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**Graph Algorithms with Hostile
Partners**

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Submitted in partial fulfilment of the requirements for
Bachelor of Science with Honours in Mathematics.

Abstract

A short description of the project goes here.

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Chapter 1

Introduction

Chapter 2

Dominating sets

We begin by listing some definitions.

Definition. The Dominating set, D , of a graph $G = (E, V)$ is any subset of V such that every vertex in V is adjacent to at least one vertex in D .

Definition. The Dominating number, $\gamma(G)$, of a graph $G = (E, V)$ is the size of the smallest dominating set of G .

Definition. Independent set, maximum independent set, independence number $\alpha(G)$

2.1 min size dominating set

Lemma 2.1. Let G be a graph.

$$\gamma(G) \geq \alpha(G)$$

Proof. Let X be a minimum dominating set in some graph $G = (V, E)$. By definition of dominating set vertex in V is adjacent to at least one vertex in

□

Recall that $\chi(G)$ is the chromatic number of the graph G .

Theorem 2.2 (Willis 2011 3.1). For any graph $G = (V, E)$ [2]

$$\alpha(G) \leq \frac{|V|}{\chi(G)}$$

Recall that $\Delta(G)$ is the maximum degree of any vertex in G .

Theorem 2.3 (Balakrishnan 2012 10.3.2). For any graph G with n vertices,

$$\left\lceil \frac{n}{1 + \Delta(G)} \right\rceil \leq \gamma(G) \leq n - \Delta(G)$$

[1]

Theorem 2.4. Let G be a graph with n vertices, such that $n \geq 4$. Then,

$$\gamma_g(G) \geq \left\lceil \frac{n}{2} \right\rceil$$

Proof. Worst case is G is minimally connected, i.e. G is a tree. Thus by 2.3

$$\gamma(G) \geq \left\lceil \frac{n}{1 + \Delta(G)} \right\rceil$$

□

Theorem 2.5. *Let G be a graph with n vertices. Then,*

$$\gamma_g(G) \leq n$$

Bibliography

- [1] BALAKRISHNAN, R. *A Textbook of Graph Theory (Universitext)*. Springer, sep 2012.
- [2] WILLIS, W. Bounds for the independence number of a graph.