A METHOD FOR THE DESIGN OF HYBRID POSITION/FORCE CONTROLLERS FOR MANIPULATORS CONSTRAINED BY CONTACT WITH THE ENVIRONMENT

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ABSTRACT

A new method for the design of hybrid position/force controllers for constrained manipulators is derived. This method can be applied to all types of constraint due to contact with the environment; including constraint due to contact at the end effector, constraint due to more than one robot manipulating a workpiece, and constraint due to the bracing of a robot arm against a work surface. The manipulator and its contact with the environment are modeled in terms of lower order pairs. From this model a general equation describing the constraint on the motion of the arm is derived. The task is modeled as a set of essential position vectors and a set of essential force vectors. A hybrid position/force controller is derived to control the position and force at the joints of a manipulator such that the motion of the the robot conforms to the constraints imposed on it due to contact with the environment; and the motion at the end effector, and the force at the contact with the environment are those required for the performance of the task. The method is illustrated by a simple three degree of freedom example.

1. INTRODUCTION

There are many operations that require a manipulator to move whilst constrained by contact with the environment. Robotic assembly, for example, is the process of mating two workpieces; one workpiece being held in the robot end effector and the other fixed in the workspace or the environment. The motion of the robot during the assembly operation is constrained because of the contact between these two workpieces. Machining operations such as grinding and drilling also require the robot arm to move whilst constrained because of the contact between the machining tool, held by the robot, and the workpiece fixed in the environment. For these tasks the motion of the robot is constrained due to contact between the end effector and the workpiece.

Apart from contact at the end effector, there are several other ways in which a robot arm can be constrained. If two robots are used to pick up one object then the motion of both robots must be coordinated since they are constrained by each other. Secondly, several authors have suggested the use of "Jig Hands" or bracing structures to brace an arm against the workpiece, notably [Hogan and Moore 1983][5], [Book, Le, and Sangveraphunsiri 1984][3], and [Asada and West 1984][1]. These bracing structures increase the stiffness and improve the accuracy of the end effector, but they also constrain the motion of the arm.

An example of a robot constrained in this way is shown in Figure 1. A planar robot with three joints is simply supported close to its end effector by a jig hand which is free to slide along a flat surface. The unconstrained arm has three degrees of freedom at the end effector. However, when the jig hand is in contact with the work surface the number of degrees of freedom is reduced to two. The motion of the end effector is constrained because of contact with the environment even though the contact with the environment is not at the end effector, but at another point on the arm. The task for which this manipulator is to be used is to pick up and move parts that are located on the same flat surface. The only motions of the end effector essential for performing this task are motions parallel and perpendicular to this flat surface. The advantage of building a robot of this type is that, because the arm is levered close to the point at which the load is applied, it can move heavier loads than would be possible without the jig hand, and position them more accurately relative to the work surface. A discussion of this approach to robot design is given in [Asada and West, 1984][1].

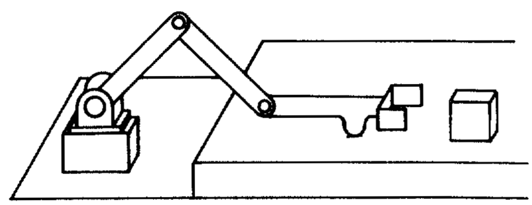

Figure 1. Example Of A Constrained Manipulator

There are many tasks for which it is necessary or advantageous to constrain a manipulator by contact with the environment, but a constrained manipulator is more complex to control. The difficulty in controlling a constrained manipulator arises in two ways. First, the constraint results in a reduction in the number of degrees of freedom of the arm so that an arbitrary motion at the end effector can no longer be specified. Secondly, the constraint can exert a reaction force[1] on the arm, and, therefore, there can be large forces in

[1]In this paper, *position* implies *position and orientation* and *force* implies *force and torque.*

the joints of the robot even when there is no load at the end effector. These joint forces must be controlled; if not they can become arbitrarily large and damage the manipulator or the surface with which the manipulator is in contact. Both the position and the force in the joints of a constrained manipulator must be controlled.

The motion at the end effector is a function of the vector of joint motions. The vector of joint motions, however, is constrained by another function due to contact between the arm and the environment. Therefore, two sets of parameters must be specified in order to control the position of the end effector. First, the degrees of freedom in which the vector of joint motions can be controlled must be identified. These are the joint motions that are allowed by the constraint, and can be used to move the end effector. Secondly, using only these joint motions, an end effector trajectory must be found that performs the desired task. If the constrained arm has less than six degrees of freedom at the end effector, then the directions of motion at the end effector which are essential to the performance of the task must be identified. The design requirements of the position controller are that it should ensure that the motion of the robot conforms to the constraints imposed on it due to contact with the environment, and that the robot achieves the essential motions at the end effector necessary to perform the task.

A parallel approach is needed to control the forces in the arm. First, the degrees of freedom of the vector of joint forces that are allowed by the constraint must be identified. Secondly, using only these joint force vectors, the forces in the joints must be controlled to achieve an appropriate force at the contact. The design requirements of the force controller are that it should ensure that the joint forces of the robot arm conform to the constraints imposed on it, and that a suitable force is applied at the contact with the environment.

Relationship To Previous Work

The purpose of this section is to summarize the several important concepts that have already been developed. The example of a robot inserting a peg into a hole, see Figure 2, will be used to illustrate these concepts [Mason 1981][6].

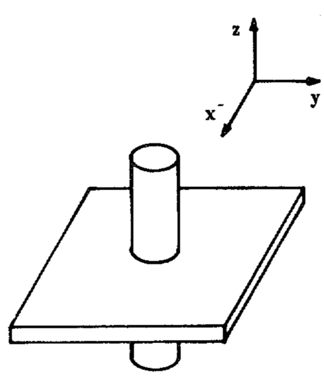

Figure 2. The Peg In The Hole Constraint

The velocity and force at the robot end effector can be described by two vectors in end effector space :

$$\mathbf{v}^T = [v(x), v(y), v(z), w(x), w(y), w(z)]$$

$$\mathbf{f}^T = [f(x), f(y), f(z), g(x), g(y), g(z)]$$

The peg in the hole problem is an example of an end effector constrained because of contact at the end effector with the environment. The constraint is due to the contact between the peg held by the end effector and the hole, and can be described as a constraint on the end effector velocity and force vectors such that:

$$\mathbf{S}\ \mathbf{v} = \mathbf{0} \tag{1}$$

and
$$[\mathbf{I} - \mathbf{S}]\ \mathbf{f} = \mathbf{0} \tag{2}$$

Where **S** is a 6x6 selection matrix, given for this example by

$$\mathbf{S} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & 0 & & & \\ & & & 1 & & \\ & 0 & & & 1 & \\ & & & & & 0 \end{bmatrix} \tag{3}$$

Notice that the directions in which the velocity vector is constrained are complementary to the directions in which the force vector is constrained. Velocity and force can be controlled in the directions in which they are not constrained.

The task is to insert the peg into the hole, and can also be specified in end effector space. The motions which must be controlled in order to perform the task are v(z) and w(z), and the forces which must be controlled are f(x), f(y), g(x), and g(y).

The form of the appropriate control method follows quite naturally; it is known as hybrid position/force control. The end effector space is divided into two orthogonal domains, a position domain and a force domain, which are complementary to the directions of the corresponding constraints at the end effector. In each of these domains position or force can be controlled independently and arbitrarily. The selection matrix **S** and its complement [**I** - **S**] are used to separate the directions in end effector space in which force and position are controlled. These directions constitute the motion and force trajectories that will be followed to perform the task. A hybrid controller is then used to regulate the motion and force at the end effector about these trajectories. This control strategy has been successfully demonstrated controlling a two degree of freedom manipulator with its end effector in contact with a flat surface [Raibert and Craig 1981][7].

The difficulty in designing a hybrid controller arises when the selection matrix, **S**, which is used to separate the position and force controllable directions, cannot be simply obtained by inspection. If the constraint on the motion of the end effector is due to contact between the manipulator and the environment at a point other than the end effector, as in the case of the robot shown in Figure 1, then the effective constraint on the motion of the end effector is not a constant but is a function of the position of the manipulator. The selection matrix, **S**, must therefore be recalculated for each position of the arm. Mason has shown for a simple planar example of two manipulators grasping an object how the effective constraint at the end effector of each manipulator can be calculated [Mason 1981][6]. However, in general this is a complicated, and ad hoc procedure. The problem is further complicated if the manipulator does not have six degrees of freedom.

Scope Of This Paper

There are many practical tasks for which manipulators, such as robots and locomotive devices, must be controlled as they move whilst constrained by contact with the environment. The problem of controlling an end effector that is constrained by contact at the end effector is well understood. However, there has been no algebraic method of kinematic analysis available for solving the more complex constraint problem in which the constraint is due to contact between the manipulator and the environment at a point other than the end effector. A more general method is needed for the design of hybrid position/force controllers for manipulators constrained by contact with the environment.

The solution presented in this paper is to select between the position controllable vectors and the force controllable vectors in joint space. The advantage of this approach is that the joint space of a manipultor is common to both the end effector and to the constraint, even if the constraint is not at the end effector. A systematic approach is derived for identifying the vectors of joint motion and the vectors of joint force which can be controlled. A method for describing the motions and forces essential to the performance of a task is shown. Combining these two pieces of analysis a solution to the problem of controlling a manipulator to perform a task whilst it is constrained by contact with the environment is derived. It is presumed that the manipulator is capable of performing the desired task and that therefore a solution to the control problem does exist. Apart from this assumption, the analysis is carried out completely generally. The illustration of the simple three degree example shown in Figure 1 is used to make the results easier to understand.

2. CONSTRAINT ANALYSIS

A Kinematic Model For Constrained Manipulators

In general the contact between a manipulator and the environment can be described by modeling any higher order contacts, joints with more than one degree of freedom, as a series of lower order kinematic pairs, joints with only one degree of freedom, [Hartenburg and Denavit 1964][4]. In the example described in Figure 1 the contact between the arm and the work surface has two degrees of freedom in the plane of the figure. This constraint can be modeled as a *virtual* arm with two joints each of which are lower order pairs. A kinematic model of the constrained robot in terms of lower order pairs is given in Figure 3. An m-jointed robot constrained by a contact with the environment of c degrees of freedom is represented as an (m+c)-jointed robot constrained by a point on the arm fixed to the environment.

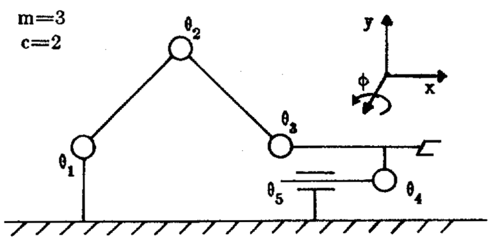

Figure 3. Kinematic Model Of Example Shown In Figure 1.

Modeling the contact with the environment in this way reveals an essential feature of mechanical constraints: *Constraints due to contact with the environment are constraints due to a closed kinematic chain.* The feature that distinguishes a constrained manipulator from an unconstrained one is the existence of a closed kinematic chain; therefore, the general properties of constrained manipulators can be derived from the analysis of closed kinematic chains.

2. Analysis Of A Closed Kinematic Chain

Small motion at the end of a serial link manipulator, $\delta\mathbf{x}$, can be described in terms of the Jacobian matrix of the manipulator, $\mathbf{J}$, and the variation in the displacement of each of the joints, $\delta\theta$:

$$\delta\mathbf{x} = \mathbf{J}\,\delta\theta \tag{4}$$

$\mathbf{x}$ is a 6x1 position vector in a coordinate system fixed to the workspace, θ is an mx1 joint vector, where m is the number of joints in the manipulator, and $\mathbf{J}$ is a a 6xm matrix relating variations in the two vectors.

If we consider a massless arm with frictionless joints then the work done by the end of the arm must be equal to the sum of the work done on the arm by each of the joints. Therefore, for a force at the end of the manipulator, represented by the 6x1 force vector, $\mathbf{q}$, defined in a coordinate system fixed to the work space, the equivalent forces in the joints of the arm, represented by the mx1 vector τ, are given by:

$$\tau = \mathbf{J}^T\,\mathbf{q} \tag{5}$$

A Jacobian matrix also exists for a manipulator forming a closed kinematic chain. Figure 4 shows a manipulator with n joints, all active, incorporating a closed kinematic chain. The 6xn Jacobian, $\mathbf{J}_c$, referred to as the contact Jacobian, relates variation in the joint positions (all joints) to variations in the position of the contact point, C. The contact point is fixed to the workspace and therefore the motion of this point relative to its base, $\delta\mathbf{x}_c$, is always zero so that:

$$\delta\mathbf{x}_c = \mathbf{J}_c\,\delta\theta = \mathbf{0} \tag{6}$$

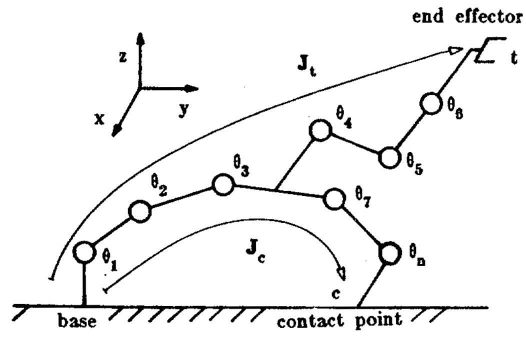

Figure 4. A Manipulator With A Closed Kinematic Chain

θ is now an nx1 vector. The solutions to this equation are given by:

$$\delta\theta = (\mathbf{I} - \mathbf{J}_c^+ \mathbf{J}_c)\ \mathbf{y} \tag{7}$$

where $\mathbf{y}$ is an arbitrary nx1 vector and $\mathbf{J}_c^+$ is a generalized inverse of $\mathbf{J}$, or 1,2-inverse, that satisfies the equations $\mathbf{J}_c\ \mathbf{J}_c^+\ \mathbf{J}_c = \mathbf{J}_c$, and $\mathbf{J}_c^+\ \mathbf{J}_c\ \mathbf{J}_c^+ = \mathbf{J}_c^+$. A more detailed discussion of generalized inverses is given in the Appendix.

For a closed kinematic chain, the work done at the end of the chain must also be zero since $\delta\mathbf{x} = \mathbf{0}$. Therefore, if again we consider a massless arm, then the sum of the work done by each of the joints must be zero. From equation (7)

$$\tau^T\ \delta\theta = \tau^T\ (\mathbf{I} - \mathbf{J}_c^+\ \mathbf{J}_c)\ \mathbf{y} = \mathbf{0} \tag{8}$$

which has the general solution:

$$\tau = (\mathbf{J}_c^+\ \mathbf{J}_c)^T\ \mathbf{z} \tag{9}$$

where $\mathbf{z}$ is an arbitrary vector, and τ is now an nx1 vector of joint forces.

The allowable motion and the allowable forces of an n-jointed closed kinematic chain are characterized by the two nxn matrices $[\mathbf{I} - \mathbf{J}_c^+\ \mathbf{J}_c]$ *and* $[\mathbf{J}_c^+\ \mathbf{J}_c]^T$. Together the two matrices span the joint space of the chain. The space spanned by each matrix is the orthogonal complement of the space spanned by the other, and hence:

$$[\mathbf{I} - \mathbf{J}_c^+\ \mathbf{J}_c] \cdot [\mathbf{J}_c^+\ \mathbf{J}_c]^T = \mathbf{0} \tag{10}$$

This result corresponds in joint space to the result shown in the introduction; that the directions in which velocity are constrained are complementary to the directions in which forces are constrained.

These two matrices are idempotent, and are therefore projectors, although not necessarily orthogonal projectors. An idempotent matrix, $\mathbf{P}$, is one that has the property $\mathbf{P}^2 = \mathbf{P}$, and is a projector on to the space spanned by $\mathbf{P}$. If $\mathbf{P}$ is also symmetric then it is an orthogonal projector (see Appendix). The matrices $[\mathbf{I} - \mathbf{J}_c^+\ \mathbf{J}_c]$ and $[\mathbf{J}_c^+\ \mathbf{J}_c]^T$ represent the projections in joint space onto the allowable position variations, and onto the allowable forces respectively. They will be abbreviated to:

$$_j\mathbf{P}_{ap} = [\mathbf{I} - \mathbf{J}_c^+\ \mathbf{J}_c] \tag{11}$$

$$_j\mathbf{P}_{af} = [\mathbf{J}_c^+\ \mathbf{J}_c]^T = [\mathbf{I} - ({}_j\mathbf{P}_{ap})^T] \tag{12}$$

The subscripts $_j$ before each projector indicate that the projector is defined in joint space.

Passive Joints

In a closed kinematic chain some of the joints may be passive. For example, the joints in the virtual link of the contact are always passive. The force in a passive joint is not controllable, it is always zero. Therefore, passive joints impose a further constraint on the allowable forces in a closed kinematic chain.

There are two approaches to the problem of controlling the allowable forces in a closed kinematic chain with passive joints. The force spanned by the active joints can be described by the nxn diagonal matrix, $\mathbf{A}$; a 1 on the diagonal indicating an active joint and a 0 indicating a passive joint. The allowable force space is then the intersect of the space spanned by $\mathbf{A}$ and the space spanned by $(\mathbf{J}_c^+\ \mathbf{J}_c)^T$. A new projector onto the intersection of these two spaces may be derived using the techniques described in the Appendix.

An alternative approach is to control the contact force at C to satisfy the requirement that the force in the passive joints is zero. The directions of force at the contact which are constrained to be zero by the passive joints are given by the columns of the matrix $\mathbf{J}_c\ [\mathbf{I} - \mathbf{A}]$. It will be shown in the next chapters how the force at the contact may be controlled.

Extension To Multiple Constraints

A manipulator may have multiple constraints due to more than one contact with the environment. For example, a multilegged walking machine, or a robot constrained both by a jig hand and at the end effector. Each constraint can be modeled as an independent closed kinematic chain. Corresponding to each chain there is a 6xn contact Jacobian, $\mathbf{J}_{ci}$; i = 1...k, where n is the number of joints in the *entire* manipulator, including the virtual joints at the contact with the environment, and k is the number of independent closed kinematic chains. Each closed chain can be characterized by the two corresponding nxn projection matrices in joint space; $_j\mathbf{P}_{api}$ and $_j\mathbf{P}_{afi}$. For a manipulator with multiple constraints the vectors of allowable position variations must be allowable for each constraint taken individually. The vectors of allowable joint forces must include the allowable forces for each constraint taken individually. The manipulator with multiple constraints can therefore be characterized by combining each of the k pairs of projection matrices; by taking the intersect of the space spanned by the allowable position projectors, and the union of the spaces spanned by the allowable force projectors. Equations for projections onto the intersect, and onto the union of two spaces are given in the Appendix.

Application To The Design Of A Hybrid Position/Force Controller

The analysis of constrained manipulators and their characterization in terms of two projection matrices suggests a new design for a hybrid position/force controller. The controller described by Raibert and Craig uses selection matrices to select between position controlled and force controlled vectors in end effector space. An alternative approach is to use the two projection matrices as *filters in joint space* to select between position controlled and force controlled vectors. It can be shown that these two approaches are equivalent if the manipulator is constrained at the end effector.

Example

In the planar example of Figure 1 the end of the manipulator has three degrees of freedom when unconstrained by contact with the environment. The end effector can be moved in three independent directions; parallel to the work surface, perpendicular to the worksurface, and it can also vary its orientation to the work surface. These freedoms can be referred to as δx, δy, and $\delta\phi$ as shown by the axes in Figure 5. When the arm is constrained due to contact with the environment, the number of degrees of freedom at the end effector is reduced to two, δa_1 and δa_2 , as shown in the figure. The end effector can be moved in the δx direction, but motion in the δy and $\delta\phi$ directions are coupled.

Let us now look at how the filter in joint space, ${}_j\mathbf{P}_{ap}$, would perform for this example. The filter projects any requested joint motion, $\delta\theta_r$, onto an allowable joint motion, $\delta\theta_r'$. At the end effector the result is that a requested motion, $\delta\mathbf{r}$, is projected onto an allowable motion $\delta\mathbf{r}'$, as shown in Figure 5. We now have to examine the result of this filter on the requested motion to determine if the result is satisfactory for performing the task.

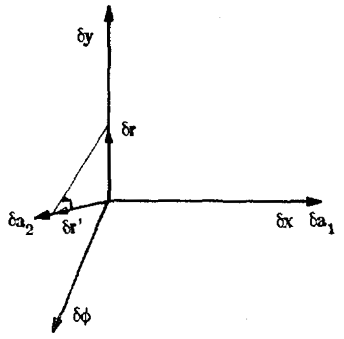

FIGURE 5. The Allowable Motion Space; $\delta\mathbf{a}_1$, $\delta\mathbf{a}_2$

3. TASK ANALYSIS

In general, the motion of an end effector necessary to perform a task can be described by specifying a vector in six dimensional space (three positions, x, y, z; and three orientations, ϕ_x , ϕ_y , ϕ_z). The desired motion can be achieved by controlling the motion of the end effector in each of these directions independently; requiring six degrees of freedom at the end effector.

For many tasks, however, it is not necessary to control motion in all six directions independently; there are some directions in which the motion of the end effector is not critical. If there are less than six degrees of freedom at the end effector it becomes important to distinguish between directions in which motion is essential for performing the task, and directions in which motion is arbitrary. In the planar example shown in Figure 1 the manipulator is used for picking up and moving objects on a flat work surface. The directions of motion of the end effector necessary for performing the task are motion parallel to the work surface, and motion perpendicular to the work surface. It is also necessary to control the force exerted by the robot on to the work surface at the constraint. The orientation of the end effector, however, is not critical.

We define essential position variables as those directions of motion *at the end effector* which must be controlled precisely in order to perform the task. Arbitrary position variables are those directions of position *at the end effector* which do not have to be controlled exactly, but which can vary over some range. Essential force variables are those directions of force *at the contact with the environment* which must be controlled in order to perform the task. Arbitrary force variables are those directions of force *at the contact with the environment* which do not have to be controlled precisely, but which can vary over some range. The total number of essential variables, position plus force, is equal to the minimum number of controllable actuators in the manipulator necessary for performing the task.

The directions of the essential position and force variables may be described by 6x1 column vectors $\mathbf{e}_{pi}$; i = 1...pα, and 6x1 column vectors $\mathbf{e}_{fi}$; i = 1...β in a coordinate system fixed to the work space. α is the number of essential position variables and β the number of essential force variables. The notation may be made more compact by defining the 6xα matrix $\mathbf{E}_p$ and the 6xβ matrix $\mathbf{E}_f$:

$$\mathbf{E}_p = [\mathbf{e}_{p1} \; .\,.\,.\,.\, \mathbf{e}_{p\alpha}] \tag{13}$$

$$\mathbf{E}_f = [\mathbf{e}_{f1} \; .\,.\,.\,.\, \mathbf{e}_{f\beta}] \tag{14}$$

Each set of column vectors is chosen as a minimum set needed to describe the task; therefore, the essential variable matrices $\mathbf{E}_p$ and $\mathbf{E}_f$ are of full column rank.

In order to perform a task it is necessary to control the motion of the end effector in the directions of the essential position variables and the force at the contact in the directions of the essential force variables. Vectors in the direction of the essential variables can be separated from vectors in the direction of the arbitrary variables by projecting onto the essential variable space. It is shown in the Appendix that the orthogonal projector onto the space spanned by the matrix $\mathbf{A}$ is given by $\mathbf{A}\ \mathbf{A}^{++}$, where $\mathbf{A}^{++}$ is the pseudo inverse of $\mathbf{A}$. The pseudo inverse of a matrix of full column rank is given by $(\mathbf{A}^T\ \mathbf{A})^{-1}\ \mathbf{A}^T$. Therefore, the orthogonal projection onto the essential position space, ${}_w\mathbf{P}_{ep}$, and the orthogonal projection onto the essential force space, ${}_w\mathbf{P}_{ep}$, are given by

$${}_w\mathbf{P}_{ep} = \mathbf{E}_p\ (\mathbf{E}_p^T\ \mathbf{E}_p)\ \mathbf{E}_p^T \tag{15}$$

$${}_w\mathbf{P}_{ef} = \mathbf{E}_f\ (\mathbf{E}_f^T\ \mathbf{E}_f)\ \mathbf{E}_f^T \tag{16}$$

The subscript $_w$ before each projector indicates that the projection matrices are defined in a coordinate system fixed to the work space. Similar expressions may be obtained in terms of the arbitrary variables.

Example

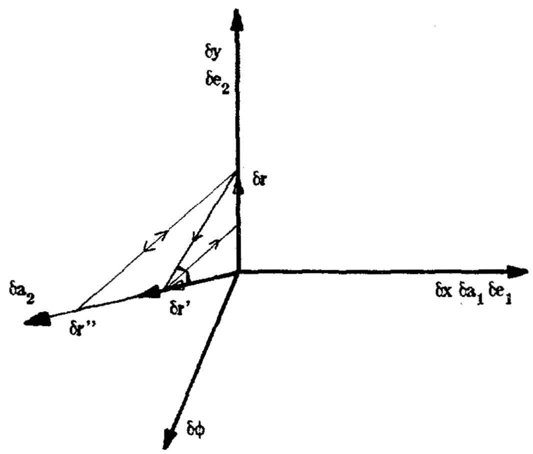

Figure 6. The Allowable Motion Space; $\delta\mathbf{a}_1$, $\delta\mathbf{a}_2$
And The Essential Position Variable Space; $\delta\mathbf{e}_1$, $\delta\mathbf{e}_2$

The essential position variables for the planar example of Figure 1 are denoted by e_1 and e_2 , and shown in Figure 6. In the example described at the end of the previous chapter, the projection matrix ${}_j\mathbf{P}_{ap}$, applied in joint space, resulted in the requested motion, $\delta\mathbf{r}$, being projected onto an allowable motion, $\delta\mathbf{r}'$, at the end effector. From the analysis of this chapter it is apparent that the resulting motion is unsatisfactory for performing the task. The projection of the allowed motion, $\delta\mathbf{r}'$, onto the essential variable plane is not equal to the projection of the requested motion onto the essential variable plane. The task that was requested has not been performed. A new type of projection from $\delta\mathbf{r}$ to $\delta\mathbf{r}''$ is needed, giving an allowable motion that still performs the task requested at the end effector.

4. A METHOD FOR THE DESIGN OF HYBRID POSITION/FORCE CONTROLLERS

Statement Of The Problem

The general control problem for a constrained manipulator, as illustrated in Figure 4, may be stated as follows:

(1) A manipulator and virtual link representing the contact with the environment is comprised of an ordered set of n joints, $\theta^T = [\theta_1, \ldots \theta_n]$

Of these joints: m are part of the manipulator
c are virtual joints representing the contact with the environment

(2) Corresponding to this set of joints are:

(a) The 6xn task Jacobian, $\mathbf{J}_t$, relating the joint motions to the motion of the end effector

(b) The 6xn contact Jacobian, $\mathbf{J}_c$, relating the joint forces to the force at the contact with the environment

(3) A task which may be described in terms of:

(a) The 6xα essential position variable matrix, ${}_w\mathbf{E}_p$

(b) The 6xβ essential force variable matrix, ${}_w\mathbf{E}_f$

(4) The task control problem is to find a trajectory for the position and force at the joints of the manipulator, such that:

(a) The motion of the robot conforms to the constraint imposed on it due to contact with the environment.

(b) The motion of the end effector and the force at the contact with the environment in the directions of the essential variables are those required for the performance of the task.

The solution to this problem is most conveniently obtained in terms of projectors. From the analysis of a closed kinematic chain, described by a contact Jacobian, $\mathbf{J}_c$, it was shown how to obtain the projectors onto the allowable motion space and allowable force space, ${}_j\mathbf{P}_{ap}$ and ${}_j\mathbf{P}_{af}$, defined in equations 11 and 12. From the analysis of the essential variables of a task, described by the essential variable matrices, ${}_w\mathbf{E}_p$ and ${}_w\mathbf{E}_f$, it was shown how to obtain the projectors onto the essential position variable space and essential force variable space, ${}_w\mathbf{P}_{ep}$ and ${}_w\mathbf{P}_{ef}$, defined in equations 15 and 16. The task control problem for a constrained manipulator is to find a trajectory that lies in the allowable motion and force spaces, and results in the desired motion and force essential for performing the task when projected onto the essential variable spaces. However, the allowable and essential projectors have been defined in different coordinate systems; in joint coordinates and work space coordinates respectively. In order to solve the constrained task control problem the allowable and essential projectors must be transformed into a common coordinate system.

Coordinate Transformation

The choice of common coordinate system depends on the number of joints in the manipulator. Let the number of joints linking the base of the manipulator to the end effector be denoted by ν, representing the number of non zero columns in $\mathbf{J}_t$, and the number of active joints linking the base of the manipulator to the constraint be denoted by λ. For most practical manipulators the number of degrees of freedom at the end effector is equal to ν, or to 6 if there are more than six joints. That is to say; the rank of $\mathbf{J}_t$ is equal to ν for $\nu \leq 6$, and equal to 6 for $\nu \geq 6$. The rank of $\mathbf{J}_c$ is similarly determined by λ. These conditions apply when the robot is not at a "singular" position. The common coordinate system can then be chosen as follows:

$\nu \leq 6 \Rightarrow$ motions can be controlled in work space coordinates

$\nu \geq 6 \Rightarrow$ motions can be controlled in joint space coordinates

$\lambda \leq 6 \Rightarrow$ forces can be controlled in work space coordinates

$\lambda \geq 6 \Rightarrow$ forces can be controlled in joint space coordinates

The physical explanation for the choice of common coordinate system is that if $\nu \leq 6$ then an arbitrary motion in joint space can be represented by a motion in work space; if $\nu \geq 6$ then an arbitrary motion in work space can be represented in joint space. Similarly, if $\lambda \leq 6$ then an arbitrary force in joint space can be represented by a force in work space; if $\lambda \geq 6$ then an arbitrary force in work space can be represented in joint space. If the number of joints is equal to 6 then either work space coordinates or joint space coordinates may be used.

For the case in which $\nu \leq 6$ the projector onto the allowable motion space is given in work space coordinates at the end effector by:

$$ {}_w\mathbf{P}_{ap} = \mathbf{J}_t \, {}_j\mathbf{P}_{ap} \, \mathbf{J}_t^{+} \tag{17} $$

The matrix ${}_w\mathbf{P}_{ap}$ projects onto the motion space allowed by both the constraint due to contact with the environment *and* the constraint due to less than six joints linking the base to the end effector. For $\lambda \leq 6$ the projector onto the allowable forces at the contact in work space coordinates is given by:

$$ {}_w\mathbf{P}_{af} = (\mathbf{J}_c^{\mathrm{T}})^{+}\ {}_j\mathbf{P}_{af}\ \mathbf{J}_c^{\mathrm{T}} \tag{18} $$

For the case in which $\nu \geq 6$ and $\lambda \geq 6$ the projectors onto the esential variables are given in joint space by

$$ {}_j\mathbf{P}_{ep} = \mathbf{J}_t^{+}\ {}_t\mathbf{P}_{ep}\ \mathbf{J}_t \tag{19} $$

$$ {}_j\mathbf{P}_{ef} = \mathbf{J}_c^{\mathrm{T}}\ {}_c\mathbf{P}_{ef}\ (\mathbf{J}_c^{\mathrm{T}})^{+} \tag{20} $$

If $\nu > 6$ and $\lambda < 6$ then the position control problem must be solved in joint space, and the force control problem solved in work space coordinates.

Solution To The Problem

The analysis will be continued for the position control problem in joint space; a similar set of equations can be derived for the force control problem. The analysis in work space coordinates follows the same form.

The problem is to find the projector in joint space that projects onto the allowable motion space, and results in the desired motion necessary for performing the task when projected onto the essential variable space. This projector can be used as a filter to separate the position controllable joint vectors from the force controllable vectors and to generate the appropriate trajectory for performing the task. A hybrid position/force controller incorporating this filter is then used to regulate the motions at the joints about this trajectory. A similar filter for force control completes the hybrid controller.

The position filter in joint space will be denoted by ${}_j\mathbf{F}_p$; it projects any requested joint motion $\delta\theta_r$ onto the appropriate joint trajectory, $\delta\theta_t$, for the constrained manipulator to perform the task.

$$ \delta\theta_t = {}_j\mathbf{F}_p\ \delta\theta_r \tag{21} $$

This trajectory must satisfy two conditions. First, the joint trajectory must lie in the allowable motion space, and therefore:

$$ {}_j\mathbf{P}_{ap}\ \delta\theta_t = \delta\theta_t \tag{22} $$

Second, the projection of the joint trajectory onto the essential variable space must equal the projection of the original requested joint motion onto the essential variable space. This indicates that the essential variables of the requested motion have been preserved. Therefore

$$ {}_j\mathbf{P}_{ep}\ \delta\theta_t = {}_j\mathbf{P}_{ep}\ \delta\theta_r \tag{23} $$

Substituting equation (21) into equations (22) and (23) gives the conditions that must be satisfied by ${}_j\mathbf{F}_p$:

$$ {}_j\mathbf{P}_{ap}\ {}_j\mathbf{F}_p = {}_j\mathbf{F}_p \tag{24} $$

$$ {}_j\mathbf{P}_{ep}\ {}_j\mathbf{F}_p = {}_j\mathbf{P}_{ep} \tag{25} $$

Projectors are idempotent, and therefore the general solution to equations (24) can be written in the form:

$$ {}_j\mathbf{F}_p = {}_j\mathbf{P}_{ap}\ \mathbf{X} \tag{26} $$

Where $\mathbf{X}$ is an arbitrary matrix. Substituting equation (26) into equation (25) gives

$$ {}_j\mathbf{P}_{ep}\ {}_j\mathbf{P}_{ap}\ \mathbf{X} = {}_j\mathbf{P}_{ep} \tag{27} $$

Solutions to equation (27), are given by

$$ \mathbf{X} = ({}_j\mathbf{P}_{ep}\ {}_j\mathbf{P}_{ap})^{+}\ {}_j\mathbf{P}_{ep} \tag{28} $$

Where $({}_j\mathbf{P}_{ep}\ {}_j\mathbf{P}_{ap})^{+}$ represents the 1-inverse of $({}_j\mathbf{P}_{ep}\ {}_j\mathbf{P}_{ap})$

If there is a unique solution to equation (27) then that is given by equation (28). If there are many solutions then the pseudo inverse may be used to give the solution with minimum norm.

The filter for generating the appropriate motion trajectory in joint space for a constrained manipulator is therfore given by the nxn matrix:

$$ {}_j\mathbf{F}_p = {}_j\mathbf{P}_{ap}\ ({}_j\mathbf{P}_{ep}\ {}_j\mathbf{P}_{ap})^{+}\ {}_j\mathbf{P}_{ep} \tag{29} $$

and similarly, the filter for generating the appropriate joint forces, ${}_j\mathbf{F}_f$, is given by the nxn matrix:

$$ {}_j\mathbf{F}_f = {}_j\mathbf{P}_{af}\ ({}_j\mathbf{P}_{ef}\ {}_j\mathbf{P}_{af})^{+}\ {}_j\mathbf{P}_{ef} \tag{30} $$

Implementation In A Hybrid Controller

The structure of a hybrid position/force controller designed using these filters is shown in Figure 7. Applying these filters to the example of Figure 1, the appropriate end effector trajectory $\delta\mathbf{r}''$ is achieved as shown in Figure 6.

Equations (29) and (30) represent the homogeneous solution to the constrained control problem for a massless arm and stationary constraint. A feed forward velocity vector, $\mathbf{J}_c^{+}\ \mathbf{v}_c$, can be added at point FFA to compensate for the velocity of the constraint. A feed forward force vector can be added at point FFB to compensate for the gravitational loading on the arm.

The joint forces in the closed chain can be controlled to exert a force, $\mathbf{f}_c$, at the contact as shown in Figure 7. Alternatively, the constraint can be used to bear part of the load at the end effector and reduce the load on the arm. The appropriate joint forces are then given by the projection of the joint forces due to the load at the end effector, $\mathbf{f}_t$, onto the force space allowed by the constraint. Therefore, the desired force trajectory, τ_d is given by

$$ \tau_d = {}_j\mathbf{P}_{af}\ \mathbf{J}_t^{\mathrm{T}}\ \mathbf{f}_t \tag{31} $$

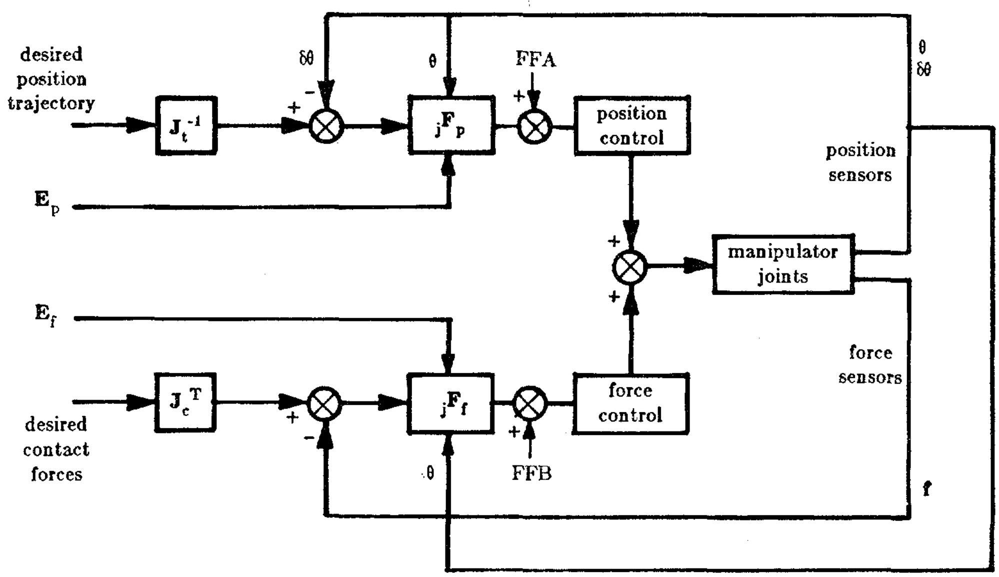

Figure 7. The Hybrid Position/Force Controller in Joint Space

5. CONCLUSION

A general method has been developed for the design of hybrid position/force controllers for manipulators constrained by contact with the environment. The method provides a solution to the problem of controlling the motion of an end effector which is constrained due to contact between the manipulator and the environment at a point other than the end effector. This method can be applied to all types of constraint due to contact with the environment including; constraint due to more than one robot manipulating a workpiece, constraint due to the bracing of a robot arm against a work surface, constraint due to contact with an obstacle in the environment, and constraint due to contact at the end effector. It is assumed in deriving this method that a solution exists, i.e. it is possible for the robot to perform the task. If this were not so the control problem would be immaterial. It is also assumed that the Jacobian for motion at the end effector, J_t, and the Jacobian which would describe motion at the constraint, J_c, are known.

The manipulator and its contact with the environment are modeled in terms of lower order pairs. From this model a general equation describing the constraint on the motion of the arm is derived, and a systematic approach developed for identifying the vectors of joint motion and the vectors of joint force which can be controlled. The task is modeled as a set of essential position vectors and a set of essential force vectors. The essential position vectors are the directions of motion at the end effector which must be controlled to achieve the task, and the essential force vectors are the directions of force at the contact with the environment which must be controlled. A hybrid position/force controller is derived to control the position and force at the joints of the manipulator such that the motion of the robot conforms to the constraints imposed on it due to contact with the environment, and the motion at the end effector and the force at the contact with the environment are those required for the performance of the task.

The method makes use of generalized 1,2-inverses of a singular matrix to generate projectors onto the allowable motion space and allowable force space of the constrained manipulator. The calculation of 1,2-inverses of a matrix is explained in an appendix. Projectors are also used to project onto the essential motion space and essential force space of the task. A filter is derived for a hybrid controller that projects onto the allowble motion and allowable force spaces in such a way that the projection onto the essential motion and essential force spaces of the task remain unchanged. The controller makes the effect of the constraint transparent to the control of the task.

A physical interpretation of the method is provided by a simple three degree of freedom example.

ACKNOWLEDGEMENTS

The authors would like to thank Daikin Industries, Ltd., Robot System Division for their support of this project.

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APPENDIX

Generalized Inverses[2]

Solutions to the problem,

$$\mathbf{A}\ \mathbf{x} = \mathbf{b} \tag{A1}$$

where the matrix $\mathbf{A}$ is not necessarily nonsingular, are given by

$$\mathbf{x} = \mathbf{X}\ \mathbf{b} \tag{A2}$$

where $\mathbf{X}$ represents a *generalized inverse* of $\mathbf{A}$.

The defining equations for generalized inverses are:

$$\mathbf{A}\ \mathbf{X}\ \mathbf{A} = \mathbf{A} \tag{D1}$$

$$\mathbf{X}\ \mathbf{A}\ \mathbf{X} = \mathbf{X} \tag{D2}$$

$$(\mathbf{A}\ \mathbf{X})^T = \mathbf{A}\ \mathbf{X} \tag{D3}$$

$$(\mathbf{X}\ \mathbf{A})^T = \mathbf{X}\ \mathbf{A} \tag{D4}$$

An inverse that satisfies equation D1 and D2 is referred to as a 1,2-inverse of $\mathbf{A}$, in general it is not a unique matrix. If there are solutions to equation A1 then a solution is given by substituting a 1,2-inverse, $\mathbf{A}^{1,2}$, for $\mathbf{X}$ in equation A2. If there is only one solution then that unique solution will be given by using the 1,2-inverse.

The advantage of using 1,2 inverses is their ease of calculation. If $\mathbf{A}$ can be written in the form:

$$\mathbf{P}\ \mathbf{A}\ \mathbf{Q} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \tag{A3}$$

Where $\mathbf{A}_{11}$ is nonsingular and $\mathbf{P}$ and $\mathbf{Q}$ are permutation matrices, then the 1,2-inverse of $\mathbf{A}$ is given by:

$$\mathbf{A}^{1,2} = \mathbf{Q} \begin{bmatrix} \mathbf{A}_{11}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{P} \tag{A4}$$

A generalized inverse that satisfies all four of the defining equations, D1 to D4, is often called a *pseudo inverse*. The pseudo inverse is a unique matrix and provides the least squares solution in equation A2. If there is more than one solution to equation A1 then equation A2 gives the $\mathbf{x}$ which has the minimum norm. If there is no solution to equation A1 then equation A2 gives the solution that minimizes the norm of $(\mathbf{A}\ \mathbf{x} - \mathbf{b})$.

If $\mathbf{A}$ is of full column rank then its pseudo inverse is given by

$$\mathbf{A}^{(1,2,3,4)} = (\mathbf{A}^T\ \mathbf{A})^{-1}\ \mathbf{A}^T \tag{A5}$$

Projection Matrices[2]

Projectors are matrices that have the property $\mathbf{P}^2 = \mathbf{P}$. If $\mathbf{P}$ is also symmetric then it is an orthogonal projector.

The expression $\mathbf{A}\ \mathbf{X}$ is a projector on to the space spanned by the matrix $\mathbf{A}$, and the expression $\mathbf{X}\ \mathbf{A}^T$ is a projector on to the space spanned by $\mathbf{A}^T$. If $\mathbf{X}$ represents the pseudo inverse then the projectors are orthogonal projectors.

If $\mathbf{P}_L$ and $\mathbf{P}_M$ represent the orthogonal projections onto the spaces $\mathbf{L}$ and $\mathbf{M}$ then the projector $\mathbf{P}_{L+M}$ which projects onto the union of $\mathbf{L}$ and $\mathbf{M}$ is given by

$$\mathbf{P}_{L+M} = (\mathbf{P}_L + \mathbf{P}_M)\ (\mathbf{P}_L + \mathbf{P}_M)^{1,2,3,4} \tag{A6}$$

The projector $\mathbf{P}_{L \cap M}$ which projects onto the intersection of $\mathbf{L}$ and $\mathbf{M}$ is given by

$$\mathbf{P}_{L \cap M} = 2\ \mathbf{P}_L\ (\mathbf{P}_L + \mathbf{P}_M)^{1,2,3,4}\ \mathbf{P}_M \tag{A7}$$