C_n p^n q^{n-n} \quad [p+q=1] \quad \text{PMF}$$

$$\text{PMF} = [p+q]^n = \binom{n}{k} = 1$$

$$\begin{cases} \text{mean} = np \\ \text{variance} = npq \end{cases}$$

binomial

$$\text{MGF (Moment generating function)} = E(e^{xt})$$

$$E(e^{xt}) = (pe^t + q)^n \quad \text{solve}$$

$$\text{Characteristic fun: } E(e^{itx}) \quad t \rightarrow it$$

characteristic function

$$\rightarrow (pe^{itx} + q)^n$$

PGF (Probability Generating Function)

$$\rightarrow E(z^n) = (zp+q)^n$$

pgf probability generating

Poisson Distribution

poisson

$n \rightarrow \infty$   $p \rightarrow 0$

but  $np$  is finite.

$$\text{PMF} = P(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

where  $\lambda = \text{mean} = np$

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$\rightarrow \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$\text{MGF} = e^{\lambda(e^z - 1)} \quad \text{PGF} = e^{\lambda(z-1)}$$

Mean = Variance

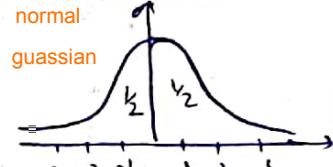
Normal Distribution / Gaussian Distribution

Mean = Median = Mode

normal

guassian

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$\text{Area: } (-1 < z < 1) : 0.68$$

$$(-2 < z < 2) : 0.95$$

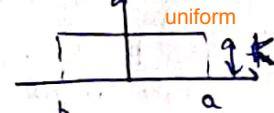
$$(-3 < z < 3) : 0.99$$

$$(-\infty < z < \infty) : 1$$

Rectangular / Uniform distribution

rectangular uniform

$$f(x) = \begin{cases} K & b < x < a \\ 0 & \text{else} \end{cases}$$



$$\text{We know } \int_a^b f(x) dx = 1 \Leftrightarrow \text{Area}$$

$$\rightarrow \text{so } K = \frac{1}{a-b} \quad \text{mgf } (e^{tb} - e^{ta}) / (b-a)t$$

$$\text{mean} = \frac{a+b}{2} = E(x)$$

~~E(x^2) Variance =  $\frac{(b-a)^2}{12}$~~

Moments Representation

about Origin  $\rightarrow \mu_0 = 0$

moments  $\mu_1 \rightarrow \mu_1^0$   $\mu_2 \rightarrow \mu_2^0$   $\mu_3 \rightarrow \mu_3^0$

Mean  $\mu_0^0 = E(x - \bar{x})^0$

Any value  $\mu_1^0 = 0$

$\mu_2^0 = E(x^2) - (\bar{x})^2$

$\mu_3^0 = 0$

$\mu_4^0 = \mu_2^1 - (\mu_1^0)^2$

$\mu_5^0 = 0$

$\mu_6^0 = \mu_3^1 - 3\mu_2^1\mu_1^0 + 2\mu_1^0$

$\mu_7^0 = 0$

$\mu_8^0 = \mu_4^1 - 4\mu_3^1\mu_1^0 + 6\mu_2^1\mu_1^0 - 3\mu_1^0$

$\mu_9^0 = 0$

$\mu_{10}^0 = \mu_5^1 - 10\mu_4^1\mu_1^0 + 30\mu_3^1\mu_1^0 - 30\mu_2^1\mu_1^0 + 5\mu_1^0$

$\mu_{11}^0 = 0$

$\mu_{12}^0 = \mu_6^1 - 15\mu_5^1\mu_1^0 + 45\mu_4^1\mu_1^0 - 45\mu_3^1\mu_1^0 + 15\mu_2^1\mu_1^0 - \mu_1^0$

$\mu_{13}^0 = 0$

$\mu_{14}^0 = \mu_7^1 - 21\mu_6^1\mu_1^0 + 63\mu_5^1\mu_1^0 - 63\mu_4^1\mu_1^0 + 21\mu_3^1\mu_1^0 - \mu_2^1\mu_1^0$

$\mu_{15}^0 = 0$

$\mu_{16}^0 = \mu_8^1 - 28\mu_7^1\mu_1^0 + 84\mu_6^1\mu_1^0 - 84\mu_5^1\mu_1^0 + 28\mu_4^1\mu_1^0 - 7\mu_3^1\mu_1^0$

$\mu_{17}^0 = 0$

$\mu_{18}^0 = \mu_9^1 - 35\mu_8^1\mu_1^0 + 105\mu_7^1\mu_1^0 - 105\mu_6^1\mu_1^0 + 35\mu_5^1\mu_1^0 - 7\mu_4^1\mu_1^0$

$\mu_{19}^0 = 0$

$\mu_{20}^0 = \mu_{10}^1 - 42\mu_9^1\mu_1^0 + 126\mu_8^1\mu_1^0 - 126\mu_7^1\mu_1^0 + 42\mu_6^1\mu_1^0 - 7\mu_5^1\mu_1^0$

$\mu_{21}^0 = 0$

$\mu_{22}^0 = \mu_{11}^1 - 56\mu_{10}^1\mu_1^0 + 168\mu_9^1\mu_1^0 - 168\mu_8^1\mu_1^0 + 56\mu_7^1\mu_1^0 - 7\mu_6^1\mu_1^0$

$\mu_{23}^0 = 0$

$\mu_{24}^0 = \mu_{12}^1 - 70\mu_{11}^1\mu_1^0 + 210\mu_{10}^1\mu_1^0 - 210\mu_9^1\mu_1^0 + 70\mu_8^1\mu_1^0 - 7\mu_7^1\mu_1^0$

$\mu_{25}^0 = 0$

$\mu_{26}^0 = \mu_{13}^1 - 84\mu_{12}^1\mu_1^0 + 252\mu_{11}^1\mu_1^0 - 252\mu_{10}^1\mu_1^0 + 84\mu_9^1\mu_1^0 - 7\mu_8^1\mu_1^0$

$\mu_{27}^0 = 0$

$\mu_{28}^0 = \mu_{14}^1 - 98\mu_{13}^1\mu_1^0 + 294\mu_{12}^1\mu_1^0 - 294\mu_{11}^1\mu_1^0 + 98\mu_{10}^1\mu_1^0 - 7\mu_9^1\mu_1^0$

$\mu_{29}^0 = 0$

$\mu_{30}^0 = \mu_{15}^1 - 112\mu_{14}^1\mu_1^0 + 336\mu_{13}^1\mu_1^0 - 336\mu_{12}^1\mu_1^0 + 112\mu_{11}^1\mu_1^0 - 7\mu_{10}^1\mu_1^0$

$\mu_{31}^0 = 0$

$\mu_{32}^0 = \mu_{16}^1 - 126\mu_{15}^1\mu_1^0 + 378\mu_{14}^1\mu_1^0 - 378\mu_{13}^1\mu_1^0 + 126\mu_{12}^1\mu_1^0 - 7\mu_{11}^1\mu_1^0$

$\mu_{33}^0 = 0$

$\mu_{34}^0 = \mu_{17}^1 - 140\mu_{16}^1\mu_1^0 + 420\mu_{15}^1\mu_1^0 - 420\mu_{14}^1\mu_1^0 + 140\mu_{13}^1\mu_1^0 - 7\mu_{12}^1\mu_1^0$

$\mu_{35}^0 = 0$

$\mu_{36}^0 = \mu_{18}^1 - 154\mu_{17}^1\mu_1^0 + 462\mu_{16}^1\mu_1^0 - 462\mu_{15}^1\mu_1^0 + 154\mu_{14}^1\mu_1^0 - 7\mu_{13}^1\mu_1^0$

$\mu_{37}^0 = 0$

$\mu_{38}^0 = \mu_{19}^1 - 168\mu_{18}^1\mu_1^0 + 510\mu_{17}^1\mu_1^0 - 510\mu_{16}^1\mu_1^0 + 168\mu_{15}^1\mu_1^0 - 7\mu_{14}^1\mu_1^0$

$\mu_{39}^0 = 0$

$\mu_{40}^0 = \mu_{20}^1 - 182\mu_{19}^1\mu_1^0 + 588\mu_{18}^1\mu_1^0 - 588\mu_{17}^1\mu_1^0 + 182\mu_{16}^1\mu_1^0 - 7\mu_{15}^1\mu_1^0$

$\mu_{41}^0 = 0$

$\mu_{42}^0 = \mu_{21}^1 - 196\mu_{20}^1\mu_1^0 + 630\mu_{19}^1\mu_1^0 - 630\mu_{18}^1\mu_1^0 + 196\mu_{17}^1\mu_1^0 - 7\mu_{16}^1\mu_1^0$

$\mu_{43}^0 = 0$

$\mu_{44}^0 = \mu_{22}^1 - 210\mu_{21}^1\mu_1^0 + 693\mu_{20}^1\mu_1^0 - 693\mu_{19}^1\mu_1^0 + 210\mu_{18}^1\mu_1^0 - 7\mu_{17}^1\mu_1^0$

$\mu_{45}^0 = 0$

$\mu_{46}^0 = \mu_{23}^1 - 224\mu_{22}^1\mu_1^0 + 728\mu_{21}^1\mu_1^0 - 728\mu_{20}^1\mu_1^0 + 224\mu_{19}^1\mu_1^0 - 7\mu_{18}^1\mu_1^0$

$\mu_{47}^0 = 0$

$\mu_{48}^0 = \mu_{24}^1 - 238\mu_{23}^1\mu_1^0 + 784\mu_{22}^1\mu_1^0 - 784\mu_{21}^1\mu_1^0 + 238\mu_{20}^1\mu_1^0 - 7\mu_{19}^1\mu_1^0$

$\mu_{49}^0 = 0$

$\mu_{50}^0 = \mu_{25}^1 - 252\mu_{24}^1\mu_1^0 + 840\mu_{23}^1\mu_1^0 - 840\mu_{22}^1\mu_1^0 + 252\mu_{21}^1\mu_1^0 - 7\mu_{20}^1\mu_1^0$

$\mu_{51}^0 = 0$

$\mu_{52}^0 = \mu_{26}^1 - 266\mu_{25}^1\mu_1^0 + 906\mu_{24}^1\mu_1^0 - 906\mu_{23}^1\mu_1^0 + 266\mu_{22}^1\mu_1^0 - 7\mu_{21}^1\mu_1^0$

$\mu_{53}^0 = 0$

$\mu_{54}^0 = \mu_{27}^1 - 280\mu_{26}^1\mu_1^0 + 972\mu_{25}^1\mu_1^0 - 972\mu_{24}^1\mu_1^0 + 280\mu_{23}^1\mu_1^0 - 7\mu_{22}^1\mu_1^0$

$\mu_{55}^0 = 0$

$\mu_{56}^0 = \mu_{28}^1 - 294\mu_{27}^1\mu_1^0 + 1038\mu_{26}^1\mu_1^0 - 1038\mu_{25}^1\mu_1^0 + 294\mu_{24}^1\mu_1^0 - 7\mu_{23}^1\mu_1^0$

$\mu_{57}^0 = 0$

$\mu_{58}^0 = \mu_{29}^1 - 308\mu_{28}^1\mu_1^0 + 1104\mu_{27}^1\mu_1^0 - 1104\mu_{26}^1\mu_1^0 + 308\mu_{25}^1\mu_1^0 - 7\mu_{24}^1\mu_1^0$

$\mu_{59}^0 = 0$

$\mu_{60}^0 = \mu_{30}^1 - 322\mu_{29}^1\mu_1^0 + 1170\mu_{28}^1\mu_1^0 - 1170\mu_{27}^1\mu_1^0 + 322\mu_{26}^1\mu_1^0 - 7\mu_{25}^1\mu_1^0$

$\mu_{61}^0 = 0$

$\mu_{62}^0 = \mu_{31}^1 - 336\mu_{30}^1\mu_1^0 + 1236\mu_{29}^1\mu_1^0 - 1236\mu_{28}^1\mu_1^0 + 336\mu_{27}^1\mu_1^0 - 7\mu_{26}^1\mu_1^0$

$\mu_{63}^0 = 0$

$\mu_{64}^0 = \mu_{32}^1 - 350\mu_{31}^1\mu_1^0 + 1302\mu_{30}^1\mu_1^0 - 1302\mu_{29}^1\mu_1^0 + 350\mu_{28}^1\mu_1^0 - 7\mu_{27}^1\mu_1^0$

$\mu_{65}^0 = 0$

$\mu_{66}^0 = \mu_{33}^1 - 364\mu_{32}^1\mu_1^0 + 1368\mu_{31}^1\mu_1^0 - 1368\mu_{30}^1\mu_1^0 + 364\mu_{29}^1\mu_1^0 - 7\mu_{28}^1\mu_1^0$

$\mu_{67}^0 = 0$

$\mu_{68}^0 = \mu_{34}^1 - 378\mu_{33}^1\mu_1^0 + 1434\mu_{32}^1\mu_1^0 - 1434\mu_{31}^1\mu_1^0 + 378\mu_{30}^1\mu_1^0 - 7\mu_{29}^1\mu_1^0$

$\mu_{69}^0 = 0$

$\mu_{70}^0 = \mu_{35}^1 - 392\mu_{34}^1\mu_1^0 + 1500\mu_{33}^1\mu_1^0 - 1500\mu_{32}^1\mu_1^0 + 392\mu_{31}^1\mu_1^0 - 7\mu_{30}^1\mu_1^0$

$\mu_{71}^0 = 0$

$\mu_{72}^0 = \mu_{36}^1 - 406\mu_{35}^1\mu_1^0 + 1566\mu_{34}^1\mu_1^0 - 1566\mu_{33}^1\mu_1^0 + 406\mu_{32}^1\mu_1^0 - 7\mu_{31}^1\mu_1^0$

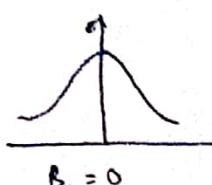
$\mu_{73}^0 = 0$

$\mu_{74}^0 = \mu_{37}^1 - 420\mu_{36}^1\mu_1^0 + 1632\mu_{35}^1\mu_1^0 - 1632\mu_{34}^1\mu_1^0 + 420\mu_{33}^1\mu_1^0 - 7\mu_{32}^1\mu_1^0$

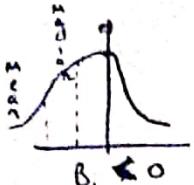
Skewness ! lack of symmetry.

$$B_1 = \frac{(\bar{u}_3)^2}{(\bar{u}_2)^3}$$

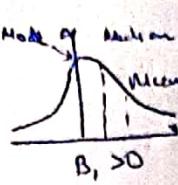
$\bar{u}_1 \rightarrow$  about mean



$$B_1 = 0$$



$B_1 < 0$



$B_1 > 0$

Mean = Median = Mode

-ve skewed.

+ve skewed

Distributions

with replacement  $\rightarrow$  independent trials  
without  $\rightarrow$  not independent

(i) Binomial!  $\rightarrow P(X=x) = {}^n C_x p^x (1-p)^{n-x}$

$$\text{mean} = np \quad \sigma^2 = npq$$

independent trials

(ii) Hypergeometric! dependent trials. hypergeometric

$n \rightarrow$  objects  $a \rightarrow$  success  $(N-a) \rightarrow$  failure

$$P(X=x) = \frac{{}^a C_x}{{}^N C_n} \frac{{}^{N-a} C_{n-x}}{{}^N C_n}$$

↓ solve normally without formula

$$\text{mean} = n \frac{a}{N}$$

$$\sigma^2 = ?$$

Sometime Ans of Binomial  $\sim$  Hypergeom ( $\approx$ )

(iii) Geometric: First success on  $n^{th}$  trial geo

i.e. (i)  $(n-1)$   $\rightarrow$  failure i.e.  $(1-p)^{n-1}$  geometric  
(ii)  $n^{th}$  must be success  $P$

$$\text{PMF} = P(X=n) = (1-p)^{n-1} p$$

$$\text{mean} = \frac{1}{p} \quad \sigma^2 = \frac{1-p}{p^2} \quad P(X \leq x) = 1 - (1-p)^x$$

Negative Binomial:

negative

(i) First  $(r-1)$  trials  $\rightarrow$   $(r-1)$  success  
(solve via Binomial)

(ii)  $n^{th}$   $\rightarrow$  success

$$\text{mean} = \frac{r}{p} \quad \sigma^2 = \frac{r(1-p)}{p^2}$$

Multinomial

$$P(X=x_1, Y=y_1, Z=z_1, \dots, N=n) = \frac{n!}{x_1! y_1! z_1! \dots n!} p_1^{x_1} q_1^{y_1} \dots p_n^{z_n} q_n^{n-z}$$

multinomial  
multi

General

$$P(X=x_1, \dots, Y=y_n) = \frac{n!}{x_1! x_2! \dots x_n!} p_1^{x_1} p_2^{x_2} \dots (p_n)^{x_n}$$

$x_i = 0, 1, 2, \dots, n$

$$\sum x_i = 1$$

mgf change origin and scale :

$y = (x-a)/h$  (origin shift of  $+a$  and scale of  $h$ )

$M_y(t) = e^{(-at/h)} M_x(t/h)$

cumulates distribution func =  $\log(MGF)$

marginal

$X=0, 1, \dots, n$	$Y=0, 1, \dots, m$	Total
Marginal X	Marginal Y	Total
0	0	$\Sigma$
1	1	$\Sigma$
2	2	$\Sigma$
$\vdots$	$\vdots$	$\vdots$
$n$	$m$	$\Sigma$

Join PMF

Conditional PMF  $\left( \frac{(x+1)(y+1)}{m+n+1} \right) = \frac{a}{B} \quad \sum \sum = 1$

→ Chebyshew's thrm is an inequality that can be applied to all data sets. It gives minimum proportion of the measurements that must lie within more than one standard deviations of mean.

chef

$$\text{for } k \text{ standard deviation, } P\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}, \quad k > 1$$

Upper bound  
↓  
at least this % of data will lie within  $k$  std dev.

$$K = \frac{X - \text{mean}}{\sigma^2}$$

inequality

~~Key has to be integer~~

→ Markov's Inequality

markovs

$$P(X \geq a) \leq \frac{E[X]}{a}$$

if only mean is given and we want to find upper bound use Markov's.

mean + s-d is given → Chebyshew's thrm.

→ Fitting a distribution mean?

(linear) Given 2 parameters ~~on~~ (x-axis, y-axis) and some points on plane to correspondingly x-y pairs. We test and analyse them and try to find the best fit curve for them. So we can do Hypothesis testing on them.

eg: plot weight of bat vs runs scored using that weighted bat. ( $y$ )

$$\text{Linear variable} = T = \beta_0 + \beta_1 X$$

$$\text{other model} = f(y) = \theta e^{-\theta y} \quad f(y) = \text{PDF.}$$

Fitting of distribution means finding  $\theta$  for given set of ( $T$ ).

Page No.:

Date:



Method of Area: Area under normal distribution will represent total no. of freq. (Table is used, mean - nahi percentage).

Steps → → Mean area

→ Variance

→ Create Table.

find mean and variance.

Use table

Given	Class Intervall	freq <sub>i</sub>	Midpoint	Area	$\Delta A$	freq <sub>i+1</sub>
$x - y$			$z = \frac{x - \bar{x}}{\sigma}$		$\Delta A_i^o =  A_{x_i} + A_{x_{i+1}} $	
$y - \infty$						$\Delta A \times N$
int						
convert to						
$x_i$ i.e. $x_i = x$						
$x_{i+1} = y$						
$x_{i+2} = z$						

depends on which side.

### \* Beta Distribution of 1st kind

beta distribution

kind1

$$\text{If pdf of } f(x) = \begin{cases} x^{m-1} (1-x)^{n-1} & : 0 \leq x \leq 1 ; m, n > 0 \\ \beta(m, n) & \\ 0 & \text{else} \end{cases}$$

then  $X$  follows  $\beta$ -dist of 1 kind

$$\begin{aligned} \text{so } \int_{-\infty}^{\infty} f(x) dx &= 1 \\ \Rightarrow \int_0^1 f(x) dx &= 1 = \int_0^1 x^{m-1} (1-x)^{n-1} dx \\ &= \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} = \frac{\beta(m, n)}{\beta(m, n)} \end{aligned}$$

Moment about origin

$$\mu'_n = E[x^n]$$

$$= \frac{\beta(m+n, n)}{\beta(m, n)} = \frac{\frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}}{\frac{\Gamma(m+n+\sigma)}{\Gamma(m)\Gamma(n)}}$$

Moment about origin  $\rightarrow$

$$\frac{m+r}{m+n}$$

$$\frac{m+r}{m+n+r} \frac{m+n}{m}$$

Now mean =  $\mu_1'$

$$\text{variance} = \mu_2' - (\mu_1')^2 = mn / ((m+n)^2(m+n+1))$$

mean  $\mu_1' = \frac{m}{m+n}$        $\mu_2' = \frac{m(m+1)}{(m+n)(m+n+1)}$

\* Beta Distribution  $\rightarrow$  2 Type

beta2

$$f(x) = \begin{cases} \frac{1}{B(m,n)} \cdot \frac{x^{m-1}}{(1+x)^{m+n}} & ; x \geq 0, m, n > 0 \\ 0 & \text{else} \end{cases}$$

Moment about origin. =  $\mu_1' = E[x]$

$$\Rightarrow \int_0^\infty x^m \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

$$\mu_1' = \frac{n-m}{\Gamma(m) \Gamma(n)}$$

mean =  $\mu_1'$

$$\text{variance} = \mu_2' - (\mu_1')^2 = m(m+n+1) / (n-2)(n-1)(n-1)$$

mean  $\mu_1 = \frac{m}{n-1}$        $\mu_2 = \frac{m(m+1)}{(n-1)(n-2)}$

\* Gamma Distribution

gamma

PDF is  $f(x) = \frac{\alpha^m x^{m-1} e^{-\alpha x}}{\Gamma(m)}$

$$G(m, \alpha) = \begin{cases} 0 & \text{for } x \leq 0; \text{ else} \end{cases}$$

## Moment about origin.

$$\text{mean } \underline{\mu}_1 = \frac{m}{a} \quad \underline{\mu}_2 = \frac{(p+1)m}{a^2}$$

$$\mu_3 = \frac{m(m+1)(m+2)}{a^2}$$

## Cauchy Distribution

## cauchy

$$\text{PDF is } f(x) = \begin{cases} \frac{1}{\pi} \cdot \frac{1}{1+x^2}, & -\infty < x < \infty \\ 0, & \text{else.} \end{cases}$$

$$\text{Let } y = \frac{x-u}{\lambda}$$

$$g(y) = \begin{cases} \frac{\lambda}{\pi} \frac{1}{\lambda^2 + (y-\mu)^2} & ; -\infty < y < \infty \\ 0 & ; -\infty < \mu < \infty \end{cases}$$

Change of origin  $\rightarrow$  not affect on moment  
change of scale  $\rightarrow$  power of  $\alpha$ .

Moment Generation Function  $M_X(t) = E(e^{tX})$

1<sup>st</sup> derivative Mean  $\Rightarrow$  Variance at t = 0.

## Weak law of large No:

WLLN

weak law of large no

WLLN

Let  $x_1, x_2, x_3 \dots x_n$  is a sequence of random variables and  $u_1, \dots, u_n$  be means. and

$$B_n = \text{Var}(x_1 + x_2 + \dots + x_n) \text{ is finite.}$$

$$\text{Then } P\left\{ \left| \frac{x_1 + x_2 + \dots + x_n}{n} - u_1 + u_2 + \dots + u_n \right| < c \right\} \geq 1 - \alpha$$

~~omega~~

Probability of averages converges to probability of their mean.

$$\frac{B_n}{n^2} \rightarrow 0, \text{ as } n \rightarrow \infty$$

CLT

## Central limit theorem

central central limit theorem

(1)  $\rightarrow$  The mean of samples means will be equal to population mean.

population

$$(2) \rightarrow \text{S.D. of sample mean} = \frac{\text{S.D. of population}}{\sqrt{n}}$$

$$\text{S.D. of sample} = \frac{\text{S.D. of population}}{\sqrt{\text{Population size}}}$$

Sample  $\rightarrow$  some section from whole population

N. Then no we have

Size of population = N

obtained using popula

$$\mu (\text{Mean}) = \frac{\sum x}{N}$$

is called parameter

$$\text{Variance} = \frac{\sum (x_i - \mu)^2}{N}$$

distribution of sample means

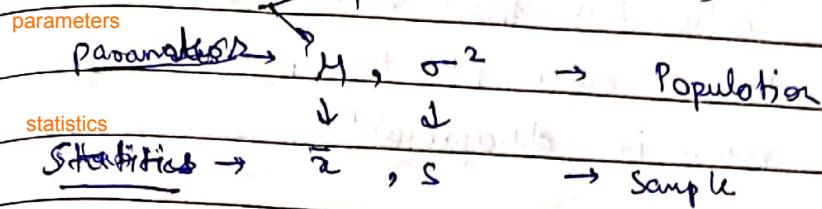
Distribution of sample means is a distribution obtained by listing the means computed from random samples of specific size taken from population.

Page No.:

Date:

## Sampling

Sometimes we may be able to decide types of probability distribution. but not the parameters (mean, variance, prob, median). so we rely on samples.



state

Simple random sampling is a commonly used sampling plan in which every sample of size  $n$  has the same chance of being selected.

Generalisation from sample to population is called

Statistical Inference.

Collect mean of various samples, group according to freq and do sampling distribution of the mean.

standard error

The standard deviation of sampling distribution is called standard error.

If  $n \geq 30$  large sample else small sample.

hypothesis

testing of hypo

Testing a hypothesis: To reach info about population on basis of sampling is called statistical hypothesis, which may or may not be true.

Testing a hypothesis means deciding whether to accept or reject hypothesis.

Error

error

type 1

1

type 2

2

Type 1: If hypo. is rejected, while it should have accepted.

Type 2: hypo is accepted → rejected.

To reduce both errors ↑ sample size.

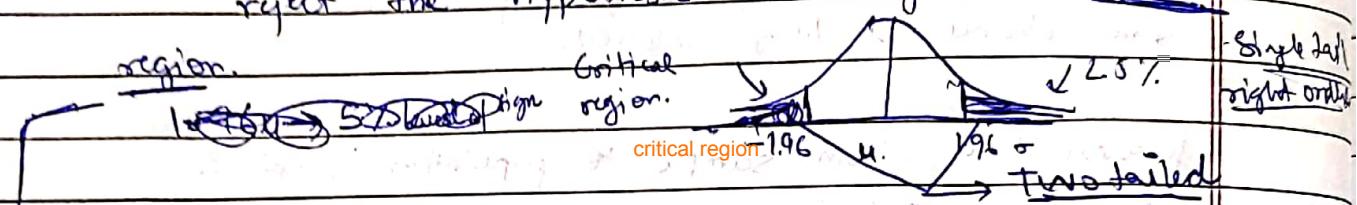
Page No.:

→ we assume these are true, unless proved false

**Null hypothesis** : hypo. formulated for sake of rejecting it under assumption 'it true.'

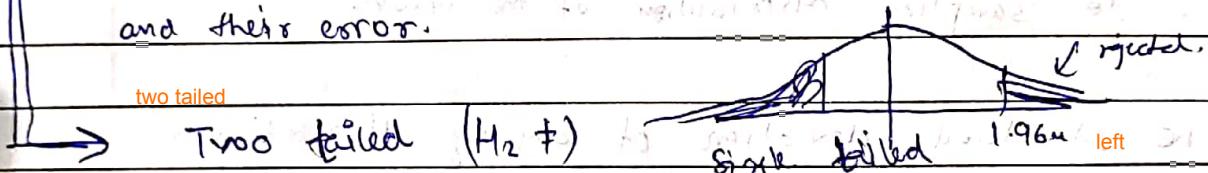
By accepting null hypothesis, we mean that on basis of statistic calculated from sample, we don't reject it. But doesn't mean its true. Also rejection doesn't mean it disapproved.

level of significance  
 $\rightarrow$  level of significance : The level below which we reject the hypothesis. The region is critical region.



test of significance → Test of significance : Test which enables us to decide whether to accept or reject hypo.

We check b/w sample value and population value and their error.



$\rightarrow$  Two tailed ( $H_2 \neq$ )  $\rightarrow$  single tailed  
 level { 5%  $\rightarrow$  1.96  $\downarrow$  (H<sub>2</sub>  $\neq$ ) 5%  $\rightarrow$  1.640  $\downarrow$   
 significant { 1%  $\rightarrow$  2.58  $\downarrow$  (H<sub>2</sub>  $\neq$ ) 1%  $\rightarrow$  2.33  $\downarrow$

confidence limits: Tells how accurate your estimate.

of mean is likely to be quite good.

Chi Square Test: goodness of fit.

The square of standard normal variate ( $z$ ) is known as chi-square variate with 1 degree of freedom.

i.e. If  $X \sim N(\mu, \sigma^2)$  then  $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$

$$X^2 = \left( \frac{X - \mu}{\sigma} \right)^2$$

2 degrees of freedom  
1 variable ( $X$ ) is unknown

$$X_i \sim N(\mu_i, \sigma_i^2)$$

definition? do next  
Find next.

$$X^2 = \sum_{i=1}^n \left( \frac{X_i - \mu_i}{\sigma_i} \right)^2 \quad n \text{ dof}$$

Pdf →

chi square

$$p(X^2) = \frac{1}{2^{\frac{n}{2}}} e^{-\frac{X^2}{2}} (X^2)^{\frac{n}{2}-1}$$

where  $v \rightarrow$  degree of freedom | unknown variables

$$\text{Mean} = v$$

$$\text{Mode} = v - 2$$

$$\text{Variance} = 2v$$

$$\text{Skewness} \rightarrow \sqrt{\frac{2}{v}}$$

Chi-Square Test is used for (Application)

→ goodness of fit.

→ independence of attributes etc.

→ To test if population has a specified value of  $\sigma^2$

## DEFINITION

chi square definition

$X^2$  afford a measure of correspondence b/w theory and observation.

It describes the magnitude of discrepancy b/w theory and observe

e.g.: toss coin 100 time

expected / theory  $\rightarrow$  50 heads

observed  $\rightarrow$  55 heads

formula:

$$X^2 = \sum_{i=1}^n \left[ \frac{(O_i - E_i)^2}{E_i} \right]. \quad O_i \rightarrow \text{observed frequency} \\ E_i \rightarrow \text{theoretical frequency}$$

$$\sum O_i = \sum E_i = N$$

degree of freedom =  $(n-1)$

$X^2 = 0 \rightarrow$  both frequency matches

theory

Page No.:	
Date:	

Degree of freedom = Total No of observations

dof

degree of freedom

(minus) numbers of independent constraints  
(subtracts) imposed on observation.

conditions

chi square

Apply  $\chi^2$  test cond<sup>n</sup>

$$\textcircled{1} \rightarrow N \geq 50$$

frequency  $\geq 50$

(3) No info of population.

$\rightarrow$  The constraint on cell frequencies should be linear. 😊

$$\textcircled{2} \rightarrow \sum E_i = \sum O_i \text{ and all } E_i, O_i > 5$$

general

$\rightarrow$  If calculated value of  $\chi^2$  is less than table value (5%) of significance, fit is considered to be good (Null hypothesis is accepted). Else reject.

accept

reject

question: Assume Null Hypothesis and find avg (mean)

problems

then do  $\bar{x} = \frac{\sum O_i}{N}$  mean

$$\rightarrow \text{i.e. } O_i - E_i$$

calculate  $\chi^2$

if  $\chi^2$  is less than 5% of table value for given dof.

Hypothesis is correct.

Test of Independence: We make hypothesis that

no two attributes are associated, and there is no

association. [two attributes are associated or not] [we make contingency table]

if  ~~$\chi^2 \geq 0$~~  or  $\chi^2 <$  table value of 5% of significance  
hypo is good.

contingency table

given

contingency table

a	b
c	d

$$\begin{array}{ccccc}
& & \textcircled{1} & & \textcircled{2} \\
& \rightarrow & \frac{(a+b)(a+c)}{a+b+c+d} & & \frac{(b+c)(b+d)}{a+b+c+d} \\
& & a+b+c+d & & a+b+c+d \\
& & \textcircled{3} & & \textcircled{4} \\
& & \frac{(c+a)(c+d)}{a+b+c+d} & & \frac{(a+b)(c+d)}{a+b+c+d} \\
& & (a+b+c+d) & & (a+b+c+d)
\end{array}$$

$$\chi^2 = \frac{(a-1)^2}{1} + \frac{(b-2)^2}{2}$$

Yates correction: is used to in certain situations

when testing for independence contingency table.

yates correction  
It aims at correcting error introduced by assuming the discrete probability of frequency in table can be approximated by continuous distribution

contingency table

we add/subtract  $\frac{1}{2}$  to  
sub/add  $\frac{1}{2}$  to a and c

a	b
c	d

## Application / Use of Chi Test

applications

chi square

attributes

(1) Independence of attributes.

independence whether two attributes are related or not we make contingency table.

(2) Good for fit good for fit

whether observed data matches theoretical data

(3) Test for homogeneity  $\rightarrow$  homogeneity whether two proportions are

interference equal or not.

(4) Test of inference Assuming we know about population. check if theorit variance = observe variance.

alternate

→ Alternative Hypo  $\rightarrow$  complementary to Null Hypo.

## Tests

tests

## Hypothesis

+ T-Test

Null

Alternative

parameters

parametric

knows about  
population  
mean, variance  
stdevs

Parametric  
Test

Non-parametric  
Test

Z Test  
F Test  
T Test

$\chi^2$ -Test  
U-Test  
M-Test

end

T-test

→ testing the significance of difference of mean values when the sample size is small

t-test

ttest

( $\mu$  & population standard deviation is not available)

Assume

→ Population dist → normal

→ Sample size  $\rightarrow$  small, random & independent

→ Population stand deviation is not known.

Z-test

determine whether the means  $\mu_1$  &  $\mu_2$  are different when a population variance is known.

z-test

ztest

It assume

→ Population dist → normal

→ Sample size  $\rightarrow$  large, random, independent

→ Population variance is known.

→ F-test → It is a test for the null hypothesis  
that two normal populations have same variance.

F-statistic = ratio of two variances

→ To test equality of means.

→ Normal population distri

→ random & independent samples.

f-test

ftest