Chapter 12 Solutions

Exercise 12.1.

$$\begin{aligned}\mathcal{F}^T \mathcal{V}_A &= ([p_A]\hat{n})^T \omega_A + \hat{n}^T v_A \\ &= -\hat{n}^T [p_A] \omega_A + \hat{n}^T v_A \\ &= \hat{n}^T [\omega_A] p_A + \hat{n}^T v_A = \hat{n}^T \dot{p}_A\end{aligned}$$

Exercise 12.2.

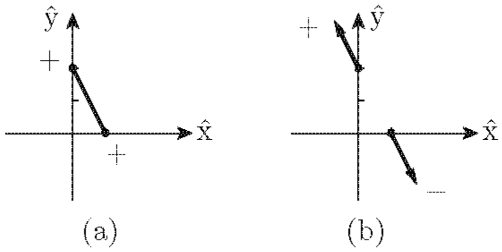

Figure 12.1

Exercise 12.3.
$-3\omega_x + \omega_z + v_y \geq 0.$

Exercise 12.4.
$p = (0, 0, 0)$ and $\hat{n} = (0, 0, 1)$.
(a) $v_z = 0$.
(b) $v = 0$.
(c) $\mathcal{V} = 0$.

Exercise 12.5.
Form closure subsets: $\{1, 3, 4, 5\}$ and $\{1, 2, 3, 5\}$.

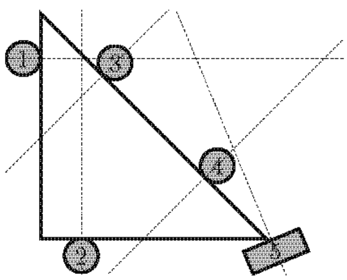

Figure 12.2

Exercise 12.6.

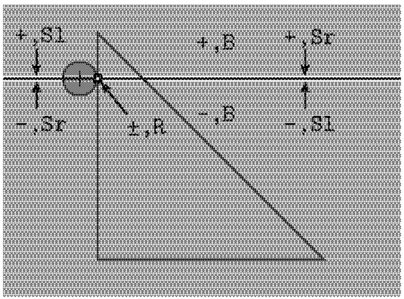

Figure 12.3

Exercise 12.7.

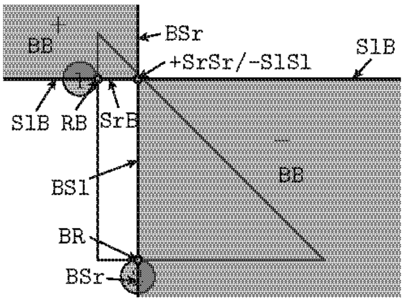

Figure 12.4

Exercise 12.8.

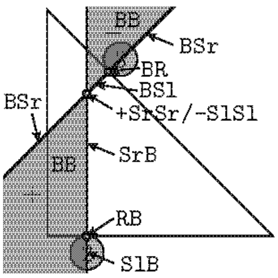

Figure 12.5

Exercise 12.9.

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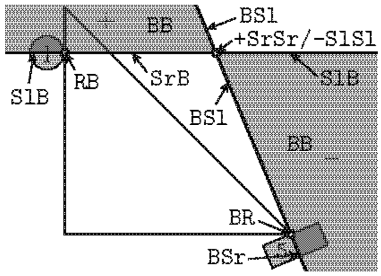

Figure 12.6

Exercise 12.10.

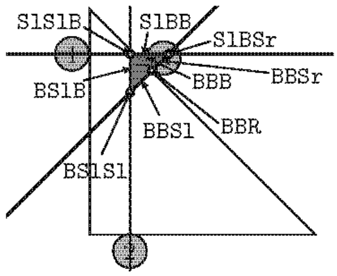

Figure 12.7

Exercise 12.11.

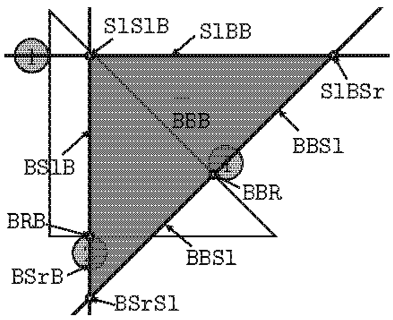

Figure 12.8

Exercise 12.12.

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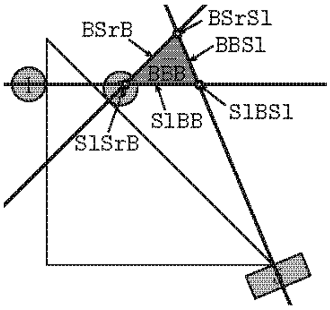

Figure 12.9

Exercise 12.13.

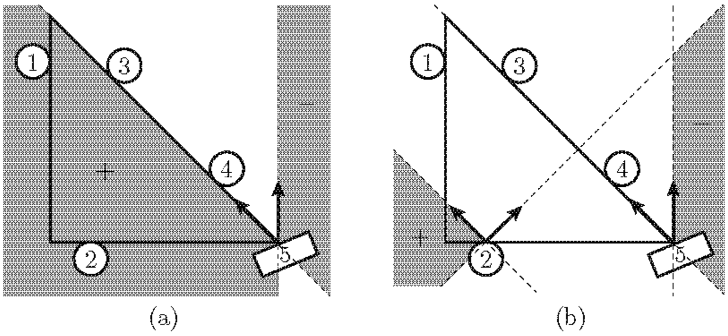

Figure 12.10

Exercise 12.14.

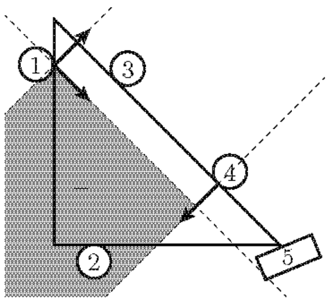

Figure 12.11

Exercise 12.15.

(a) The conditions for force closure of Figure 12.12(a) can be arranged into a linear equation of the form $Ax = b$:

$$f_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} c_1,\ f_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} c_2,\ f_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} c_3,\ f_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} c_4,\ f_5 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} c_5$$

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$$\begin{bmatrix} -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & -\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ M_z \end{bmatrix}$$

Matrix A can be reduced to the following row echelon form via Gauss-Jordan elimination:

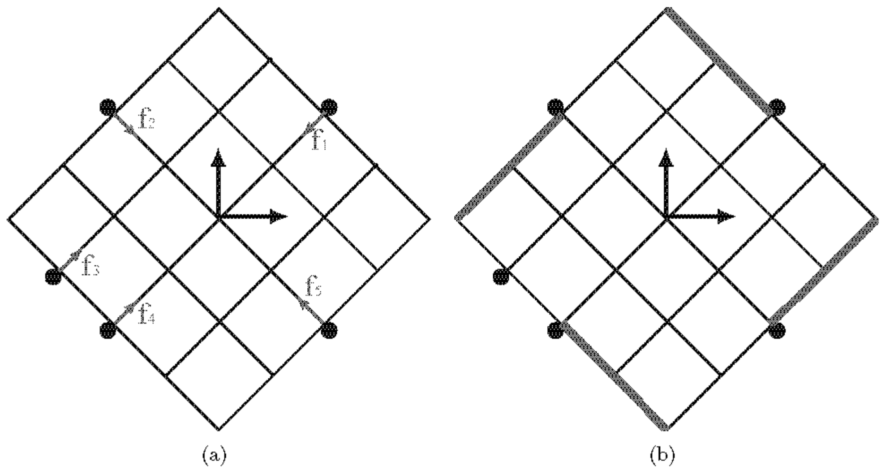

Figure 12.12

$$[I \quad S] = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

There does not exist any $w \geq 0$ satisfying $Sw < 0$. Thus, this is not force closure.

(b) In order to make force closure by adding one frictionless point contact, new point contact should be able to exert positive moment to planar square. Figure 12.12(b) represents all possible locations of this contact as red line.

Exercise 12.16.

We can express the force applied to the center by a vector sum of c_1, c_2 as shown in Figure 12.13:

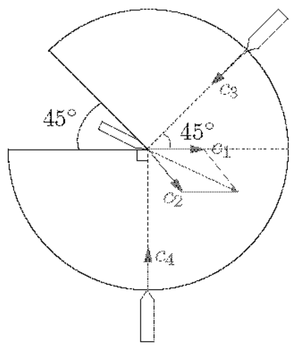

Figure 12.13

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If we place the reference frame at the center, the grasp matrix is a 3×4 matrix of the form:

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ M_z \end{bmatrix}.$$

Since the four vectors have no z-component (moment term), the convex hull lies on the X-Y plane, and the grasp is not force-closure. For force closure we need 2 more vectors, one with a positive moment component, another with a negative moment component:

$$\begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ -L_2 \end{bmatrix},$$

where $L_1, L_2 > 0$. The additional two frictionless point contacts can be placed between the concave edges, e.g. one at the upper edge, another at the lower.

Exercise 12.17.

(a) From Nguyen's theorem, this is a force closure grasp.
(b) From Nguyen's theorem, this is NOT a force closure grasp.
(c) From Nguyen's theorem, all positions x that ensure that the grasp is force closure are of the form $0 < x < L$.

Exercise 12.18.
The conditions for force closure of Figure 12.14 can be arranged into a linear equation of the form $Ax = b$:

$$f_{1a} = \begin{bmatrix} -\mu \\ 1 \end{bmatrix} c_1,\ f_{1b} = \begin{bmatrix} \mu \\ 1 \end{bmatrix} c_2,\ f_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} c_3,\ f_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} c_4$$

$$\begin{bmatrix} -\mu & \mu & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

Matrix A can be reduced to the following row-echelon form via Gauss-Jordan elimination:

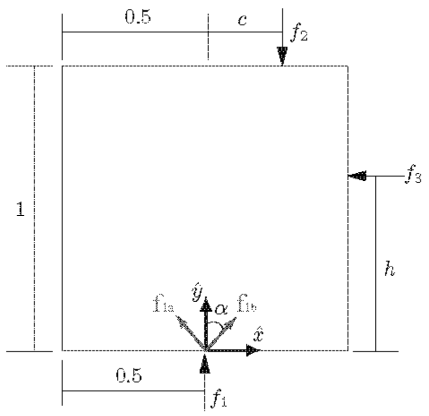

Figure 12.14

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$$\begin{bmatrix} -\mu & \mu & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1+\frac{1}{2\mu} \\ 0 & 1 & 0 & -1-\frac{1}{2\mu} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$[I \quad S] = \begin{bmatrix} 1 & 0 & 0 & -1+\frac{1}{2\mu} \\ 0 & 1 & 0 & -1-\frac{1}{2\mu} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

In order to be force closure, all elements of S should be negative. In other words, $-1+\frac{1}{2\mu} < 0$ and $-1-\frac{1}{2\mu} < 0$.

$$\therefore \mu > \frac{1}{2}$$

Exercise 12.19.

(a) The conditions for force closure of Figure 12.15(a) can be arranged into a linear equation of the form $Ax = b$:

$$f_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} c_1, f_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} c_2, f_3 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} c_3, f_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} c_4, f_5 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} c_5$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 1 \\ 1 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ M_z \end{bmatrix}$$

Matrix A can be reduced to the following row-echelon form via Gauss-Jordan elimination:

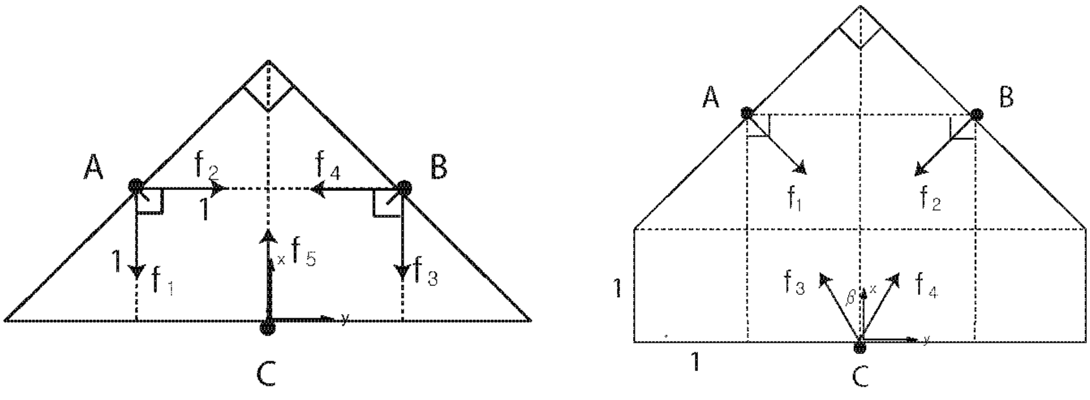

Figure 12.15

$$[I \quad S] = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Since there exists w satisfying $Sw < 0$, this grasp is force closure.

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(b) The conditions for force closure of Figure 12.15(b) can be arranged into a linear equation of the form $Ax = b$:

$$f_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} c_1, f_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} c_2, f_3 = \begin{bmatrix} -\mu \\ 1 \end{bmatrix} c_3, f_4 = \begin{bmatrix} \mu \\ 1 \end{bmatrix} c_4$$

$$\begin{bmatrix} 1 & -1 & -\mu & \mu \\ -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

Matrix A can be reduced to the following row-echelon form via Gauss-Jordan elimination:

$$\begin{bmatrix} 1 & -1 & -\mu & \mu \\ -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & \frac{\mu}{2}-\frac{1}{2} & -\frac{\mu}{2}-\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$[I \quad S] = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Regardless of μ, this grasp is force closure except when $\mu = 0$. When $\mu = 0$, there are only three columns in the matrix, so it is impossible to construct a fourth column with all elements negative.

$$\therefore 0 < \beta \leq \frac{\pi}{2}$$

Exercise 12.20.

(a) $\mu > \tan^{-1}(\pi/4n)$.

(b) $\mu > 0$.

Exercise 12.21.

(a) $4-3 = 1$. The three constraints come from force balance in the normal direction and moment balances about two horizontal directions.

(b) $4 \times 3 - 3 - 3 = 6$. Each contact force is three dimensional, and there are three constraints in forces and three in moments.

Exercise 12.22.

As shown in Figure 12.16, when place the finger on the bottom, then the location of the finger $x < 0$.

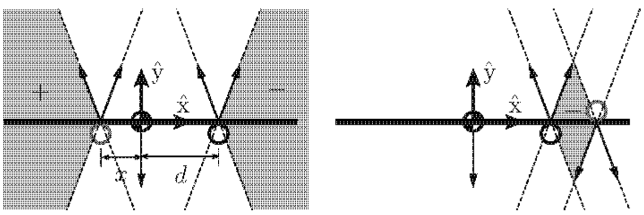

Figure 12.16

And when the finger is placed on the top of the rod, let $x > d$ can balance the gravity force of the rod. The magnitude of the second contact normal forces when placed on the top are larger than on the bottom since when on the top the normal force need to balance the gravity force plus the normal force of the stationary finger.

Exercise 12.23.
(a) See Figure 12.17(a).
(b) From Figure 12.17(a), the box will tip right.
(c) If we reduce the support friction a little, as in Figure 12.17(c), the box will slide right.

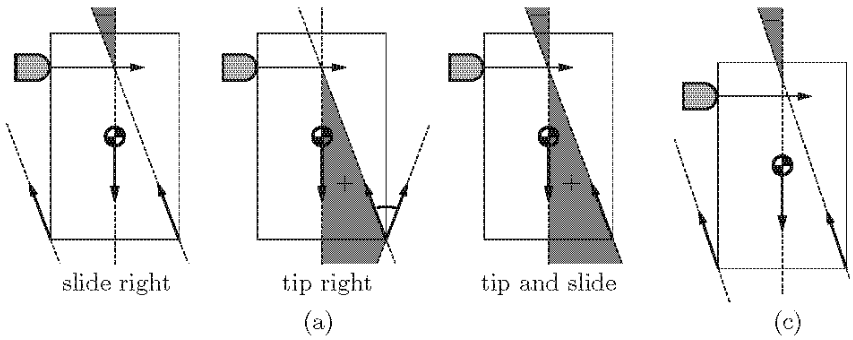

Figure 12.17

Exercise 12.24. The contact wrenches are labelled as in Figure 12.18.

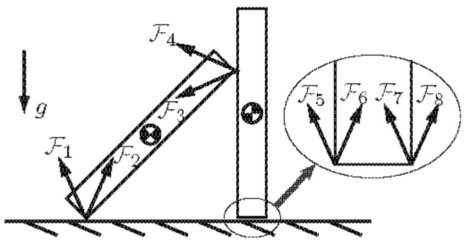

Figure 12.18

And we have

$$
\begin{aligned}
\mathcal{F}_1 &= (0, -\mu, 1)^T \\
\mathcal{F}_2 &= (0, \mu, 1)^T \\
\mathcal{F}_3 &= (y - \mu x_L, -1, -\mu)^T \\
\mathcal{F}_4 &= (y + \mu x_L, -1, \mu)^T \\
\mathcal{F}_5 &= (x_L + \mu, -\mu, 1)^T \\
\mathcal{F}_6 &= (x_L - \mu, \mu, 1)^T \\
\mathcal{F}_7 &= (x_R + \mu, -\mu, 1)^T \\
\mathcal{F}_8 &= (x_R - \mu, \mu, 1)^T \\
\mathcal{F}_{g1} &= (-m_1 g x_1, 0, -m_1 g)^T \\
\mathcal{F}_{g2} &= (-m_2 g x_2, 0, -m_2 g)^T
\end{aligned}
$$

The conditions to stay standing are

$$k_1\mathcal{F}_1 + k_2\mathcal{F}_2 + k_3\mathcal{F}_3 + k_4\mathcal{F}_4 + k_5\mathcal{F}_5 + k_6\mathcal{F}_6 + k_7\mathcal{F}_7 + k_8\mathcal{F}_8 + \mathcal{F}_{g1} + \mathcal{F}_{g2} = 0$$

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where $k_i \geq 0$. There are 3 equations and 8 unknowns.

Exercise 12.25.
Programming assignment.

Exercise 12.26.
Programming assignment.

Exercise 12.27.
Programming assignment.

Exercise 12.28.
Programming assignment.

Exercise 12.29.
Programming assignment.

Exercise 12.30.
Programming assignment.

Exercise 12.31.
Programming assignment.

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