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REGULAR ARTICLE

Planning for a bus-based evacuation

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Abstract Planning for a bus-based regional evacuation is essential for emergency preparedness, especially for regions threatened by hurricanes that have large numbers of transit-dependent people. While this difficult planning problem is a variant of the vehicle routing problem, it differs in some key aspects, including the objective and the network structure (e.g., capacitated shelters). This problem is not well studied. In this paper we introduce a model specifically designed for bus-based evacuation planning, along with two mathematical programming formulations, which are used to develop a heuristic algorithm. Using these models, we analyze the differences in the structural properties of optimal solutions between this problem and traditional vehicle routing problems.

Keywords Evacuation planning · Disaster management · Vehicle routing · Mixed-integer programming

1 Introduction

This paper introduces and studies a novel model for bus-based evacuation planning when advance notice of a threat is available (e.g., in response to a hurricane) that explicitly considers the unique characteristics of such evacuations and the limited resources available. We incorporate these unique characteristics into a mathematical programming formulation, study its properties, and discuss the differences between this problem and the well-studied, related, vehicle routing problem (VRP). We also

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provide an effective heuristic algorithm that produces close-to-optimal solutions, and use it to discuss some of the solution issues.

Evacuation planning is an important aspect of regional emergency preparedness, especially for coastal regions threatened by hurricanes; over 35 million people in the USA live in the coastal regions from North Carolina to Texas, which are most threatened by hurricanes (United States Census Bureau 2008). Evacuation planning has predominantly focused on evacuation by car, which is certainly important, but insufficient when transit-dependent people are present. Many regions threatened by hurricanes have significant numbers of transit-dependent people, but few have transit-based evacuation plans (Renne and Sanchez 2006). Thus, transit-dependent people are often left out of the planning process. See (Renne et al. 2008) for an extensive literature review and discussion of the issues involved with the evacuation of transit-dependent populations.

To illustrate the importance of evacuation planning in general, and of bus-based evacuation planning in particular, consider the example of New Orleans. New Orleans had an estimated 200,000 transit-dependent people (Wolshon 2002), many of whom had special needs, i.e., they were elderly, had very limited financial resources, had medical or mobility issues, or were not proficient in English. The car-based evacuation of New Orleans in response to Hurricane Katrina is deemed a success (see Wolshon et al. 2006, for instance), because those with a car were able, if they wished, to evacuate the area while encountering an acceptable level of congestion. One reason for the successful evacuation was that the evacuation plan (for cars) was well developed. In fact, it was tested just a year earlier, when New Orleans was evacuated in response to Hurricane Ivan. That evacuation illustrated deficiencies as the evacuation produced unacceptable levels of congestion, which prompted planning improvements. In contrast, as Hurricane Ivan missed New Orleans, those remaining in the city were not adversely affected, thus problems evacuating transit-dependent people were less noticeable and did not prompt any planning changes. Unfortunately, Hurricane Katrina did not miss New Orleans, and the lack of proper planning for the transit-dependent population was made obvious. Buses were not used to transport residents out of the city before the hurricane made landfall (Litman 2006); in fact, many buses were left behind and flooded (Murdock 2005). In summary, it was much more difficult for the transit-based population to evacuate the city, and many of them did not leave! In Louisiana, there were approximately 1,446 deaths attributed to Katrina, most of these from New Orleans, and most of the victims were elderly (Louisiana Department of Health and Hospitals 2006). Furthermore, an estimated 60,000 people were rescued in the aftermath of Hurricane Katrina; the Coast Guard alone rescued about 24,000 people and helped evacuate 9,400 medical patients to safety (United States Coast Guard 2006). Sadly, the search and rescue in the aftermath of Katrina was undoubtedly more logistically complex, expensive, and dangerous than the bus-based evacuation would have been.

This unfortunate event highlights the importance of evacuation planning for transit-dependent people. The objective of this paper is to study models for bus-based evacuation planning in response to threats with advance notice. Advance notice evacuations are common; for example, a hurricane is observed, its landfall is forecasted, and coastal regions evacuated before the destructive forces of the hurricane arrive. Other threats having advanced notice include floods and tsunamis (with the proper warning system).[1] For such advance-notice evacuations, an important objective is to minimize the total duration of the evacuation, that is, the network clearance time. For example, it is of utmost importance for the evacuation to be complete before the hurricane strikes, after which the remaining population is exposed to high risks. Thus, a hurricane evacuation is always constrained by risk-based deadlines (e.g., when tropical storm force winds are forecasted to occur). Conversely, forecasts have uncertainty, and evacuating too early increases the chance that the evacuation ends up being unnecessary. So, minimizing the duration of the evacuation, thorough careful planning and execution, allows the evacuation to start later (when more reliable and accurate forecasts are available), yet still complete the evacuation before the region is exposed to risk. Planners must explicitly consider the limited available resources. For example, New Orleans had about 500 buses; using a fleet of that size would have required each bus to serve at least four bus-loads of evacuees to transport all residents who required help to safety (Litman 2006). Most often, the number of available buses is insufficient to transport all those in need without multiple trips (Wolshon et al. 2001), even in rural areas, which can still have a significant carless population (Hess and Gotham 2007).

In this paper, we introduce, formulate, and analyze a unique variant of the VRP, which we refer to as the bus evacuation problem (BEP). For the BEP, a set of bus yards, pickup locations, and shelters is given. The objective is to transport evacuees from the pickup locations to the shelters in the minimal amount of time, thus, minimizing the duration of the evacuation, by routing and scheduling a fleet of homogeneous and capacitated buses, which are initially located at one or more yards. We define the duration as the time span between when the first bus leaves its yard until the last evacuee is sheltered. Each pickup location, which either serves a "neighborhood"or alternatively a facility that requires evacuation services, has a known number of evacuees to be transported. The number of evacuees at each pickup location, however, can exceed the capacity of a single bus. Each shelter has a capacity that limits the number of evacuees it can serve.

The VRP literature is vast (see Desrochers et al. 1990; Eksioglu et al. 2009 for taxonomies and overviews) and includes papers that explore insights into the VRP, describe variations of the problem, introduce exact solution techniques, and explore heuristic solution techniques. The body of research that uses operations research methodology for evacuation management is relatively sparse and mainly focuses on the car-based aspects of an evacuation (see Hobeika and Kim 1998; Cova and Johnson 2003; Chiu et al. 2007; Bish et al. 2010, for instance) or building evacuations (see Hamacher and Tjandra 2001; Chen and Miller-Hooks 2008, for instance). The only modeling paper considering transit-based evacuation is (Sayyady and Eksioglu 2010), which examines no-notice evacuations (we discuss this paper in more detail below). This raises the question of whether the existing VRP knowledge can be leveraged for the BEP. In this paper, we demonstrate that the unique characteristics of the BEP make it quite

[1] A "no-notice" evacuation, in contrast, has no warning before the population experiences risk, e.g., nuclear and chemical accidents. Of course, not all threats are easily categorized; for instance, wild fires usually offer some notice before impacting populated areas, but with relatively short notice.

a different problem from the VRP, to the extent that the properties that are shown to hold for the VRP do not necessarily hold for the BEP.

The remainder of this paper is organized as follows. In Sect. 2 we describe the BEP and provide an mixed-integer programming formulation. We also compare the BEP with the VRP and its variants. In Sect. 3, we explore how the BEP's objective function affects the properties of an optimal solution, while in Sect. 4 we examine the unique network structure of the BEP. In Sect. 5, we present an alternative formulation and develop an effective heuristic to solve the BEP. Finally, in Sect. 6, we present some concluding remarks.

2 Problem description and modeling formulation

In this section, we describe the BEP in detail, propose a novel mixed-integer programming formulation for the BEP, and compare it with the well-known VRP. While the VRP has attracted a significant research effort, the BEP has not been well studied. Thus, it is important to first understand if and how the existing VRP research can be leveraged for the study of the BEP. As a first step, we describe the BEP in detail.

2.1 The BEP

Consider a network (N, A), where N and A respectively denote the set of nodes and arcs. N is composed of three subsets of nodes: Y, a set of yard nodes where buses are initially located and dispatched from; P, a set of demand nodes, each of which represents a pickup location serving a neighborhood or facility requiring evacuation services; and S, a set of shelter nodes. The set of available buses, V, each having a capacity Q, is subdivided into the subsets V_i, $i \in Y$, where bus $j \in V_i$ is initially located at yard i. Demand node j has a demand D_j, $j \in P$, and shelter i has a capacity C_i, $i \in S$. Each arc (i, j) has a non-negative travel cost of τ_{ij}, $(i, j) \in A$. For the arc set, we adopt three assumptions commonly made in the VRP literature: (1) travel cost is proportional to the travel time and distance; hence, we use the terms "cost", "time" and "distance" interchangeably throughout; (2) all costs in the network are symmetric, that is, $\tau_{ij} = \tau_{ji}$, and (3) all travel costs satisfy the triangular inequality, that is, $\tau_{ij} \leq \tau_{ik} + \tau_{kj}$, $\forall (i, j),\ (i, k),\ (k, j) \in A$. The objective is to route and schedule the buses to minimize the duration of the evacuation, while satisfying all evacuee demand and without violating both shelter and vehicle capacity constraints. In addition, split delivery service is allowed; in fact, the number of evacuees at a demand node might be greater than the capacity of a bus, Q, thus requiring split service.

Figure 1 illustrates an example of a BEP network with one yard (Y1), three demand nodes (P1, P2, and P3), and two capacitated shelters (S1 and S2). (This network corresponds to our smallest test scenario, Scenario Y1P3S2Vv, as detailed below). The BEP network is not fully connected because the yards require only outgoing arcs. Furthermore, because the triangle inequality holds and buses start out empty, an optimal solution does not require a bus to initially visit a shelter before a demand node, allowing us to eliminate yard to shelter arcs. Thus, yards only define the initial location of the buses, and play no further role in the evacuation process—buses do not return to

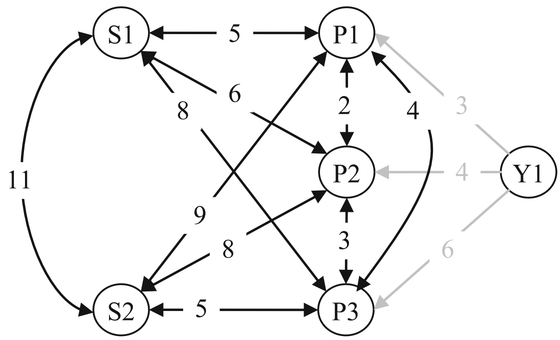


Fig. 1 Example of a BEP-type network (Scenario Y1P3S2Vv)

the yards. This is to ensure the safety of the bus drivers, as it could be risky to return to the yard under a threat; in addition, yards may not be the best places to store the vehicles during the threat. (Bus yards in New Orleans were flooded in the aftermath of Hurricane Katrina.)

The BEP network has an inherently different structure than that of the traditional VRP, which necessitates a novel mathematical programming formulation. We now introduce the decision variables; each arc traversal, the basic travel decision, is referred to as a trip, and we keep track of consecutive trips with an index.

Decision variables:

x_{ij}^{mt} : binary variable that equals 1 if trip t for bus m traverses arc (i, j), else 0, $\forall (i, j) \in A,\ m \in V,\ t = 1, \ldots, T$.

b_j^{mt} : number of evacuees from node j assigned to (or, if j is a shelter, released from) bus m after trip t, $\forall j \in N,\ m \in V,\ t = 1, \ldots, T$.

T_{evac} : the duration of the evacuation.

A formulation for the BEP is as follows:

Model 1:

$$\text{Minimize } T_{\text{evac}} \tag{1}$$

$$\text{subject to: } T_{\text{evac}} \geq \sum_{(i,j)\in A} \sum_{t=1}^{T} \tau_{ij} x_{ij}^{mt}, \quad \forall\, m \in V \tag{2}$$

$$\sum_{i:(i,j)\in A} x_{ij}^{mt} = \sum_{k:(j,k)\in A} x_{jk}^{m(t+1)}, \quad \forall j \in P, m \in V,\ t = 1, \ldots, T-1 \tag{3}$$

$$\sum_{i:(i,j)\in A} x_{ij}^{mt} \geq \sum_{k:(j,k)\in A} x_{jk}^{m(t+1)}, \quad \forall j \in S, m \in V, t = 1, \ldots, T-1 \tag{4}$$

$$\sum_{(i,j)\in A} x_{ij}^{mt} \leq 1, \quad \forall\, m \in V,\ t = 1, \ldots, T \tag{5}$$

$$x_{ij}^{m1} = 1, \quad \forall\, i \in Y,\ j : (i, j) \in A,\ m \in V_i \tag{6}$$

$$x_{ij}^{mt} = 0, \quad \forall\, i \in Y,\ j : (i, j) \in A,\ m \in V,\ t = 2, \ldots, T \tag{7}$$

$$x_{ij}^{mT} = 0, \quad \forall\, j \in P,\ i : (i, j) \in A,\ m \in V \tag{8}$$

$$b_j^{mt} \leq \sum_{(i,j)\in A} Q x_{ij}^{mt}, \quad \forall\, j \in N,\ m \in V,\ t = 1, \ldots, T \tag{9}$$

$$0 \leq \sum_{j\in P} \sum_{l=1}^{t} b_j^{ml} - \sum_{k\in S} \sum_{l=1}^{t} b_k^{ml} \leq Q, \quad \forall\, m \in V,\ t = 1, \ldots, T \tag{10}$$

$$\sum_{m\in V} \sum_{t=1}^{T} b_j^{mt} \leq C_j, \quad \forall\, j \in S \tag{11}$$

$$\sum_{m\in V} \sum_{t=1}^{T} b_j^{mt} = D_j, \quad \forall\, j \in P \tag{12}$$

$$\sum_{j\in P} \sum_{t=1}^{T} b_j^{mt} = \sum_{k\in S} \sum_{t=1}^{T} b_k^{mt}, \quad \forall\, m \in V \tag{13}$$

$$x_{ij}^{mt} \in \{0, 1\}, \quad \forall\, (i, j) \in A,\ m \in V,\ t = 1, \ldots, T \tag{14}$$

$$b_j^{mt} \geq 0, \quad \forall\, (i, j) \in A,\ m \in V,\ t = 1, \ldots, T. \tag{15}$$

Constraint (2) requires T_{evac} to be greater than or equal to the maximum cost incurred by any bus, which is then minimized by the objective function (1), thus minimizing the duration of the evacuation, which is defined by the bus with the highest travel cost. Because of this, we will refer to this objective function as the "min–max" objective. Constraint (3) is the flow-balance constraint for the demand nodes; it ensures that a bus traveling to demand node j on trip t leaves node j on trip $t + 1$; Constraint (4) is the flow-balance constraint for the shelters; it does not require the bus to leave the shelter i, that is, the last trip a bus makes can end at a shelter. Constraint (5) allows a bus to make at most one trip at a time. Constraint (6) specifies that the first trip of each bus start from its yard, and Constraint (7) ensures that the buses do not leave the yard for later trips. Constraint (8) does not allow the last trip a bus can make to end at a demand node. Constraint (9) dictates that a bus can only pick up evacuees from node j if it has in fact traveled to node j. Constraint (10) is the bus capacity constraint. Constraint (11) is the shelter capacity constraint, while (12) ensures that all evacuees are picked up. Constraint (13) ensures all evacuees are delivered to a shelter. Constraints (14) and (15) are the logical binary and non-negativity restrictions on the x and b variables, respectively.

As we have already alluded, the BEP differs from a traditional VRP. The VRP variant that is closest to the BEP is the split delivery multi-depot vehicle routing problem with inter-depot routes (SDMDVRPI). To the best of our knowledge, the SDMDVRPI has not been studied in the VRP literature, but conceptually, it combines the multi-depot vehicle routing problem (MDVRP) with inter-depot routes (MDVRPI) (see Crevier et al. 2007) and the well-studied split delivery vehicle routing problem (SDVRP). The

Table 1 The characteristics of the SDMDVRPI and the BEP

	SDMDVRPI	BEP
Objective function	Minimize total cost	Minimize evacuation duration
		≡ Minimize maximum vehicle cost
Network structure	Multi-depot (uncapacitated)	Multi-yard (uncapacitated)
		Multi-shelter (capacitated)
	Multiple demand nodes	Multiple demand nodes
Vehicles	Capacitated	Capacitated
	Multiple routes assigned	Multiple routes assigned
Routes	Tours and/or non-tours	Non-tour (from the yard) then tours and/or non-tours
Demand	Satisfy all demand	All evacuees sheltered
	Split deliveries allowed	Split deliveries allowed (may even be required)

MDVRPI differs from the MDVRP (see Polacek et al. 2004) in that vehicles can be replenished at any depot. (This is similar to BEP, where a bus can deliver evacuees to any shelter.) The evacuation model in (Sayyady and Eksioglu 2010) does not allow vehicles to serve multiple routes, unlike the BEP or MDVRPI. The BEP, like the SDVRP, allows more than one vehicle to serve a demand node (split delivery), which, not surprisingly, can lead to lower costs (Dror and Trudeau 1989, 1990). The BEP, unlike some of the SDVRP literature, might require split service because the demand at a node can be larger than the capacity of a bus. The SDMDVRPI is likely to be difficult to solve, as both the MDVRPI and SDVRP are already quite difficult to solve. Next, we describe the SDMDVRPI, and then in Table 1 summarize the characteristics of the SDMDVRPI and the BEP.

2.2 The SDMDVRPI

A set of customers, each with known demand, has to be served by a fleet of identical, capacitated vehicles, which are initially located at the depots. Each vehicle serves a subset of customers, returning to any depot for replenishment (not necessarily the one it started from, thus the inter-depot designation). Vehicles can be assigned multiple routes, and each customer can be served by multiple vehicles (the split delivery part). The objective is to find a set of routes for the vehicles that minimize the total cost, while satisfying all customer demand and without violating the vehicle capacity constraints.

The first major difference between the SDMDVRPI and the BEP (see Table 1) is the objective function used. The BEP objective minimizes the duration of evacuation, which is equivalent to minimizing the routing cost for the bus with the maximum routing cost. On the other hand, the VRP objective minimizes the total routing cost for all vehicle. For an evacuation, minimizing cost may be desirable, but it is *not* the primary concern, especially for such an infrequent event. In (16) we formulate the standard VRP objective function using our notation; we will refer to this as the "cost"

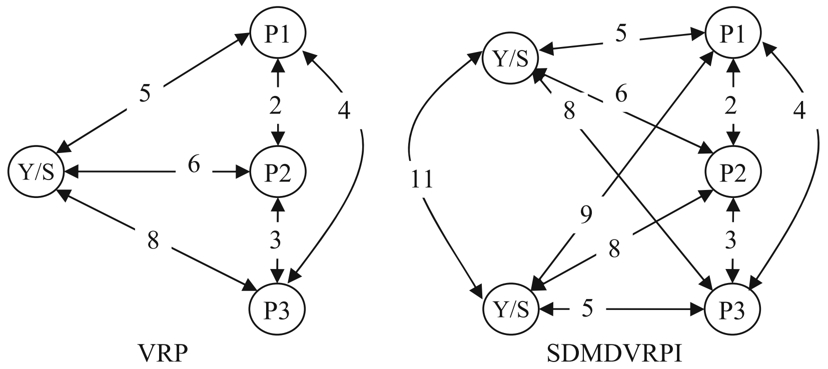


Fig. 2 Example of VRP-type networks (Y/S designates a depot node)

objective.

$$\text{Minimize} \sum_{(i,j)\in A} \sum_{m\in V} \sum_{t=1}^{T} \tau_{ij} x_{ij}^{mt}. \tag{16}$$

We note that there are other objective functions besides the cost objective considered in the VRP literature, and (Campbell et al. 2008) study the most relevant one to the BEP, a min–max objective for the delivery of supplies in a disaster relief effort. Specifically, the objective is to minimize the time until the last delivery is made (i.e., the last visit to a demand node). While this objective also has a min–max structure, it differs because the BEP objective includes the cost of the final shelter trip when calculating the evacuation duration. As discussed above, this is to ensure the safety of the drivers and evacuees, which is of crucial importance. By including the shelter trip in the duration, the BEP- min–max objective behaves quite differently from the objective in (Campbell et al. 2008). Furthermore, the BEP can assign multiple routes to a bus, thus requiring the consideration of the return to the shelter for earlier routes. The model in (Campbell et al. 2008) uses a standard VRP-type network. The evacuation model in (Sayyady and Eksioglu 2010) maximizes the number of evacuees served, assuming that a complete evacuation is not possible, and thus the objective function is quite different from the min–max objective.

The second major difference between the SDMDVRPI and BEP (see Table 1) is the network structure. While the SDMDVRPI has a network structure that is similar to the BEP, by comparing Figs. 1 and 2 we observe some of the differences. The depots in SDMDVRPI are replaced by yards and shelters in the BEP. The yards indicate the initial location of the vehicles (much like a depot does), and the shelters, where the evacuees are taken, have capacity limits, unlike depots. Furthermore, in the BEP, the vehicles do not return to their yards as discussed above. Thus in the BEP, one needs to consider two sets of routes: the initial set of routes that start at the yard(s) (the initial location of the buses) and end at a shelter, and a second set of routes that includes tours that start and end at the same shelter and non-tours that start and end at different shelters.

Table 2 Test results for the BEP for three scenarios

Scenario descriptor	Objective function	Run time (s)	Variables		Duration	Cost
			Binary	Continuous		
Y1P3S2V4	Min–max (1)	95.3	552	146	30	106
	Cost (16)	5.6	552	145	30	95
Y1P4S2V4	Min–max (1)	305.2	816	170	25	99
	Cost (16)	41.2	816	169	30	78
Y1P5S3V6	Min–max (1)	Time limit	1,980	326	–	–
	Cost (16)	189.8	1,980	325	41	149

We will illustrate issues and concepts using selected test scenarios. We describe scenarios using a descriptor such as $YiPjSkVv$, where i, j, k, and v, respectively, denote the number of yards, number of demand nodes, number of shelters, and number of vehicles in the fleet. For instance, the network in Fig. 1, when the fleet size is four vehicles, would be labeled Y1P3S2V4, for one yard, three demand nodes, two shelters, and four vehicles. In all our scenarios, the bus capacity is taken to be 20 evacuees. All computational runs are made using CPLEX Version 12.1 on a PC with a 3.20 GHz Pentium D CPU. In the next section, we study the impact of using the min–max objective.

3 The min–max objective

As discussed above, the min–max objective is a departure from the traditional VRP cost objective. In this section, we examine the impact that the min–max objective has on the optimal solution structure and other problem characteristics. Table 2 shows the results for some illustrative scenarios from our numerical study of this problem. These results indicate that, not surprisingly, the run-time for the BEP, with either objective, grows quickly with problem size. Furthermore, they suggest that the min–max objective is significantly more difficult to solve than the cost objective. We set a run-time limit of 18,000 s, and were not able to find an optimal solution to the Y1P5S3V6 scenario when using the min–max objective within this limit (even when we extended the limit to 24 h, we still did not obtain an optimal solution). For these runs, we use $T = 6$.

To illustrate the structural differences between these two objectives, consider the following observation for the cost objective, where we define R^*_{cost} as the maximum number of routes in any optimal solution.

Observation 1 *For the cost objective on a VRP-type network, if* $|V| \leq R^*_{\text{cost}}$*, that is, if the fleet size is less than or equal to the maximum number of routes in any optimal solution, then the fleet size does not impact optimality.*

Since vehicles can be used on multiple routes, one vehicle covering all the routes would yield the same cost as multiple vehicles covering those same routes. Intuitively, we see that this is quite different from the min–max objective because the evacuation duration is determined by the bus (or buses) assigned to the maximum-cost set of routes. We refer to this bus as the bottleneck vehicle. Thus, when considering the cost objective the problem is a route selection problem, while, with the min–max objective, the problem is both a route selection and route-to-vehicle assignment problem, which is more complex.

3.1 Objective function enhancements: lexicographic ordering

Both the min–max and cost objective can have multiple optimal solutions, some of which are "better" than others. For instance, the optimal min–max solution often displays inefficiencies, which, while not increasing the duration, are nevertheless undesirable. This is because, for the min–max objective, the non-bottleneck vehicles often have flexibility regarding the selection and assignment of routes; the route cost for each of these vehicle is only required to be less than or equal to that of the bottleneck vehicle. This can lead to the selection of routes for the non-bottleneck vehicles that are inefficient or obviously flawed. For example, we commonly observed optimal solutions where non-bottleneck buses traverse routes but do not pick up evacuees. In other words, when there are multiple optimal solutions, some of the solutions can include such undesirable or unnecessary routes (leading to more trips and larger costs). To remedy this situation, we introduce an alternative lexicographic min–max objective formulation:

$$\text{Minimize } T_{\text{evac}} + \frac{1}{L}\left(\sum_{(i,j)\in A}\sum_{m\in V}\sum_{t=1}^{T}\tau_{ij}x_{ij}^{mt}\right), \tag{17}$$

where L is a lexicographic constant that ensures that the first term, the evacuation duration, lexicographically dominates the second term, the total evacuation cost, that is, the cost objective. Setting $L = \sum_{m\in V}\sum_{t=1}^{T}\max\{\tau_{ij} : (i,j)\in A\}$ ensures the lexicographic nature of this objective, as this value is equal to the maximum possible value of the second term (Sherali 1982). Thus (17) produces a solution that is optimal for (1), and given multiple optimal solutions, it selects the one that requires the lowest evacuation cost. This objective requires (2) as a support constraint.

The cost objective can have multiple optimal solutions that represent a range of evacuation durations. Similarly, we can also lexicographically minimize costs and duration with objective function (18), which follows:

$$\text{Minimize } \sum_{(i,j)\in A}\sum_{m\in V}\sum_{t=1}^{T}\tau_{ij}x_{ij}^{mt} + \frac{1}{L}T_{\text{evac}}, \tag{18}$$

where the second term corresponds to the min–max objective, which is scaled by an appropriate lexicographic constant (which we again denote as L). Constraint (2) is required to support (18).

Table 3 Test results for the BEP with the lexicographic versions of the min–max and cost objectives

Scenario descriptor	Objective function	Run time (s)	Variables		Duration	Cost
			Binary	Continuous		
Y1P3S2V4	Min–max (17)	30.7	552	146	30	95
	Cost (18)	7.6	552	146	30	95
Y1P4S2V4	Min–max (17)	84.1	816	170	25	79
	Cost (18)	29.3	816	170	27	78
Y1P5S3V6	Min–max (17)	10,211.8	1,980	326	27	157
	Cost (18)	193.9	1,980	326	31	149

Table 3 displays the results obtained using the lexicographic versions of the objective functions. The lexicographic min–max solution required less run-time and produced a lower cost evacuation than the solution to (1), see Table 2. The cost reduction is partially due to the elimination of extraneous routes and yields a more desirable solution. On the other hand, the lexicographic cost objective, while producing an evacuation of shorter duration than (16) (see Table 2), does not provide any run-time benefit. The shorter duration is accomplished by better distributing the routes to the vehicles, which can involve the selection of different routes (but with the same total cost) that can be packed more efficiently. The results from Table 3 also illustrate that the optimal min–max and cost solutions do not necessarily use the same set of routes, that is, there is often more to the minimum duration solution than an optimal assignment of the minimum cost routes to the vehicles. Figure 3 depicts the cost per vehicle in the optimal solution to objectives (1), (16), (17), and (18) for the Y1S2P4V4 test scenario. The evacuation duration is determined by the tallest bar for each objective function's solution, and the evacuation cost is determined by the sum of the heights of all bars for a solution (we have numbered the vehicles from highest cost to the lowest cost assignment).In the remainder of this work, when referring to the min–max and cost objective, we are referring to the lexicographic versions, (17) and (18), respectively.

3.2 Cost-based bounds on the duration

Because the BEP is easier to solve with the cost objective than with the min–max objective, we ask, would a cost solution produce a near-optimal evacuation duration? If this were so, then the techniques developed to solve the VRP might be used to solve the BEP heuristically. To help answer this question, we examine the bounds on the optimal duration that the cost solution produces.

Let T^*_{evac} and T^C_{evac}, respectively, denote the optimal evacuation duration and the duration obtained from the cost objective. As Remark 1 indicates, when considering a single vehicle the min–max and cost solutions are interchangeably optimal.

Remark 1 The min–max and cost objectives have equivalent optimal solutions when the fleet consists of only one vehicle.

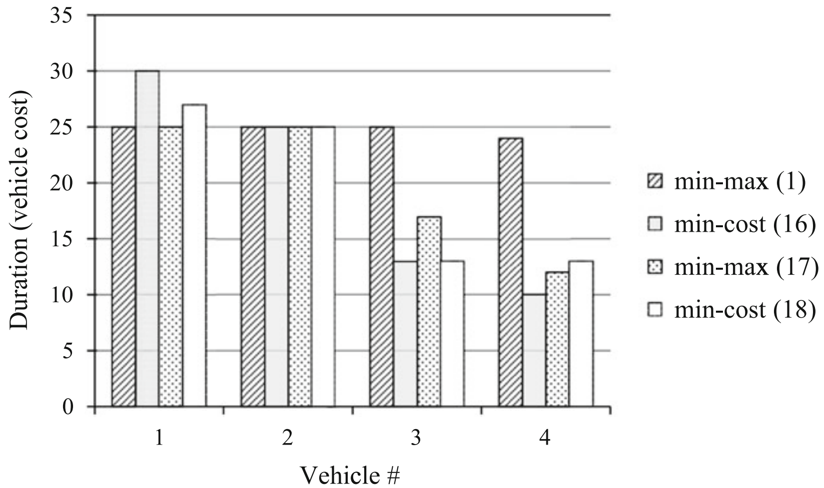


Fig. 3 The total usage of each individual vehicle for the Y1P4S2V4 test scenario

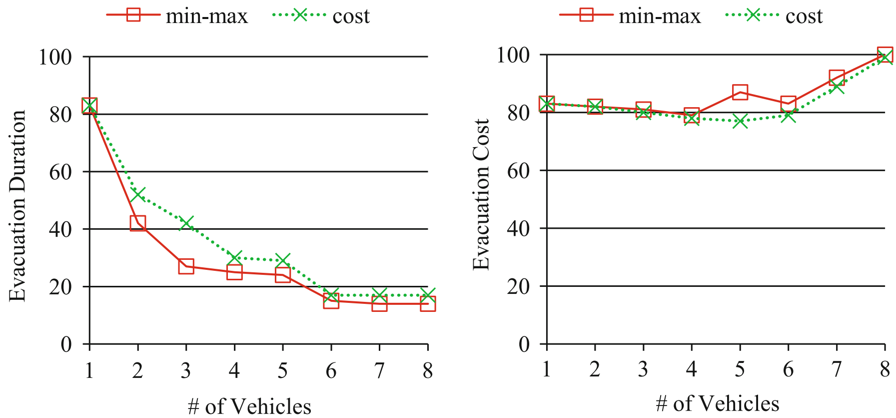


Fig. 4 The duration and cost of an evacuation as the fleet size increases for objectives (17) and (18) on a Y1P4S2 network

For the more common case where the fleet consists of multiple vehicles, our numerical study suggests that the gap between T^*_{evac} and T^C_{evac} is small. Figure 4 shows the optimality gap for various fleet sizes for the Y1P4S2Vv scenario.

Despite the empirical evidence provided, the worst-case analysis of Proposition 1 shows that the ratio $\frac{T^C_{\text{evac}}}{T^*_{\text{evac}}}$ cannot be bounded from above by a constant.

Proposition 1 *The ratio of $\frac{T^C_{evac}}{T*_{evac}}$ can be arbitrarily large, that is, solving the BEP with the cost objective can produce an evacuation with a duration, T^C_{evac}, which is arbitrarily larger than the minimum duration of the evacuation, $T*_{evac}$.*

Proof Referring to Fig. 5, we define a petal as the type of sub-structure formed by nodes P1 and P2 and the three arcs that connect them to each other and the yard/shelter

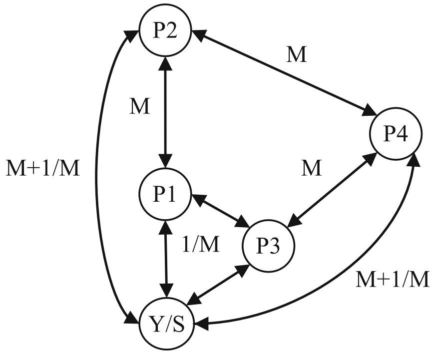


Fig. 5 An network to illustrate the relationship between the min–max and cost objectives

(P3 and P4 also form a petal). Now, consider a network like the one displayed in Fig. 5, but expanded to have Q petals along with the required connecting arcs (in Fig. 5 arcs $(P1, P3)$ and $(P2, P4)$ connect the two petals). Assume that arcs that connect non-adjacent petals offer no cost reduction when traveling between petals. In this network, each outer node has one evacuee and each inner node has $Q - 1$ evacuees, and a fleet of Q vehicles is available. The cost solution will send one vehicle on a tour that visits each petal's outer node, and this tour will determine the duration of this solution, which will be $(Q + 1)M + 2/M$. The minimum duration solution will assign a vehicle to each petal and thus have a duration of $2M + 2/M$. Thus, $\frac{T^C_{\text{evac}}}{T^*_{\text{evac}}} = \frac{(Q+1)M+2/M}{2M+2/M}$, which, by properly scaling the M and Q parameters, can be made arbitrarily large. □

Figure 5 is an example where $Q = 2$. The optimal min–max solution is to send the first bus on route Y/S-P1-P2-Y/S and the second bus on the route Y/S-P3-P4-Y/S, which has a duration of $2M + 2/M$ (and a cost of $4M + 4/M$). The optimal cost solution is to send the first bus on the route Y/S-P2-P4-Y/S and the second bus on the route Y/S-P1-P3-Y/S, which has a cost of $3M + 5/M$ (and a duration of $3M + 2/M$). We note that the proof for Proposition 1 does not rely on Observation 1, which can be manipulated to produce poor worst-case bounds because using fewer vehicles does not impact on the cost solution's optimality, but does have a potentially large impact on duration.

3.3 Fleet size and the objective functions

In this section, we study how fleet size affects the optimal min–max solution. We start with an observation about the fleet size for the min–max objective.

Observation 2 *For the min–max objective there is an optimal threshold fleet size. Increasing the fleet size beyond this threshold does not impact optimality.*

Observation 2 is intuitive. However, interestingly, we observe the following as the fleet size is increased toward the threshold.

Observation 3 *For the min–max objective, the evacuation duration does not always decrease in a convex manner with the number of vehicles.*

Observation 3 infers that the concept of diminishing returns does not strictly apply to the BEP. Figure 4 illustrates this observation; when the fleet size increases from four to five vehicles, the duration decreases by only one time unit, but increasing the fleet size again, from five to six vehicles, decreases the duration by nine time units. This behavior occurs because adding a vehicle allows the redistribution of routes among the vehicles and also potentially the selection of new optimal routes. For instance, we see from Fig. 3 that when there are four vehicles, two of them are bottleneck vehicles, that is, vehicles that define the duration of the evacuation. In this case, adding another vehicle might have little impact, which is the case here as the duration only decreases by one. Observation 3 has important **management** implications, because there is not a strict diminishing return when adding more vehicles; the problem of determining the appropriate fleet size is more complex.

Estimating the optimal threshold fleet size can help in the fleet size decision. The following is an upper bound for the threshold, for a special case.

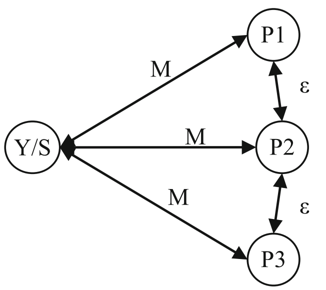
Remark 2 For the special case of the BEP with uncapacitated shelters, the threshold on the number of vehicles is bounded from above by $\sum_{i \in P} \lceil D_i/Q \rceil$. Note that this number serves as an upper bound on the threshold, as it allows an evacuation without multiple routes or split deliveries for all vehicles.

The min–max objective and its interplay with fleet size also impacts the split delivery aspect of this problem. To illustrate, consider Example 1 (adapted from Dror and Trudeau 1989, 1990), which illustrates the cost savings that can be obtained by allowing split deliveries.

Example 1 Consider an instance of the BEP on the network displayed in Fig. 6 with $Q = 5$, and nodes P1, P2, and P3 having demands of 3, 4, and 3, respectively. Without split deliveries, the evacuation cost would be $6M$, while allowing split deliveries would reduce this cost to $4M + 2\varepsilon$, which yields a bound of 1.5. This bound does not change as we increase the fleet size, although no more than two vehicles will be utilized. This bound holds for duration when the fleet consists of one vehicle (see Remark 1). When the fleet size increases to two vehicles, the duration without split deliveries is $4M$ and with split deliveries is $2M + 2\varepsilon$, which yields a bound of 2. If we increase the number of vehicles to three, then the duration is $2M$ and split deliveries do not improve the solution.

Thus, once again we see the importance of fleet size to the min–max objective. Archetti et al. (2006) proves that the worst-case bound is 2 for the cost objective, and

Fig. 6 Split delivery network example (adapted from Dror and Trudeau 1989, 1990)



it shows that this bound is tight, using a more complex network than that shown in Fig. 6.

Next, we examine how the network structure impacts the problem.

4 The BEP network structure

The BEP network structure adds complexity to the solution because it must not only indicate which demand nodes to initially serve from each yard, but also which shelter each vehicle should use considering shelter capacities and their location relative to the demand nodes. This decision is not an issue in the VRP, and even the SDMDVRPI does not have the added complexity of shelter capacities. The next question then is whether some intuitive properties would hold for the shelter selection decision in an optimal solution to the BEP. If this were the case, this would help with the development of an effective heuristic. However, the next property shows that the complex network structure can evade such properties. Even for the case where the shelters are uncapacitated (alternatively, have sufficient capacities for all evacuees), sending each vehicle to its closest shelter (i.e., the shelter closest to its last pickup node on the assigned route) is not necessarily optimal, as the following example demonstrates (although this, probably, is what the naive evacuation planner would have done, in the absence of research on optimal decisions).

Observation 4 *Even without shelter capacity constraints, it is not always optimal for the BEP to allocate each vehicle to the nearest shelter (i.e., the shelter closest to its last pickup node).*

Such a counter-example is given in Example 2

Example 2 Even without capacity constraints, a bus might not use the nearest shelter if the bus is scheduled to serve additional routes. Consider the following example that uses the network fragment displayed in Fig. 7. A bus has picked up a full load of evacuees at node P1 and must still pick up a full load of evacuees at P2. The bus can transport the current load of evacuees to either shelter S1 or S2. If the closest shelters are used, then the bus would take the following route: P1–S1–P2–S2, which has a cost of 17, while the optimal solution would be P1–S2–P2–S2, which has a cost of 13.

Thus, as this example demonstrates well, the network topology, along with assignment of multiple routes to a bus, requires that the optimal solution consider not only

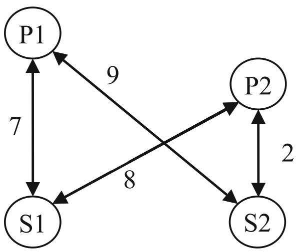


Fig. 7 A network fragment for the uncapacitated shelter example

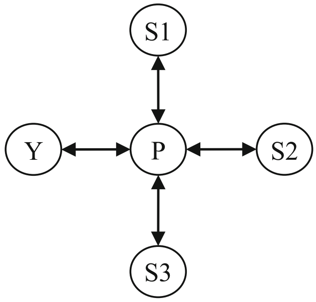


Fig. 8 A network for the threshold example

the distance to the alternative shelters, but also the impact the selected shelter has on the cost of the vehicle's next route. Next, we examine how the network structure impacts fleet size considerations.

4.1 Fleet size and the network structure

In Sect. 3.3 we discussed the threshold fleet size for an optimal solution to the min–max objective, and in Remark 2 we give an upper bound on this threshold value of $\sum_{i \in P} \lceil D_i / Q \rceil$. This is for the special case when shelters are uncapacitated. If shelters do have capacity limitations (which, of course, is the case in practice), then the upper bound in Remark 2 no longer holds, as the following example shows.

Example 3 Consider the network in Fig. 8, where each arc shown has a cost of M; arcs between shelters are of no consequence in this example and we could, without violating our assumptions, give them costs of $2M$. Consider a problem instance on this network with demand at node P of 30, vehicle capacity of 20, and shelter capacity of 10. In this case, the optimal evacuation duration will require three vehicles, more than the upper bound of two for the uncapacitated case.

Next, we observe that the network structure of the BEP changes the relationship between the cost objective and the fleet size. Figure 4 shows one example (among many others we have solved) that illustrates that the dependence of the BEP on fleet size remains, even with the cost objective, because of the network structure. In particular, as the number of vehicles increase, the cost first decreases, then increases, as summarized below.

Observation 5 *The dependence of the cost objective on fleet size occurs on a BEP-type network; specifically, our numerical study indicates that the optimal cost first decreases, then increases with the fleet size.*

The depot node found in the VRP is replaced by a combination of yards and shelter nodes, which can impact on the cost objective. For instance, if the distance between the various yards and the pickup nodes are all larger than the distances between the pickup locations and the shelters, the optimal solution to the cost objective will only utilize one bus. The optimal number of buses will depend on which nodes are better served (i.e., at a lower cost) from the yards. The scenario Y1P4S2, whose results are displayed in Fig. 4, has three pickup nodes that are better served from the yard, and five vehicle loads are required to serve all of the evacuees at these nodes. Thus, five vehicles will minimize the cost. Using less than five vehicles will require substituting a lower cost route starting from the yard with a higher cost route starting from a shelter (this would of course not be the first route assigned to a vehicle). Using more than five vehicles will require substituting some lower cost route from the shelters with higher cost route from the yards. This accounts for the cost increase shown in Fig. 4 when more than five vehicles are available. Due to the nature of the min–max objective, this interesting network effect does not occur, (sending buses over higher cost routes from the yards will not increase the duration as more vehicles are almost always beneficial when considering duration).

5 An alternative formulation for BEP and an effective heuristic algorithm

In this section, we introduce an alternative formulation for the BEP, which relies on the idea of separating the route construction step from the route assignment step. We also discuss a heuristic that uses this formulation as its basis and study its performance through a numerical study.

The alternative formulation for the BEP requires a set of routes, R, as an input. Thus instead of building routes from arc traversals, as the first formulation did, routes are given. When discussing Model 1, we used the term trip to refer to an arc traversal, the most fundamental travel decision. Here, we recycle the term and use trip to refer to a route, this formulation's most fundamental travel decision. So the trip number is used to describe sequential routes assigned to a vehicle. To describe the routes in set R, we use three binary indicator parameters, $s_{rj}, \forall r \in R, j \in Y \cup S$, which is 1 if route r starts from node j, $v_{rj}, \forall r \in R, j \in P$, 1 if route r visits node j, and $e_{rj}, \forall r \in R, j \in S$, 1 if route r ends from node j. As this implies, a route might start at one shelter and end at another. We will use the same basic notation to denote variables of similar function, but use a hat (e.g. $\widehat{x}$) to signify that a variable belongs to the second formulation. The variables are as follows:

Variables:

$\hat{x}_r^{mt}$: binary variable that equals 1 if the bus m is assigned to route r for trip t, else $0, \forall\, r \in R,\ m \in V,\ t = 1, \ldots, T$.

$\hat{b}_j^{mt}$: number of evacuees from node j served by (or, if j is a shelter, released from) bus m in trip t, $\forall\, j \in P \cup S,\ m \in V,\ t = 1, \ldots, T$.

Also, instead of arc costs, we now use route costs, which we denote as $\hat{\tau}$. The alternative BEP formulation, which we denote as Model 2, is as follows:

Model 2:

$$\text{Minimize}\ \ T_{\text{evac}} + \frac{1}{L} \sum_{r \in R} \sum_{m \in V} \sum_{t=1}^{T} t \hat{x}_r^{mt} \tag{19}$$

$$\text{subject to: } T_{\text{evac}} \geq \sum_{r \in R} \sum_{t=1}^{T} \hat{\tau}_r \hat{x}_r^{mt}, \quad \forall\, m \in V \tag{20}$$

$$\sum_{r \in R} s_{ri} \hat{x}_r^{m1} = 1, \quad \forall\, i \in Y, m \in V_i \tag{21}$$

$$\sum_{r \in R} s_{ri} \hat{x}_r^{mt} = 0, \quad \forall\, i \in Y,\ m \in V,\ t = 2, \ldots, T \tag{22}$$

$$\sum_{r \in R} \hat{x}_r^{mt} \leq 1, \quad \forall\, m \in V,\ t = 1, \ldots, T \tag{23}$$

$$\sum_{r \in R} s^{sr} \hat{x}_r^{m(t+1)} \leq \sum_{r \in R} e^{sr} \hat{x}_r^{mt}, \quad \forall\, s \in S, m \in V, t = 1, \ldots, T-1 \tag{24}$$

$$\hat{b}_j^{mt} \leq Q \sum_{r \in R} v^{jr} x_r^{mt}, \quad \forall\, j \in P,\ m \in V,\ t = 1, \ldots, T \tag{25}$$

$$\hat{b}_j^{mt} \leq Q \sum_{r \in R} e^{jr} x_r^{mt}, \quad \forall\, j \in S,\ m \in V,\ t = 1, \ldots, T \tag{26}$$

$$\sum_{j \in P} \hat{b}_j^{mt} \leq Q, \quad \forall\, m \in V,\ t = 1, \ldots, T \tag{27}$$

$$\sum_{J \in S} \hat{b}_j^{mt} \leq Q, \quad \forall\, m \in V,\ t = 1, \ldots, T \tag{28}$$

$$\sum_{m \in V} \sum_{t=1}^{T} \hat{b}_j^{mt} \leq C_j, \quad \forall\, j \in S \tag{29}$$

$$\sum_{m \in V} \sum_{t=1}^{T} \hat{b}_j^{mt} = D_j, \quad \forall\, j \in P \tag{30}$$

$$\sum_{s \in S} \hat{b}_s^{mt} = \sum_{j \in P} \hat{b}_j^{mt}, \quad \forall\, m \in V,\ t = 1, \ldots, T \tag{31}$$

$$\hat{x}_r^{mt} \in \{0, 1\}, \quad \forall\, r \in R,\ m \in V,\ t = 1, \ldots, T \tag{32}$$

$$\hat{b}_j^{mt} \geq 0, \quad \forall\, j \in P \cup S,\ m \in V,\ t = 1, \ldots, T. \tag{33}$$

The min–max objective is formulated as (19), where (20) is a supporting constraint. The second term in (19) eliminates unneeded routes from the solution, much like the second term in (17). Equation (21) requires a bus's first trip be assigned to a route that starts from the yard at which the bus is located, while (22) prohibits a bus from being assigned to routes that start from the yard for later trips. Constraint (23) limits a bus to serve one route at a time. Constraint (24) enforces a logical connection between routes; a bus can only be assigned to a route that begins at the shelter where its last route ended. Constraints (25) and (26) require a bus to be assigned to a route that visits a pickup node or shelter in order to serve or drop off evacuees at that node. Constraints (27) and (28) are the vehicle capacity constraints. Constraint (29) is the shelter capacity constraint. Constraint (30) ensures that all evacuees are served, while

(31) insures all evacuees are dropped off at the shelter. Constraints (32) and (33) are the logical binary and non-negativity restrictions on the $\hat{x}$ and $\hat{b}$ variables, respectively.

The cost objective for Model 2 is formulated as follows.

$$\text{Minimize} \sum_{r \in R} \sum_{m \in V} \sum_{t=1}^{T} \hat{\tau}_r \hat{x}_r^{mt} \tag{34}$$

This type of route-based formulation can be difficult to solve to optimality, as it can be prohibitively expensive to derive all possible routes. Instead of deriving all possible routes, this type of formulation can be adapted to a column generation approach, see (Azi et al. 2010) as an example, where a sub-problem is used to generate routes that can improve a solution generated using an abbreviated, sub-set of routes. Next we will explore using this formulation in a heuristic algorithm to solve the BEP.

5.1 A heuristic algorithm for the BEP

In this section, we describe two heuristic algorithms for solving the BEP; the first is based on a simple search strategy and the second makes use of Model 2. We demonstrate these heuristics on our previous test scenarios and then expand the analysis to much larger scenarios, scenarios that are too large to solve using optimization techniques alone. Heuristic 1, which has two phases, is described next. Phase 1 is used to quickly produce a feasible solution, while Phase 2 uses a search technique to improve the solution by route swapping and route reassignment.

Heuristic 1- Phase 1

Step 1 Produce a list of buses sequenced in nondecreasing order of their total number of movements from a yard or shelter to a pickup location plus the movements from a pickup location to a shelter, and then further sequenced by the travel costs of the routes assigned to each bus, breaking ties arbitrarily. (Observe that the number of movements and cost for each bus is initially zero.)

Step 2 Denote the first bus in the list as bus i. If bus i is at a pickup location, route it to the nearest shelter with sufficient remaining capacity, and if bus i is at a yard or shelter, route it to the nearest pickup location that has remaining evacuees. Once there, pick up as many evacuees as the vehicle capacity allows. If there is remaining capacity, go to the nearest pickup location having remaining evacuees and continue in this manner until the capacity of the bus i is exhausted. Update the list of buses.

Step 3 Go to Step 1 and continue until all evacuees are in shelters.

This first step provides us with a feasible solution for the problem. This is done using a layering technique. Each bus completes a layer, a movement to a set of pickup locations from a shelter or yard, or a movement to a shelter from the pickup locations, before any bus does a next layer. After all buses complete a layer, the bus with the highest travel cost selects first, choosing the lowest cost option available. This tends to keep the layers somewhat even. Of course, this type of layering is not necessarily optimal; we observe many optimal solutions that do not follow this strategy. Furthermore the layers can still be fairly uneven. The next phase of the heuristic is a search procedure for improving the solution found in Phase 1. In this phase we will try to swap partial or complete routes from the bottleneck bus, i.e., the bus with the highest travel cost, to another bus, or move a route entirely from the bottleneck bus, such that the current T_{evac} decreases, while preserving feasibility.

Heuristic 1- Phase 2

Step 1 Select the bottleneck bus, i.e., the bus with the highest travel cost (breaking ties arbitrarily); this bus determines the current T_{evac}. Let the bottleneck swap route be the first route assigned to the bottleneck bus, let the swap bus be the bus with the lowest travel cost, and let the swap route be the first route assigned to the swap bus.

Step 2 Check if a swap of (a) the complete routes, (b) the set of pickup locations, or (c) the ending shelter location (in that order) between the bottleneck swap route and the swap route will reduce the overall T_{evac} while maintaining feasibility. If so perform the swap, update costs and go to Step 1. Else go to Step 3.

Step 3 Let the next route on the swap bus be the swap route. If there is such a route go to Step 2, or else go to Step 4.

Step 4 Check if the bottleneck swap route can be assigned as the last route of the swap bus to reduce T_{evac}. If so, perform the route re-assignment, update costs, and go to Step 1, or else go to Step 5.

Step 5 Let the next lowest cost bus be the swap bus and let the swap route be the first route assigned to the swap bus. If there is such a bus go to Step 2, or else go to Step 6.

Step 6 Let the next route on the bottleneck bus be the bottleneck swap route. If there is such a bottleneck swap route let the swap bus be the bus with the lowest travel cost, let the swap route be the first route assigned to the swap bus and then go to Step 2, or else stop.

The feasibility testing for most of these swaps or reassignments is straightforward and easy to accomplish. We next examine the second heuristic, which uses Model 2. Here, we show how it can be used in conjunction with Heuristic 1 to improve the Heuristic 1 solution.

Heuristic 2

Step 1 Solve Heuristic 1, or any heuristic that provides a feasible solution to the BEP, and determine the routes used.

Step 2 Solve a relaxation of Model 1 where the x-variables are allowed to take continuous values. Take the solution to this linear program (LP) and determine the routes used.

Step 3 Solve Model 2 using the route sets generated in Steps 1 and 2, eliminating any duplicates, and use a warm-start procedure to provide Model 2 with the feasible solution generated in Step 1. When solving Model 2 use an appropriate run-time limit.

Table 4 displays the results for these heuristics compared to the optimal solutions for our previously described test scenarios.

Table 4 Results for Heuristic 1, Heuristic 2, and the optimal min–max solution

Scenario descriptor	Heuristic 1		Heuristic 2			Optimal (min-max)	
	Duration	Cost	Duration	Cost	Run-time (s)	Duration	Cost
Y1P3S2V4	30	95	30	95	0.54	30	95
Y1P4S2V4	26	82	25	79	0.46	25	79
Y1P5S3V6	27	158	27	157	1.54	27	157

Table 5 Results from Heuristic 1 and Heuristic 2 for larger test scenarios

Scenario descriptor	Heuristic 1		Heuristic 2	
	Duration	Cost	Duration	Cost
Y2P12S3V6	150	912	149	882
Y2P22S4V10	110	1,085	108	1,080
Y2P32S5V18	106	1,816	104	1,821

As the results in Table 4 illustrate, these heuristics are very effective on these small size problems. Heuristic 1 did not find the optimal solutions in each case, but it found a near-optimal solution quickly. Heuristic 2 was able to find the optimal solution (the run-times reported for Heuristic 2 are only for Model 2; they neglect Steps 1 and 2, which are negligible for these small problem sizes). To test how these heuristics scale, we introduce some much larger test scenarios, scenarios for which we are not able to find optimal solutions using Model 1. The largest scenario shown has two yards, 32 pickup locations, five shelters, and 18 vehicles. Table 5 shows some representative results from the analysis performed using the larger scenarios (for each scenario's network we tested various instances using different demand parameters, shelter capacities, and fleet sizes).

Heuristic 1 was able to solve these scenarios in a few minutes at most, but the heuristic has not been coded for efficiency, and better results might be attainable. For these larger scenarios, we used a 2,000 second time limit for Model 2 in Heuristic 2. We note that solving a linear relaxation of Model 1 (Step 2 of Heuristic 2) no longer takes a negligible amount of time, the Y2P32S5V18 scenario requires 121.3 s to solve. From our analysis, we find that the warm-start is quite effective, for instance, without it we could not find the solution shown in Table 5 for scenario Y2P32S5V18 within the 2,000 s limit. We note that since the min–max objective has an objective value set by a single or small set of buses, it has some ability to absorb cost inefficiencies without increasing the evacuation duration. We also note that when producing routes for Heuristic 2 using more than one source, the number of new routes added by each new route generating procedure tends to decrease. For instance, consider scenario Y2P12S3V6, which has a schematic displayed in Fig. 9. Heuristic 1 produced 31 routes, of which 18 are distinct, while the relaxation of Model 1 produced 49 routes, of which 28 are distinct. Combining the routes from both route generating procedures and eliminating duplicates produces 32 distinct routes. In contrast, the set of all single-pickup routes would number 180 distinct routes, and not include the routes found using Heuristic 1 and the Model 1 LP that stop at multiple pickup locations. We found that for the larger problems, we cannot run Heuristic 2 until an optimal solution is found (for the given set

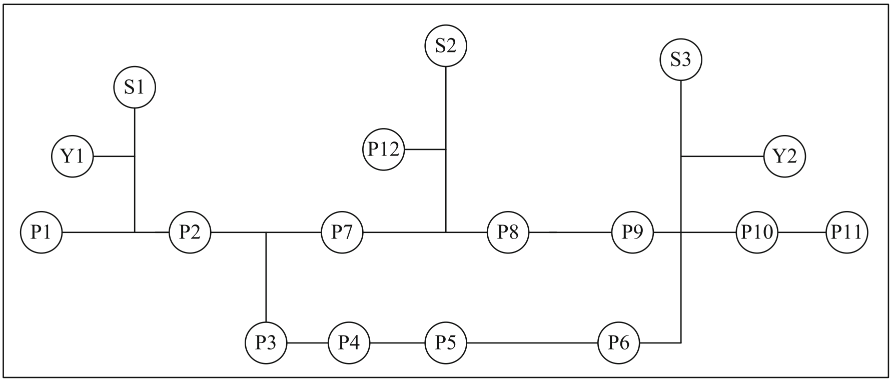


Fig. 9 A network schematic for the Y2P12S3V6 scenario

of routes), but that it is still useful for improving the solution found in Heuristic 1. One advantage of a heuristic based on an optimization model is that there is a large body of work on methods to improve the performance of these optimization models, and any improvements in the underlying model can improve the heuristic. For instance, both Models 1 and 2 have various symmetry issues that complicate the problem; breaking these symmetries might improve the solution run-time performance.

We cannot easily determine if the solutions derived from Heuristic 2 are optimal (even after running Model 1 for long intervals the optimality gap was too large to be of use for the larger scenarios). The linear relaxation of Model 1 provides a lower bound on the solution, but we find that it is not very tight and thus not so useful for estimating an optimality gap. We can consider a rule of thumb based on the travel costs for each bus: consider the results from the scenario Y2P12S3V6 instance obtained using Heuristic 2. The cost vector for the buses is 149, 147, 147, 143, 149, 147; as a rule of thumb, we have more confidence that a solution is close to optimality when there is a narrow cost gap between the bottleneck vehicle and the lowest cost vehicle, especially when multiple routes per vehicle are required. Of course; some optimal solutions might have a large gap, but without such a gap it is less likely that the solution can be improved. For this scenario, each bus is assigned between four and six tours in the solution obtained.

For scenario Y2P12S3V6, we perform a sensitivity analysis on the shelter locations and on the consolidation of pickup locations. The average distance from each pickup location to each shelter, S1, S2, and S3, is 25, 20, and 21 time intervals, respectively, and the evacuee-weighted distance is 23.5, 19.3, and 22.3. Table 6 shows the results of removing shelters and redistributing their capacity, as obtained using Heuristic 2. Interestingly, the increase in evacuation duration caused by removing shelter S3 is larger than that caused by removing shelter S1, despite the rough symmetry of the network. Just using the central shelter, S2, greatly increases the duration, as expected, because buses now have costs based on the average travel distance between S2 and the pickup locations.

Table 6 also displays the results from consolidating various pickup locations. This is the type of analysis an emergency planner might want to use when developing the

Table 6 Comparison of results for scenario Y2P12S3V6 for various shelter configurations and pickup location consolidations

Shelter used	Duration	Cost	Consolidated pickup locations	Duration	Cost
S1, S2	178	1,052	(P3, P4)	143	856
S1, S3	174	1,026	(P5, P6)	147	862
S2, S3	166	976	(P8, P9)	151	896
S2	183	1,090	(P10, P11)	147	862

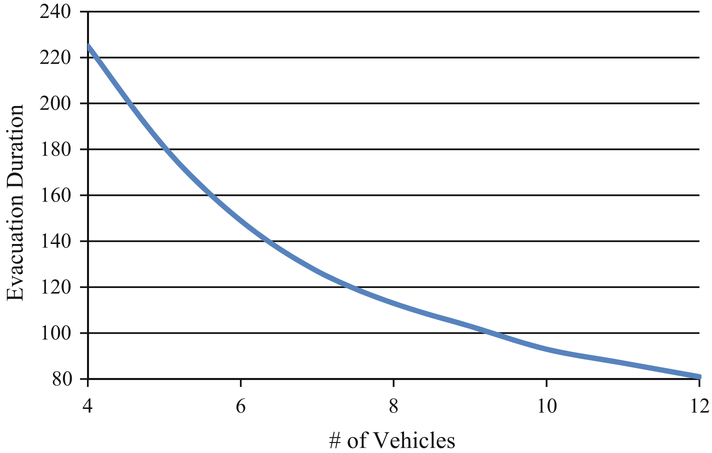


Fig. 10 The duration of an evacuation as the fleet size increases for the Y2P12S3Vv scenario

network. From an evacuee perspective, the more the pickup locations the better, as the evacuees have to travel shorter distances. Unfortunately, this might have a negative impact on the overall duration. For instance, we can see that combining P8 and P9 does not reduce the evacuation duration, thus there is no benefit from consolidating them, unlike the other pairs considered. The emergency manager would have to consider these trade-offs when developing a plan.

Figure 10 shows the evacuation duration as a function of the number of vehicles. The results is a convex curve; we see none of the non-convex effects displayed in scenario Y1P4S3V4 (see Fig. 4). The durations in Fig. 10 are averages for the given number of vehicles. This is because there are two yards in this scenario, and thus when changing the number of vehicles we must decide from which yard to add or delete them. Here we keep the fleets in each yard more or less balanced. We found that for this particular scenario, there is a slight advantage when placing more vehicles in yard Y1. This is logical, as shelter S1 is, on average, the most distant shelter from the pickup locations, so vehicles starting in Y1, which is close to S1, but closer still to the pickup locations, can negate some of the cost associated with using shelter S1.

6 Conclusions

The BEP is a variant of the VRP that is designed for planning bus-based evacuations. It differs from its closest VRP relatives in two important aspects: (1) the objective function, which has a min-max structure that minimizes the duration of the evacuation, that is, how long it takes to transport evacuees to shelters, and (2) the network structure, in which the traditional VRP depot is divided into yards and shelters; the yards represent the initial location of the buses, and the shelters, which have capacity restrictions, are where evacuees are delivered. These differences make the BEP significantly more difficult to solve. This is due to the interaction among various decisions that need to be made: (a) construction of routes, (b) assignment of routes to the multiple vehicles, and (c) selection of a shelter for each route. Furthermore, the properties of an optimal solution of the VRP do not necessarily hold for the BEP, for example, the optimal solution has a different relationship with the fleet size, an important planning parameter.

We have provided two mathematical programming formulations for the BEP. Due to its computational complexity, for realistic size problems a heuristic technique is required. We provide two heuristics, one based on a simple search technique and the other based one one of the mathematical programming formulations provided. While the heuristic presented are effective, further research into improvements and other heuristics is warranted. We observed that heuristics potentially perform differently for the BEP because of the min–max objective.

Planning a bus-based evacuation is an extremely rich problem, and several extensions to the model proposed here should be investigated. To help motivate further research, we list and discuss some of these extensions here.

- We assume a symmetric network structure, but this assumption depends how the transit based evacuation is integrated into a larger regional evacuation that includes many modes, most significantly cars, which share the same infrastructure. If the car-based evacuation causes excessive congestion, this assumption might no longer be valid. Furthermore, if network modification strategies are used, like extensive contraflow, then this assumption might also no longer hold.
- We assume a model where people gather to be picked up, but the population we are serving has a high percentage of special needs people. We might have to modify the model such that the buses (or perhaps other secondary vehicles) pick people up from their homes. In this case, vehicles might have to be assigned based on special needs (e.g., can the bus accommodate a wheelchair), and the pickup service time should be considered.
- The special needs of the population can also impact on shelter considerations. Those evacuees from assisted-care facilities would probably not be sent to a general shelter, but to a shelter suited to their needs. Certain shelters might be designated for those with medical needs. This can impact on the evacuation planning considerably.
- There might be relevant behavioral issues, which, when accounted for in the modeling framework, can improve the performance of the evacuation plan. For example, evacuees might not all be ready at the same time, and acknowledging this in the model might improve the resulting evacuation plan. Of course, just implementing an evacuation plan is an improvement over the existing state of affairs as it gives potential evacuees an evacuation option.

Evacuation planning in general is challenging. Unlike commercial logistics systems, there is less experience with the actual working system and very little opportunity for experimentation. Thus, plans must be carefully analyzed, and every evacuation should be used as an opportunity to collect information and access the quality of the plan.

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