Charles A. Klein
Bruce E. Blaho
Department of Electrical Engineering
The Ohio State University
Columbus, Ohio 43210

Dexterity Measures for the Design and Control of Kinematically Redundant Manipulators

Abstract

In this paper, we have proposed a number of measures for the quantification of dexterity of manipulators. The use of such measures is especially important for kinematically redundant manipulators since they can satisfy secondary criteria in addition to satisfying a specification of end-effector motion. We will compare several measures for the problems of finding an optimal configuration for a given end-effector position, finding an optimal workpoint, and designing the optimal link lengths of an arm.

1. Introduction

Kinematically redundant manipulators, those with more than the minimum number of degrees of freedom, are currently being studied by a number of investigators because of their potential to optimize secondary while also satisfying the primary specification of end-effector trajectory tracking. A seven-degree-of-freedom manipulator would be redundant because the usual specification of end-effector position and orientation requires six degrees of freedom. However, for tasks requiring fewer degrees of freedom, a manipulator with fewer joints would be redundant. Secondary criteria that can be optimized over the extra degrees of freedom include obstacle avoidance (Maciejewski and Klein 1985), energy minimization, keeping joints within their mechanical limits (Liegeois 1977), and increasing dynamic response (Trevelyan, Kovesi, and Ong 1984; Hollerbach and Suh 1985; Salisbury and Abramowitz 1985).

An additional consideration for the use of the redundancy of a robotic system is to make the system as dexterous as possible. Dexterity is a desirable goal for almost all robotic systems, whether or not they are redundant. While the intuitive motivation of the term *dexterity* applied to a robotic system is clear, it has been interpreted to mean different physical concepts. One concept of dexterity is a measure of the kinematic extent over which a manipulator can reach all orientations, such as in the description of dexterous workspaces (Gupta and Roth 1982; Vijaykumar, Tsai, and Waldron 1985). This concept would be useful in specifying a manipulator when a task description involves relatively large dimensions.

Another concept of dexterity involves the operation of manipulators with multiple, actively powered fingers (Salisbury and Roth 1983; Jacobsen et al. 1985) and thus measures the "goodness" of grasping motion (Kobayashi 1985). The same concept is also needed in evaluating the best locations for supports in active jigging (Asada and By 1985). Dexterity has also been interpreted as a specification of the dynamic response of a manipulator (Yoshikawa 1985*a*). Asada (1983) has described the design of an arm so that the moment of inertia matrix is as uniform as possible. Another consideration of dexterity involves a global measure

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applied over a complete end-effector trajectory. Uchiyama, Shimizu, and Hakomori (1985) have planned trajectories that minimize the integral of the reciprocal of the determinant of the Jacobian matrix.

Each of these concepts of dexterity has validity for a measure of the performance of a manipulator doing a certain type of task. The concept of dexterity that we will consider in this paper is an attempt to measure kinematic performance of fine end-effector motion in a local sense. We feel that this concept is most applicable for designing and controlling manipulators that perform accurate motion at a relatively slow speed. Because of the slow, accurate motion at the workpoint, dynamic effects are minimized. Also, in some dynamic formulations, the kinematic dexterity problem appears to be a separable part (Yoshikawa 1985*a*). Since accurate hand motion at the workpiece is much more important than accuracy along the trajectory leading there, local kinematic criteria are more important for this situation than global ones. Within the concept of local kinematic dexterity, a number of quantitative measures have been proposed and will be examined in this paper.

The redundancy of the system will be treated through pseudoinverse techniques applied to the Jacobian control method (Whitney 1972; Paul 1981). In this formulation, the end effector and joint velocities are related by

$$\mathbf{J}\dot{\boldsymbol{\theta}} = \dot{\mathbf{x}}, \tag{1}$$

where $\mathbf{J}$ is the Jacobian matrix. For a redundant system, a general solution for $\dot{\boldsymbol{\theta}}$ is given by

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^{+}\dot{\mathbf{x}} + (\mathbf{I} - \mathbf{J}^{+}\mathbf{J})\mathbf{z}, \tag{2}$$

where the plus superscript indicates the pseudoinverse and $\mathbf{z}$ is an arbitrary vector (Rao and Mitra 1971; Nakamura and Hanafusa 1985). The first term of Eq. (2) represents the *minimum-norm solution* to Eq. (1). The second term, which can be used to optimize secondary criteria, will be called the *homogeneous solution,* since it represents a homogeneous solution to Eq. (1).

In Section 2, we will describe a number of measures that have been proposed to describe dexterity. In Section 3, we will examine how these different measures can determine an optimal solution for the joint configuration of a given end-effector position. Since the designer may be able to choose the position of the workpiece within the reach of the arm, Section 4 will show how the dexterity measures can be used to evaluate the choice of a workpoint. Assuming the lengths are adjustable, dexterity measures can be used in arm design, as presented in Section 5.

2. Dexterity Measures

Even for the sense of dexterity as local kinematic accuracy, a number of mathematical measures have been proposed for quantification (Klein 1985). The four measures to be examined here are the determinant, condition number, minimum singular value, and joint range availability.

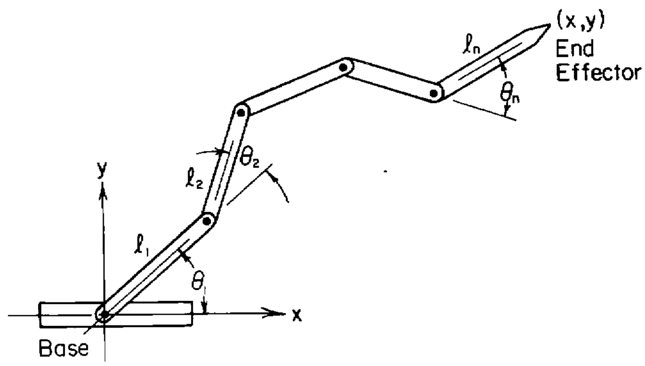
The first measure to be considered is the *determinant.* For nonredundant manipulators, the determinant has been used to evaluate wrist configurations (Paul and Stevenson 1983). For redundant manipulators, Yoshikawa (1984) has extended the definition of a determinant to nonsquare matrices by using the square root of the determinant of $\mathbf{J}\mathbf{J}^T$, which he terms *manipulability.* By the singular value decomposition representation (Klema and Laub 1980), it can be seen that for both square and nonsquare matrices of full rank,

$$\det(\mathbf{J}) = \mu_1 \quad \mu_2 \ . \ . \ . \ \mu_r. \tag{3}$$

For many arm geometries, the determinant can be explicitly written as a function of joint angles, and therefore the gradient of this measure can also be explicitly calculated and used as the vector $\mathbf{z}$ in Eq. (2).

While a determinant going to zero marks the presence of a singularity, the actual value of the determinant cannot be used as a practical measure of the degree of ill-conditioning (Forsythe and Moler 1967). Instead, the matrix *condition number* has been recommended by numerical analysts because of its quantitative accuracy estimates (Isaacson and Keller 1966). When the pseudoinverse is used for the solution of linear systems, the expression for the Euclidean condition number can be extended logically to nonsquare

Fig. 1. Planar revolute manipulator with relative joint angle actuators.



matrices as

$$\mathrm{cond}_2\,(\mathbf{J}) = \mu_1/\mu_r, \tag{4}$$

where μ_1 and μ_r are the largest and smallest singular values, respectively. Thus the condition number indicates the uniformity of the Jacobian transformation with respect to direction. When the condition number has an optimal value of 1, the configuration has been termed *isotropic* (Salisbury and Craig 1982).

Examining both the determinant and the condition number in terms of their singular values shows both measures' critical dependence on the *minimum singular value.* The minimum singular value changes more radically near singularities than the other singular values, thus dominating the behavior of the condition number and the determinant. This trend suggests that the minimum singular value itself can be used as a measure of dexterity. The minimum singular value forms the norm of the pseudoinverse part of Eq. (2) and can be used to put an upper bound on required joint velocities.

$$\|\dot{\boldsymbol{\theta}}\| \leq (1/\mu_r)\|\dot{\mathbf{x}}\| \tag{5}$$

Thus the minimum singular value directly shows situations where excessive joint velocities would be required, which is a characteristic of nearness to singularities.

A fourth dexterity measure is *joint range availability,* a measure used by Liegeois (1977) for automatically reconfiguring redundant arms to minimize the possibility that a joint will reach a stop. Quantitatively for the Euclidean norm,

$$\mathrm{JRAE} = \sum (\theta_i - \theta_{ci})^2, \tag{6}$$

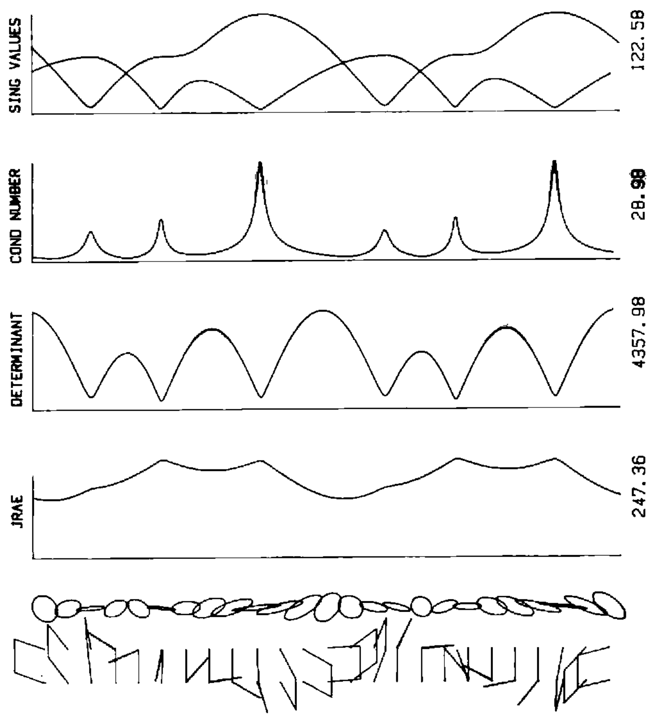
where θ_{ci} is the center of the range of travel. Of course, angles can be scaled to adjust to different ranges of travel. The use of the infinite or maximum norm instead of a Euclidean norm has also been examined (Klein and Huang 1983). By properly setting the center angle, this measure can be used to judge the "naturalness" or evenness of a joint-angle distribution. Using either norm, this measure is advantageous computationally since its gradient can be computed easily and used as the vector **z** in Eq. (2). Baillieul (1985) discussed an extended Jacobian approach where a condition using the gradient of a desired secondary criterion is included as an additional row in the Jacobian matrix. When the gradient of Eq. (6) is used with a gain of .5 in Eq. (2), the solution will have the same conservative property (Klein and Chirco, in press).

3. Optimal Posture for a Redundant Arm

Some simulations were performed to compare the effects of using the four different dexterity measures for arm control. For most of the experiments, a simple planar manipulator with three links and three revolute joints was used. (See Fig. 1.) The details of the Jacobian formulation can be found in Klein and Huang (1983). While the results presented here are extendable to more general robotic systems, the study of planar manipulators themselves is a topic of increasing interest, especially for use in mechanical fingers and hands (Salisbury and Craig 1982; Jacobsen et al. 1985). For these applications, precise manipulations are desired, so manipulator dexterity is a primary consideration.

The first question to be answered for controlling a redundant manipulator is to determine the optimal joint angle distribution for a given end-effector position. This *posture-control* question can be examined by starting with an arbitrary set of joint angles yielding a given position and then adding homogeneous joint-velocity solutions to generate new joint-angle solu-

Fig. 2. Dexterity measures plotted along the homogeneous solution path for a three-link manipulator with equal-length links. This graph shows the variation of the measures over the entire range of possible postures. The number to the right of each plot is the maximum value of that measure. The end-effector reach is 1.01 times the link length from the base.



tions. The advantage of the three-link, planar problem is that the homogeneous solutions have only one degree of freedom and can be calculated simply as the cross product of the two rows of the Jacobian.

In Klein (1985), figures show a visualization of homogeneous solutions in joint space. For the equal length case, where the length of each link is ℓ, when the reach is between ℓ and 3ℓ, the locus of possible joint configurations forms a closed loop in joint space. As the reach approaches 3ℓ, the loop becomes smaller until it is a single point at 3ℓ. For reaches between 0 and ℓ, the loci form spirals that continue indefinitely in the θ_1 direction.

To accurately trace the locus of joint angles for a given end-effector position, a variation on Jacobian control has been used (Klein and Huang 1983). Suppose the desired end-effector position is (x_c, y_c), but at the current time it is (x_a, y_a). Let $\mathbf{h}$ be the cross product of the two rows of $\mathbf{J}$, which is then normalized to unit length. The vector is in the direction of the null space, since it is orthogonal to the row space. Conceptually one could generate the locus of joint angles by successively adding $\epsilon\mathbf{h}$ to the present joint angle where ϵ is the desired increment size, but eventually the end-effector position would drift due to approximation error. A better numerical procedure is to apply Gaussian elimination to the following square system:

$$\left[\frac{\mathbf{J}}{\mathbf{h}^T}\right]\Delta\theta = \begin{bmatrix} x_c - x_a \\ y_c - y_a \\ \epsilon \end{bmatrix}. \tag{7}$$

This system simultaneously finds a change in joint angles that moves a distance ϵ along the locus of possible joint angles and corrects any small amount of drift. The procedure starts with an initial set of angles placing the end effector at (x_c, y_c) and stops when either the path returns to the starting point or when any angle has changed by 2π.

Figure 2 shows the dexterity measures plotted as a function of displacement around the path of possible joint angles for a fixed end-effector position. Both singular values are plotted, together with the ellipse they form and a drawing of the manipulator configuration. In the calculation of the JRAE measure, Eq. 6, zero has been used as the center angle θ_c and the joint angles have been considered to be in the range $-\pi$ to $+\pi$. This modularity should not be applied in summing squares of joint angles for a manipulator joint with a limited range over 2π radians. The relative minimums of the present curves do show local behavior of the JRAE measure when the joint angles are in the principle range of the joints' motion.

In Fig. 2, the reach is slightly beyond one link length. At this reach, some joint configurations make the arm's condition number nearly equal to the optimal value of 1, and some configurations are nearly singular. The singular value curves appear to cross. However, a detailed study (Blaho 1985) shows that the two singular value curves actually reach extrema and turn away from each other. A consequence is that the condition number is not exactly 1 unless the reach is exactly one link length. The curves do verify the claim made earlier that the variation in the minimum singular value is much more dramatic than the variation in the maximum singular value.

Examining the postures for both the optimal and worst values of the various dexterity measures shows that both the determinant and condition number indi-

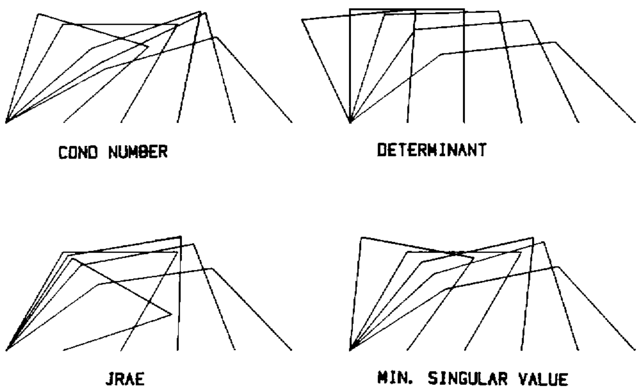


Fig. 3. Optimal postures using four different dexterity measures for an equal-length, three-link manipulator operating at reaches of 0.5, 1.0, 1.5, 2.0, and 2.5 times the link length from the base.

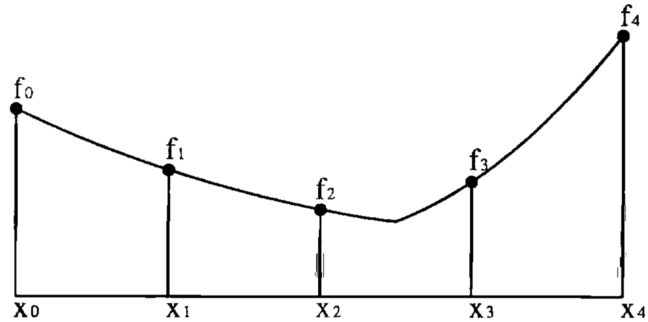


Fig. 4. Illustration of method of robust, one-dimensional minimization technique using no derivatives.

cate where the system approaches a singularity but that the dexterity measures disagree on the best posture. To examine more carefully the differences between the optimal postures as determined by different dexterity measures, an optimization procedure was applied. Figure 3 shows the resulting optimal postures for a discrete number of reaches. The optimization can be considered an optimization of a dexterity measure along the one-dimensional curve of joint-angle solution in joint space.

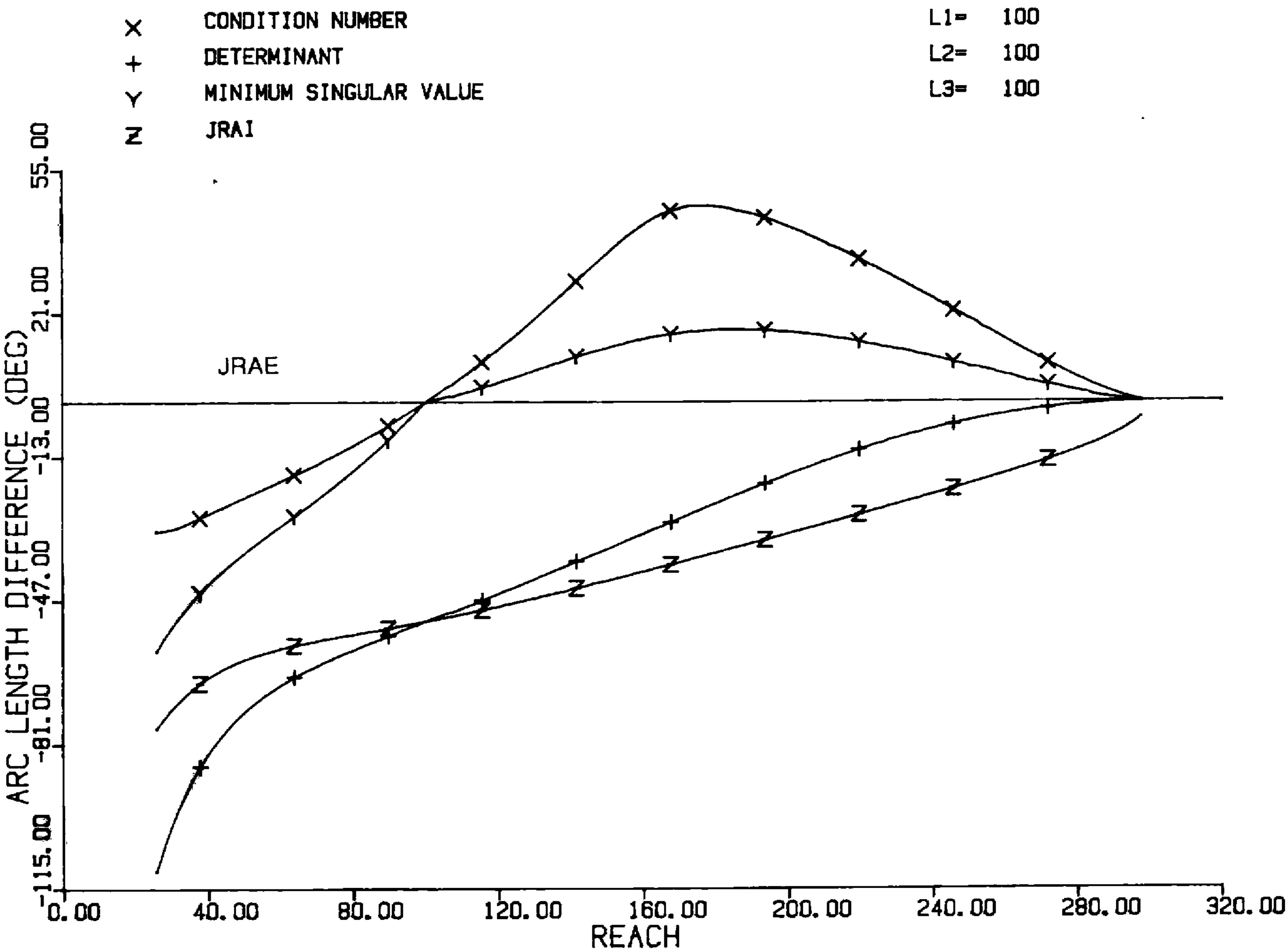
It is desirable to be able to compare quantitatively the differences in postures. One possibility would be to calculate the norm of joint-angle differences. However, this would require comparing the optimal posture produced by each measure against the posture produced by every other measure, since the norms correspond to direct distances between points in joint space. A better method for comparison is to compare the difference in arc length along the one-degree-of-freedom, homogeneous solution path for the different postures. Since the arc length is a one-dimensional quantity, a single graph can be used to show how the posture produced by each dexterity measure differs from the postures of all other measures.

To calculate the difference in postures as a function of reach, a careful numerical procedure must be followed to avoid difficulties (Blaho 1985). To calculate arc-length differences, a reference point is needed. The point on the homogeneous solution curve that minimizes the JRAE measure has been arbitrarily chosen as the reference because of its smooth behavior with reach. Although the optimization is a one-dimensional search along the path, since not all the measures are smooth, a five-point subdivision scheme must be used in place of methods using derivatives (Klein 1984). To generate curves as a function of reach, a small change is made in the x-coordinate of the end-effector location and the procedure is repeated. Extrema on previous reaches must be remembered in order to avoid jumping to other local extrema. An original reach should be well within the possible limits, and the joint difference curves must be extended both inward and outward from this point to insure that the same local extrema are followed as a function of distance. For reaches both close to the base and at maximum distance, the variations in measures are ill-conditioned and represent poor starting points. More detailed studies show that compared with the total path arc length, the optimal postures found are relatively close and that the arc-length differences are therefore a valid measure of posture variances (Blaho 1985).

Figure 4 illustrates the principles of the five-point subdivision scheme, which is a one-dimensional, minimization procedure using no derivatives. Suppose that stepping along the one-dimensional path shows that the minimum is in a given region x_0–x_4. Calculate the function at points x_1, x_2, and x_3. Either f_1, f_2, or f_3 will be the minimum value, in which case x_0–x_2, x_1–x_3, or x_2–x_4, respectively, will be set to be the new internal x_0–x_4. In each case, the interval containing the minimum has been halved, and thus the procedure bears a resemblance to the binary chop algorithm for the solution of nonlinear equations.

Examining the differences in optimal postures in Fig. 5, where the links are equal length, shows a number of interesting trends. At a reach one link away, the condition number, minimum singular value, and JRAE measure recommend the same posture while

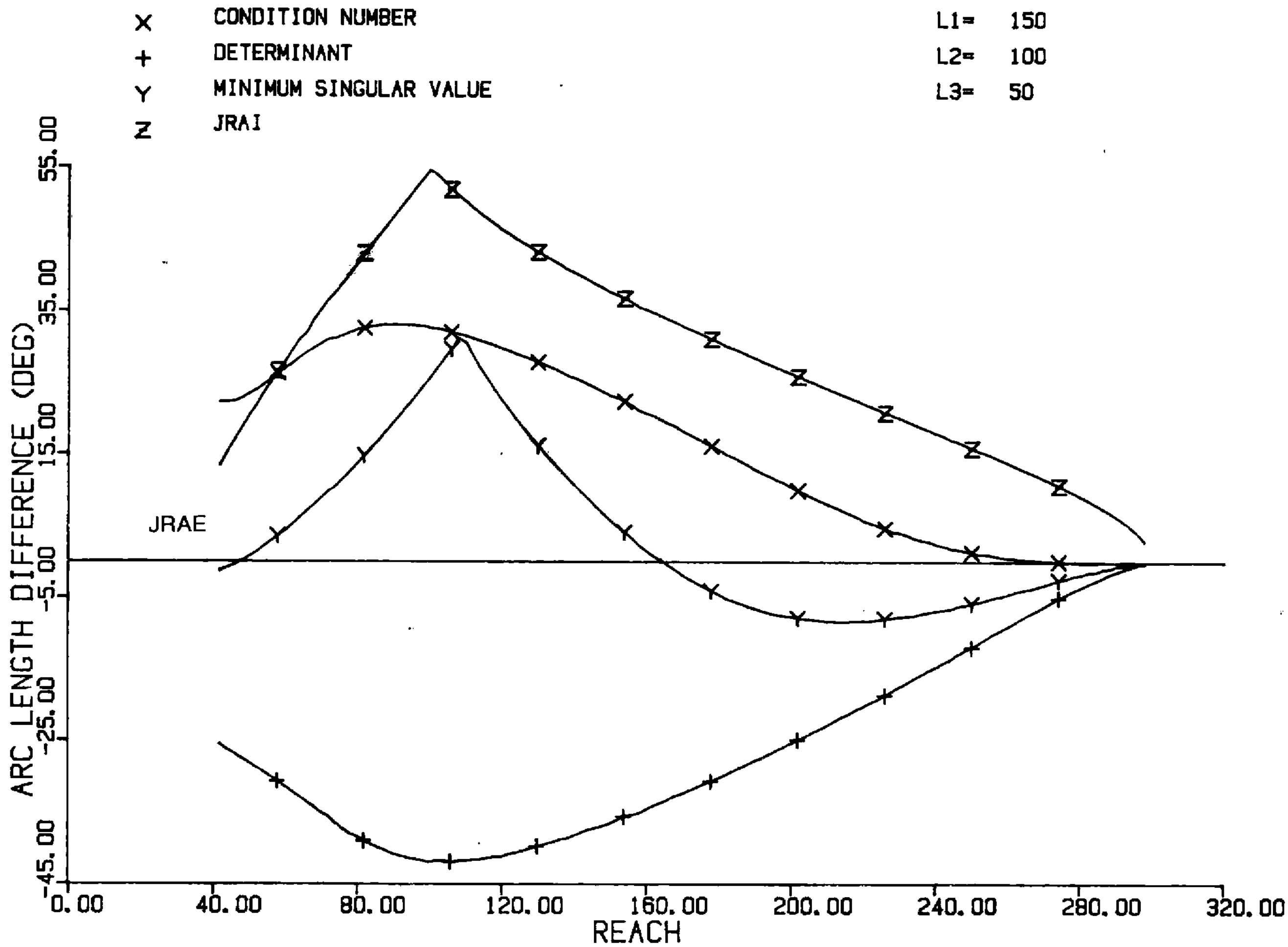
Fig. 5. Differences in postures optimized by different dexterity measures as a function of end-effector location from the base. Posture differences are measured by the arc length along the homogeneous solution path from a reference point. The reference point for each reach, shown as the solid line, is the one with the minimum JRAE measure.



the determinant recommends a different posture. As pointed out by Yoshikawa (1985*b*), the posture produced by the determinant has a uniform joint distribution. The concept can be quantified by using the infinite vector norm on the joint range availability (Klein and Huang 1983). We define this new measure as JRAI. When this measure is minimized, the maximum angles decrease, forcing the smaller angles to increase, and thus the angles become more uniform. Comparing this curve of minimax joint-angle postures to the determinant curve shows that the curves are relatively close and that they exactly meet at a reach of one link.

However, when one examines the results for an unequal link length case, many of the trends shown in the equal length case disappear (see Fig. 6). There still is a reach where the condition number and the minimum singular value curves meet that represents an isotropic condition. However, this posture does not have a minimum JRAE measure. An even more striking difference from the equal length case is that the infinite norm (JRAI) and the determinant curves are now the farthest apart and the condition number curve yields a posture closer to the minimax angle case. For some other link lengths there will not even be an isotropic case, and the minimum singular value curve will not reach an extreme posture at the condition number curve (Blaho 1985).

Fig. 6. Differences in postures optimized by different dexterity measures as a function of end-effector location from the base. Note that for the unequal length geometry of this manipulator many of the trends for the equal-length manipulator are no longer present.



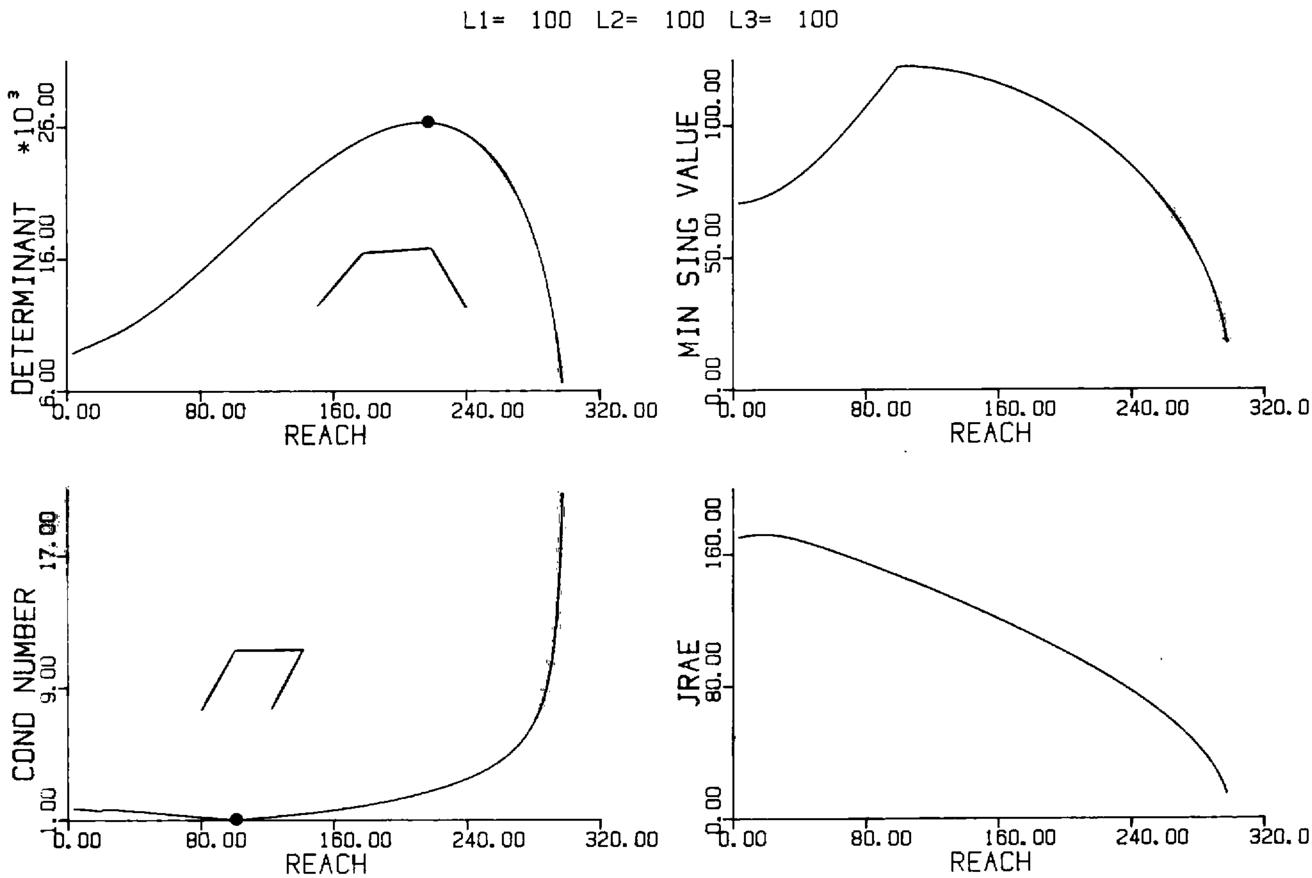
4. Optimal Working Point

In the last section, optimal postures or joint configurations for a given end-effector reach were examined based on different dexterity measures. For situations where a manipulator does small-range, accurate motion on a workpiece, the engineer may have a choice where the workpiece should be located in the arm's workspace. This section will examine the use of the described dexterity measures to compare the resulting optimal workpoints.

In order to compare the dexterity of one workpoint to another, the dexterities of the optimal postures must be compared. This information is easily generated by the posture optimization procedure described in Section 3. Figure 7 shows results for the four dexterity measures plotted against the end-effector position on the x-axis.

By noting the best value obtained for each of the measures over the range of operating points, the best working point for a given manipulator design can be ascertained. It can be seen immediately that the JRAE measure is of no use in choosing an optimal operating point. Since any vector norm is an indication of the size of the elements in the vector, the optimal operating point will always correspond to the point with all joint angles at their designated zero positions. For the case at hand, this corresponds to a fully extended manipulator. Since this position is arbitrarily chosen, it

Fig. 7. Optimal values of dexterity measures plotted against end-effector location for an equal-length manipulator. At each reach, the measure has been optimized over posture. Insets show appearances of the resulting posture at the optimal reach for the determinant and condition numbers.



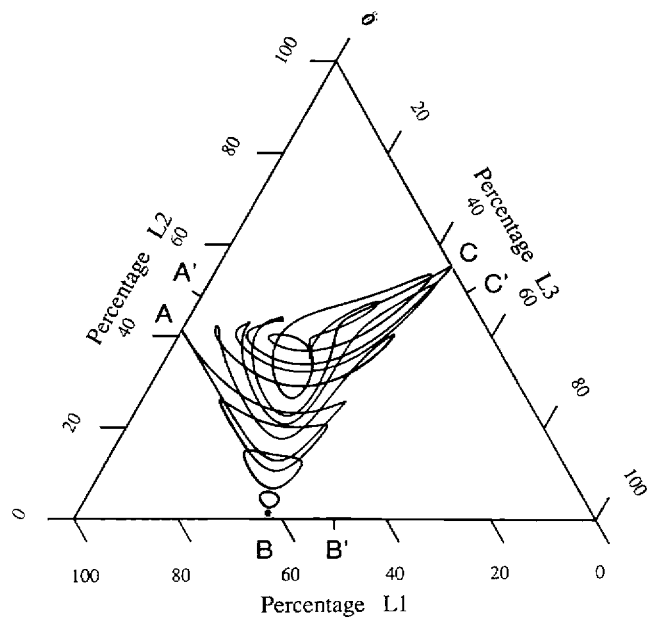
furnishes no new information about which postures are best. Using a different joint-center angle specification would similarly choose the corresponding position as optimal.

Comparing the optimal workpoints for the other three measures in Fig. 7 shows that in reach, as well as in posture, the optimal condition number and minimum singular value agree at the isotropic configuration. All three curves vary relatively slowly except near the point of maximum reach, which is shown as a singularity by all three measures. The slowly varying nature of these measures means that dexterity is not a resonant function of reach and that the designer has a high degree of flexibility.

Perhaps the most interesting result is that the condition number and minimum singular value recommend a substantially different working point than the determinant. Figure 7 includes a comparison of the configurations and working points selected.

When examining results for different arm geometries, fairly similar trends develop. When an isotropic situation is possible, the minimum singular value peaks at this situation. When an isotropic condition is not possible, the minimum singular value curves have a broad maximum that is slightly displaced from the condition number optimum. If a given arm cannot reach its base, then, of course, a singularity at the minimum reach is apparent.

Fig. 8. *Loci of link lengths that can assume an isotropic configuration plotted in a chart of relative link lengths. Each curve corresponds to a fixed end-effector position.*



5. Manipulator Design

In Sections 3 and 4, we discussed the use of dexterity measures to determine optimal postures and workpoints for fixed manipulator designs. In this section, we will consider the link lengths as variable so that the manipulator design can be optimized. In the preceding sections, certain manipulator designs had isotropic configurations: i.e., where the condition number was the optimal value of 1. A more specific design issue to be considered here is how to generate redundant manipulator designs that can assume isotropic configurations. We consider only the condition number for several reasons. First, the optimal condition number is a fixed number 1 with a quantitative interpretation. Secondly, as will be shown, a simple numerical procedure can be employed that uses this fact in optimizing manipulator designs.

To generate isotropic configurations numerically, a more tractable formulation for the condition number being 1 is needed. For a nonredundant manipulator, the Jacobian is square and for the system to be isotropic, the rows or alternatively the columns, must have the same vector magnitude and be orthogonal (Salisbury and Craig 1982). The above condition on the columns can be interpreted simply as the end effector being an equal distance from the joint axes, and the incremental rectilinear motions of the end effector caused by each joint must be orthogonal to the motion caused by the other joints.

For a redundant manipulator, isotropic configurations are not as easy to visualize geometrically. For the two-dimensional, three-link case, the Jacobian is 2×3; it is easily shown that the rows, but not necessarily the columns, must be equal length and orthogonal. This property can be used in a variation of Jacobian control to trace curves of manipulator designs in isotropic configurations. In this scheme, the link lengths as well as the joint angles are considered independent degrees of freedom. Define the following new functions:

$$P = \mathbf{r}_1^T\mathbf{r}_2, \tag{8}$$

$$M = \mathbf{r}_1^T\mathbf{r}_1 - \mathbf{r}_2^T\mathbf{r}_2, \tag{9}$$

$$L = \ell_1 + \ell_2 + \ell_3, \tag{10}$$

where $\mathbf{r}_1^T$ and $\mathbf{r}_2^T$ are the two rows of the original 2×3 Jacobian $\mathbf{J}$, which relates rectilinear motion to changes in joint angles, and ℓ_1, ℓ_2, ℓ_3 are the link lengths. The three functions are used to constrain the orthogonality and equal magnitude of the original rows (hence the isotropy of the system) and to maintain a constant total arm length. Applying the three functions in Jacobian control yields the following system:

$$\begin{bmatrix} \nabla^T x \\ \nabla^T y \\ \nabla^T P \\ \nabla^T M \\ \nabla^T L \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta\ell_1 \\ \Delta\ell_2 \\ \Delta\ell_3 \end{bmatrix} = \begin{bmatrix} x_c - x_a \\ y_c - y_a \\ 0 - P_a \\ 0 - M_a \\ L_c - L_a \end{bmatrix} \tag{11}$$

In the above matrix, the gradients are with respect to both angles and lengths. Details are given in Blaho (1985). $\nabla^T L$ is simply (0, 0, 0, 1, 1, 1).

Given a total arm length L_c and an end-effector position (x_c, y_c), Eq. (11) is a 5×6 system and has a one-degree-of-freedom solution space. This curve of possible manipulator designs that have an isotropic configuration at a fixed working point for a fixed total length can be generated by a technique similar to the one described in Section 3. Basically, in addition to the pseudoinverse solution to Eq. (1), a component in the null space that represents motion around the path of solutions is added. The null space for the first case is generated by projecting an arbitrary vector **z** into the null space by Eq. (2). However, on subsequent iterations, the previously determined null space direction is used as **z** in order to keep the system moving in a consistent direction around the path. As before, by having the right-hand side contain restoring terms, the possible small drift can be easily corrected. In the right-hand side of Eq. (11), the first term is the desired value and the second is the actual value calculated after the last iteration.

The results of these trajectories are shown in Fig. 8. Here, we used a trilinear chart to show the relative percentages of the three link lengths since the total length has been fixed (Hoelscher and Springer 1961). With increasing reach, the curves generally slide down this figure until the value of 51.8% of the total length is obtained, which corresponds to the single point B. Mathematically, the reach can be increased, but the numerical procedures generate negative lengths for some links. This can be simply reconsidered as the angle being 180 degrees different, but when the true total length is again normalized back to $\mathbf{L}_c$, the points are inside the area already traced.

The traced area of isotropic designs has three cases where a link goes to zero. However, only the case of the final link going to zero corresponds to a two-link manipulator. This case can be seen to have relative lengths $\sqrt{2}:1$, which was shown by Salisbury and Craig (1982) to be isotropic. The other two cases, links 1 or 2 being zero, are not true two-link designs since the zero length link does not remove the effect of the corresponding angle.

It can be noted that the area of isotropic designs is roughly but not exactly the area of designs for which the manipulator can reach its own base. The area of the figure that satisfies this last property is a triangle marked by the points A′, B′, and C′.

6. Summary and Conclusions

Several dexterity measures have been examined for optimizing posture for a given end-effector position, for optimizing the end-effector position, and for finding optimal manipulator designs using a particular dexterity measure. The different measures yield different results, yet to some extent it is not possible to conclude that one is superior to the others since the measures have slightly different purposes.

The determinant has the advantage that for some geometries it can be expressed analytically as a function of the joint angles and therefore its gradient can be used with a projection operator in real-time control. However, it also has a number of disadvantages compared to the other measures. If uniformity of joint angles for a given end-effector position is important, then it is best to use the JRA measures, with either the Euclidean or infinite norms, since this measure can be more easily calculated for a wider range of geometries. Also, except for certain special cases, the determinant does not yield the most uniform distribution.

The condition number, unlike the determinant, provides a quantitative interpretation of relative force or positional accuracy. Although it cannot be expressed analytically, this is not a problem in determining optimal postures or designing the workpiece position, which can be calculated in advance. Fortunately, the results of Fig. 7 show that the curves of determinant and condition number results are fairly broad and that a configuration optimized for one will have a reasonable value for the other.

The minimum singular value also has a quantitative interpretation that complements the condition number. This measure indicates magnitude of possible joint velocity response under pseudoinverse control, as compared to relative magnitudes of velocities. Although in many cases maximizing the minimum singular value gives the same configuration as minimizing the condition number, in other problems—particularly those with more degrees of freedom, such as the manipulator design problem described in Section 5—the minimum singular value could be used to choose between cases with equal optimal condition numbers.

In Section 3 it was shown that the norm of the joint-range availability chooses an optimal workpoint as a circular reflection of the designed joint-angle center points. This property can be used to advantage, however, for practical control. The condition number does not have an analytical gradient that can be applied in real-time in Eq. (2). However, the optimal configuration and workpoint based on the condition number or any other dexterity measure can be calculated off line. By saving the joint angles corresponding to these optimal situations and using them as a specification for the joint-center angles in the JRAE measure, a fast, real-time measure can be designed with identical effects to a desired measure. The gradient of the JRAE measure is simple and can be calculated easily for a more general, 3-D geometry.

In this paper a simple three-link planar manipulator was examined. Results for 3-D redundant manipulators should be similar since many of the proposed seven-degree-of-freedom designs also have one redundant degree of freedom. One important consideration for 3-D manipulators is properly comparing accuracy in both position and orientation.

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