### VICTORIA UNIVERSITY OF WELLINGTON Te Whare Wānanga o te Ūpoko o te Ika a Māui



#### School of Mathematics and Statistics Te Kura Mātai Tatauranga

PO Box 600 Wellington New Zealand

Tel: +64 4 463 5341 Fax: +64 4 463 5045 Internet: sms-office@vuw.ac.nz

# Graph Algorithms with Hostile Partners

Matthew Askes

Supervisor: Rod Downey

Submitted in partial fulfilment of the requirements for Bachelor of Science with Honours in Mathematics.

#### **Abstract**

A short description of the project goes here.

### **Contents**

1	Introduction			
2	Dominating sets	3		
	2.1 min size dominating set	3		

## **Figures**

### **Chapter 1**

### Introduction

### **Chapter 2**

### **Dominating sets**

We begin by listing some definitions.

**Definition.** The Dominating set, D, of a graph G = (E, V) is any subset of V such that every vertex in V is adjacent to at least one vertex in D.

**Definition.** The Dominating number,  $\gamma(G)$ , of a graph G = (E, V) is the size of the smallest dominating set of G.

**Definition.** Independent set, maximum independent set, independence number  $\alpha(G)$ 

#### 2.1 min size dominating set

**Lemma 2.1.** *Let G be a graph.* 

$$\gamma(G) \geq \alpha(G)$$

*Proof.* Let X be a minimum dominating set in some graph G = (V, E). By definition of dominating set vertex in V is adjacent to at least one vertex in

Recall that  $\chi(G)$  is the chromatic number of the graph G.

**Theorem 2.2** (Willis 2011 Theorem 3.1). For any graph G = (V, E) [1]

$$\alpha(G) \le \frac{|V|}{\chi(G)}$$

**Theorem 2.3.** Let G be a graph, such that the number of vertices in G, n, is  $\geq 4$ . Then for any G,

$$\gamma_g(G) \ge \left\lceil \frac{n}{2} \right\rceil$$

*Proof.* asd □

## **Bibliography**

[1] WILLIS, W. Bounds for the independence number of a graph.