Chapter 13 Solutions

Exercise 13.1.
For robots with greater than three wheels, the constraints from the wheels mean that slipping will occur in general if each wheel velocity is specified arbitrarily, so it makes sense to start with the chassis velocity then determine the consistent wheel velocities. This modeling approach does not work if the rank of the matrix $H(0)$ is not full (rank three). The matrix $H(0)$ depends on the wheel locations and driving directions.

Exercise 13.2.
The kinematic modeling may be unintuitive because it does not consider forces or accelerations. Under the kinematic velocity modeling, each extra wheel only provides motion constraints (in terms of its max velocity), not extra forces or torques that increase the acceleration of the robot.

Exercise 13.3.
No it is not possible to drive the wheels so that they skid. The matrix $H(0)$ is invertible, and therefore there is a 1:1 mapping from the wheel velocities to the motion of the robot.

Exercise 13.4.
Yes it is possible for the wheels to slip. Any choice of u that does not have a valid solution $\mathcal{V}_b$ means slipping is occurring. By choosing $u_1 = u_2 = u_3 = 1$, we can solve for the unique values of ω_{bz}, ν_{bx}, and ν_{by} which are $\{0, r, 0\}$. Multiplying $H(0)\mathcal{V}_b$ gives $u_4 = 1$. Therefore if $u_4 \neq 1$, slipping occurs.

Exercise 13.5.
Assuming $r = l = w = 1$,

$$H(0) = \begin{pmatrix} -1-\sqrt{3} & 1 & -\sqrt{3} \\ 1+\sqrt{3} & 1 & \sqrt{3} \\ 1+\sqrt{3} & 1 & -\sqrt{3} \\ -1-\sqrt{3} & 1 & \sqrt{3} \end{pmatrix},$$

which is rank three.

Exercise 13.6.
If the three-omniwheel robot has its wheels replaced with 45° mecanum wheels,

$$H(0) = \begin{pmatrix} -\frac{d}{r_i} & \frac{1}{r_i} & \frac{1}{r_i} \\ -\frac{d}{r_i} & \frac{-1+\sqrt{3}}{2r_i} & -\frac{1+\sqrt{3}}{2r_i} \\ -\frac{d}{r_i} & -\frac{1+\sqrt{3}}{2r_i} & \frac{-1+\sqrt{3}}{2r_i} \end{pmatrix}$$

which is rank three, so it is still a properly constructed omnidirectional mobile robot.

Exercise 13.7.
If the triangular, three-omniwheel robot in Exercise 6 has wheel one replaced with a $-45°$ mecanum wheel and wheels two and three replaced with 45° mecanum wheels aligned with the $\hat{x}_b$ axis,

$$H(0) = \begin{pmatrix} -\frac{d}{r_i} & \frac{1}{r_i} & -\frac{1}{r_i} \\ \frac{(1+\sqrt{3})d}{2r_i} & \frac{1}{r_i} & \frac{1}{r_i} \\ -\frac{(-1+\sqrt{3})d}{2r_i} & \frac{1}{r_i} & \frac{1}{r_i} \end{pmatrix}$$

which is rank three. Therefore it is a properly constructed omnidirectional mobile robot.

Exercise 13.8.
Programming assignment.

Exercise 13.9.

Programming assignment.

Exercise 13.10.
For the described robot,

$$H(0) = \begin{pmatrix} \sqrt{2} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \sqrt{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \sqrt{2} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \sqrt{2} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix},$$

which is rank three. Therefore it is a properly constructed omnidirectional mobile robot.

Exercise 13.11.
Programming assignment.

Exercise 13.12.
Programming assignment.

Exercise 13.13.
Programming assignment.

Exercise 13.14.
For the unicycle problem the wheel cannot slip perpendicular to the velocity direction which results in the Pfaffian constraint,

$$A(q)\dot{q} = \begin{bmatrix} 0 & \sin(\phi) & -\cos(\phi) & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$$

Can also assume that there is no slipping in the x or y directions at the contact between the wheel and the ground leading to the two constraints: $\dot{x} - r\cos\phi\dot{\theta} = 0$, and $\dot{y} - r\sin\phi\dot{\theta} = 0$

which can be expressed as, $A(q)\dot{q} = \begin{bmatrix} 0 & 1 & 0 & -r\cos\phi \\ 0 & 0 & 1 & -r\sin\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$

Exercise 13.15.
For the differential drive robot, the constraints that there is no slipping in the x or y direction at each wheel are given by: $A(q)\dot{q} = \begin{bmatrix} -d\cos\phi & 1 & 0 & -r\cos\phi & 0 \\ -d\sin\phi & 0 & 1 & -r\sin\phi & 0 \\ d\cos\phi & 1 & 0 & 0 & -r\cos\phi \\ d\sin\phi & 0 & 1 & 0 & -r\cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\theta}_{\mathrm{L}} \\ \dot{\theta}_{\mathrm{R}} \end{bmatrix} = 0$

Exercise 13.16.
For a car we have the Pfaffian constraints for no slipping at the back wheel, and no slipping at the virtual wheel located between the front two wheels,

$$A(q)\dot{q} = \begin{bmatrix} -l\cos\psi & \sin(\phi+\psi) & -\cos(\phi+\psi) & 0 \\ 0 & \sin(\phi) & -\cos(\phi) & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = 0$$

Exercise 13.17.
A space shuttle with two rear thrusters is STLA. A robot arm with no joint limits, but with motors that

can only move in one direction is also STLA.

Exercise 13.18.

$$\begin{aligned}
q(3\epsilon) = F_\epsilon^{-g_i}(q(2\epsilon)) &= q(2\epsilon) - \epsilon g_i(q(2\epsilon)) + \frac{1}{2}\epsilon^2 \frac{\partial g_i}{\partial q} g_i(q(2\epsilon)) + O(\epsilon^3) \\
&= q(0) + \epsilon g_i(q(0)) + \epsilon g_j(q(0)) + \epsilon^2 \frac{\partial g_j}{\partial q} g_i(q(0)) + \frac{1}{2}\epsilon^2 \frac{\partial g_i}{\partial q} g_i(q(0)) + \frac{1}{2}\epsilon^2 \frac{\partial g_j}{\partial q} g_j(q(0)) \\
&\quad - \epsilon g_i(q(0)) - \epsilon^2 \frac{\partial g_i}{\partial q} g_i(q(0)) - \epsilon^2 \frac{\partial g_i}{\partial q} g_j(q(0)) + \frac{1}{2}\epsilon^2 \frac{\partial g_i}{\partial q} g_i(q(0)) + O(\epsilon^3) \\
&= q(0) + \epsilon g_j(q(0)) + \epsilon^2 \frac{\partial g_j}{\partial q} g_i(q(0)) + \frac{1}{2}\epsilon^2 \frac{\partial g_j}{\partial q} g_j(q(0)) - \epsilon^2 \frac{\partial g_i}{\partial q} g_j(q(0)) + O(\epsilon^3)
\end{aligned}$$

$$\begin{aligned}
q(4\epsilon) = F_\epsilon^{-g_j}(q(3\epsilon)) &= q(3\epsilon) - \epsilon g_j(q(3\epsilon)) + \frac{1}{2}\epsilon^2 \frac{\partial g_j}{\partial q} g_j(q(3\epsilon)) + O(\epsilon^3) \\
&= q(0) + \epsilon g_j(q(0)) + \epsilon^2 \frac{\partial g_j}{\partial q} g_i(q(0)) + \frac{1}{2}\epsilon^2 \frac{\partial g_j}{\partial q} g_j(q(0)) - \epsilon^2 \frac{\partial g_i}{\partial q} g_j(q(0)) \\
&\quad - \epsilon g_j(q(0)) - \epsilon^2 \frac{\partial g_j}{\partial q} g_i(q(0)) - \epsilon^2 \frac{\partial g_j}{\partial q} g_j(q(0)) + \epsilon^2 \frac{\partial g_j}{\partial q} g_i(q(0)) + \frac{1}{2}\epsilon^2 \frac{\partial g_j}{\partial q} g_j(q(0)) + O(\epsilon^3) \\
&= q(0) + \epsilon^2 \left(\frac{\partial g_j}{\partial q} g_i(q(0)) - \frac{\partial g_i}{\partial q} g_j(q(0)) \right) + O(\epsilon^3)
\end{aligned}$$

Exercise 13.19.

If we draw the body frame $\{b\}$ as in Figure 13.1, then $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$.

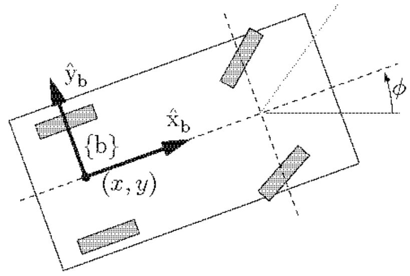

Figure 13.1

Exercise 13.20.

We have $A_1 = (0, \cos\phi \sin\phi)^T$ and $A_2 = (1, 0, 0)^T$. So $A_3 = [A_1, A_2] = \begin{bmatrix} [\omega_1] & 0 \\ [v_1] & [\omega_1] \end{bmatrix} \begin{bmatrix} \omega_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\phi \\ -\cos\phi \end{bmatrix}$.

Exercise 13.21.
Programming assignment.

Exercise 13.22.
We have $g_1 = (-\frac{r}{2d}, \frac{r}{2}\cos\phi, \frac{r}{2}\sin\phi, 1, 0)^T$ and $g_2 = (\frac{r}{2d}, \frac{r}{2}\cos\phi, \frac{r}{2}\sin\phi, 0, 1)^T$.
Then $g_3 = [g_1, g_2] = (0, \frac{r^2}{2d}\sin\phi, -\frac{r^2}{2d}\cos\phi, 0, 0)^T$, and $g_4 = [g_1, g_3] = (0, -\frac{r^4}{4d^2}\cos\phi, -\frac{r^4}{4d^2}\sin\phi, 0, 0)^T$.

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The Pfaffian constraint is $A(q)\dot{q} = (0, \sin\phi, \cos\phi, 0, 0)\dot{q} = 0$.

Exercise 13.23.
Programming assignment.

Exercise 13.24.
Programming assignment.

Exercise 13.25.
Programming assignment.

Exercise 13.26.
Programming assignment.

Exercise 13.27.
Programming assignment.

Exercise 13.28.
Programming assignment.

Exercise 13.29.
Programming assignment.

Exercise 13.30.

$$\begin{aligned}
\mathcal{V}_e &= J_e(\theta_1)\begin{bmatrix} u_L \\ u_R \\ \dot{\theta}_1 \end{bmatrix} = [J_{base} J_{arm}] \begin{bmatrix} u_L \\ u_R \\ \dot{\theta}_1 \end{bmatrix}, \\
J_{base} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1+\phi) & \sin(\theta_1+\phi) \\ h & -\sin(\theta_1+\phi) & \cos(\theta_1+\phi) \end{bmatrix} \begin{bmatrix} -\frac{r}{2d} & \frac{r}{2d} \\ \frac{r}{2}\cos\phi & \frac{r}{2}\cos\phi \\ \frac{r}{2}\sin\phi & \frac{r}{2}\sin\phi \end{bmatrix} \\
&= \begin{bmatrix} -\frac{r}{2d} & \frac{r}{2d} \\ \frac{r}{2}\cos\theta_1 & \frac{r}{2}\cos\theta_1 \\ -\frac{r(h+d\sin\theta_1)}{2d} & \frac{r(h-d\sin\theta_1)}{2d} \end{bmatrix}, \\
J_{arm} &= (1, 0, 0)^T,
\end{aligned}$$

where $h = \sqrt{L_1^2 + x_r^2 + 2x_r L_1 \cos(\theta_1)}$. And $\det(J_e) = \frac{hr^2\cos\theta_1}{2d}$. So J_e is not full rank when $x_r = 0,\ L_1 = 0$ or $\theta_1 = \pi/2 + k\pi,\ k \in \mathbb{Z}$.

Exercise 13.31.
Programming assignment.

Exercise 13.32.
Programming assignment.

Exercise 13.33.
Programming assignment.

Exercise 13.34.
Programming assignment.