Non-Existence of Linear–Quartic Factorization for the Second Cuboid Quintic

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January 5, 2026

Abstract

Let $Q_{p,q}(t) \in \mathbb{Z}[t]$ be Sharipov's even monic degree-10 *second cuboid polynomial* depending on coprime integers $p \neq q > 0$. Writing $Q_{p,q}(t)$ as a quintic in t^2 produces an associated monic quintic polynomial. After the weighted normalization $r = p/q$ and $s = r^2$ we obtain a one-parameter family $P_s(x) \in \mathbb{Q}[x]$ such that

$$Q_{p,q}(t) = q^{20}\, P_s\left(\frac{t^2}{q^4}\right) \qquad \text{with} \qquad s = \left(\frac{p}{q}\right)^2.$$

We show that for every rational $s > 0$ with $s \neq 1$ the equation $P_s(x) = 0$ has no rational solutions. Equivalently, P_s admits no $1 + 4$ factorization over $\mathbb{Q}$.

The proof uses an explicit quotient by the inversion involution $(s, y) \mapsto (1/s, 1/y)$ and reduces the rational-root problem for P_s to rational points on the fixed genus-2 hyperelliptic curve

$$C: \quad w^2 = t^5 + 21t^4 + 26t^3 + 10t^2 + 5t + 1 = (t + 1)(t^4 + 20t^3 + 6t^2 + 4t + 1).$$

Using Magma and Chabauty's method on the Jacobian of C, we compute $C(\mathbb{Q})$ exactly and conclude that the only parameter value producing a rational root is the excluded case $s = 1$ (equivalently $p = q$).

As a consequence, for coprime $p \neq q > 0$ the polynomial $Q_{p,q}(t)$ has no rational roots (hence no linear factor over $\mathbb{Q}$, and in particular no linear factor over $\mathbb{Z}$).

Keywords: perfect cuboid; cuboid polynomials; rational points; hyperelliptic curves; Jacobians; Chabauty method; Magma; computer-assisted proof.

Mathematics Subject Classification: 11D41, 11G30, 14H25, 12E05, 11Y16.

1 Introduction

The perfect cuboid problem asks for a rectangular box with integer edges such that all three face diagonals and the space diagonal are integers. In a framework due to R. A. Sharipov [1, 2], one is led to explicit parameter-dependent even polynomials whose irreducibility is conjectured for coprime parameters. In particular, Sharipov defines the *second cuboid polynomial* $Q_{p,q}(t) \in \mathbb{Z}[t]$ of degree 10 and formulates the conjecture that $Q_{p,q}(t)$ is irreducible over $\mathbb{Z}$ for coprime $p \neq q > 0$.

Goal. We close the *linear–quartic* case for the normalized associated quintic $P_s(x)$: we prove that for every rational $s > 0$ with $s \neq 1$ the quintic P_s has no rational root (equivalently, it admits no $1+4$ factorization over $\mathbb{Q}$). As a consequence, for coprime $p \neq q > 0$ the second cuboid polynomial $Q_{p,q}(t)$ has no rational roots (and hence no linear factor).

Method. The key observation is an explicit inversion symmetry and a quotient reduction that transforms the rational-root condition for a one-parameter family of quintics into the computation of rational points on a fixed genus-2 hyperelliptic curve. We then compute all rational points on that curve using Magma [3] (rank bound plus Chabauty on the Jacobian), yielding a certificate-style closure of the $1+4$ case.

2 The second cuboid polynomial and its associated quintic

2.1 Sharipov's second cuboid polynomial $Q_{p,q}(t)$

Let $p, q \in \mathbb{Z}_{>0}$ be coprime and $p \neq q$. The second cuboid polynomial is the even monic degree-10 polynomial

$$\begin{aligned} Q_{p,q}(t) = t^{10} &+ (2q^2+p^2)(3q^2-2p^2)\,t^8 \\ &+ (q^8+10p^2q^6+4p^4q^4-14p^6q^2+p^8)\,t^6 \\ &- p^2q^2\,(q^8-14p^2q^6+4p^4q^4+10p^6q^2+p^8)\,t^4 \\ &- p^6q^6\,(q^2+2p^2)(-2q^2+3p^2)\,t^2 - p^{10}q^{10} \in \mathbb{Z}[t]. \end{aligned} \tag{1}$$

This is the polynomial denoted $Q_{p,q}(t)$ in [1, 2].

2.2 Weighted normalization to $Q_r(u)$

The polynomial (1) is weighted-homogeneous of total weight 20 for

$$\deg(p) = \deg(q) = 1, \qquad \deg(t) = 2,$$

hence one may normalize to a one-parameter family.

Lemma 1 (Normalization). *Let $q \neq 0$ and set*

$$r := \frac{p}{q} \in \mathbb{Q}, \qquad u := \frac{t}{q^2} \in \mathbb{Q}.$$

Then

$$Q_{p,q}(t) = q^{20}\,Q_r(u), \tag{2}$$

where

$$\begin{aligned} Q_r(u) = u^{10} &+ (2+r^2)(3-2r^2)\,u^8 + \big(1+10r^2+4r^4-14r^6+r^8\big)\,u^6 \\ &- r^2\big(1-14r^2+4r^4+10r^6+r^8\big)\,u^4 - r^6(1+2r^2)(-2+3r^2)\,u^2 - r^{10} \in \mathbb{Q}[u]. \end{aligned} \tag{3}$$

Proof. Substitute $p = rq$ and $t = q^2u$ into (1) and factor out q^{20}. $\square$

2.3 The associated quintic $P_s(x)$

Since $Q_r(u)$ is even, it is a quintic in $x = u^2$.

Definition 1 (Second cuboid quintic)**.** Let $s := r^2 \in \mathbb{Q}_{\geq 0}$. Define $P_s(x) \in \mathbb{Q}[x]$ by the identity

$$Q_r(u) = P_s(u^2). \tag{4}$$

Equivalently, $P_s(x)$ is the monic quintic

$$\begin{aligned} P_s(x) = x^5 &+ (2+s)(3-2s)\,x^4 + (1+10s+4s^2-14s^3+s^4)\,x^3 \\ &- s(1-14s+4s^2+10s^3+s^4)\,x^2 - s^3(1+2s)(-2+3s)\,x - s^5. \end{aligned} \tag{5}$$

Derivation. Substitute $x = u^2$ into (3) and set $s = r^2$. □

Definition 2 ($1+4$ factorization for P_s)**.** Let K be a field of characteristic 0. We say that a monic quintic $P(x) \in K[x]$ admits a $1+4$ *factorization over* K if it has a root in K (equivalently $P(x) = (x - x_0)H(x)$ with $\deg H = 4$).

Lemma 2 (Roots and even quadratic factors)**.** *Let $p, q \in \mathbb{Z}_{>0}$, $q \neq 0$, and set $r = p/q$ and $s = r^2$. Then the following are equivalent:*

1. *$P_s(x)$ has a rational root $x_0 \in \mathbb{Q}$;*
2. *$Q_r(u)$ is divisible in $\mathbb{Q}[u]$ by the even quadratic factor $u^2 - x_0$;*
3. *$Q_{p,q}(t)$ is divisible in $\mathbb{Q}[t]$ by the even quadratic factor $t^2 - q^4 x_0$.*

Moreover, if $Q_{p,q}(t)$ has a rational root $t_0 \in \mathbb{Q}$, then P_s has a rational root $x_0 = (t_0/q^2)^2 \in \mathbb{Q}$.

Proof. The equivalence of (1) and (2) follows from (4). Indeed, if $P_s(x_0) = 0$ then $(x - x_0) \mid P_s(x)$ in $\mathbb{Q}[x]$, hence $(u^2 - x_0) \mid P_s(u^2) = Q_r(u)$ in $\mathbb{Q}[u]$. Conversely, if $(u^2 - x_0) \mid P_s(u^2)$, then in the quotient ring $\mathbb{Q}[u]/(u^2 - x_0)$ we have $P_s(u^2) = P_s(x_0) = 0$, hence $P_s(x_0) = 0$ in $\mathbb{Q}$.

The equivalence of (2) and (3) follows from the normalization (2): substitute $u = t/q^2$ and clear denominators to see that $u^2 - x_0$ divides $Q_r(u)$ if and only if $t^2 - q^4 x_0$ divides $Q_{p,q}(t)$.

For the final claim, if $Q_{p,q}(t_0) = 0$ with $t_0 \in \mathbb{Q}$ then $Q_r(u_0) = 0$ for $u_0 = t_0/q^2 \in \mathbb{Q}$, hence $P_s(u_0^2) = 0$ and $x_0 = u_0^2 = (t_0/q^2)^2$ is a rational root of P_s. □

Remark 1. In the original cuboid setting we have $p, q > 0$, hence $r > 0$ and $s = r^2 \in \mathbb{Q}_{>0}$. Moreover $p \neq q$ is equivalent to $r \neq 1$, i.e. $s \neq 1$.

3 Normalized root equation and inversion symmetry

Fix $s \in \mathbb{Q}_{>0}$. A $1+4$ factorization of P_s is equivalent to the existence of $x \in \mathbb{Q}$ such that $P_s(x) = 0$. Since $P_s(0) = -s^5 \neq 0$ for $s \neq 0$, any root x satisfies $x \neq 0$. It is convenient to scale by s.

Lemma 3 (Scaled root equation)**.** *Let $s \in \mathbb{Q} \setminus \{0\}$ and set $x = sy$. Then*

$$P_s(x) = 0 \quad \Longleftrightarrow \quad F(s, y) = 0,$$

where $F(s, y) \in \mathbb{Z}[s, y]$ is

$$\begin{aligned} F(s, y) = s^2y^5 + (-2s^3 - s^2 + 6s)y^4 + (s^4 - 14s^3 + 4s^2 + 10s + 1)y^3 \\ + (-s^4 - 10s^3 - 4s^2 + 14s - 1)y^2 + (-6s^3 + s^2 + 2s)y - s^2. \end{aligned} \tag{6}$$

Proof. Substitute $x = sy$ into (5) and divide the resulting identity by s^3 (valid for $s \neq 0$). □

Lemma 4 (Inversion symmetry)**.** *The curve $F(s, y) = 0$ is invariant under the involution*

$$(s, y) \longmapsto \left(\frac{1}{s}, \frac{1}{y}\right).$$

More precisely,

$$F\left(\frac{1}{s}, \frac{1}{y}\right) = -\frac{1}{s^4y^5} F(s, y). \tag{7}$$

Proof. This is a direct verification from (6) by substitution and simplification. □

4 Quotient by inversion and a rational parametrization

By Lemma 4, it is natural to pass to invariants of the inversion involution.

Definition 3 (Invariants)**.** Define

$$U := s + \frac{1}{s}, \qquad V := y + \frac{1}{y}.$$

Equivalently, s and y satisfy the quadratics

$$s^2 - Us + 1 = 0, \qquad y^2 - Vy + 1 = 0. \tag{8}$$

Proposition 1 (Elimination to a plane curve $G(U, V) = 0$)**.** *Let $(s, y) \in (\mathbb{Q} \setminus \{0\})^2$ satisfy $F(s, y) = 0$. Then $(U, V) \in \mathbb{Q}^2$ defined by Definition 3 satisfies*

$$G(U, V) = 0, \tag{9}$$

where $G \in \mathbb{Z}[U, V]$ is the degree-5 polynomial in V

$$\begin{aligned} G(U, V) = V^5 + (4U - 2)V^4 + (-10U^2 - 8U + 64)V^3 \\ + (4U^3 - 108U^2 + 384)V^2 + (U^4 - 8U^3 - 192U^2 + 768)V \\ + (-2U^4 - 128U^2 + 512). \end{aligned} \tag{10}$$

Proof. By (8), the existence of (s, y) with given (U, V) is equivalent to the vanishing of the double resultant

$$\mathrm{Res}_y\big(\mathrm{Res}_s\big(F(s, y),\ s^2 - Us + 1\big),\ y^2 - Vy + 1\big) \in \mathbb{Z}[U, V].$$

A direct elimination yields that this resultant equals $G(U, V)^2$ (no additional factors occur), and hence $G(U, V) = 0$ for any solution (s, y). □

Proposition 2 (Rationality and parametrization). *The affine curve $G(U,V)=0$ has a singular point at $(U,V)=(2,-2)$ of multiplicity 4, hence it is a rational curve. Moreover, the family of lines through $(2,-2)$*

$$V+2=t(U-2), \qquad t\in\mathbb{P}^1,$$

parametrizes $G=0$ as

$$U=U(t), \qquad V=V(t), \tag{11}$$

where

$$U(t)=\frac{2\left(t^5+6t^4+30t^3+16t^2+9t+2\right)}{t\,(t-1)^2(t^2+6t+1)}, \tag{12}$$

$$V(t)=\frac{2\left(t^4+36t^3+22t^2+4t+1\right)}{(t-1)^2(t^2+6t+1)}. \tag{13}$$

Proof. Substitute $V=t(U-2)-2$ into $G(U,V)$. A direct expansion and factorization gives

$$G\big(U,\,t(U-2)-2\big)=(U-2)^4\cdot\Big(U\cdot\big(t^5+4t^4-10t^3+4t^2+t\big)-\big(2t^5+12t^4+60t^3+32t^2+18t+4\big)\Big).$$

This shows that $(U,V)=(2,-2)$ is a multiplicity-4 singular point and that the remaining intersection is governed by a linear equation in U. Solving for U yields (12), and then $V=t(U-2)-2$ gives (13). □

5 Lifting back to (s,y) and a genus-2 obstruction curve

The parametrization (11) describes all rational points on the quotient curve $G(U,V)=0$. However, we must also enforce that U and V lift to rational s and y via (8).

Lemma 5 (Square conditions). *Let $(s,y)\in(\mathbb{Q}\setminus\{0\})^2$ and define (U,V) as in Definition 3. Then*

$$U^2-4=\left(s-\frac{1}{s}\right)^2\in(\mathbb{Q}^\times)^2, \qquad V^2-4=\left(y-\frac{1}{y}\right)^2\in(\mathbb{Q}^\times)^2.$$

Conversely, given $U,V\in\mathbb{Q}$, the quadratics (8) have solutions $s,y\in\mathbb{Q}$ if and only if U^2-4 and V^2-4 are squares in $\mathbb{Q}$.

Proof. This follows by completing the square in (8). □

Proposition 3 (Reduction to a genus-2 hyperelliptic curve). *Let $(s,y)\in(\mathbb{Q}\setminus\{0\})^2$ satisfy $F(s,y)=0$ and assume $s\neq 1$. Then there exists $t\in\mathbb{Q}$ and $w\in\mathbb{Q}$ such that*

$$w^2=t^5+21t^4+26t^3+10t^2+5t+1. \tag{14}$$

Equivalently, there exists a rational point on the genus-2 hyperelliptic curve

$$C:\quad w^2=(t+1)(t^4+20t^3+6t^2+4t+1). \tag{15}$$

Proof. Let $U = s + 1/s$ and $V = y + 1/y$. By Proposition 1 we have $G(U, V) = 0$. Since $s \neq 1$, we have $U \neq 2$ and therefore we can define

$$t := \frac{V + 2}{U - 2} \in \mathbb{Q}.$$

By Proposition 2, this t corresponds to the line through the singular point $(2, -2)$ and (U, V), hence (U, V) is represented by $U = U(t)$ and $V = V(t)$.

Now compute (using (12)–(13)) that

$$U(t)^2 - 4 = \frac{16\,(t+1)^5(t^4 + 20t^3 + 6t^2 + 4t + 1)}{t^2(t-1)^4(t^2 + 6t + 1)^2}, \tag{16}$$

$$V(t)^2 - 4 = \frac{256\,t^2(t+1)(t^4 + 20t^3 + 6t^2 + 4t + 1)}{(t-1)^4(t^2 + 6t + 1)^2}. \tag{17}$$

The denominators in (16) and (17) are squares in $\mathbb{Q}$, and so are the factors 16, 256, $(t+1)^4$, and t^2. Therefore $U^2 - 4$ and $V^2 - 4$ being squares (Lemma 5) implies that

$$(t + 1)(t^4 + 20t^3 + 6t^2 + 4t + 1)$$

is a square in $\mathbb{Q}$, i.e. there exists $w \in \mathbb{Q}$ satisfying (15). Expanding the right-hand side gives (14). □

Remark 2 (Special values $t = 0$ and $t = 1$). The parametrization (12)–(13) has poles at $t = 0$ and $t = 1$; geometrically these correspond to directions through the singular point $(2, -2)$ for which the line does not meet the curve $G = 0$ at any other finite point. In our application $s \neq 1$ implies $U \neq 2$, so $t = (V + 2)/(U - 2)$ is well-defined and does not correspond to the singular point.

6 Rational points on C and closure of the $1 + 4$ case

Proposition 4 (Rational points on C)**.** *Let C be the hyperelliptic curve* (15)*. Then*

$$C(\mathbb{Q}) = \{\infty,\ (-1, 0),\ (0, \pm 1),\ (1, \pm 8)\}.$$

Computational proof in Magma. This is certified by Script 02 in Appendix A. The script computes a rank bound RankBound$(J) = 1$ for the Jacobian $J = \mathrm{Jac}(C)$, constructs a Jacobian point of infinite order (verified by `Order(pt_J) eq 0`), and then applies `Chabauty(pt_J)` (based on the method of Chabauty [4] and Coleman [5]) to obtain all rational points on C. The complete output list agrees with the set above. □

Lemma 6 (No admissible parameters from $C(\mathbb{Q})$)**.** *Let $(t, w) \in C(\mathbb{Q})$ and let $U = U(t)$ be as in* (12) *whenever defined. Then the only positive rational value of s satisfying $U = s + 1/s$ is $s = 1$.*

Proof. By Proposition 4, the rational points on C correspond to $t \in \{-1, 0, 1, \infty\}$. We determine the admissible values of s by solving $s + 1/s = U(t)$ using the expression (12).

Case $t = \infty$. Comparing the degrees of the numerator and denominator in (12), we find $\lim_{t\to\infty} U(t) = 2$. The equation $s + 1/s = 2$ is equivalent to $(s-1)^2 = 0$, yielding the unique solution $s = 1$. This corresponds to the excluded case $p = q$.

Case $t = -1$. Substituting $t = -1$ into (12) yields

$$U(-1) = \frac{2\,(-1 + 6 - 30 + 16 - 9 + 2)}{(-1)\,(-2)^2\,(1 - 6 + 1)} = \frac{-32}{16} = -2.$$

The equation $s + 1/s = -2$ implies $(s+1)^2 = 0$, so $s = -1$. However, the cuboid parameter $s = (p/q)^2$ must be positive, so this solution is inadmissible.

Cases $t = 0$ and $t = 1$. The denominator of $U(t)$ in (12) contains the factors t and $(t-1)^2$, so it vanishes at $t = 0$ and $t = 1$. Evaluating the numerator at these points yields 4 (for $t = 0$) and 128 (for $t = 1$), which are non-zero. Consequently, $t = 0$ and $t = 1$ are poles of the rational function $U(t)$, implying $|U(t)| \to \infty$. The relation $s + 1/s = U$ implies that $s \to 0$ or $s \to \infty$. Since we require $s \in \mathbb{Q}_{>0}$ (a finite non-zero rational), these points yield no admissible parameters.

Conclusion. The only positive rational s arising from $C(\mathbb{Q})$ is $s = 1$. □

Theorem 1 (No $1 + 4$ factorization for P_s; no rational roots of $Q_{p,q}(t)$)**.** *Let $s \in \mathbb{Q}_{>0}$ with $s \neq 1$. Then the quintic $P_s(x)$ has no rational root (equivalently, it admits no $1 + 4$ factorization over $\mathbb{Q}$).*

In particular, for coprime $p, q \in \mathbb{Z}_{>0}$ with $p \neq q$, setting $s = (p/q)^2$, the second cuboid polynomial $Q_{p,q}(t)$ has no rational roots (hence no linear factor over $\mathbb{Q}$, and in particular no linear factor over $\mathbb{Z}$).

Proof. Assume, for contradiction, that $P_s(x)$ has a rational root for some $s \in \mathbb{Q}_{>0}$ with $s \neq 1$. Let $x = sy$ as in Lemma 3. Then $F(s, y) = 0$ for some $y \in \mathbb{Q}$. By Proposition 3 (using $s \neq 1$) there exists a rational point $(t, w) \in C(\mathbb{Q})$ on the hyperelliptic curve (15). By Proposition 4 this point is among $\{\infty, (-1, 0), (0, \pm 1), (1, \pm 8)\}$. Lemma 6 shows that the only positive rational s arising from these points is $s = 1$, contradicting $s \neq 1$.

Therefore P_s has no rational root for $s > 0$ with $s \neq 1$.

For the stated consequence for $Q_{p,q}(t)$, suppose that $Q_{p,q}(t)$ had a rational root $t_0 \in \mathbb{Q}$. Then by Lemma 2 (final sentence) the corresponding normalized parameter $s = (p/q)^2$ would yield a rational root of P_s, contradicting what we have just proved. Hence $Q_{p,q}(t)$ has no rational roots. □

Acknowledgments

The author would like to thank Randall L. Rathbun for helpful discussions and correspondence regarding the problem.

A Magma scripts and transcripts

All computer-assisted steps in this note are executed in Magma. Script 01 is a diagnostic computation showing that the naive plane curve $S(x, r) = 0$ (encoding rational roots directly) has genus 6. Script 02 is the certificate computation used in the main proof: it computes the rational points of the genus-2 curve C via a rank bound and Chabauty on the Jacobian.

Script 01: Genus of the naive rational-root curve $S(x, r) = 0$

Code.

```
// Setup Ring and Polynomial
Q := Rationals();
R<x, r> := PolynomialRing(Q, 2);

// Coefficients
A := (2 + r^2)*(3 - 2*r^2);
B := 1 + 10*r^2 + 4*r^4 - 14*r^6 + r^8;
C := -r^2*(1 - 14*r^2 + 4*r^4 + 10*r^6 + r^8);
D := -r^6*(1 + 2*r^2)*(-2 + 3*r^2);
Const := -r^10;

// The polynomial S(x, r) itself
S := x^5 + A*x^4 + B*x^3 + C*x^2 + D*x + Const;
print "Polynomial S(x, r) constructed.";

// If S(x) has a root, then the point (x,r) lies on the curve S(x,r) = 0.
// Define Affine Space and Curve
A2 := AffineSpace(R);
C_aff := Curve(A2, S);

// Take Projective Closure to compute the genus correctly
C_proj := ProjectiveClosure(C_aff);

// Compute the geometric genus, resolving singularities internally if the curve is
↪  singular.
g_root := Genus(C_proj);

printf "Curve equation defined by S(x, r) = 0\n";
printf "Geometric Genus (1+4 case): %o\n", g_root;
```

Transcript.

```
Polynomial S(x, r) constructed.
Curve equation defined by S(x, r) = 0
Geometric Genus (1+4 case): 6
```

Script 02: Rational points on C via Jacobian rank bound and Chabauty

Code.

```
// Setup Polynomial ring and Hyperelliptic curve
Q<t> := PolynomialRing(Rationals());
```

```
f := t^5 + 21*t^4 + 26*t^3 + 10*t^2 + 5*t + 1;
C := HyperellipticCurve(f);
J := Jacobian(C);

// 1. Find "naive" rational points (heuristic search)
pts := RationalPoints(C : Bound := 1000);
print "Found points:", pts;

// 2. Compute the Rank Bound of the Jacobian
r := RankBound(J);
print "Rank Bound:", r;

if r eq 1 then
    print "Rank is 1. Using Chabauty (requires a generator)...";

    // We need a point of infinite order on the Jacobian.
    // We take the difference of two points on the curve: P_rat - P_inf

    // Point (1:0:0) represents Infinity on this model.
    P_inf := C![1, 0, 0];

    // Select a rational point (0 : 1 : 1) from the found list 'pts'
    // Note: Avoid points with y=0 (Weierstrass points) as they are torsion.
    P_rat := C![0, 1, 1];

    // Construct a point on the Jacobian: D = [P_rat - P_inf]
    // Note: We use a sequence [P1, P2] to define the divisor class P1-P2
    pt_J := J ! [P_rat, P_inf];

    // Verify the point is not torsion (Order must be 0 for infinite order)
    if Order(pt_J) eq 0 then
        // Run classical Chabauty
        final_points := Chabauty(pt_J);
        print "All proven rational points:", final_points;
    else
        print "Error: Selected point has finite order (torsion). Try a different
        ↪ P_rat.";
    end if;

elif r eq 0 then
    print "Rank is 0. Using Chabauty0...";
    // Chabauty0 is specifically for Rank 0 cases
    print Chabauty0(J);
else
    print "Rank >= Genus. Chabauty method is not applicable.";
end if;
```

Transcript.

```
Found points: {@ (1 : 0 : 0), (-1 : 0 : 1), (0 : -1 : 1), (0 : 1 : 1), (1 : -8 :
1), (1 : 8 : 1) @}
Rank Bound: 1
Rank is 1. Using Chabauty (requires a generator)...
All proven rational points: { (1 : -8 : 1), (0 : -1 : 1), (1 : 8 : 1), (-1 : 0 :
1), (0 : 1 : 1), (1 : 0 : 0) }
```

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