Integrated nested Laplace approximations for extended latent Gaussian models with application to the Naomi HIV model

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Summary

- Approximate Bayesian inference method using adaptive Gauss-Hermite quadrature and Laplace approximation
- Implemented as a part of the `aghq` package, allowing easy use for any model with a `TMB` C++ user template
- Motivated by a challenging problem in evidence synthesis for small-area estimation of HIV

The Naomi HIV model

- Spatio-temporal small-area estimation model of HIV indicators which combines data from household surveys, antenatal care clincs, and routine service provision of antiretroviral therapy (Eaton et al. 2021)
- Yearly estimation process where the model is run interactively by country teams using the web-app `naomi.unaids.org`
- Inference currently conducted with empirical Bayes and a Gaussian approximation to the latent field using Template Model Builder `TMB` (Kristensen et al. 2016)
- Model incompatible with the `R-INLA` implementation of integrated nested Laplace approximations (Rue, Martino, and Chopin 2009) and takes days to get accurate answers with Markov chain Monte Carlo via `tmbstan` (Monnahan and Kristensen 2018)
- Fits into the recently described class of extended latent Gaussian models (Stringer, Brown, and Stafford 2022)

Inference procedure

Laplace approximation

- Integrate out variables using a Gaussian approximation in the denominator

$$p(\theta, y) \approx \tilde{p}_{\mathrm{LA}}(\theta, y)$$

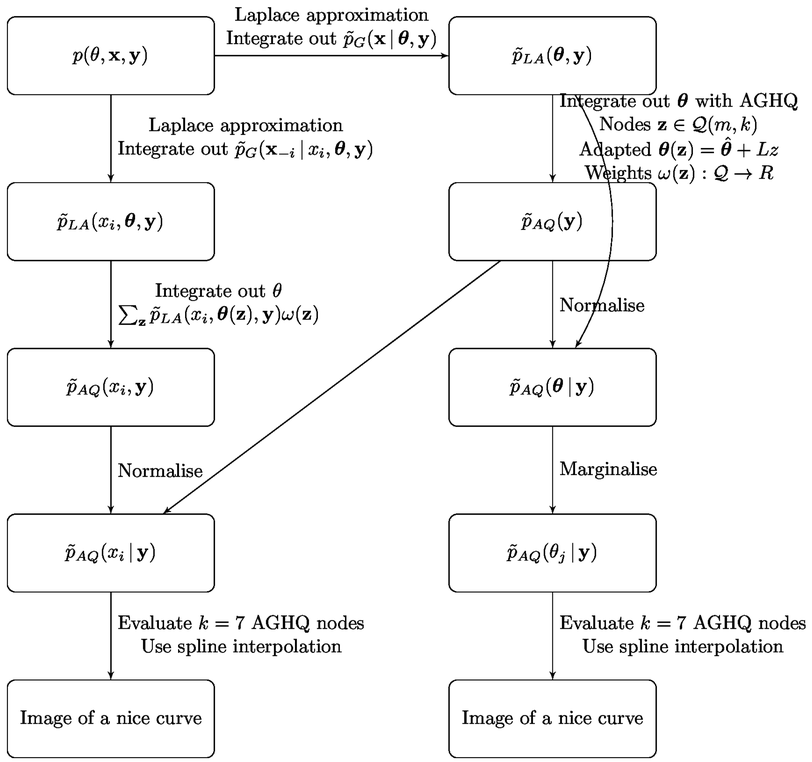
where

Adaptive Gauss-Hermite Quadrature

- Approximate integrals by $\int_\Theta p(\theta)\mathrm{d}\theta \approx |L| \sum_{z \in \mathcal{Q}(m,k)} p(\hat{\theta} + Lz)\omega(z)$ with Gauss-Hermite quadrature rule $z \in \mathcal{Q}(m,k)$ adapted based upon the mode $\hat{\theta} = \mathrm{argmax}_{\theta \in \Theta} \in p(\theta)$ and lower Cholesky $LL^\top = -\partial^2_\theta \log p(\theta)|_{\theta=\hat{\theta}}$ of the target

Our algorithm

Given C++ user template for $-\log p(y, x, \theta)$:



Inference comparison

- Compare posterior inferences from `TMB`, `aghq` and `tmbstan` using Kolmogorov-Smirnov tests on posterior marginals
- Find that...

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References

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