

Definitions

finite automata

Fintite automata

Finite Automata(FA) is the simplest machine to recognize patterns. The finite automata or finite state machine is an abstract machine which have five elements or tuple. It has a set of states and rules for moving from one state to another but it depends upon the applied input symbol. Basically it is an abstract model of digital computer.

Alphabet.

Definition – An **alphabet** is any finite set of symbols.

Kleen closure \rightarrow^* (according to wiki and book)

Kleene Star

- **Definition** – The Kleene star, Σ^* , is a unary operator on a set of symbols or strings, Σ , that gives the infinite set of all possible strings of all possible lengths over Σ including λ .
- **Representation** – $\Sigma^* = \Sigma_0 \cup \Sigma_1 \cup \Sigma_2 \cup \dots$ where Σ_p is the set of all possible strings of length p.

Kleene Closure / Plus

- **Definition** – The set Σ^+ is the infinite set of all possible strings of all possible lengths over Σ excluding λ .
- **Representation** – $\Sigma^+ = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \dots$

$$\Sigma^+ = \Sigma^* - \{\lambda\}$$

Language

- **Definition** – A language is a subset of Σ^* for some alphabet Σ . It can be finite or infinite.

language

dfa

Deterministic Finite Automaton (DFA)

In DFA, for each input symbol, one can determine the state to which the machine will move. Hence, it is called **Deterministic Automaton**. As it has a finite number of states, the machine is called **Deterministic Finite Machine** or **Deterministic Finite Automaton**.

Formal Definition of a DFA

A DFA can be represented by a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where –

- **Q** is a finite set of states.
- **Σ** is a finite set of symbols called the alphabet.
- **δ** is the transition function where $\delta: Q \times \Sigma \rightarrow Q$
- **q_0** is the initial state from where any input is processed ($q_0 \in Q$).
- **F** is a set of final state/states of Q ($F \subseteq Q$).

Regular language regular language

A language is said to be regular if some fsm recognises it

Regular Expressions

Regular Expressions are used to denote regular languages. An expression is regular if: **regular expression**

- ϕ is a regular expression for regular language ϕ .
- ϵ is a regular expression for regular language $\{\epsilon\}$.
- If $a \in \Sigma$ (Σ represents the input alphabet), a is regular expression with language $\{a\}$.
- If a and b are regular expression, $a + b$ is also a regular expression with language $\{a,b\}$.
- If a and b are regular expression, ab (concatenation of a and b) is also regular.
- If a is regular expression, a^* (0 or more times a) is also regular.

regular grammar

Regular Grammar : A grammar is regular if it has rules of form $A \rightarrow a$ or $A \rightarrow aB$ or $A \rightarrow \epsilon$ where ϵ is a special symbol called NULL.

closure properties

Closure Properties of Regular Languages

union

- **Union :** If L1 and L2 are two regular languages, their union $L1 \cup L2$ will also be regular. For example, $L1 = \{a^n \mid n \geq 0\}$ and $L2 = \{b^n \mid n \geq 0\}$ $L3 = L1 \cup L2 = \{a^n \cup b^n \mid n \geq 0\}$ is also regular.

intersection

- **Intersection :** If L1 and L2 are two regular languages, their intersection $L1 \cap L2$ will also be regular. For example,
 $L1 = \{a^m b^n \mid n \geq 0 \text{ and } m \geq 0\}$ and $L2 = \{a^m b^n \cup b^n a^m \mid n \geq 0 \text{ and } m \geq 0\}$
 $L3 = L1 \cap L2 = \{a^m b^n \mid n \geq 0 \text{ and } m \geq 0\}$ is also regular.

Kleen closure

- **Concatenation :** If L1 and L2 are two regular languages, their concatenation $L1.L2$ will also be regular. For example,
 $L1 = \{a^n \mid n \geq 0\}$ and $L2 = \{b^n \mid n \geq 0\}$
 $L3 = L1.L2 = \{a^m . b^n \mid m \geq 0 \text{ and } n \geq 0\}$ is also regular.

- **Kleene Closure :** If L1 is a regular language, its Kleene closure $L1^*$ will also be regular. For example,
 $L1 = (a \cup b)$
 $L1^* = (a \cup b)^*$
- **Complement :** If $L(G)$ is regular language, its complement $L'(G)$ will also be regular. Complement of a language can be found by subtracting strings which are in $L(G)$ from all possible strings. For example,
 $L(G) = \{a^n \mid n > 3\}$
 $L'(G) = \{a^n \mid n \leq 3\}$

equivalent

Two regular **expressions are equivalent** if languages generated by them are same. For example, $(a+b^*)^*$ and $(a+b)^*$ generate same language. Every string which is generated by $(a+b^*)^*$ is also generated by $(a+b)^*$ and vice versa.

NDFA/NFA

nfa

In NDFA, for a particular input symbol, the machine can move to any combination of the states in the machine. In other words, the exact state to which the machine moves cannot be determined. Hence, it is called **Non-deterministic Automaton**. As it has finite number of states, the machine is called **Non-deterministic Finite Machine or Non-deterministic Finite Automaton**.

Formal Definition of an NDFA

An NDFA can be represented by a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where –

- **Q** is a finite set of states.
- **Σ** is a finite set of symbols called the alphabets.
- **δ** is the transition function where $\delta: Q \times \Sigma \rightarrow 2^Q$

(Here the power set of Q (2^Q) has been taken because in case of NDFA, from a state, transition can occur to any combination of Q states)

- q_0 is the initial state from where any input is processed ($q_0 \in Q$).
- F is a set of final state/states of Q ($F \subseteq Q$).

Unreachable state: a state is said to be unreachable if there is no way it can be reached from initial state.

Epsilon nfa

enfa

The NFA with epsilon-transition is a finite state machine in which the transition from one state to another state is allowed without any input symbol i.e. empty string ϵ . Adding the transition for the empty string doesn't increase the computing power of the finite automata but adds some flexibility to construct then DFA and NFA.

Identities of regular expression

regx

regex

Identities of Regular Expression

- | | |
|---|--|
| 1) $\emptyset + R = R$ | 7) $RR^* = R^*R$ |
| 2) $\emptyset R + R\emptyset = \emptyset$ | 8) $(R^*)^* = R^*$ |
| 3) $\epsilon R = R\epsilon = R$ | 9) $\epsilon + RR^* = \epsilon + R^*R = R^*$ |
| 4) $\epsilon^* = \epsilon$ and $\emptyset^* = \epsilon$ | 10) $(PQ)^*P = P(QP)^*$ |
| 5) $R + R = R$ | 11) $(P + Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$ |
| 6) $R^*R^* = R^*$ | 12) $(P + Q)R = PR + QR$ and

$R(P + Q) = RP + RQ$ |

Arden's theorem state that:

"If P and Q are two regular expressions over sigma, and if P does not contain epsilon, then the following equation in R given by $R = Q + RP$ has an unique solution i.e., $R = QP^*$."

ardens theorem

Equivalence of 2 finite automata

The two finite automata (FA) are said to be equivalent if both the automata accept the same set of strings over an input set Σ .

When two FA's are equivalent then, there is some string x over Σ . On acceptance of that string, if one FA reaches to the final state, the other FA also reaches to the final state.

Method equivalence fsm

The method for comparing two FA's is explained below –

Let M and M_1 be the two FA's and Σ be a set of input strings.

Step 1 – Construct a transition table that has pairwise entries (q, q^1) where $q \in M$ and $q^1 \in M^1$ for each input symbol.

Step 2 – If we get in a pair as one final state and other non-final state then we terminate the construction of transition table declaring that two FA's are not equivalent

Step 3 – The construction of the transition table gets terminated when there is no new pair appearing in the transition table.

Mealy Machine

A Mealy Machine is an FSM whose output depends on the present state as well as the present input.

mealy

It can be described by a 6 tuple $(Q, \Sigma, O, \delta, X, q_0)$ where –

- **Q** is a finite set of states.
- Σ is a finite set of symbols called the input alphabet.
- **O** is a finite set of symbols called the output alphabet.
- δ is the input transition function where $\delta: Q \times \Sigma \rightarrow Q$
- X is the output transition function where $X: Q \times \Sigma \rightarrow O$
- q_0 is the initial state from where any input is processed ($q_0 \in Q$).

Moore Machine

Moore machine is an FSM whose outputs depend on only the present state.

A Moore machine can be described by a 6 tuple $(Q, \Sigma, O, \delta, X, q_0)$ where –

- **Q** is a finite set of states.
- Σ is a finite set of symbols called the input alphabet.

moore

tuples

- **O** is a finite set of symbols called the output alphabet.
- **δ** is the input transition function where $\delta: Q \times \Sigma \rightarrow Q$
- **X** is the output transition function where $X: Q \rightarrow O$
- **q_0** is the initial state from where any input is processed ($q_0 \in Q$).

Mealy Machine	Moore Machine
Output depends both upon the present state and the present input	Output depends only upon the present state.
Generally, it has fewer states than Moore Machine.	Generally, it has more states than Mealy Machine.
The value of the output function is a function of the transitions and the changes, when the input logic on the present state is done.	The value of the output function is a function of the current state and the changes at the clock edges, whenever state changes occur.
Mealy machines react faster to inputs. They generally react in the same clock cycle.	In Moore machines, more logic is required to decode the outputs resulting in more circuit delays. They generally react one clock cycle later.

Both have same power.

Moore \rightarrow mealy = no. of states same

mealy

moore

Mealy \rightarrow moore = no. of states increases(states*output at max)

states

Difference between dfa and nfa(both have same power)

Deterministic Finite Automata	Non-Deterministic Finite Automata
Each transition leads to exactly one state called as deterministic	A transition leads to a subset of states i.e. some transitions can be non-deterministic.
Accepts input if the last state is in Final	Accepts input if one of the last states is in Final.
Backtracking is allowed in DFA.	Backtracking is not always possible.
Requires more space. table2	Requires less space.
Empty string transitions are not seen in DFA.	Permits empty string transition.
For a given state, on a given input we reach a deterministic and unique state.	For a given state, on a given input we reach more than one state.
DFA is a subset of NFA.	Need to convert NFA to DFA in the design of a compiler.
$\delta : Q \times \Sigma \rightarrow Q$ For example – $\delta(q_0, a) = \{q_1\}$	$\delta : Q \times \Sigma \rightarrow 2^Q$ For example – $\delta(q_0, a) = \{q_1, q_2\}$
DFA is more difficult to construct.	NFA is easier to construct.

Deterministic Finite Automata	Non-Deterministic Finite Automata
DFA is understood as one machine.	NFA is understood as multiple small machines computing at the same time.

Kleene theorem

Kleene's Theorem-I :

kleene theorem

For any Regular Expression r that represents Language $L(r)$, there is a Finite Automata that accepts same language.

clean

pumping lemma

Pumping lemma for regular language

Pumping Lemma (For Regular Languages)

» Pumping Lemma is used to prove that a Language is NOT REGULAR

» It cannot be used to prove that a Language is Regular

If A is a Regular Language, then A has a Pumping Length ' P ' such that any string ' S ' where $|S| \geq P$ may be divided into 3 parts $S = xyz$ such that the following conditions must be true:

- (1) $xy^iz \in A$ for every $i \geq 0$
- (2) $|y| > 0$
- (3) $|xy| \leq P$

noam chomsky hierarchy chomsky hierarchy TYPE

Noam Chomsky gave a Mathematical model of Grammar which is effective for writing computer languages

The four types of Grammar according to Noam Chomsky are:

Grammar Type	Grammar Accepted	Language Accepted	Automaton
TYPE-0	Unrestricted Grammar	Recursively Enumerable Language	Turing Machine
TYPE-1	Context Sensitive Grammar	Context Sensitive Language	Linear Bounded Automaton
TYPE-2	Context Free Grammar	Context Free Language	Pushdown Automata
TYPE-3	Regular Grammar	Regular Language	Finite State Automaton

Grammar :

It is a finite set of formal rules for generating syntactically correct sentences or meaningful correct sentences.

Constitute Of Grammar :

Grammar is basically composed of two basic elements –

1. Terminal Symbols –

Terminal symbols are those which are the components of the sentences generated using a grammar and are represented using small case letter like a, b, c etc.

2. Non-Terminal Symbols –

Non-Terminal Symbols are those symbols which take part in the generation of the sentence but are not the component of the sentence. Non-Terminal Symbols are also called Auxiliary Symbols and Variables. These symbols are represented using a capital letter like A, B, C, etc.

grammar

4tuples

Formal Definition of Grammar :

Any Grammar can be represented by 4 tuples – $\langle N, T, P, S \rangle$

- **N** – Finite Non-Empty Set of Non-Terminal Symbols.
- **T** – Finite Set of Terminal Symbols.
- **P** – Finite Non-Empty Set of Production Rules.
- **S** – Start Symbol (Symbol from where we start producing our sentences or strings).

production rules

Production Rules :

A production or production rule in computer science is a rewrite rule specifying a symbol substitution that can be recursively performed to generate new symbol sequences. It is of the form $\alpha \rightarrow \beta$ where α is a Non-Terminal Symbol which can be replaced by β which is a string of Terminal Symbols or Non-Terminal Symbols.

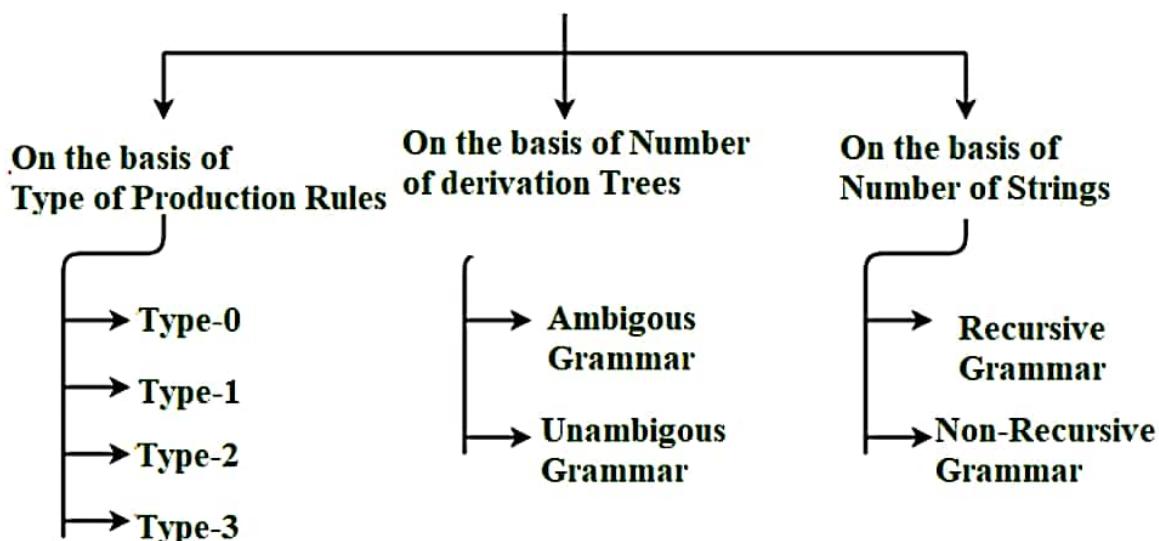
Equivalent Grammars : types

equivalent

Grammars are said to be equivalent if they produce the same language.

Types of grammar

Types of Grammar



regular grammar

Regular Grammar : A grammar is regular if it has rules of form $A \rightarrow a$ or $A \rightarrow aB$ or $A \rightarrow \epsilon$ where ϵ is a special symbol called NULL.

Right linear grammar: $A \rightarrow xB$ or $A \rightarrow x$

linear

Left linear grammar: $A \rightarrow Bx$ or $A \rightarrow x$

The set of all strings generated from a is said to be language of that grammar.

Context free grammar

cfg

Definition – A context-free grammar (CFG) consisting of a finite set of grammar rules is a quadruple (N, T, P, S) where

- N is a set of non-terminal symbols.

- **T** is a set of terminals where $N \cap T = \text{NULL}$. tuples
- **P** is a set of rules, $P: N \rightarrow (N \cup T)^*$, i.e., the left-hand side of the production rule **P** does have any right context or left context.
- **S** is the start symbol.

Derivation tree

derivation tree

A derivation tree or parse tree is an ordered rooted tree that graphically represents the semantic information a string derived from a context-free grammar.

Representation Technique

- **Root vertex** – Must be labeled by the start symbol.
- **Vertex** – Labeled by a non-terminal symbol.
- **Leaves** – Labeled by a terminal symbol or ϵ .

Sentential Form and Partial Derivation Tree

sentential

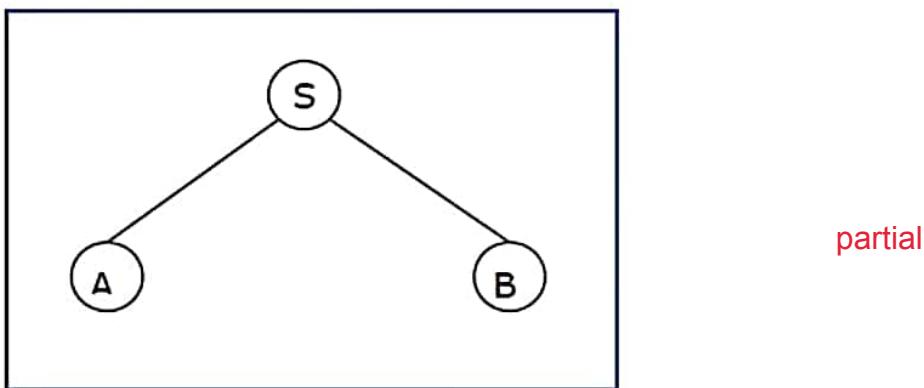
A partial derivation tree is a sub-tree of a derivation tree/parse tree such that either all of its children are in the sub-tree or none of them are in the sub-tree.

Example

If in any CFG the productions are –

$$S \rightarrow AB, A \rightarrow aaA \mid \epsilon, B \rightarrow Bb \mid \epsilon$$

the partial derivation tree can be the following –



If a partial derivation tree contains the root S, it is called a **sentential form**. The above sub-tree is also in sentential form.

Leftmost and Rightmost Derivation of a String

tree

leftmost

rightmost

derivation

Ambiguous grammar ambiguous

A CFG is said to be ambiguous if there exists more than one derivation tree for the given input string i.e., more than one **LeftMost Derivation Tree (LMDT)** or **RightMost Derivation Tree (RMDT)**.

Simplification of cfg

simplification of cfg

In a CFG, it may happen that all the production rules and symbols are not needed for the derivation of strings. Besides, there may be some null productions and unit productions. Elimination of these productions and symbols is called **simplification of CFGs**. Simplification essentially comprises of the following steps -

- Reduction of CFG
- Removal of Unit Productions
- Removal of Null Productions

Reduction of cfg

REDUCTION OF CFG

CFG are reduced in two phases

Phase 1: Derivation of an equivalent grammar G' , from the CFG, G , such that each variable derives some terminal string

Derivation Procedure:

Step 1: Include all Symbols W_1 , that derives some terminal and initialize $i = 1$
 Step 2: Include symbols W_{i+1} , that derives W_i
 Step 3: Increment i and repeat Step 2, until $W_{i+1} = W_i$
 Step 4: Include all production rules that have W_i in it

Phase 2: Derivation of an equivalent grammar G'' , from the CFG, G' , such that each symbol appears in a sentential form

Derivation Procedure:

Step 1: Include the Start Symbol in Y_1 and initialize $i = 1$
 Step 2: Include all symbols Y_{i+1} , that can be derived from Y_i and include all production rules that have been applied
 Step 3: Increment i and repeat Step 2, until $Y_{i+1} = Y_i$

reduction



Removal of unit productions

removal of unit

Simplification of Context Free Grammar

Removal of Unit Productions

Any Production Rule of the form $A \rightarrow B$ where $A, B \in \text{Non Terminals}$ is called Unit Production
Procedure for Removal

Step 1: To remove $A \rightarrow B$, add production $A \rightarrow x$ to the grammar rule whenever $B \rightarrow x$ occurs in the grammar. [$x \in \text{Terminal}$, x can be Null]

Step 2: Delete $A \rightarrow B$ from the grammar.

Step 3: Repeat from Step 1 until all Unit Productions are removed.

Example: Remove Unit Productions from the Grammar whose production rule is given by

P: $S \rightarrow XY, X \rightarrow a, Y \rightarrow Z|b, Z \rightarrow M, M \rightarrow N, N \rightarrow a$

$Y \rightarrow Z, Z \rightarrow M, M \rightarrow N$

i) Since $N \rightarrow a$, we add $M \rightarrow a$

P: $S \rightarrow XY, X \rightarrow a, Y \rightarrow Z|b, Z \rightarrow M, M \rightarrow a, N \rightarrow a$



Removal of null productions

removal null

Simplification of Context Free Grammar

Removal of Null Productions

In a CFG, a Non-Terminal Symbol 'A' is a nullable variable if there is a production $A \rightarrow \epsilon$ or there is a derivation that starts at 'A' and leads to ϵ . (Like $A \rightarrow \dots \rightarrow \epsilon$)

Procedure for Removal:

Step 1: To remove $A \rightarrow \epsilon$, look for all productions whose right side contains A

Step 2: Replace each occurrences of 'A' in each of these productions with ϵ

Step 3: Add the resultant productions to the Grammar

Example: Remove Null Productions from the following Grammar

$S \rightarrow ABAC, A \rightarrow aA|\epsilon, B \rightarrow bB|\epsilon, C \rightarrow c$

$A \rightarrow \epsilon, B \rightarrow \epsilon$

i) To eliminate $A \rightarrow \epsilon$

\downarrow



Chomsky normal form

cnf

In Chomsky Normal Form (CNF) we have a restriction on the length of RHS; which is; elements in RHS should either be two variables or a Terminal.

A CFG is in Chomsky Normal Form if the productions are in the following forms:

$$A \rightarrow a$$

$$A \rightarrow BC$$

where A, B and C are non-terminals and a is a terminal

Steps

Steps to convert a given CFG to Chomsky Normal Form:

- Step 1: If the Start Symbol S occurs on some right side, create a new Start Symbol S' and a new Production S' → S.
- Step 2: Remove Null Productions. (Using the Null Production Removal discussed in previous Lecture)
- Step 3: Remove Unit Productions. (Using the Unit Production Removal discussed in previous Lecture)
- Step 4: Replace each Production $A \rightarrow B_1 \dots B_n$ where $n > 2$, with $A \rightarrow B_1 C$ where $C \rightarrow B_2 \dots B_n$. Repeat this step for all Productions having two or more Symbols on the right side.
- Step 5: If the right side of any Production is in the form $A \rightarrow aB$ where 'a' is a terminal and A and B are non-terminals, then the Production is replaced by $A \rightarrow XB$ and $X \rightarrow a$. Repeat this step for every Production which is of the form $A \rightarrow aB$

Gnf greibach normal form

gnf

Greibach Normal Form

A CFG is in Greibach Normal Form if the productions are in the following forms:

$$A \rightarrow b$$

$$A \rightarrow bC_1 C_2 \dots C_n$$

where A, C₁, ..., C_n are Non-Terminals and b is a Terminal

Steps to convert a given CFG to GNF:

- Step 1: Check if the given CFG has any Unit Productions or Null Productions and Remove if there are any (using the Unit & Null Productions removal techniques discussed in the previous lecture)
- Step 2: Check whether the CFG is already in Chomsky Normal Form (CNF) and convert it to CNF if it is not. (using the CFG to CNF conversion technique discussed in the previous lecture)
- Step 3: Change the names of the Non-Terminal Symbols into some A_i in ascending order of i

Removal of left recursion in gnf

left recursion

Step 5: Remove Left Recursion

Introduce a New Variable to remove the Left Recursion

$$A_4 \rightarrow b \mid b A_3 A_4 \mid A_4 A_4 A_4$$

$$Z \rightarrow A_4 A_4 Z \mid A_4 A_4$$

$$A_4 \rightarrow b \mid b A_3 A_4 \mid \underset{\downarrow}{b Z} \mid b A_3 A_4 Z$$

Now the grammar is:

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b \mid b A_3 A_4 \mid b Z \mid b A_3 A_4 Z$$

$$Z \rightarrow A_4 A_4 \mid A_4 A_4 Z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

$$A_1 \rightarrow b A_3 \mid b A_4 \mid b A_3 A_4 A_4 \mid b Z A_4 \mid b A_3 A_4 Z A_4$$

$$A_4 \rightarrow b \mid b A_3 A_4 \mid b Z \mid b A_3 A_4 Z$$

$$Z \rightarrow b A_4 \mid b A_3 A_4 A_4 \mid b Z A_4 \mid b A_3 A_4 Z A_4 \mid$$

$$b A_4 Z \mid b A_3 A_4 A_4 Z \mid b Z A_4 Z \mid b A_3 A_4 Z A_4 Z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

regular grammar or context free grammar me kya difference hai?

Regular Expressions

Context-free grammar

table3

Lexical rules are quite simple in case of Regular Expressions.

Lexical rules are difficult in case of Context free grammar.

Notations in regular expressions are easy to understand.

Notations in Context free grammar are quite complex.

Regular Expressions

A set of string is defined in case of Regular Expressions.

It is easy to construct efficient recognizer from Regular Expressions.

There is proper procedure for lexical and syntactical analysis in case of Regular Expressions.

Regular Expressions are most useful for describing the structure of lexical construct such as identifiers, constant etc.

Context-free grammar

In Context free grammar the language is defined by the collection of productions.

By using the context free grammar, it is very difficult to construct the recognizer.

There is no specific guideline for lexical and syntactic analysis in case of Context free grammar.

Context free grammars are most useful in describing the nested chain structure or syntactic structure such as balanced parenthesis, if else etc. and these can't be define by Regular Expression.

Pumping Lemma for Context-free Languages (CFL)

pumping lemma

Pumping Lemma for CFL states that for any Context Free Language L, it is possible to find two substrings that can be 'pumped' any number of times and still be in the same language. For any language L, we break its strings into five parts and pump second and fourth substring.

Pumping Lemma, here also, is used as a tool to prove that a language is not CFL. Because, if any one string does not satisfy its conditions, then the language is not CFL.

Thus, if L is a CFL, there exists an integer n, such that for all $x \in L$ with $|x| \geq n$, there exists $u, v, w, x, y \in \Sigma^*$, such that $x = uvwxy$, and

- (1) $|vwx| \leq n$
- (2) $|vx| \geq 1$
- (3) for all $i \geq 0$: $uv^iwx^iy \in L$

pushdown automata

pda

pushdown automata

Pushdown Automata is a finite automata with extra memory called stack which helps Pushdown automata to recognize Context Free Languages.

A Pushdown Automata (PDA) can be defined as :

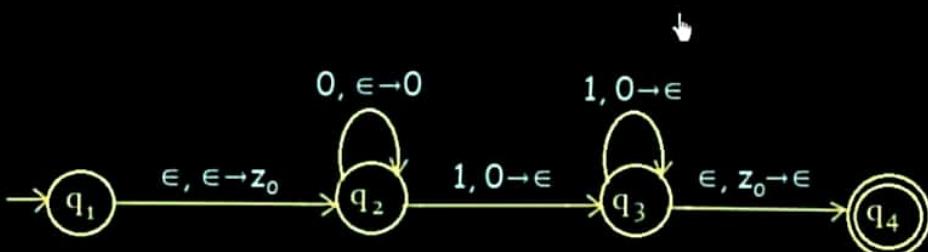
- Q is the set of states
- Σ is the set of input symbols
- Γ is the set of pushdown symbols (which can be pushed and popped from stack)
- q_0 is the initial state
- Z is the initial pushdown symbol (which is initially present in stack)
- F is the set of final states
- δ is a transition function which maps $Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma$ into $Q \times \Gamma^*$. In a given state, PDA will read input symbol and stack symbol (top of the stack) and move to a new state and change the symbol of stack.

gamma

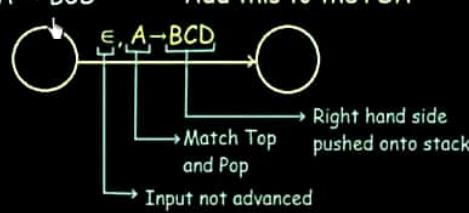
tuples

Pda example

Example: Construct a PDA that accepts $L = \{ 0^n 1^n \mid n \geq 0 \}$



Rule: $A \rightarrow BCD$ Add this to the PDA



pda

stack

δ = The Transition Function

q_0 = The Start State

Press Esc to exit full screen

z_0 = The Start Stack Symbol

F = The set of Final / Accepting States

δ takes as argument a triple $\delta (q, a, X)$ where:

- (i) q is a State in Q
- (ii) a is either an Input Symbol in Σ or $a = \epsilon$
- (iii) X is a Stack Symbol, that is a member of Γ

The output of δ is finite set of pairs (p, γ) where:

p is a new state

γ is a string of stack symbols that replaces X at the top of the stack

Eg. If $\gamma = \epsilon$ then the stack is popped

If $\gamma = X$ then the stack is unchanged

If $\gamma = YZ$ then X is replaced by Z and Y is pushed onto the stack



stack

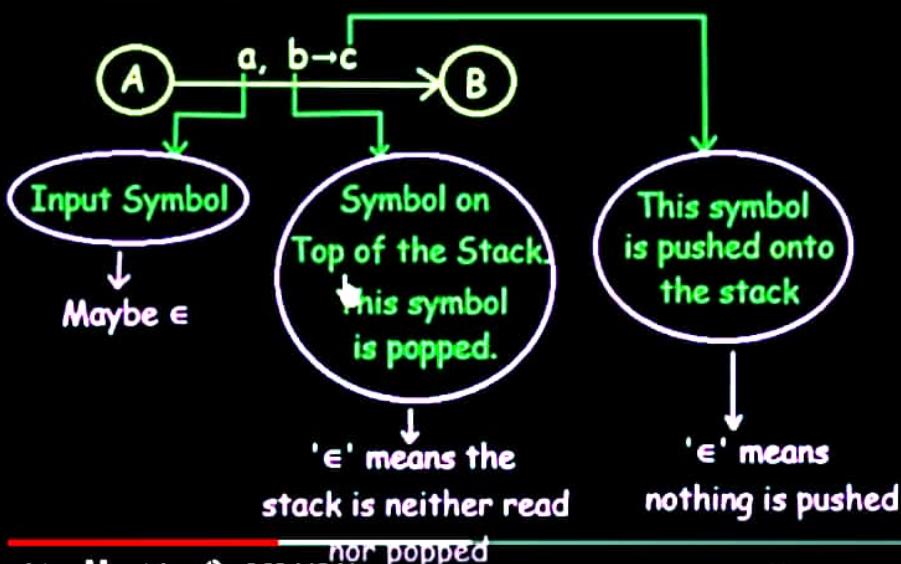
transition

Pushdown Automata (Graphical Notation)

Finite State Machine



Pushdown Automata



Equivalence of pda and cfg

Equivalence of CFG and PDA (Part-1) (From CFG to PDA)

Theorem: A language is Context Free iff some Pushdown Automata recognizes it.

Proof: Part 1: Given a CFG, show how to construct a PDA that recognizes it.

Part 2: Given a PDA, show how to construct a CFG that recognizes the same language.

equivalence

pda to cfg

cfg

Pda to cfg

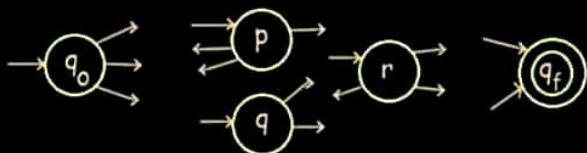
Equivalence of CFG and PDA (Part-2a) (From PDA to CFG)

Theorem: A language is Context Free iff some Pushdown Automata recognizes it.

Proof: Part 1: Given a CFG, show how to construct a PDA that recognizes it.

Part 2: Given a PDA, show how to construct a CFG that recognizes the same language.

Given a PDA \rightarrow Build a CFG from it



Step 1: Simplify the PDA

Step 2: Build the CFG



There will be a Non-Terminal for every pair of states : A_{pq} , A_{qr} , A_{rq_0} ,

The starting Non-Terminal will be : $A_{q_0 q_f}$



Simplifying the PDA:

1) The PDA should have only one final/accept state.

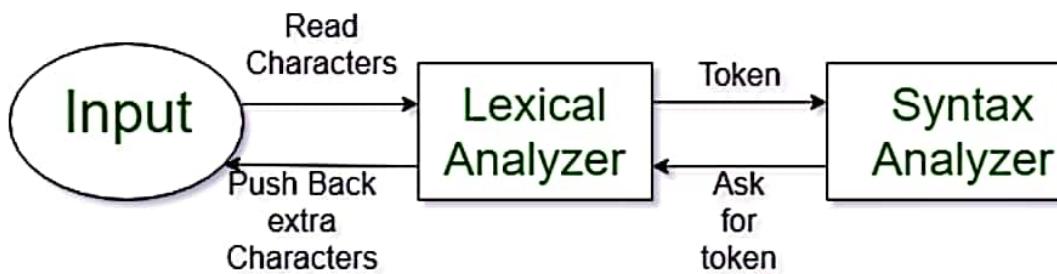


lexical analysis

Lexical analysis

Lexical Analysis is the first phase of the compiler also known as a scanner. It converts the High level input program into a sequence of **Tokens**.
tokens

- Lexical Analysis can be implemented with the Deterministic finite Automata.
- The output is a sequence of tokens that is sent to the parser for syntax analysis



practical applications of finite automata

The **Applications** of these Automata are given as follows:

1. Finite Automata (FA) -

practical application

- For the designing of lexical analysis of a compiler.
- For recognizing the pattern using regular expressions.
- For the designing of the combination and sequential circuits using Mealy and Moore Machines.
- Used in text editors.
- For the implementation of spell checkers.

2. Push Down Automata (PDA) -

- For designing the parsing phase of a compiler (Syntax Analysis).
- For implementation of stack applications.
- For evaluating the arithmetic expressions.

- For solving the Tower of Hanoi Problem.

3. Linear Bounded Automata (LBA) –

- For implementation of genetic programming.
- For constructing syntactic parse trees for semantic analysis of the compiler.

4. Turing Machine (TM) –

- For solving any recursively enumerable problem.
- For understanding complexity theory.
- For implementation of neural networks.
- For implementation of Robotics Applications.
- For implementation of artificial intelligence.

Recursive Enumerable (RE) or Type -0 Language re recursive enumerable

RE languages or type-0 languages are generated by type-0 grammars. An RE language can be accepted or recognized by Turing machine which means it will enter into final state for the strings of language and may or may not enter into rejecting state for the strings which are not part of the language. It means TM can loop forever for the strings which are not a part of the language. RE languages are also called as Turing recognizable languages.

Recursive Language (REC) recursive language rec

A recursive language (subset of RE) can be decided by Turing machine which means it will enter into final state for the strings of language and rejecting state for the strings which are not part of the language. e.g.; $L = \{a^n b^n c^n \mid n \geq 1\}$ is recursive because we can construct a turing machine which will move to final state if the string is of the form $a^n b^n c^n$ else move to non-final state. So the TM will always halt in this case. REC languages are also called as Turing decidable languages. The relationship between RE and REC languages can be shown in Figure 1.

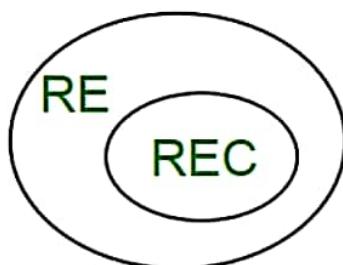


Figure 1

True or false ?

Ki humare paas ek automata h A^P where P is prime

Yes using turing machine prime

Prove ye cfl h ya nhi with pumping lemma

Nhi hai

use of NFA in UNIX commands

unix

union of two dfas

is possible, as the regular expression is closed under union.

union

cellular automata

cellular automata

different normal form(dekh lena bs cnf ya gnf hai ya kuch aur bhi)

cfg pr union wagerah se kaunsi grammar bnti hai

union, concat, kleene closure of cfg = cfg,

a^p where p is prime is cfg or not using pumping lemma

Recursive enumerable set k subsets saare recursive honge ?



Figure 1

True or false ? (false ig)

emptiness in dfa

Grammar for palindrome (cfg imo)

wwr

wawr

is dfa valid without final states

yes

palindrome

Computable properties of regular language

1) How many states will be there in a DFA that accepts 3 or 5

2

What is membership problem?

Given a word w and a language L, we want to check if $w \in L$. This is called the membership problem.

membership problem

3) How to solve it (CYK)

4) Explain it in detail (CYK)

cyk

CYK algorithm is a parsing algorithm for context free grammar.

In order to apply CYK algorithm to a grammar, it must be in Chomsky Normal Form. It uses a dynamic programming algorithm to tell whether a string is in the language of a grammar.

Time and Space Complexity :

Time Complexity –

$O(n^3 \cdot |G|)$

Where $|G|$ is the number of rules in the given grammar.

Space Complexity –

$O(n^2)$

[CYK Algorithm for Context Free Grammar - GeeksforGeeks](#)

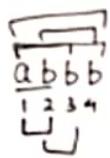
similar to MCM DP

→ CYK Algorithm | Membership Algorithm in CFG || TOC



CYK Algorithm: Check whether a string 'abbb' is a valid member of following CFG.

$$\begin{cases} S \rightarrow AB \\ A \rightarrow BB/a \\ B \rightarrow AB/b \end{cases}$$



(a b) ✓
(bbb) ✓

* CYK applicable on CNF
Only

CNF: $A \rightarrow BC$
 $A \rightarrow a$ or

* It is Universal (applicable on all CNF)

	4	3	2	1
1	S, B	A	S, B	A
2	S, B	A	B	
3	A	B		
4	B			

12
(11) (2,2)
A, B

23
(2,2) (3,3)
BB

31
(3,3) (4,4)
BB

24
2/3,
22
B, 34
A

23
34
44
φ

13
1|2|3

(11) (23)
AA φ

(12) (33)
(S, B)(B)

SB φ

BB φ

φ φ

14
1|2|3|4
(11) (24)
A, S, B

(12) (34)
S, B, A

(13) (44)
A, B

AS X
AB

* Time Complexity $\Theta(n^3)$

* Space Complexity $\Theta(n^2)$

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