Exercise 3

A lightweight 6R robot with a spherical wrist operates in a working environment where a human is occasionally present. During normal operation, the robot task is to track accurately a desired smooth trajectory $\boldsymbol{p}_d(t)$ for the end-effector position $\boldsymbol{p} = \boldsymbol{f}_p(\boldsymbol{q}) \in \mathbb{R}^3$ and an associated desired trajectory $\boldsymbol{\phi}_d(t)$ for a minimal representation of the end-effector orientation $\boldsymbol{\phi} = \boldsymbol{f}_\phi(\boldsymbol{q}) \in \mathbb{R}^3$. Assume that:

- The complete dynamic model of the robot in free motion is perfectly known, and is described (with the usual notations) by the following equations

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} - \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}), \tag{1}$$

  where the friction term $\boldsymbol{\tau}_f$ denotes a dissipative action at the joints.
- The direct kinematic functions $\boldsymbol{f}_p$ and $\boldsymbol{f}_\phi$ are known, as well as the 6×6 analytic Jacobian associated to the end-effector task

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{pmatrix} \frac{\partial \boldsymbol{f}_p(\boldsymbol{q})}{\partial \boldsymbol{q}} \\ \frac{\partial \boldsymbol{f}_\phi(\boldsymbol{q})}{\partial \boldsymbol{q}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_p(\boldsymbol{q}) \\ \boldsymbol{J}_\phi(\boldsymbol{q}) \end{pmatrix}, \tag{2}$$

  where the two matrices $\boldsymbol{J}_p$ and $\boldsymbol{J}_\phi$ have dimension 3×6, and matrix $\boldsymbol{J}$ is nonsingular in the region of interest.
- The robot is equipped only with encoders at the joints, and the environment is *not* monitored by any external sensor.

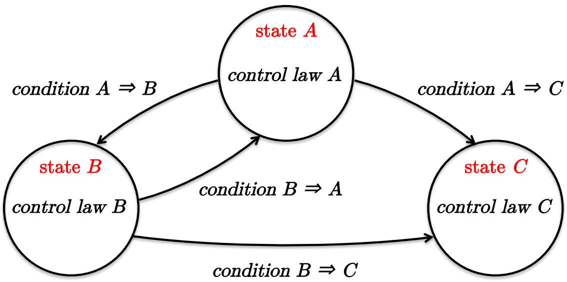

Figure 2: A state diagram of robot control operation in collision-aware tasks.

With reference to the state diagram in Fig. 2, the following collision-aware behavior for safe Human-Robot Interaction (HRI) should be realized through a suitable set of robot control laws and conditions for the transitions:

- During normal operation (state A in the diagram), if a mild contact occurs and is detected, the robot keeps the three-dimensional position task but relaxes the orientation task, trying to accommodate in this way a reflex reaction to the contact (state B).
- Instead, when a severe collision occurs during normal operation, the robot abandons the task completely by bouncing away from the collision area (state C) ad then stops.
- While in state B, the robot may either switch back to normal operation when the contact is no longer present, or abandon also the orientation task and switch to state C in case the interaction forces will increase further.

Specify the control laws and the transition conditions to be used in the state diagram of Fig. 2.

Exercise 3

The problem can be solved by using the residual vector $\boldsymbol{r}$ as a collision monitoring signal, together with a number of ordered positive thresholds on its norm $\|\boldsymbol{r}\|$ to be used in the switching conditions, and suitable control laws for each state.

Based on the known model (1), the residual $\boldsymbol{r} \in \mathbb{R}^6$ can be defined as

$$\boldsymbol{r}(t) = \boldsymbol{K} \left(\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \int_0^t \left(\boldsymbol{\tau} + \boldsymbol{C}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}) + \boldsymbol{r} \right) ds \right), \quad \text{with } \boldsymbol{K} > 0 \text{ (diagonal)}, \tag{9}$$

where $\boldsymbol{\tau}$ is the actual control torque applied in any of the robot states A, B, or C. Using (1), equation (9) implies the dynamic behavior

$$\dot{\boldsymbol{r}} = \boldsymbol{K}\left(\boldsymbol{\tau}_c - \boldsymbol{r}\right), \tag{10}$$

where $\boldsymbol{\tau}_c \in \mathbb{R}^6$ is the joint torque resulting from a collision force/moment occurring anywhere along the robot structure. Indeed, if at some time t the torque τ_c returns to zero, then each component of $\boldsymbol{r}$ will decay exponentially to zero as well. Moreover, for a sufficiently large $\boldsymbol{K}$, from (10) we can use the approximation $\boldsymbol{\tau}_C \simeq \boldsymbol{r}$ and use the residual vector $\boldsymbol{r}$ as a proxy of the unknown joint torque $\boldsymbol{\tau}_c$ due to collision.

With reference to Fig. 2, in the following suitable control laws will be defined for each state.

- **Control in state** A. Define the desired task trajectory as $\boldsymbol{x}_d(t) = \left(\boldsymbol{p}_d^T(t)\ \boldsymbol{\phi}_d^T(t)\right)^T \in \mathbb{R}^6$. In order to accurately follow this smooth trajectory, we use the Cartesian feedback linearization controller

$$\begin{aligned}\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q})\left(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_D\left(\dot{\boldsymbol{x}}_d - \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}\right) + \boldsymbol{K}_P\left(\boldsymbol{x}_d - \boldsymbol{f}(\boldsymbol{q})\right) - \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}\right)\\ + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}),\end{aligned} \tag{11}$$

  with 6×6 (typically diagonal) gain matrices $\boldsymbol{K}_P > 0$ and $\boldsymbol{K}_D > 0$. Within this law, the presence of a PD action on the task error allows to recover exponentially transient errors. This is necessary, e.g., when the complete task is partially abandoned and then resumed (in case we are coming back to state A from state B).

- **Control in state** B. In this case, the orientation part of the desired task will be relaxed, while the position task $\boldsymbol{p}_d(t) \in \mathbb{R}^3$ for the robot end-effector should be kept. Therefore, the robot becomes kinematically redundant since the task has dimension $m = 3$ while the robot has $n = 6$ control commands available; the degree of redundancy is thus $n - m = 3$. We continue to achieve Cartesian position tracking, e.g., by using a dynamically consistent redundancy resolution scheme. This control scheme uses the 3×6 Jacobian $\boldsymbol{J}_p$ in a partially feedback linearizing law that is weighted by the inverse of the task inertia matrix $\boldsymbol{\Lambda}(\boldsymbol{q})$ and adds a suitable torque $\boldsymbol{\tau}_0 \in \mathbb{R}^6$ projected in the dynamic null space of the task. We have thus

$$\begin{aligned}\boldsymbol{\tau} = \boldsymbol{J}_p^T(\boldsymbol{q})\boldsymbol{\Lambda}(\boldsymbol{q})\Big(\ddot{\boldsymbol{p}}_d + \boldsymbol{K}_{D,p}\left(\dot{\boldsymbol{p}}_d - \boldsymbol{J}_p(\boldsymbol{q})\dot{\boldsymbol{q}}\right) + \boldsymbol{K}_{P,p}\left(\boldsymbol{p}_d - \boldsymbol{f}_p(\boldsymbol{q})\right) - \dot{\boldsymbol{J}}_p(\boldsymbol{q})\dot{\boldsymbol{q}}\\ + \boldsymbol{J}_p(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\left(\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\dot{\boldsymbol{q}})\right)\Big),\\ + \left(\boldsymbol{I} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{\Lambda}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\right)\boldsymbol{\tau}_0,\end{aligned} \tag{12}$$

  with 3×3 (typically diagonal) gain matrices $\boldsymbol{K}_{P,p} > 0$ and $\boldsymbol{K}_{D,p} > 0$, and the 3×3 inertia matrix reduced to the task

$$\boldsymbol{\Lambda}(\boldsymbol{q}) = \left(\boldsymbol{J}_p(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{J}_p^T(\boldsymbol{q})\right)^{-1}.$$

  In (12), the torque $\boldsymbol{\tau}_0 = \boldsymbol{K}_r\boldsymbol{r}$ is used, with $\boldsymbol{K}_r > 0$, so as to obtain a reaction to the collision torque $\boldsymbol{\tau}_c \simeq \boldsymbol{r}$ which is consistent with the remaining Cartesian position task.

- **Control in state** C. In this case, the complete original task is abandoned. The robot reacts to the collision in a stronger or weaker way depending on the intensity (and direction in the joint space) of $\boldsymbol{r}$, which is a proxy of the severity of the collision. Moreover, to avoid bias in the reaction due to the gravity, this term should be cancelled. As a result

$$\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_r \boldsymbol{r} \tag{13}$$

with $\boldsymbol{K}_r > 0$. Once the contact is lost, $\boldsymbol{r}$ will go to zero. As a result, thanks of the presence of friction, the robot will come to a stop in a zero-gravity condition. Joint velocity damping can be added so as to anticipate the instant when the robot is finally at rest, but this will limit quick reaction to collisions.

Transitions between the states in Fig. 2 will be driven by the actual value of $\|\boldsymbol{r}\| \geq 0$. To this end, define a sequence of positive thresholds for this variable:

$$0 < r_{low} < r_{mild} < r_{severe}.$$

The value r_{low} is the minimum threshold that should be crossed by $\|\boldsymbol{r}\|$ in order to detect reliably contact/collision events (i.e., obtaining few false positives, or eliminating them). The detection instant $t_{detect} \geq 0$ is the first instant at which $\|\boldsymbol{r}(t_{detect})\| \geq r_{low}$. For the choice of this lowest threshold, one takes into account the presence of noise in position sensing and in the generation of an estimate of the velocity $\dot{\boldsymbol{q}}$ by numerical differentiation of the position measures $\boldsymbol{q}$, as well as the remaining model uncertainties. For the two other thresholds, the rationale is that mild collisions will generate small values of the norm of the residual and, conversely, severe collisions will be associated to large values of $\boldsymbol{r}$. The value r_{mild} is chosen only slightly above r_{low}, so that the control system may detect a contact but not yet consider it as a collision, letting thus the robot continue the original motion task. With this in mind, the following switching conditions correctly realize the desired behavior:

- **condition** $A \Rightarrow B$: $r_{mild} \leq \|\boldsymbol{r}\| < r_{severe}$;
- **condition** $A \Rightarrow C$: $\|\boldsymbol{r}\| \geq r_{severe}$;
- **condition** $B \Rightarrow C$: $\|\boldsymbol{r}\| \geq r_{severe}$;
- **condition** $B \Rightarrow A$: $\|\boldsymbol{r}\| < r_{low}$.

Note that the last condition may be replaced also by $\|\boldsymbol{r}\| < r_{mild}$. However, using the more conservative value r_{low} introduces some hysteresis, so that the robot will avoid switching several times between the states A and B when the norm of the residual is oscillating around r_{mild}.

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