Fast approximate Bayesian inference for small-area estimation of HIV indicators using the Naomi model

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Summary

- Approximate Bayesian inference method combining Laplace approximations and adaptive Gauss-Hermite quadrature
- Motivated by a challenging, policy-relevant problem in small-area estimation of HIV indicators in sub-Saharan Africa
- Implemented as a part of the `aghq` R package, allowing use for any model with a `TMB` C++ user template

The Naomi HIV model

- District-level model of HIV indicators which synthesises data from household surveys, antenatal care (ANC) clincs, and routine service provision of antiretroviral therapy (ART) (Eaton et al. 2021)
- Yearly estimation process: model run interactively by country teams using the web-app `naomi.unaids.org`
- Inference currently conducted with empirical Bayes and a Gaussian approximation to the latent field using Template Model Builder `TMB` (Kristensen et al. 2016)
- Days to get accurate answers with Markov chain Monte Carlo via `tmbstan` (Monnahan and Kristensen 2018)
- Motivates looking for a fast, approximate approach

Extended latent Gaussian models

- In a latent Gaussian model (LGM) (Rue, Martino, and Chopin 2009) the conditional mean depends on exactly one structured additive predictor $\mu_i = g(\eta_i)$ with $g : \mathbb{R} \to \mathbb{R}$
  - The `R-INLA` implementation of integrated nested Laplace approximations applies only to LGMs
- Extended latent Gaussian models (ELGM) remove this requirement such that $\mu_i = g(\eta_{\mathcal{J}_i})$ where $g_i : \mathbb{R}^{|\mathcal{J}_i|} \to \mathbb{R}$
- Naomi is an ELGM, and not an LGM, because in our model:
  - ANC indicators are offset from household survey indicators
  - Incidence depends on prevalence and ART coverage
  - Observed data may be aggregated
  - Individuals may attend ART clinics outside their home district

Inference procedure

Background

- **Laplace approximation** Integrate out variables using a Gaussian approximation to the denominator e.g.

$$p(\theta, y) \approx \tilde{p}_{\text{LA}}(\theta, y) = \frac{p(y, x, \theta)}{\tilde{p}_{\text{G}}(x \,|\, \theta, y)} \Big|_{x = \hat{x}(\theta)}$$

  where $\tilde{p}_{\text{G}}(x \,|\, \theta, y) = \mathcal{N}(x \,|\, \hat{x}(\theta), \mathbf{H}(\theta)^{-1})$
- **Adaptive Gauss-Hermite Quadrature** Approximate integrals by $\int_\Theta p(\theta) \text{d}\theta \approx |L| \sum_{z \in \mathcal{Q}(m,k)} p(\hat{\theta} + Lz)\omega(z)$ with Gauss-Hermite quadrature rule $z \in \mathcal{Q}(m,k)$ adapted based upon the mode $\hat{\theta} = \text{argmax}_{\theta \in \Theta} \in p(\theta)$ and lower Cholesky $LL^\top = -\partial_\theta^2 \log p(\theta)|_{\theta = \hat{\theta}}$ of the target

Our algorithm

- Given a C++ user template for $-\log p(y, x, \theta)$ our algorithm is summarized by Figure 1

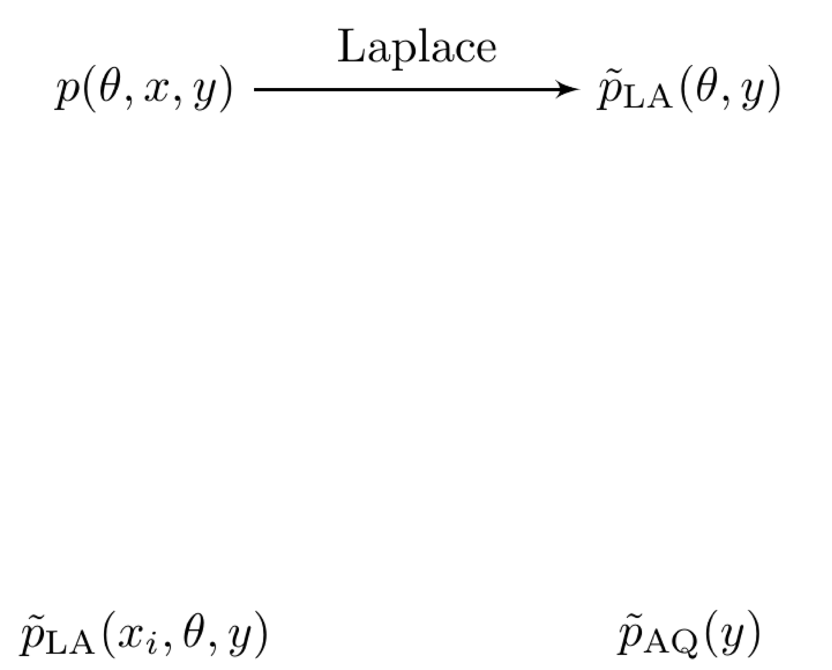

Figure 1: Flowchart describing the algorithm

Inference comparison

- Compare posterior inferences from `TMB`, `aghq`, `adam` and `tmbstan` using Kolmogorov-Smirnov tests on posterior marginals of hyperparameters, latent field, and outputs
- Figure 2 compares the distances between `TMB` and `adam` to `tmbstan`

Future work

- More comprehensive inference comparison e.g. maximum mean discrepancy, Pareto-smoothed importance sampling
- Implement Laplace matrix algebra approximations e.g. Wood (2020)

Interested to read more? Working notebooks and R code available are available from `github.com/athowes/multi-agyw`.

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