VICTORIA UNIVERSITY OF WELLINGTON
Te Whare Wānanga o te Ūpoko o te Ika a Māui

School of Mathematics and Statistics
Te Kura Mātai Tatauranga

PO Box 600
Wellington
New Zealand

Tel: +64 4 463 5341
Fax: +64 4 463 5045
Internet: sms-office@vuw.ac.nz

Graph Algorithms with Hostile Partners

Matthew Askes

Supervisor: Rod Downey

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Abstract

A short description of the project goes here.

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Dominating sets

We begin by listing some definitions.

Definition. The Dominating set, D, of a graph $G = (E, V)$ is any subset of V such that every vertex in V is adjacent to at least one vertex in D.

Definition. The Dominating number, $\gamma(G)$, of a graph $G = (E, V)$ is the size of the smallest dominating set of G.

Definition. Independent set, maximum independent set, independence number $\alpha(G)$

2.1 min size dominating set

Lemma 2.1. *Let G be a graph.*

$$\gamma(G) \geq \alpha(G)$$

Proof. Let X be a minimum dominating set in some graph $G = (V, E)$. By definition of dominating set vertex in V is adjacent to at least one vertex in

□

Recall that $\chi(G)$ is the chromatic number of the graph G.

Theorem 2.2 (Willis 2011 3.1)**.** *For any graph $G = (V, E)$ [2]*

$$\alpha(G) \leq \frac{|V|}{\chi(G)}$$

Recall that $\Delta(G)$ is the maximum degree of any vertex in G.

Theorem 2.3 (Balakrishnan 2012 10.3.2)**.** *For any graph G with n vertices,*

$$\left\lceil \frac{n}{1 + \Delta(G)} \right\rceil \leq \gamma(G) \leq n - \Delta(G)$$

[1]

It is obvious that in the case when $\gamma(G)$ is known that $\gamma(G) > \gamma_g(G)$.

Theorem 2.4. *Let G be a graph if x is a tight upper bound for the domination number ($\gamma(G)$) then*

$$\gamma_g(G) \geq x$$

Proof. Let G be a graph where $\gamma(G) = x$

Thus for G we are unable to find a dominating set with $< x$ vertices.

Therefore there cannot be a winning strategy for Alice with $< x$ vertices.

Therefore $\gamma_g(G) \geq x$ $\square$

Theorem 2.5. *Let G be a graph with n vertices, such that $n \geq 4$. Then,*

$$\gamma_g(G) > \left\lfloor \frac{n}{2} \right\rfloor$$

Proof. Worst case is G is minimally connected, i.e. G is a tree. Thus by 2.3

$$\gamma(G) \geq \left\lceil \frac{n}{1 + \Delta(G)} \right\rceil$$

$\square$

Theorem 2.6. *Let G be a graph with n vertices. Then,*

$$\gamma_g(G) \leq \frac{2n}{3}$$

Bibliography

[1] BALAKRISHNAN, R. *A Textbook of Graph Theory (Universitext)*. Springer, sep 2012.

[2] WILLIS, W. Bounds for the independence number of a graph.