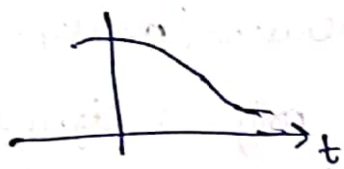
- Sampling Theory
  - ↳ process ⇒ continous time → discrete time
  - ↳ no. of samples depends on max freq component of signal

- Sampling Theorem
  - ↳ A band limited signal of finite energy which has no freq component higher than $f_m$ (Hz) is completely by its sample values at uniform interval less than or equal to $\frac{1}{2f_m}$

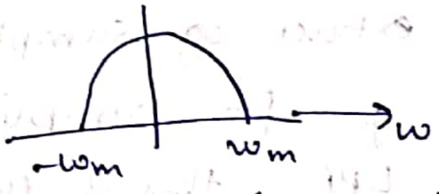
$$\boxed{T_s \leq \frac{1}{2f_m}} \quad \text{or} \quad \boxed{f_s \geq 2f_m}$$

Note: if signal is band limited to $f_m$ then,

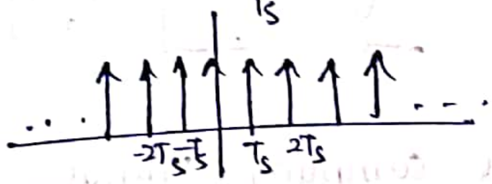
$X(\omega) = 0$ for $\omega > \omega_m$




⇒




$$\delta_{T_s}(t) = \frac{1}{T_s}(1 + 2\cos\omega_s t + 2\cos 2\omega_s t + \dots)$$

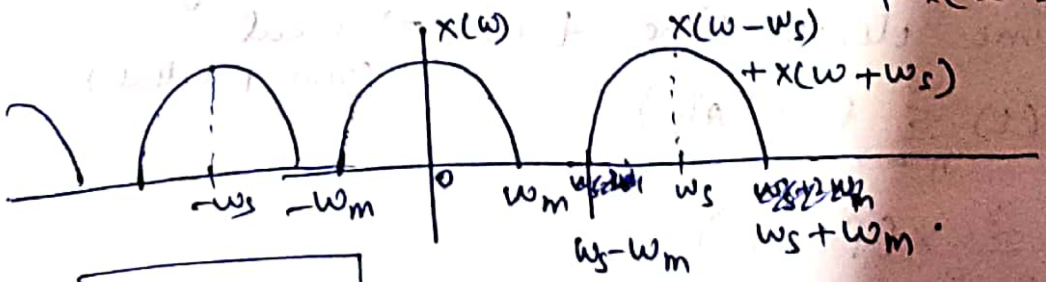




$g(t) = x(t)\,\delta_{T_s}(t)$


$$g(t) = x(t)\left[\frac{1}{T_s}(1 + 2\cos\omega_s t + 2\cos 2\omega_s t \dots)\right.$$

$$G(\omega) = \frac{1}{T_s}\big(X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots\big)$$




$$\boxed{\omega_s \geq 2\omega_m}$$

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- if $f_s > 2f_m$ ⇒ no overlapping and periodically repetition.
- At receiver end we place low pass filter of frequency $\omega_m$ so we can extract original information.

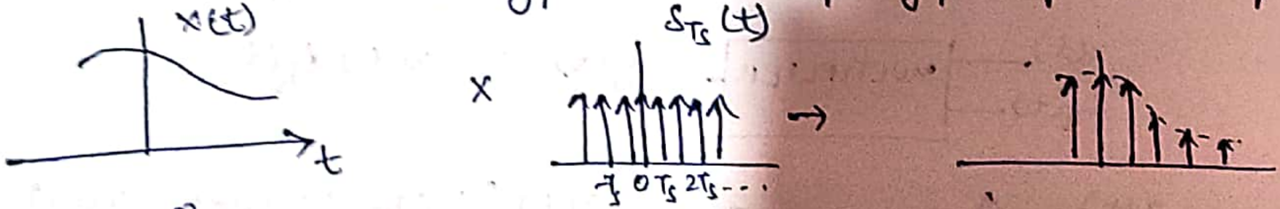
$f_s > 2f_m$ → no overlapping
$f_s = 2f_m$ → touch (nyquist rate)
$f_s < 2f_m$ → overlapping (distortion)
$T_s = \frac{1}{2f_m}$ → Nyquist time interval.

# Effect of under sampling → Aliasing

if $f_s < 2f_m$ when we apply LPF we won't be able to extract og sample bcos of aliasing (overlapping).

↳ If we go for sampling, we pass original signal through LPF. this is even referred to as pre alias filter other name is band limit filter. → Avoid aliasing
or
$f_s \geq 2f_m$

# Instantaneous sampling / Ideal sampling / Impulse sampling)




$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(T - nT_s)$$

↳ To generate ideal samples we use switching sampler
↳ If we assume closing time $t \to 0$ ⇒ ideal (not practical)

$$g(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$G(\omega) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

demerit
↳ not practical
↳ high noise interference. (bcos signal energy is low)

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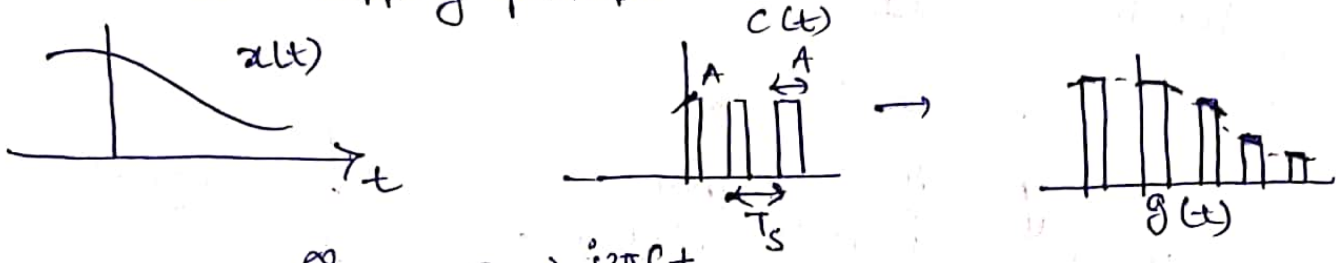
# Natural Sampling
↳ It uses chopping principle




$$c(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(f_n \tau) e^{i 2\pi f_s t}$$

$$g(t) = x(t) \times c(t)$$

$$g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \, \text{sinc}(f_n \tau) \, e^{i 2\pi f_s t}$$

$$G(\omega) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(n f_s \tau) \times (f - n f_s)$$

This method is used practically
noise interference is less.

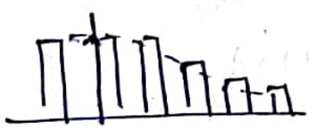
# flat top sampling
↳ uses sample and hold circuit
↳ flat top is easier than natural.
↳ has high noise interference.

$x(t)$ = 

$\delta_{T_s}$ → 

$s(t)$ = 

$h(t)$ = 

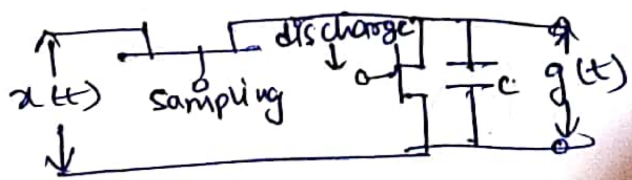
$g(t)$ = 

$$g(t) = s(t) \times h(t)$$

$$\sum_{n=-\infty}^{\infty} x(t) \, h(t - nT_s)$$

$$G(\omega) = \sum_{n=-\infty}^{\infty} x(f - n f_s) \, H(f)$$

Same as PAM (Pulse amplitude modulation.

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Time domain $\longrightarrow$ frequency domain
using Fourier transform

FT exist for $\rightarrow$ Energy signal
Power signal
impulse related signal $\Big] \rightarrow$ integrable

using property

$$x(t) \rightleftharpoons X(j\omega) \quad \text{or} \quad X(f)$$

rad/s  Hz

$$x(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) e^{j\omega t} dt$$

properties

1) linearity

$x_1(t) \rightleftharpoons X_1(j\omega) \qquad x_2(t) \rightleftharpoons X_2(j\omega)$

$a x_1(t) + b x_2(t) \rightleftharpoons a X_1(j\omega) + b X_2(j\omega)$

2) conjugation

$x(t) \rightleftharpoons X(j\omega)$

$x^*(t) \rightleftharpoons X^*(-j\omega)$

3) Area under $x(t) = X(j\omega)\big|_{\omega=0}$

4) Area under $X(j\omega) = 2\pi\, x(t)\big|_{t=0}$

5) time reversal

$x(t) \rightleftharpoons X(j\omega)$

$x(-t) \rightleftharpoons X(-j\omega)$

6) time scaling

$x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$

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7) time shifting

$$x(t \pm t_0) \rightleftharpoons X(j\omega)e^{\pm j\omega t_0}$$

8) frequency shifting

$$X[j(\omega \mp \omega_0)] \rightleftharpoons e^{\pm j\omega_0 t} x(t)$$

9) convolution in time

$$x_1(t) \rightleftharpoons X_1(j\omega)$$
$$x_2(t) \rightleftharpoons X_2(j\omega)$$

$$x_1(t) * x_2(t) \rightleftharpoons X_1(j\omega) \cdot X_2(j\omega)$$

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$$

10) multiplication in time / convolution in freq

$$x_1(t) \cdot x_2(t) \rightleftharpoons \frac{[X_1(j\omega) * X_2(j\omega)]}{2\pi}$$

11) differentiation in time

$$\frac{d(x(t))}{dt} = (j\omega)\, X(j\omega)$$

$$\frac{d^n(x(t))}{dt^n} = (j\omega)^n X(j\omega)$$

12) integration in time

$$x(t) \rightleftharpoons X(j\omega)$$

$$\int_{-\infty}^{\tau} x(\tau)\, d\tau \rightleftharpoons \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

13) differentiation in frequency

$$t \cdot x(t) = j \frac{dX(j\omega)}{d\omega}$$

$$t^n x(t) = j^n \frac{d^n X(j\omega)}{d\omega^n}$$

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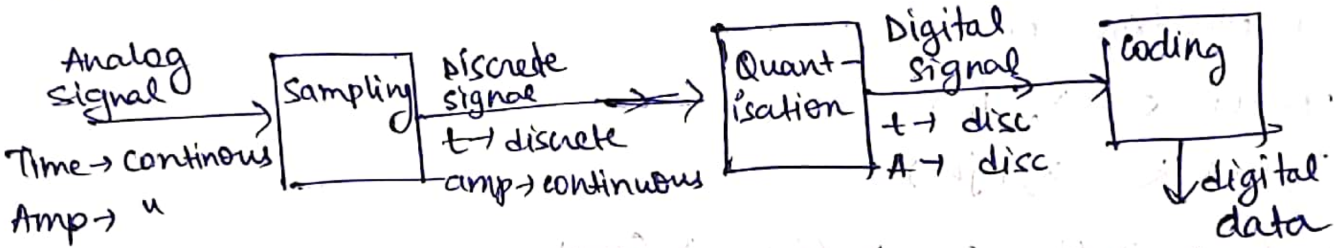
14) Modulation

$x(t) \rightleftharpoons X(\omega)$ or $X(j\omega)$

i) $x(t) \cdot \cos \omega_0 t \rightleftharpoons \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$

ii) $x(t) \cdot \sin \omega_0 t \rightleftharpoons \frac{j}{2}[X(\omega+\omega_0) - X(\omega-\omega_0)]$

# Quantisation

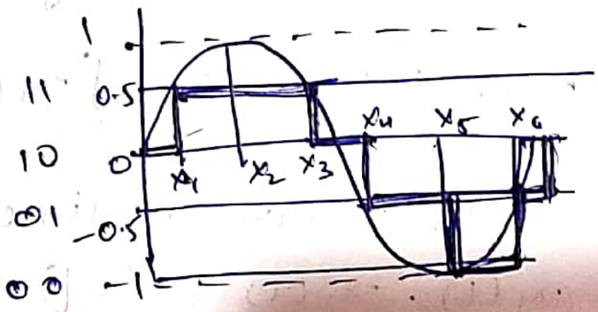



no. of bits $= n = 2$ (say)

" " levels $= 2^n \Rightarrow 4$

Step size $\Delta = \frac{A_{max} - A_{min}}{L}$

$= 0.5$



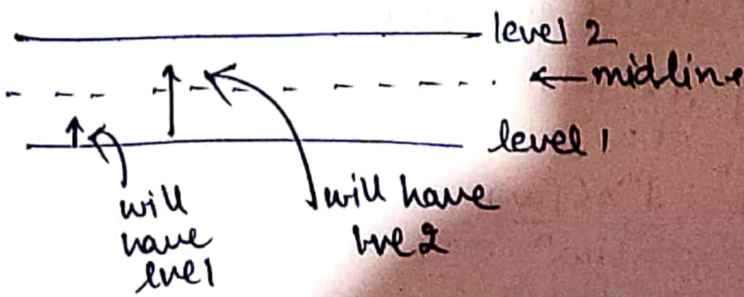

| points | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | ~~$x_7$~~ |
|---|---|---|---|---|---|---|---|
| sampling | 0.57 | 0.8 | 0.6 | -0.6 | -0.9 | -0.5 | |
| quantisation | 0.5 | 0.5 | 0.5 | 0.5 | 1 | 0.5 | |
| quantisation error | 0.07 | 0.3 | 0.1 | 0.1 | 0.1 | 0 | |

features of quantisation:

1) continous → ~~lim~~ finite set
2) infinite precision → finite precision
3) Rounding off samples to nearest quantised level.

to choose a level of quantisation divide the 2 levels ~~an~~ and chose the level acc to region

ex:



dynamic range of quantisation

↳ ratio of largest to smallest measurable amplitude.

$$= 20 \log \left[\frac{\text{largest Amp}}{\text{Smalles Amp}}\right]$$

$$= 20 \log \left[\frac{2^{n-1}\Delta}{2^{-1}\Delta}\right] = 20 \log 2^n = 20\, n \log 2$$

$$= \cancel{6.02n} \quad \underline{6.02n}$$

# SNR of quantisation

$$SNR = 20 \log \left[\frac{\text{signal rms voltage}}{\text{noise rms voltage}}\right]$$

$$q_e^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\,dq = \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{\Delta^2}{12}$$

$$q_e = \frac{\Delta}{2\sqrt{3}} \leftarrow \text{noise rms}$$

full scale signal $= V_{max} - V_{min} = V_{fs}$

Peak Voltage $= V_f/2$

Rms of Peak voltage $= \dfrac{V_f}{2\sqrt{2}} = \dfrac{2^n\Delta}{2\sqrt{2}}$

$$\Delta = \frac{V_{max} - V_{min}}{2^n} \Rightarrow V_f = 2^n\Delta$$

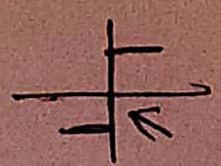
$$SNR = 20 \log \left[\frac{\frac{2^n\cancel{\Delta}}{\cancel{2}\sqrt{2}}}{\frac{\cancel{\Delta}}{\cancel{2}\sqrt{3}}}\right] = 20 \log \left(\frac{2^n\sqrt{3}}{2}\right)$$

$$= 6.02n + 1.76$$

$$\boxed{SNR = 6.02n + 1.76}$$ (same for SQNR)

Quantisation → uniform (step size = same) ; non unifor

uniform (step size = same) → midrise ; midtread

midrise  midtread 

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Quantisation noise / error (for uniform step size)

PDF of quantisation error

$$f_Q(q) \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} < q < \frac{\Delta}{2} \\ 0 & \text{else} \end{cases} \Bigg\} \text{ uniform distribution}$$

mean of quantisation error

$$= \int_{-\Delta/2}^{\Delta/2} q \left(\frac{1}{\Delta}\right) dq = \frac{1}{\Delta}\left[\frac{q^2}{2}\right]_{-\Delta/2}^{\Delta/2} = 0$$

Variance of quantisation

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} q^2 \left(\frac{1}{\Delta}\right) dq = \frac{1}{3\Delta}\left[q^3\right]_{-\Delta/2}^{\Delta/2}$$

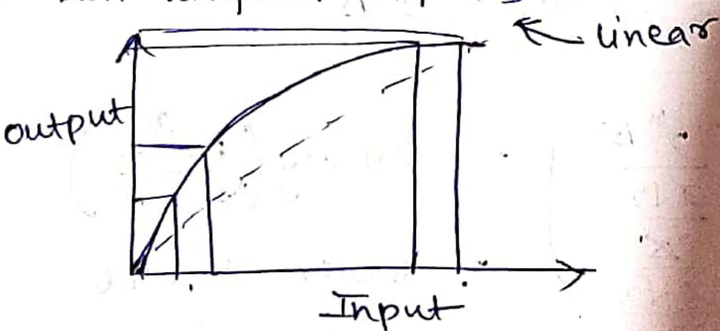
$$= \frac{\Delta^2}{12}$$

(Same as SNR)

- If step size ↑ → Quantisation noise ↑
  If step size ↓ → channel noise ↑

∴ for large variance → large step size
  for small variance → small step size.

∴ non uniform step size.

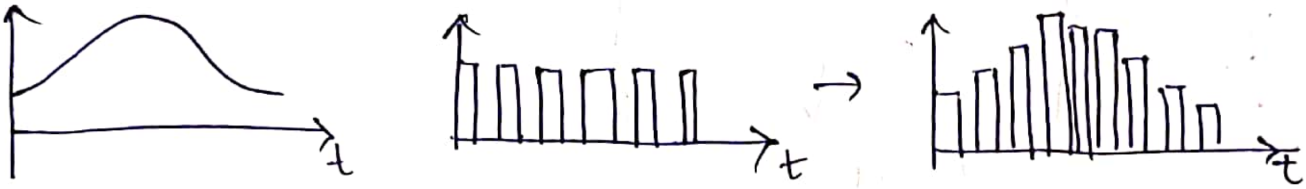


Pulse modulation
↳ Process where one parameter of carrier signal varies according to modulating signal.

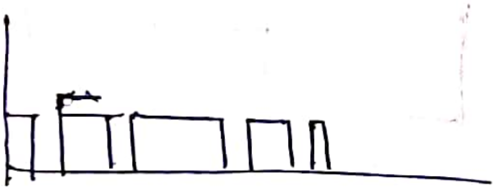
Modulating signal → low freq which we want to transfer
carrier signal → High freq.

size of antenna → $\frac{\lambda}{4}$

1) Pulse Amp modulation (Flat top)

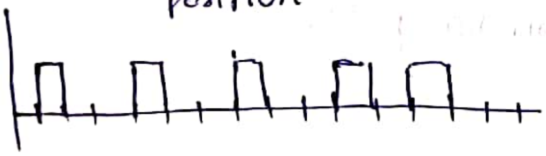



2) Pulse width modulation



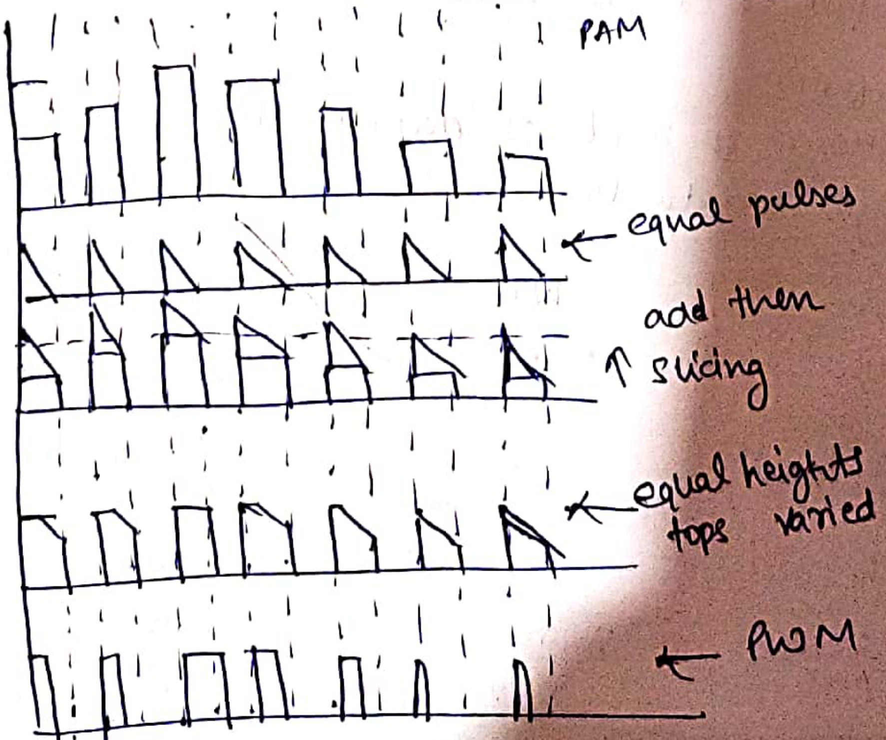
where amp is high
↳ width is high

3) Pulse ~~phase~~ position modulation

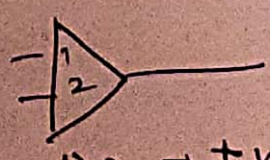


width is same
where the pulse ends in PWM the pulse rise in PWM and drop ~~a~~ ~~is~~ ~~that~~ ~~a~~ with same width

Generation




for slicing we use comparator




1>2 → +ve
1<2 → 0

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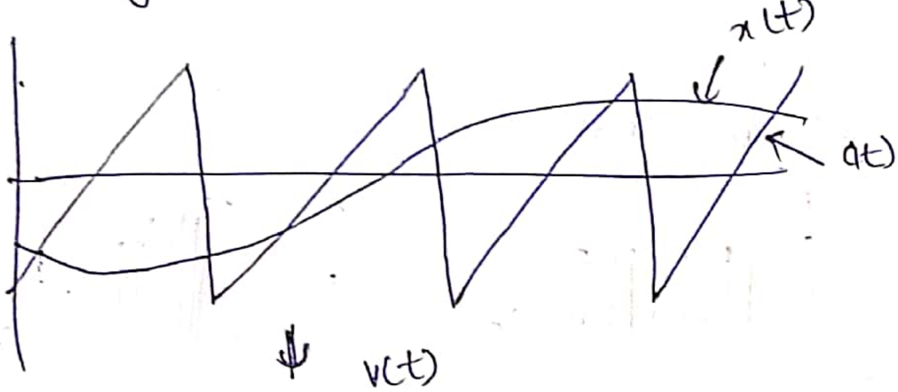
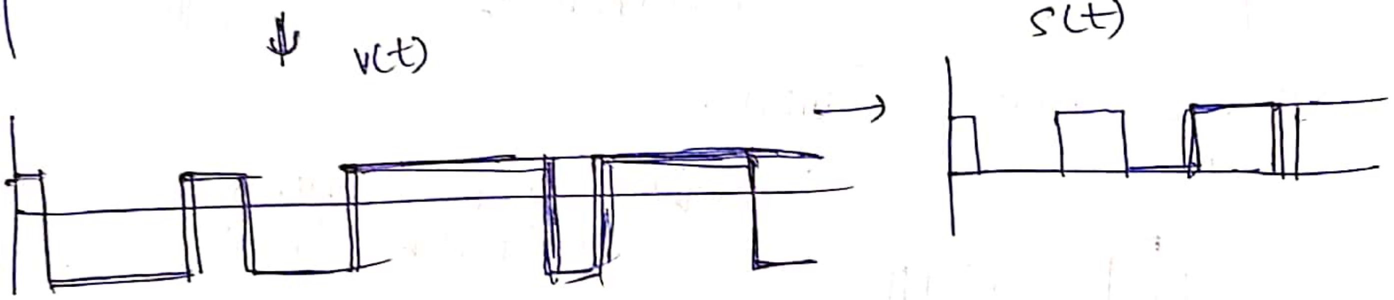
After.


s(t) → 1>2 → +ve
1<2 → 0

↳ saw tooth signal.

v(t) → 1>2 → +ve
1<2 → -ve





↑ generate fixed duration pulse

advantages of PPM
↳ less affected by noise
↳ noise contaminated can be reconstructed
↳ transmitted power is constant.

disadvantages of PPM
↳ synchronisation is reqd at demodulation
↳ requires high bandwidth.

| | PAM | PWM | PPM |
|---|---|---|---|
| B.W Band with | width | rise time | rise time |
| Transmitted power | wrt time | wrt time | constant |
| nois | high | min | low |
| | similar to AM | similar to FM | Similar to PM |

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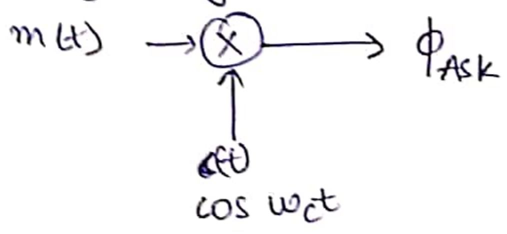
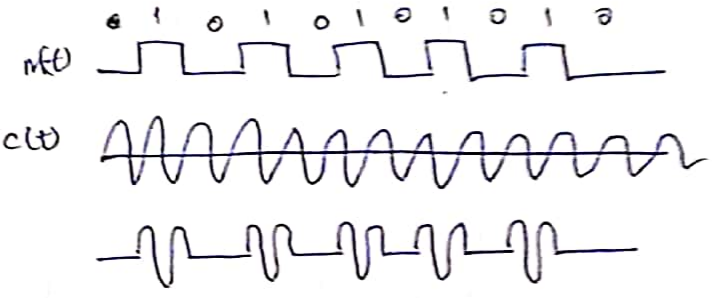
# Digital modulation

Digital → Analog

1) <u>Amplitude shift keying</u>

Amplitude of carrier signal varies wrt amplitude of msg signal.




for Binary, $n=1$

↳ BASK, OOK
on-off keying

Bandwidth :

$BW \propto r$ (baud rate)

$BW = (1+d)r$

$BW = (1+d)\frac{R}{n}$

} R → data Rate
n → bits reqd for ~~sa~~ elements
d → factor for modulation and filtering process
∈ (0, 1)
↑ ideal ↑ worst

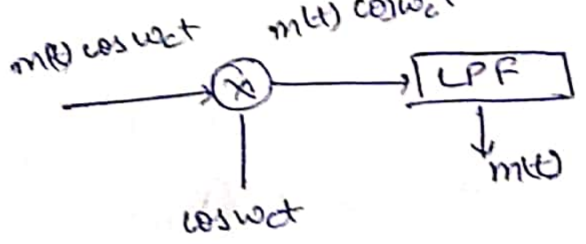
- modulation of ASK ( generation of ASK)

  $m(t) \times c(t)$  m(t) { 1 +ve, 0 no voltage

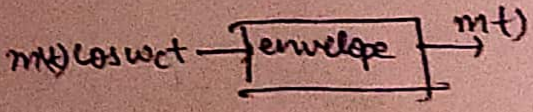
- demodulation ASK

Synchronous (coherant)

efficient but costly




non synchronous (non coherant)




low cost

poor performance with less SNR received signal.

Transmitted signal.

$x_1(t) = A \cos 2\pi f_c t$ ← bit 1

$x_2(t) = 0$ bit 2

$E_B$ = energy per bit

$$= \int_0^{T_B} x_1^2(t)\,dt = \int_0^{T_B} (A \cos 2\pi f_c t)^2 dt$$

$$= \frac{A^2}{2}[T_B - 0]$$

$$E_B = \frac{A^2 T_B}{2}$$

According to GSOP

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{E_B}} \Rightarrow \sqrt{\frac{2}{T_B}} \cos 2\pi f_c t$$

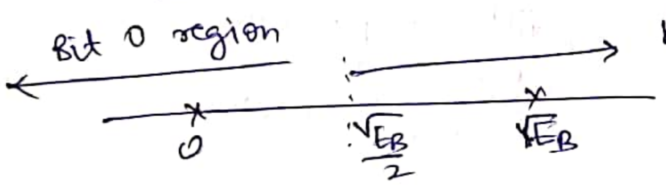
$$\phi_2(t) = 0$$

$M=2$ ← transmitted waveform

$N=1$ ← basis fn reqd.

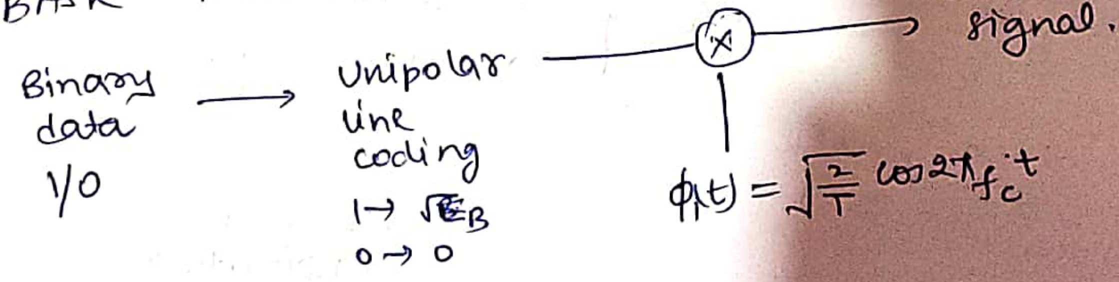
as $x_1(t) = \sqrt{E_B}\,\phi_1(t)$ ← bit 1

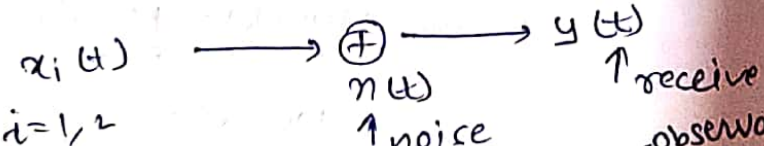
$x_2(t) = 0\,\phi_1(t)$ ← bit 2



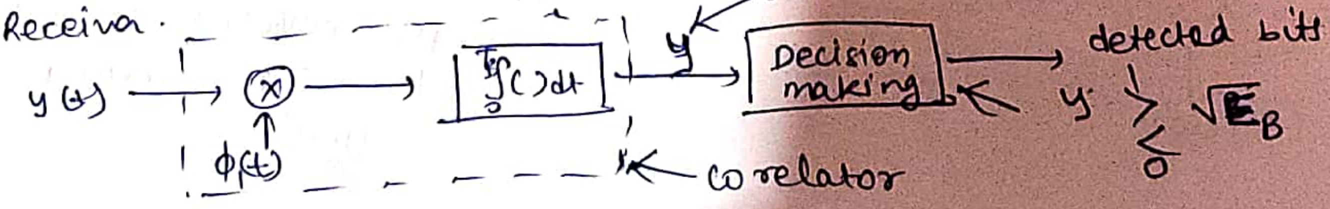

(signal space diagram)

BASK Transmitter.




Receiver.

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Probability of error

$P_e \rightarrow \dfrac{P_e(1) + P_e(0)}{2}$ $\quad$ $\left.\begin{matrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{matrix}\right\}$ error

$y(t) \begin{cases} x(t) + n(t) & \text{bit } 1 \\ 0 + n(t) & \text{bit } 2 \end{cases}$

corelation receiver output

$$y = \int_0^{T_B} y(t)\,\phi_1(t)\,dt$$

for bit 1

$$y = \int_0^{T_B} \left(x_1(t) + n(t)\right)\phi_1(t)\,dt$$

$$= \int_0^{T_B} \left(\sqrt{E_B}\,\phi_1(t) + n(t)\right)\phi_1(t)\,dt$$

$$= \sqrt{E_B}\underbrace{\int_0^{T_B} \phi_1^2(t)\,dt}_{\text{orthogonal } \Rightarrow 1} + \underbrace{n}_{\text{Gaussian as } n(t) \text{ is gaussian}}$$

$\Rightarrow y = \sqrt{E_B} + n$ $\quad$ bit 1

$y = n$ $\quad$ bit 0

$n$ is Gaussian with mean 0 and varian $\frac{N_0}{2}$

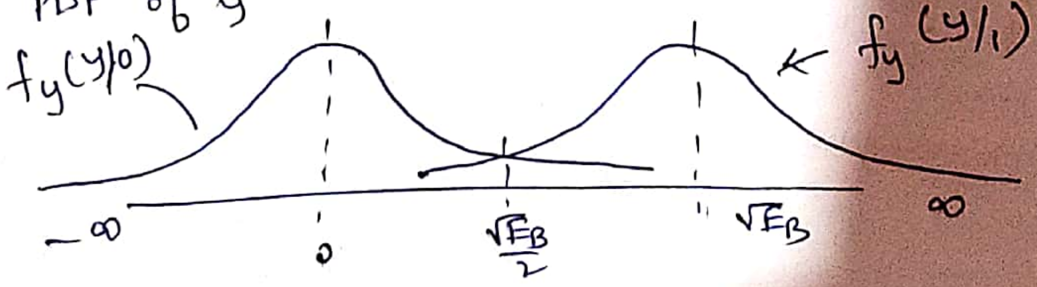
Mean of $y$ = $E(y)$ $\begin{cases} \sqrt{E_B} & \text{bit } 1 \\ 0 & \text{bit } 0 \end{cases}$

variance of $y$

$E(y-\bar{y})^2 = E\left\{\left((\sqrt{E_B}+n) - \sqrt{E_B}\right)^2\right\} = E\{n^2\} = \frac{N_0}{2}$

$E(y-\bar{y}^2) = E(n-0)^2 = E(n^2) = \frac{N_0}{2}$

PDF of $y$

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$$f_y(y/0) = \frac{1}{\sqrt{\pi N_0}} e^{-y^2/N_0} \qquad\qquad f_y(y) = \frac{1}{\sqrt{2\pi \sigma_y^2}} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}}$$

$$f_y(y/1) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y-\sqrt{E_B})^2}{N_0}}$$

Likelihood fn.

$$P_e(0) = \int_{\frac{\sqrt{E_B}}{2}}^{\infty} f_y(y/0)\,dy = \int_{\sqrt{E_B}/2}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-y^2/N_0}\,dy$$

$$\boxed{\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-z^2}\,dz}$$

$$= \quad z = \frac{y}{\sqrt{N_0}}$$

$$\Rightarrow \frac{1}{\sqrt{\pi \cancel{N_0}}} \int_{\frac{1}{2}\sqrt{\frac{E_B}{N_0}}}^{\infty} e^{-z^2} \sqrt{\cancel{N_0}}\,dz$$

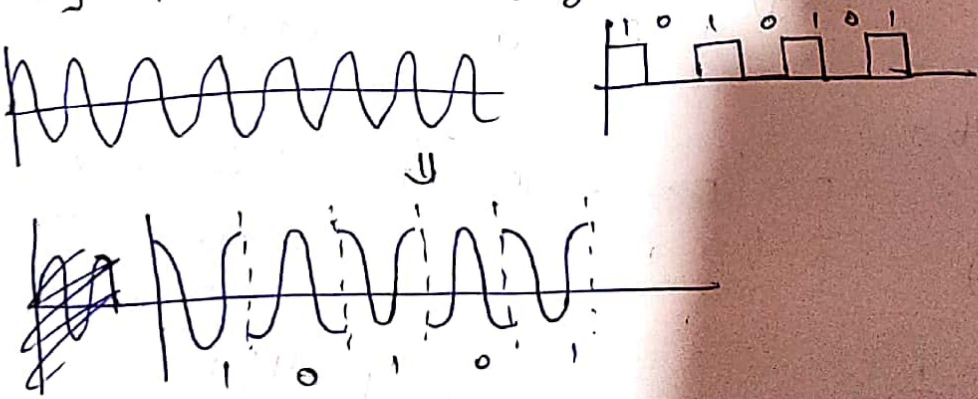
$$= \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{\frac{1}{2}\sqrt{\frac{E_B}{N_0}}}^{\infty} e^{-z^2}\,dz\right)$$

$$\Rightarrow P_e(0) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_B}{N_0}}\right)$$

$$P_e(1) = \int_{-\infty}^{\frac{\sqrt{E_B}}{2}} f_y(y/1)\,dy = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_B}{N_0}}\right)$$

$$\Rightarrow P_e = \frac{P_e(0) + P_e(1)}{2} = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_B}{N_0}}\right)$$

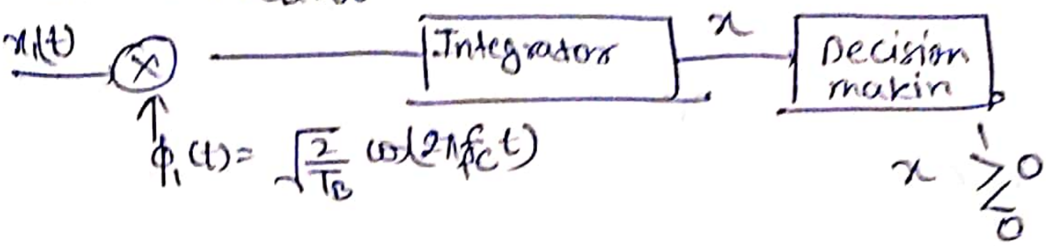
#) Binary phase shift keying



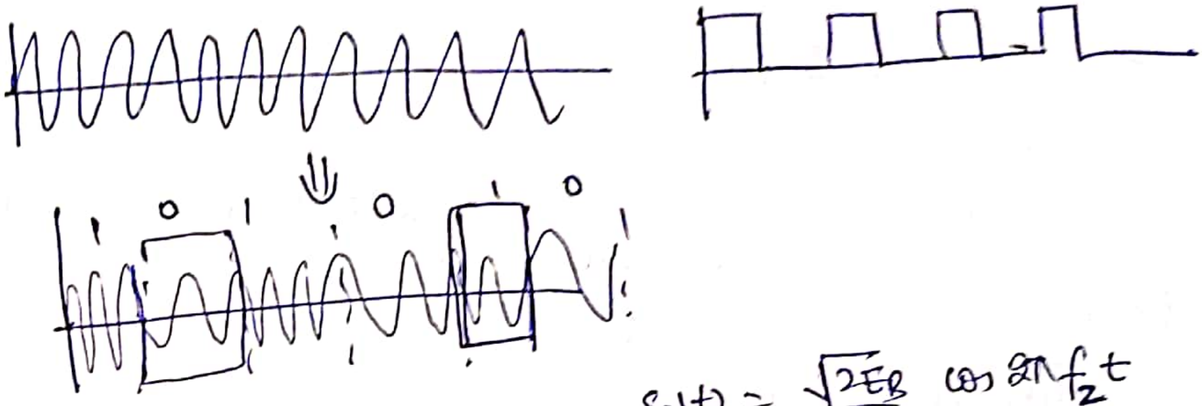

1 → phase1

0 → opposite phase.

circuit at receiver




# BFSK




$$s_1(t) = \sqrt{\frac{2E_B}{T_B}} \cos 2\pi f_1 t \qquad s_2(t) = \sqrt{\frac{2E_B}{T_B}} \cos 2\pi f_2 t$$

Basis fn

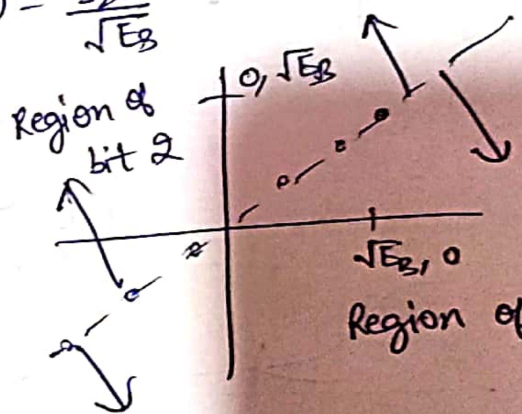
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_B}} \qquad \phi_2(t) = \frac{s_2(t)}{\sqrt{E_B}}$$

$$s_{11} = \int s_1 \phi_1 = \sqrt{E_B}$$

$$s_{12} = \int s_1 \phi_2 = 0$$

$$s_{21} = \int s_2 \phi_1 = 0$$

$$s_{22} = \int s_2 \phi_2 = \sqrt{E_B}$$




error is BFSK is less than BPSK b'cos distance b/w $(\sqrt{E_B}, 0)$ and $(\sqrt{E_B}, 0)$ is less than $2\sqrt{E_B}$ (easily understandable by graph)

if received signal is $x(t)$

$$x_1 = \int_0^{T_B} x(t)\,\phi_1(t)\,dt$$

$$x_2 = \int_0^{T_B} x(t)\,\phi_2(t)\,dt$$

$$x_1 \underset{z_2}{\overset{z_1}{\gtrless}} x_2$$

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Type → Binary ( single bit is modulated)
Quadrature (2 bit) → 4 combination ( $\phi = \pi/2$ )
m-array (general)

no. of level ↑ → $P_e$ ↑

$$s_1(t) = \sqrt{\frac{2E_B}{T_B}} \cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_B}{T_B}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_B}{T_B}} \cos(2\pi f_c t)$$

orthogonal signals → signals lie in diff direction ∴ min noise interference

$$\underbrace{\int \phi_1 \phi_2 = 0}_{\uparrow \text{ orthonormal fn}} \quad \text{as} \quad \phi_1 \perp \phi_2$$

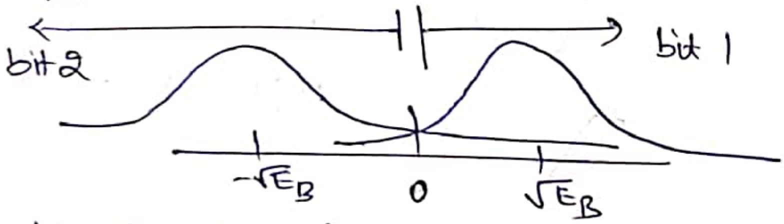
$$\phi_1 = \cancel{\sqrt{E_B} \cos} \sqrt{\frac{2}{T_B}} \cos(2\pi f_c t)$$

$$S_{11} = \int_0^{T_B} s(t)\, \phi_1(t) = \int_0^{T_B} \sqrt{E_B}\, \phi_1^2(t)\, dt$$

$$\int \phi_1 \cdot \phi_1 = 1$$

$$\int \phi_i\, \phi_j = 0 \quad i \neq j$$

$$\left.\begin{aligned} S_{11} &= \sqrt{E_B} \\ S_{21} &= \int s_2\, \phi_2 = -\sqrt{E_B} \end{aligned}\right\} \leftarrow \text{mean}$$




likelihood fn when 0 is transmitted.

$$f_x(x|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_B})^2\right]$$

$$P_e(0) = \int_0^{+\infty} f_x(x|0)\, dx = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_B}{N_0}}\right)$$

$$P_e(1) = \int_{-\infty}^{0} f_x(x|1)\, dx = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_B}{N_0}}\right)$$

$$P_e = \frac{P_e(0) + P_e(1)}{2} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_B}{N_0}}\right)$$

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far error calculation

error $= x_1 - x_2$

let $x_1 - x_2 = l$

$$l \underset{x_2}{\overset{x_1}{\gtrless}} 0$$

expectation is linear fn

expectation value

$$E[L|1] = E[x_1|1] - E[x_2|1]$$

$$= \sqrt{E_B} - 0 = \sqrt{E_B}$$

$$E[L|0] = E[x_1|0] - E[x_2|0]$$

$$= 0 - \sqrt{E_B} = -\sqrt{E_B}$$

$$Var[L] = var[x_1] + Var[x_2] = \frac{N_0}{2} + \frac{N_0}{2} = N_0$$

$L \rightarrow$ mean $= 0$
var $= N_0$

conditional probability

$$f_L(l|0) = \frac{1}{2\pi N_0} e^{\left(\frac{-l+\sqrt{E_B}}{2N_0}\right)^2}$$

$$P(l|0) = \int_0^{\infty} f_L(l|0)\, dl$$

$$= \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_B}{2N_0}}\right)$$

$||^{ly}$ $P(l|1) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_B}{2N_0}}\right)$

$$P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_B}{2N_0}}\right)$$

clearly error is BFSK < BPSK

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# Error detecting codes
↳ To minimise effect of noise in channel

parity code
↳ for 1 bit only (or odd bits)

parity → even or odd times of 1

| P | data (msg) | Tx | corruption → | Rx |
|---|---|---|---|---|
| 0 | 1001 | 01001 | | 01101 |
| 1 | 1011 | 11011 | | 11010 |
| 0 | 0101 | 00101 | | 10101 |
| 0 | 1111 | 01111 | | 11111 |

new bit → P

if Tx has even parity and Rx has odd or vice versa then odd no. of bits are corrupted

⇒ not for even corrupted bits (draw back)

# Hamming code
↳ we send data along with parity bits / redundant bits
↳ represented an $(n, k)$

$n$ — Total bits, $k$ — msg bits

Parity bits $= n-k = P$

cond$^n$ for parity bit $= 2^P \geq P + K + 1$

$P = 3$ satisfy

# Linear block code
↳ A block code satisfy cond$^n$ that sum of any 2 code word = another code word

$C_p = C_i + C_j$

property
↳ all 0 [0 . . . 0] is always a code word
↳ $C_p = C_i + C_k$

$d(C_i, C_k) = W(C_p)$

$d$ — Hamming distance, $W$ — weigh

↳ $d_{min} = W_{min}$

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Weight = Total no. of 1

~~distance~~ min hamming dist = min weight.

(7, 4) Hamming code = Linear block code

↑ satisfy the condn.

$[C] = [i][G]$ ← matrices

. $C$ = code word

$i$ = information word

$G$ = Generator matrix

$G \to$ ~~k×n~~ $n \times k$ matrix.

$[G] = [I : P]$

for (7, 4) systematic code.

$$[G] = \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & \vdots & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & \vdots & 0 & 1 & 1 \end{bmatrix}$$

↑ Identity ↑ Parity

example:

$[i] = 1110$

$[G] \to$ as above

$[C] = [i][G] = [1\ 1\ 1\ 0\ 1\ 0\ 0]$

[we do mod2 sum
$1+1=0$
$1+0=1$
$0+0=0$

in code word.

$[C] = [m : P_c]$

↑ always msg bits bcos $G = [I : P]$

$m \times I = m$

for $P_c$ we can also do

$[P_c] = [m][P]$ (simple calculation)

Parity check matrix

$[H] = [P^T : I_{n-k}]$

$n, k, P$ from Generator matrix.

Properties

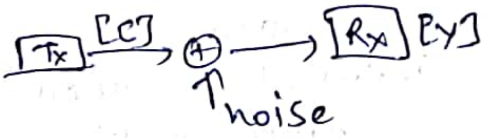
$$HG^T = GH^T = CH^T = 0$$

$$C = [i][q]$$

$$CH^T = [i][q][H^T] = 0$$

Error syndromes in linear block code

$R_x = [Y]$




Error syndrome

$$[S] = [Y][H^T]$$

$$[Y] = [c] + [e]$$

$$[S] = ([c] + [e]) H^T$$

$$[S] = eH^T$$

| Syndrome | error pattern | |
|---|---|---|
| 000 | 0000000 | (1st bit corrupted) |
| 100 | 1000000 | (2nd bit corrupted) |
| 010 | 0100000 | |
| 001 | 0010000 | |
| 110 | 0001000 | |
| 011 | 0000100 | |
| 111 | 0000010 | |
| 101 | 0000001 | |

Syndrome depends only on e

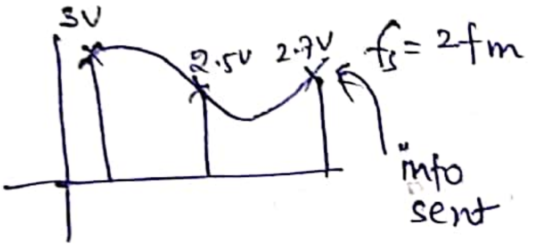
# DPCM

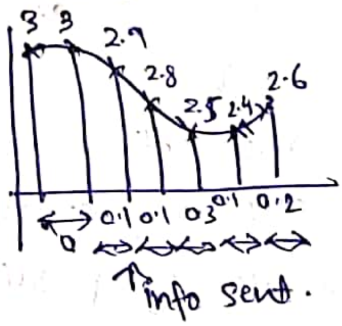
Differential pulse code modulation

↳ when $f_s > 2f_m$ (sampling) then successive samples become more correlated.

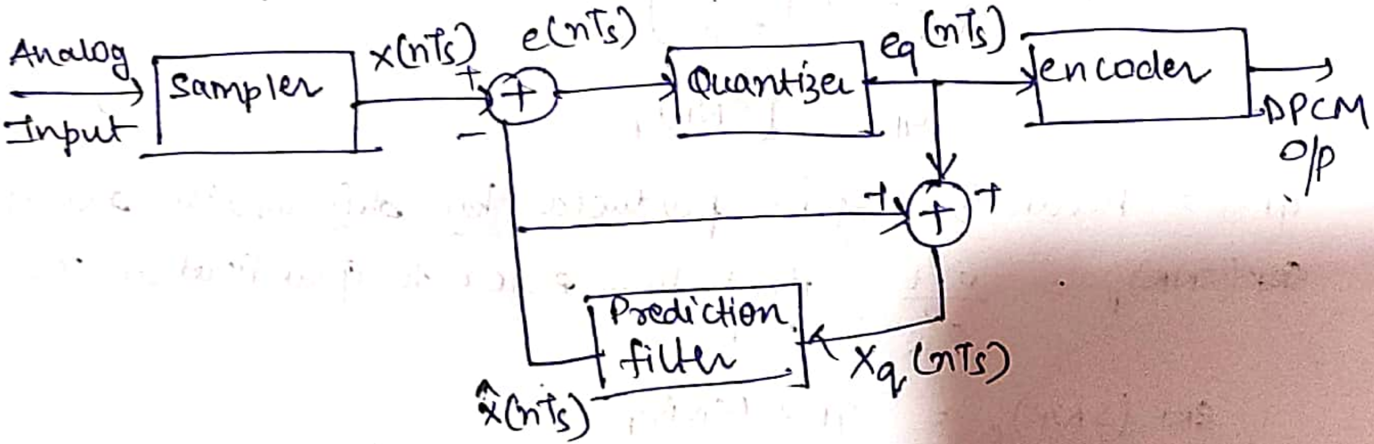
↳ Instead of sending data for individual sample (like in PCM) we send data abt successive difference in pulse voltage.

∴ no. of levels ↓ → bit rate ↓




DPCM encoder




$e(nT_s)$ = Error signal

$= x(nT_s) - \hat{x}(nT_s)$

Quantised error signal.

$e_q(nT_s) = e(nT_s) + q_e(nT_s)$

Input to prediction filter

$$x_q(nT_s) = e_q(nT_s) + \hat{x}(nT_s)$$

$$= e(nT_s) + q_e(nT_s) + \hat{x}(nT_s)$$

$$= x(nT_s) - \hat{x}(nT_s) + q_e(nT_s)$$

$$x_q(nT_s) = x(nT_s) + q_e(nT_s)$$

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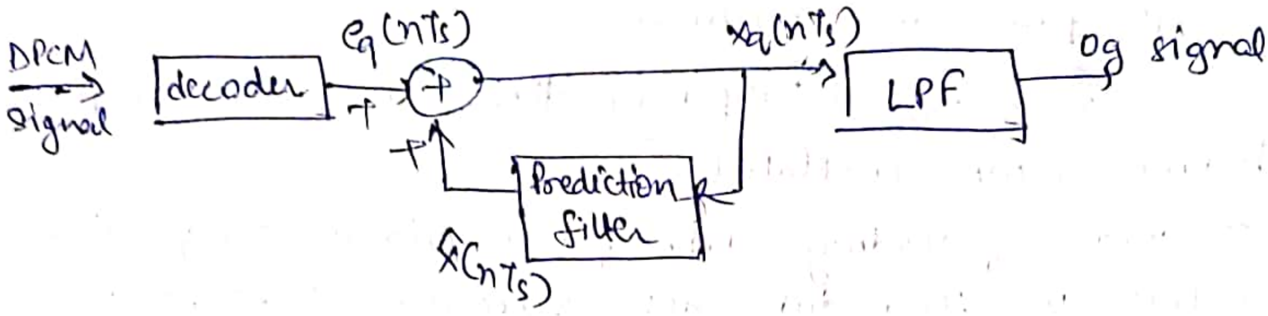
DPCM decoder.




# SQNR of DPCM

$$(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2} \quad \text{at o/p of quantiser}$$

$\sigma_x^2$ = variance of original $V_p$ $x(nT_s)$

$\sigma_Q^2$ = variance of quantised error $e_q(nT_s)$

$\sigma_E^2$ = Variance of prediction error $e(nT_s)$

$$(SNR)_o = \underbrace{\left(\frac{\sigma_x^2}{\sigma_E^2}\right)}_{GP} \underbrace{\left(\frac{\sigma_E^2}{\sigma_Q^2}\right)}_{(SNR)_p}$$

GP → Prediction gain produced by differential quantisation

~~SRP~~ $(SNR)_p = \frac{\sigma_E^2}{\sigma_Q^2}$ = Prediction error to quantisation noise ratio.

~~SR~~ $(SNR)_o = GP \times (SNR)_p$

$GP > 1 \Rightarrow \sigma_x^2 > \sigma_E^2$

we can't change $\sigma_x^2$ but can change $\sigma_E^2$

our goal is to minimise $\sigma_E^2$ (better prediction filter)

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