CS301 Assignment 1

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1 Problem 1

1.1 Part a)

Solved by Master Theorem. $a = 2 \geq 1, b = 2 > 1, f(n)$ asymptotically positive.
$n^{\log_2 2}$ so that $f(n) = \Omega(n^{1+\epsilon})$ which $\epsilon > 1$ and $\epsilon = 2$
Therefore by the master theorem $T(n) = \Theta(n^3)$

1.2 Part b)

Solved by Master Theorem. $a = 7 \geq 1, b = 2 > 1, f(n)$ asymptotically positive.
$n^{\log_2 7}$ so that $f(n) = O(n^{\log_2 7-\epsilon})$ which $\epsilon > 1$ and $\epsilon = 2$
Therefore by the master theorem $T(n) = \Theta(n^{\log_2 7})$

1.3 Part c)

Solved by Master Theorem. $a = 2 \geq 1, b = 4 > 1, f(n)$ asymptotically positive.
$n^{\log_4 2}$ so that $f(n) = \Theta(n^{1-\epsilon})$ which $\epsilon > 1$
Therefore by the master theorem $T(n) = \Theta(\sqrt{n} \lg n)$

1.4 Part d)

Solved by Iteration Method.
$i = 1 : T(n) = T(n-1) + n \quad -> \quad T(n-1) = T(n-2) + n - 1$
$i = 2 : T(n) = T(n-2) + 2n - 1 \quad -> \quad T(n-2) = T(n-3) + n - 2$
$i = 3 : T(n) = T(n-3) + 3n - 3 \quad -> \quad T(n-3) = T(n-4) + n - 4$
$i = 4 : T(n) = T(n-4) + 4n - 6 \quad -> \quad \ldots$
.
.
.
$i = k : T(n) = T(n-k) + kn - \frac{k \times (k-1)}{2}$
$base : n - k = 1 \quad -> \quad T(n) = T(1) + (n-1) \times n - \frac{(n-1) \times (n-2)}{2}$
$T(n) = T(1) + n^2) - n - \frac{n^2}{2} + \frac{3n}{2} \quad -> \quad T(n) = \Theta(n^2)$

2 Problem 2

2.1 Part a)

2.1.1 i)

2.1.2 ii)

2.2 Part b)

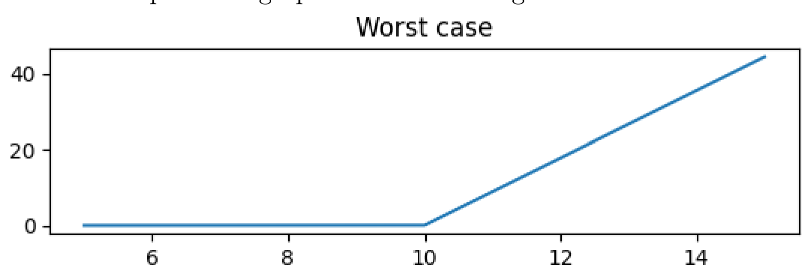
2.2.1 i)

CPU: Intel(R) Core(TM) i7-10700K CPU @ 3.80GHz 3.79 GHz
RAM 32GB OS: Windows 10 21H1 19043.2130

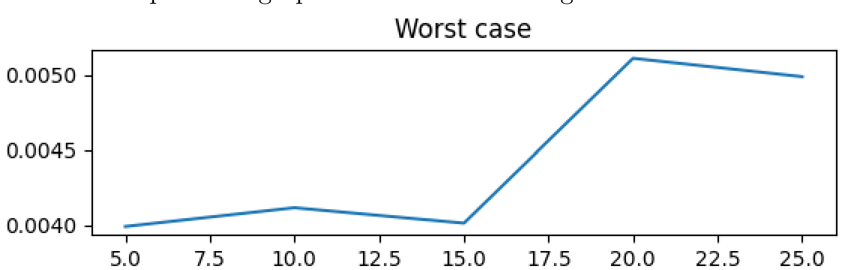
Algorithm	m = n = 5	m = n = 10	m = n = 15	m = n = 20	m = n = 25
Naive	0.00398	0.05785	44.42287	Too Long	Too Long
Memorization	0.00398	0.00411	0.00401	0.00510	0.00498

2.2.2 ii)

Worst case input-time graph of Recursive Algorithm



Worst case input-time graph of Memorization Algorithm



2.2.3 iii)

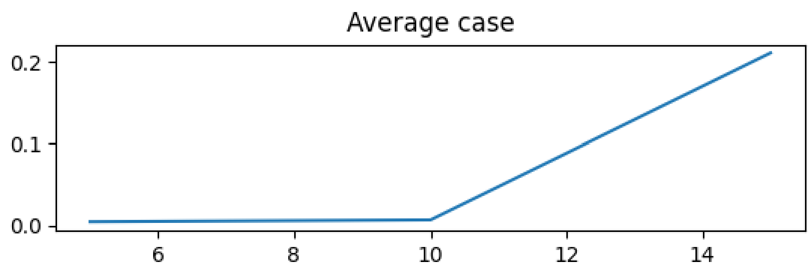
2.3 Part c)

2.3.1 i)

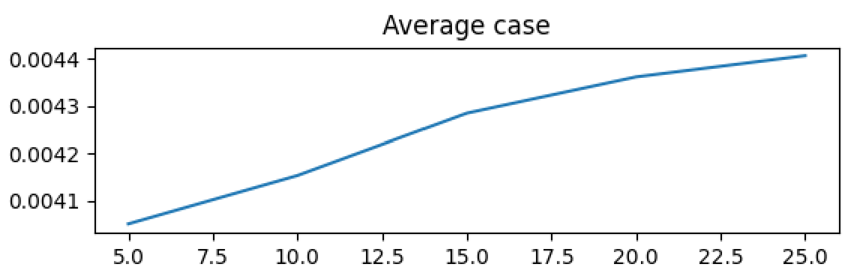
Algorithm	m = n = 5	m = n = 10	m = n = 15	m = n = 20	m = n = 25
Naive μ	0.004387	0.006628	0.211047	Too Long	Too Long
Naive σ	0.000784	0.002101	0.174624	-	-
Memo μ	0.004050	0.004153	0.004284	0.004361	0.004406
Memo σ	0.000696	0.000692	0.000647	0.000765	0.000865

2.3.2 ii)

Average case input-time graph of Recursive Algorithm



Average case input-time graph of Memorization Algorithm



2.3.3 iii)

as the graphs indicate and also prior analysis, naive algorithm has a exponential growth but memorization algorithm has growth of log-linear type.