

### DD and PD Method 3:

Method 3 is an iterative method of solving for the value of the firm ( $V$ ) and volatility of that value ( $\sigma V$ ). It assumes an initial value of  $\sigma V$  as the volatility of equity ( $\sigma E$ ). Using the estimated value of  $\sigma V$ , the value of  $V$  is calculated on a daily basis using the black-scholes equation used in method 2. The returns on value per day are then calculated, and from there a standard deviation on value returns per year are determined. This is the next estimate of  $\sigma V$ . The method is then repeated till the error between iterations of  $\sigma V$  fall within a specified value. From here, DD and PD are calculated using the equation used in method 1, with  $V$  serving as equity ( $E$ ) + face value of debt ( $F$ ). This method was then compared to the previous two methods, naïve and direct.

### Correlation Statistics:

- The distance to default (DD) had a negative correlation with probability of default (PD). This implies that as the DD increases, the PD decreases.
- The correlation between DD and PD for method 3 is nearly identical to that of methods 1 and 2. This similarity is expected, as the mathematical relationship between DD and PD is practically identical between methods.
- The DD for method 1, 2 and 3 had nearly 1 to 1 correlation. This implies that the values follow similar trends and values. However, due to the varying accuracy of the methods used to calculate the DD values, the values are slightly different. That leads to the imperfect 1 to 1 correlation between methods.
- The PD values are similarly correlated between methods as DD. This should be expected, as PD is calculated directly from the standard normal distribution of  $-DD$ , regardless of method.
- However, it was odd the naïve and iterative methods had correlations of roughly -0.4, while the direct method has only a correlation of about -0.2. It would have been expected that the PD's would have had more similar correlations between each method.

### Mean/Percentile and Comparison Plots:

- For all methods, the trends of the mean and percentiles of the DD matched near perfectly.
- This implies that companies at each percentile of the DD are values are subject to the same trends as all others. Therefore, you can assume that most companies follow the general macro trends represented by the average DD.
- Each of the percentile and mean plots for each method are near identical with one another.
- For the mean comparisons, the naïve method rests in between the two others. The direct method has a higher DD average, while the iterative method has a lower DD average. Each method does follow near identical trends.
- Since the iterative method has the lowest DD values on average, it also has the highest PD values as expected.

### Recession Descriptive Statistics:

- For normal (non-recession) years, the distance to default had maximum's in the thousands for the direct method, and only the low hundreds for the iterative and naïve method. For recession years, the direct method maximum fell, while the iterative and naïve methods rose.
- Similarly, the minimum values for are larger in recession years for all methods. This doesn't fall under expectations, as it would be assumed the lowest performers would exist in recession years.
- The percentile values for DD shifted downward for each method roughly 1 point during recession years. The PD values increased by orders of magnitude varying from 10 to  $10^6$ . The iterative method had the smallest magnitude of change between percentiles in recession and non-recession years.
- On average, normal years had higher distance to default values. Since recession years tend to shock the assets of firms, it would make sense that they have a marked decrease in their DD for those periods.
- Similarly, probability of default, on average, rises during recession years, in large part due to the decrease in average DD values.
- The standard deviation of the probability of default rises in recession years. Because PD is determined on a standard normal curve ( $PD = CDF[\text{"normal"}, -DD]$  in the SAS code), it is reasonable that should there be a marked decrease in DD, an exponentially larger portion of the distribution will be captured. Therefore, as the DD decreases in recession years, it should lead to greater variance in the PD.
- On average, the iterative method produced the lowest values of DD and the highest values of PD for recession and non-recession years.

### Recession Plots:

- Distance of default would often form a local minimum during recession years. During the mid-1970's and late 2000's, the DD plummeted, likely indicating an increase in bankruptcies and market wide failure. However, during recessions in 1981, the DD barely formed a local minimum. This may indicate the recession only hit firms in a few industries (such as energy/natural resources) rather than the entire market.
- Probability of default spikes, creating local maximum, often times recession years. PD followed an inverted trend with DD, with the largest spikes occurring during 1974, 2001 and 2007-2008. This is expected due to the relationship previously stated between DD and PD. As DD decreases, the PD will increase exponentially.
- Between each method, the minimum and maximums for recession years appear to line up. However, the iterative method shows lower DD values and higher PD values for recession years than method 1 and 2. The direct method appears to have the highest DD values for recession years.

Stress Index:

- The distance to default and stress index appear to be inversely correlated. Therefore, it makes sense that the Stress Index and PD appear to rise and fall at the same time, and at similar relative magnitudes. As DD decreases, the Stress Index and PD both increase.
- This relationship holds with each method.

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- The corporate bond rate doesn't appear to follow any trend with the DD values in the 70's and 80's.
- After 1990, the DD and corporate bond rate appear to be inversely related, with many of the drops in DD occurring at the same time as sharp rises in bond rates.
- Often times, the PD appears to spike right before a spike in the bond rate. This holds with each method. Since these spikes often occurred in recession years, as shown in previous graphs, it may be implied that the bond rate goes up during the years immediately following periods of economic downturn.