Chapter 4 Solutions

Exercise 4.1.
Programming assignment.

Exercise 4.2.
By inspection M can be obtained as

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \ell_1 + \ell_2 \\ 0 & 0 & 1 & \ell_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The screw axes $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

i	ω_i	v_i
1	(0, 0, 1)	(0, 0, 0)
2	(0, 0, 1)	$(\ell_1, 0, 0)$
3	(0, 0, 1)	$(\ell_1 + \ell_2, 0, 0)$
4	(0, 0, 0)	(0, 0, 1)

The screw axes $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

i	ω_i	v_i
1	(0, 0, 1)	$(-\ell_1 - \ell_2, 0, 0)$
2	(0, 0, 1)	$(-\ell_2, 0, 0)$
3	(0, 0, 1)	(0, 0, 0)
4	(0, 0, 0)	(0, 0, 1)

The end-effector configuration $T \in SE(3)$ can be found, using the `FKinSpace` and the `FKinBody` functions, as

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Exercise 4.3.

$$T(\theta) \quad = \quad M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} e^{[\mathcal{B}_3]\theta_3}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & L_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(-1, 0, 0)$	$(0, 0, -L_1)$	$(0, L_1, 0)$
2	$(0, -1, 0)$	$(-L_2, 0, 0)$	$(0, 0, L_2)$
3	$(0, 0, 1)$	$(0, 0, 0)$	$(0, 0, 0)$

Exercise 4.4.

$$T(\theta) = Me^{[\mathcal{B}_1]\theta_1}e^{[\mathcal{B}_2]\theta_2}\cdots e^{[\mathcal{B}_6]\theta_6}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1+L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(0,0,1)$	$(0,-L_1-L_2,0)$	$(-L_1-L_2,0,0)$
2	$(1,0,0)$	$(0,-L_1-L_2,0)$	$(0,0,L_1+L_2)$
3	$(0,0,0)$	-	$(0,1,0)$
4	$(0,1,0)$	$(0,0,0)$	$(0,0,0)$
5	$(1,0,0)$	$(0,-L_2,0)$	$(0,0,L_2)$
6	$(0,1,0)$	$(0,0,0)$	$(0,0,0)$

Exercise 4.5.
Screw axes in the body frame for UR5 $\mathcal{B}_i$:

i	ω_i	v_i
1	$(0, 1, 0)$	$(W_1+W_2, 0, L_1+L_2)$
2	$(0, 0, 1)$	$(H_2, -L_1-L_2, 0)$
3	$(0, 0, 1)$	$(H_2, -L_2, 0)$
4	$(0, 0, 1)$	$(H_2, 0, 0)$
5	$(0, -1, 0)$	$(-W_2, 0, 0)$
6	$(0, 0, 1)$	$(0, 0, 0)$

Exercise 4.6.
Screw axes in the body frame for the WAM arm $\mathcal{S}_i$:

i	ω_i	v_i
1	$(0, 1, 0)$	$(-H_1+H_2, 0, L_1+L_2)$
2	$(0, 0, 1)$	$(-L_1-L_2-L_3+W_1+W_2, -L_1-L_2, 0)$
3	$(0, 1, 0)$	$(-H_1+H_2, 0, L_1+L_2)$
4	$(0, 0, 1)$	$(-L_2-L_3+W_1+W_2, -L_1-L_2+W_1, 0)$
5	$(0, 1, 0)$	$(-H_1+H_2, 0, L_1+L_2)$
6	$(0, 1, 0)$	$(-H_1+H_2-L_3, 0, L_1+L_2)$
7	$(0, 1, 0)$	$(-H_1+H_2, 0, L_1+L_2)$

Exercise 4.7.
By inspection M can be obtained as

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1+L_2+L_3+L_4 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The screw axes $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

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i	ω_i	v_i
1	(0, 0, 0)	(0, 1, 0)
2	(0, 0, 1)	(L_1, 0, 0)
3	(-1, 0, 0)	(0, $-h$, L_1)
4	(-1, 0, 0)	(0, $-h$, $L_1 + L_2$)
5	(-1, 0, 0)	(0, $-h$, $L_1 + L_2 + L_3$)
6	(0, 1, 0)	($-h$, 0, 1)

The screw axes $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

i	ω_i	v_i
1	(0, 0, 0)	(0, 1, 0)
2	(0, 0, 1)	($-L_2 - L_3 - L_4$, 0, 0)
3	(-1, 0, 0)	(0, 0, $-L_2 - L_3 - L_4$)
4	(-1, 0, 0)	(0, 0, $-L_3 - L_4$)
5	(-1, 0, 0)	(0, 0, $-L_4$)
6	(0, 1, 0)	(0, 0, 0)

Exercise 4.8.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_6]\theta_6} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} \cdots e^{[\mathcal{B}_6]\theta_6} \end{aligned}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_3 + L_4 \\ 0 & 0 & 1 & -L_5 - L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(1,0,0)$	$(0,0,0)$	$(0,0,0)$
2	$(0,0,-1)$	$(L_1,0,0)$	$(0,L_1,0)$
3	$(0,1,0)$	$(L_1,0,L_2)$	$(-L_2,0,L_1)$
4	$(1,0,0)$	$(0,L_3,0)$	$(0,0,-L_3)$
5	$(0,0,0)$	-	$(0,1,0)$
6	$(0,1,0)$	$(L_1,0,-L_5)$	$(L_5,0,L_1)$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(1,0,0)$	$(0,-L_3-L_4,L_5+L_6)$	$(0,L_5+L_6,L_3+L_4)$
2	$(0,0,-1)$	$(0,-L_3-L_4,0)$	$(L_3+L_4,0,0)$
3	$(0,1,0)$	$(0,0,L_2+L_5+L_6)$	$(-L_2-L_5-L_6,0,0)$
4	$(1,0,0)$	$(0,-L_4,L_5+L_6)$	$(0,L_5+L_6,L_4)$
5	$(0,0,0)$	-	$(0,1,0)$
6	$(0,1,0)$	$(0,0,L_6)$	$(-L_6,0,0)$

Exercise 4.9.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_6]\theta_6} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} \cdots e^{[\mathcal{B}_6]\theta_6} \end{aligned}$$

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The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$
2	$(1,0,0)$	$(0,0,-2L)$	$(0,-2L,0)$
3	$(0,0,0)$	-	$(0,1,0)$
4	$(0,0,0)$	-	$(0,0,1)$
5	$(0,1,0)$	$(0,0,-L)$	$(L,0,0)$
6	$(0,0,-1)$	$(0,3L,0)$	$(-3L,0,0)$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,0,-1)$	$(-3L,0,0)$	$(0,-3L,0)$
2	$(0,1,0)$	$(-3L,0,0)$	$(0,0,-3L)$
3	$(0,0,0)$	-	$(1,0,0)$
4	$(0,0,0)$	-	$(0,0,-1)$
5	$(1,0,0)$	$(0,0,-L)$	$(0,-L,0)$
6	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$

Exercise 4.10.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_6]\theta_6} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} \cdots e^{[\mathcal{B}_6]\theta_6} \end{aligned}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & (2+\sqrt{3})L \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & (1+\sqrt{3})L \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,0,1)$	$(L,0,0)$	$(0,-L,0)$
2	$(0,1,0)$	$(L,0,0)$	$(0,0,L)$
3	$(0,1,0)$	$((1+\sqrt{3})L,0,-L)$	$(L,0,(1+\sqrt{3})L)$
4	$(0,1,0)$	$((2+\sqrt{3})L,0,(\sqrt{3}-1)L)$	$((1-\sqrt{3})L,0,(2+\sqrt{3})L)$
5	$(0,0,0)$	-	$(0,0,1)$
6	$(0,0,1)$	$((2+\sqrt{3})L,0,0)$	$(0,-(2+\sqrt{3})L,0)$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,0,1)$	$(-(1+\sqrt{3})L,0,0)$	$(0,(1+\sqrt{3})L,0)$
2	$(0,1,0)$	$(-(1+\sqrt{3})L,0,-(1+\sqrt{3})L)$	$((1+\sqrt{3})L,0,-(1+\sqrt{3})L)$
3	$(0,1,0)$	$(-L,0,-(2+\sqrt{3})L)$	$((2+\sqrt{3})L,0,-L)$
4	$(0,1,0)$	$(0,0,-2L)$	$(2L,0,0)$
5	$(0,0,0)$	-	$(0,0,1)$
6	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$

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Exercise 4.11.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_5]\theta_5} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} \cdots e^{[\mathcal{B}_5]\theta_5} \end{aligned}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$
2	$(0,0,0)$	-	$(1,0,0)$
3	$(0,0,1)$	$(1,0,0)$	$(0,-1,0)$
4	$(0,-1,0)$	$(1,0,-1)$	$(-1,0,-1)$
5	$(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}})$	$(1,0,0)$	$(0,-\frac{1}{\sqrt{2}},0)$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(0,0,1)$	$(-3,0,0)$	$(0,3,0)$
2	$(0,0,0)$	-	$(1,0,0)$
3	$(0,0,1)$	$(-2,0,0)$	$(0,2,0)$
4	$(0,-1,0)$	$(-2,0,-1)$	$(-1,0,2)$
5	$(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}})$	$(-2,0,0)$	$(0,\sqrt{2},0)$

Exercise 4.12.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_6]\theta_6} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} \cdots e^{[\mathcal{B}_6]\theta_6} \end{aligned}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & 0 & (4+\sqrt{2})L \\ 0 & 0 & 1 & -\sqrt{2}L \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,1,0)$	$(0,0,0)$	$(0,0,0)$
2	$(0,0,1)$	$(0,L,0)$	$(L,0,0)$
3	$(0,0,0)$	-	$(0,1,0)$
4	$(0,1,0)$	$(0,0,L)$	$(-L,0,0)$
5	$(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$	$(0,4L,L)$	$(-\frac{5}{\sqrt{2}}L,0,0)$
6	$(1,0,0)$	$(0,(4+\sqrt{2})L,-\sqrt{2}L)$	$(0,-\sqrt{2}L,-(4+\sqrt{2})L)$

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The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,1,0)$	$(-L,0,\sqrt{2}L)$	$(-\sqrt{2}L,0,-L)$
2	$(0,0,1)$	$(-L,-(3+\sqrt{2})L,0)$	$(-(3+\sqrt{2})L,L,0)$
3	$(0,0,0)$	-	$(0,1,0)$
4	$(0,1,0)$	$(-L,0,(1+\sqrt{2})L)$	$(-(1+\sqrt{2})L,0,-L)$
5	$(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$	$(-L,0,L)$	$(-\frac{1}{\sqrt{2}}L,-\frac{1}{\sqrt{2}}L,-\frac{1}{\sqrt{2}}L)$
6	$(1,0,0)$	$(0,0,0)$	$(0,0,0)$

Setting $\theta_5 = \pi$ and all other joint variables to zero, $e^{[\mathcal{S}_1]\theta_1} = e^{[\mathcal{S}_2]\theta_2} = e^{[\mathcal{S}_3]\theta_3} = e^{[\mathcal{S}_4]\theta_4} = e^{[\mathcal{S}_6]\theta_6} = I$, while $e^{[\mathcal{S}_5]\theta_5}$ can be expressed by following formula:

$$e^{[\mathcal{S}]\theta} = \begin{bmatrix} e^{[\omega]\theta} & \left(I\theta + (1-\cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2\right) v \\ 0 & 1 \end{bmatrix}.$$

Then $e^{[\mathcal{S}_5]\theta_5}$ becomes

$$e^{[\mathcal{S}_5]\theta_5} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 5L \\ 0 & -1 & 0 & 5L \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Hence T_{06} becomes

$$T_{06} = e^{[\mathcal{S}_5]\theta_5} M = \begin{bmatrix} -1 & 0 & 0 & -L \\ 0 & 0 & -1 & (5+\sqrt{2})L \\ 0 & -1 & 0 & (1-\sqrt{2})L \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and T_{60} is T_{06}^{-1}.

Exercise 4.13.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_6]\theta_6} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} \cdots e^{[\mathcal{B}_6]\theta_6} \end{aligned}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	w_i	q_i	v_i
1	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$
2	$(1,0,0)$	$(0,0,2)$	$(0,2,0)$
3	$(1,0,0)$	$(0,1,2)$	$(0,2,-1)$
4	$(0,0,0)$	-	$(0,1,0)$
5	$(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$	$(0,1,0)$	$(\frac{1}{\sqrt{2}},0,0)$
6	$(0,0,-1)$	$(0,4,0)$	$(-4,0,0)$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

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frame i	w_i	q_i	v_i
1	$(0,0,-1)$	$(0,-4,0)$	$(4,0,0)$
2	$(-1,0,0)$	$(0,-4,-1)$	$(0,1,-4)$
3	$(-1,0,0)$	$(0,-3,-1)$	$(0,1,-3)$
4	$(0,0,0)$	-	$(0,1,0)$
5	$(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$	$(0,-1,-1)$	$(\sqrt{2},0,0)$
6	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$

Exercise 4.14.
By inspection M can be obtained as:

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_0+L_2 \\ 0 & 0 & -1 & L_1-L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The screw axes $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

i	ω_i	v_i
1	(0, 0, 1)	$(L_0, 0, 0)$
2	(0, 0, 0)	(0, 1, 0)
3	(0, 0, −1)	$(-L_0-L_2, 0, h)$

where $L_0 = 4$, $L_1 = 3$, $L_2 = 2$, $L_3 = 1$, and $h = 0.1$.
The screw axes $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

i	ω_i	v_i
1	(0, 0, −1)	$(L_2, 0, 0)$
2	(0, 0, 0)	(0, 1, 0)
3	(0, 0, 1)	$(0, 0, -h)$

Using `FKinSpace` and `FKinBody` should give the following configuration T:

$$T = \begin{bmatrix} 0 & 1 & 0 & -3-L_2 \\ 1 & 0 & 0 & L_0 \\ 0 & 0 & -1 & \pi h + L_1 - L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -5 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 2.314 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Exercise 4.15.

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} M \\ &= M e^{[\mathcal{B}_1]\theta_1} e^{[\mathcal{B}_2]\theta_2} e^{[\mathcal{B}_3]\theta_3} \end{aligned}$$

The end-effector zero position configuration M is given by

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_2+L_4 \\ 0 & 0 & 1 & -L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Note that the first joint of the robot is a screw joint with nonzero pitch h. Hence, for frame {1} we should use $v = -\omega \times q + h\omega$.
The values of the screw parameters $\mathcal{S}_i = (\omega_i, v_i)$ are listed in the following table:

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frame i	ω_i	q_i	v_i
1	$(0,0,1)$	$(0,0,0)$	$(0,0,h)$
2	$(0,1,0)$	$(0,0,0)$	$(0,0,0)$
3	$(1,0,0)$	$(0,L_2,-L_3)$	$(0,-L_3,-L_2)$

The values of the screw parameters $\mathcal{B}_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(0,0,1)$	$(0,-L_2-L_4,0)$	$(-L_2-L_4,0,h)$
2	$(0,1,0)$	$(0,0,L_3)$	$(-L_3,0,0)$
3	$(1,0,0)$	$(0,-L_4,0)$	$(0,0,L_4)$

Exercise 4.16.
The forward kinematics of a four-dof open chain manipulator in its zero position is written in the following exponential form:

$$T_{04}(\theta_1,\theta_2,\theta_3,\theta_4) = e^{[A_1]\theta_1}e^{[A_2]\theta_2}Me^{[A_3]\theta_3}e^{[A_4]\theta_4}.$$

Substituting $\theta_i = \theta'_i + \alpha_i$ $(i = 1,\cdots,4)$ into the above formula, the forward kinematics is of the form:

$$\begin{aligned}
T_{04}(\theta'_1,\theta'_2,\theta'_3,\theta'_4) &= e^{[A_1](\theta'_1+\alpha_1)}e^{[A_2](\theta'_2+\alpha_2)}Me^{[A_3](\theta'_3+\alpha_3)}e^{[A_4](\theta'_4+\alpha_4)} \\
&= e^{[A_1]\theta'_1}e^{[A_1]\alpha_1}e^{[A_2]\theta'_2}e^{[A_2]\alpha_2}Me^{[A_3]\alpha_3}e^{[A_3]\theta'_3}e^{[A_4]\alpha_4}e^{[A_4]\theta'_4} \\
&= e^{[A_1]\theta'_1}e^{[A'_2]\theta'_2}e^{[A_1]\alpha_1}e^{[A_2]\alpha_2}Me^{[A_3]\alpha_3}e^{[A_4]\alpha_4}e^{[A'_3]\theta'_3}e^{[A_4]\theta'_4},
\end{aligned}$$

where $A'_2 = [\mathrm{Ad}_{e^{[A_1]\alpha_1}}]A_2$ and $A'_3 = [\mathrm{Ad}_{e^{-[A_4]\alpha_4}}]A_3$.

$$\begin{aligned}
\therefore [A'_1] &= [A_1] \\
[A'_2] &= e^{[A_1]\alpha_1}[A_2]e^{-[A_1]\alpha_1} \\
[A'_3] &= e^{-[A_4]\alpha_4}[A_3]e^{[A_4]\alpha_4} \\
[A'_4] &= [A_4] \\
M' &= e^{[A_1]\alpha_1}e^{[A_2]\alpha_2}Me^{[A_3]\alpha_3}e^{[A_4]\alpha_4}
\end{aligned}$$

Exercise 4.17.

(a) As given in the problem, the forward kinematics of the manipulator is expressed as $T_{b_1b_2} = e^{[S_1]\theta_1}e^{[S_2]\theta_2}\cdots e^{[S_5]\theta_5}M$, with the end-effector 1 grasping the tree. Let S_i be the screw parameter of the i^{th} joint expressed in the $\{b_1\}$ frame, and $M \in SE(3)$ be the displacement from the $\{b_1\}$ frame to the $\{b_2\}$ frame. M and S_i can be derived as follows:

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4L \\ 0 & 0 & -1 & L \\ 0 & 1 & 0 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$\begin{aligned}
S_1 &: w_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_1 = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} 0 \\ -L \\ 0 \end{bmatrix} \\
S_2 &: w_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \\
S_3 &: w_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, q_3 = \begin{bmatrix} L \\ 0 \\ L \end{bmatrix}, v_3 = \begin{bmatrix} L \\ 0 \\ -L \end{bmatrix} \\
S_4 &: w_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_4 = \begin{bmatrix} 2L \\ L \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} L \\ -2L \\ 0 \end{bmatrix} \\
S_5 &: w_5 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, q_5 = \begin{bmatrix} 3L \\ 0 \\ L \end{bmatrix}, v_5 = \begin{bmatrix} L \\ 0 \\ -3L \end{bmatrix}.
\end{aligned}$$

(b) Here end-effector 2 is rigidly grasping a tree; the forward kinematics of the manipulator is expressed as

$$T_{b_2b_1} = e^{[A_5]\theta_5} e^{[A_4]\theta_4} e^{[A_3]\theta_3} N e^{[A_2]\theta_2} e^{[A_1]\theta_1}. \tag{4.1}$$

This equation can be modified as follows:

$$T_{b_2b_1} = e^{[A_5]\theta_5} e^{[A_4]\theta_4} e^{[A_3]\theta_3} e^{[A_2']\theta_2} e^{[A_1']\theta_1} N.$$

$A_1' - A_5$ are the screw parameters of the i^{th} joint expressed in the $\{b_2\}$ frame, and $N \in SE(3)$ is the displacement from the $\{b_2\}$ frame to the $\{b_1\}$ frame. Therefore, N and $A_1' - A_5$ can be derived as follows:

$$N = \begin{bmatrix} 1 & 0 & 0 & -4L \\ 0 & 0 & 1 & -L \\ 0 & -1 & 0 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
A_5 &: w_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_5 = \begin{bmatrix} -L \\ 0 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix} \\
A_4 &: w_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, q_4 = \begin{bmatrix} -2L \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ -2L \end{bmatrix} \\
A_3 &: w_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_3 = \begin{bmatrix} -3L \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 3L \\ 0 \end{bmatrix} \\
A_2' &: w_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ -L \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix} \\
A_1' &: w_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, q_1 = \begin{bmatrix} -3L \\ 0 \\ L \end{bmatrix}, v_1 = \begin{bmatrix} -L \\ 0 \\ 3L \end{bmatrix}.
\end{aligned}$$

Now if we move N as below and compare it with Equation (4.1), we get the following:

$$T_{b_2b_1} = e^{[A_5]\theta_5} e^{[A_4]\theta_4} e^{[A_3]\theta_3} N e^{N^{-1}[A_2']N\theta_2} e^{N^{-1}[A_1']N\theta_1}$$

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$$[A_2] = N^{-1}[A'_2]N$$
$$[A_1] = N^{-1}[A'_1]N$$

$$A_2 \ : \ w_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix}$$
$$A_1 \ : \ w_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_1 = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} 0 \\ -L \\ 0 \end{bmatrix}.$$

Exercise 4.18.

(a) As given in the problem, the forward kinematics of the robot A is expressed as

$$T_{Aa} = e^{[\mathcal{S}_1]\theta_1}e^{[\mathcal{S}_2]\theta_2}e^{[\mathcal{S}_3]\theta_3}e^{[\mathcal{S}_4]\theta_4}e^{[\mathcal{S}_5]\theta_5}M_a,$$

$\mathcal{S}_i$ is the screw parameter of the i^{th} joint expressed in the $\{A\}$ frame, and $M_a \in SE(3)$ is the displacement from the $\{A\}$ frame to the $\{a\}$ frame at the zero position. Therefore, M_a and $\mathcal{S}_i$ are derived as follows:

$$M = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\mathcal{S}_1 \ : \ w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$\mathcal{S}_2 \ : \ w_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\mathcal{S}_3 \ : \ w_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, q_3 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}, v_3 = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix}$$
$$\mathcal{S}_4 \ : \ w_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$\mathcal{S}_5 \ : \ w_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, q_5 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ L_1 \\ 0 \end{bmatrix}.$$

(b) Because robots A and B are the same robot, T_{Bb} can be expressed as

$$T_{Bb} = e^{[\mathcal{S}_1]\phi_1}e^{[\mathcal{S}_2]\phi_2}e^{[\mathcal{S}_3]\phi_3}e^{[\mathcal{S}_4]\phi_4}e^{[\mathcal{S}_5]\phi_5}M_a,$$

where $\mathcal{S}_i$ and M_a are the same as in (a). The displacement T_{AB} from the {A} frame to the {B} frame, and the displacement T_{ab} from the {a} frame to the {b} frame, are derived as follows:

$$T_{AB} = \begin{bmatrix} -1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & L_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$T_{ab} = T_{ba} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

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Using the relation $T_{Aa} = T_{AB}T_{Bb}T_{ba}$,

$$\begin{aligned}
T_{Aa} &= T_{AB}T_{Bb}T_{ba} \\
e^{[\mathcal{S}_1]\theta_1}e^{[\mathcal{S}_2]\theta_2}\cdots e^{[\mathcal{S}_5]\theta_5}M_a &= T_{AB}e^{[\mathcal{S}_1]\phi_1}e^{[\mathcal{S}_2]\phi_2}\cdots e^{[\mathcal{S}_5]\phi_5}M_aT_{ab}, \\
e^{[\mathcal{S}_1]\theta_1}e^{[\mathcal{S}_2]\theta_2}\cdots e^{[\mathcal{S}_5]\theta_5}M_a &= e^{T_{AB}[\mathcal{S}_1]T_{AB}^{-1}\phi_1}\cdots e^{T_{AB}[\mathcal{S}_5]T_{AB}^{-1}\phi_5}T_{AB}M_aT_{ba}, \\
(\because Pe^{S} &= e^{PSP^{-1}}P) \\
e^{[\mathcal{S}_1]\theta_1}e^{[\mathcal{S}_2]\theta_2}\cdots e^{[\mathcal{S}_5]\theta_5}M_a &= e^{[\mathcal{B}_1]\phi_1}e^{[\mathcal{B}_2]\phi_2}\cdots e^{[\mathcal{B}_5]\phi_5}T_{AB}M_aT_{ba}, \\
e^{-[\mathcal{B}_5]\phi_5}\cdots e^{-[\mathcal{B}_1]\phi_1}e^{[\mathcal{S}_1]\theta_1}\cdots e^{[\mathcal{S}_5]\theta_5} &= T_{AB}M_aT_{ba}M_a^{-1}.
\end{aligned}$$

where $[\mathcal{B}_i] = T_{AB}[\mathcal{S}_i]T_{AB}^{-1}$ or $\mathcal{B}_i = Ad_{T_{AB}}(\mathcal{A}_i)$, and $M = T_{AB}M_aT_{ba}M_a^{-1}$. The $\mathcal{B}_i$ are the screw parameters of robot B as seen from {A}:

$$\begin{aligned}
\mathcal{B}_1 &: w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\
\mathcal{B}_2 &: w_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, q_2 = \begin{bmatrix} L_3 \\ L_4 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} L_4 \\ -L_3 \\ 0 \end{bmatrix} \\
\mathcal{B}_3 &: w_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, q_3 = \begin{bmatrix} L_3 \\ 0 \\ L_1 \end{bmatrix}, v_3 = \begin{bmatrix} -L_1 \\ 0 \\ L_3 \end{bmatrix} \\
\mathcal{B}_4 &: w_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\
\mathcal{B}_5 &: w_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, q_5 = \begin{bmatrix} 0 \\ L_4 \\ L_1 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ -L_1 \\ L_4 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
M &= T_{AB}M_aT_{ba}M_a^{-1} \\
&= \begin{bmatrix} -1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & L_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & -2L_2 + L_3 \\ 0 & 1 & 0 & L_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned}$$

Exercise 4.19.

(a) To derive the forward kinematics in the given form, it is convenient to use the Denavit-Hartenberg parameters. However, we are not able to find the Denavit-Hartenberg parameters for the given link reference frames. Correct link frames for finding the Denavit-Hartenberg parameters are given in Figure (4.1). Using frame {3} of the third link, the corresponding Denavit-Hartenberg parameters are as follows:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	$-\frac{\pi}{2}$	0	$2L$	θ_1
2	$-\frac{\pi}{2}$	L	0	$\theta_2 - \frac{\pi}{2}$
3	$-\frac{\pi}{2}$	0	$L + \theta_3$	$\frac{\pi}{2}$
4	$\frac{\pi}{2}$	0	$-2L$	θ_4
5	$-\frac{\pi}{2}$	0	L	$\theta_5 - \frac{\pi}{2}$

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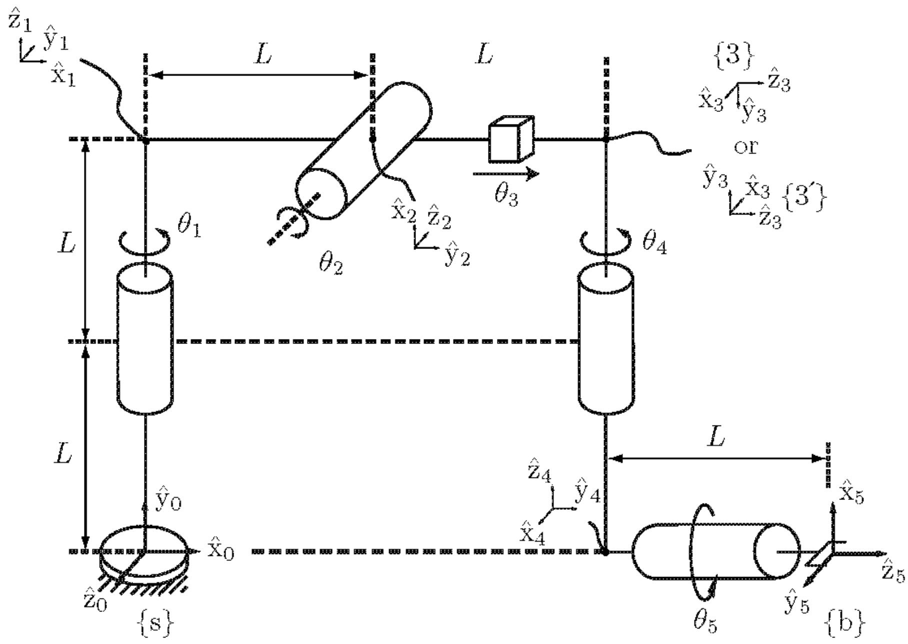

Figure 4.1

Otherwise, using the frame $\{3'\}$ of the third link, the corresponding Denavit-Hartenberg parameters for $i = 3, 4$ transform as follows:

i	α_{i-1}	a_{i-1}	d_i	θ_i
3	$-\frac{\pi}{2}$	0	$L + \theta_3$	$-\frac{\pi}{2}$
4	$-\frac{\pi}{2}$	0	$-2L$	$\theta_4 + \pi$

Using the Denavit-Hartenberg parameters derived above,

$$\begin{aligned} M_2 &= \mathrm{Rot}\left(\hat{\mathrm{x}}, -\frac{\pi}{2}\right) \cdot \mathrm{Trans}\left(\hat{\mathrm{x}}, L\right) \cdot \mathrm{Rot}\left(\hat{\mathrm{z}}, -\frac{\pi}{2}\right) \\ &= \begin{bmatrix} 0 & 1 & 0 & L \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\ M_3 &= \mathrm{Rot}\left(\hat{\mathrm{x}}, -\frac{\pi}{2}\right) \cdot \mathrm{Trans}\left(\hat{\mathrm{z}}, L\right) \cdot \mathrm{Rot}\left(\hat{\mathrm{z}}, \frac{\pi}{2}\right) \\ &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & L \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(\text{or using the frame } \{3'\}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right), \\ \mathcal{A}_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T, \\ \mathcal{A}_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T. \end{aligned}$$

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(b) From the given forward kinematics in (a),

$$\begin{aligned}
T_{sb} &= M_1 e^{[\mathcal{A}_1]\theta_1} M_2 e^{[\mathcal{A}_2]\theta_2} \cdots M_5 e^{[\mathcal{A}_5]\theta_5} \\
&= \left(M_1 e^{[\mathcal{A}_1]\theta_1} M_1^{-1}\right)\left(M_1 M_2 e^{[\mathcal{A}_2]\theta_2} M_2^{-1} M_1^{-1}\right) \\
&\quad \cdots \left(M_1 \cdots M_5 e^{[\mathcal{A}_5]\theta_5} M_5^{-1} \cdots M_1^{-1}\right)(M_1 \cdots M_5) \\
&= e^{M_1[\mathcal{A}_1]M_1^{-1}\theta_1} e^{(M_1M_2)[\mathcal{A}_2](M_1M_2)^{-1}\theta_2} \cdots e^{(M_1\cdots M_5)[\mathcal{A}_5](M_1\cdots M_5)^{-1}\theta_5}(M_1 \cdots M_5) \\
&= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \cdots e^{[\mathcal{S}_5]\theta_5} M.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\mathcal{S}_1 &= \mathrm{Ad}_{M_1}(\mathcal{A}_1), \\
\mathcal{S}_2 &= \mathrm{Ad}_{M_1M_2}(\mathcal{A}_2), \\
&\vdots \\
\mathcal{S}_5 &= \mathrm{Ad}_{M_1\cdots M_5}(\mathcal{A}_5), \\
M &= M_1 \cdots M_5.
\end{aligned}$$

Exercise 4.20.
The end-effector frame {b} as seen from the fixed frame {0} is

$$M = \begin{bmatrix} 0 & 1 & 0 & -3L \\ 1 & 0 & 0 & -L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The values of the screw parameters $S_i = (\omega_i, v_i)$ are listed in the following table:

frame i	ω_i	q_i	v_i
1	$(0,0,0)$	-	$(0,0,1)$
2	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$
3	$(0,-1,0)$	$(-L,0,L)$	$(L,0,L)$
4	$(0,0,0)$	-	$(-1,0,0)$
5	$(1,0,0)$	$(0,0,L)$	$(0,L,0)$
6	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$

Notice that the last row of above table corresponds to the screw parameters as seen from the end-effector frame {b}. The forward kinematics is written in the following exponential form:

$$\begin{aligned}
T_{06} &= e^{[S_1]\theta_1} \cdots e^{[S_5]\theta_5} e^{[S_6']\theta_6} M = e^{[S_1]\theta_1} \cdots e^{[S_5]\theta_5} M e^{M^{-1}[S_6']M\theta_6} \\
&= e^{[S_1]\theta_1} \cdots e^{[S_5]\theta_5} M e^{[S_6]\theta_6}
\end{aligned}$$

where $[S_6] = M^{-1}[S_6']M$.
$[S_6] = M^{-1}[S_6']M$ can be verified using following matrices:

$$[S_6] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad [S_6'] = \begin{bmatrix} 0 & 1 & 0 & L \\ -1 & 0 & 0 & -3L \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad M^{-1} = \begin{bmatrix} 0 & 1 & 0 & L \\ 1 & 0 & 0 & 3L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

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Exercise 4.21.

$$\begin{aligned} T &= \mathrm{Rot}(\hat{\mathrm{x}},\alpha)\mathrm{Trans}(\hat{\mathrm{x}},a)\mathrm{Trans}(\hat{\mathrm{z}},d)\mathrm{Rot}(\hat{\mathrm{z}},\phi) && (4.2) \\ &= \begin{bmatrix} \cos\phi & -\sin\phi_i & 0 & a \\ \sin\phi\cos\alpha & \cos\phi\cos\alpha & -\sin\alpha & -d\sin\alpha \\ \sin\phi\sin\alpha & \cos\phi\sin\alpha & \cos\alpha & d\cos\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix} && (4.3) \end{aligned}$$

(a)

$$T = \begin{bmatrix} 0 & 1 & 1 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_a & p_a \\ 0 & 1 \end{bmatrix}$$

Since $R_a \notin SO(3)$, there is no solution for this T.

(b)

$$T = \begin{bmatrix} \cos\beta & \sin\beta & 0 & 1 \\ \sin\beta & -\cos\beta & 0 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_b & p_b \\ 0 & 1 \end{bmatrix}$$

By a correspondence with Equation (4.3),

$$\begin{aligned} \alpha &= \pi \\ a &= 1 \\ d &= 2 \\ \phi &= -\beta \quad (\because \cos\phi = \cos\beta, \quad \sin\phi = -\sin\beta). \end{aligned}$$

(c)

$$T = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_c & p_c \\ 0 & 1 \end{bmatrix}$$

By a correspondence with Equation (4.3),

$$\begin{aligned} \cos\alpha &= 0 \\ d\cos\alpha &= 2, \end{aligned}$$

from which it follows that there is no solution for this T.