Adaptive Algebraic Multigrid with Graph Modularity Matching for Symmetric Positive Definite Matrices

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1 Introduction

This library implements some tools to explore techniques of adaptive AMG. In classical AMG, the performance of the solver depends on the construction of the relaxation and coarse-grid correction process (cite M. BREZINA). Convergence of the method slows when the residual approaches the rhs of the system (the iterate is in the 'near null space' of the system). This happens when the current iterate is an approximation of a minimal eigenvalue of the matrix. Using this near null component and utilizing graph modularity, we can construct a new relaxation and coarse-grid correction. This provides the adaptivity when convergence suffers. Combining multiple of these adaptive relaxation steps can provide a composite adaptive multigrid method.

2 Library Details

The library is broken into 3 main modules: partitioner, preconditioner, and solver.

2.1 Partitioner module

Here is implemented an algorithm described in (quiring 2019) which is graph pairwise matching algorithm that only merges vertices where the change in modularity is positive. The only requirement for this algorithm is that the matrix has positive row sums. To convert our system to have positive rowsums, we construct a new system with a matching sparsity pattern using a 'near null' component. The modularity matching algorithm applied to this new system then gives a hierarchy of interpolation matrices that can be applied the the original system to create the coarse problem.

2.2 Preconditioner module

This module is called preconditioner but maybe a better name could be smoothers or methods. In this module there are functions that take a system and provide the inverse action of some method than can be used in an iterative solver. Currently, one can build L1 smoother, symmetric/forward/backward Gauss-Seidel, multilevel L1, and multilevel Gauss-Seidel. All of these can be used as methods for the stationary solver but only the symmetric methods can be used as a preconditioner in the conjugate gradient method.

2.3 Solver module

This module implements different iterative solvers that accept linear systems, a method to base the solver on, and some parameters for the solver. Currently, only the classical stationary iterative method and preconditioned conjugate gradient is implemented. Soon I will be adding an adaptive solver to this module.

3 Example Usage

The library is currently packaged with a CLI binary that can show how the library can be used to test different solvers on your systems.

4 Goals

Classical AMG often breaks down on highly anisotropic problems often found in finite element methods. To address this failure, sometimes specific knowledge of the system can be used to carefully construct a preconidtioner. The goal of this research is to test some adaptive methods that don't require any knowledge about the system and measure the computational complexity and memory demands of these methods on systems where classical AMG performs poorly.