NumPDE

Series 9/1

Christian Lehmann

Philibert Pinkert

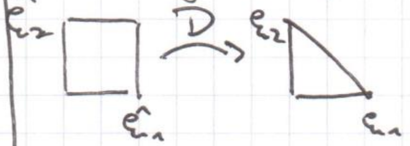
$$\int_K \lambda_1^{\beta_1}\lambda_2^{\beta_2}\lambda_3^{\beta_3}\,dx \overset{|D\varphi|}{=} 2|K| \int_{\hat K} \xi_1^{\beta_1}\xi_2^{\beta_2}(1-\xi_1-\xi_2)^{\beta_3}\,d\xi_1\,d\xi_2$$

$\lambda(x) = \lambda(\varphi(\xi))$

$\cancel{= 2|K| \int_0^1\int_0^{1-\xi_2} \dots}$

$x \in K,\ \xi \in \hat K$

→ Duffy Transformation



$$D \begin{cases} \xi_2 = \hat\xi_2 \\ \xi_1 = \hat\xi_1(1-\hat\xi_2) \end{cases}$$

$\underline{D([0,1]^2) = \hat K}$

$$\overset{|DD|}{=} 2|K| \int_0^1\int_0^1 \hat\xi_1^{\beta_1}(1-\hat\xi_2)^{\beta_1}\hat\xi_2^{\beta_2}\underbrace{(1-\hat\xi_1(1-\hat\xi_2)-\hat\xi_2)}_{=(1-\hat\xi_1)(1-\hat\xi_2)}^{\beta_3}(1-\hat\xi_2)\,d\hat\xi_1\,d\hat\xi_2$$

$$= 2|K| \int_0^1 \hat\xi_1^{\beta_1}(1-\hat\xi_1)^{\beta_3}\,d\hat\xi_1 \cdot \int_0^1 \hat\xi_2^{\beta_2}(1-\hat\xi_2)^{\beta_1+\beta_3+1}\,d\hat\xi_2$$

$$= 2|K| \cdot B(\beta_1+1,\beta_3+1)\cdot B(\beta_2+1,\beta_1+\beta_3+2)$$

$$= 2|K| \frac{\Gamma(\beta_1+1)\,\Gamma(\beta_3+1)}{\cancel{\Gamma(\beta_1+1+\beta_3+1)}} \cdot \frac{\Gamma(\beta_2+1)\,\cancel{\Gamma(\beta_1+\beta_3+2)}}{\Gamma(\beta_1+\beta_2+\beta_3+3)}$$

$$= 2|K| \frac{\beta_1!\,\beta_2!\,\beta_3!}{(\beta_1+\beta_2+\beta_3+2)!}$$