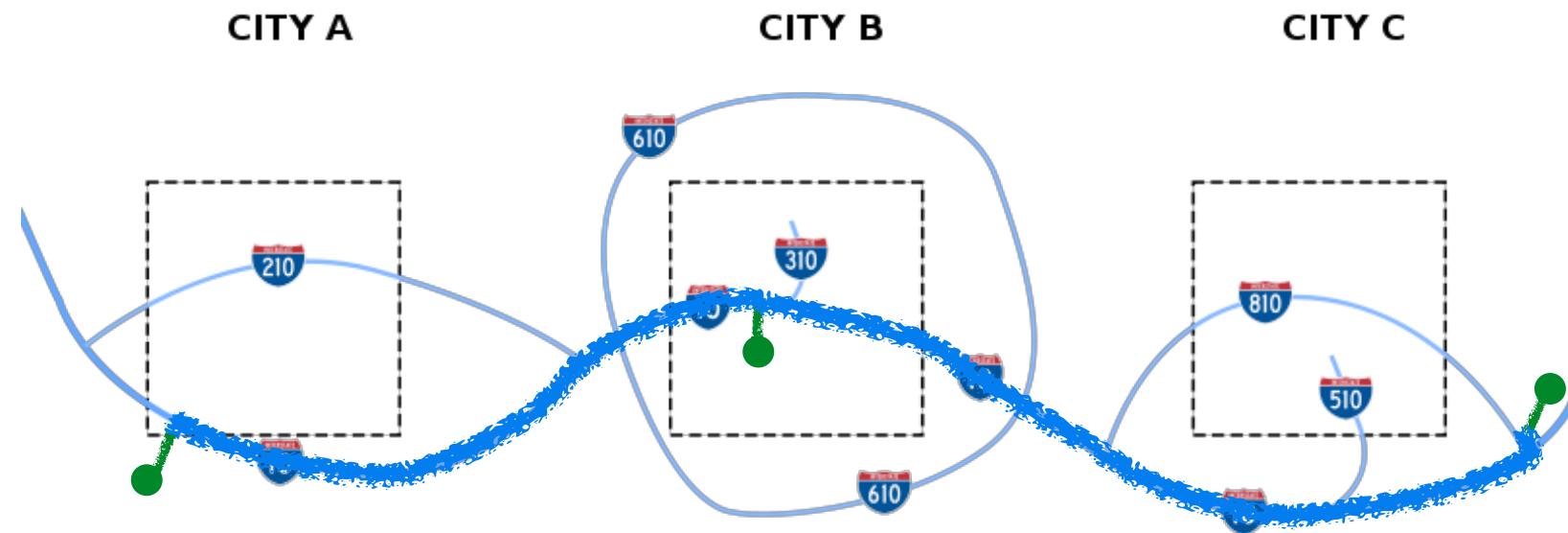


Planning with Sampling Roadmaps



Building a path to goal

Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
 - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- **Sampling-based Search (build graph):**
 - **Probabilistic Road Maps, Rapidly-exploring Random Trees**
- Optimization and local search:
 - Gradient descent, Potential fields, Simulated annealing, Wavefront

Where is this?



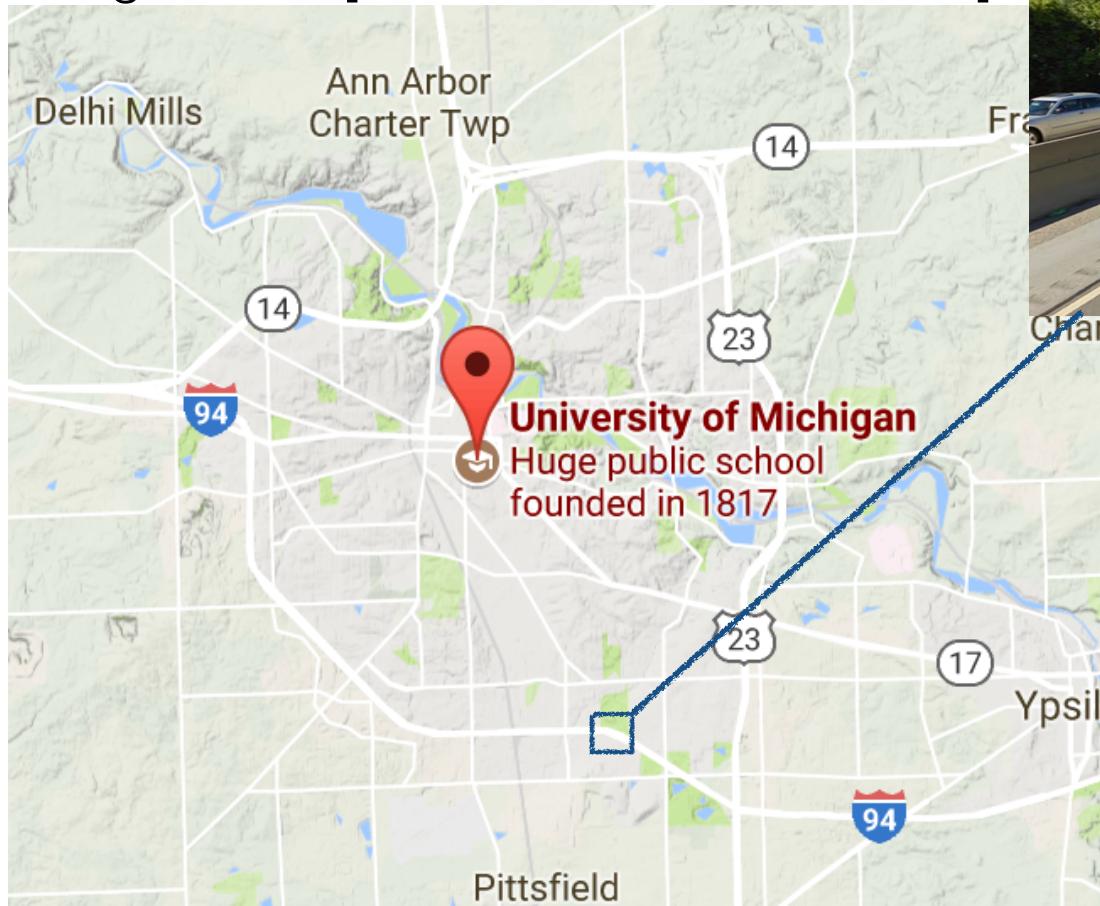
©2010 Google
©2015 Google

EXIT 177
State St
1 MILE



Configuration (lat,lon): [42.237631, -83.7147289]

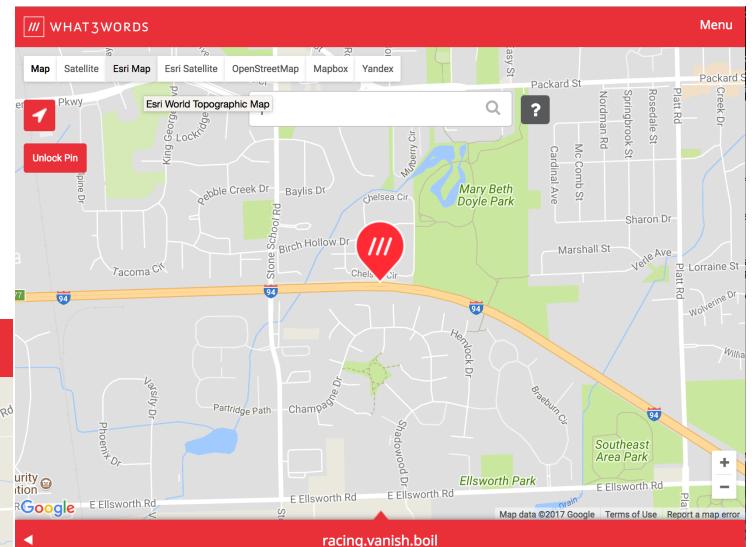
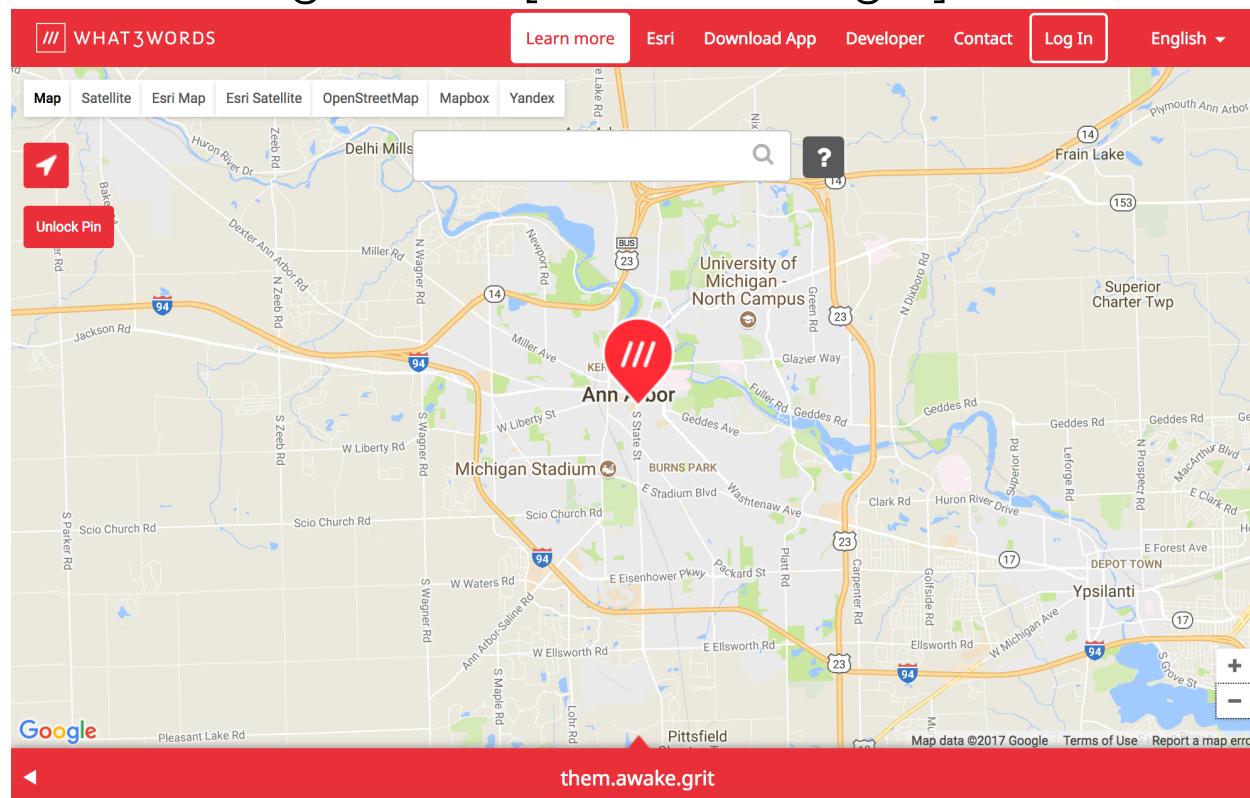
Configuration: [42.2780436, -83.7404128]



Configuration (lat,lon): [42.237631, -83.7147289]

Alternatively...

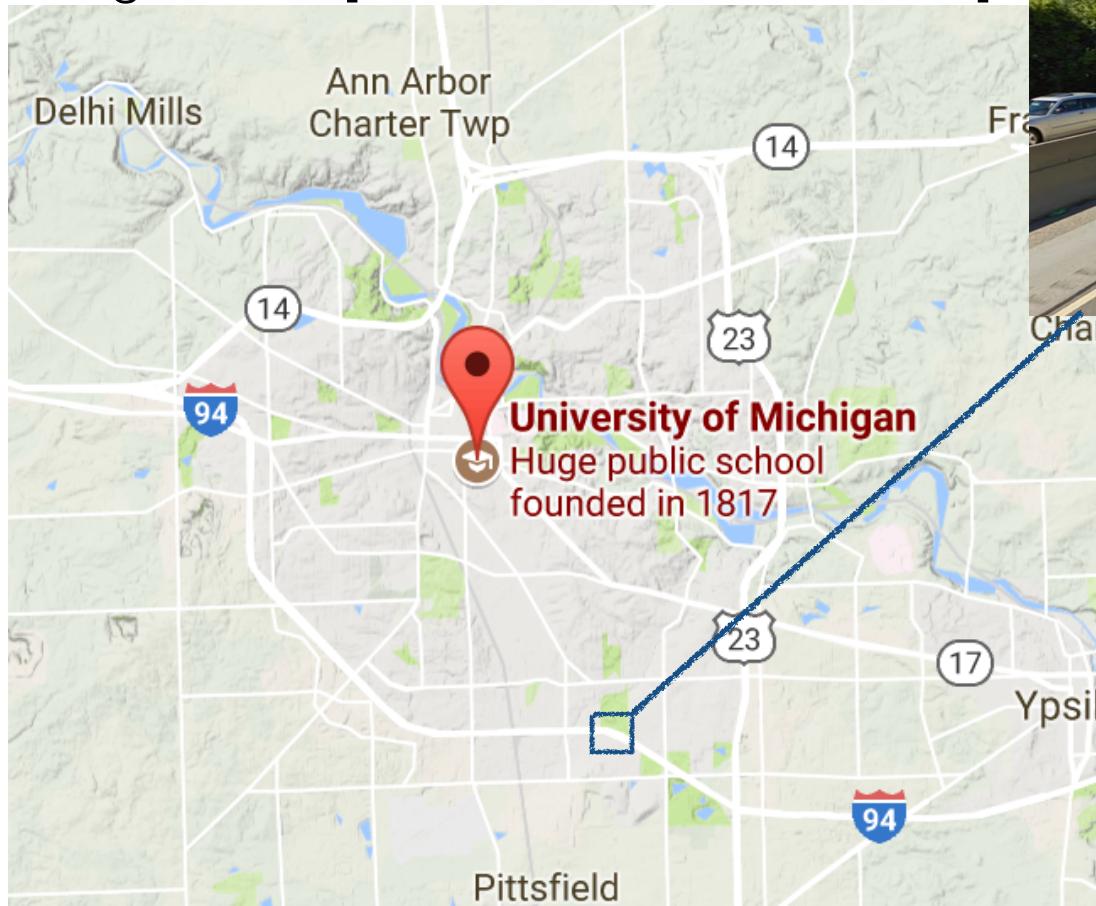
Configuration: [them.awake.grit]



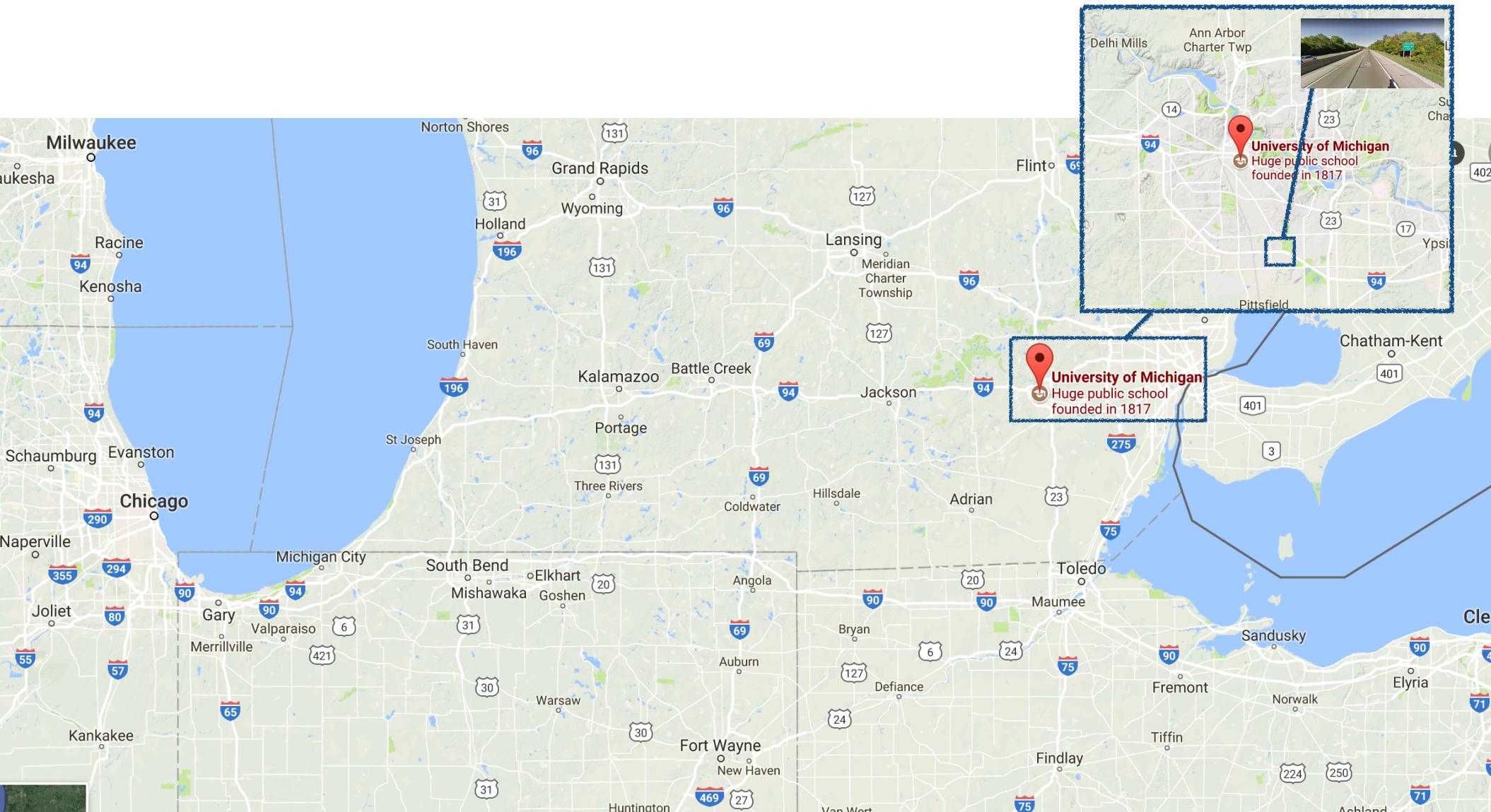
Configuration (lat,lon): [racing.vanish.boil]

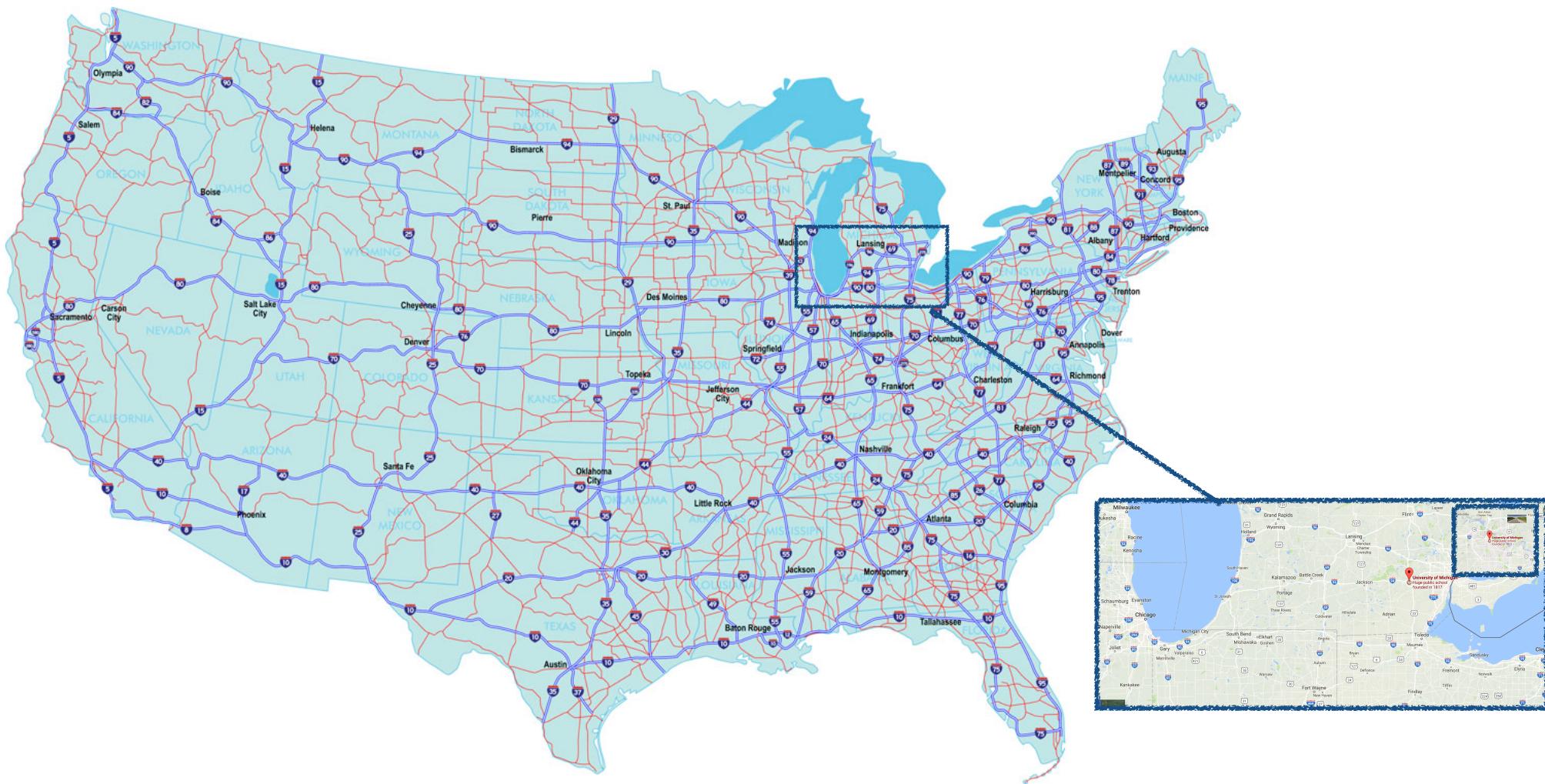
Is this a usable
representation for
configuration?

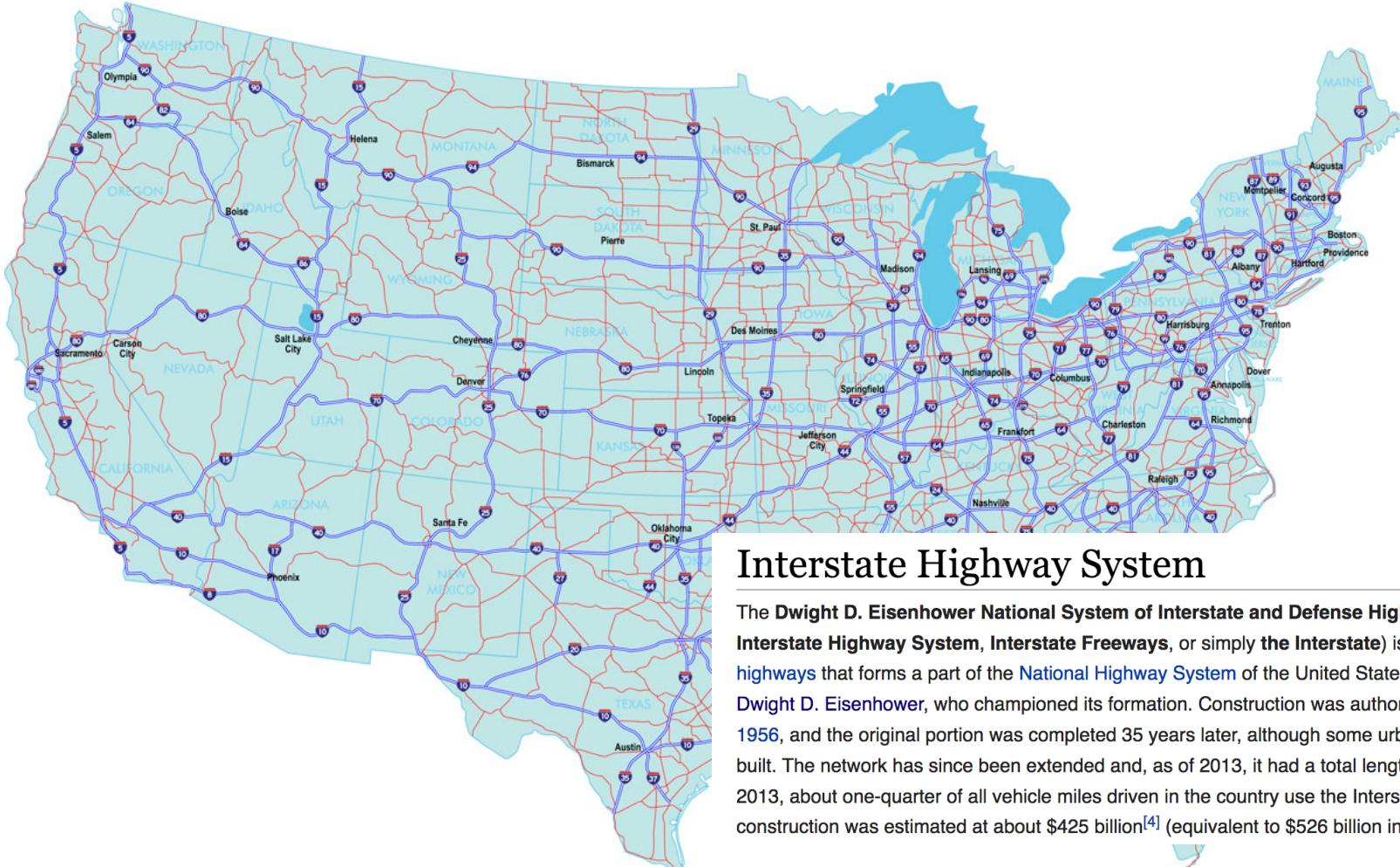
Configuration: [42.2780436, -83.7404128]



Configuration (lat,lon): [42.237631, -83.7147289]

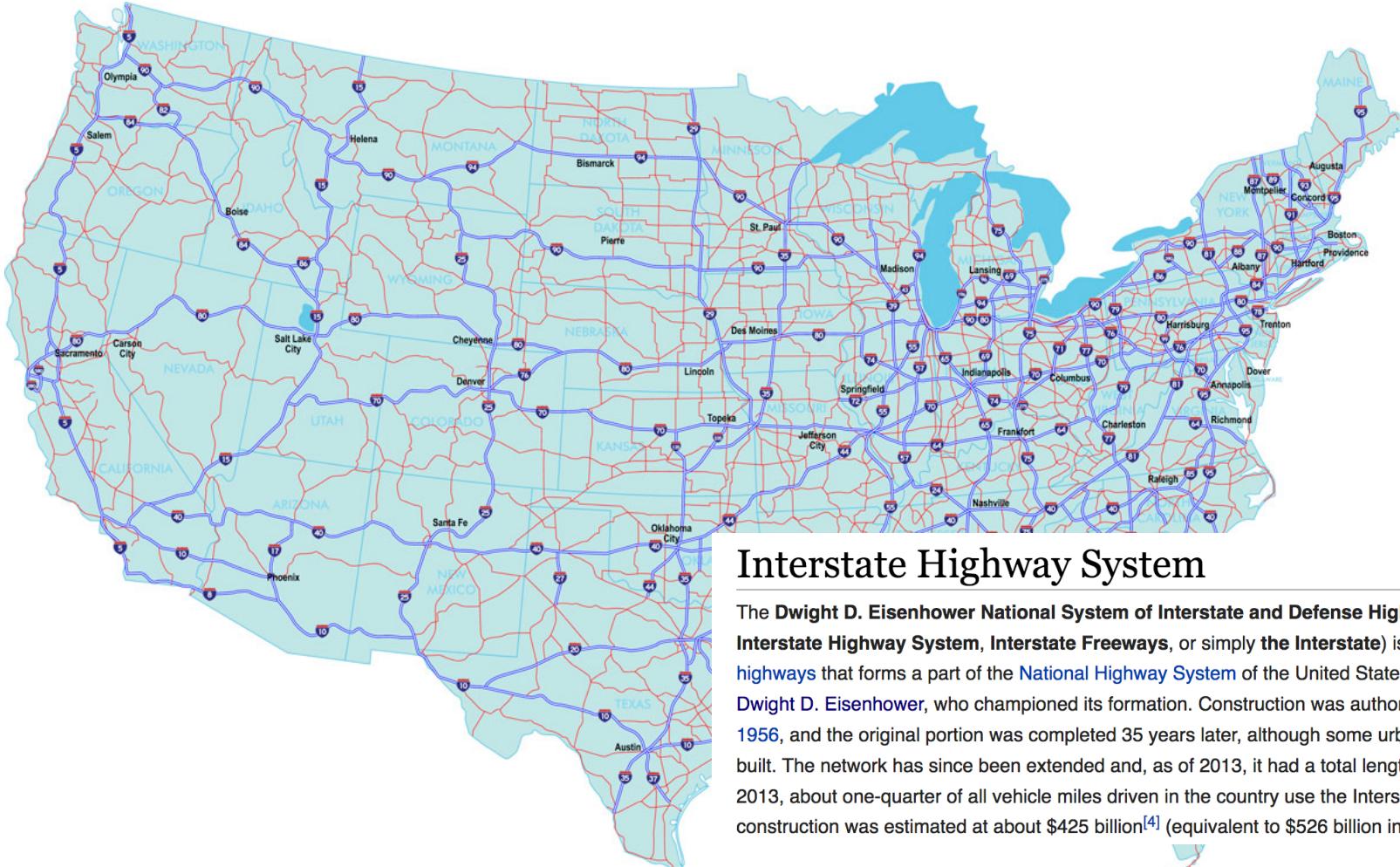




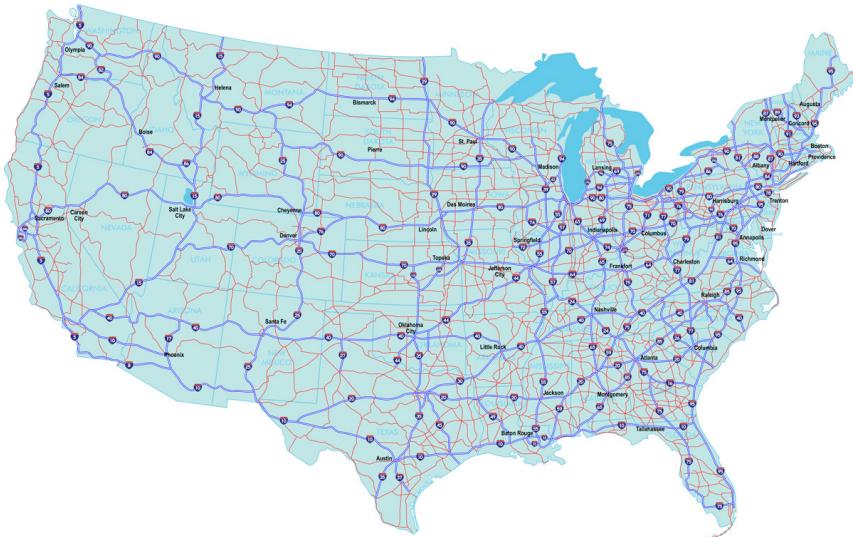


CHINA'S PROPOSED BELT AND ROAD INITIATIVE

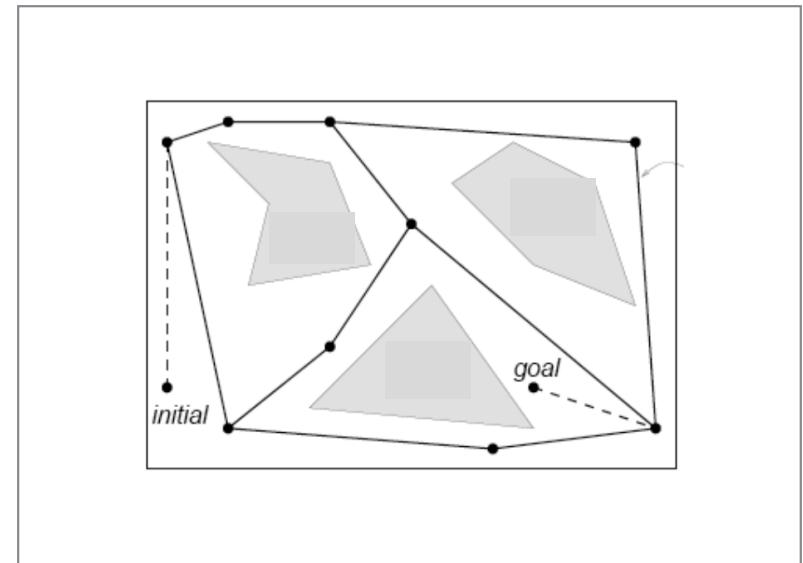




Roadmaps



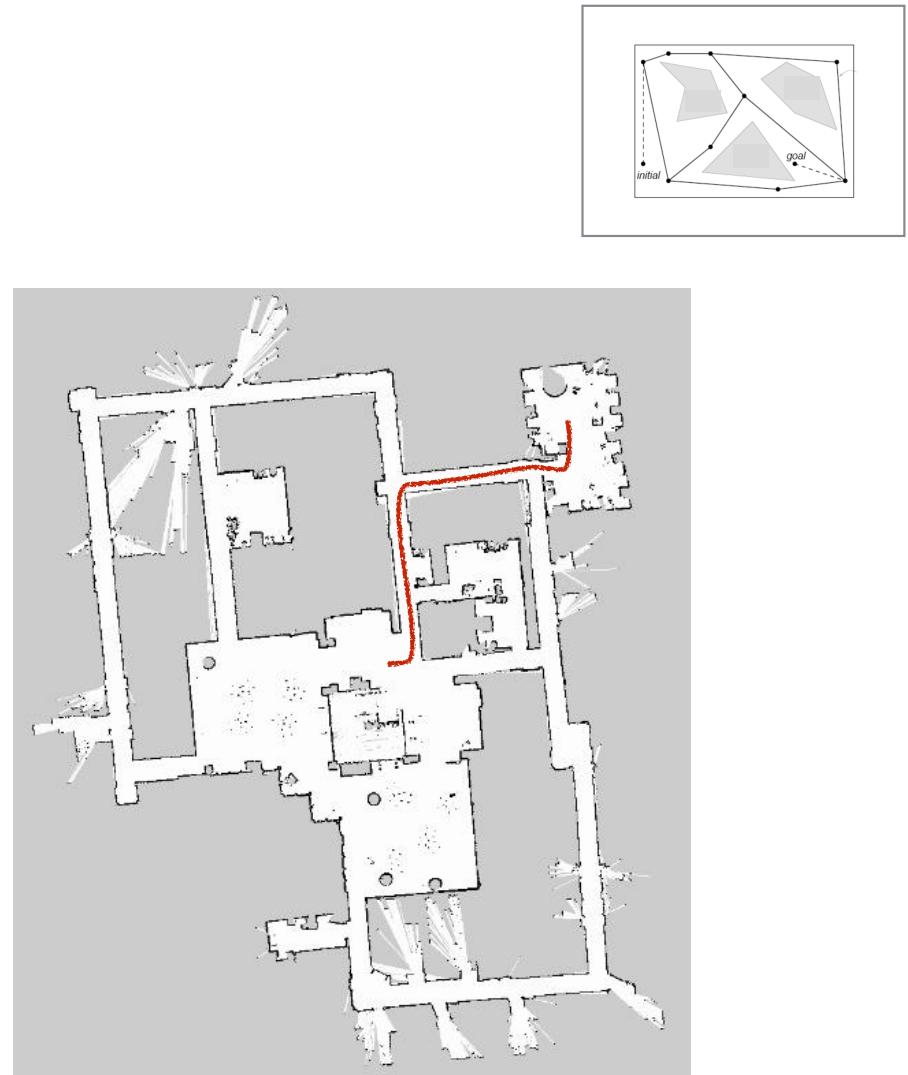
Roadmap over geolocations



Roadmap over robot configurations

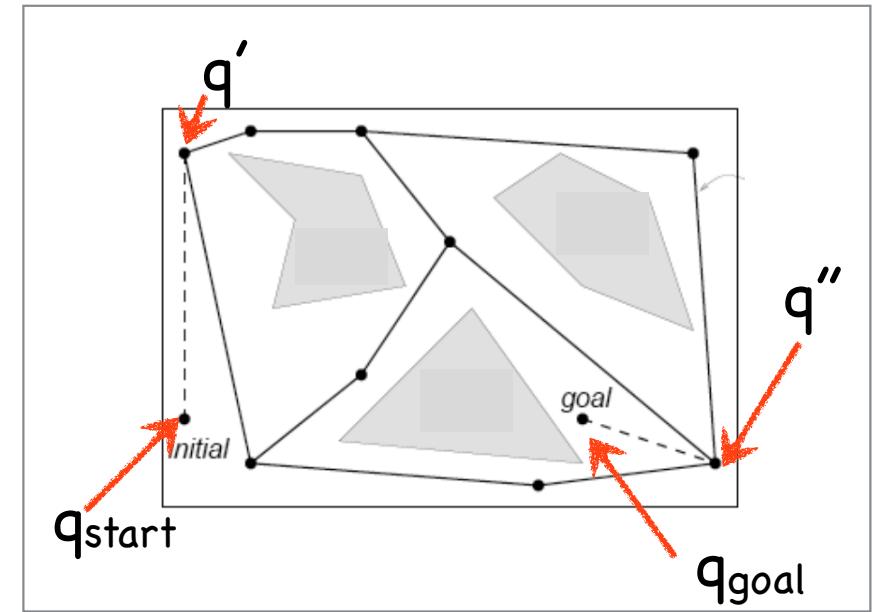
Roadmaps

- Graph search assumed C-space as a fixed uniform grid
 - finite set of discretized cells
- How does this scale beyond planar navigation?
 - curse of dimensionality
- Roadmaps are a more general notion of graphs in C-space



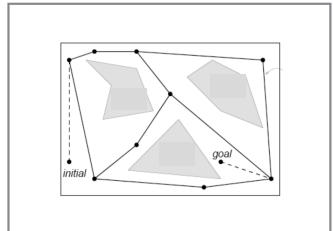
Roadmap Definition

- A roadmap RM is a union of curves s.t. all start and goal points in C-space (Q_{free}) can be connected by a path
- Roadmap properties:



- Accessibility: There is a path from $q_{start} \in Q_{free}$ to some $q' \in RM$
- Departability: There is a path from $q'' \in RM$ to $q_{goal} \in Q_{free}$
- Connectivity: there exists a path in RM between q' and q''

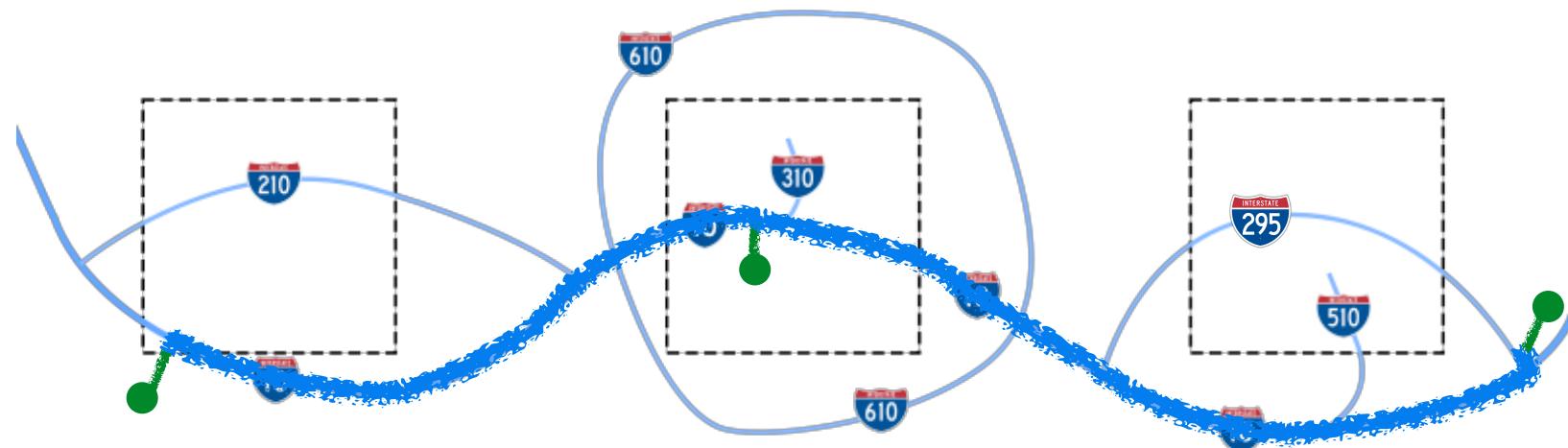
What cities? What universities?



????????

????????

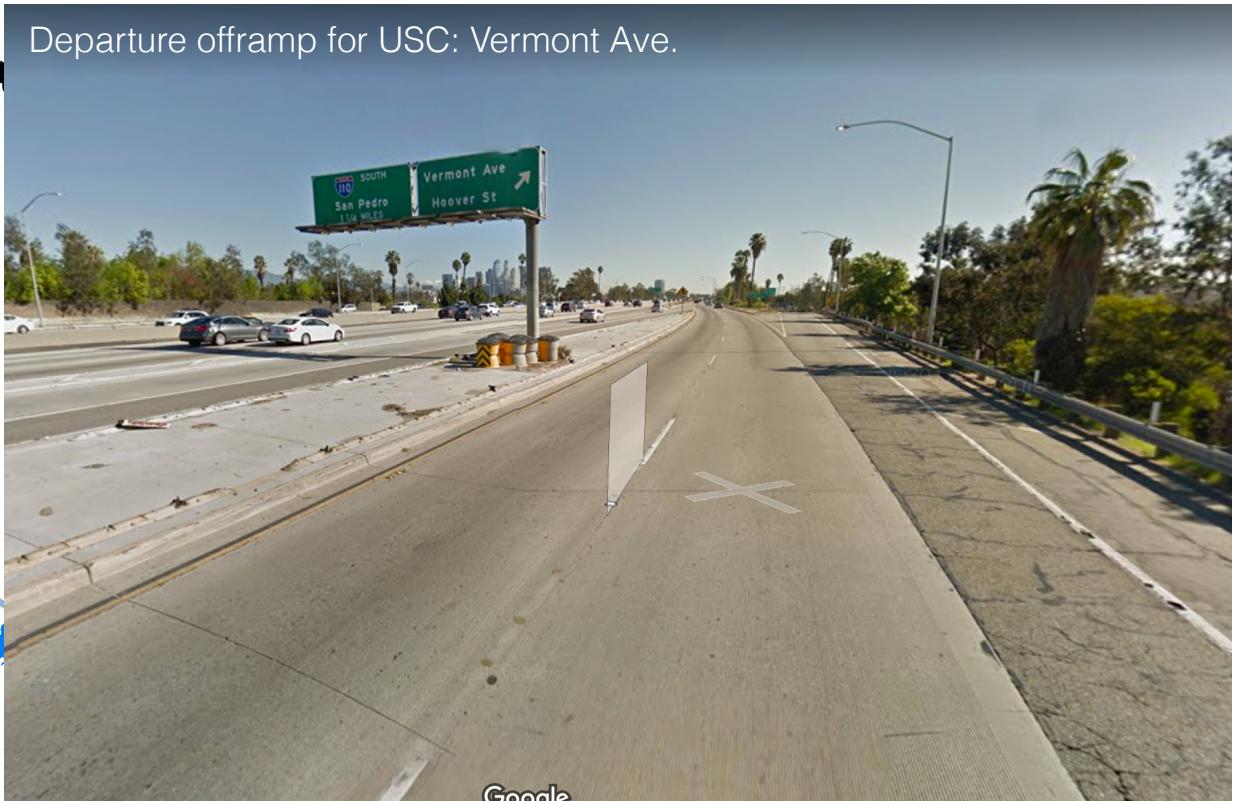
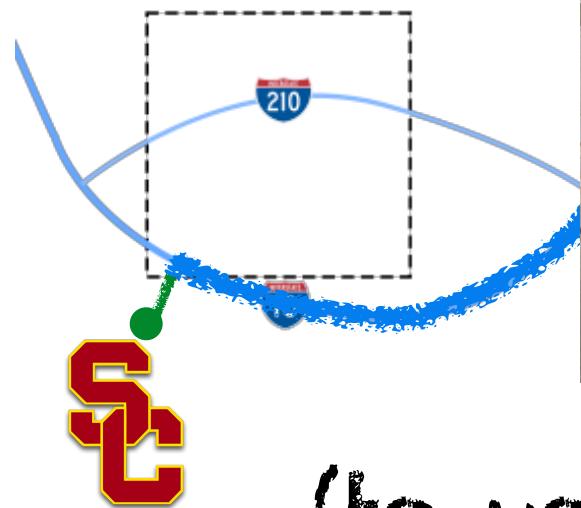
?????????



(to your best approximation)

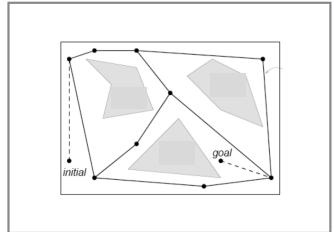
What cities? ▶

“Los Angeles”



(to your best approximation)

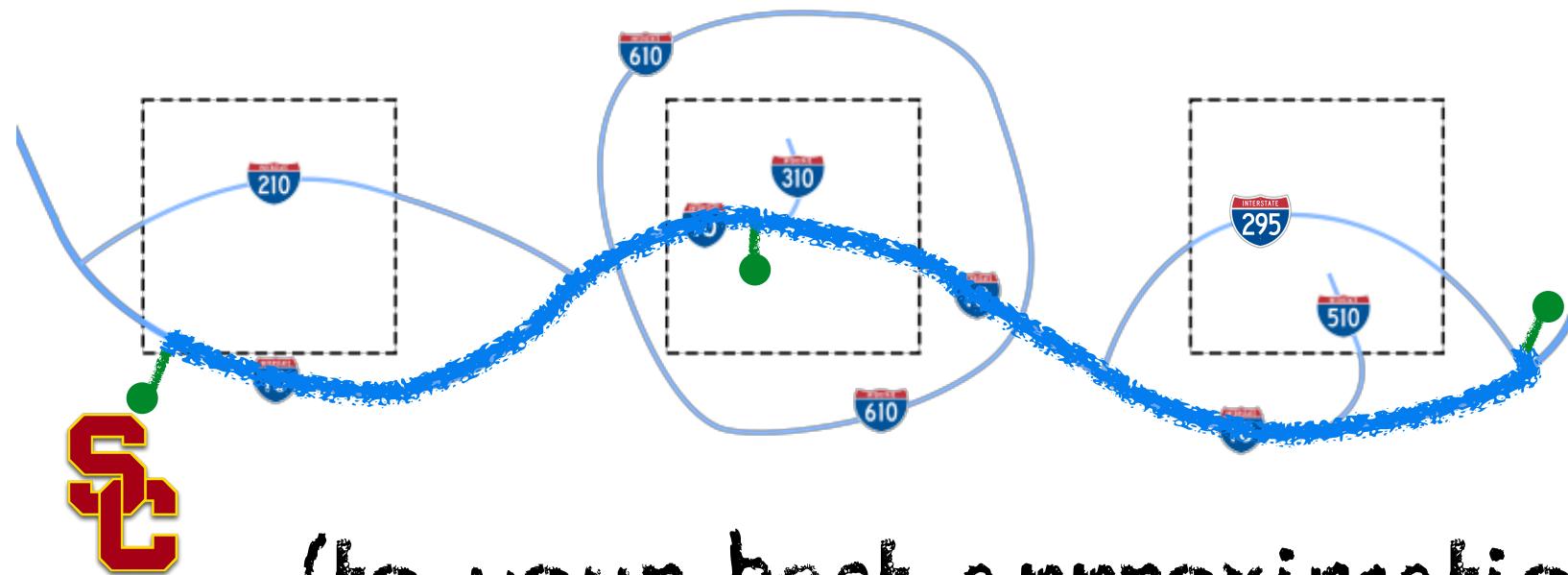
What cities? What universities?



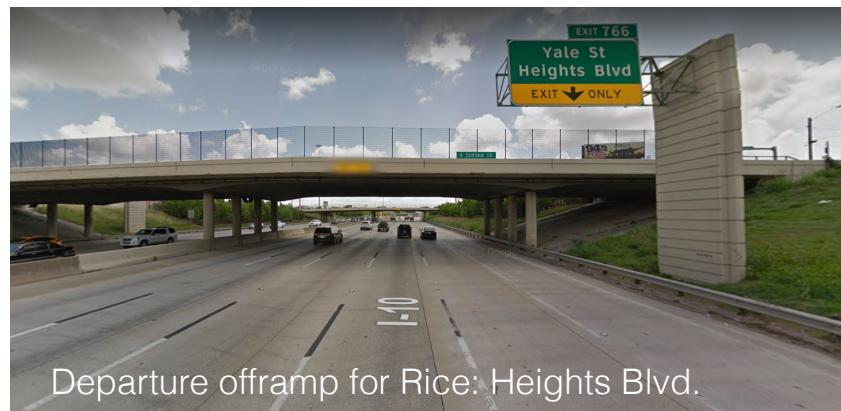
“Los Angeles”

???????

?????????

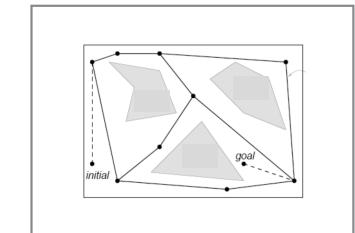


(to your best approximation)



Departure offramp for Rice: Heights Blvd.

that universities?



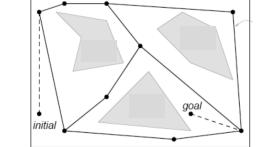
"Houston"

?????????



(to your best approximation)

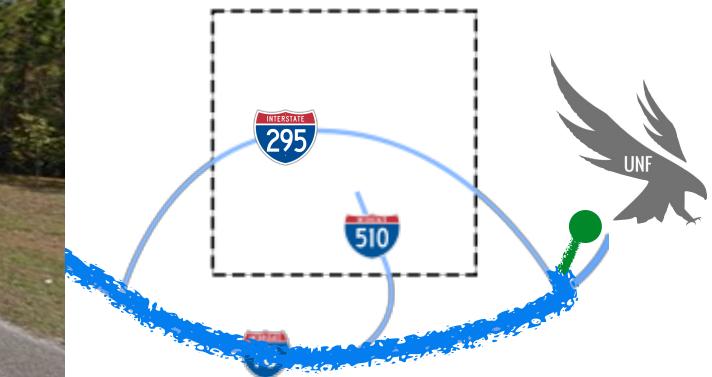
What cities? What universities?



Departure offramp for North Florida: Town Center Pkwy.



“Jacksonville”



(to your best approximation)

Departure offramp for Michigan: State St.

@2016 Google



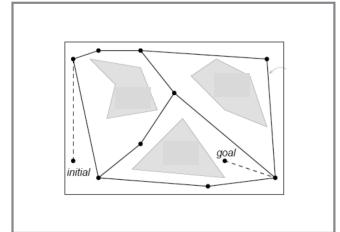
EXIT 177
State St
1 MILE

Better departure offramp for Michigan: Plymouth Rd.

@2016 Google



Basic Roadmap Planner



1) Build the roadmap RM as graph $G(V,E)$

- ④ V : nodes are “valid” in C-space in Q_{free}
 - a configuration q is valid if it is not in collision and within joint limits
- ④ E : an edge $e(q_1, q_2)$ connects two nodes if a free path connects q_1 and q_2
 - all configurations along edge assume to be valid

2) Connect start and goal configurations to RM at q' and q'' , respectively

3) Find path in RM between q' and q''

2 Approaches to Roadmaps

Deterministic:

complete algorithms

- Visibility Graph
- Voronoi
- Silhouette Method

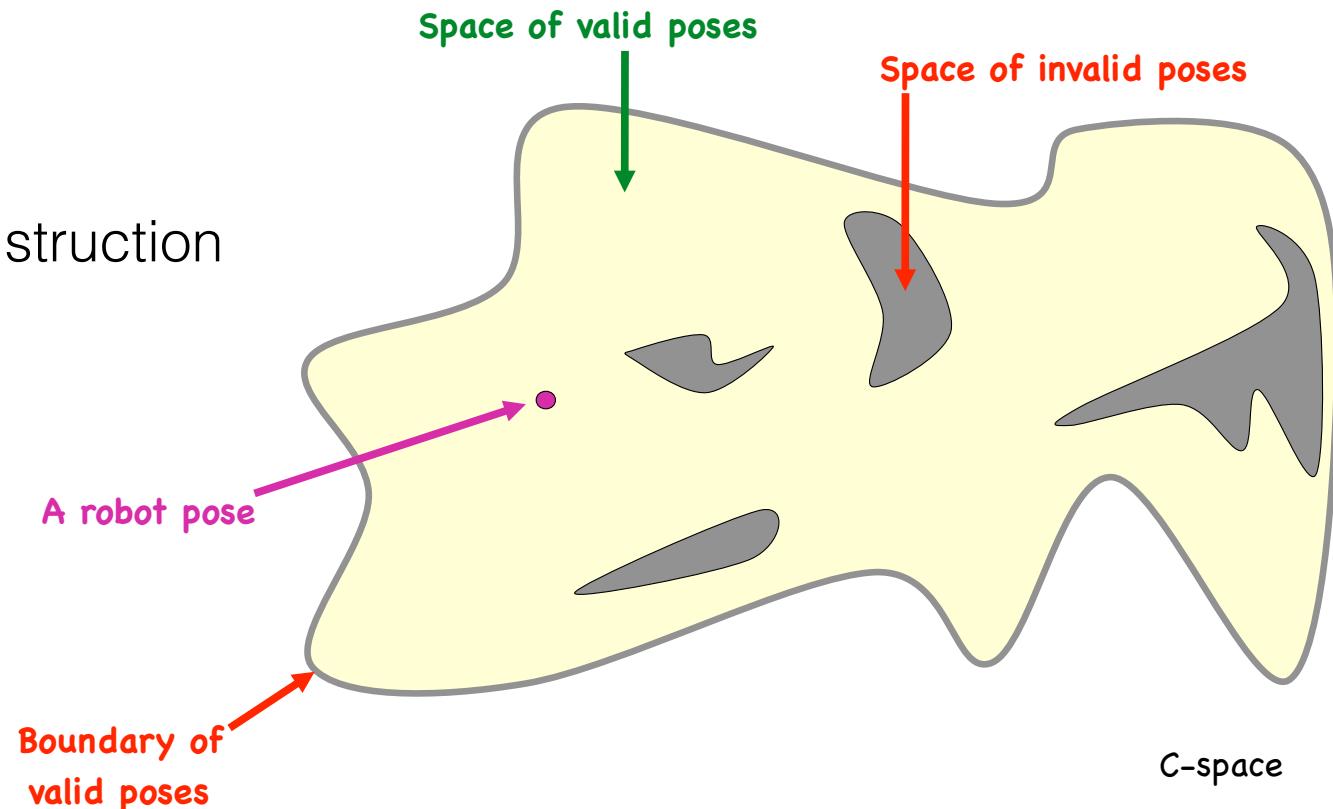
Probabilistic:

C-space sampling

- PRM (multi-query)
- RRT (single query)

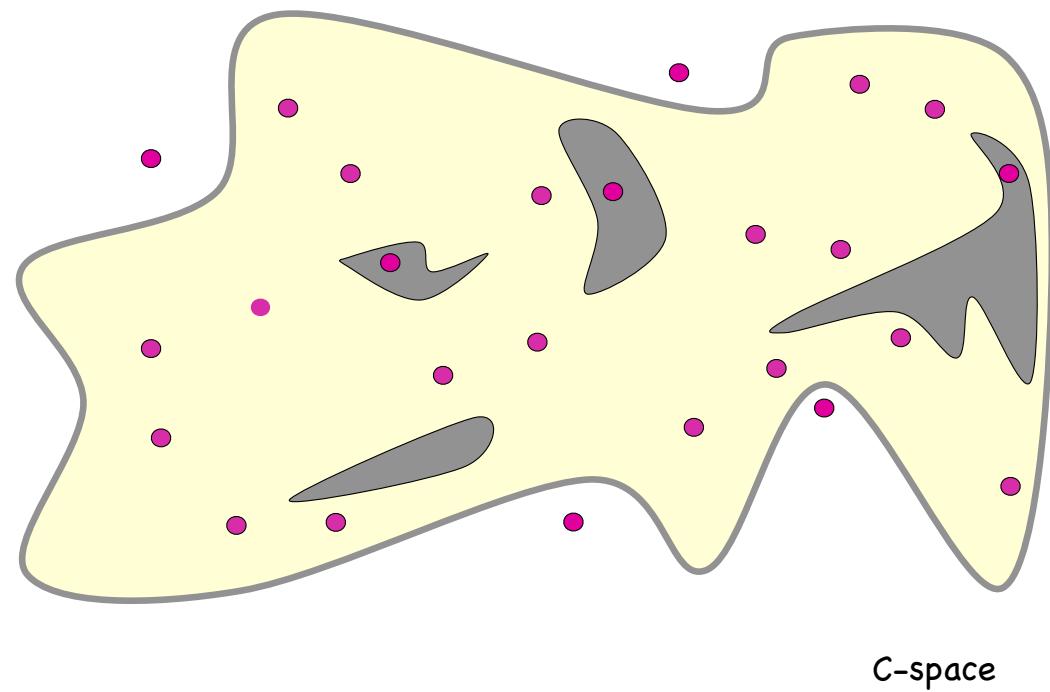
Probabilistic road maps

- Two phases
 - Roadmap construction
 - Path Query



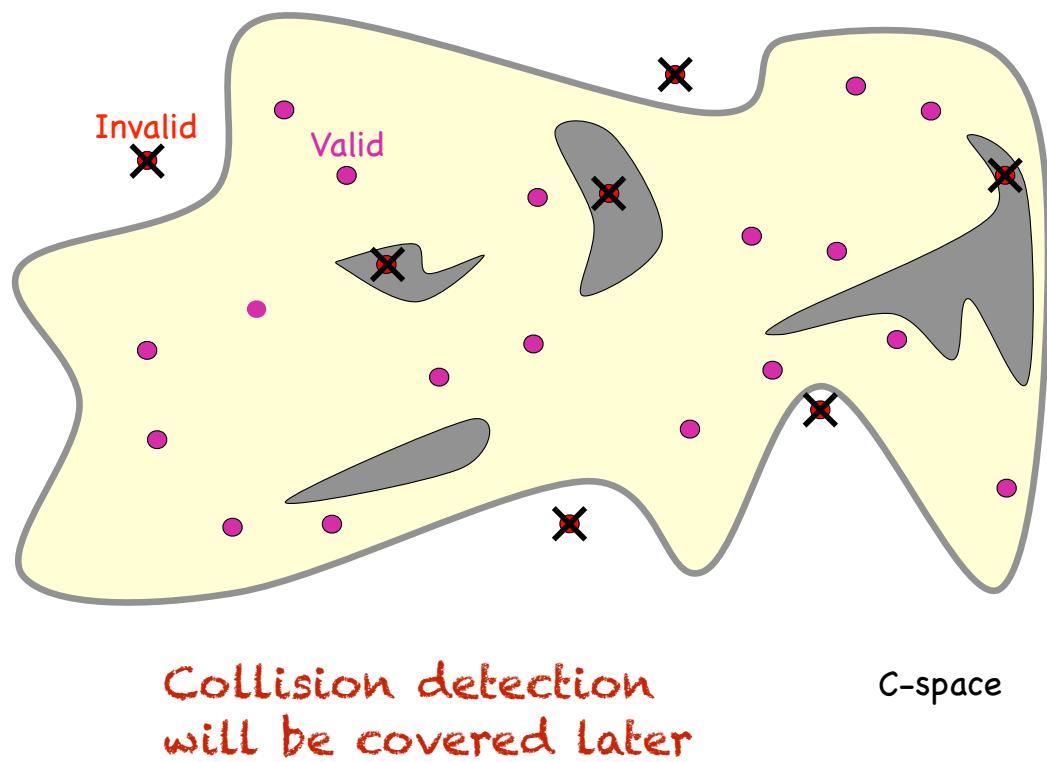
PRM: construction phase

- 1) Select N sample poses at random
- 2) Eliminate invalid poses
- 3) Connect neighboring poses



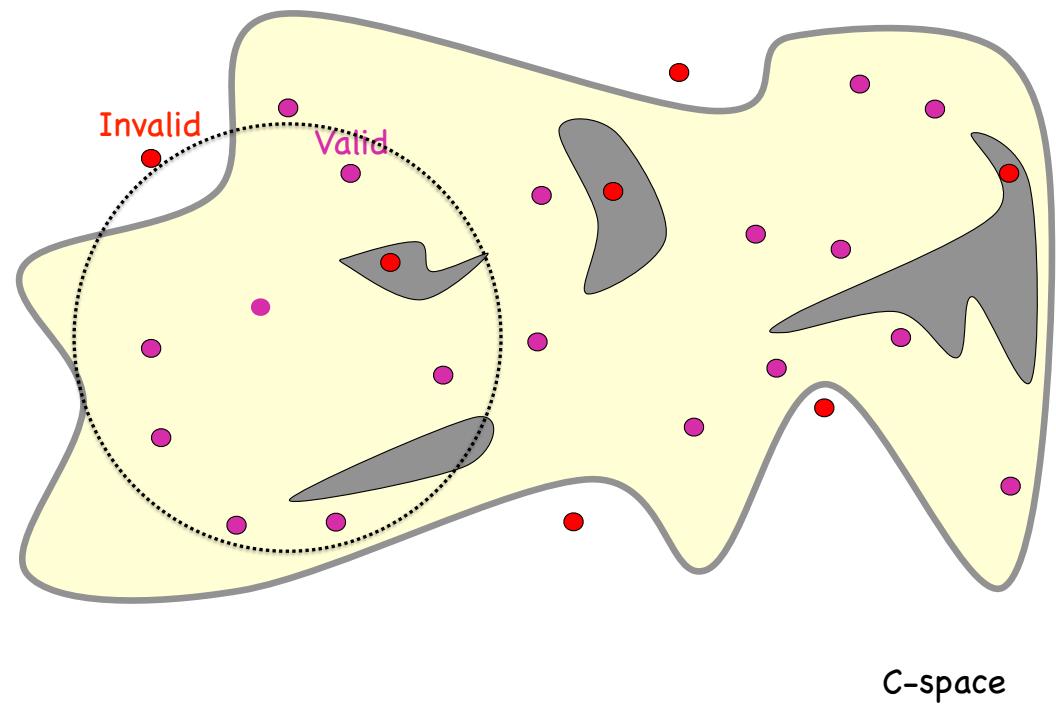
PRM: construction phase

- 1) Select N sample poses at random
- 2) **Eliminate invalid poses**
- 3) Connect neighboring poses



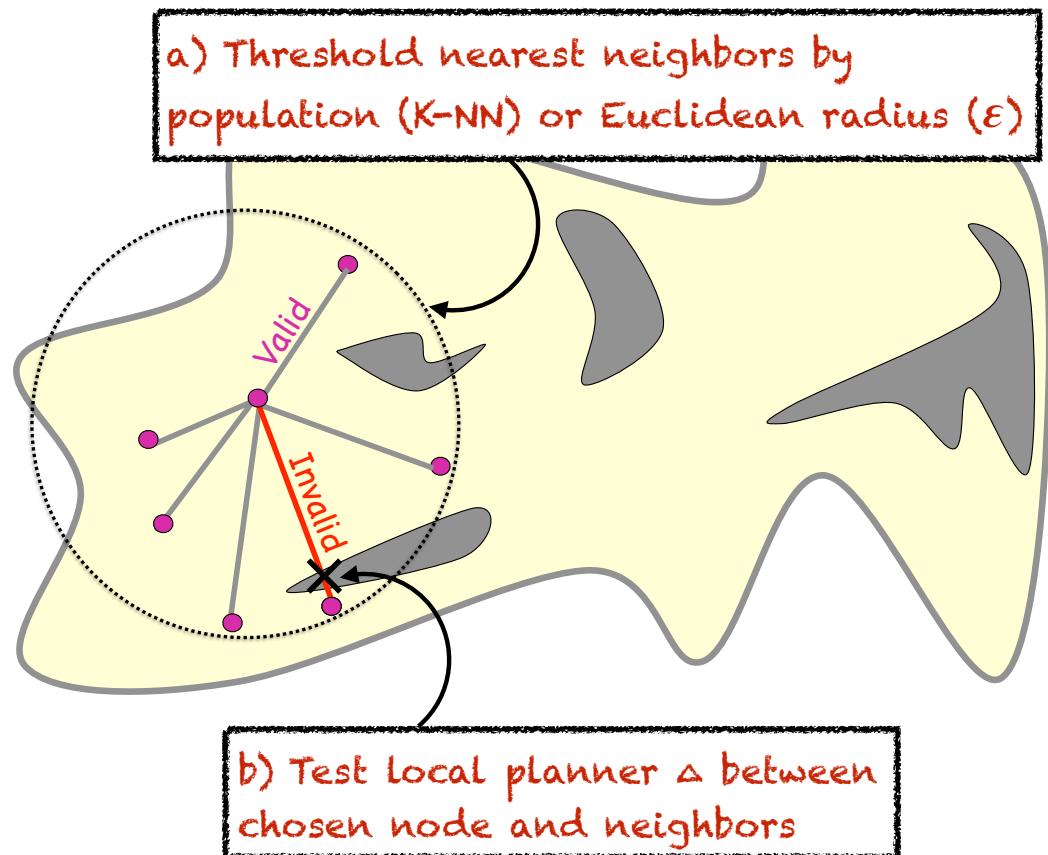
PRM: construction phase

- 1) Select N sample poses at random
- 2) Eliminate invalid poses
- 3) **Connect neighboring poses**



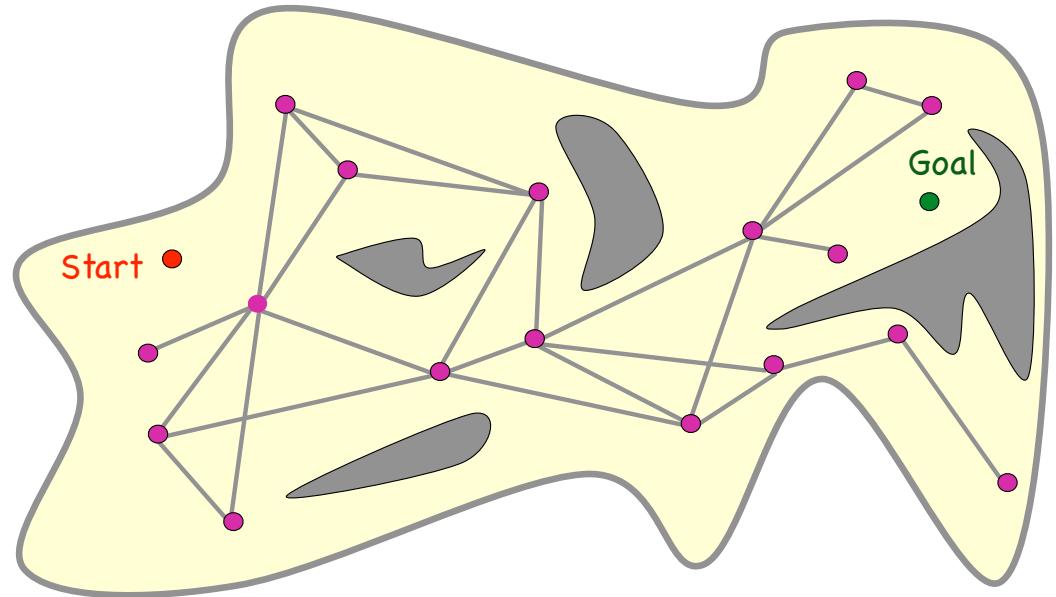
PRM: construction phase

- 1) Select N sample poses at random
- 2) Eliminate invalid poses
- 3) Connect neighboring poses



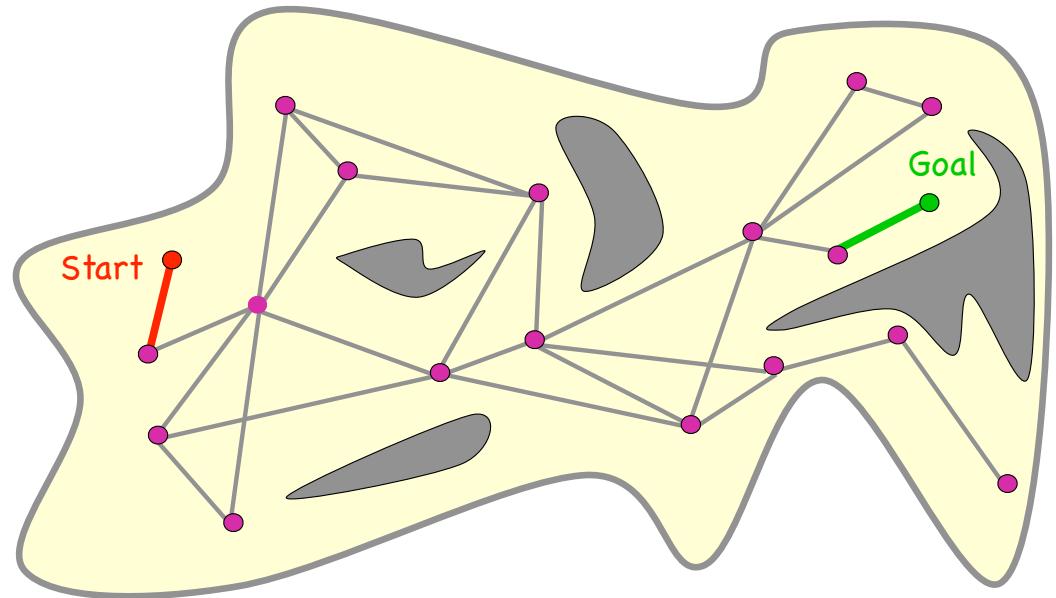
PRM: query phase

- 1) Given constructed roadmap, start pose, and goal pose
- 2) Attach goal and start to nearest roadmap entry nodes
- 3) Search for path between roadmap entry nodes
- 4) Return path with entry and departure edges



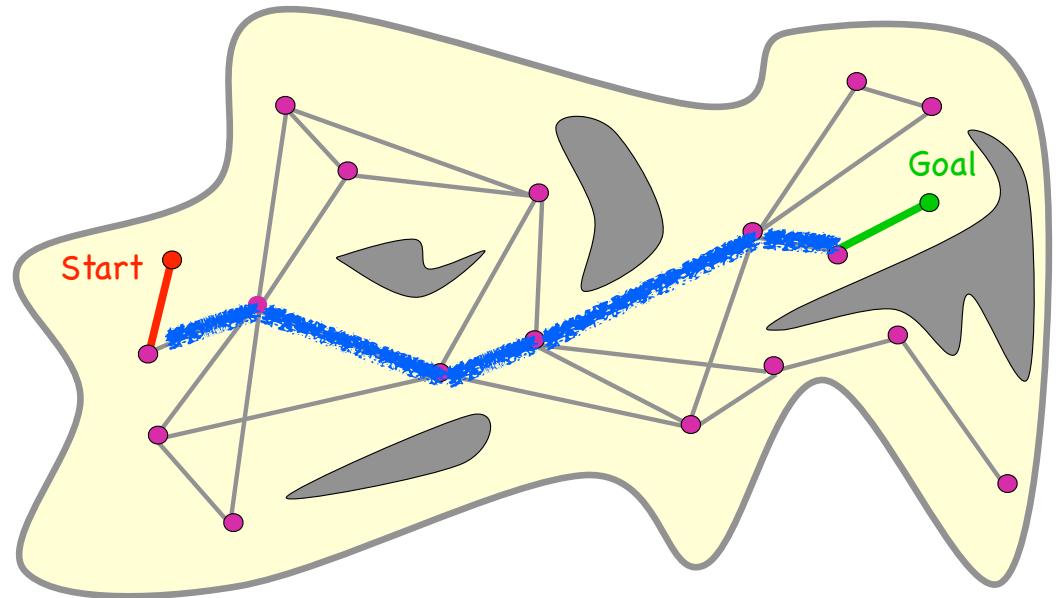
PRM: query phase

- 1) Given constructed roadmap, start pose, and goal pose
- 2) **Attach goal and start to nearest roadmap entry nodes**
- 3) Search for path between roadmap entry nodes
- 4) Return path with entry and departure edges



PRM: query phase

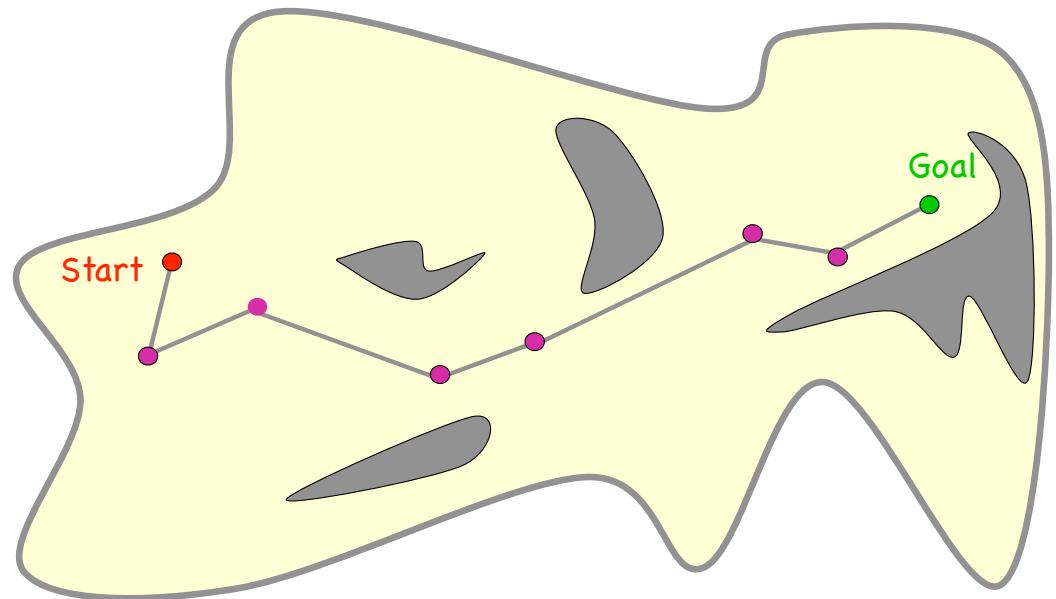
- 1) Given constructed roadmap, start pose, and goal pose
- 2) Attach goal and start to nearest roadmap entry nodes
- 3) **Search for path between roadmap entry nodes**
- 4) Return path with entry and departure edges



Remember: graph search algorithms
A*, Dijkstra, BFS, DFS

PRM: query phase

- 1) Given constructed roadmap, start pose, and goal pose
- 2) Attach goal and start to nearest roadmap entry nodes
- 3) Search for path between roadmap entry nodes
- 4) **Return path with entry and departure edges**



Considerations

- Number of samples wrt. C-space dimensionality
- Balanced sampling over C-space
- Choice of distance (e.g., Euclidean)
- Choice of local planner (e.g., line subdivision)
- Selecting neighbors: (e.g., K-NN, kd-tree, cell hashing)

Single Query Planning

- Given specific start and goal configurations
- Grow trees from start and goal towards each other
- Path is found once trees connect
- Focus sampling in unexplored areas of C-space and moving towards start/goal
- Common algorithms:
 - ESTs (expansive space trees)
 - **RRTs (rapidly exploring random trees)**

RRT Algorithm

Extend graph towards a random configuration and repeat

```
BUILD_RRT( $q_{init}$ )
1    $T.init(q_{init})$ ;
2   for  $k = 1$  to  $K$  do
3        $q_{rand} \leftarrow \text{RANDOM.CONFIG}()$ ;
4       EXTEND( $T, q_{rand}$ );
5   Return  $T$ 
```

RRT Algorithm

Extend graph towards a random configuration and repeat

```
BUILD_RRT( $q_{init}$ )
1    $T.init(q_{init})$ ;
2   for  $k = 1$  to  $K$  do
3        $q_{rand} \leftarrow \text{RANDOM.CONFIG}()$ ;
4       EXTEND( $T, q_{rand}$ );
5   Return  $T$ 
```

```
EXTEND( $T, q$ )
1    $q_{near} \leftarrow \text{NEAREST.NEIGHBOR}(q, T)$ ;
2   if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3        $T.add\_vertex(q_{new})$ ;
4        $T.add\_edge(q_{near}, q_{new})$ ;
5       if  $q_{new} = q$  then
6           Return Reached;
7       else
8           Return Advanced;
9   Return Trapped;
```

Extend graph towards a random configuration

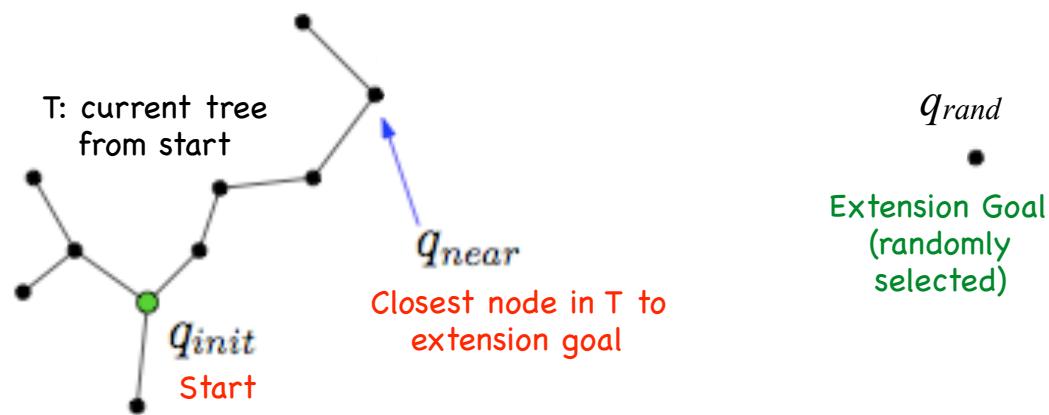


Figure 3: The EXTEND operation.

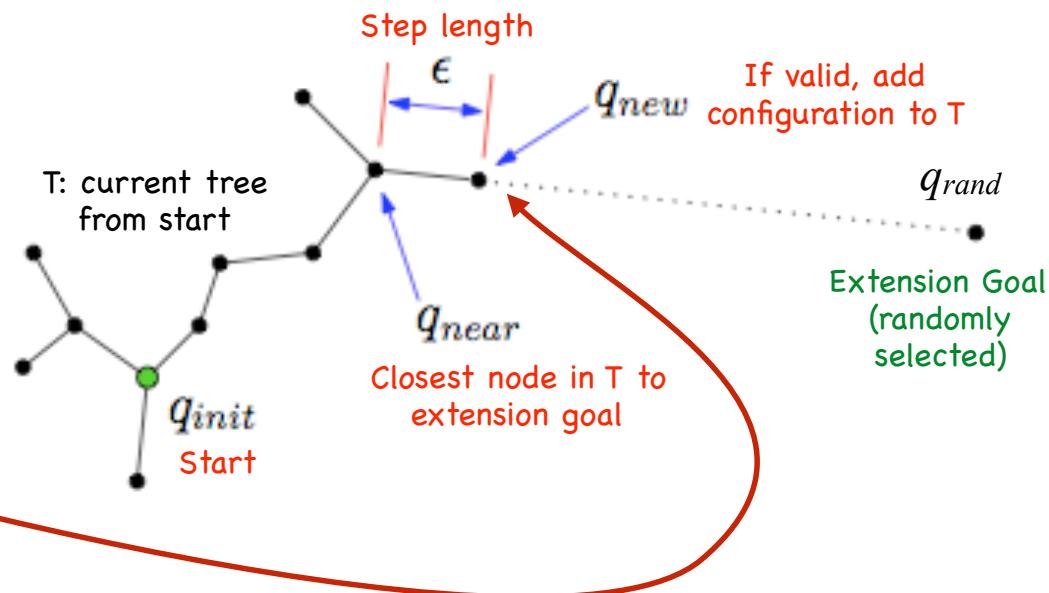
RRT Algorithm

Extend graph towards a random configuration and repeat

```
BUILD_RRT( $q_{init}$ )
1    $T.init(q_{init})$ ;
2   for  $k = 1$  to  $K$  do
3        $q_{rand} \leftarrow RANDOM.CONFIG()$ ;
4       EXTEND( $T, q_{rand}$ );
5   Return  $T$ 
```

```
EXTEND( $T, q$ )
1    $q_{near} \leftarrow NEAREST.NEIGHBOR(q, T)$ ;
2   if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3        $T.add\_vertex(q_{new})$ ;
4        $T.add\_edge(q_{near}, q_{new})$ ;
5       if  $q_{new} = q$  then
6           Return Reached;
7       else
8           Return Advanced;
9   Return Trapped;
```

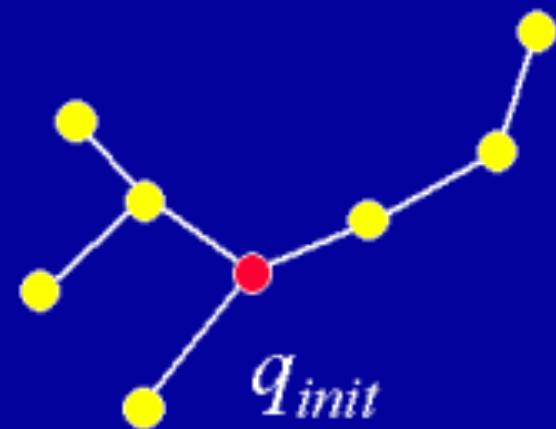
Extend graph towards a random configuration



Generate and test new configuration along vector in C-space from q_{near} to q_{rand}

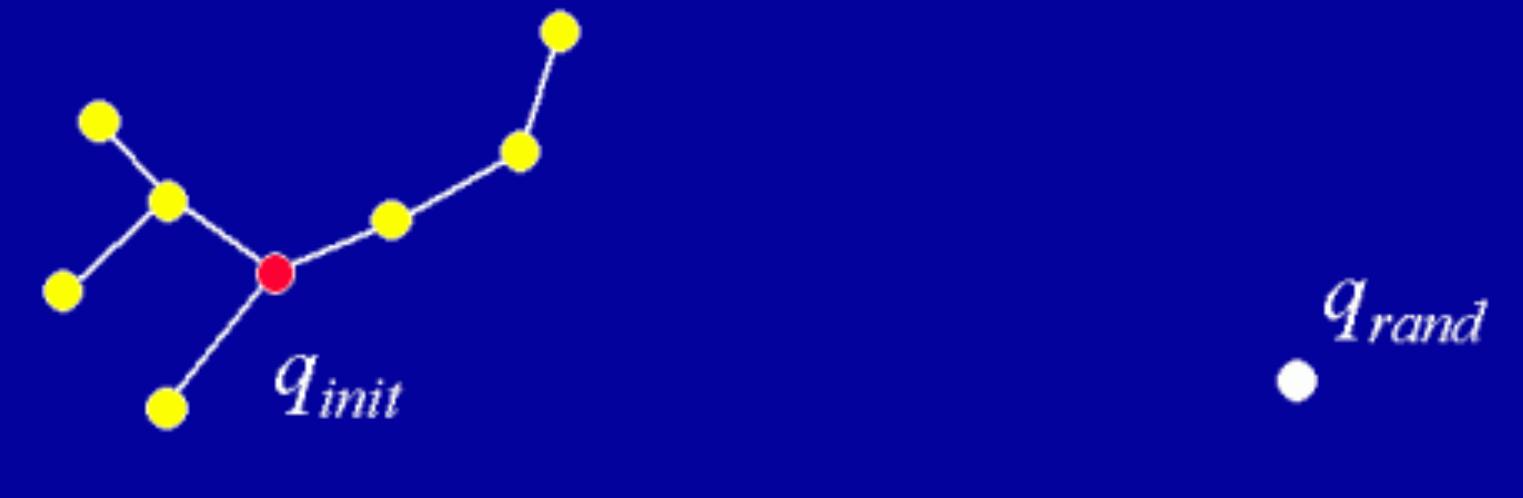
RRT Extend animation

Existing RRT is “grown” as follows...



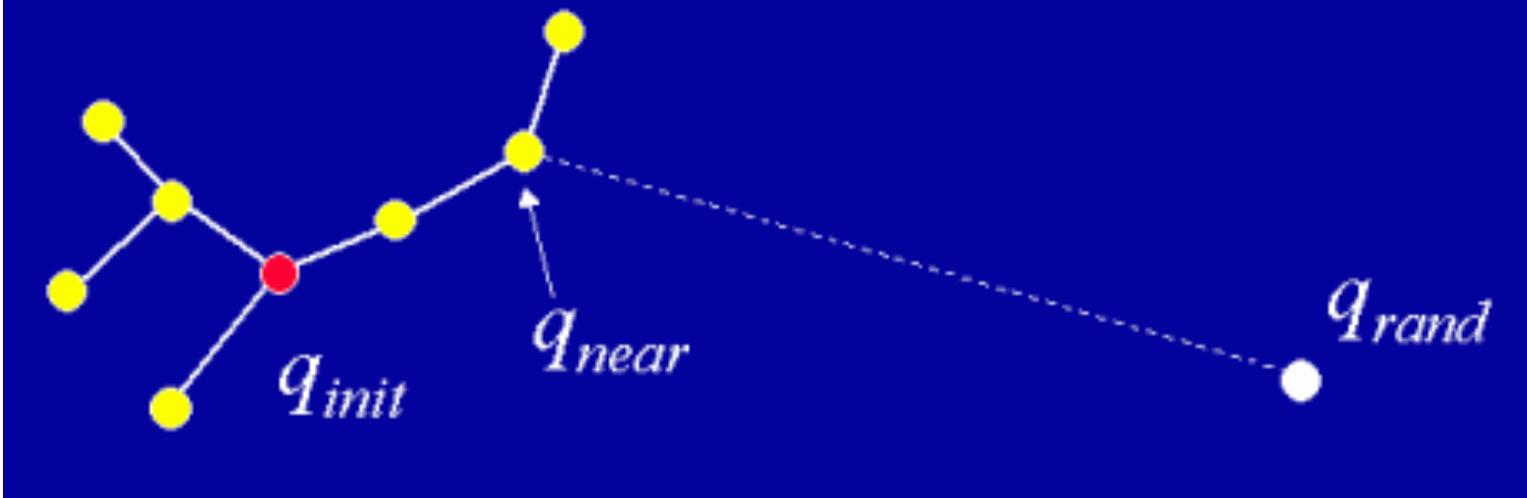
RRT Extend animation

- 1) Select a random “target” node



RRT Extend animation

2) Calculate “nearest” node in tree

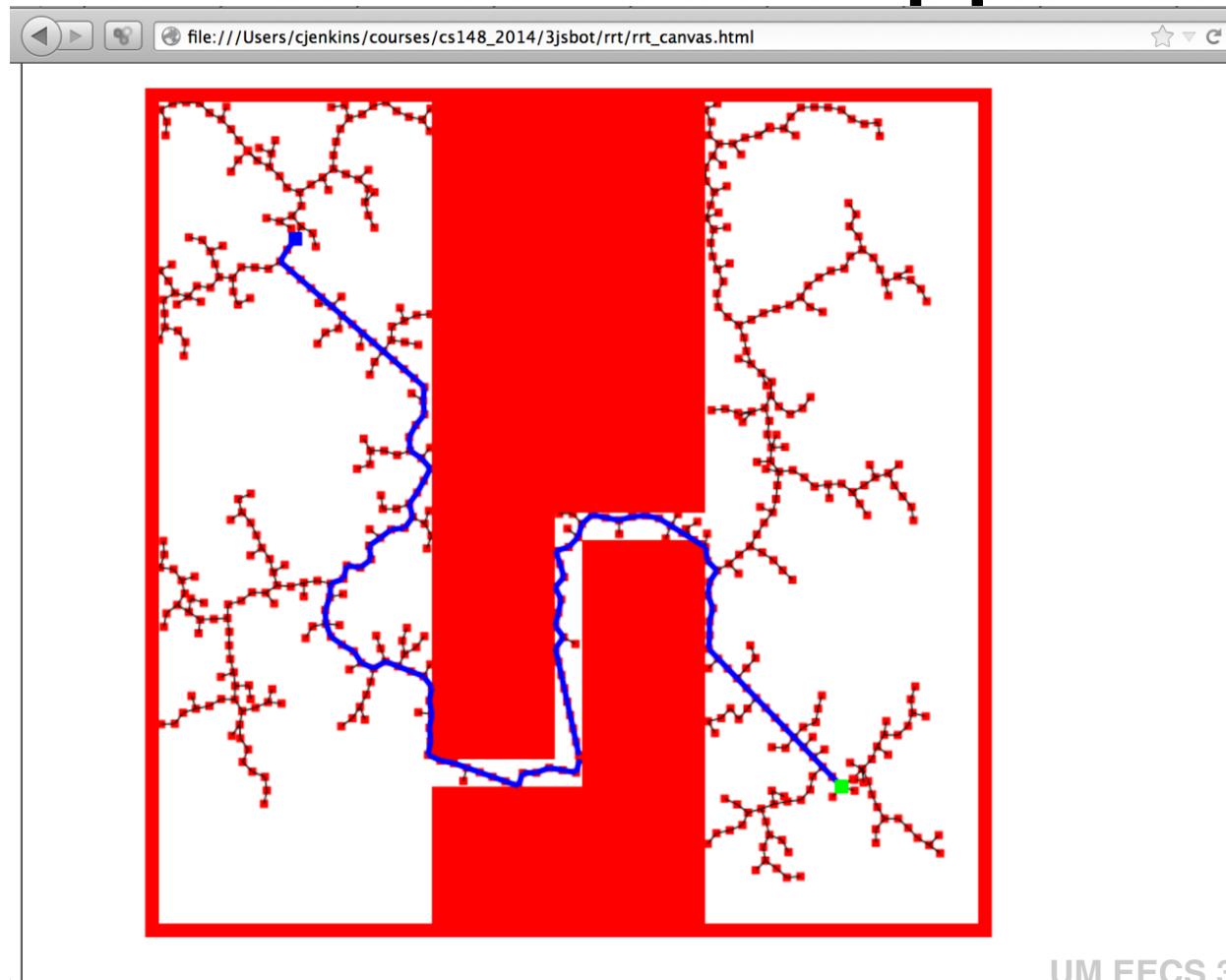


RRT Extend animation

3) Try to add new collision-free branch



Let's see what happens



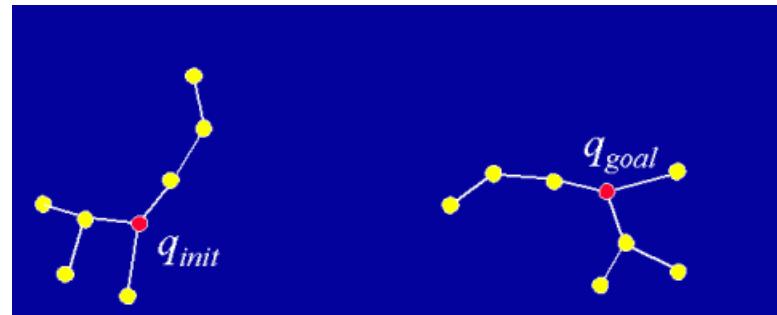
RRT Connect

- 0) Use 2 trees (A and B) rooted at start and goal configurations

```

RRT_CONNECT_PLANNER( $q_{init}, q_{goal}$ )
1    $\mathcal{T}_a.init(q_{init}); \mathcal{T}_b.init(q_{goal});$ 
2   for  $k = 1$  to  $K$  do
3        $q_{rand} \leftarrow \text{RANDOM\_CONFIG}();$ 
4       if not ( $\text{EXTEND}(\mathcal{T}_a, q_{rand}) = \text{Trapped}$ ) then
5           if ( $\text{CONNECT}(\mathcal{T}_b, q_{new}) = \text{Reached}$ ) then
6               Return PATH( $\mathcal{T}_a, \mathcal{T}_b$ );
7           SWAP( $\mathcal{T}_a, \mathcal{T}_b$ );
8   Return Failure

```



RRT Connect

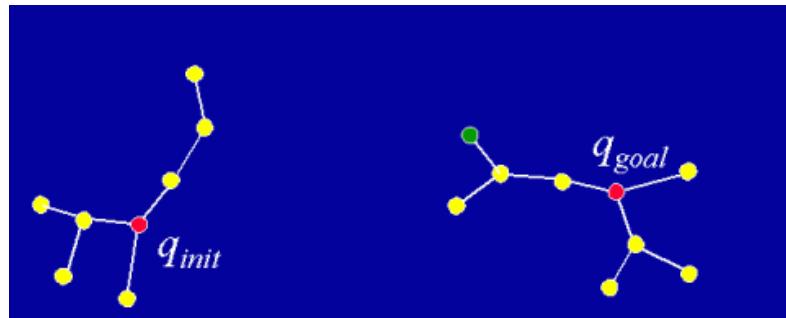
- 0) Use 2 trees (A and B) rooted at start and goal configurations

```
RRT_CONNECT_PLANNER( $q_{init}, q_{goal}$ )
1  $T_a.init(q_{init}); T_b.init(q_{goal})$ ;
2 for  $k = 1$  to  $K$  do
3    $q_{rand} \leftarrow RANDOM\_CONFIG()$ ;
4   if not (EXTEND( $T_a, q_{rand}$ ) = Trapped) then
5     if (CONNECT( $T_b, q_{new}$ ) = Reached) then
6       Return PATH( $T_a, T_b$ );
7     SWAP( $T_a, T_b$ );
8   Return Failure
```

EXTEND(T, q)

```
1  $q_{near} \leftarrow NEAREST\_NEIGHBOR(q, T)$ ;
2 if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3    $T.add\_vertex(q_{new})$ ;
4    $T.add\_edge(q_{near}, q_{new})$ ;
5   if  $q_{new} = q$  then
6     Return Reached;
7   else
8     Return Advanced;
9 Return Trapped;
```

- 1) Extend tree A towards a random configuration



RRT Connect

0) Use 2 trees (A and B) rooted at start and goal configurations

```
RRT_CONNECT_PLANNER( $q_{init}$ ,  $q_{goal}$ )
1  $T_a.init(q_{init})$ ;  $T_b.init(q_{goal})$ ;
2 for  $k = 1$  to  $K$  do
3    $q_{rand} \leftarrow RANDOM\_CONFIG()$ ;
4   if not (EXTEND( $T_a$ ,  $q_{rand}$ ) = Trapped) then
5     if (CONNECT( $T_b$ ,  $q_{new}$ ) = Reached) then
6       Return PATH( $T_a$ ,  $T_b$ );
7     SWAP( $T_a$ ,  $T_b$ );
8   Return Failure
```

EXTEND(T, q)

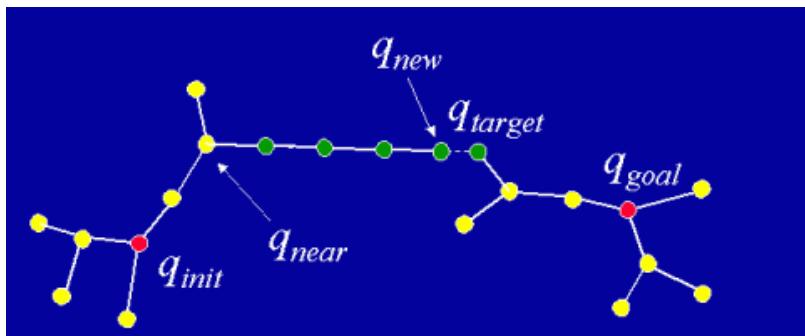
```
1  $q_{near} \leftarrow NEAREST\_NEIGHBOR(q, T)$ ;
2 if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3    $T.add\_vertex(q_{new})$ ;
4    $T.add\_edge(q_{near}, q_{new})$ ;
5   if  $q_{new} = q$  then
6     Return Reached;
7   else
8     Return Advanced;
9 Return Trapped;
```

1) Extend tree A towards a random configuration

CONNECT(T, q)

```
1 repeat
2    $S \leftarrow EXTEND(T, q)$ ;
3 until not ( $S = Advanced$ )
4 Return  $S$ ;
```

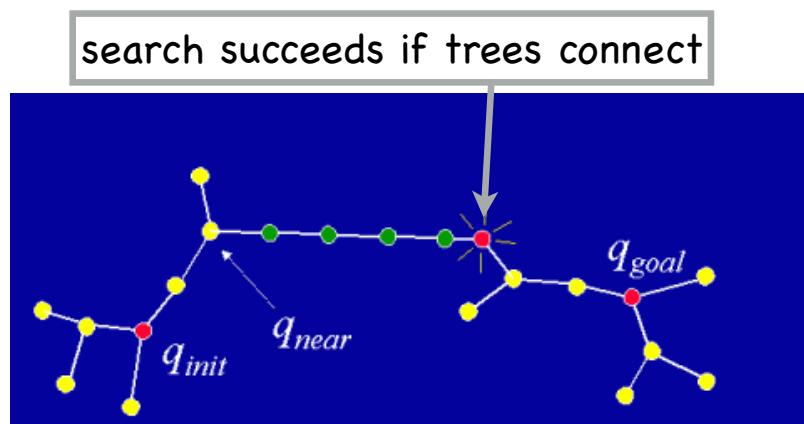
2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor



RRT Connect

0) Use 2 trees (A and B) rooted at start and goal configurations

```
RRT_CONNECT_PLANNER( $q_{init}, q_{goal}$ )
1  $T_a.init(q_{init}); T_b.init(q_{goal})$ ;
2 for  $k = 1$  to  $K$  do
3    $q_{rand} \leftarrow RANDOM\_CONFIG()$ ;
4   if not (EXTEND( $T_a, q_{rand}$ ) = Trapped) then
5     if (CONNECT( $T_b, q_{near}$ ) = Reached) then
6       Return PATH( $T_a, T_b$ );
7     SWAP( $T_a, T_b$ );
8   Return Failure
```



EXTEND(T, q)

```
1  $q_{near} \leftarrow NEAREST\_NEIGHBOR(q, T)$ ;
2 if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3    $T.add\_vertex(q_{new})$ ;
4    $T.add\_edge(q_{near}, q_{new})$ ;
5   if  $q_{new} = q$  then
6     Return Reached;
7   else
8     Return Advanced;
9 Return Trapped;
```

1) Extend tree A towards a random configuration

CONNECT(T, q)

```
1 repeat
2    $S \leftarrow EXTEND(T, q)$ ;
3 until not ( $S = Advanced$ )
4 Return S;
```

2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

RRT Connect

- 0) Use 2 trees (A and B) rooted at start and goal configurations

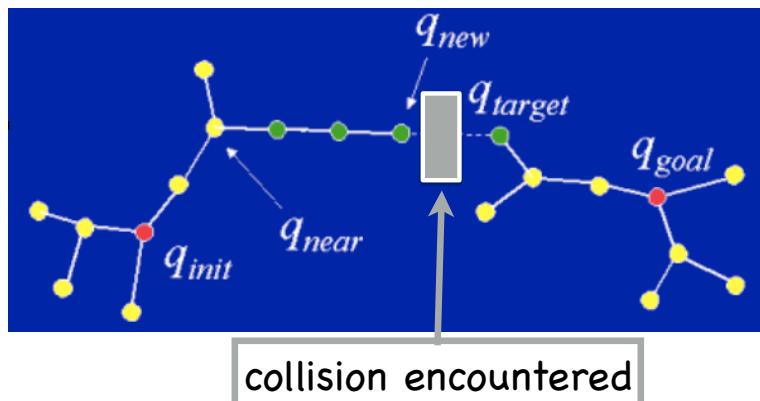
RRT_CONNECT_PLANNER(q_{init}, q_{goal})

```

1  $T_a.init(q_{init}); T_b.init(q_{goal});$ 
2 for  $k = 1$  to  $K$  do
3    $q_{rand} \leftarrow RANDOM\_CONFIG();$ 
4   if not (EXTEND( $T_a, q_{rand}$ ) = Trapped) then
5     if (CONNECT( $T_b, q_{near}$ ) = Reached) then
6       Return PATH( $T_a, T_b$ );
7     SWAP( $T_a, T_b$ );
8   Return Failure

```

- 3) reverse roles for trees A and B and repeat



EXTEND(\mathcal{T}, q)

```

1  $q_{near} \leftarrow NEAREST\_NEIGHBOR(q, \mathcal{T});$ 
2 if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3    $\mathcal{T}.add\_vertex(q_{new});$ 
4    $\mathcal{T}.add\_edge(q_{near}, q_{new});$ 
5   if  $q_{new} = q$  then
6     Return Reached;
7   else
8     Return Advanced;
9 Return Trapped;

```

- 1) Extend tree A towards a random configuration

CONNECT(\mathcal{T}, q)

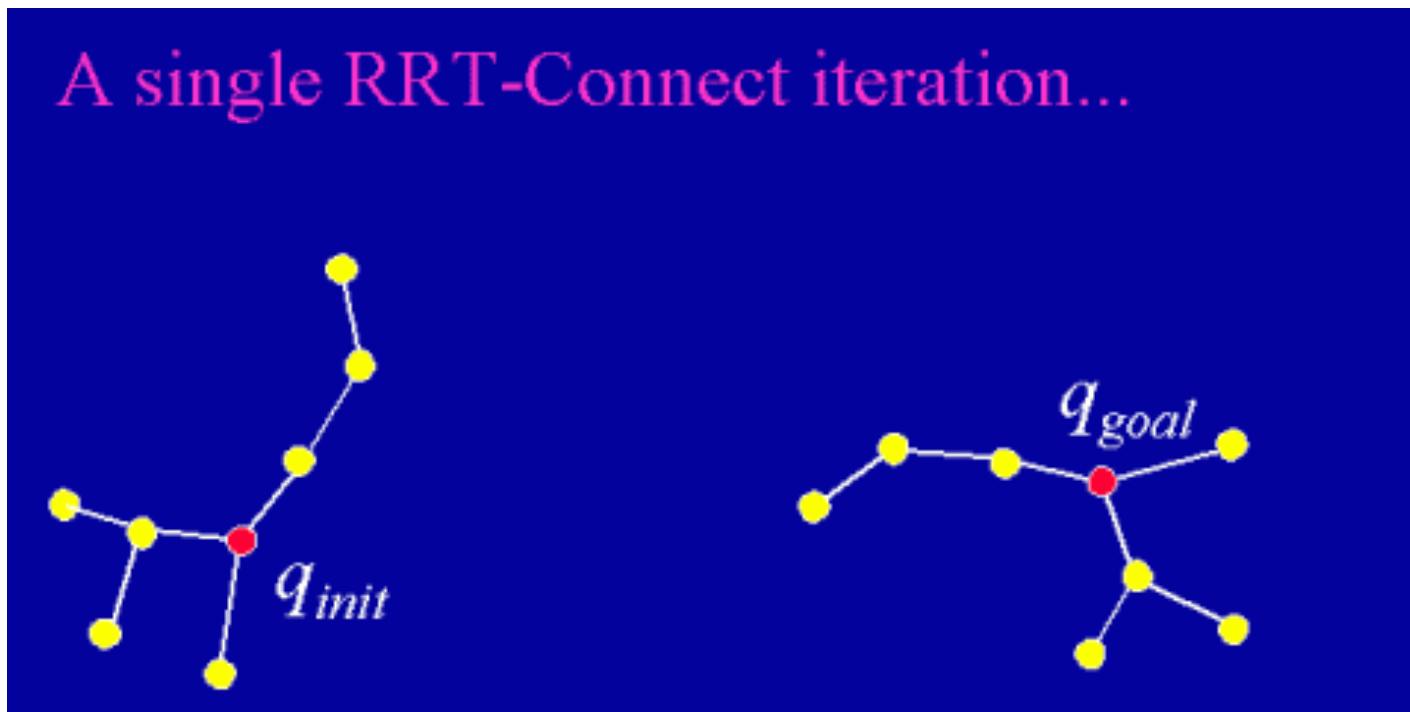
```

1 repeat
2    $S \leftarrow EXTEND(\mathcal{T}, q);$ 
3 until not ( $S = Advanced$ )
4 Return S;

```

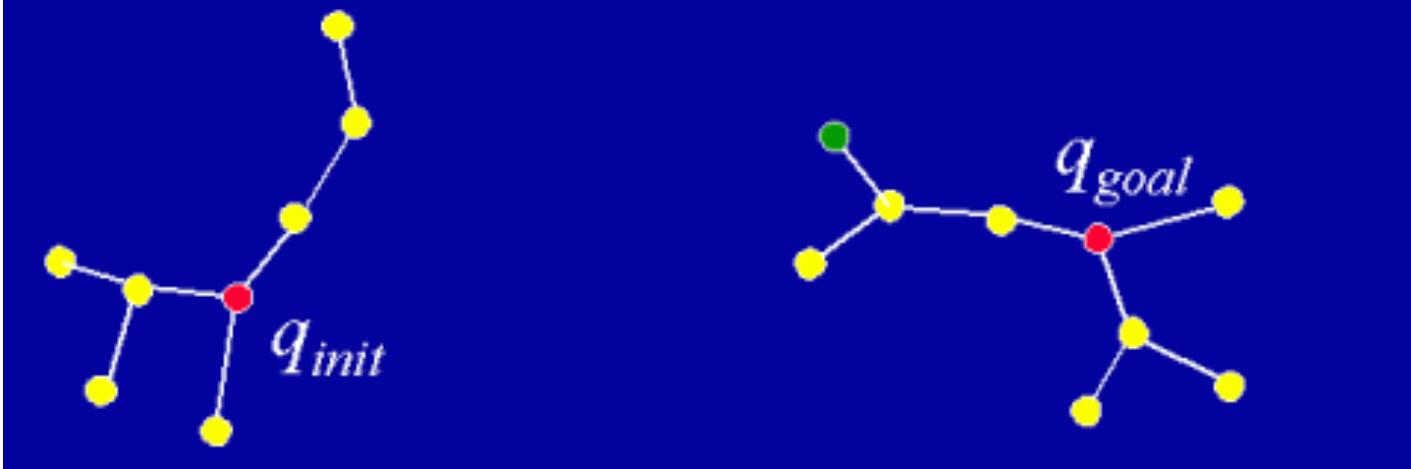
- 2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

RRT-Connect animation

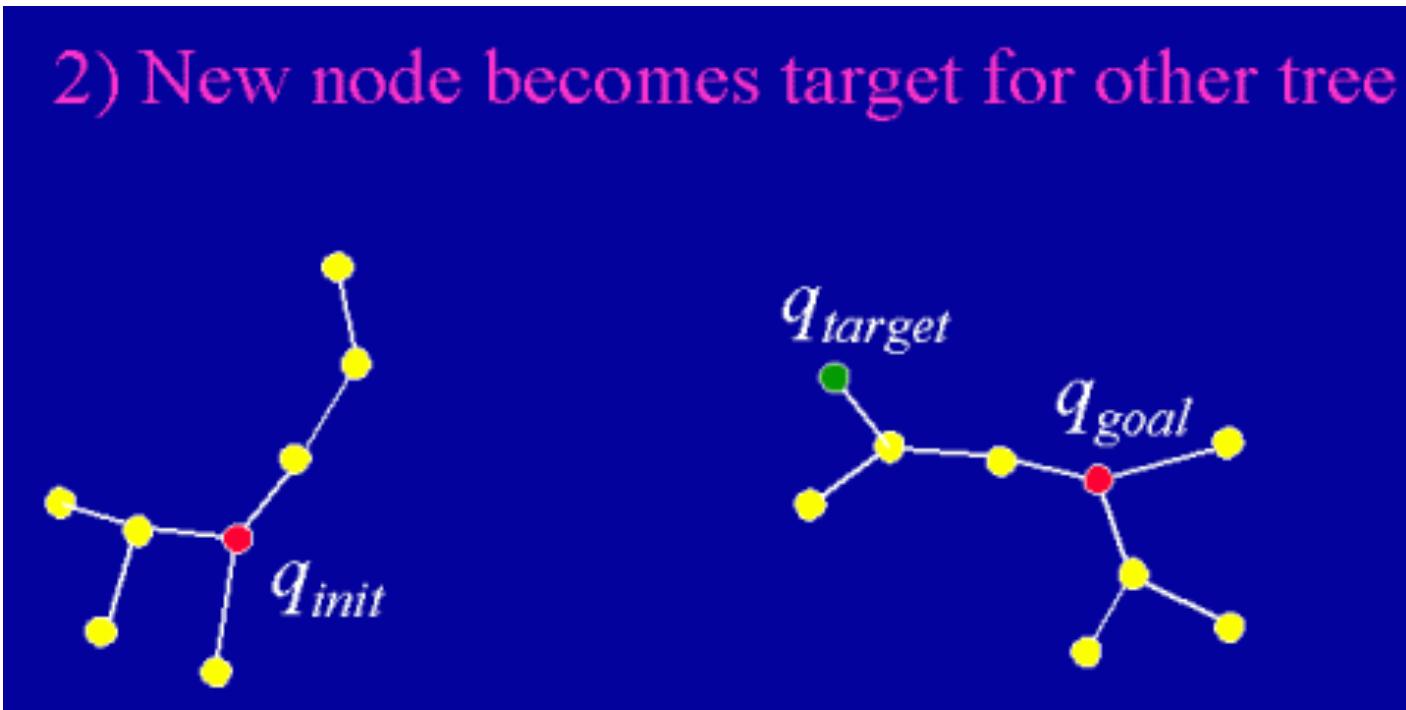


RRT-Connect animation

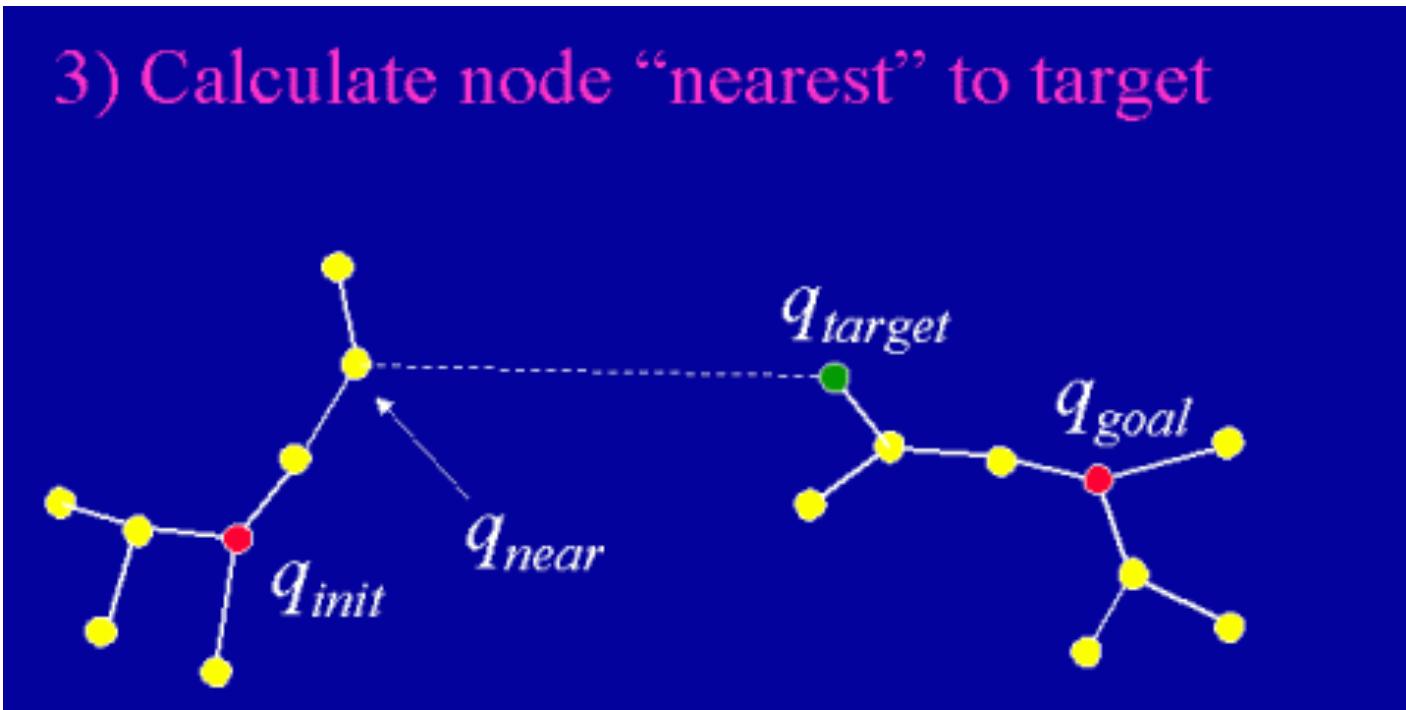
1) One tree grown using random target



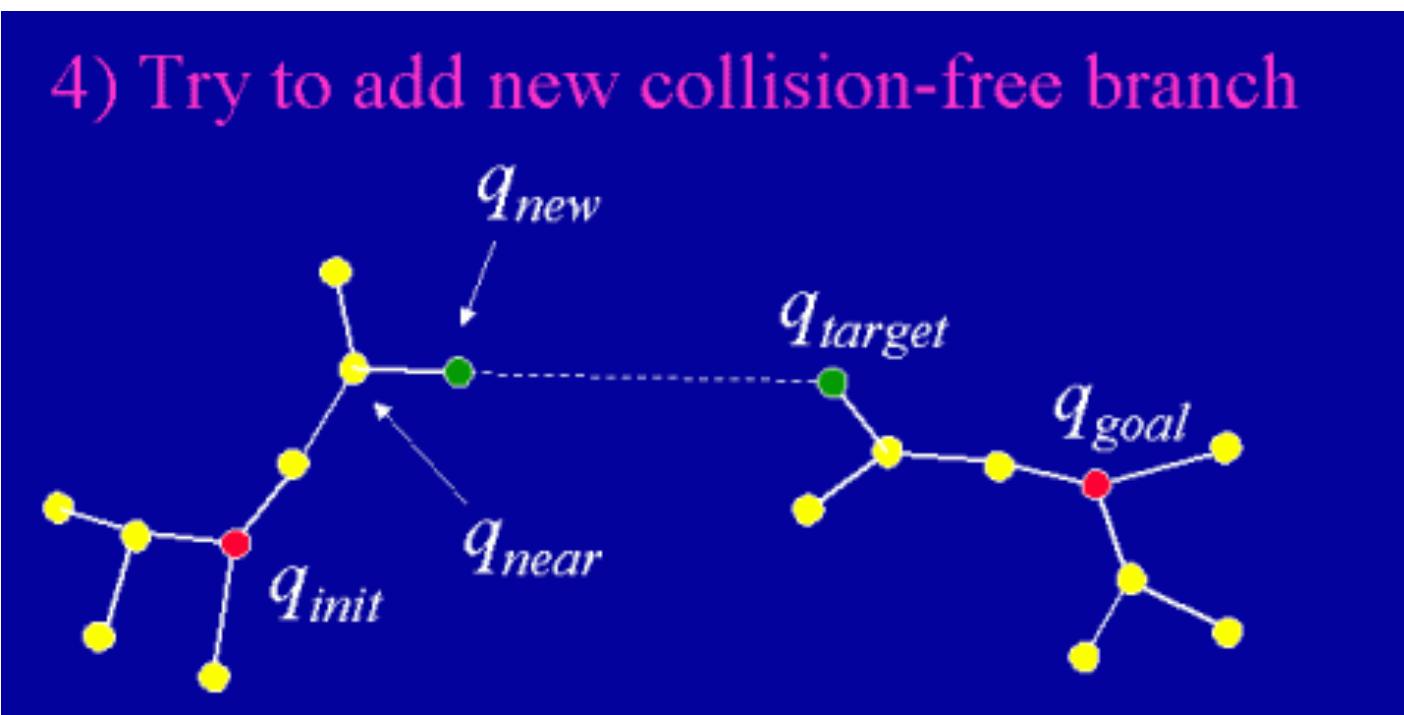
RRT-Connect animation



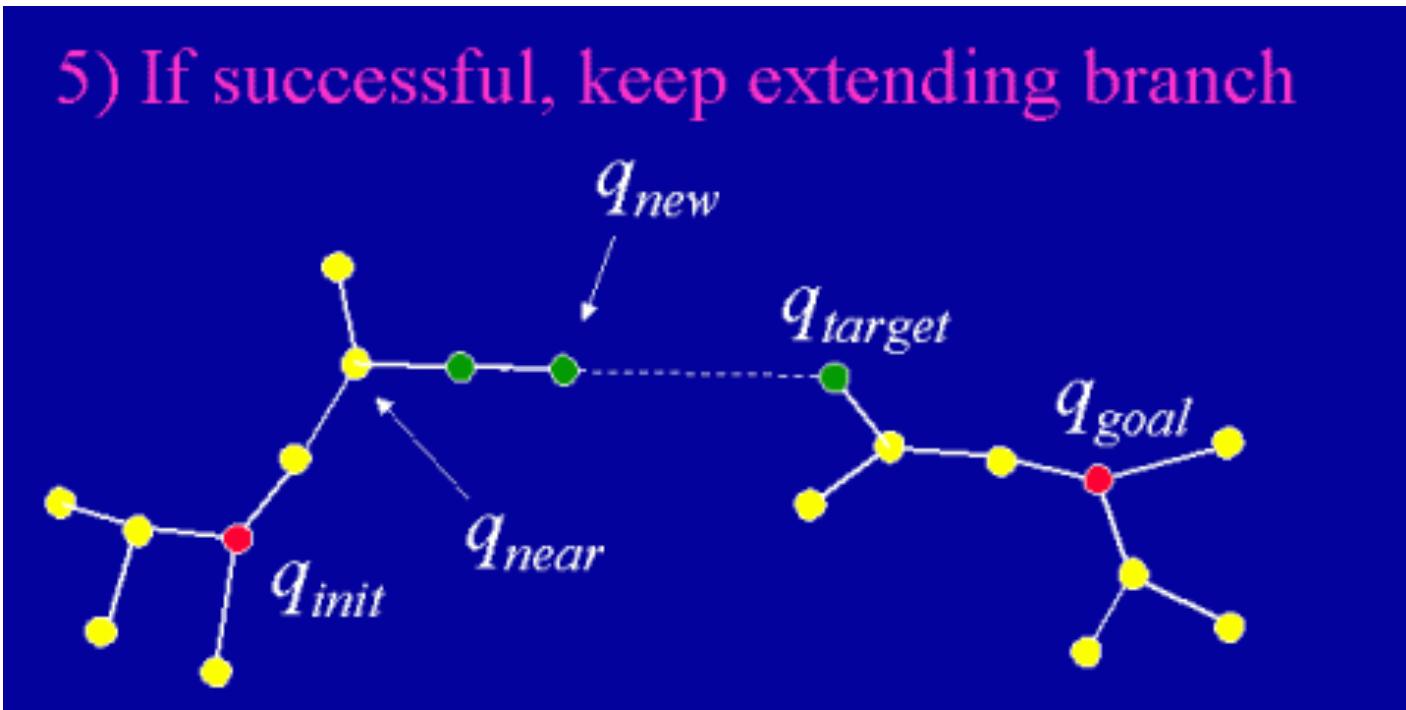
RRT-Connect animation



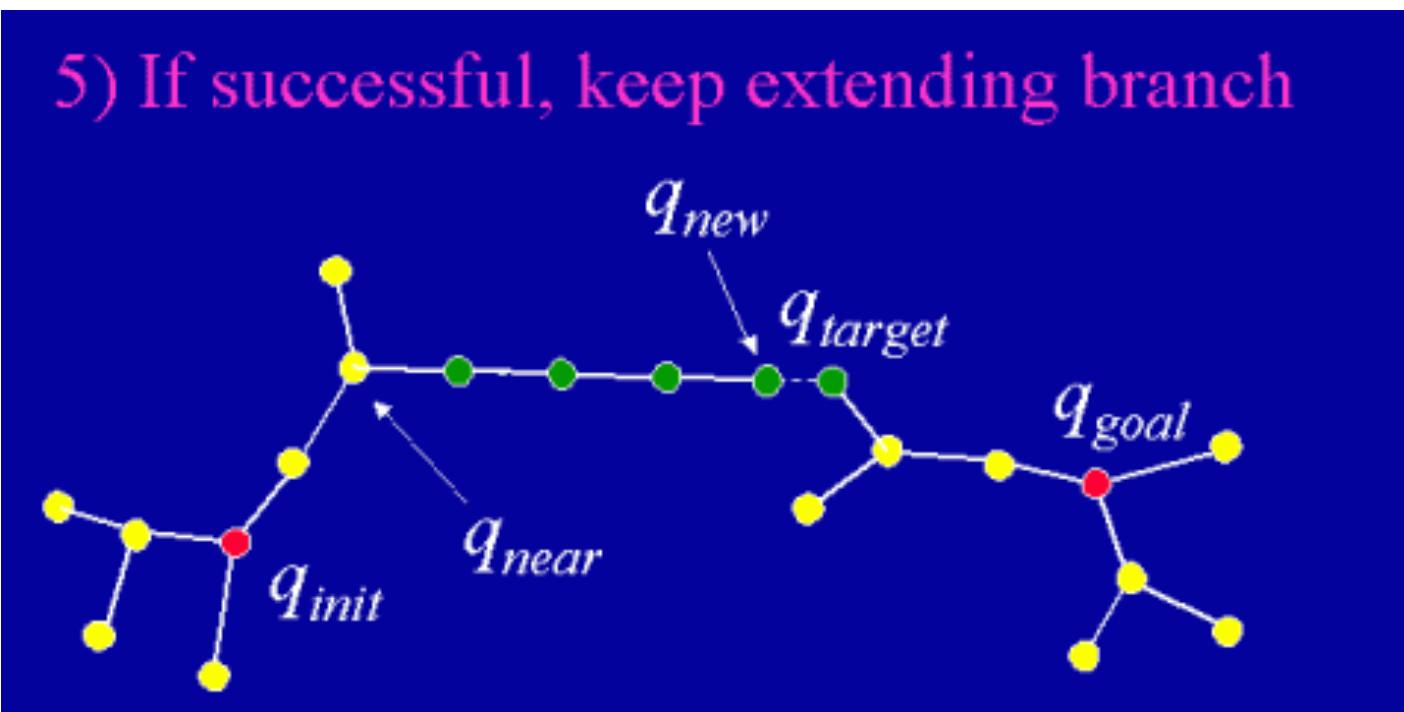
RRT-Connect animation



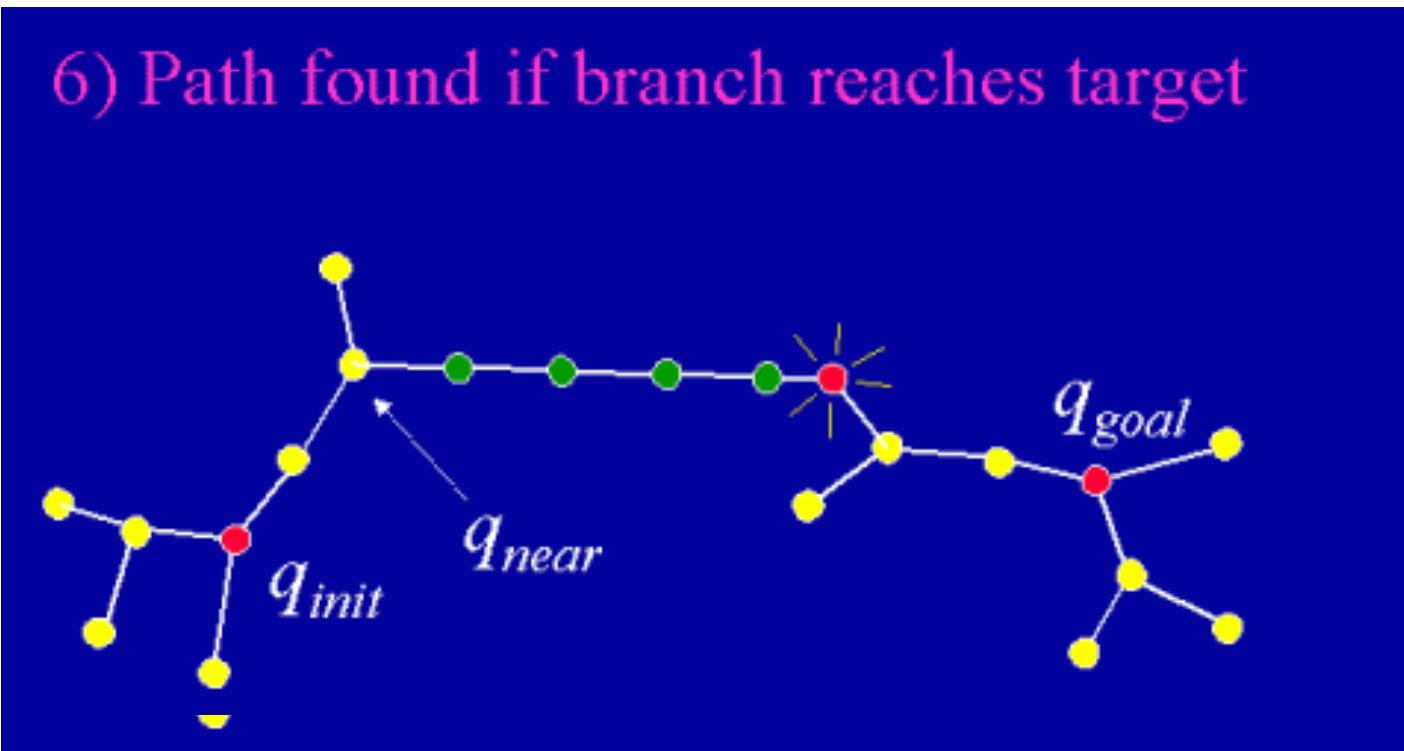
RRT-Connect animation



RRT-Connect animation

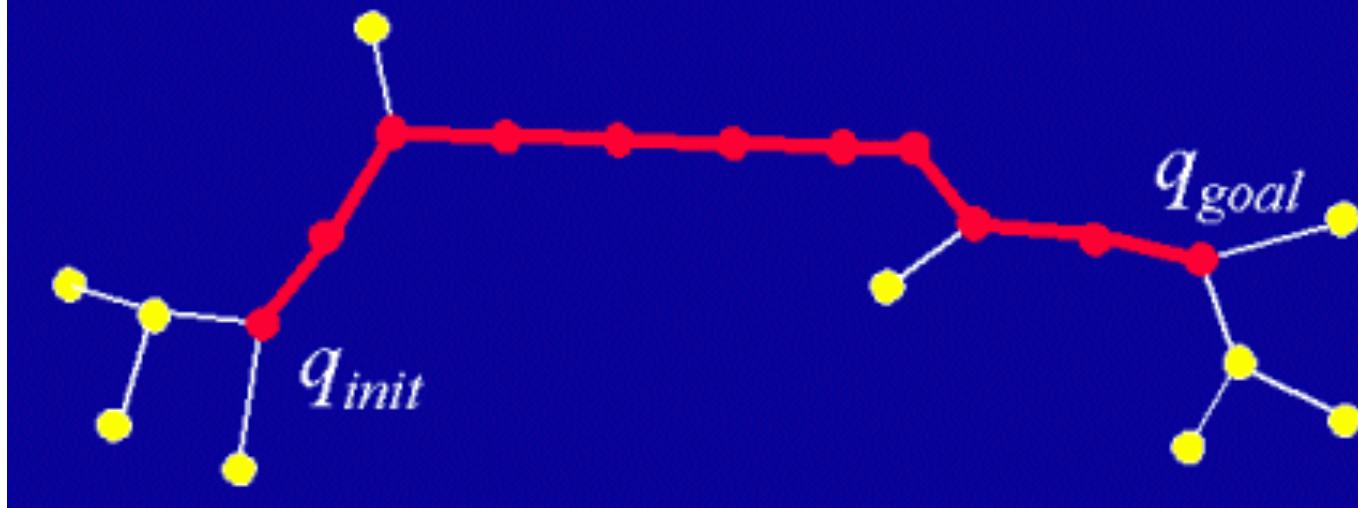


RRT-Connect animation

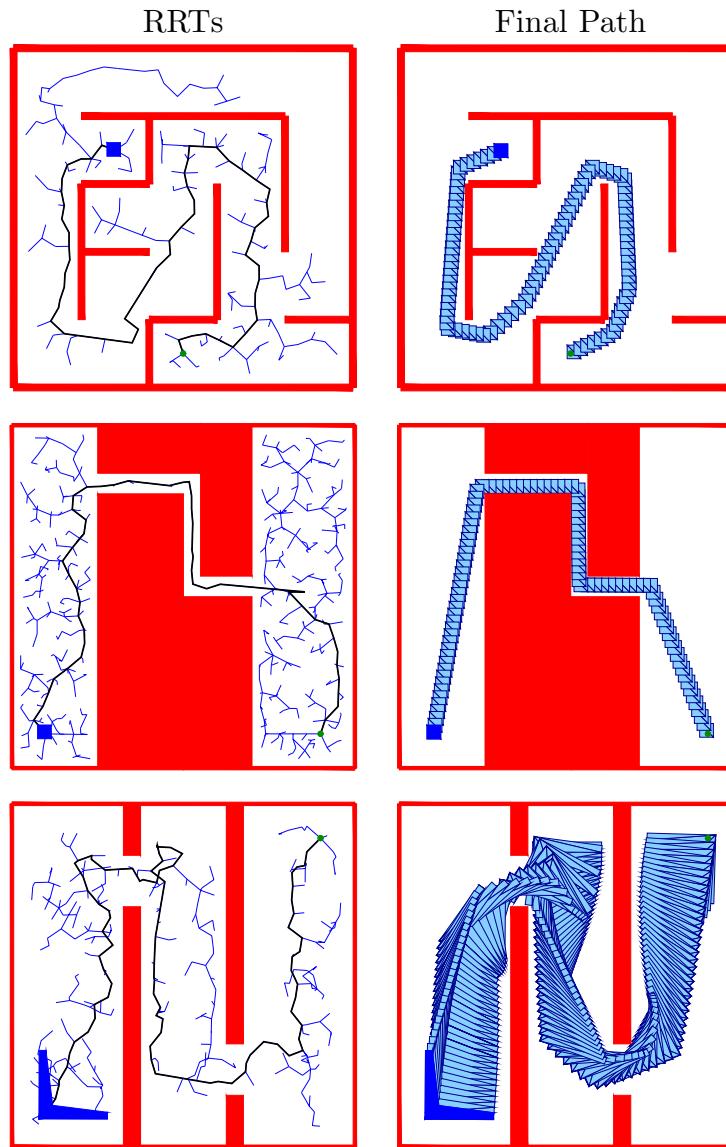


RRT-Connect animation

7) Return path connecting start and goal



Examples of RRT-Connect

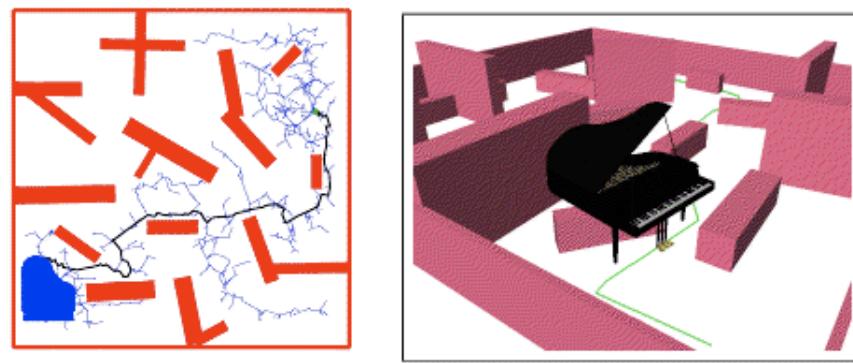


2 DOF maze

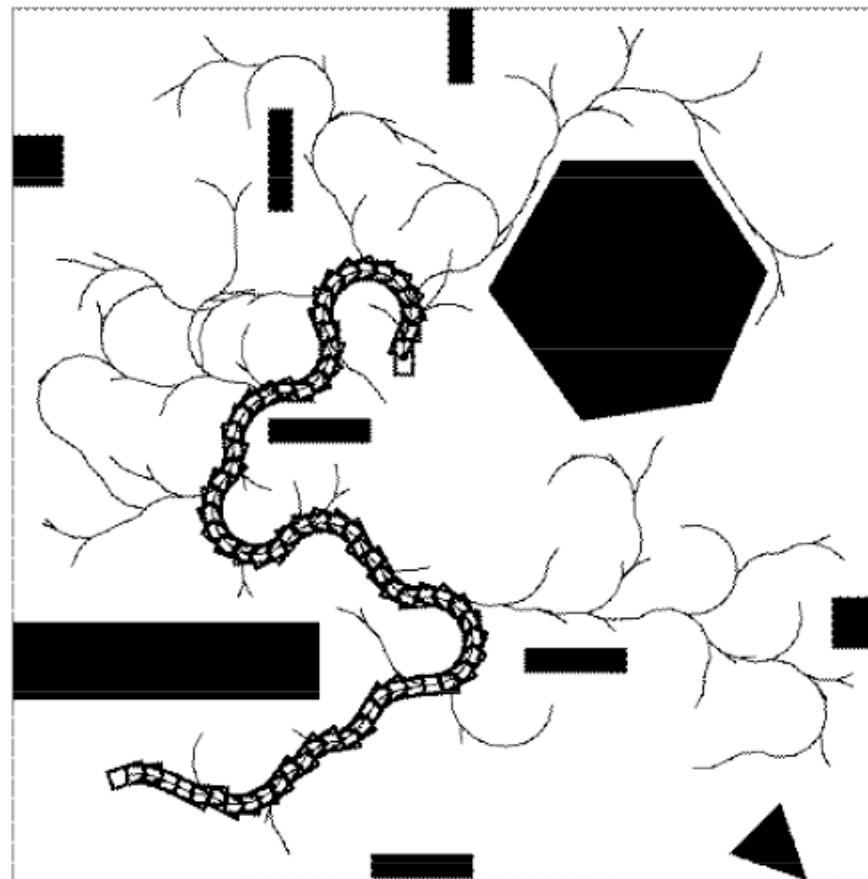
2 DOF single passway

3 DOF single passway
(with non-point geometry)

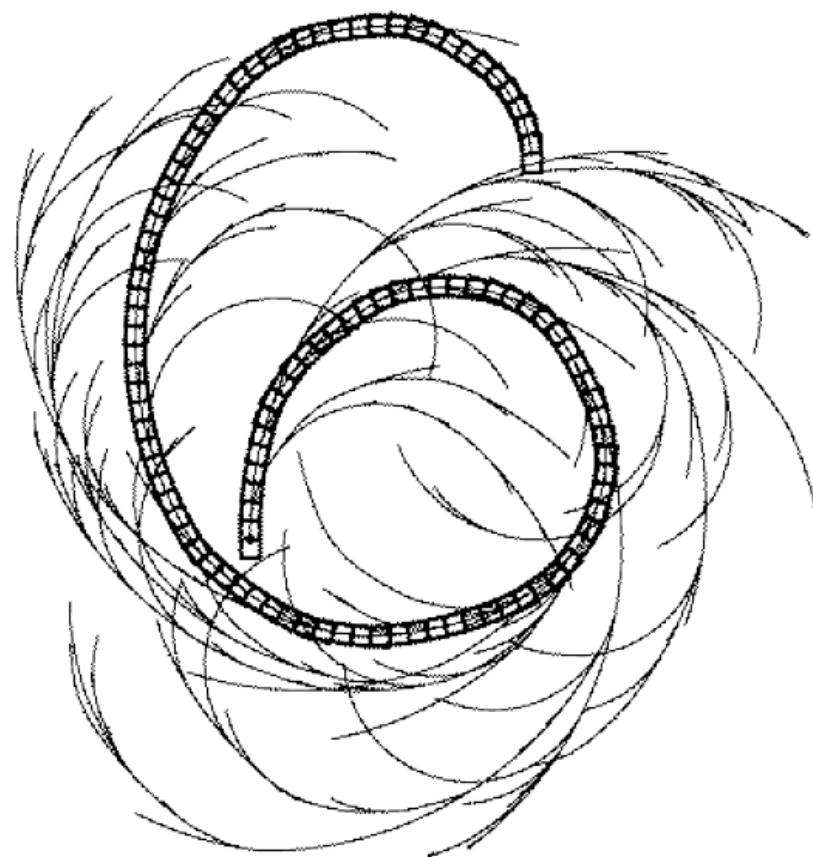
Moving a Piano



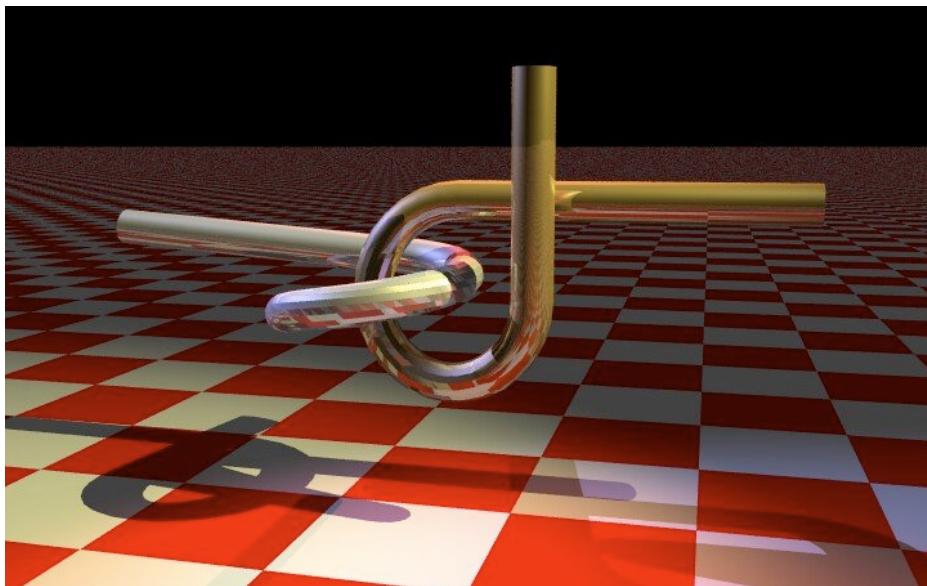
A Car-Like Robot



A Right-Turn Only Car

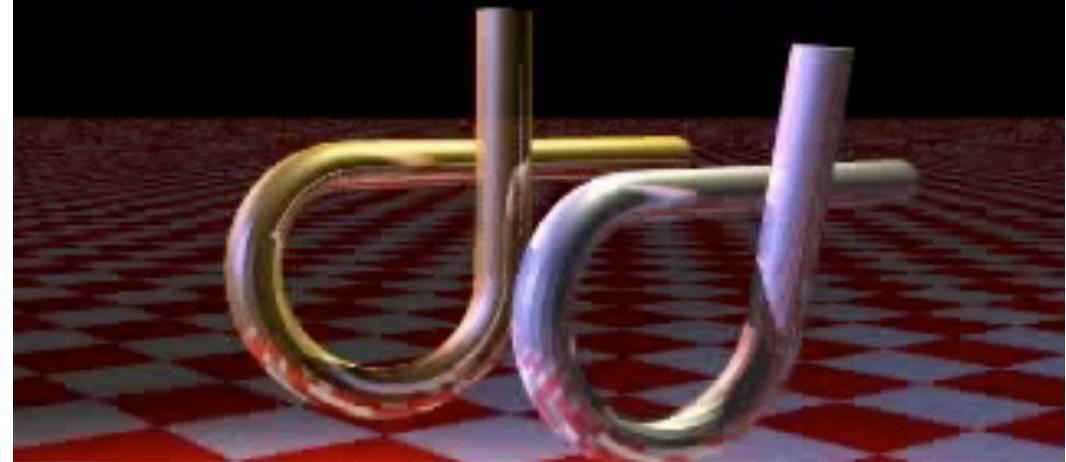


Alpha Puzzle



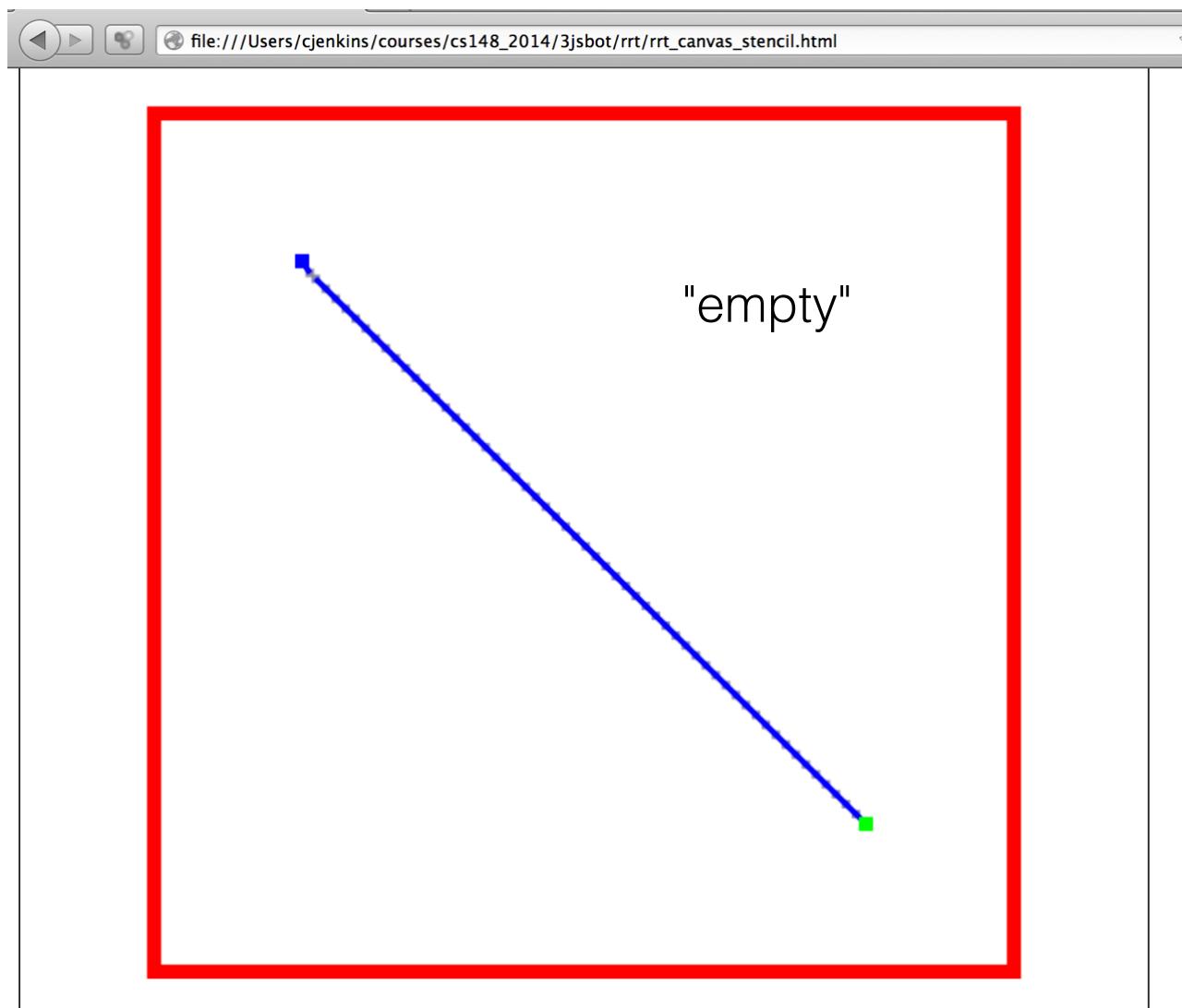
Alpha Puzzle 1.0 Solution

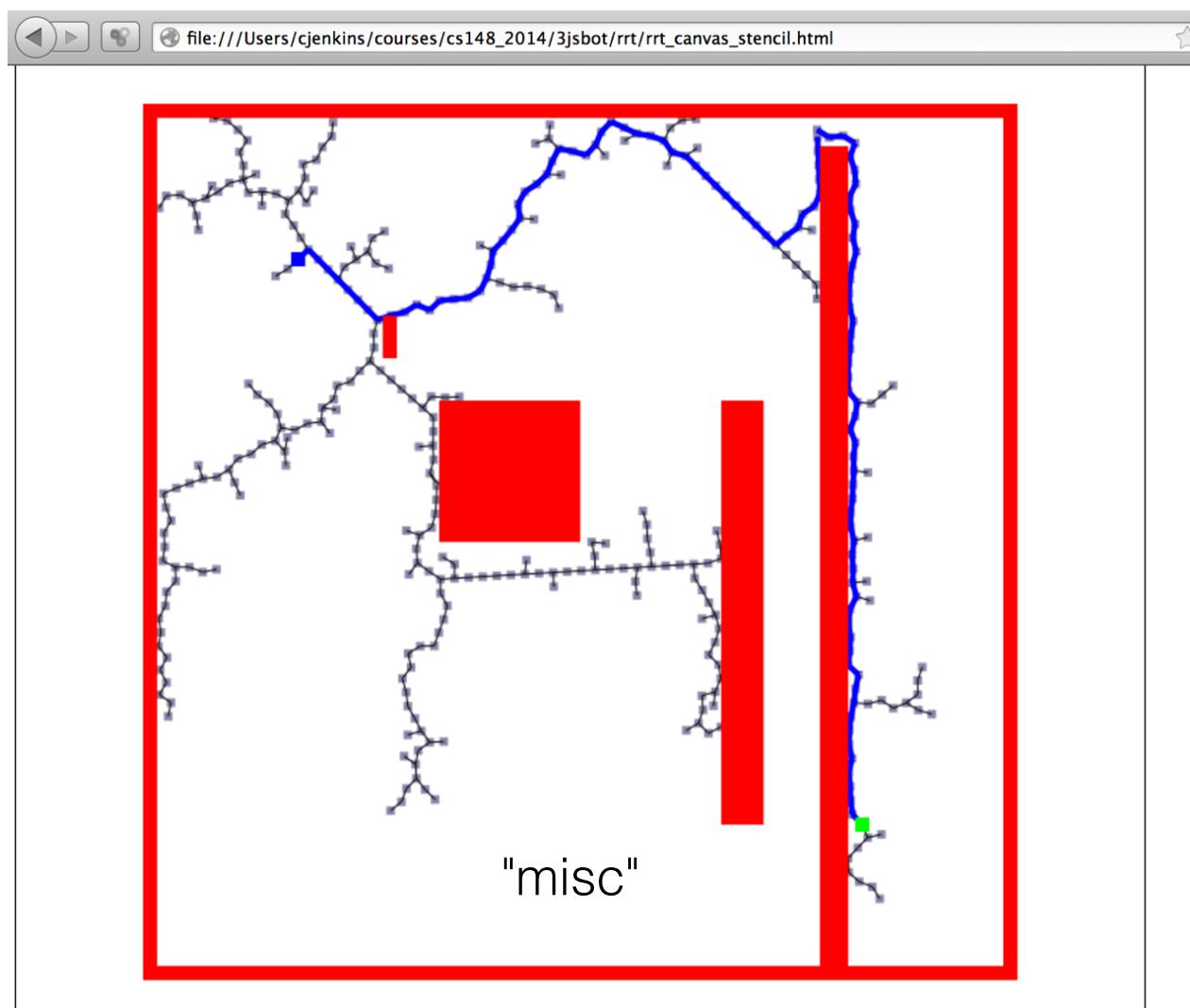
James Kuffner, Feb. 2001

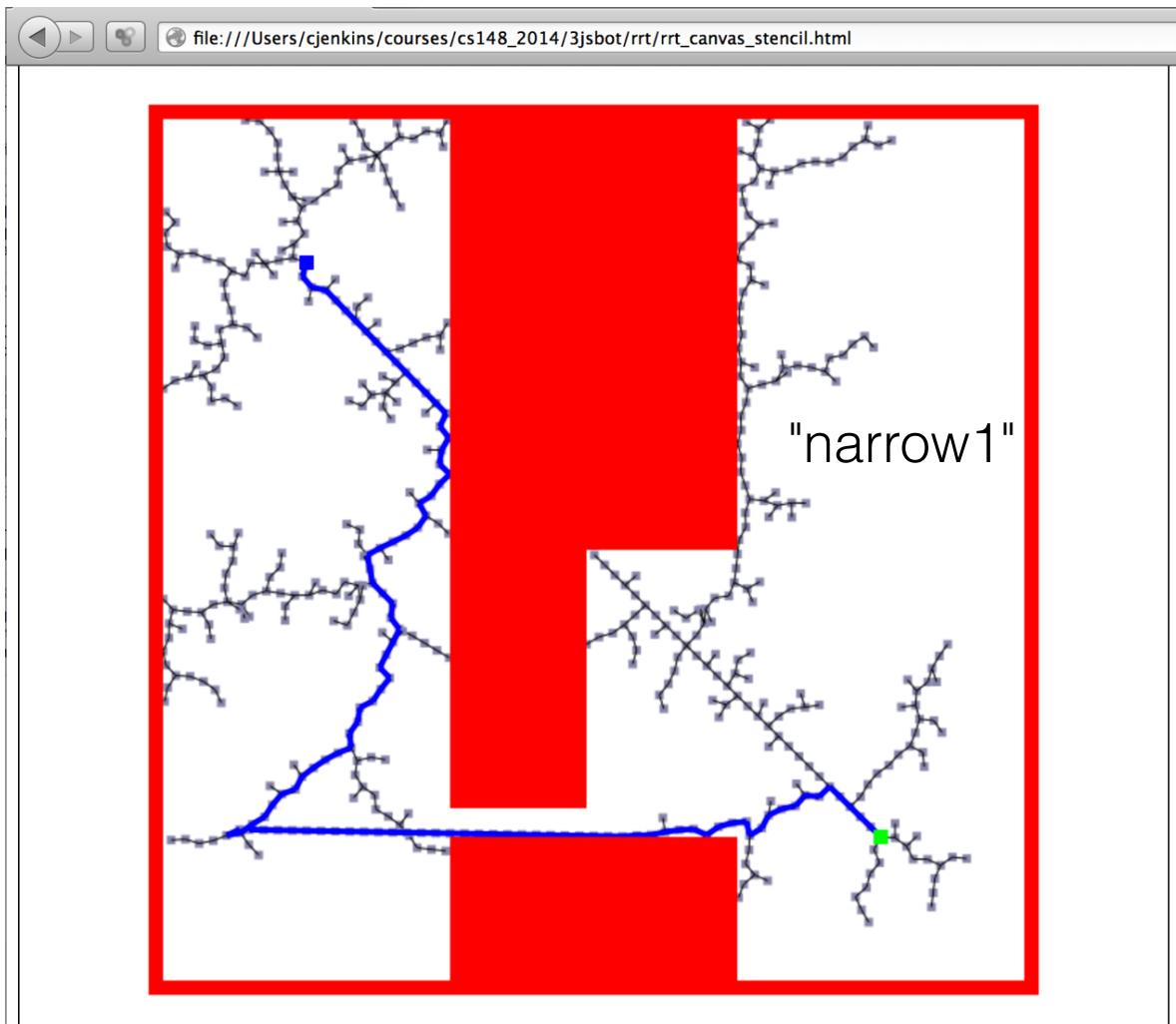


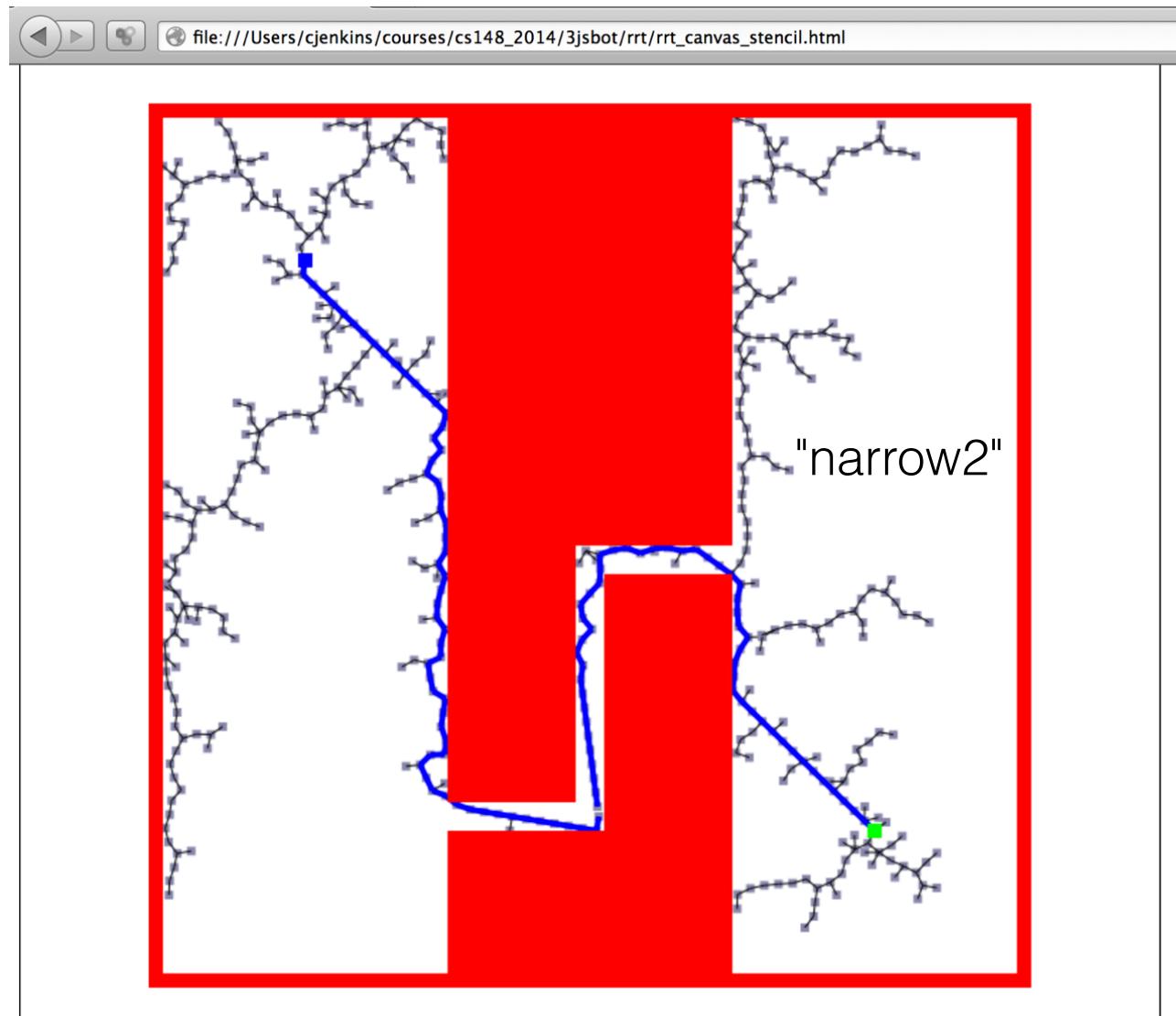
model by DSMFT group, Texas A&M Univ.
original model by Boris Yamrom, GE

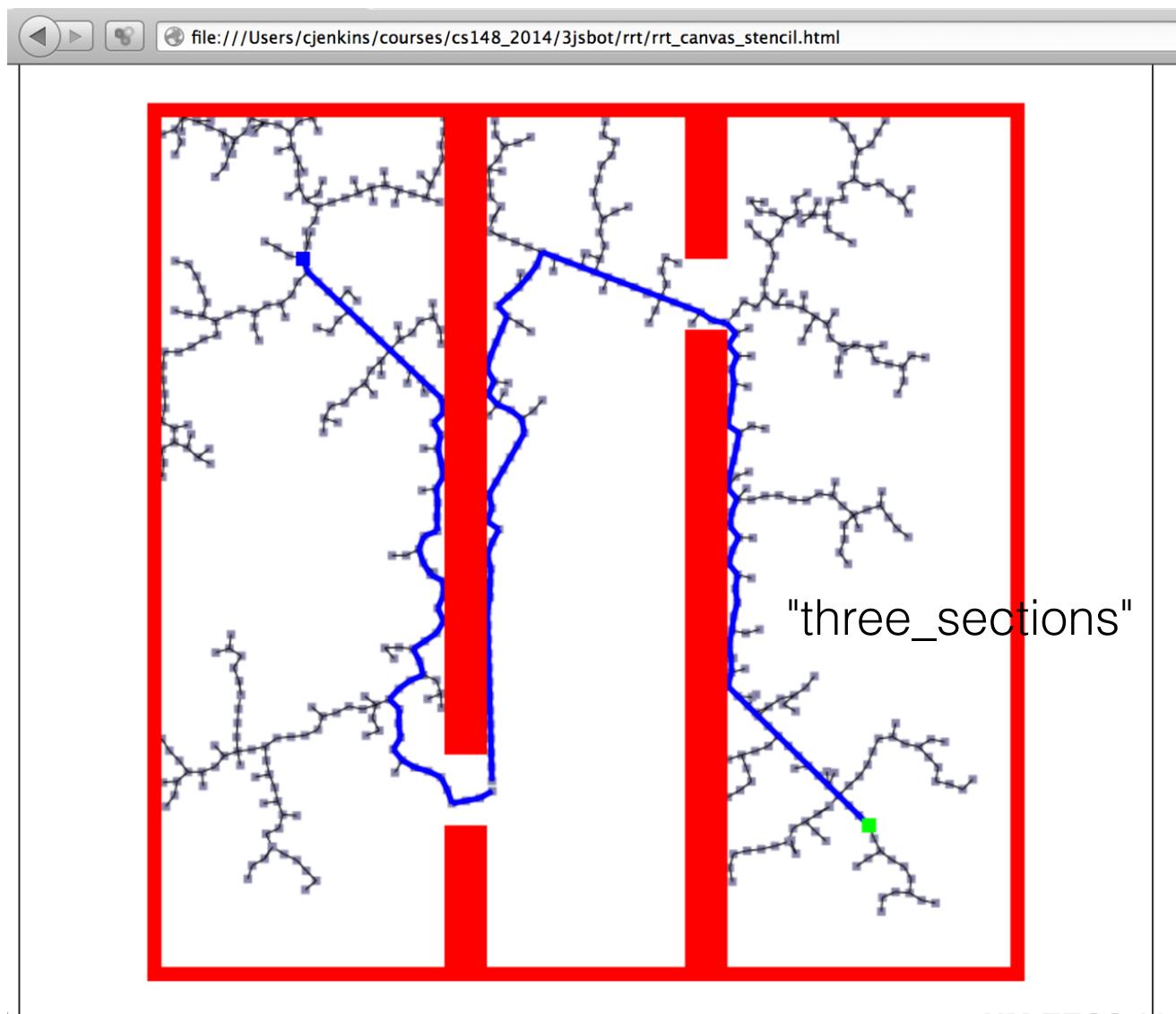
Canvas Stencil Examples



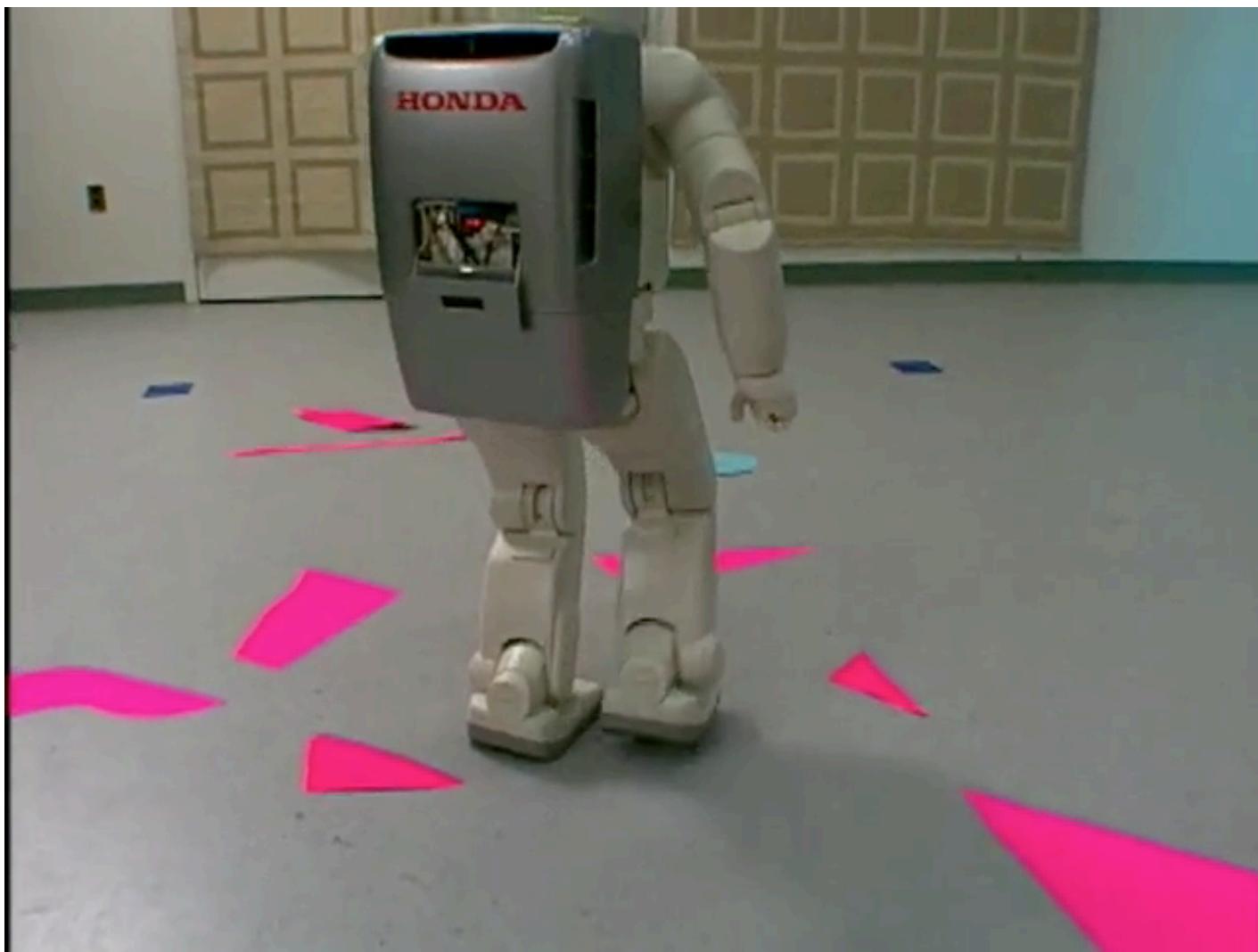








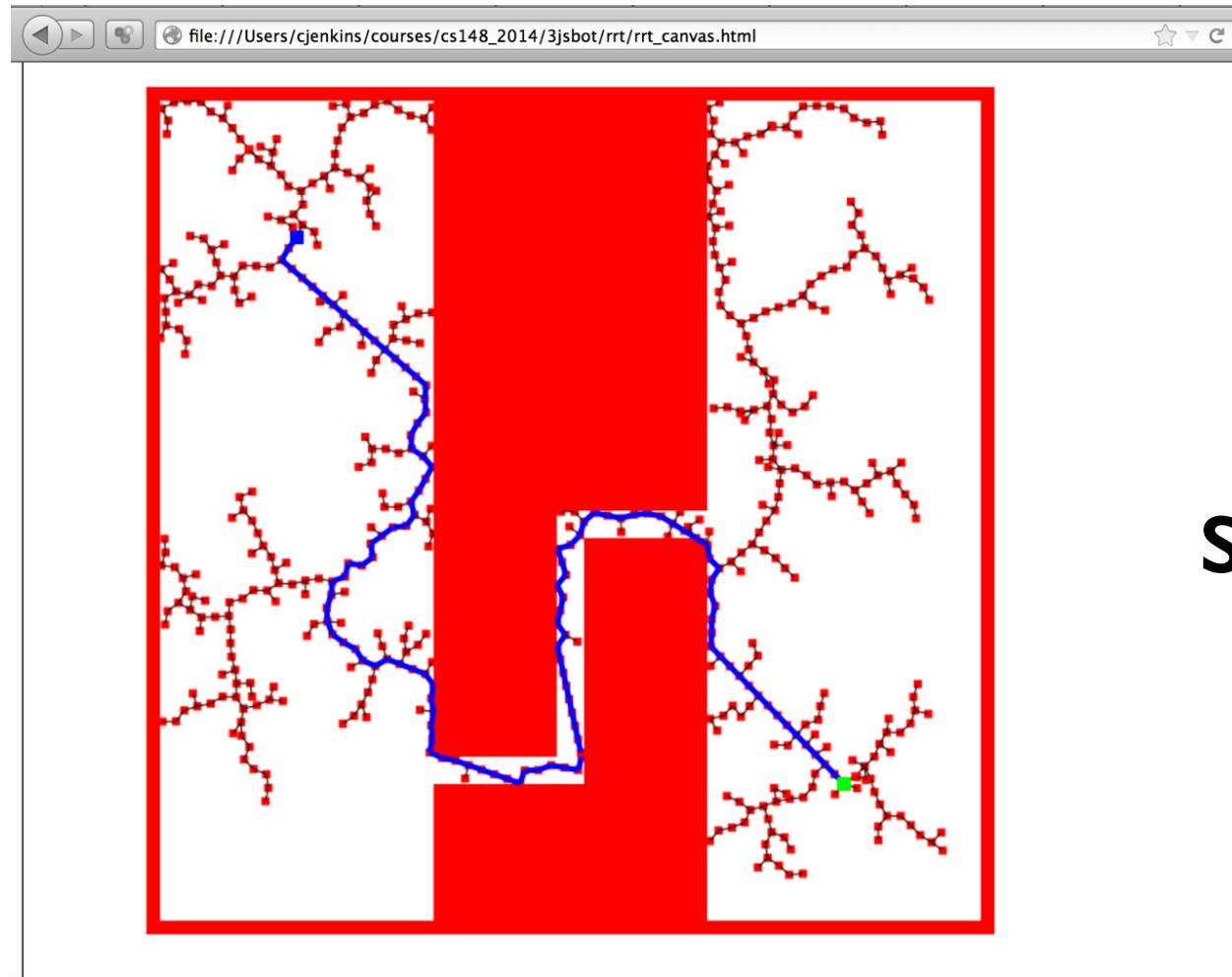
“We’ve made robot history”



Kuffner/Asimo Discovery Channel feature - <http://vimeo.com/9911572>

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RRTs can take a lot of time...



Is there a
simpler way?



Potential fields

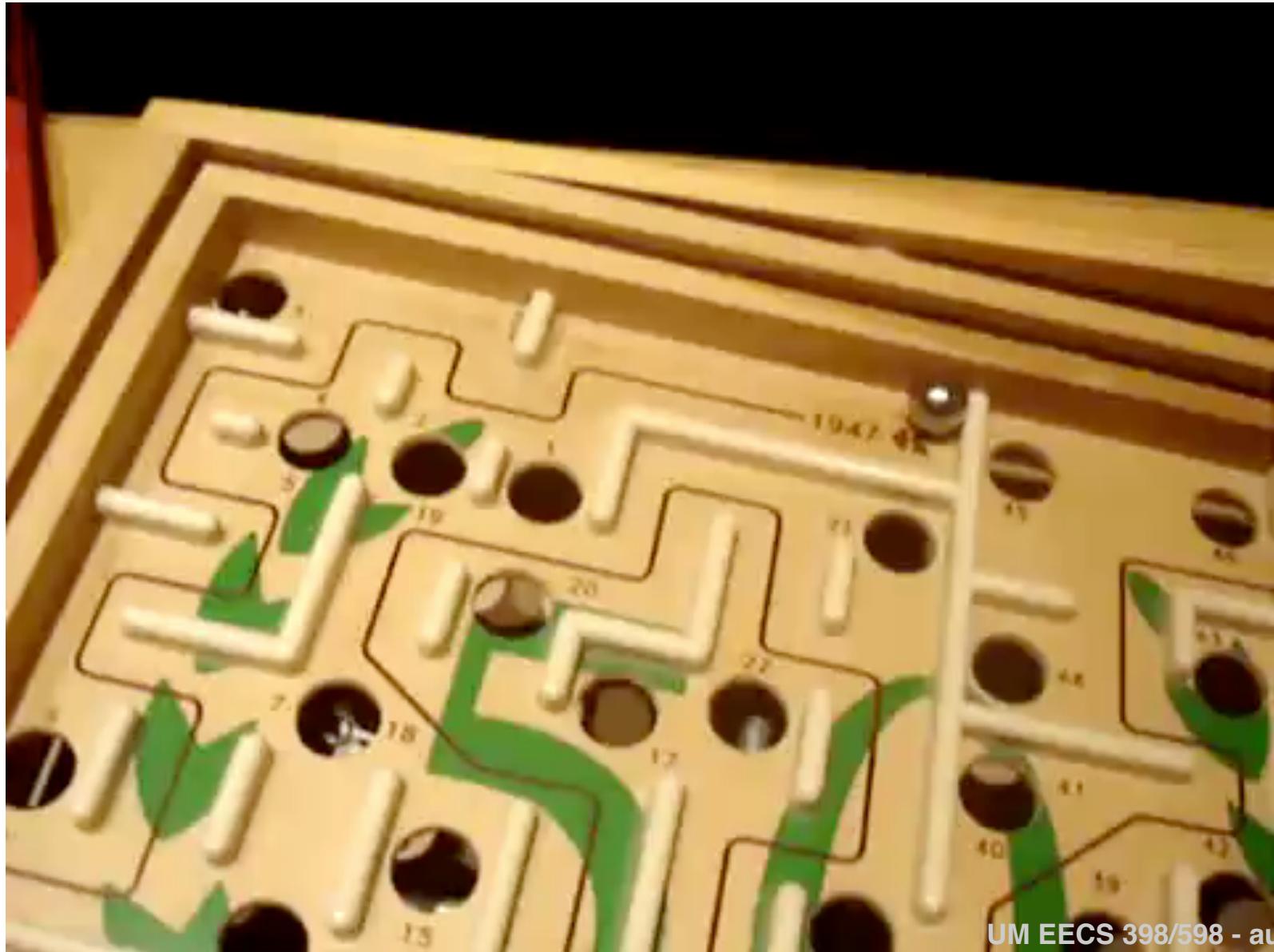
follow your
potential

Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
 - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- Sampling-based Search (build graph):
 - Probabilistic Road Maps, Rapidly-exploring Random Trees
- **Optimization and local search:**
 - **Gradient descent, Potential fields, Simulated annealing, Wavefront**

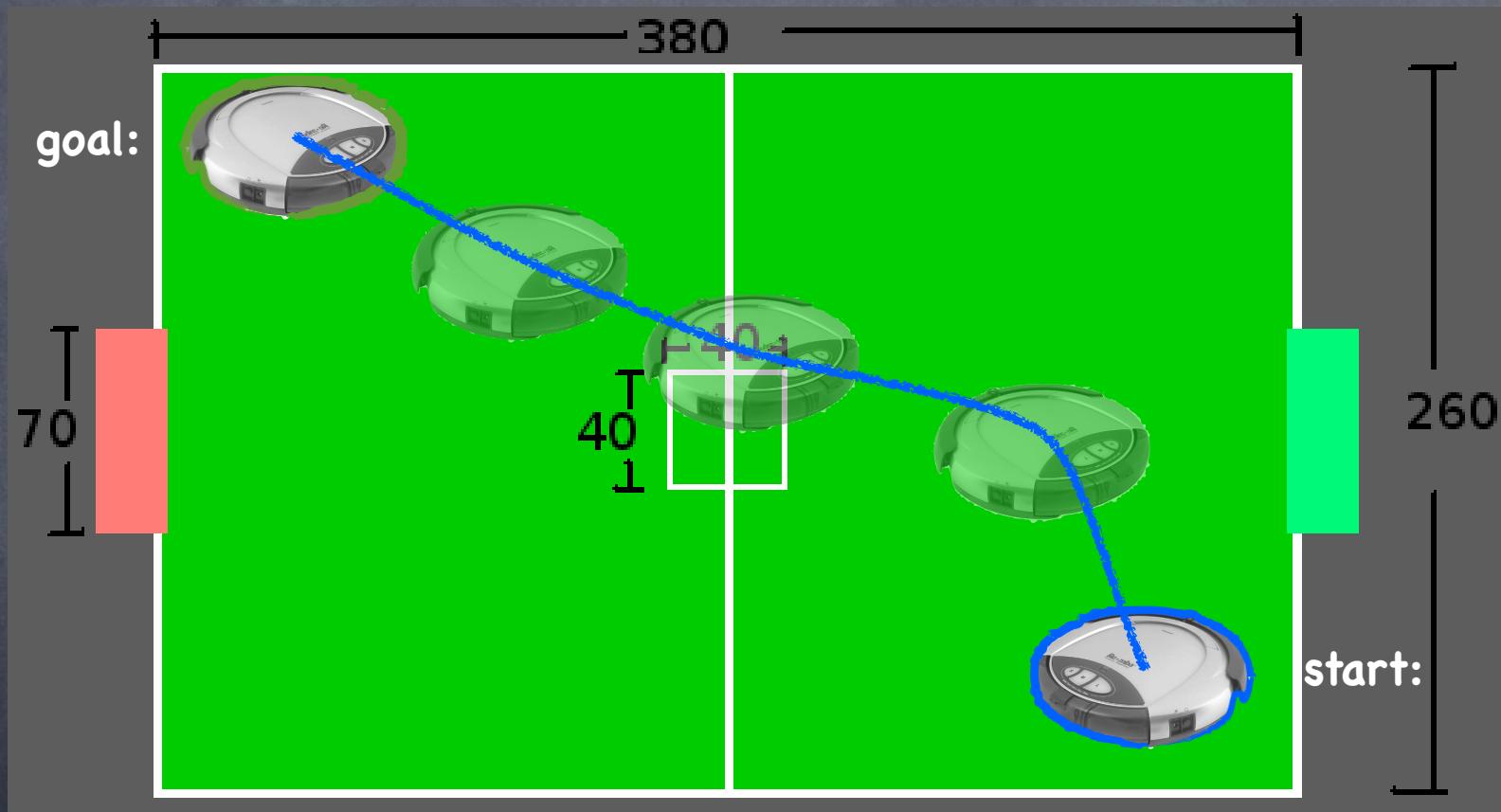


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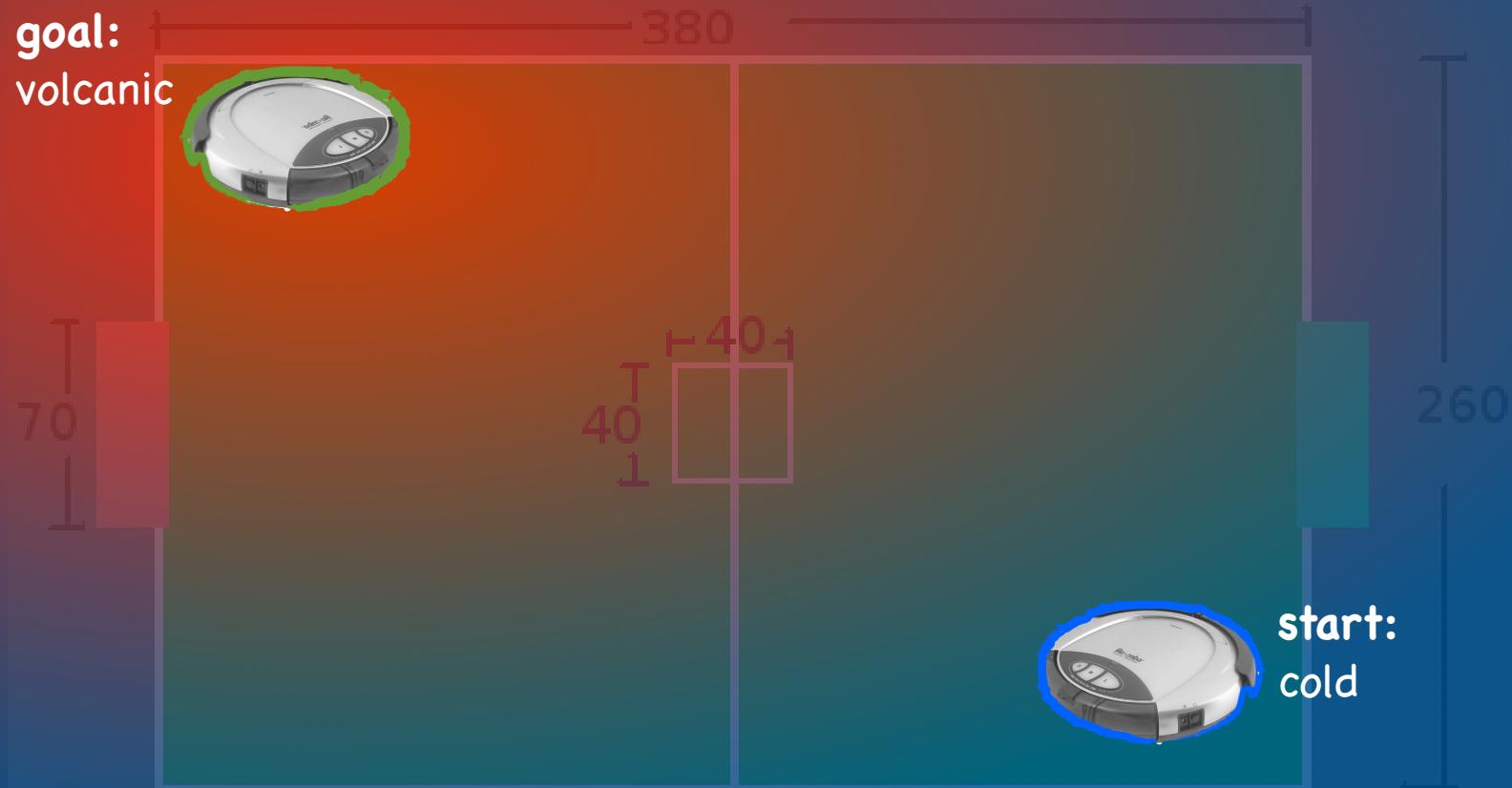
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Navigation (again)



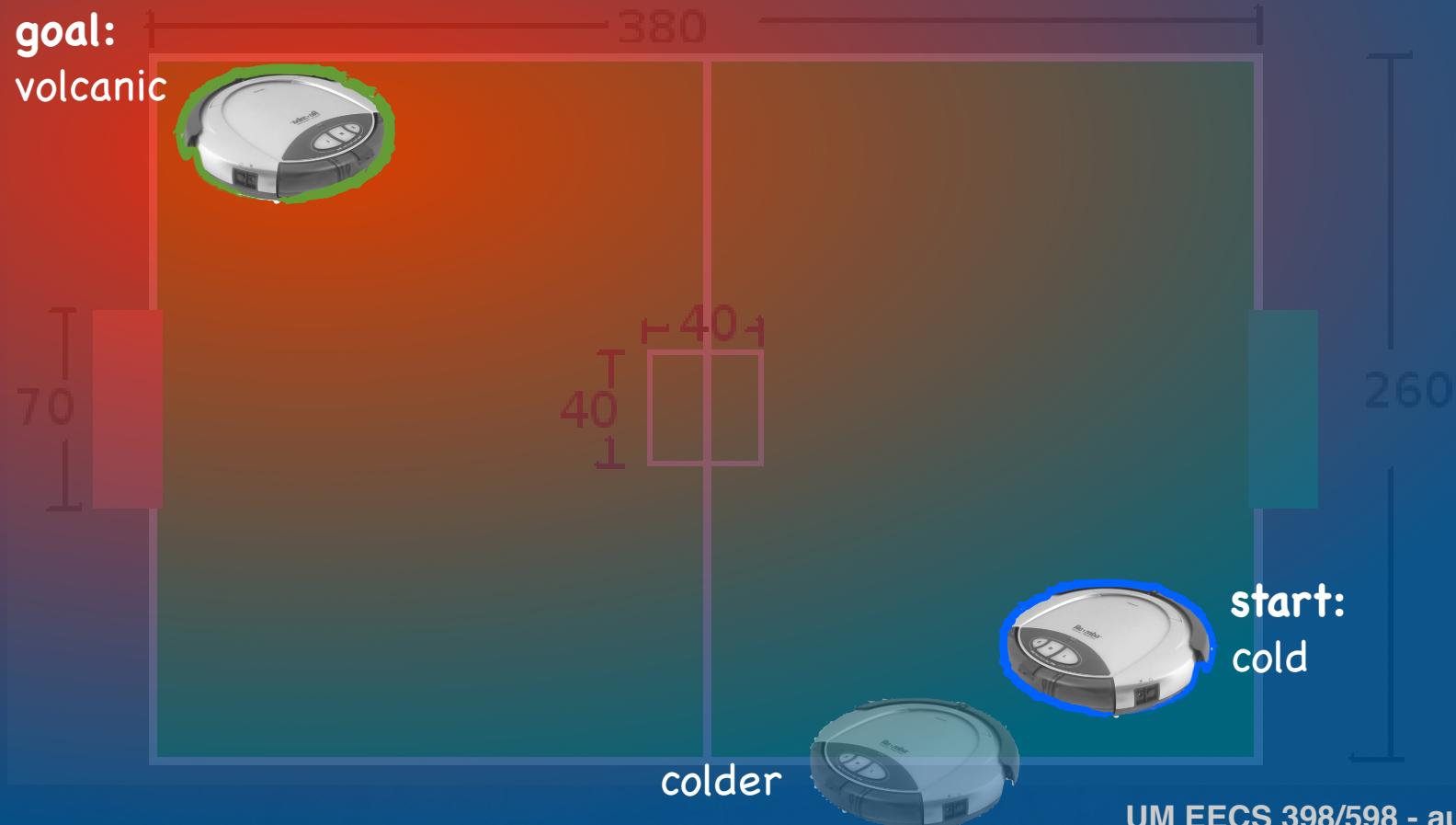
Potential field

(like a game of “warmer-colder”)



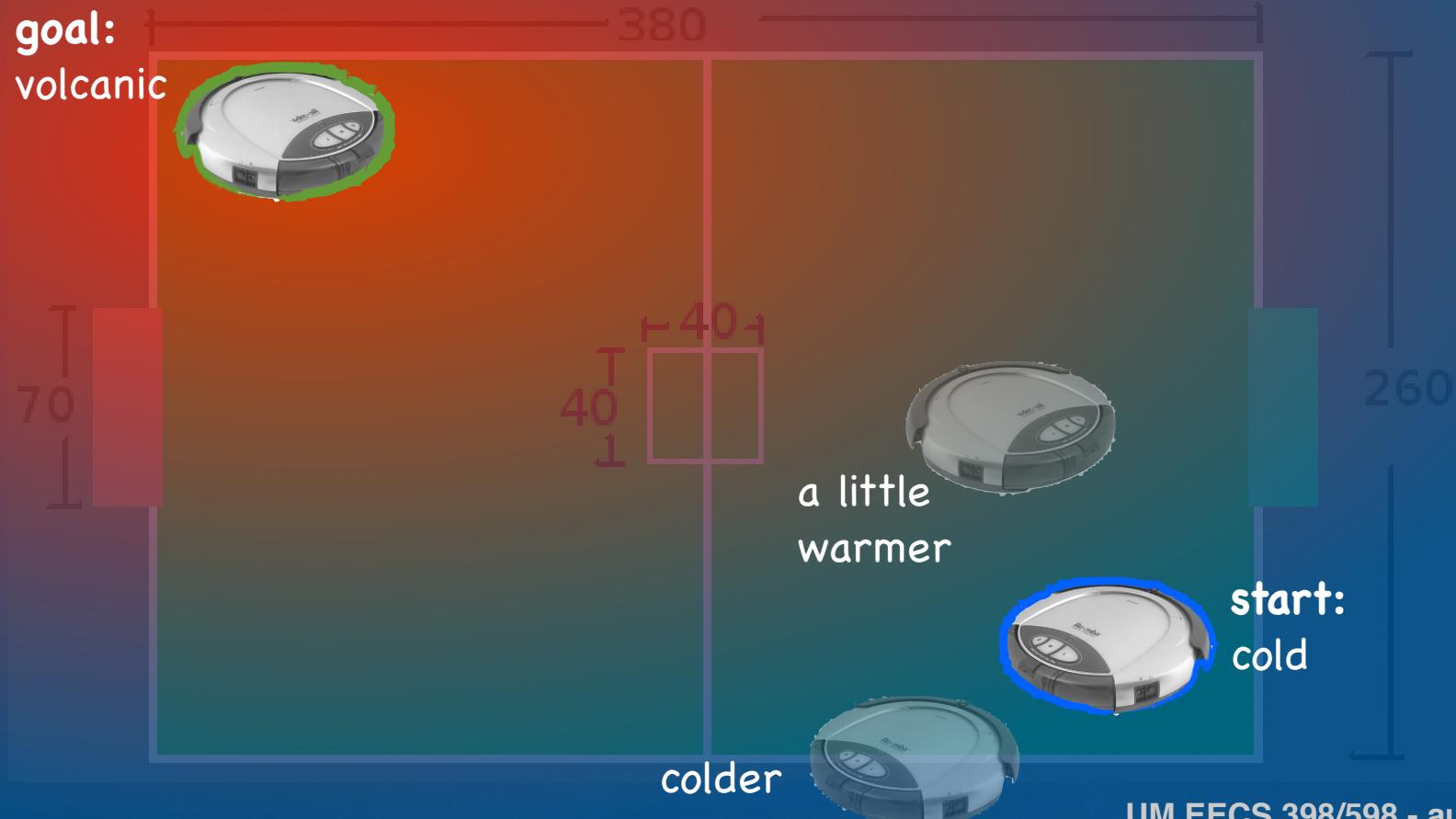
Potential field

(like a game of “warmer-colder”)



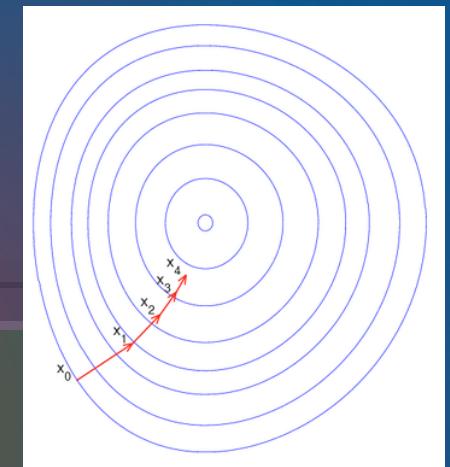
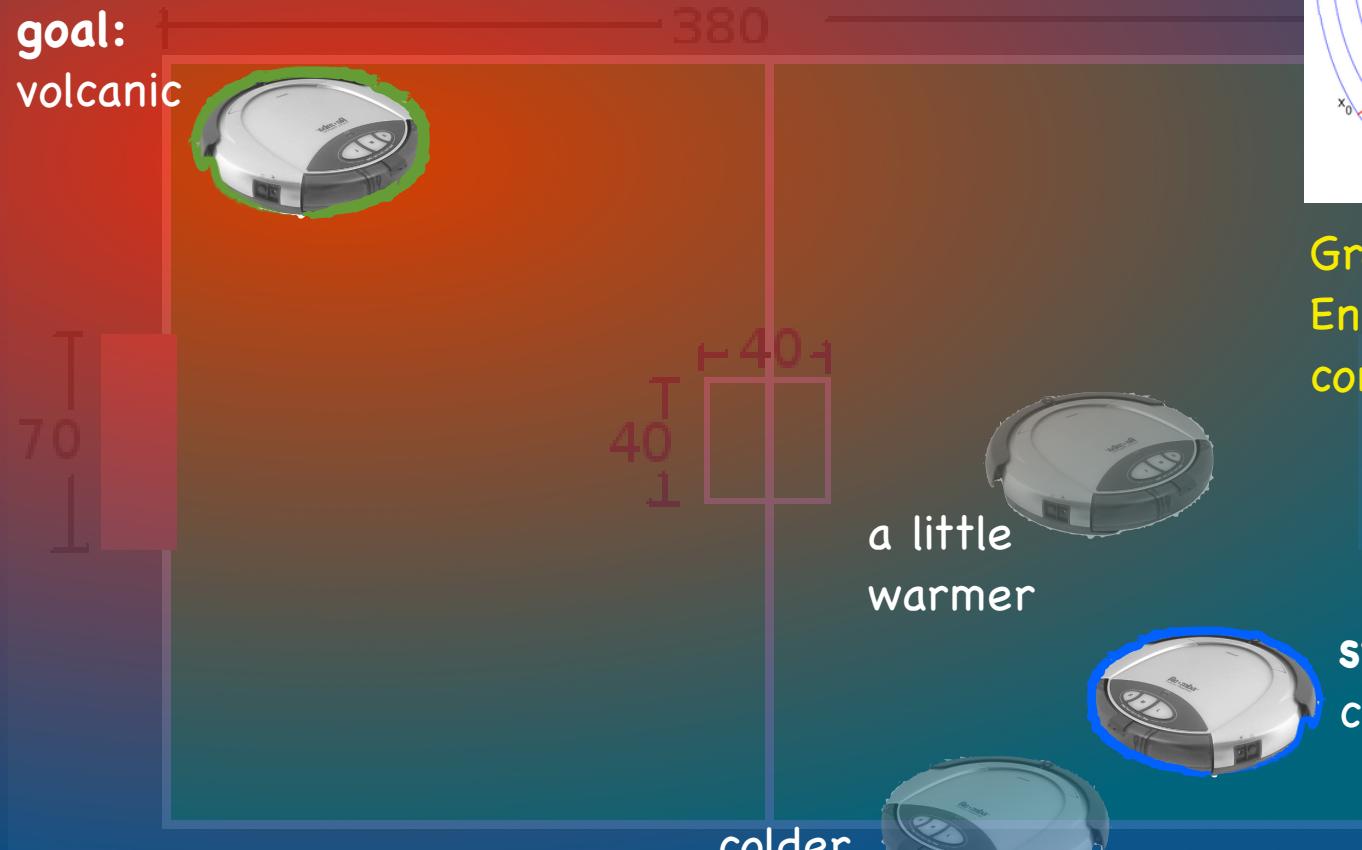
Potential field

(like a game of “warmer-colder”)



Potential field

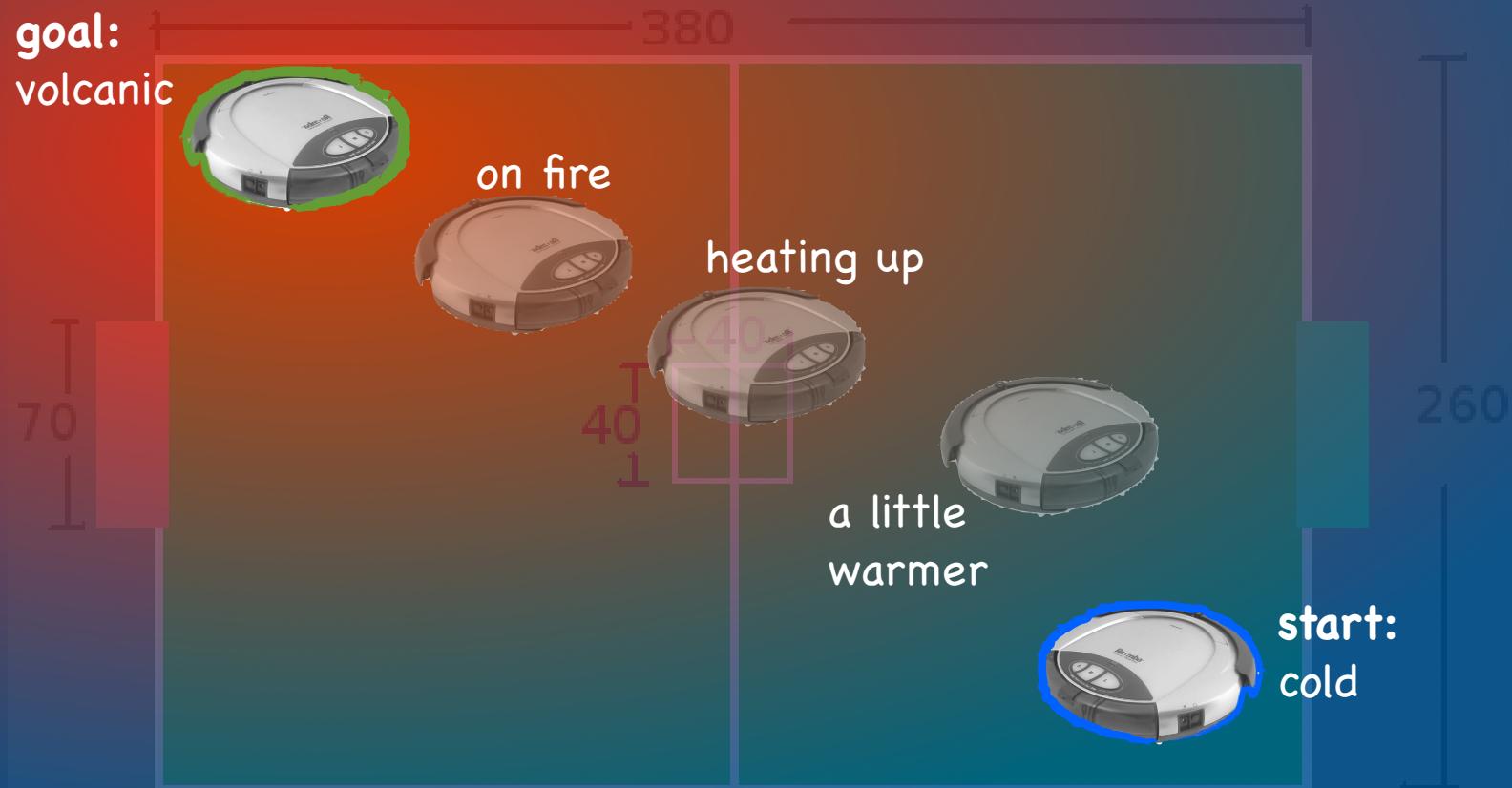
(like a game of “warmer-colder”)



Gradient descent:
Energy potential
converges at goal

Potential field

(like a game of “warmer-colder”)



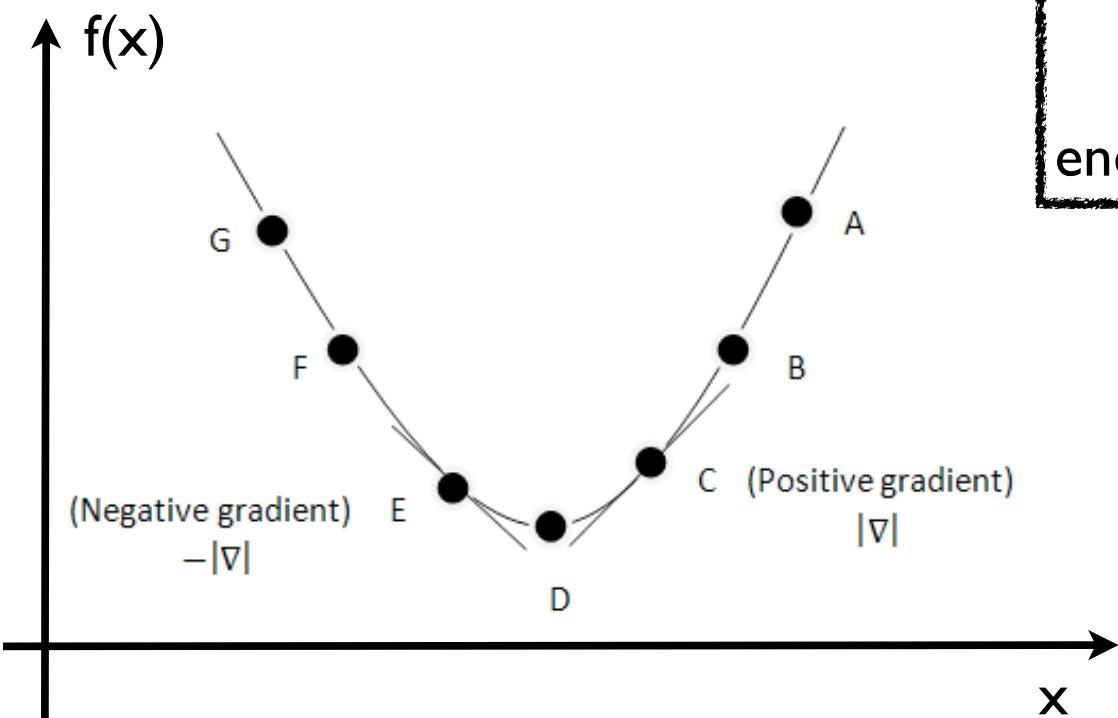
How do we define a
potential field?

Potential Field

- A potential field is a differentiable function $U(q)$ that maps configurations to scalar “energy” value
- At any q , gradient $\nabla U(q)$ is the vector that maximally increases U
- At goal q_{goal} , energy is minimized such that $\nabla U(q_{goal}) = 0$
- Navigation by descending field $-\nabla U(q)$ to goal

Gradient descent

From Wikipedia, the free encyclopedia



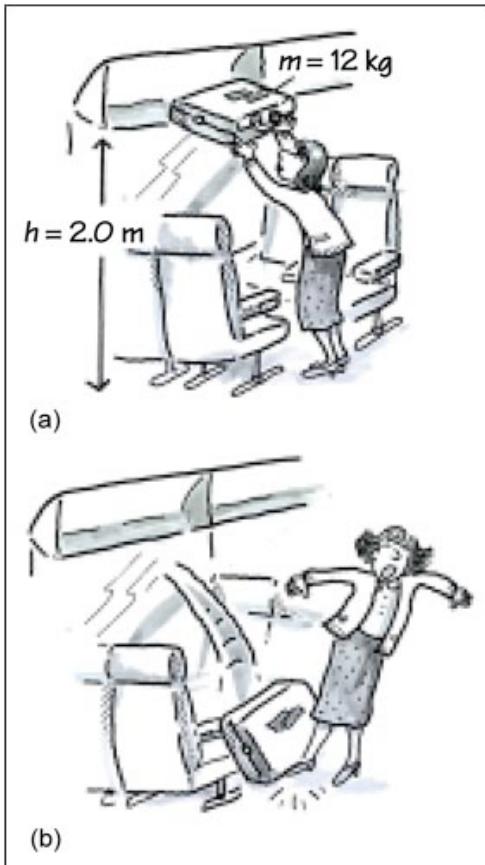
Gradient Descent Algorithm:

```
qpath[0] ← qstart
i ← 0
while (||∇U(q[i])|| > ε)
    qpath[i+1] ← qpath[i] - α ∇U(qpath[i])
    i ← i+1
end
```

Derivative assumed to be direction
of steepest ascent away from goal

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \gamma_n \nabla F(\mathbf{x}_n)$$

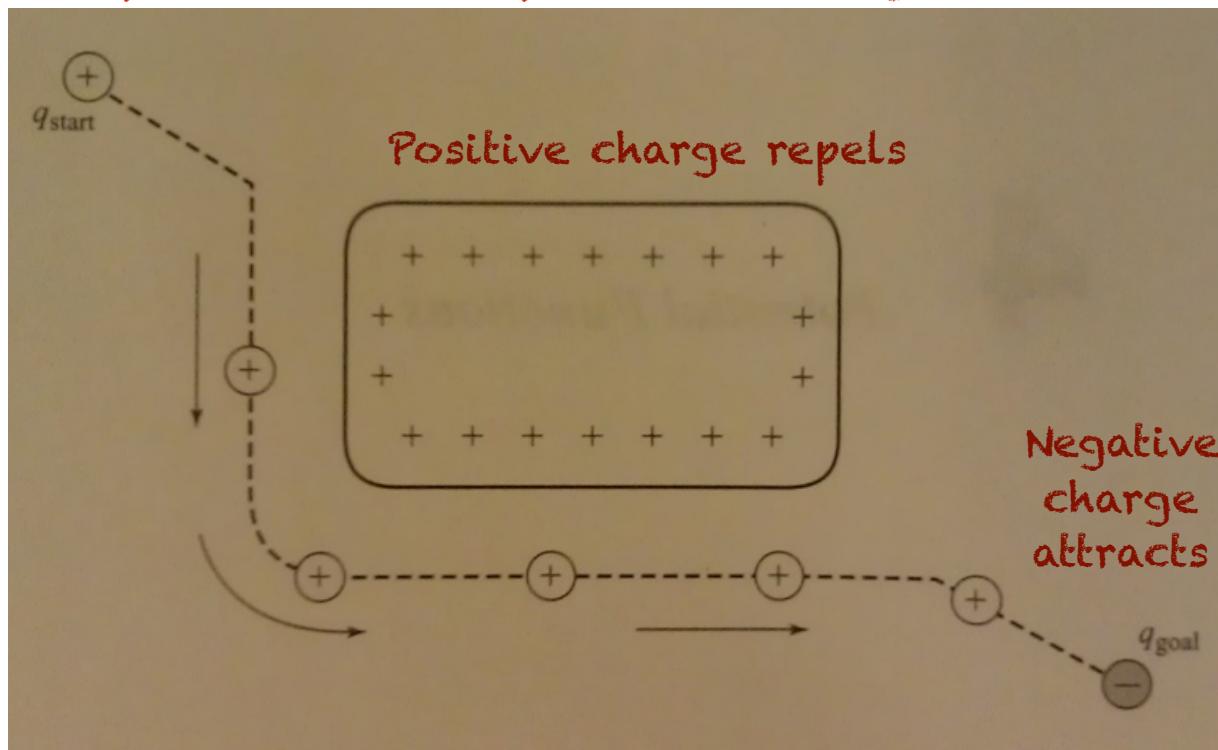
Potential Energy



- Energy stored in a physical system
- Kinetic motion caused by system moving to lower energy state
- For objects acting only w.r.t. gravity
- $\text{potential_energy} = \text{mass} * \text{height} * \text{gravity}$

Charged Particle Example

Positively charged particle follows potential energy to goal



Convergent Potentials

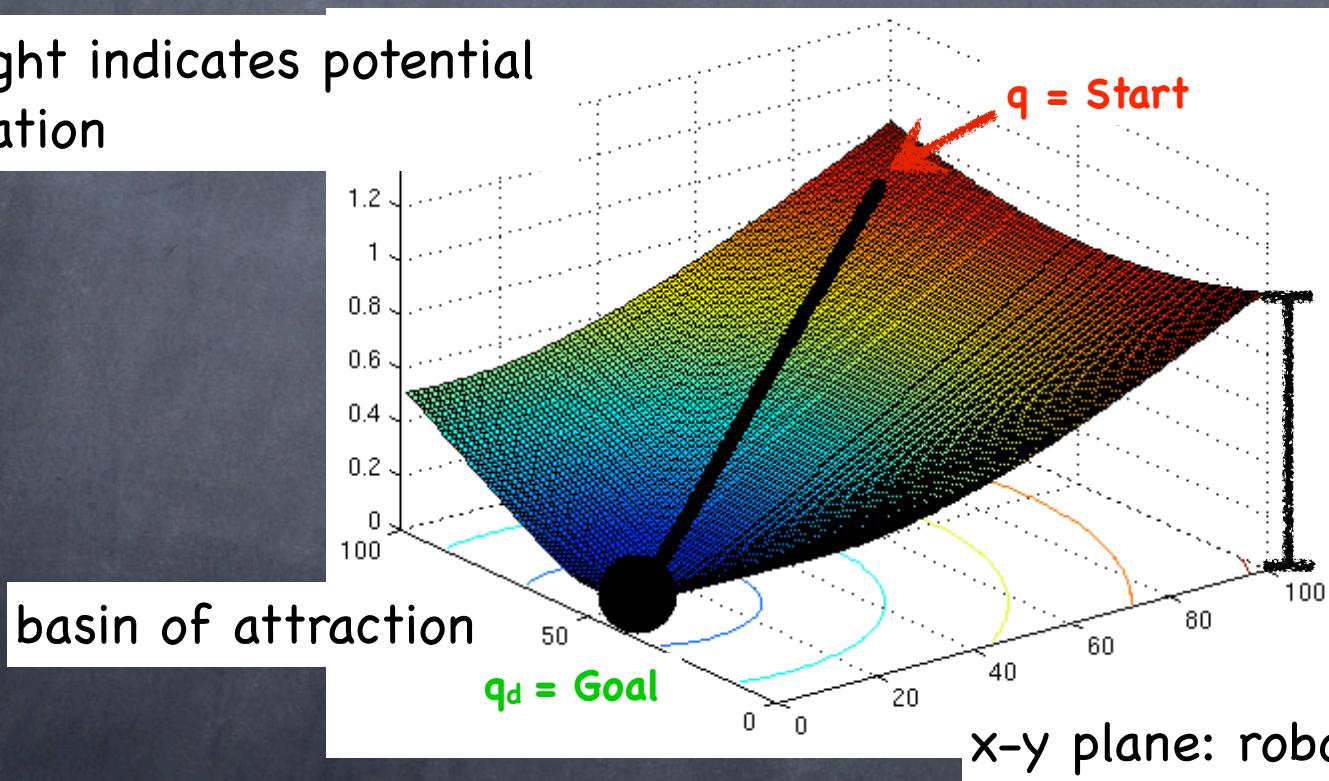
let's call these "attractor landscapes"



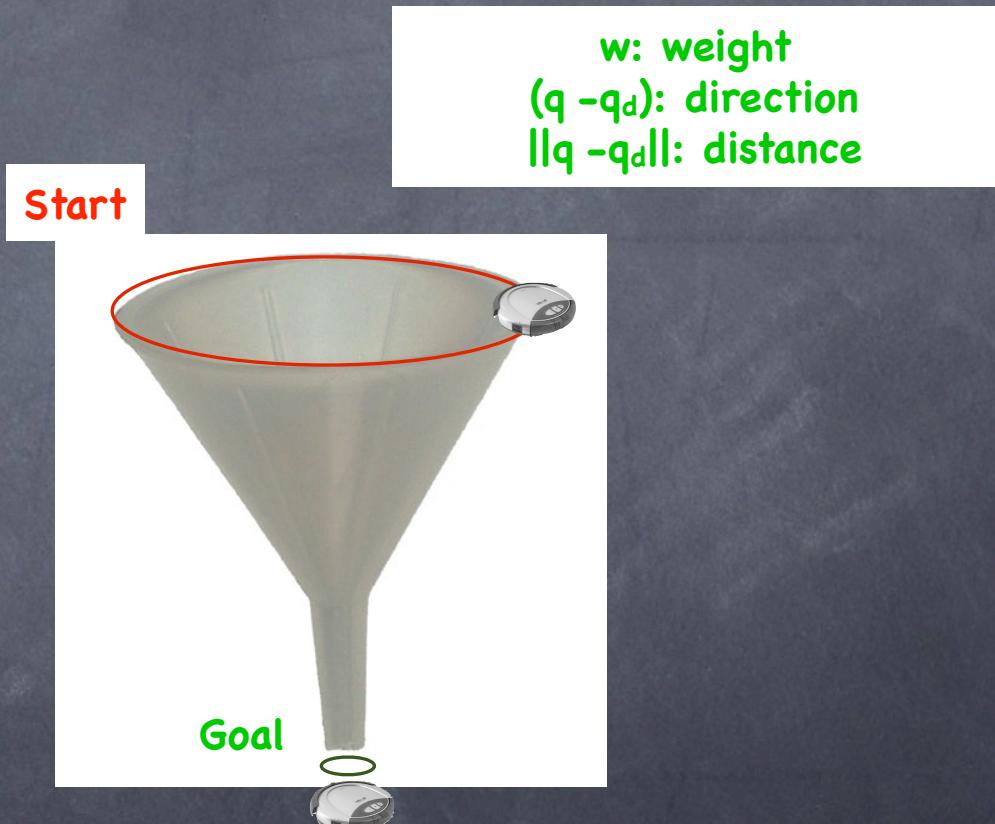
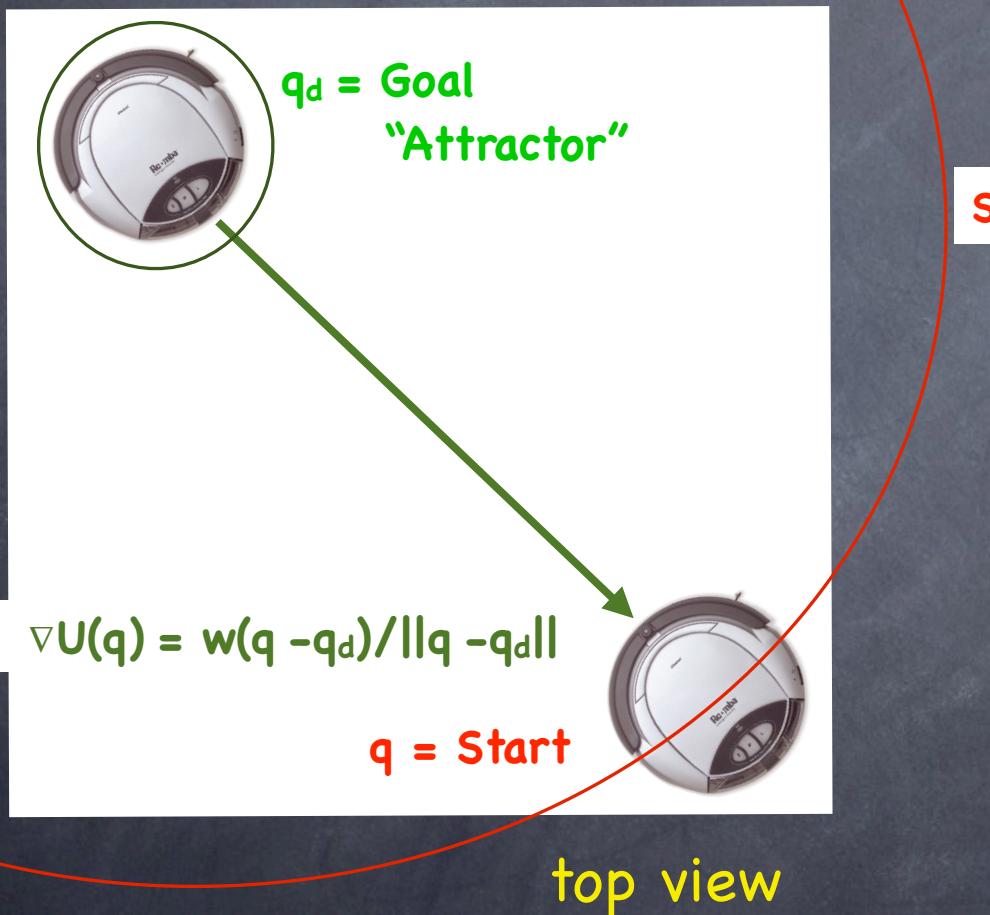
basin of attraction

2D potential navigation

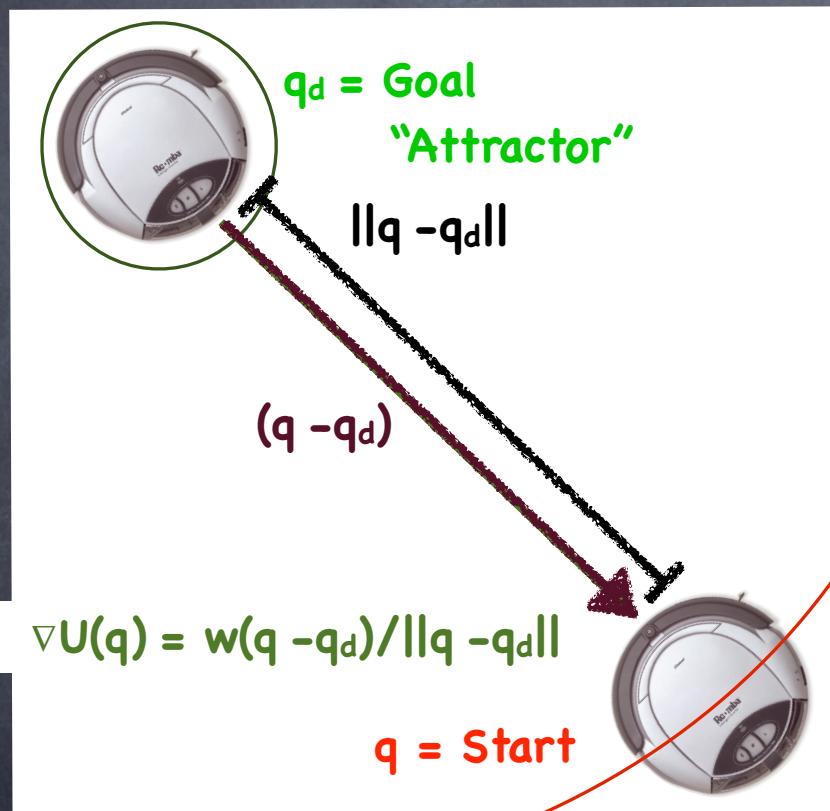
z : height indicates potential at location



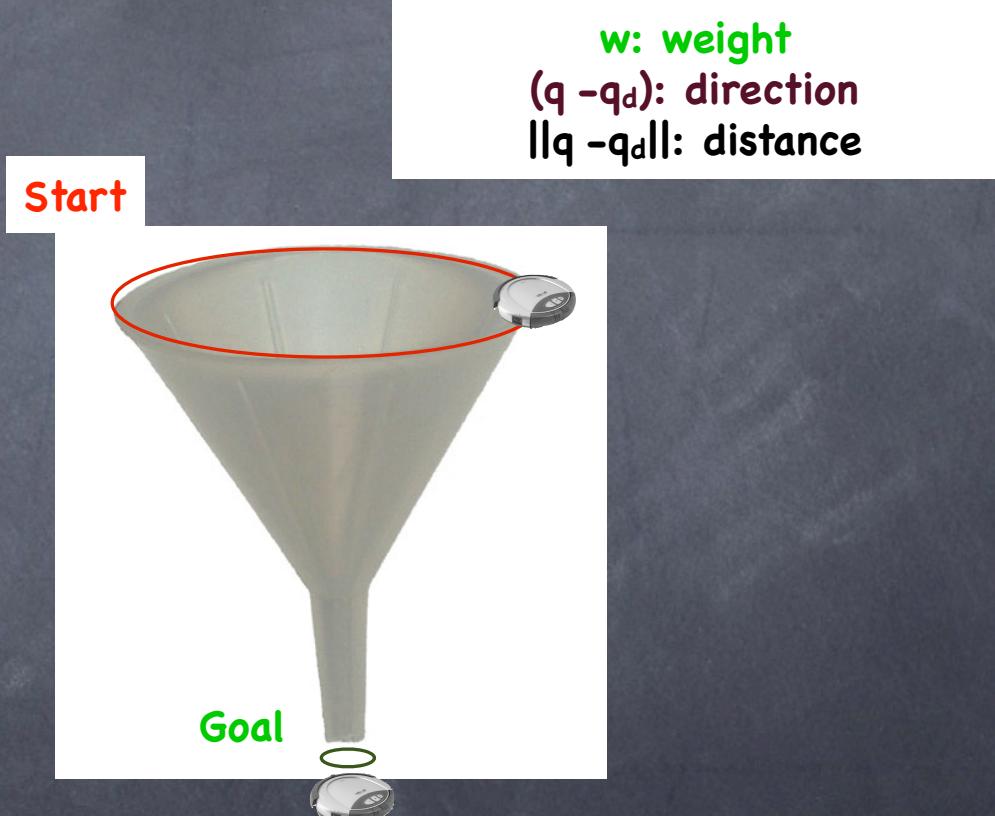
“Cone” Attractor



“Cone” Attractor

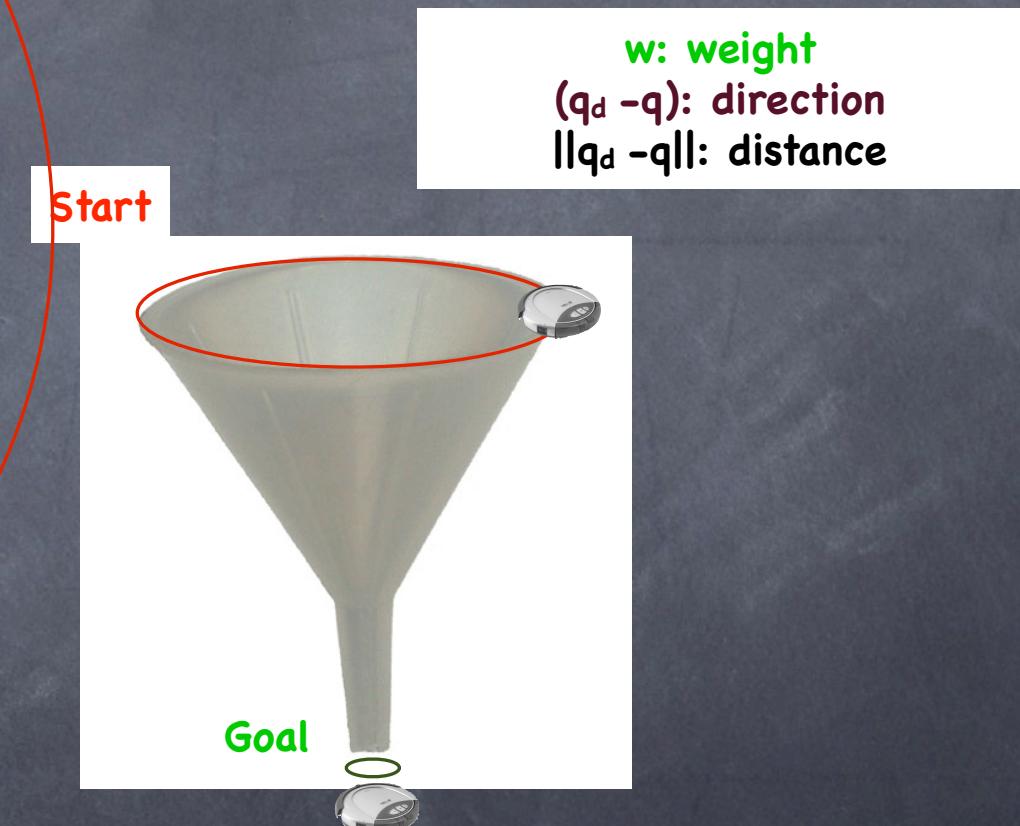
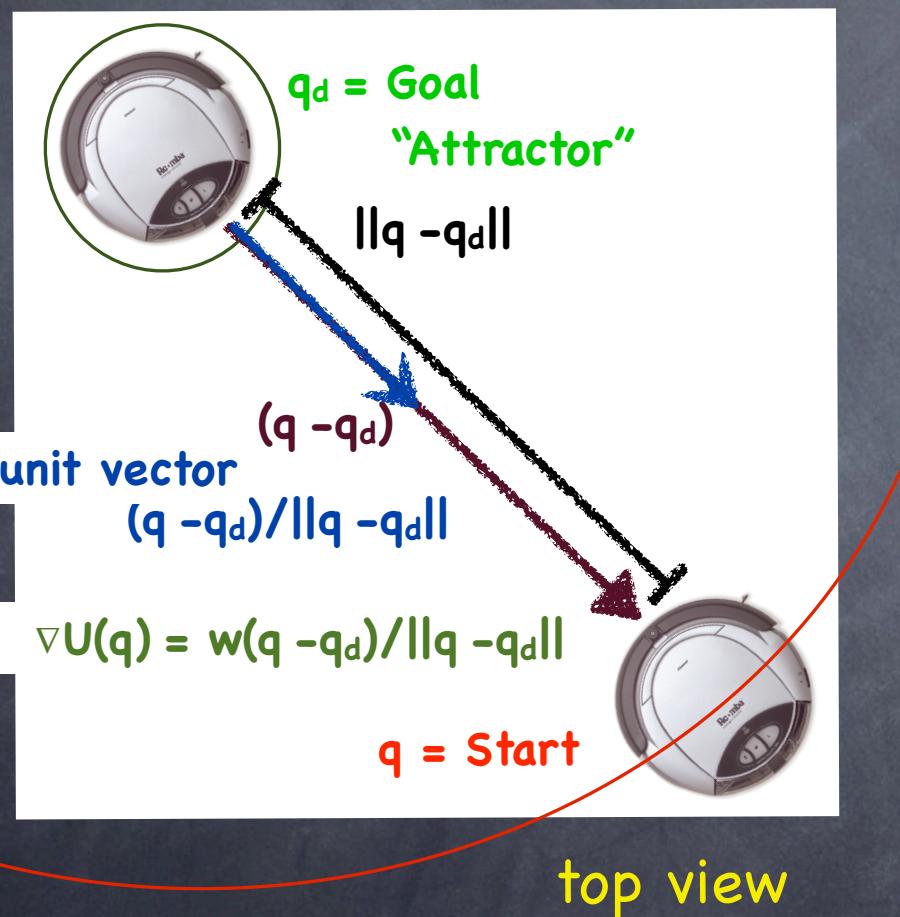


top view

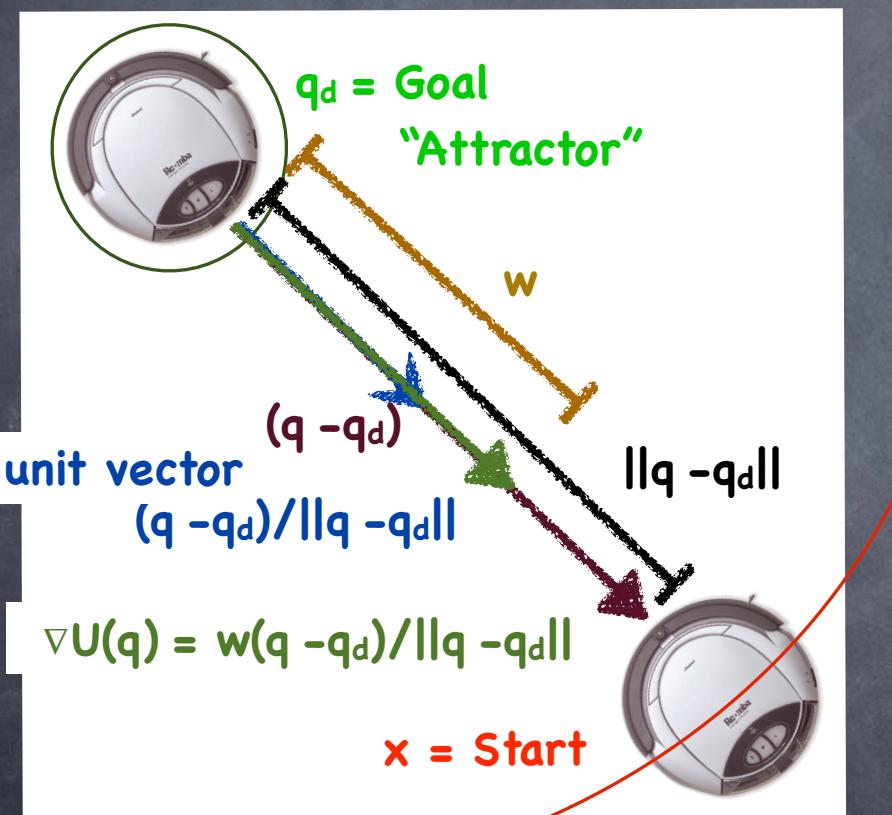


side view

“Cone” Attractor

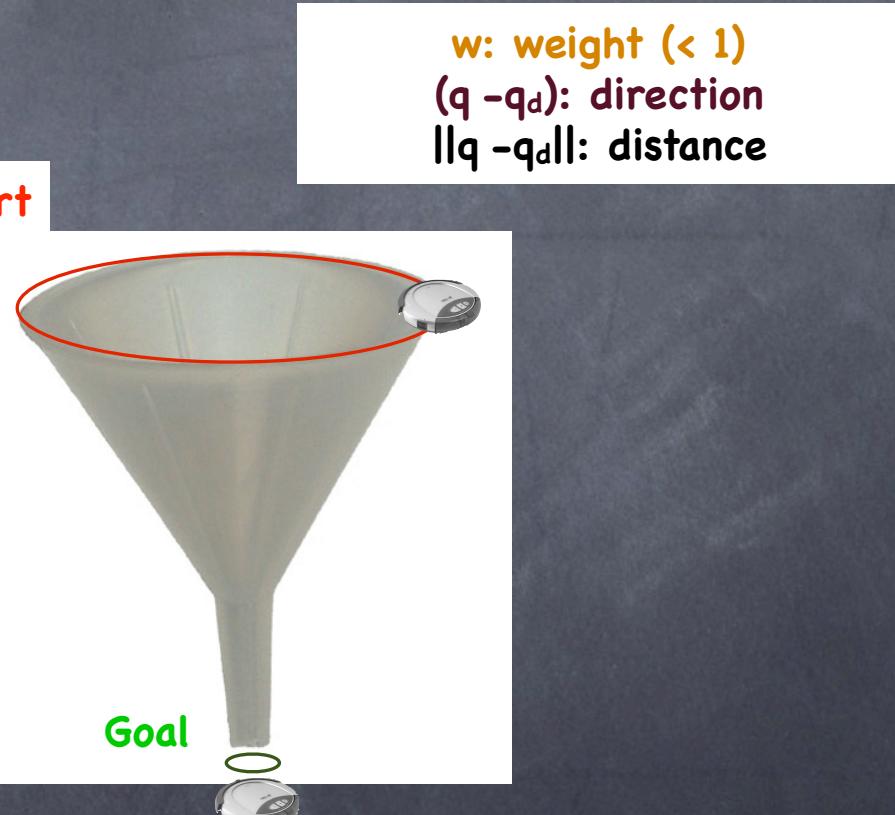


“Cone” Attractor



top view

Start

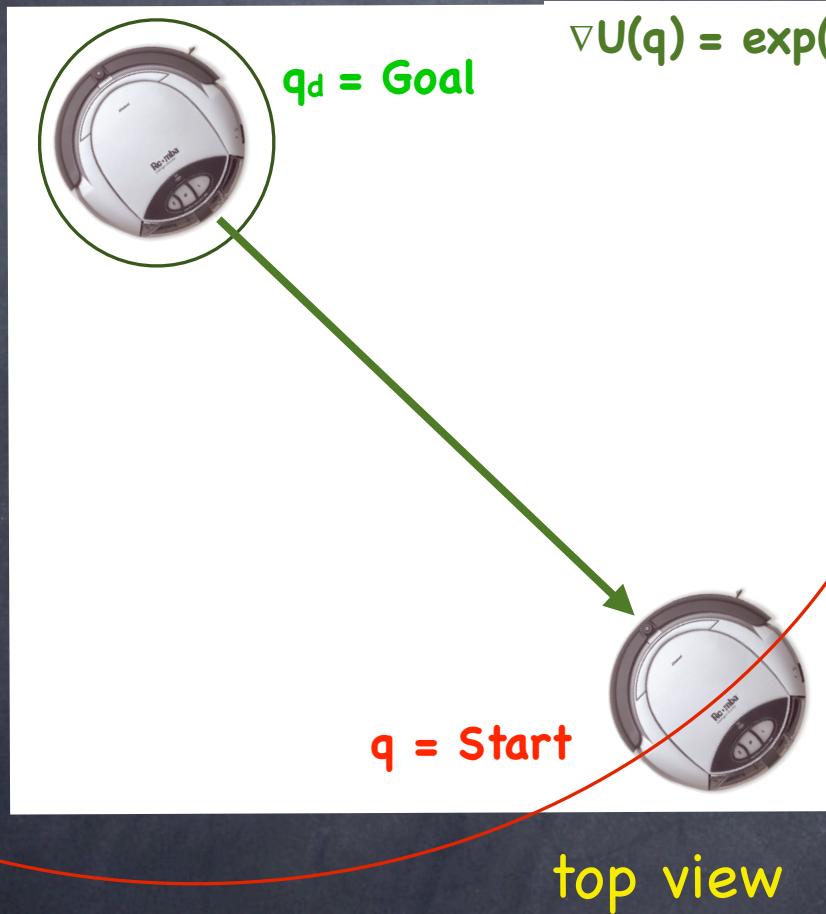


side view

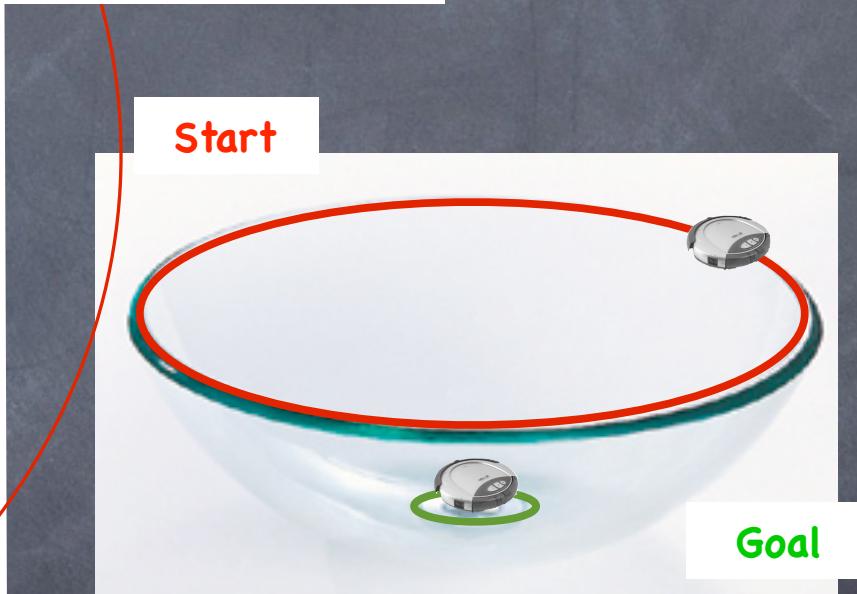
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w: weight (< 1)
 $(q - q_d)$: direction
 $\|q - q_d\|$: distance

“Bowl” Attractor

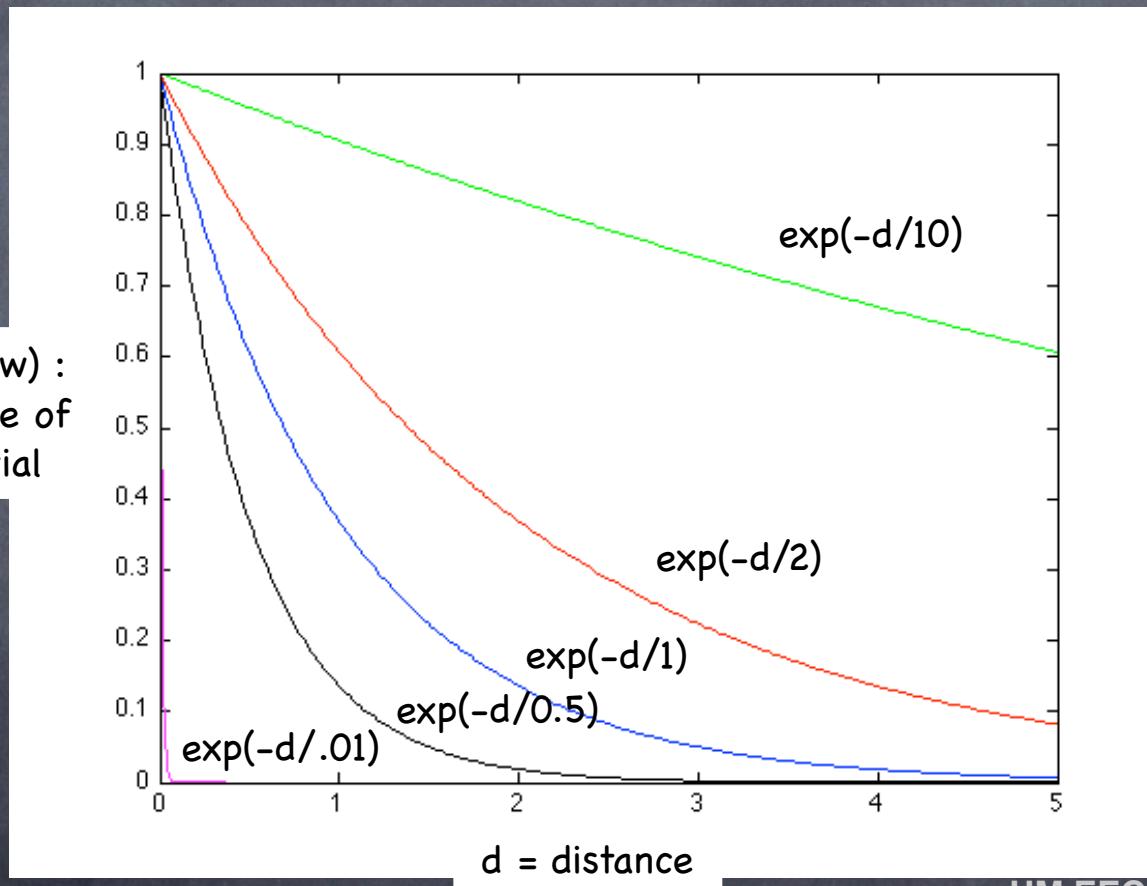


$$\nabla U(q) = \exp(-\|q - q_d\|/w) (q - q_d)$$

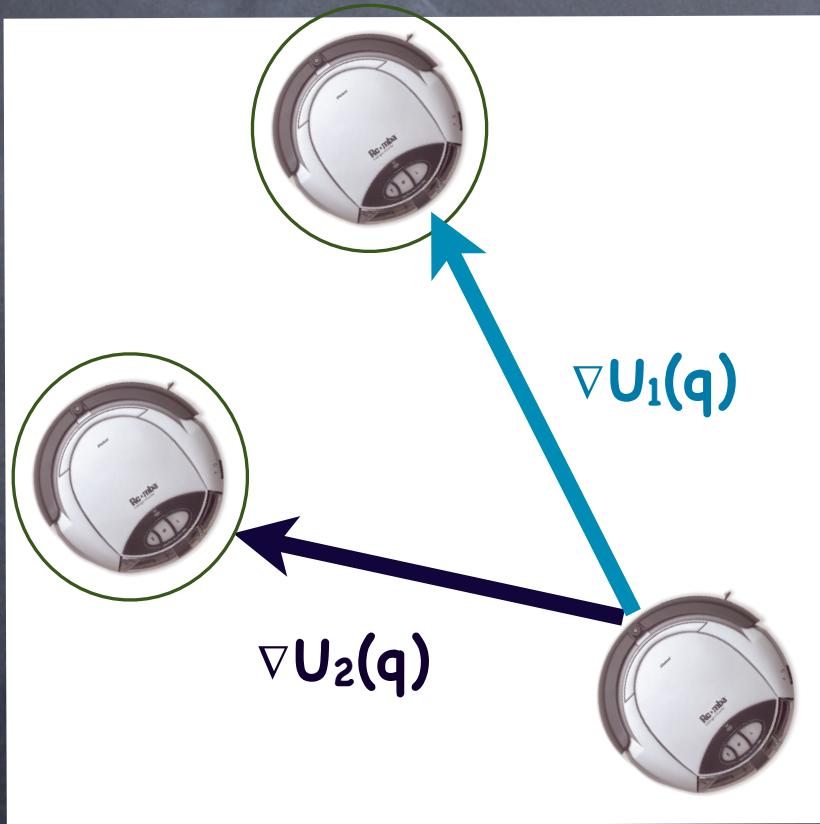


$\exp(-d/w)$

$\exp(-d/w)$:
influence of
potential

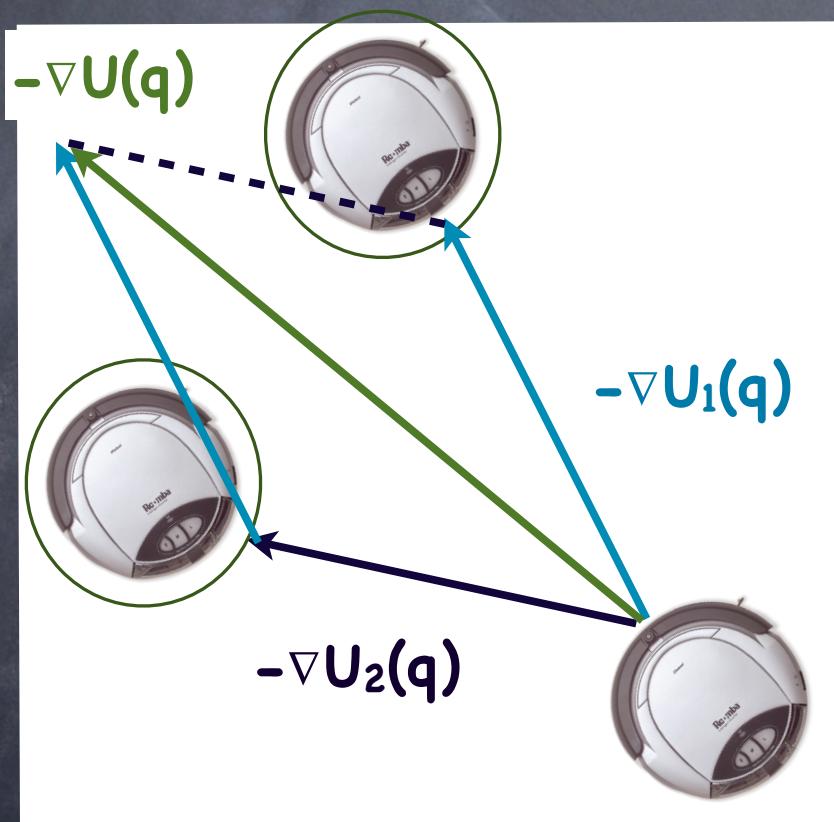


Multiple potentials



- Output of potential field is a vector
- How to combine or select between multiple potentials?

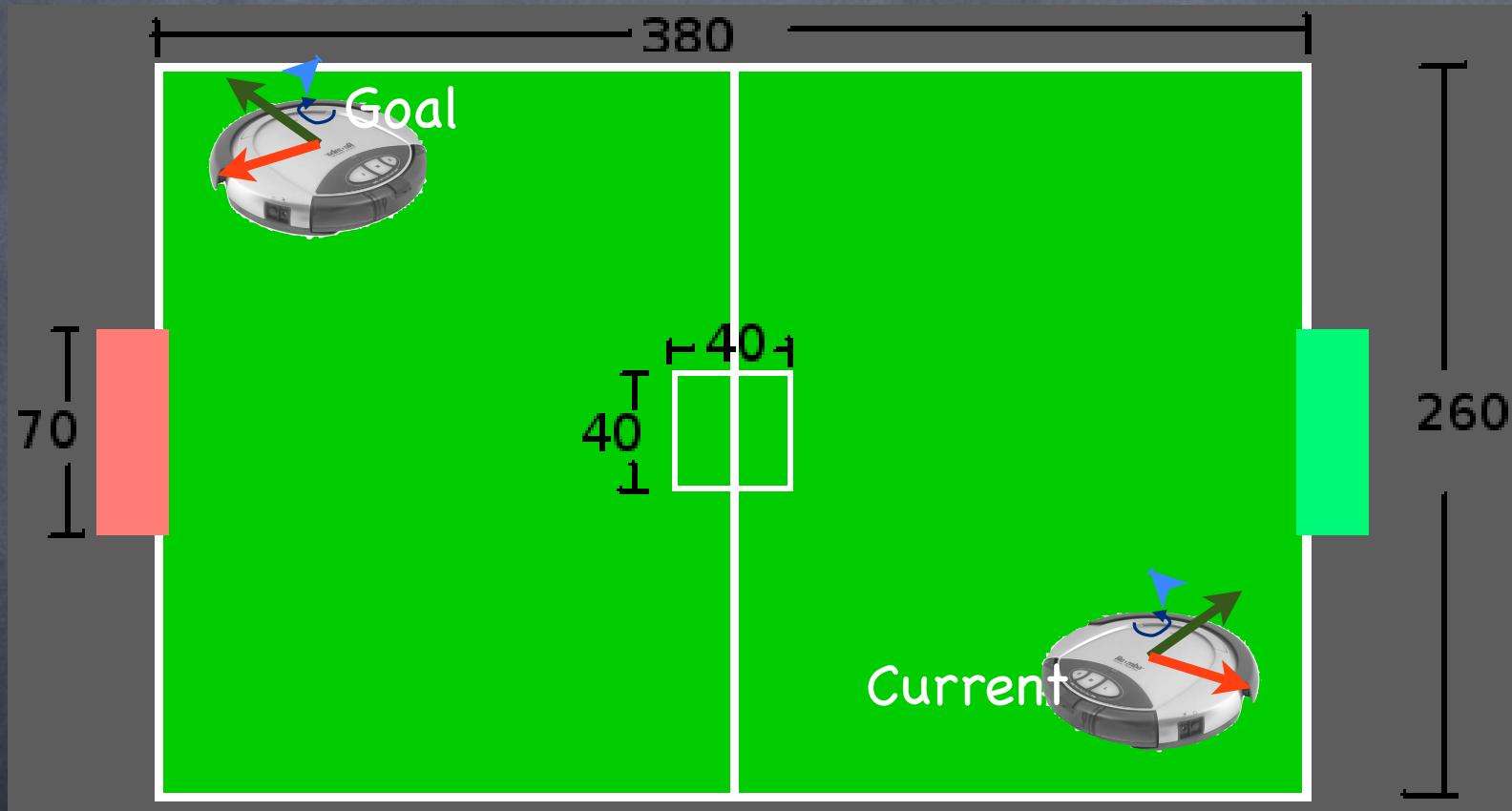
Multiple potentials



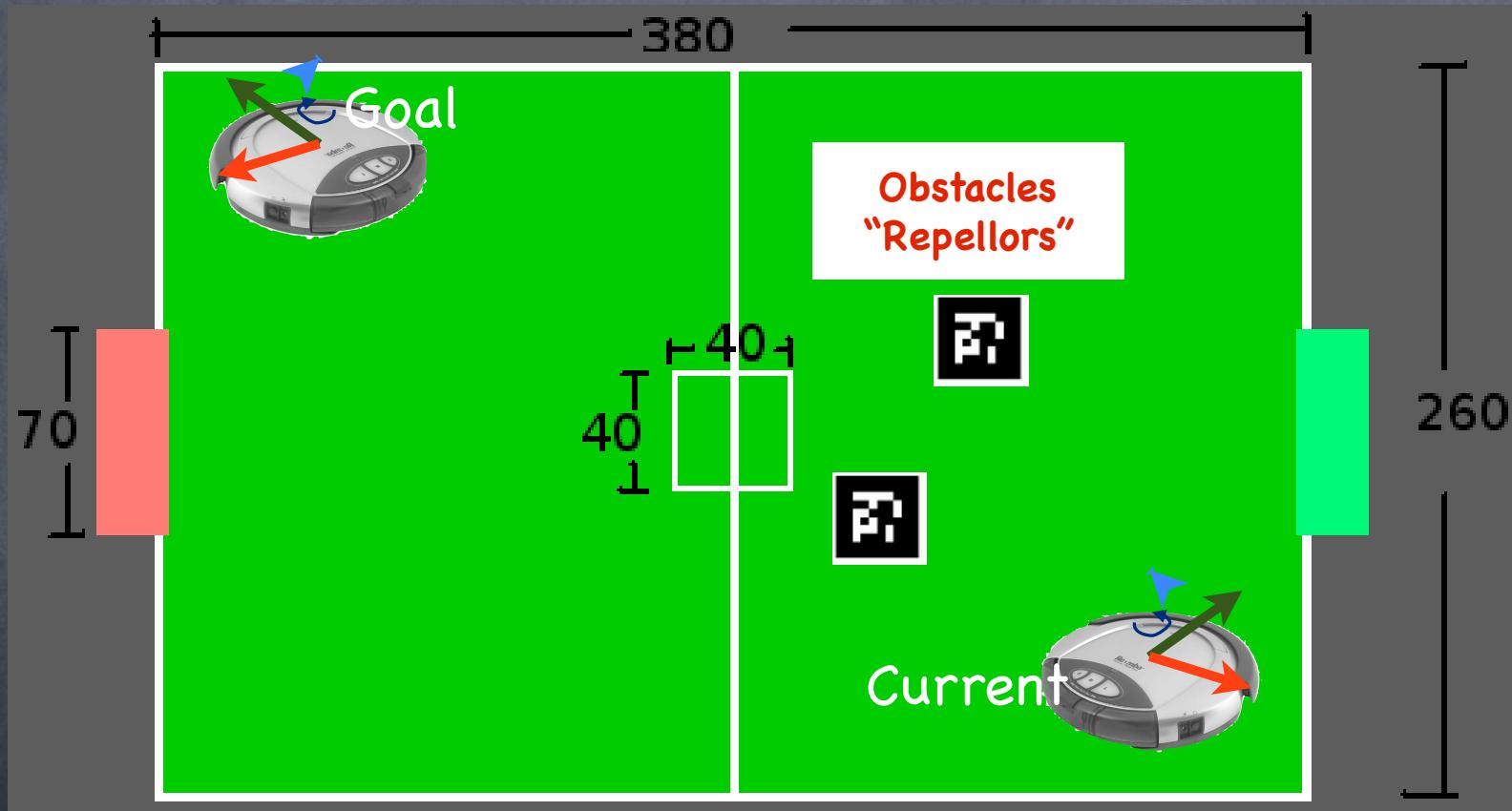
- Output of potential field is a vector
- Combine multiple potentials through vector summation

$$U(q) = \sum_i U_i(q)$$

describe performance for this case

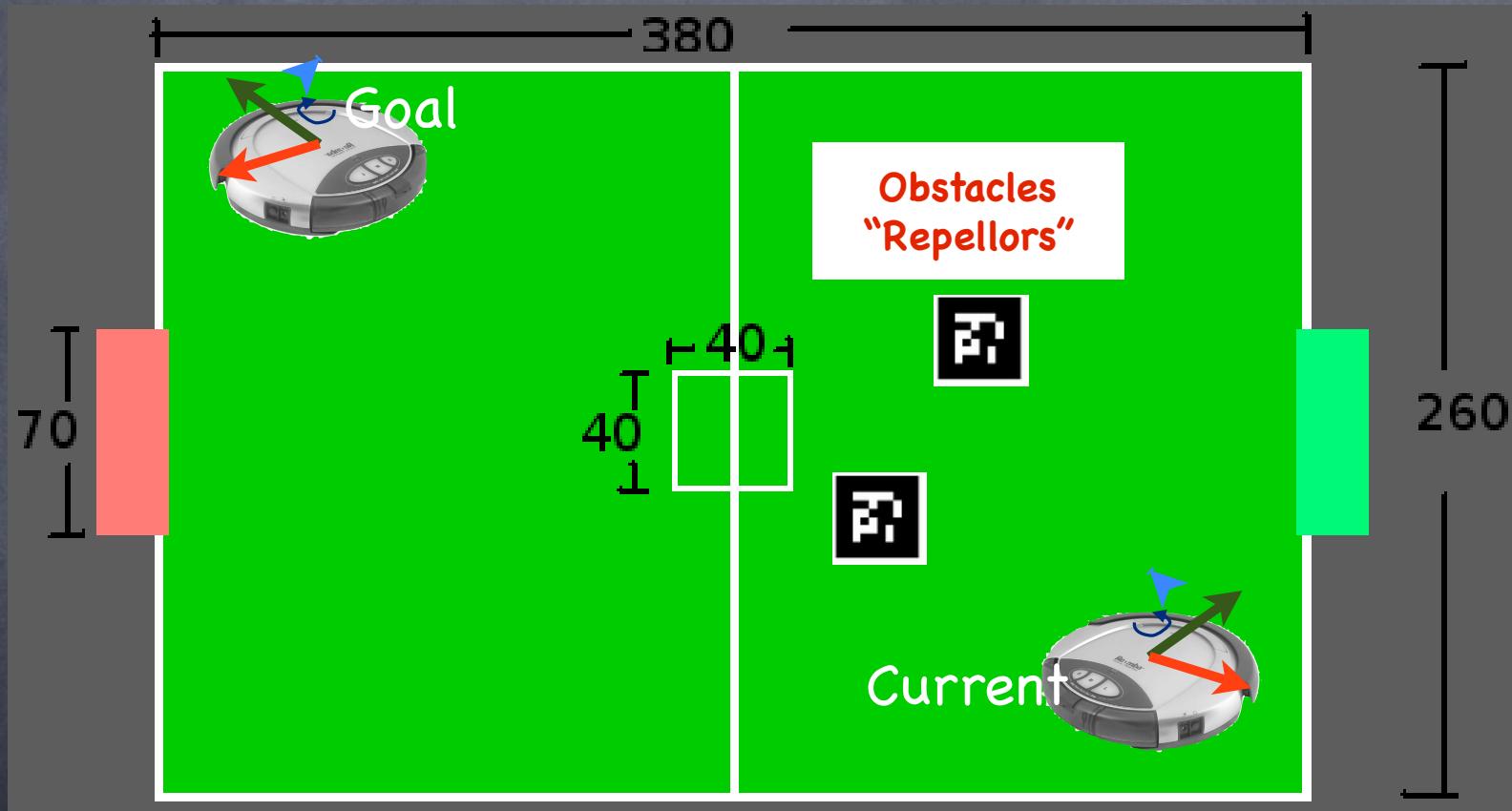


describe performance for this case



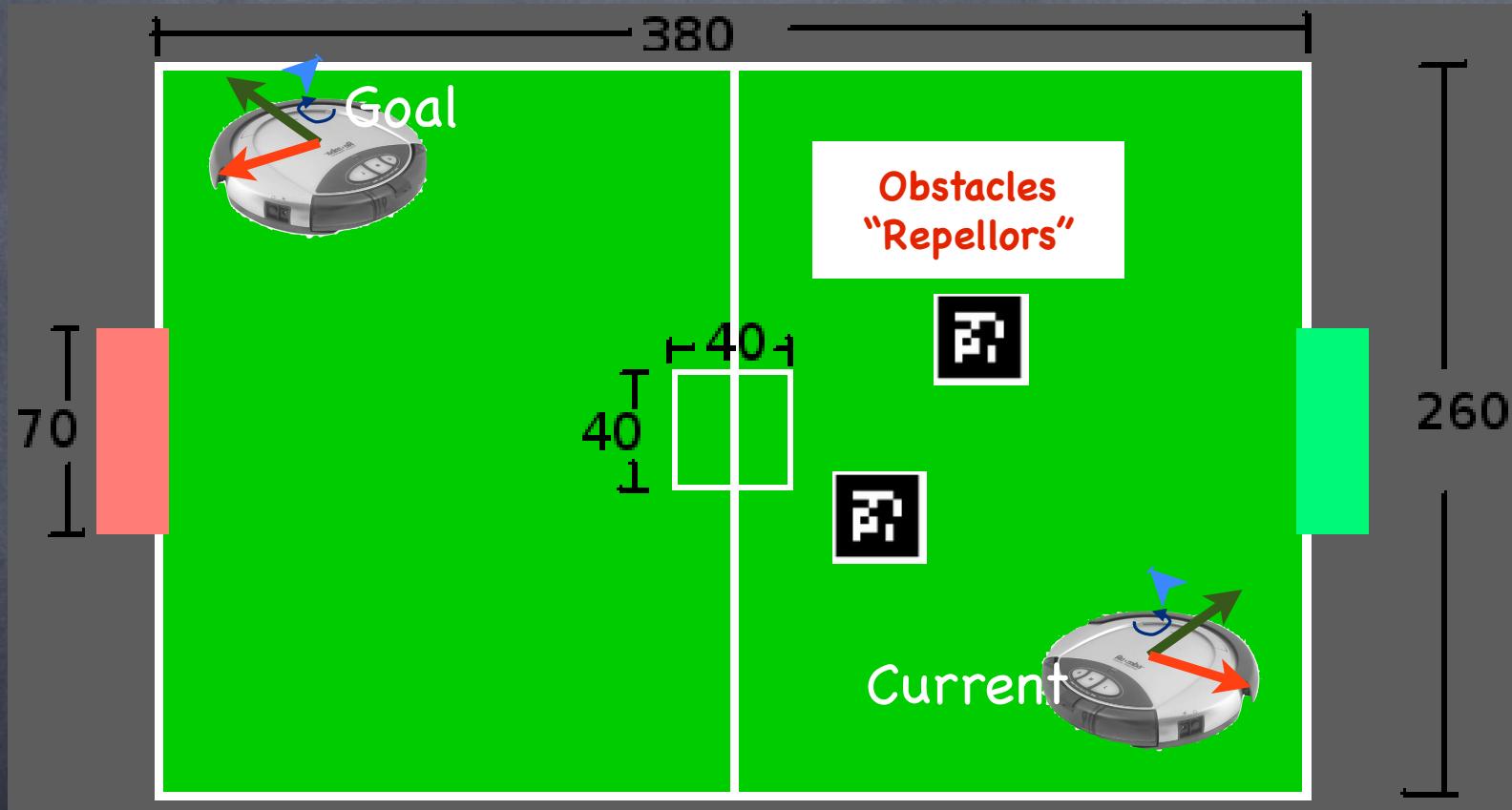
describe performance for this case

how do we deal with repellors?

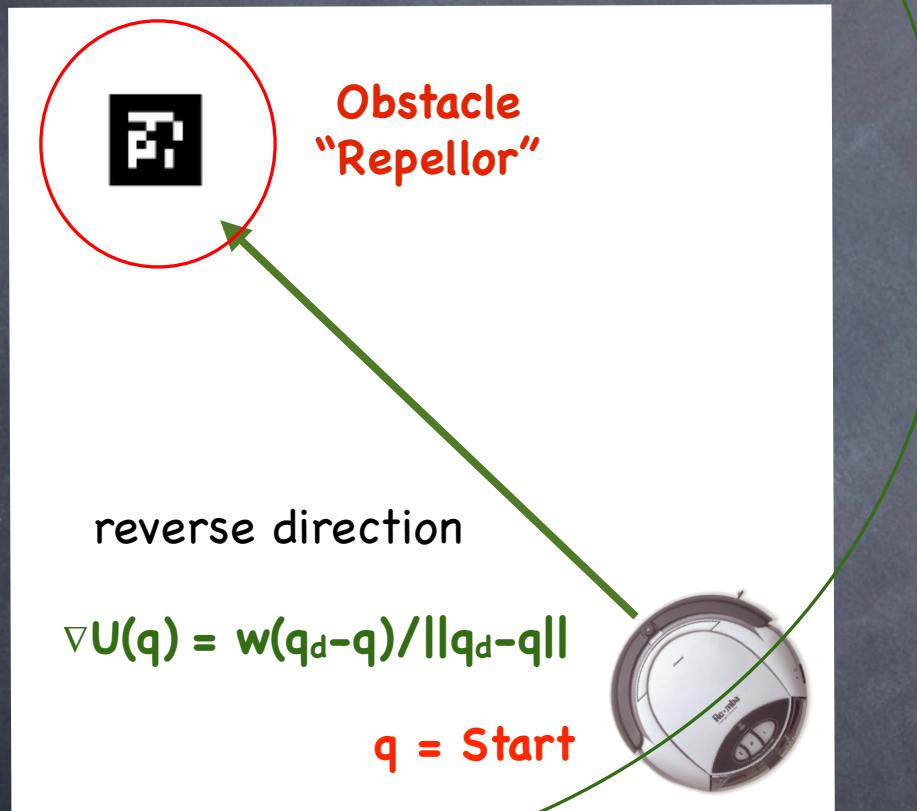


add sum of repulsive potentials

$$U(q) = U_{\text{attracts}}(q) + U_{\text{repellors}}(q)$$



“Cone” Repellor



potential problems?

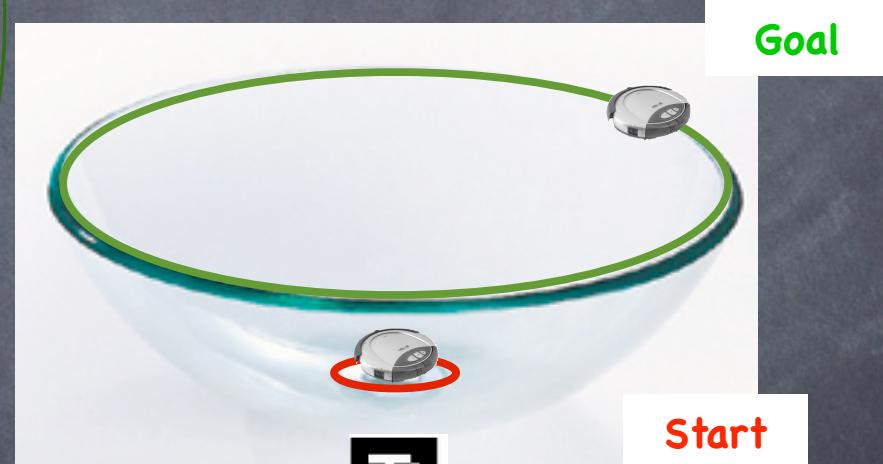
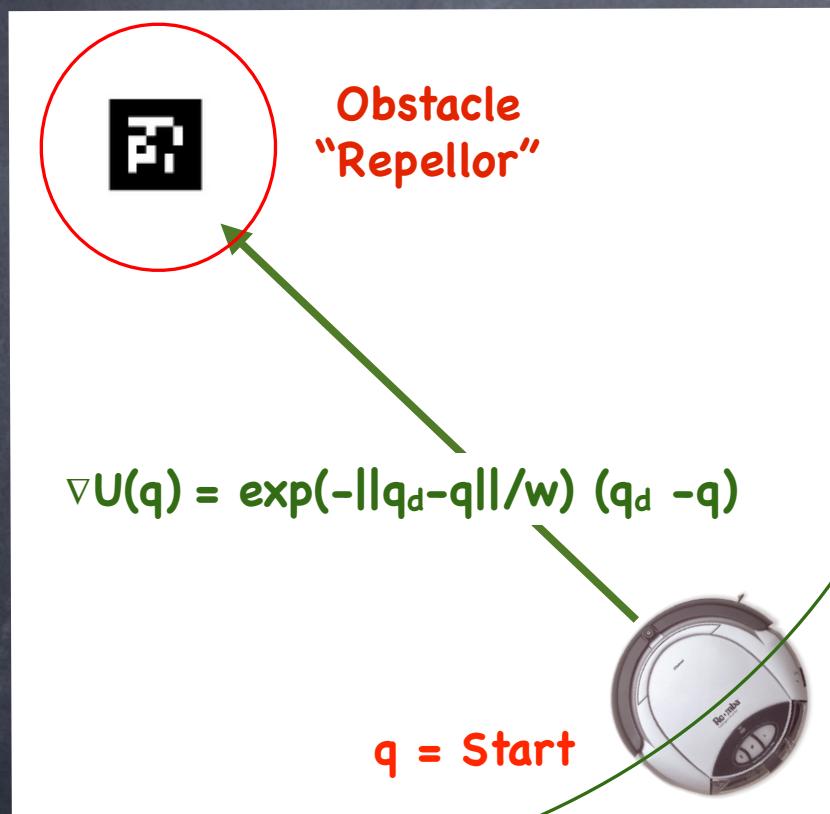


top view

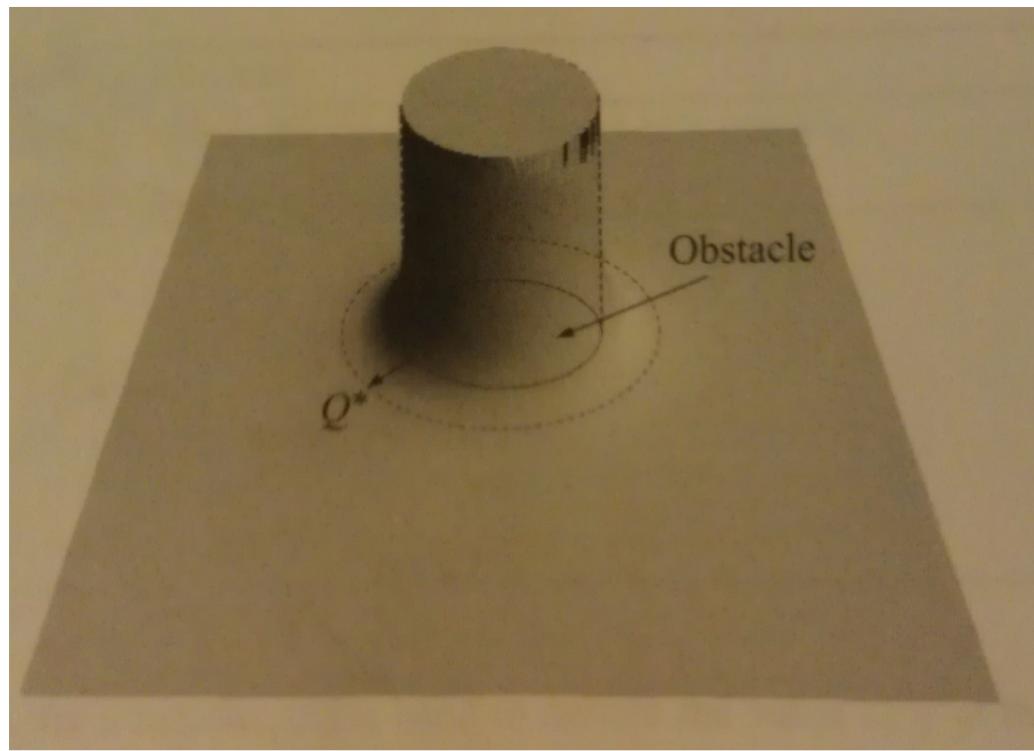
side view

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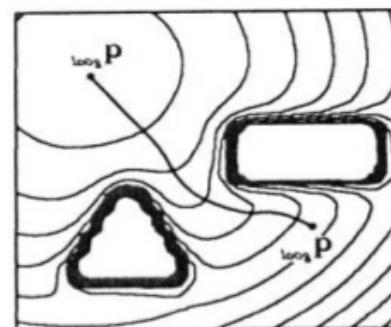
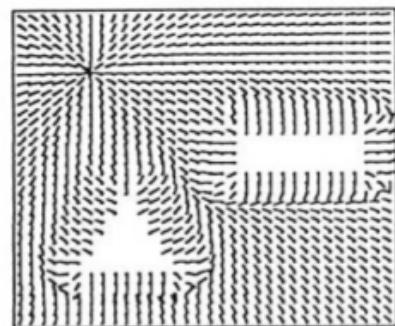
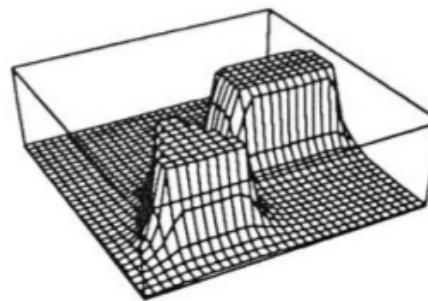
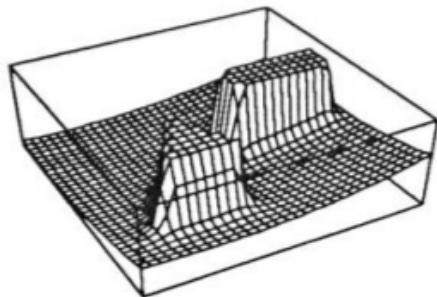
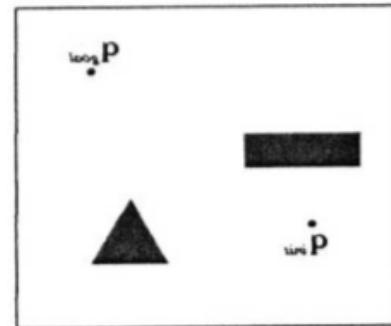
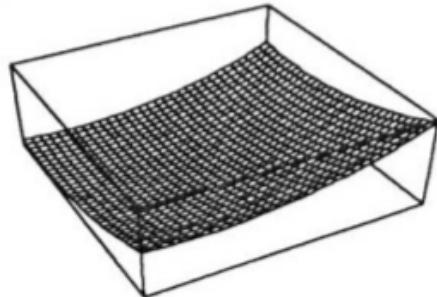
“Bowl” Repellor



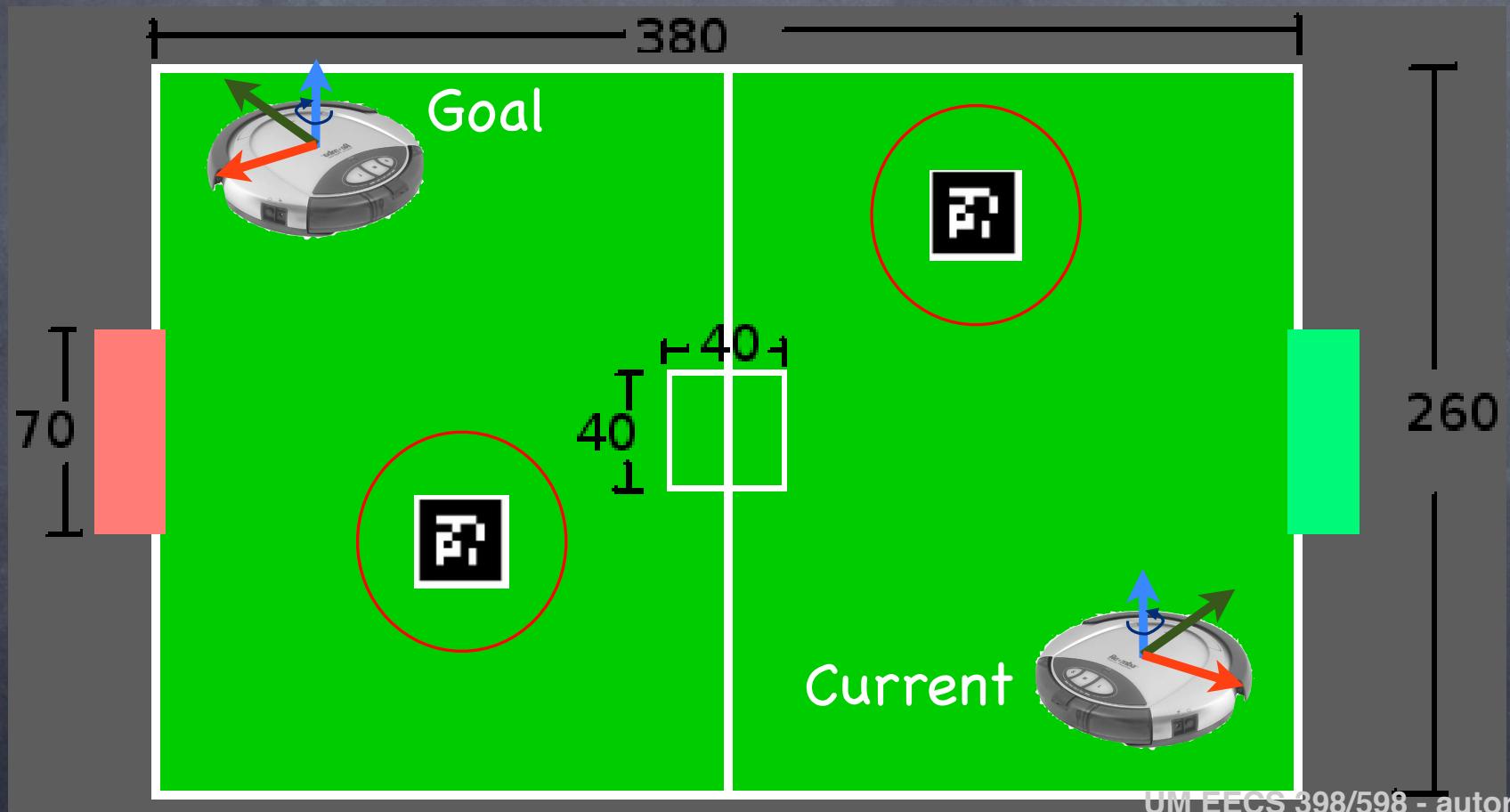
repellor should only have local influence,
repelling only around boundary improves path



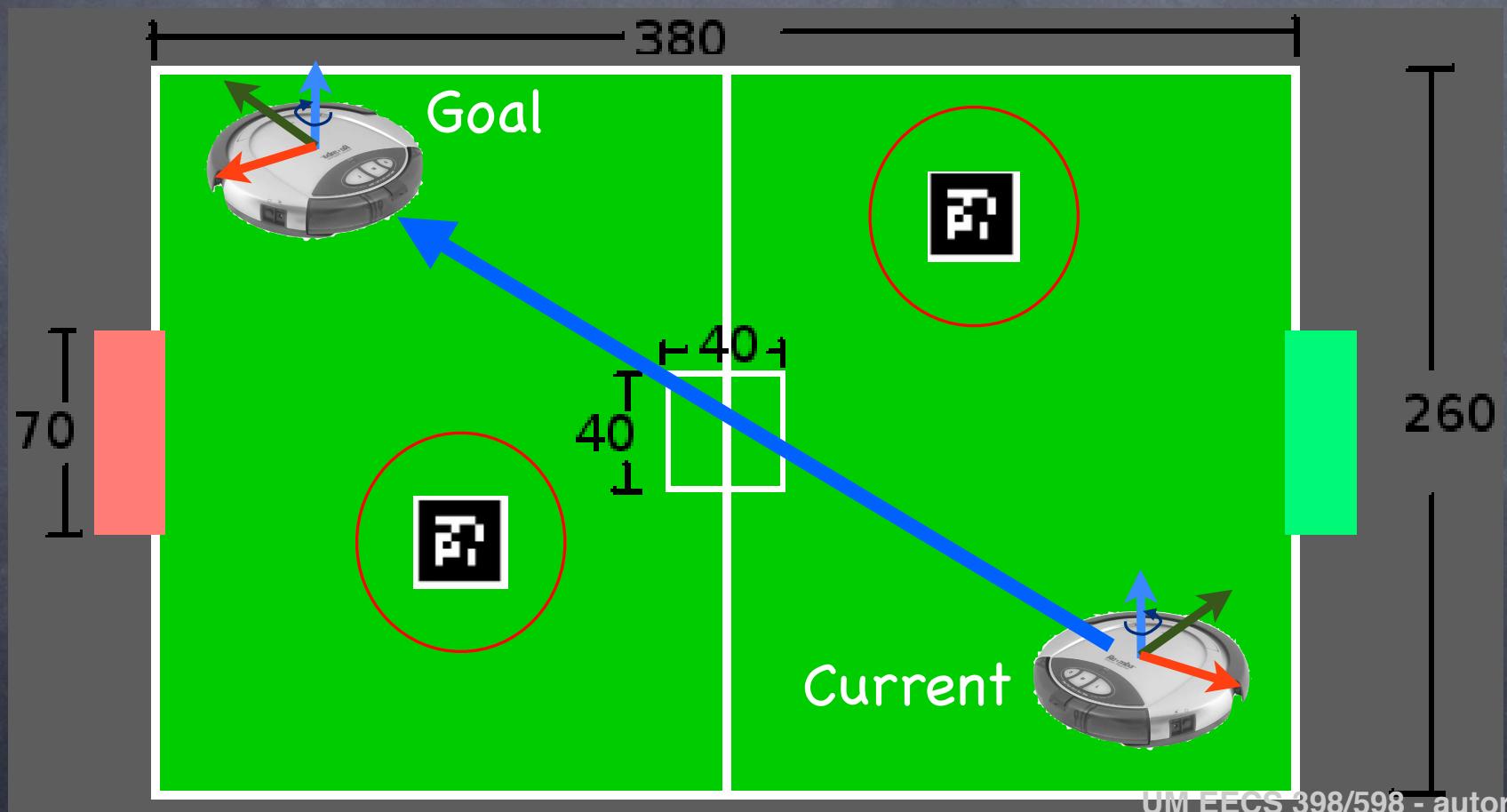
2 Obstacle example



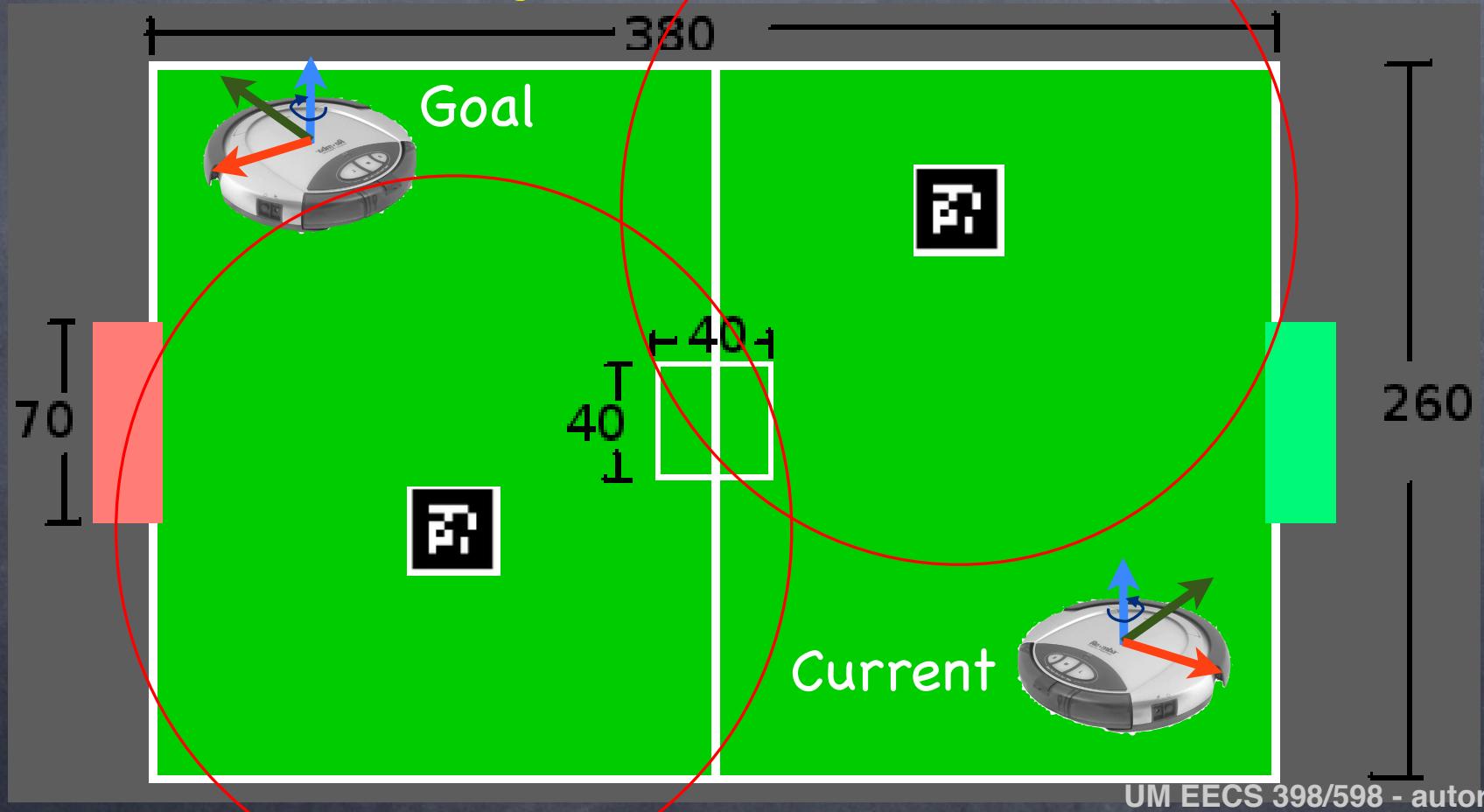
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



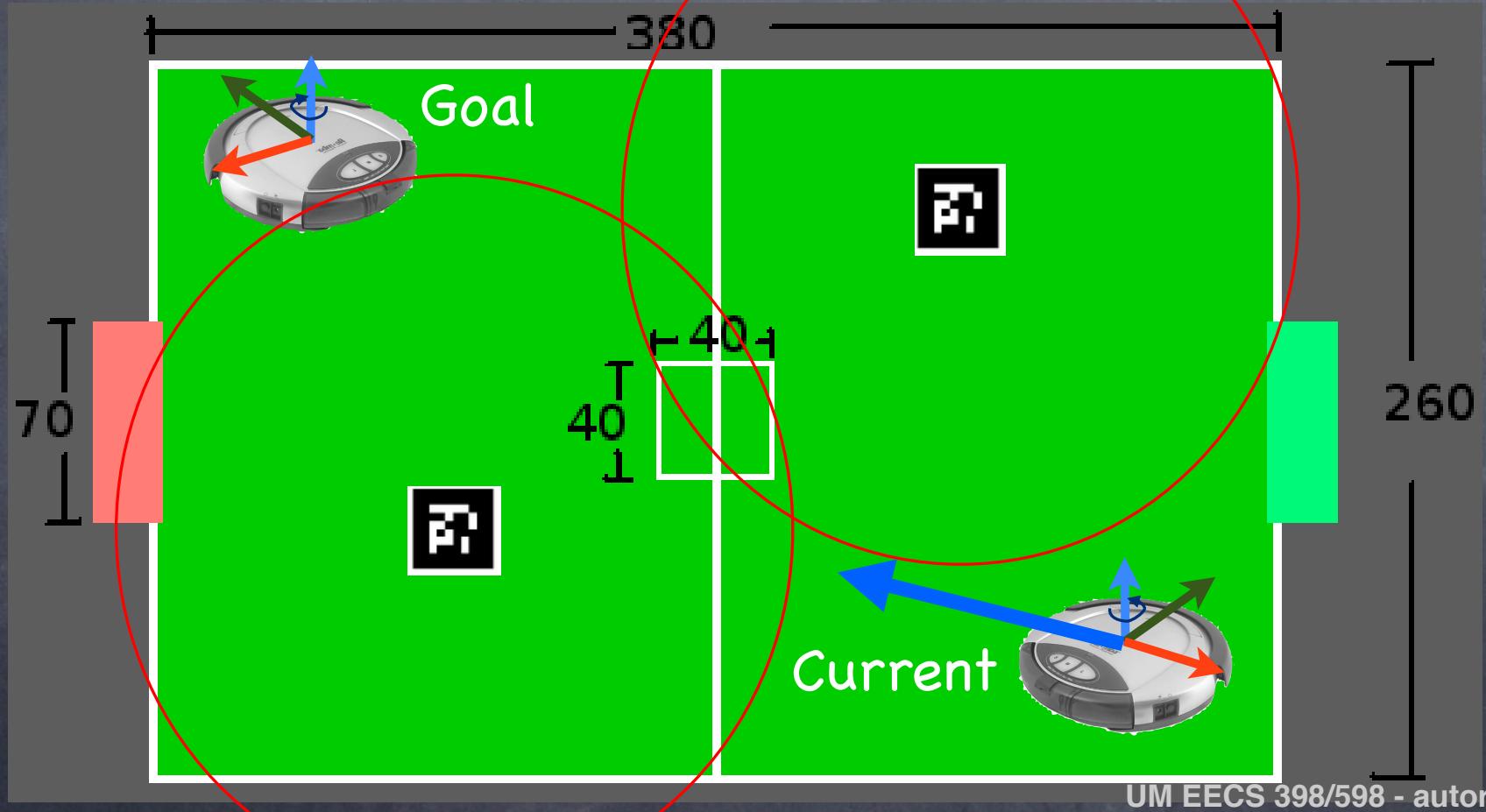
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



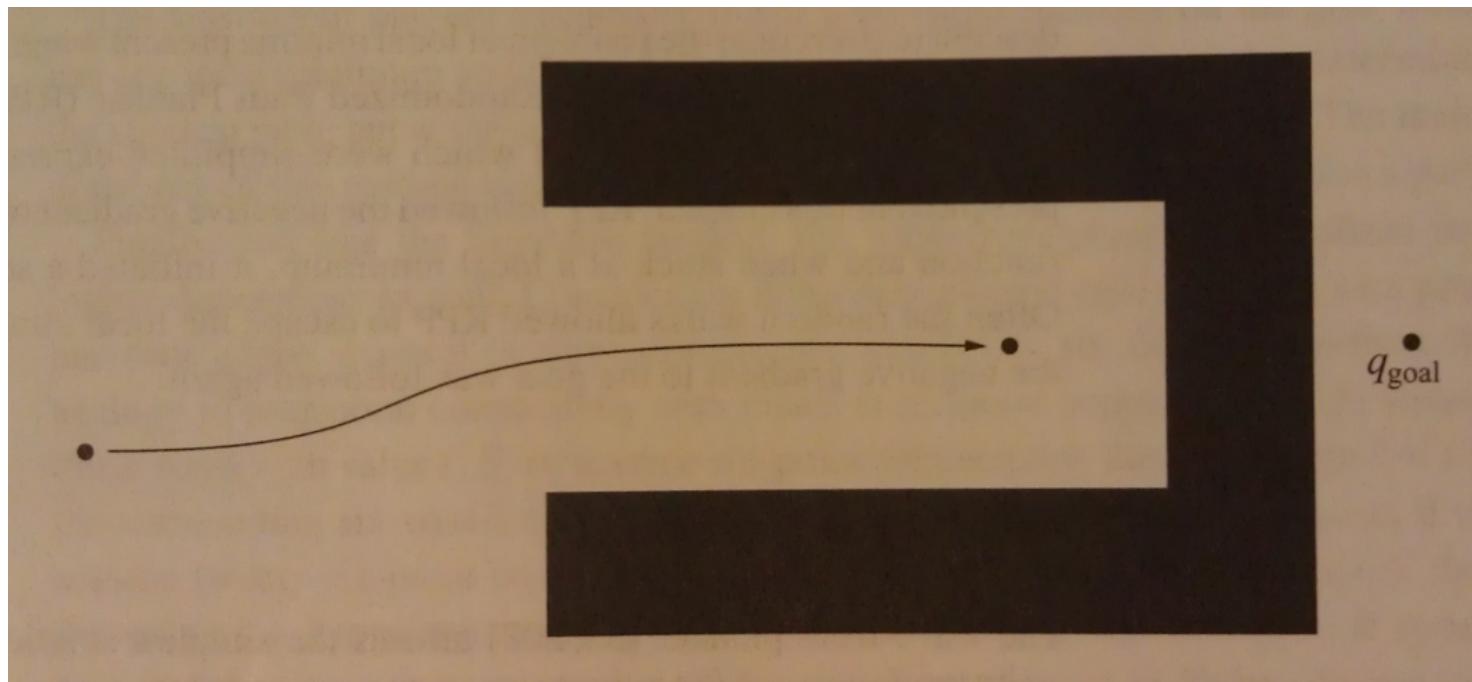
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



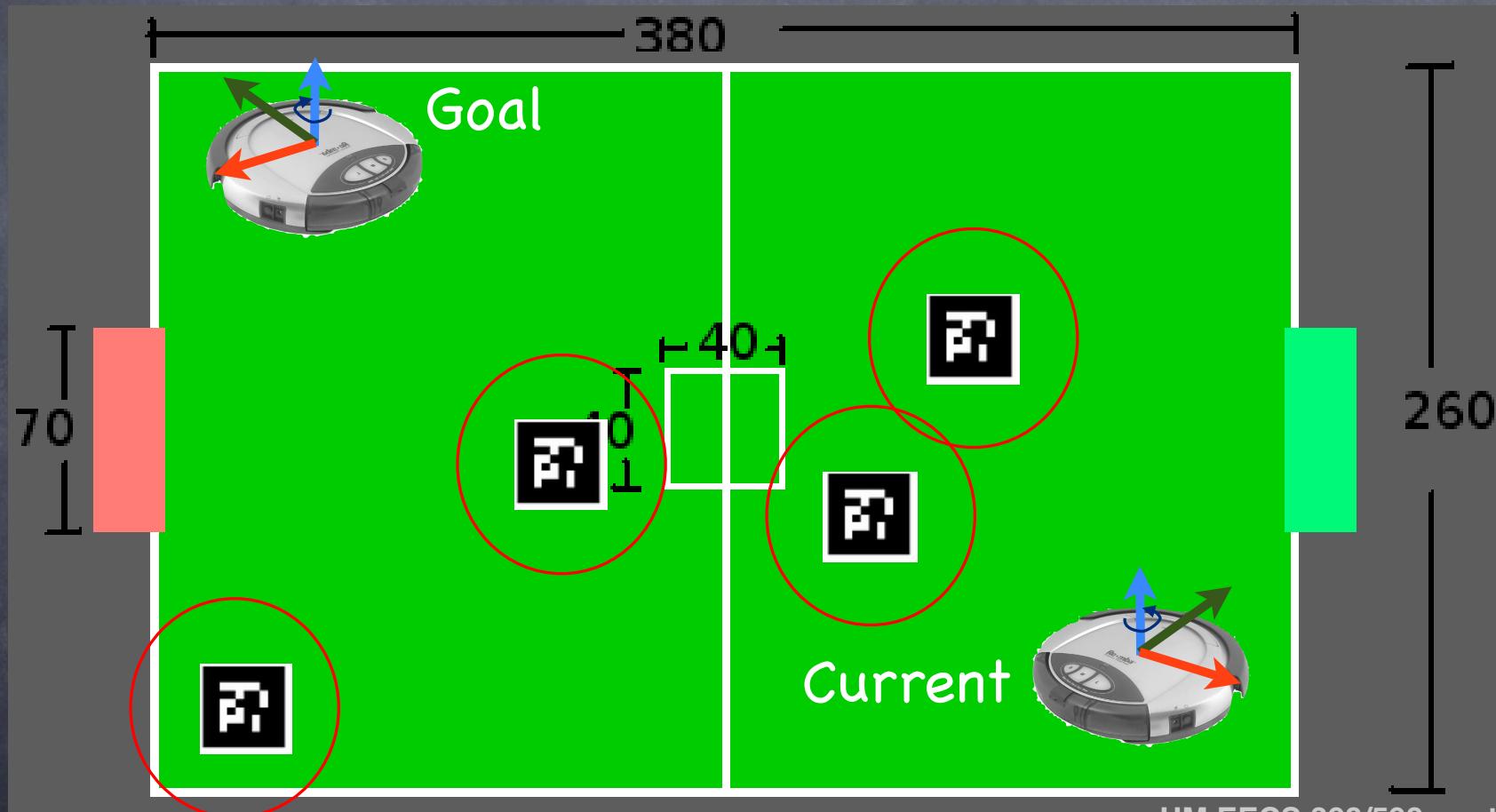
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



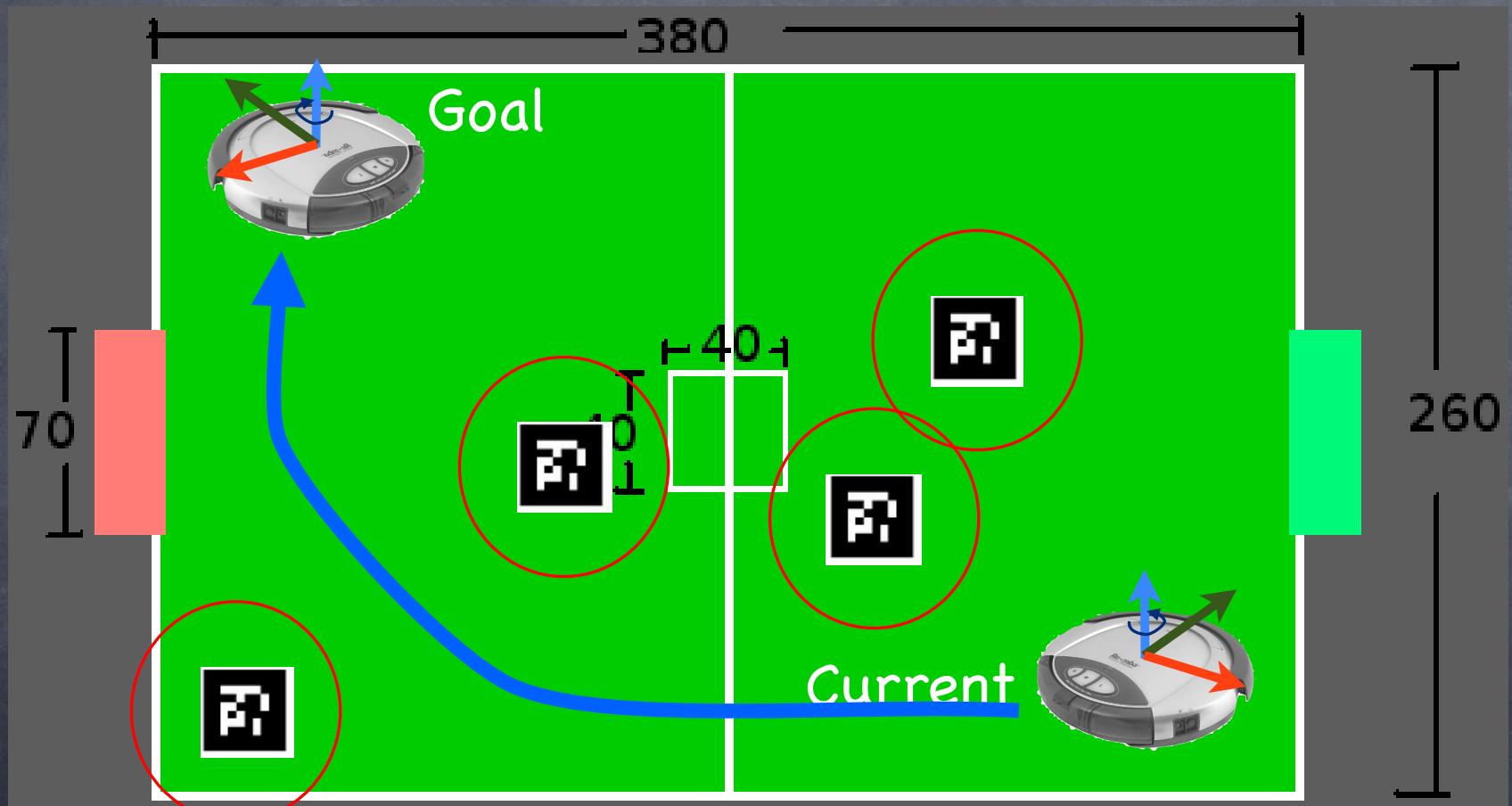
Local Minima



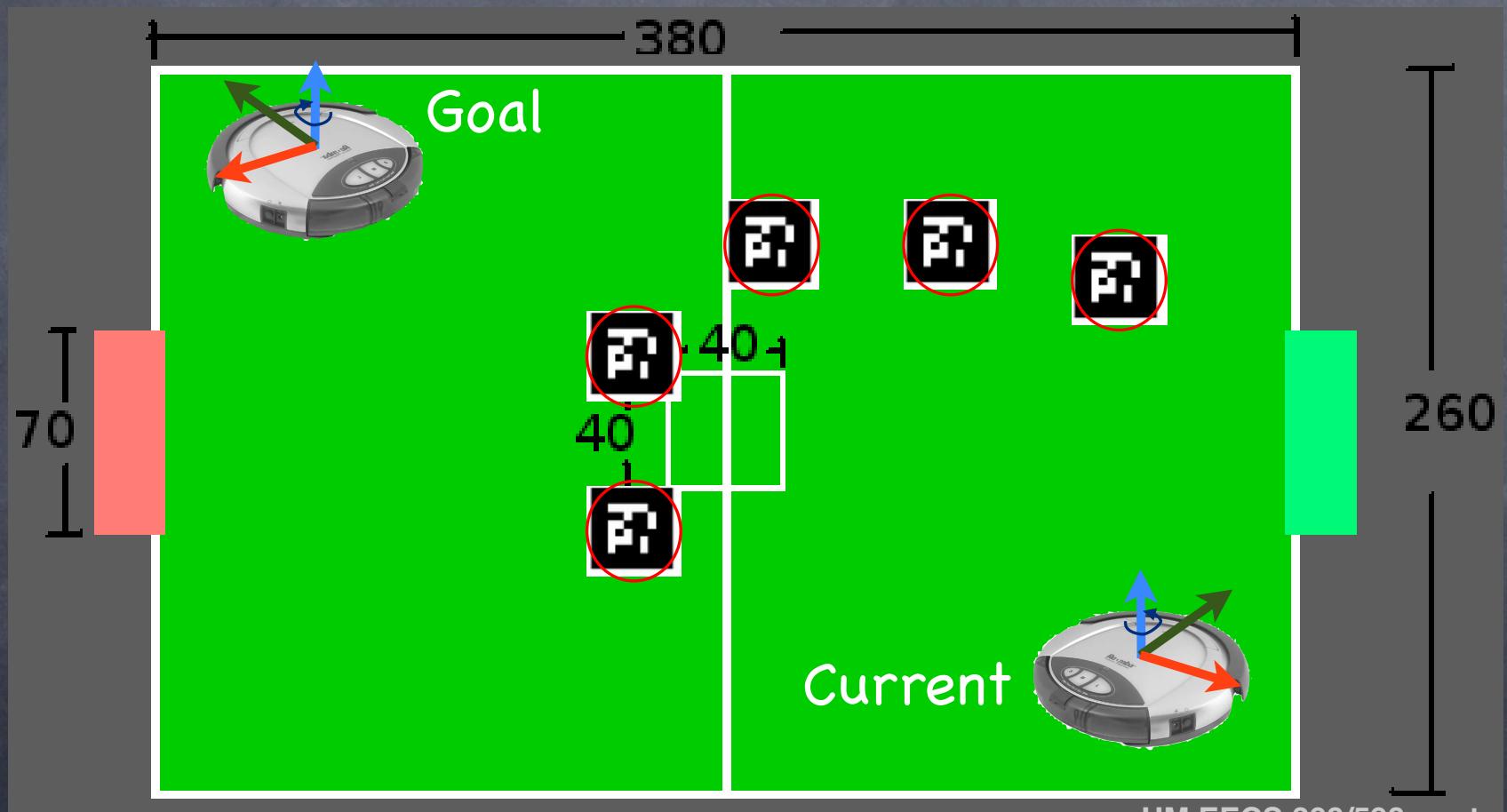
describe performance for this case



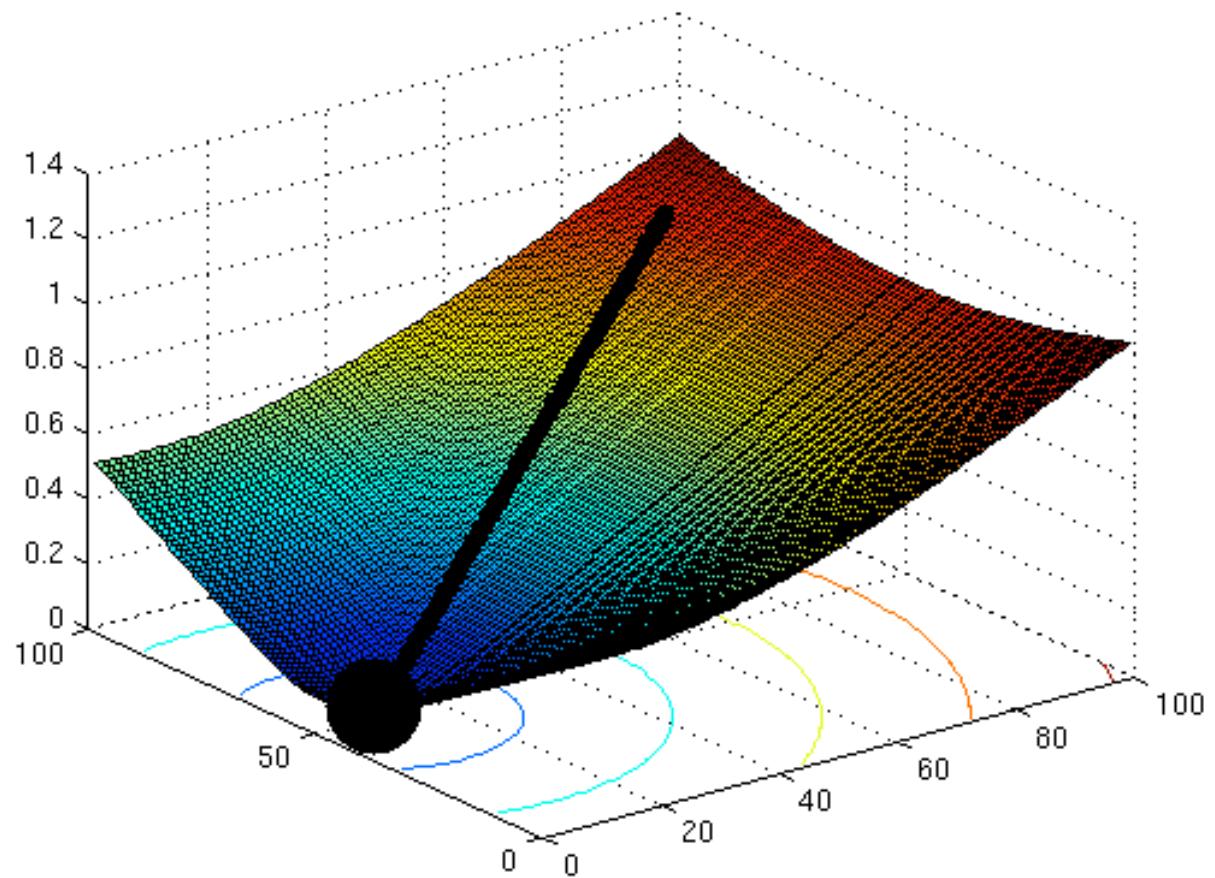
describe performance for this case



describe performance for this case



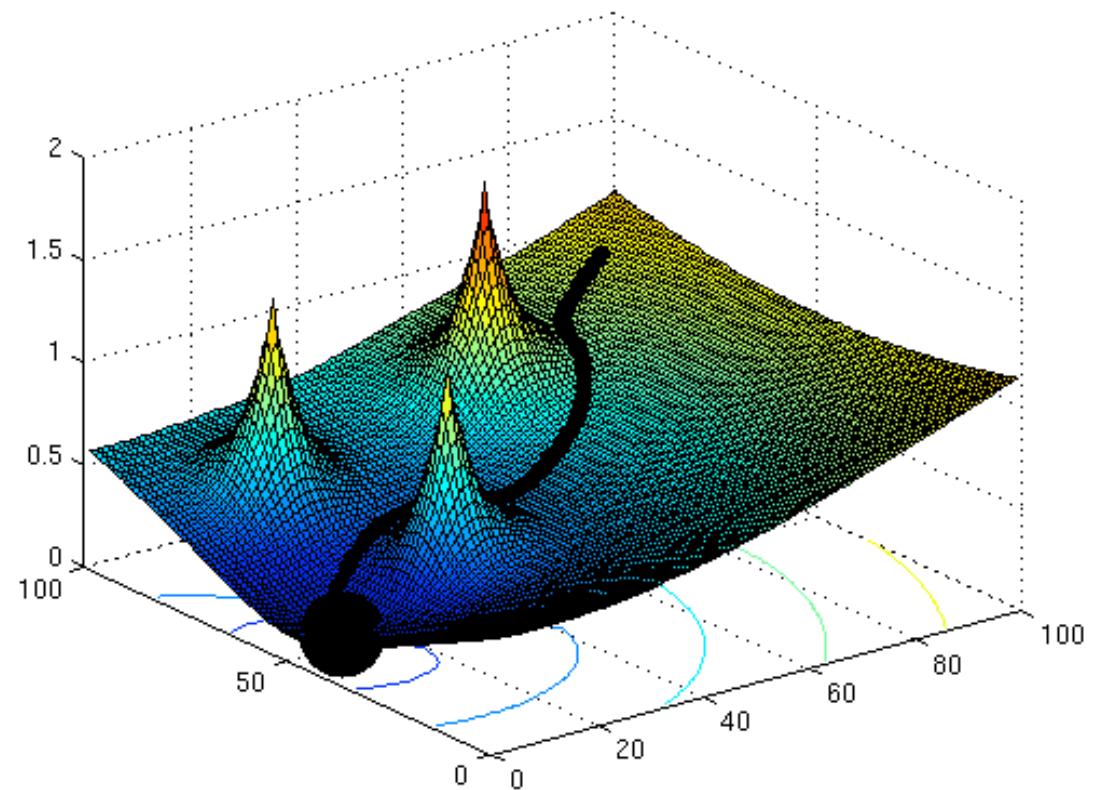
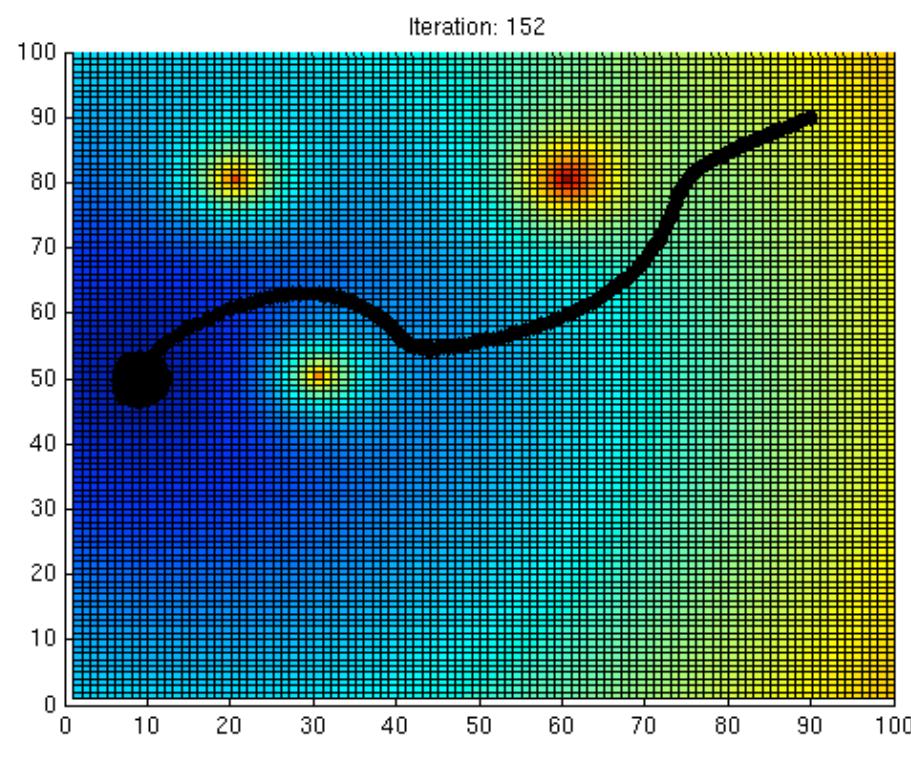
matlab example



pfield.m [1 5 8 12]

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matlab example

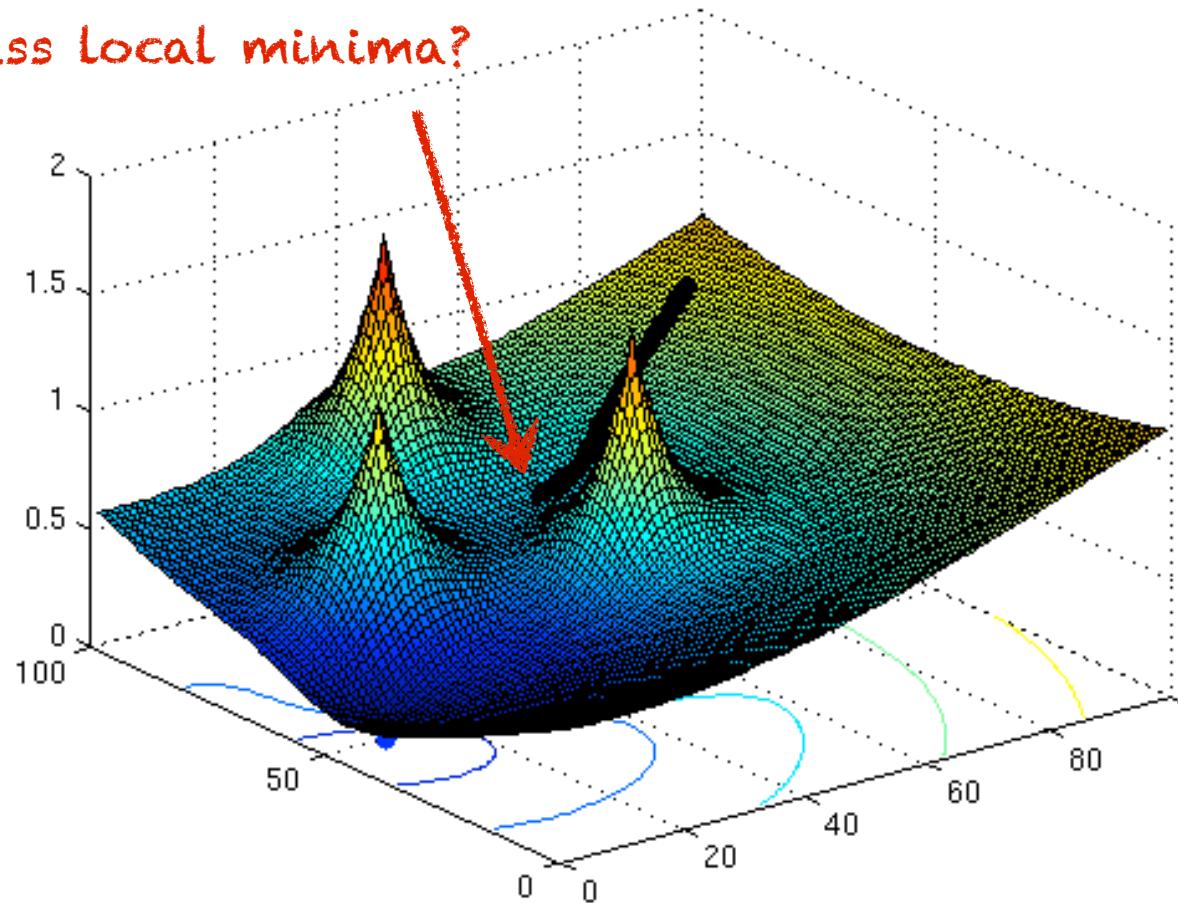


pfield.m [1 5 8 12]

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matlab example

How to address Local minima?



pfield.m [1 5 8 12]

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How can we get out of
local minima?

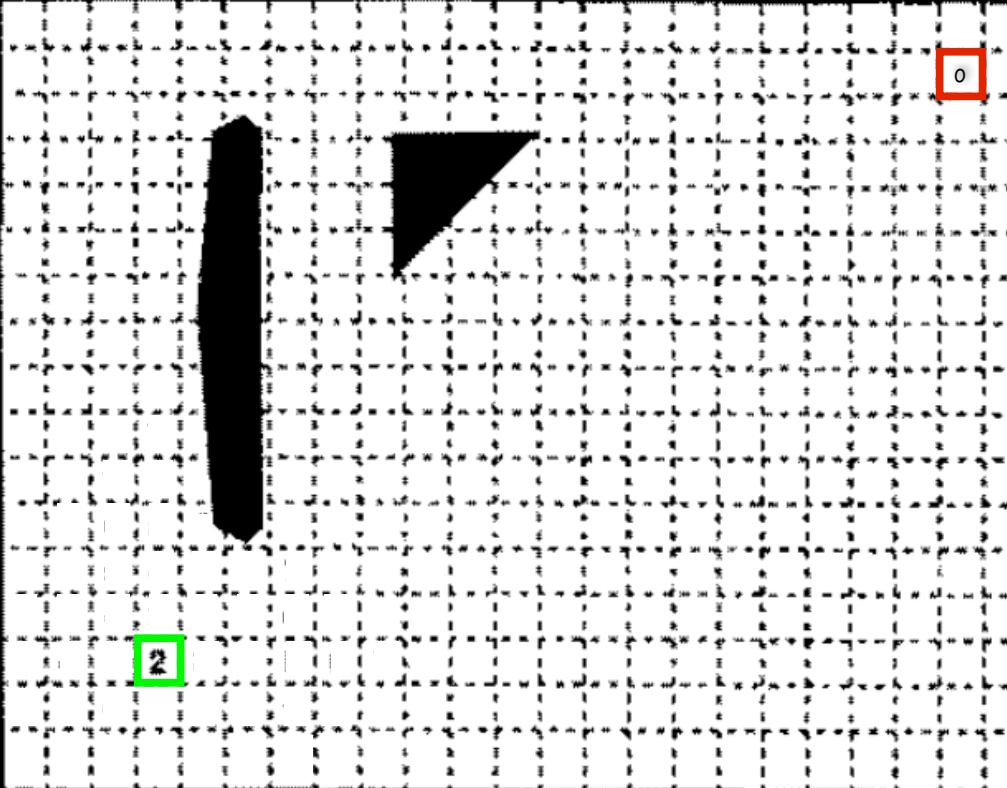
How can we get out of
local minima?

Go back to planning.

Wavefront Planning

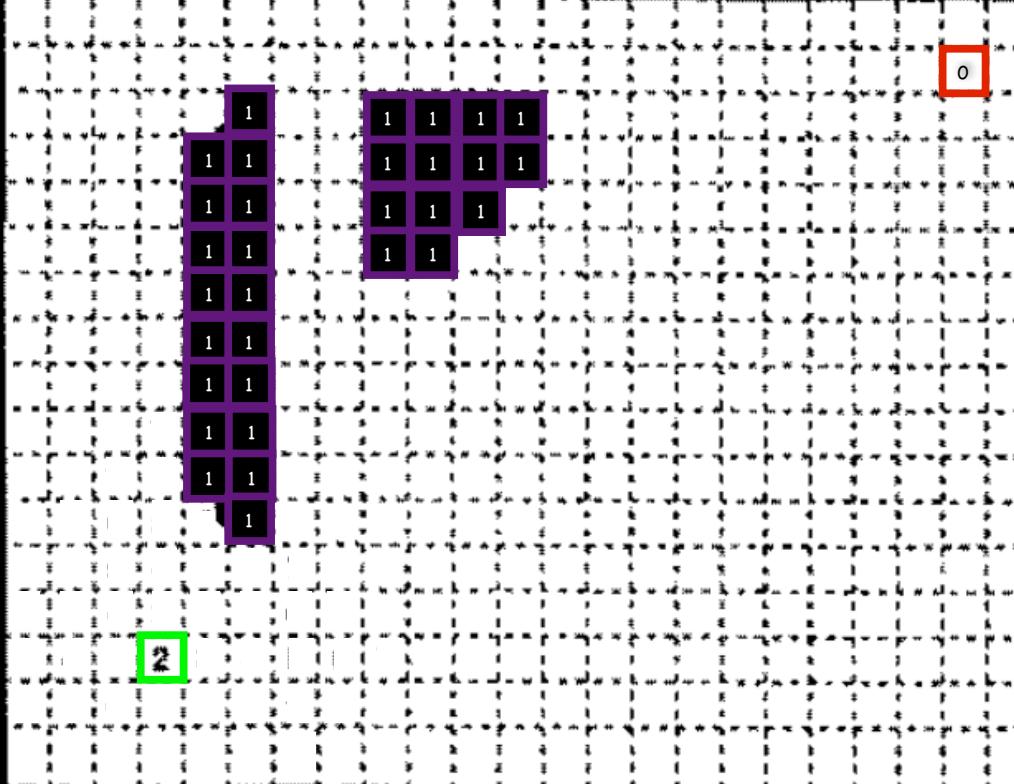
- Discretize potential field into grid
 - Cells store cost to goal with respect to potential field
 - Computed by Brushfire algorithm (essentially BFS)
- Grid search to find navigation path to goal

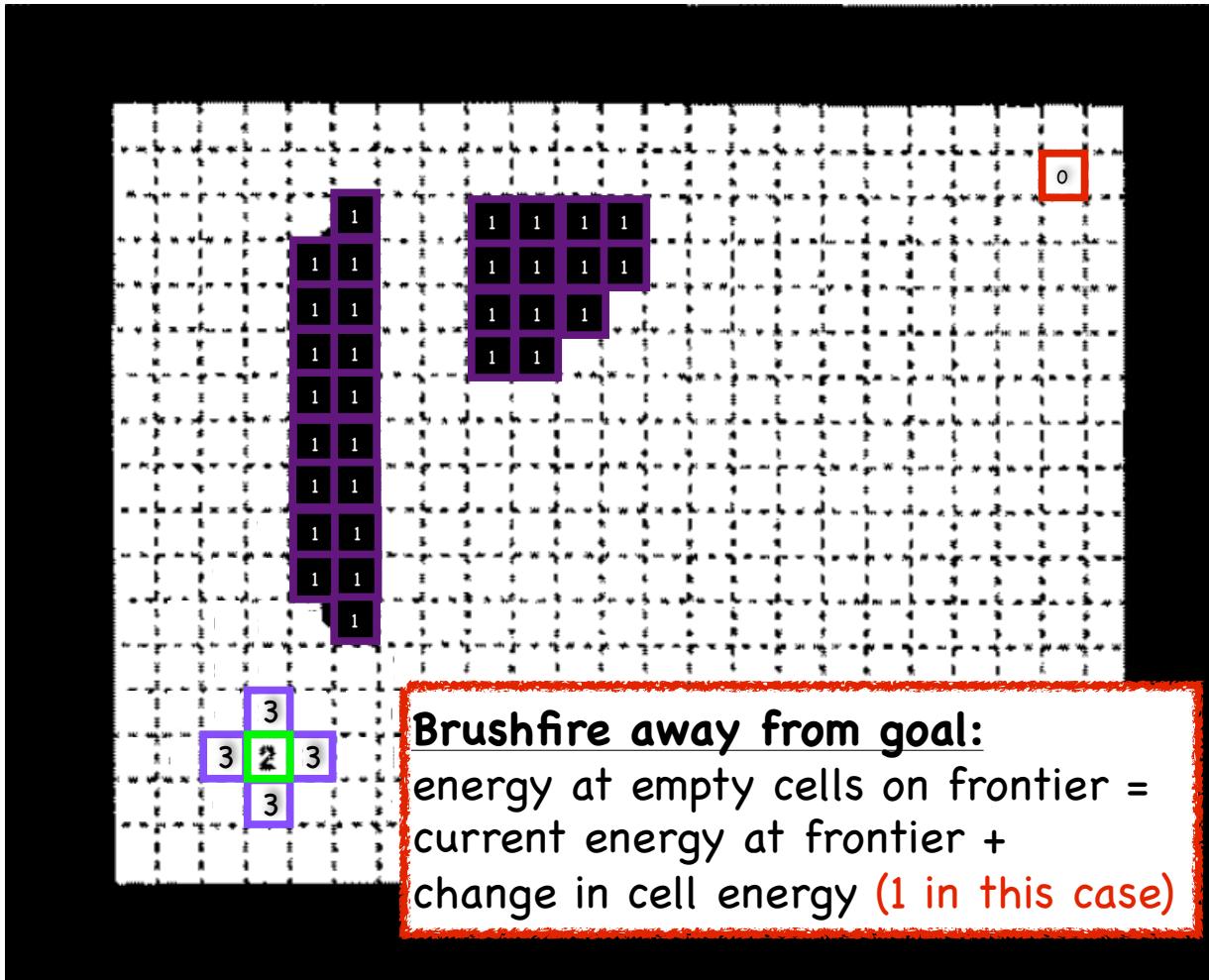
Start: mark with 0

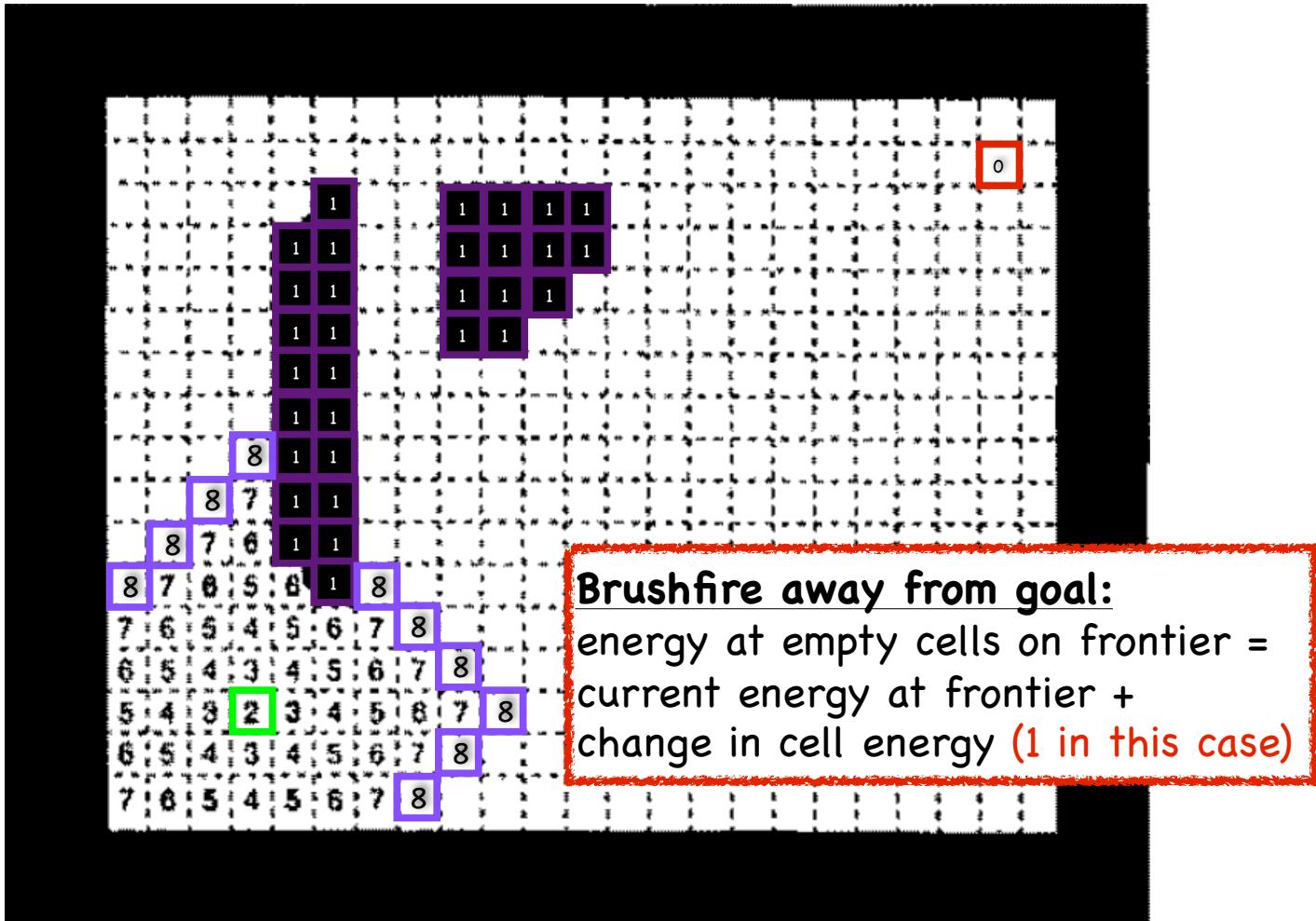


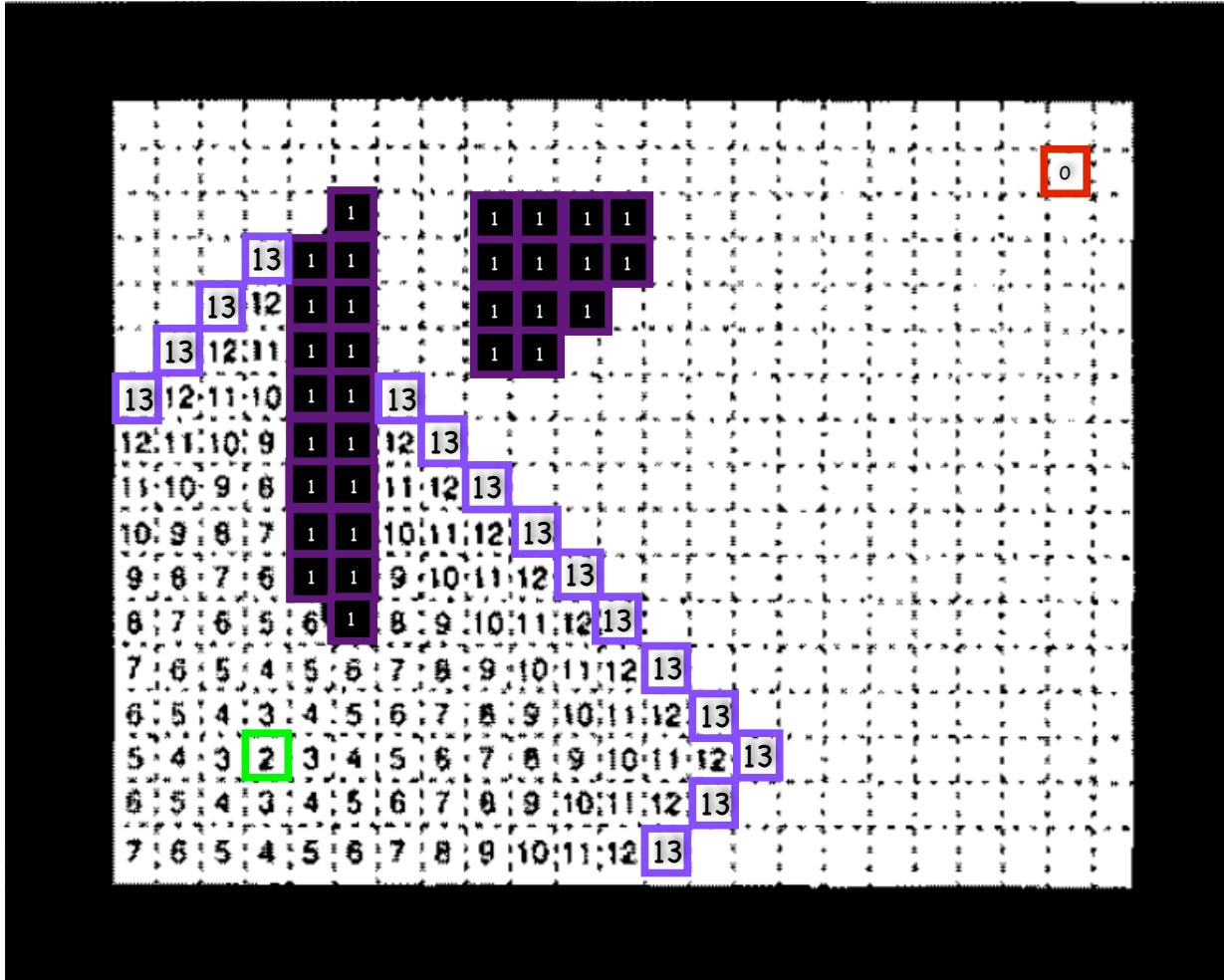
Goal: mark with 2

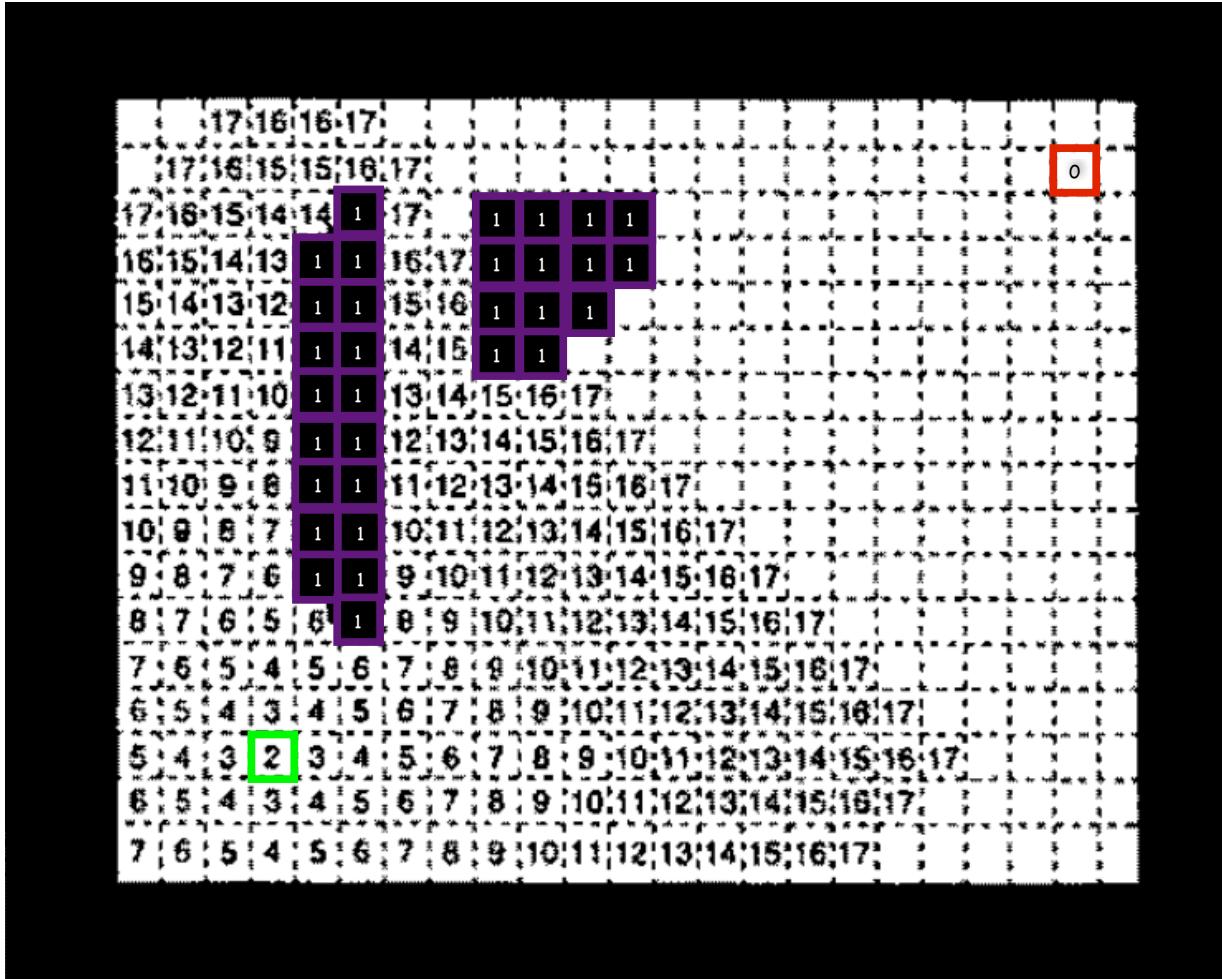
Obstacles: mark with 1

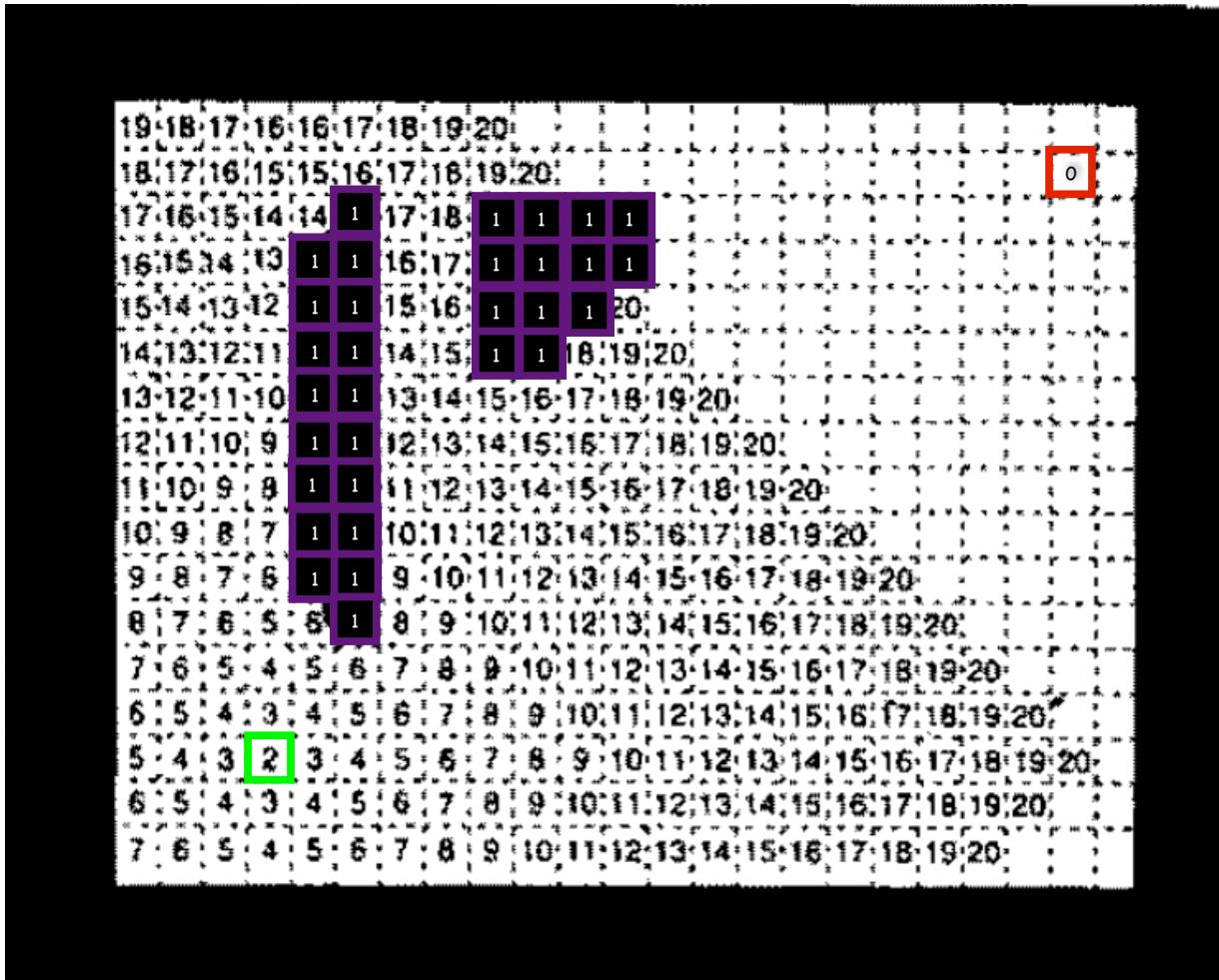


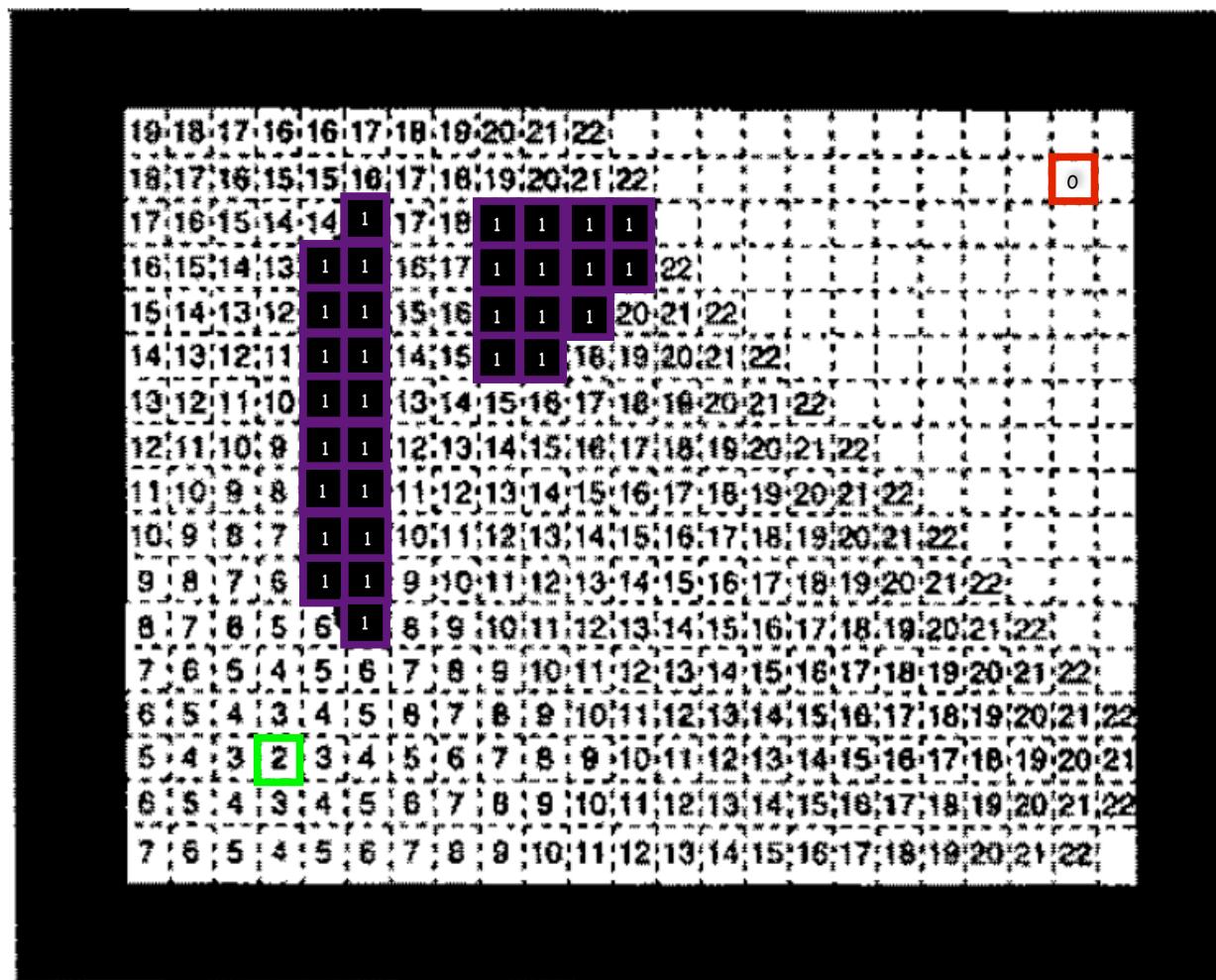






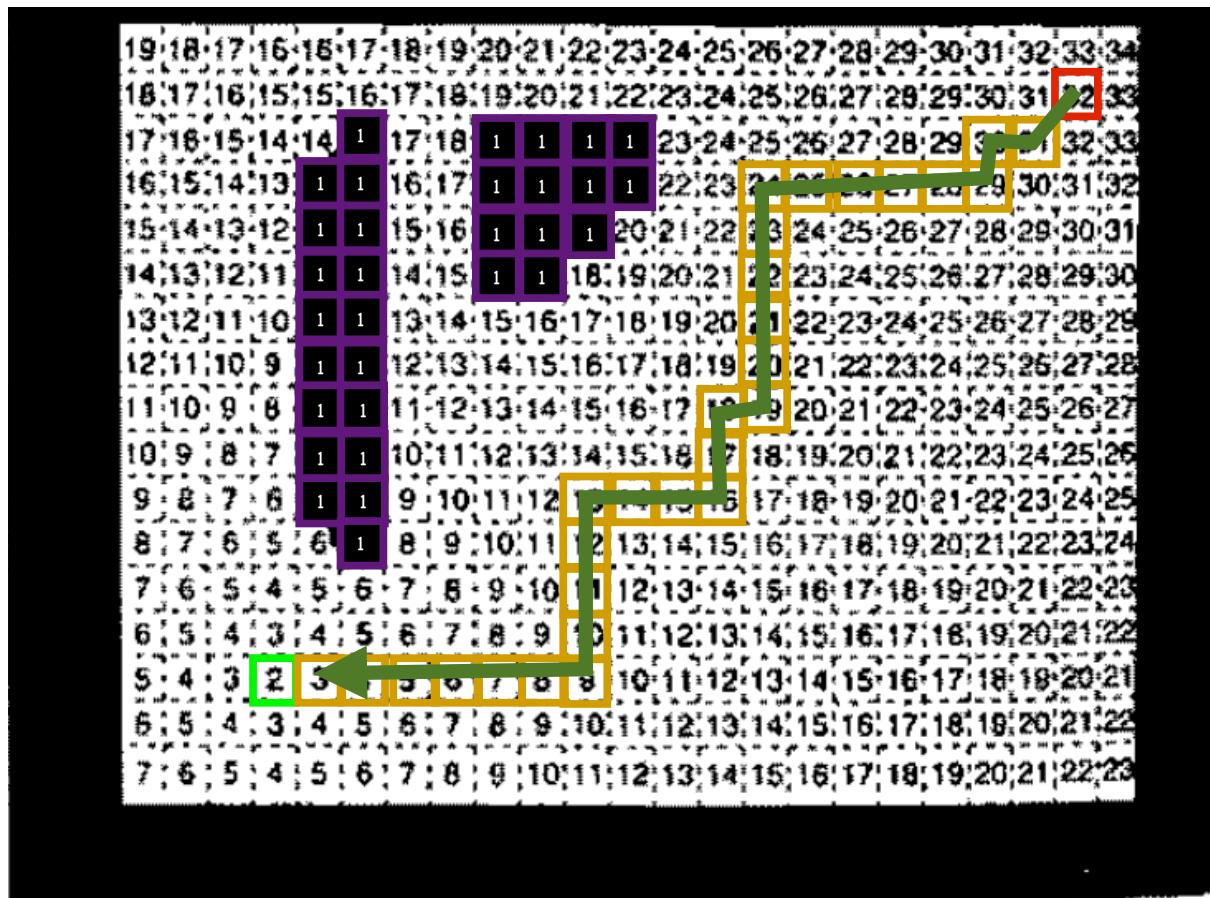






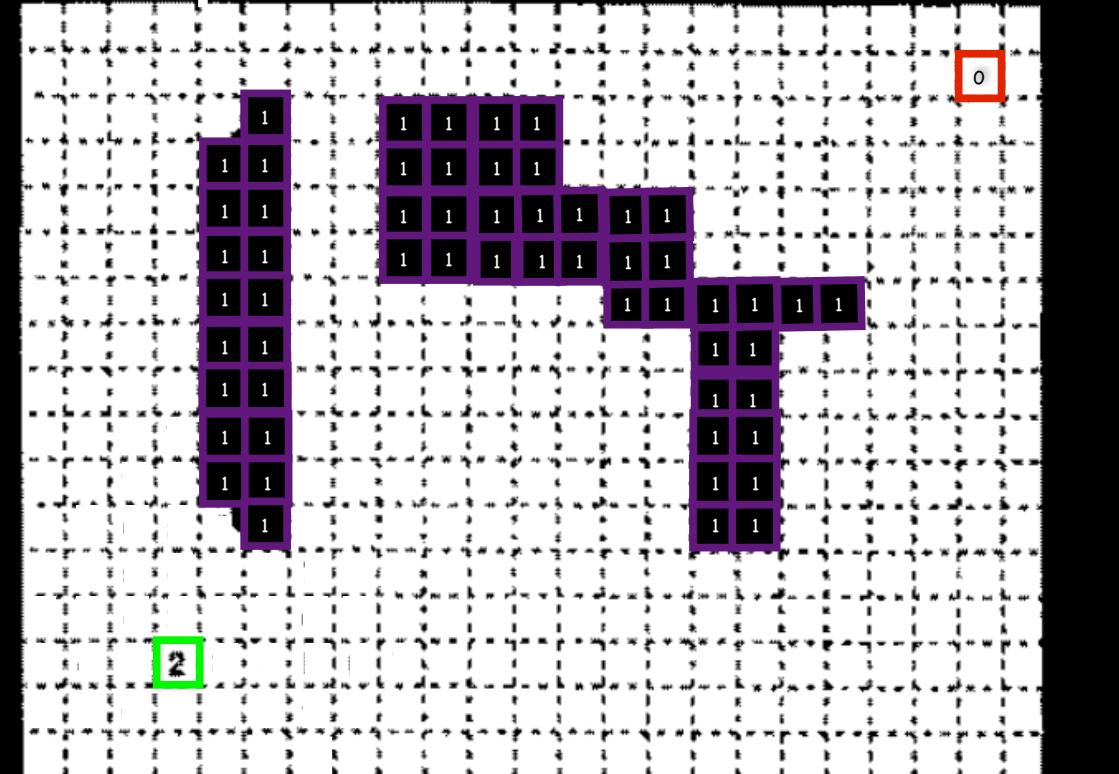
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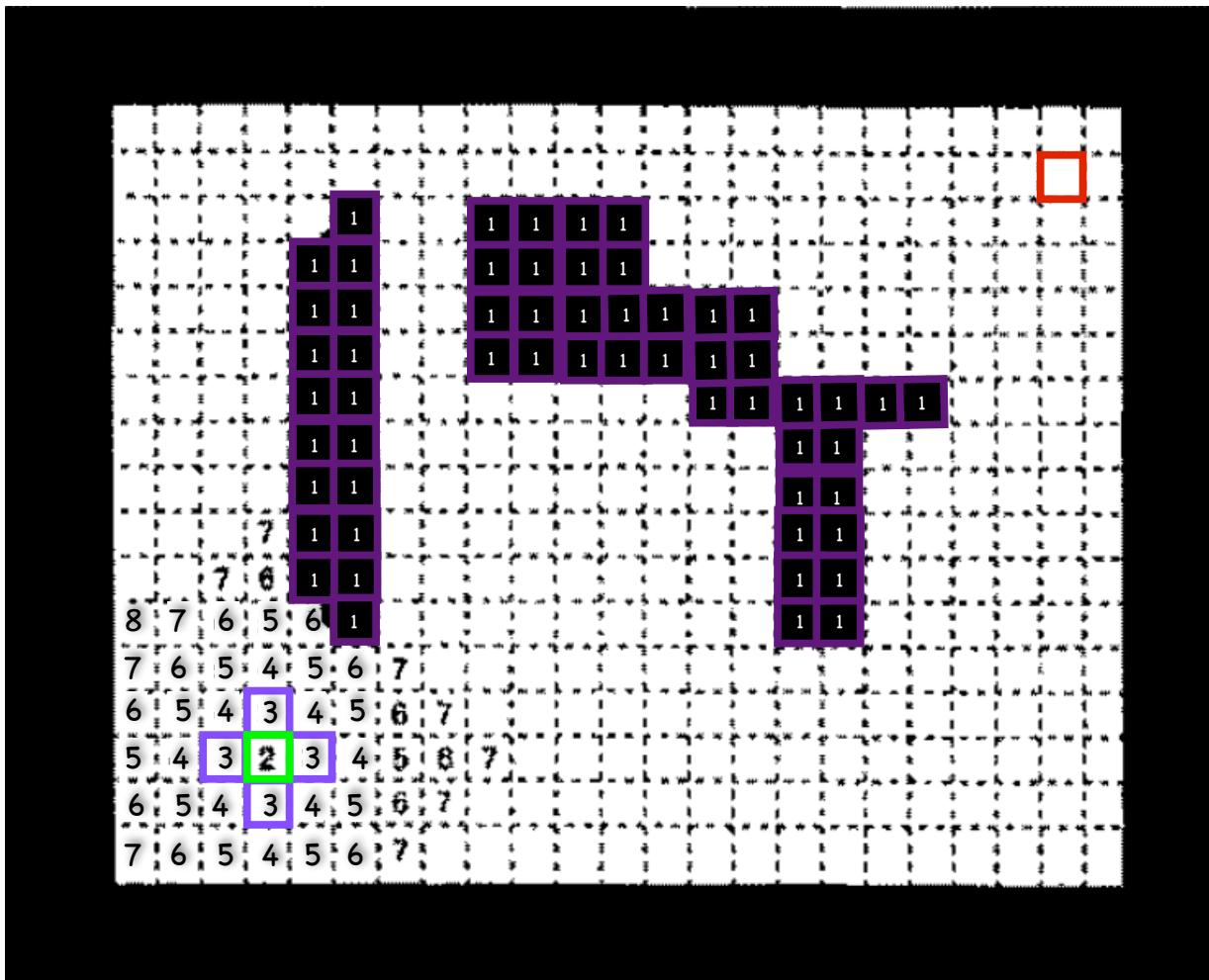
Once start reached,
follow brushfire potential to goal

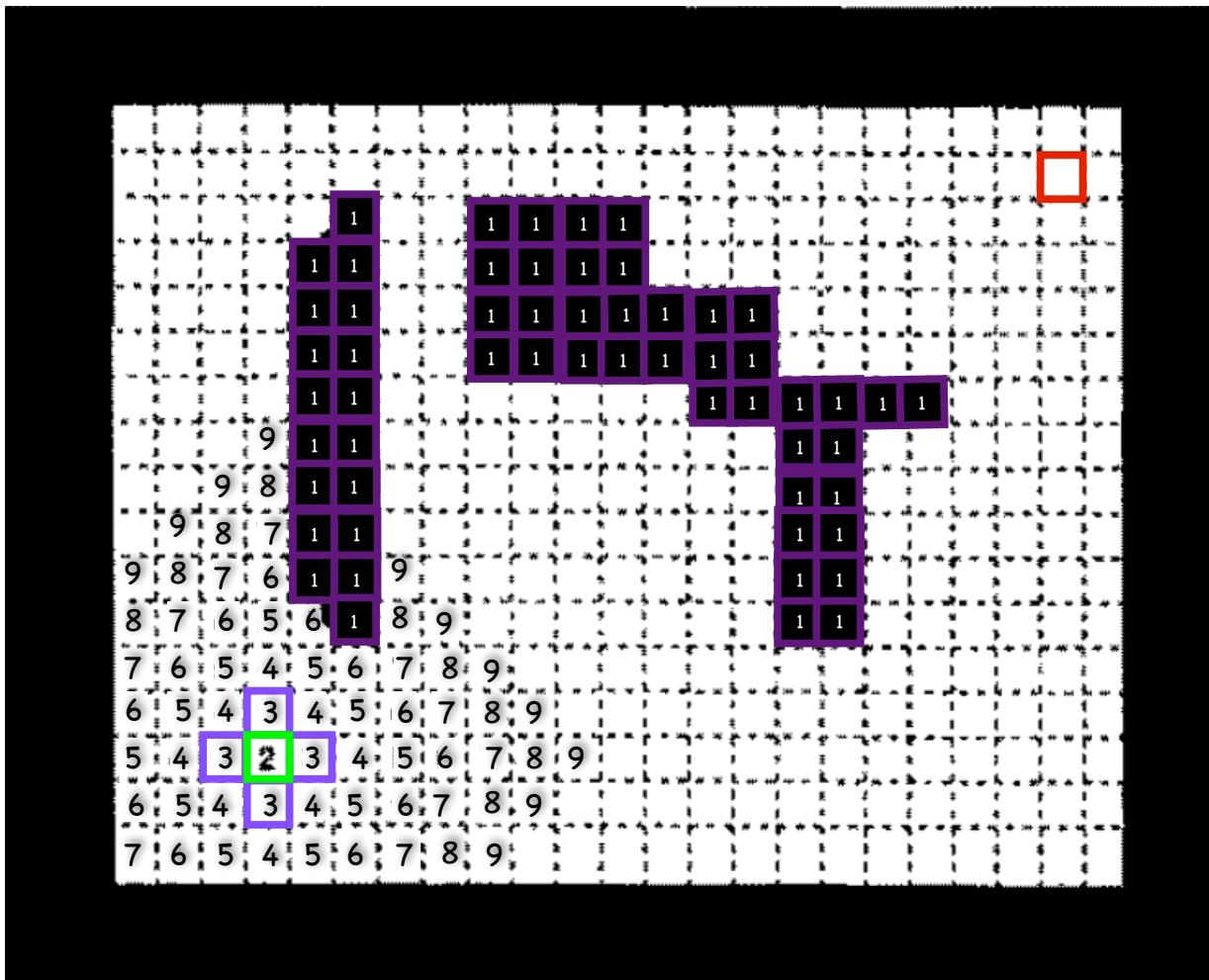


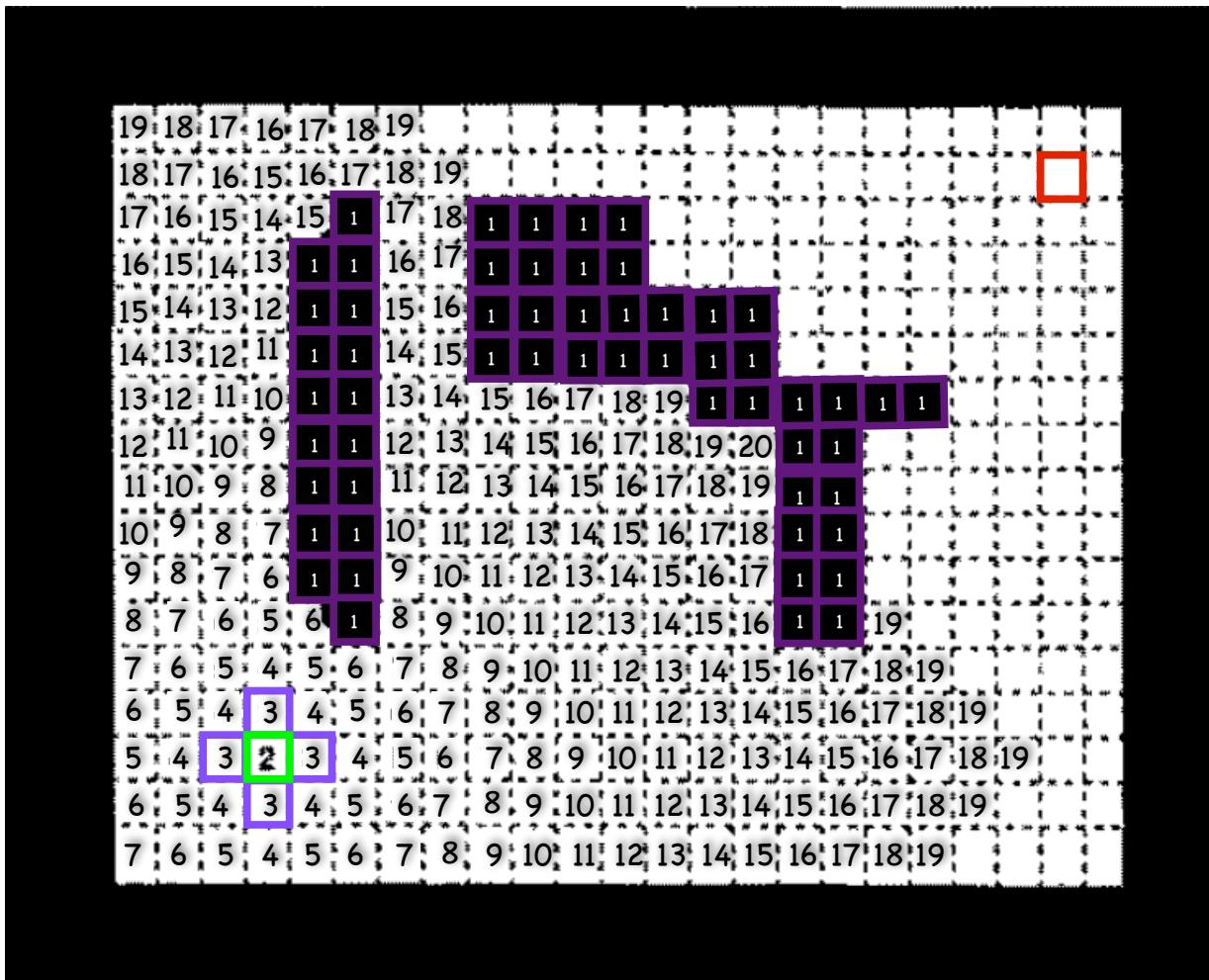
Example with Local Minima

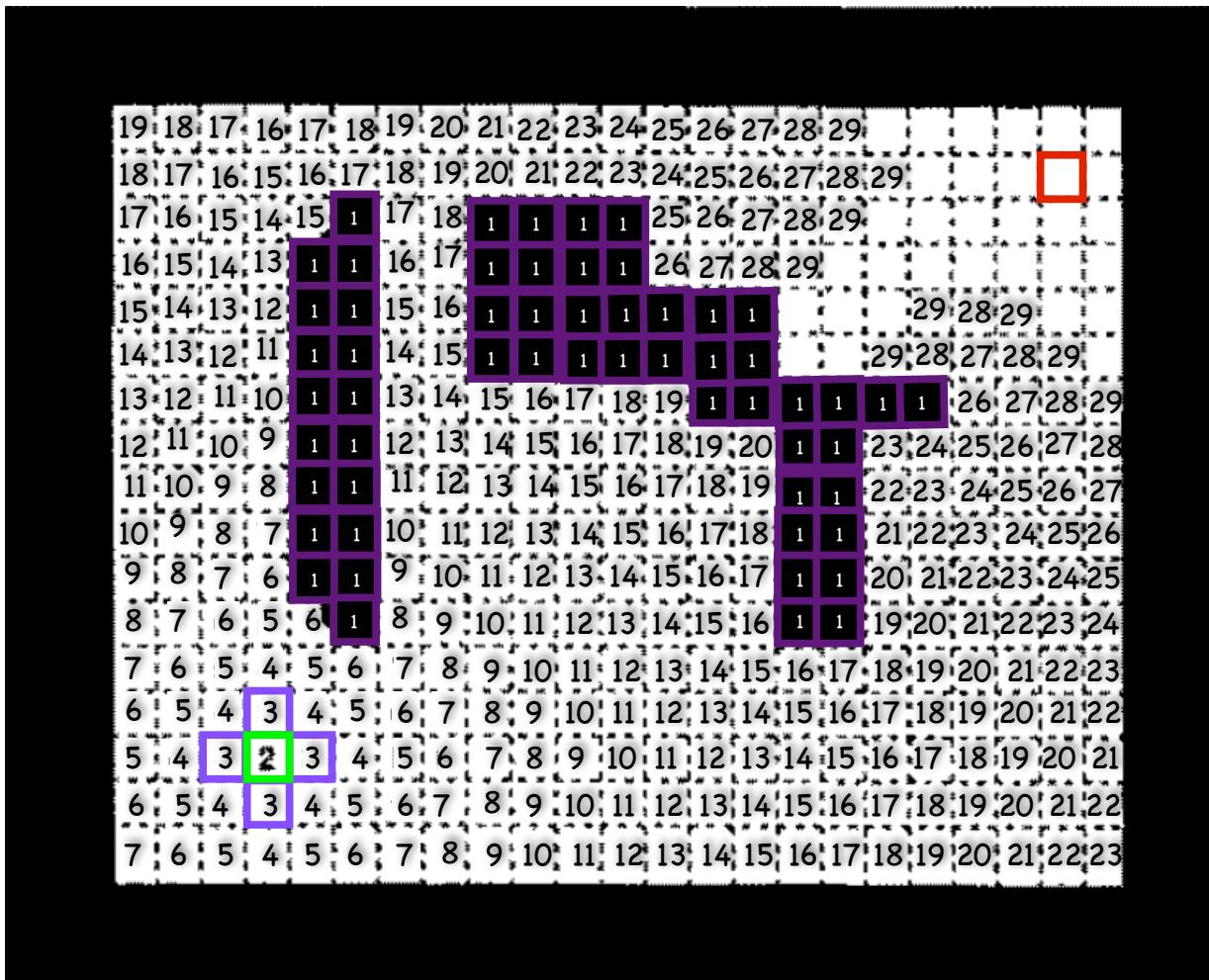
Example with Local Minima



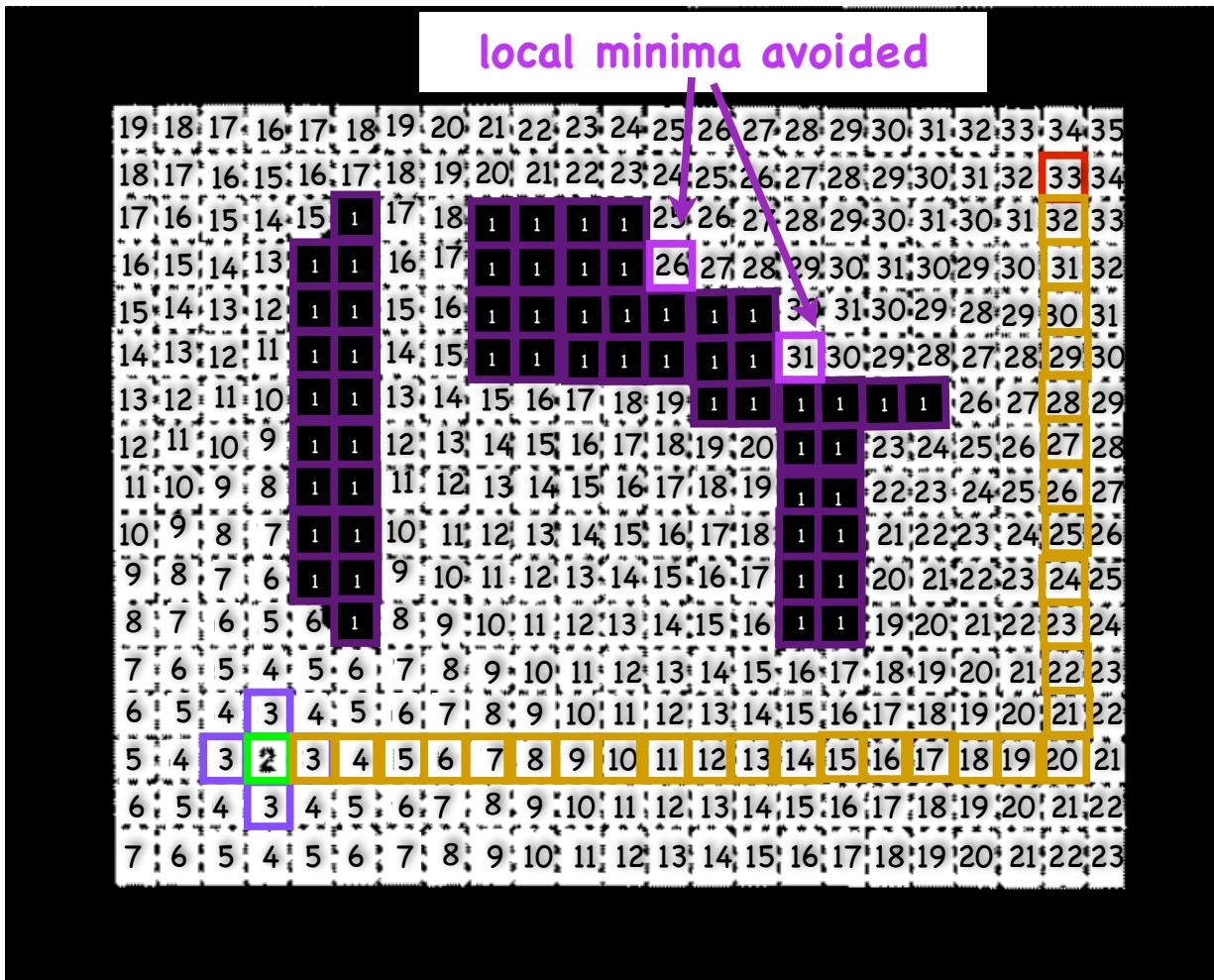








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7	6	5	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	



Can we extend potential fields
for arm navigation?

Potential Fields for Robot Arm

- Define endeffector goal as the attractive potential with cone potential
- Define repulsive potentials wrt. collision objects
 - Select points on robot links with “bowl” potential from nearest object
- Use manipulator Jacobian to transform potential at each point into velocities at robot joints
- Weighted sum of transformed velocities to generate control

Navigation Recap

Navigation Recap

Bug X

- Complete
- Non-optimal
- Planar

Subsumption and FSMs

- Fast but not adaptive
- Emphasis on good design

Potential Fields

- Complete in special cases
- Non-optimal
- General C-spaces
- Scales w/dimensionality

Grid Search/Wavefront

- Complete
- General C-spaces
- Limited dimensionality

Random walk

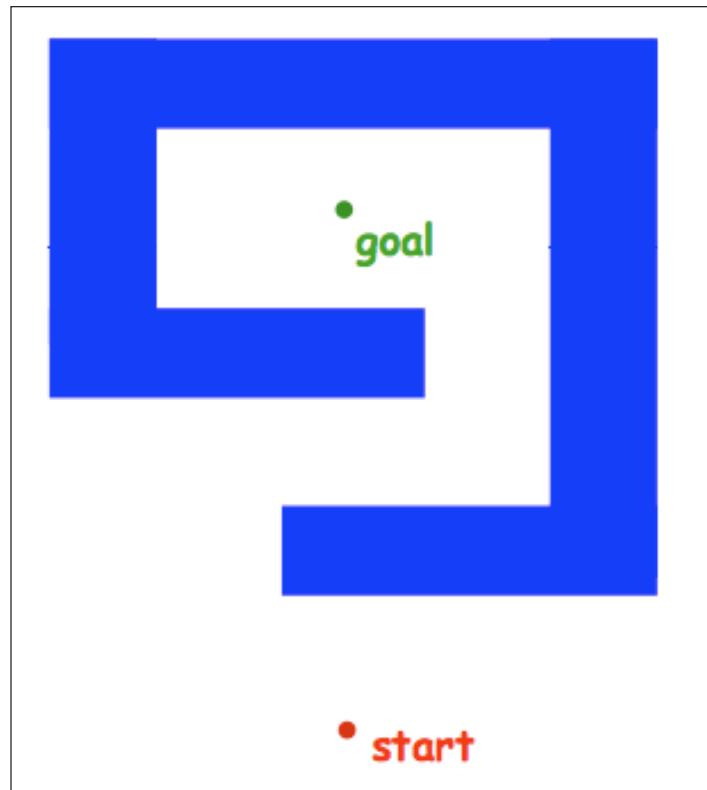
- Will find path eventually

Sampling roadmaps/RRT

- Probabilistically complete
- General
- Tractable (with good sampling)
- Scales w/dimensionality
- Not necessarily optimal

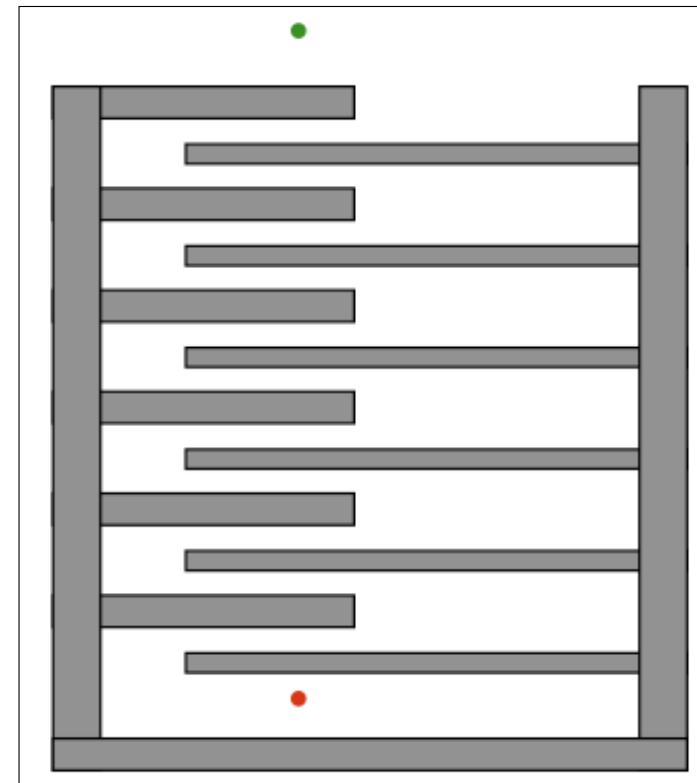
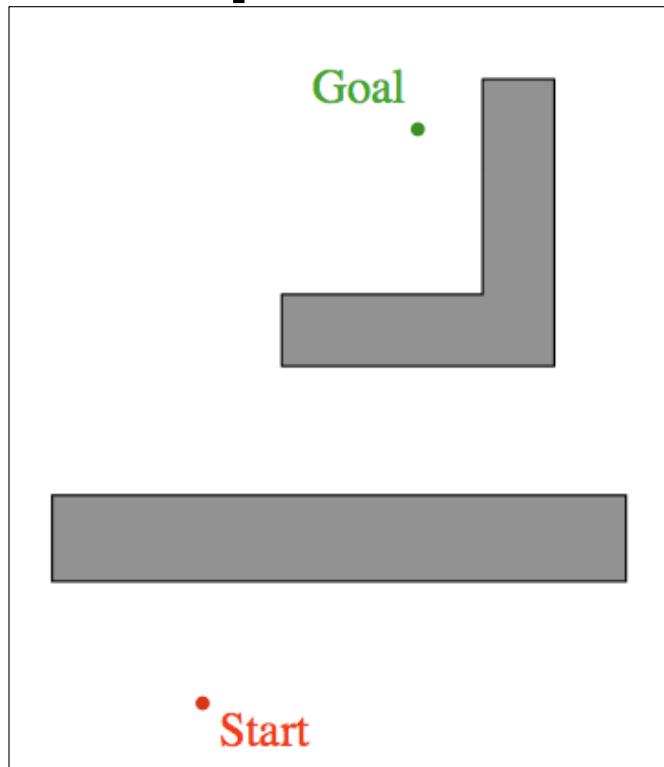
Compare: BugX examples

Describe likely paths from potential field and wavefront planners



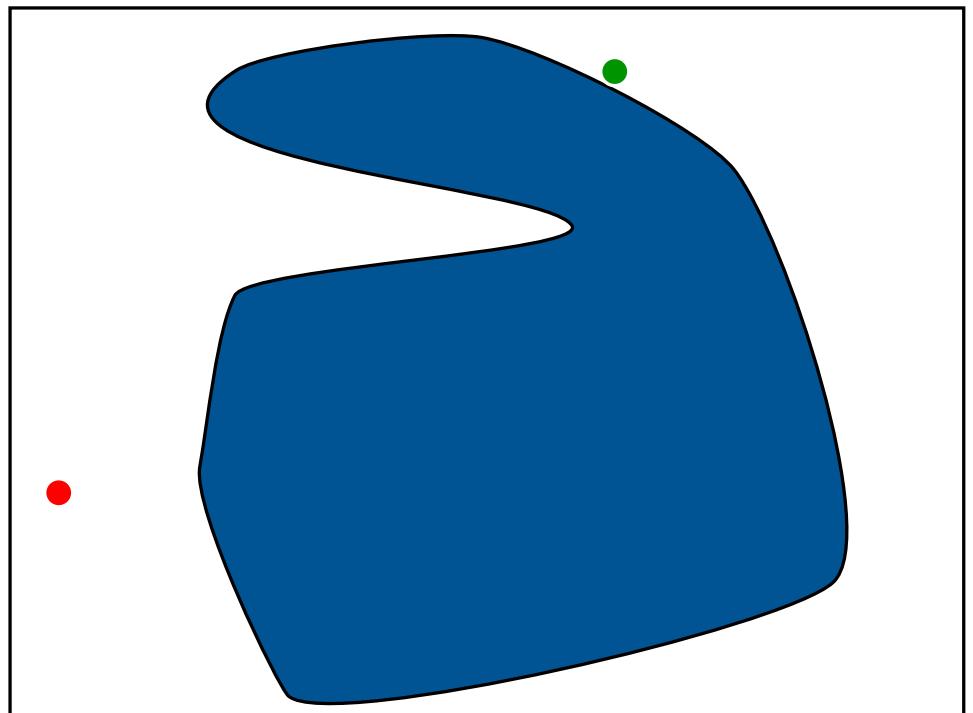
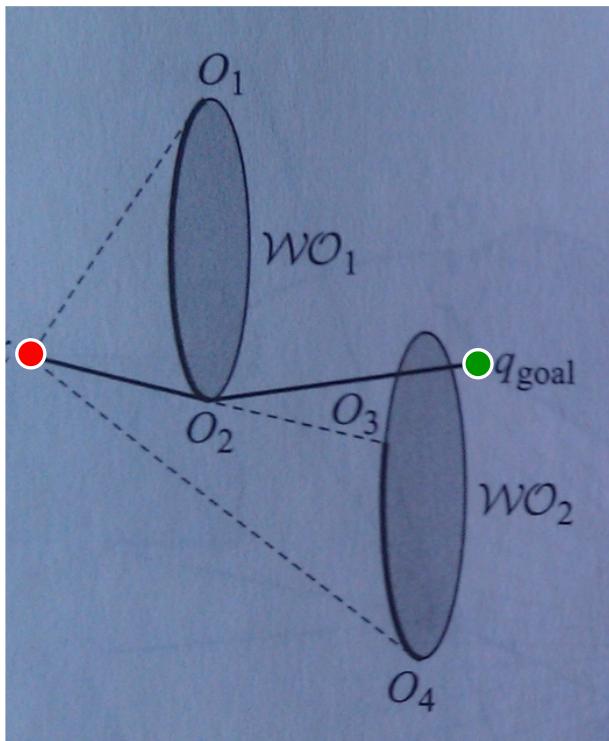
Compare: BugX examples

Describe likely paths from potential field and wavefront planners



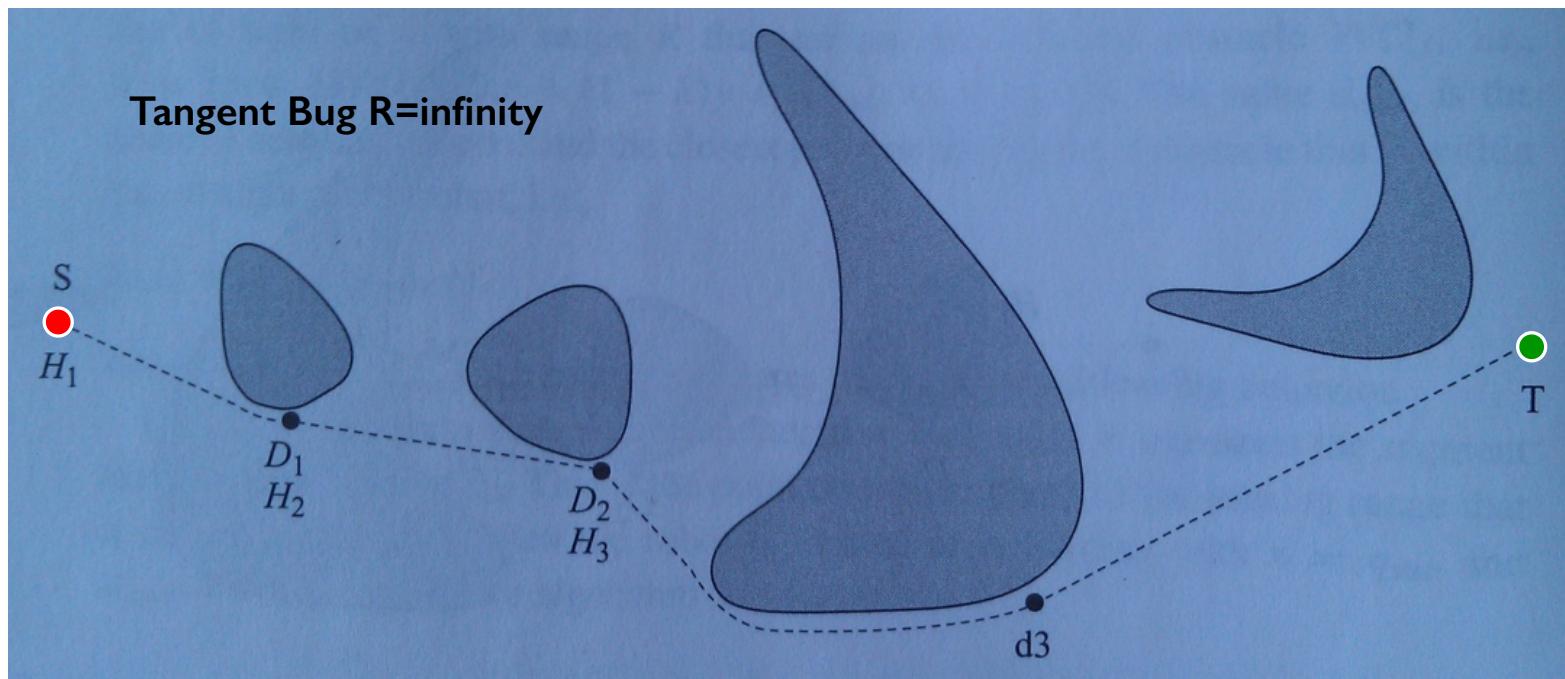
Compare: BugX examples

Describe likely paths from potential field and wavefront planners



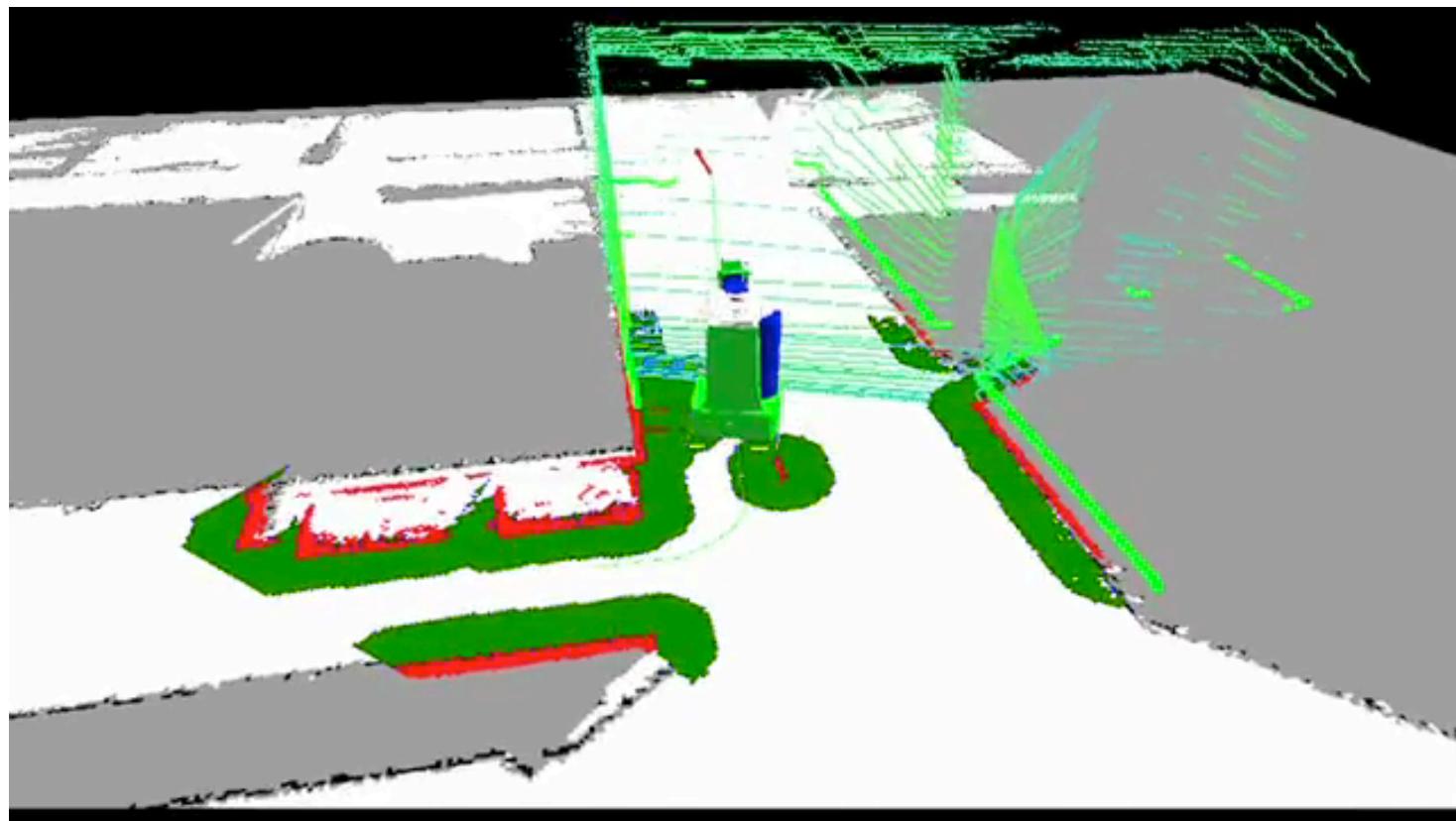
Compare: BugX examples

Describe likely paths from potential field and wavefront planners



Navigation in ROS

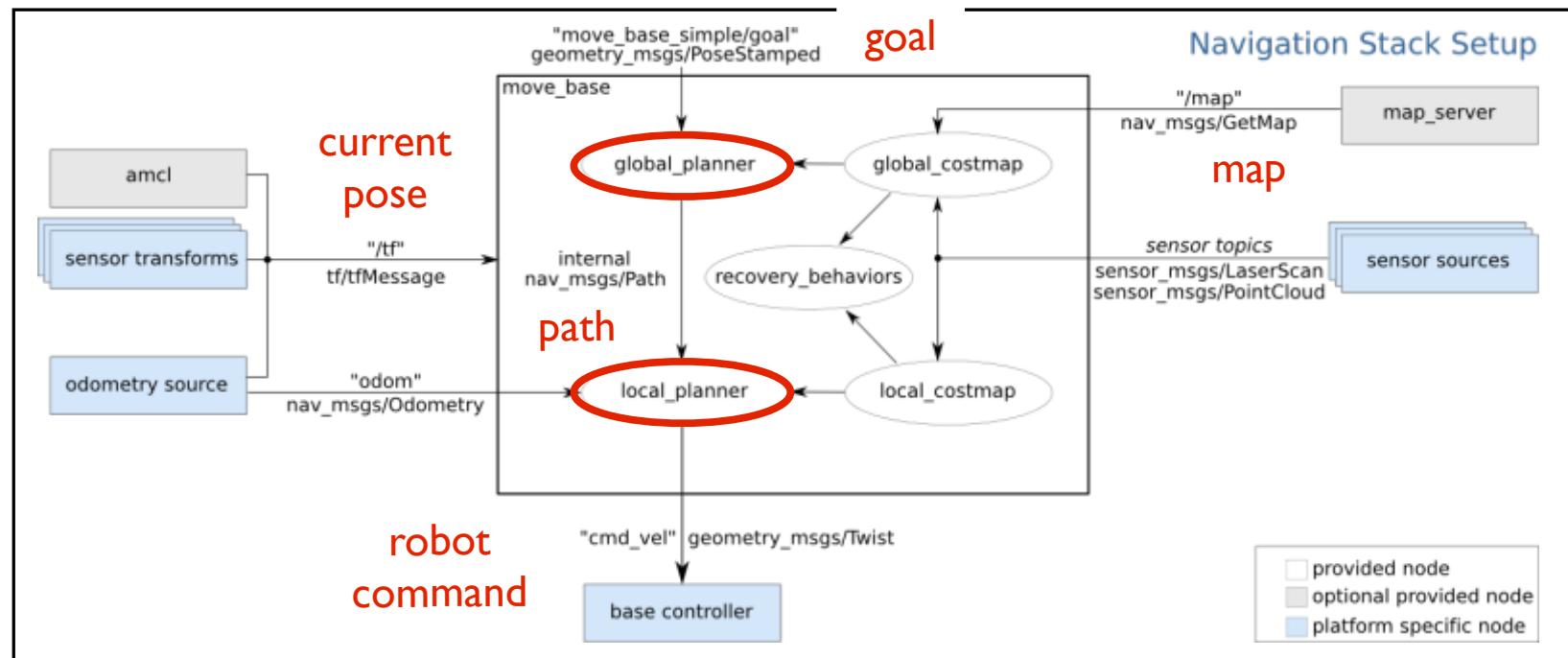
ROS Navigation Stack



<http://www.ros.org/wiki/navigation>

UM EECS 398/598 - autorob.github.io

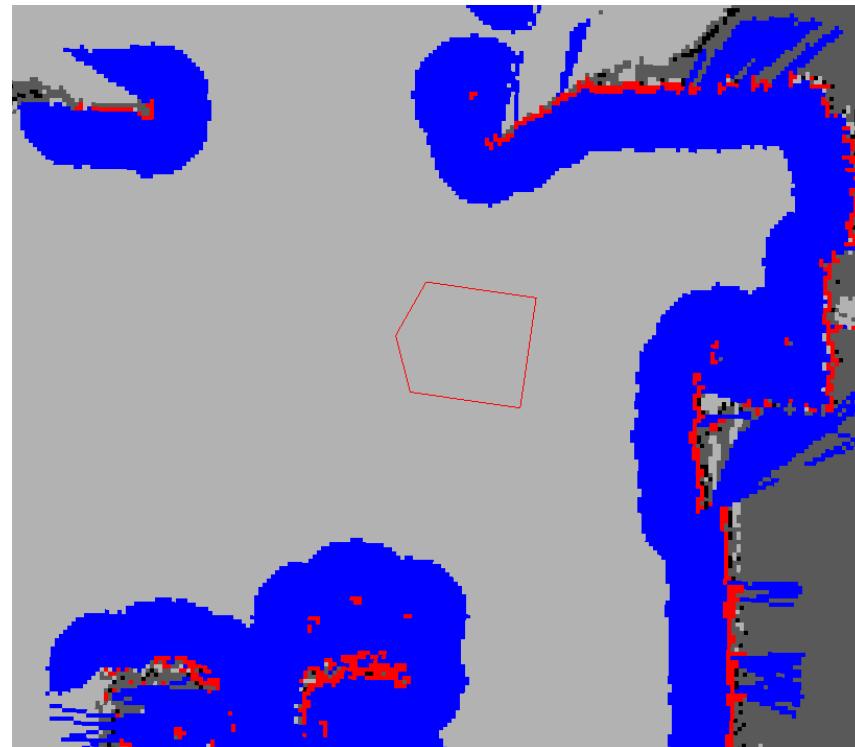
move_base



- ④ Core of nav_stack for planning and motion control

Costmap in ROS

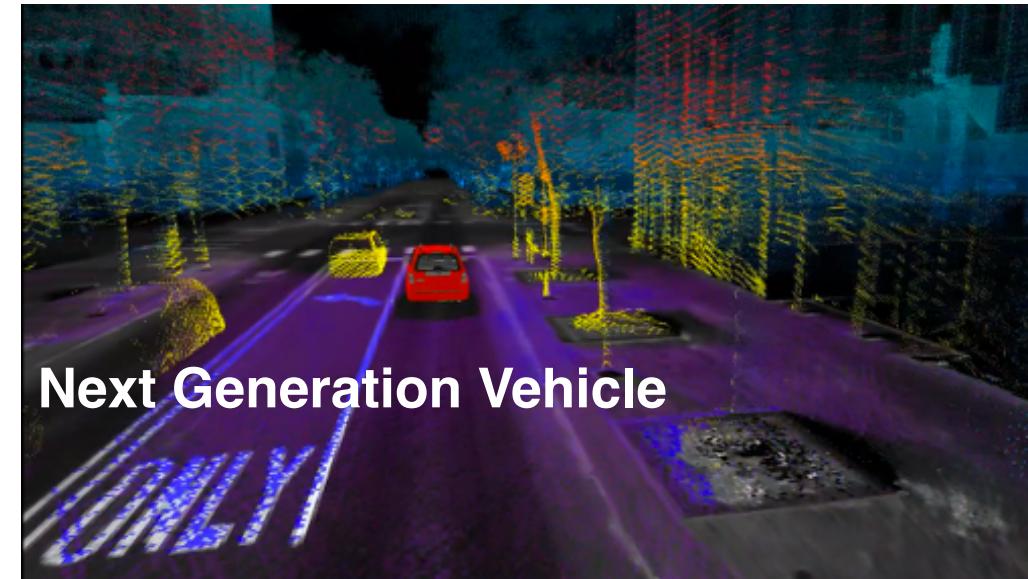
- Treat as a Minkowski sum over the given map



http://www.ros.org/wiki/costmap_2d

UM EECS 398/598 - autorob.github.io

Navigation in LCM



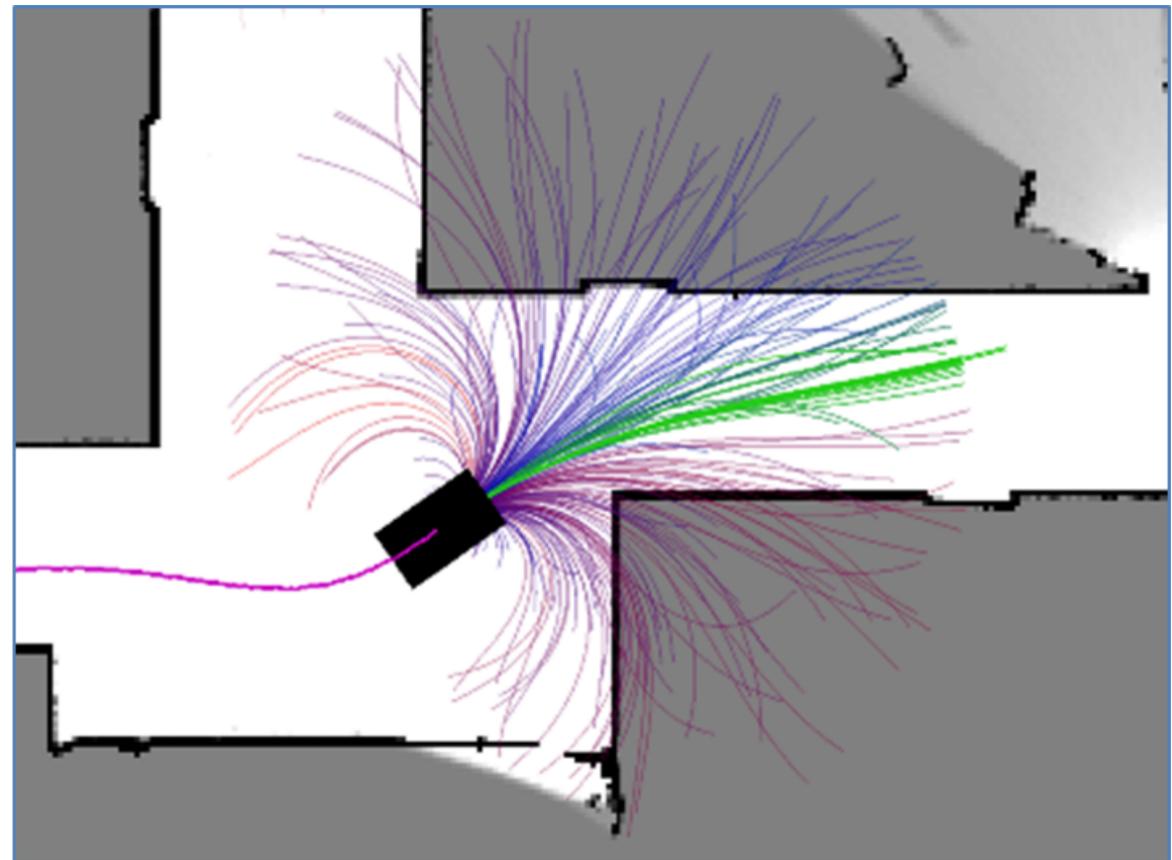
BotLab EECS 467 / ROB 550



MagicBots

[Park, Johnson, Kuipers 2012]

Vulcan Navigation



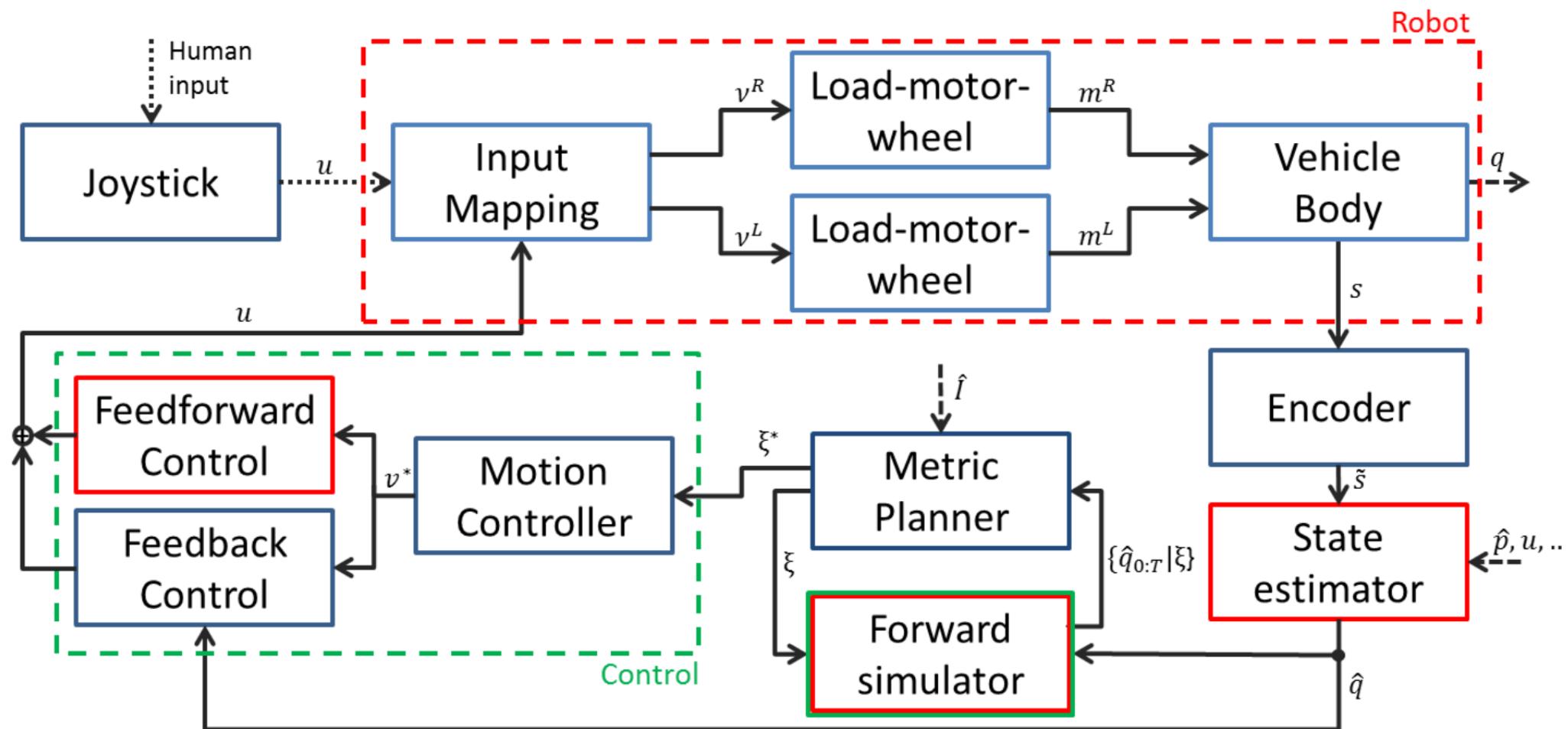


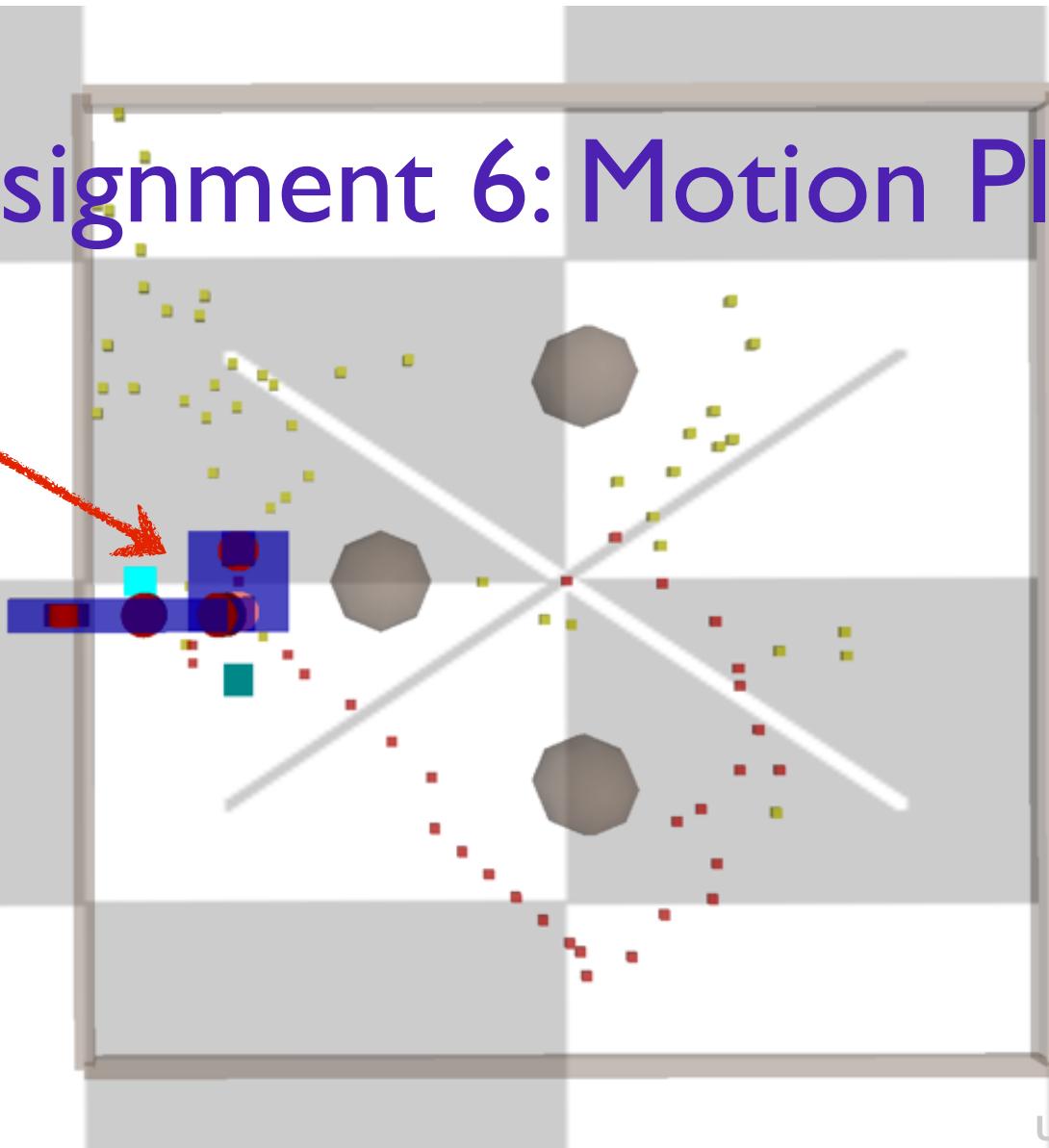
Figure A.1: Vulcan system diagram.

Assignment 6: Motion Planning

- Generate a collision free motion plan to the world origin and zero joint angle configuration

Assignment 6: Motion Planning

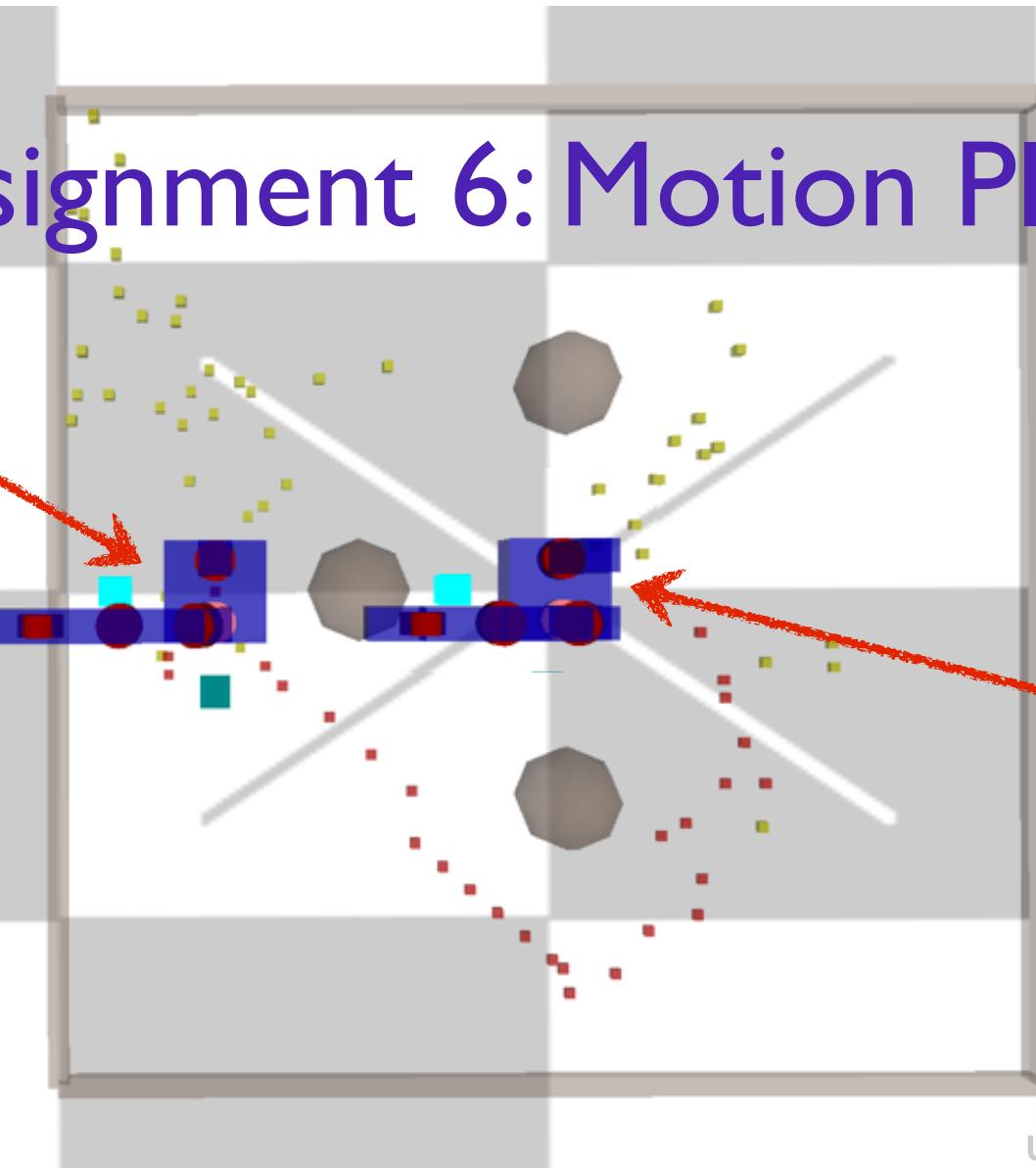
Start: random
non-colliding
configuration



Assignment 6: Motion Planning

Start: random
non-colliding
configuration

Goal: zero
configuration at
world origin

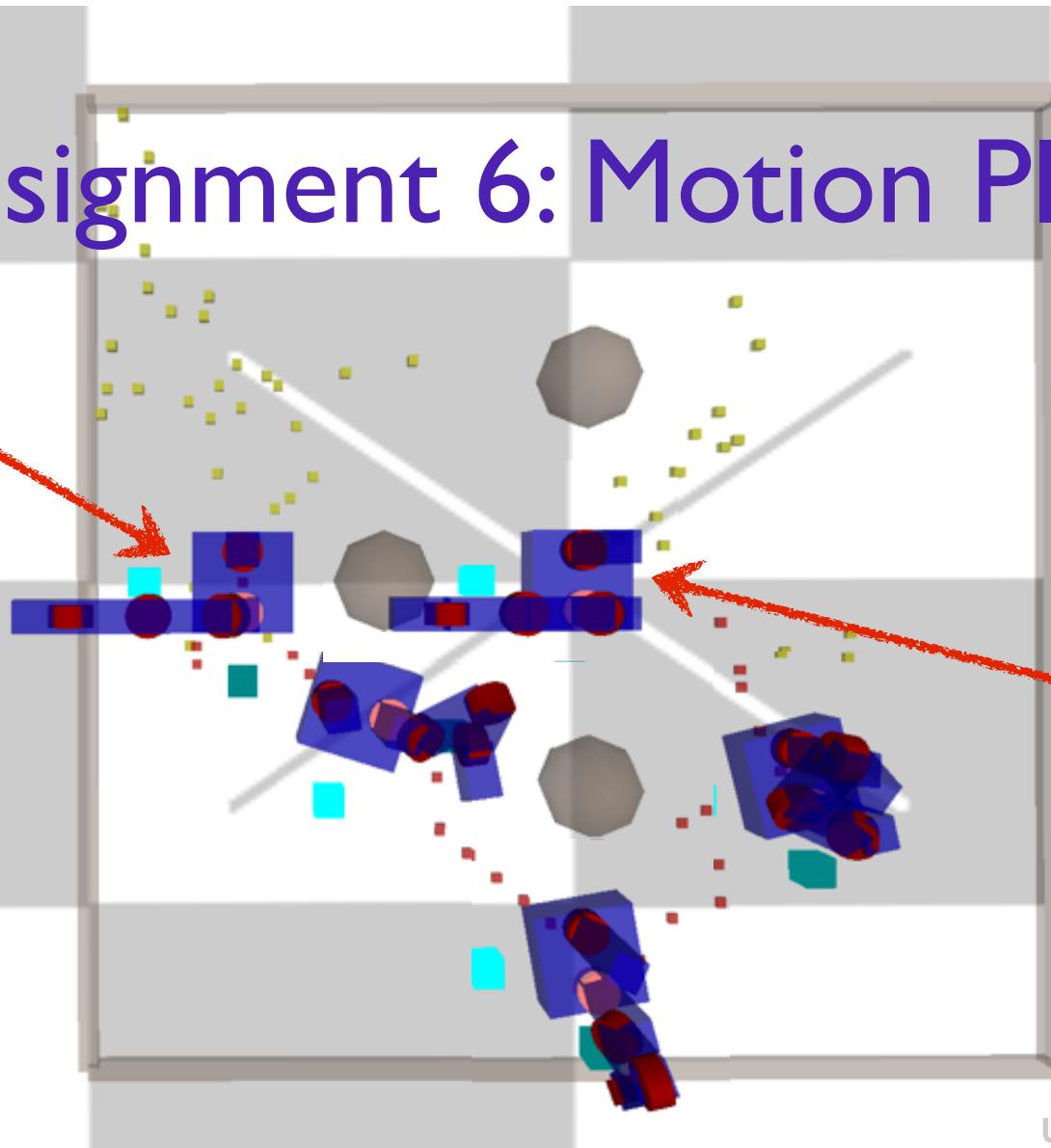


Assignment 6: Motion Planning

Start: random
non-colliding
configuration

Goal: zero
configuration at
world origin

Generate
collision-free
motion plan



GitHub, Inc. (US) https://github.com/ohseejay/kineval-stencil-fall16

ohseejay / kineval-stencil-fall16

Code Issues Pull requests Projects

Stencil code for KinEval (Kinematic Evaluator) for robots

2 commits 1 branch

Branch: master New pull request

odestc initial commit

js
kineval
project_pathplan ←
project_pendular
robots
tutorial_heapsort
tutorial_js
worlds ←
README.md
home.html

rrt connect

Watch 1 Star 0 Fork 0

Recommended starting point:
Update code stencil from Path Planning project

initial commit 26 days ago
initial commit 26 days ago

initial commit 26 days ago
initial commit 26 days ago
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initial commit 26 days ago

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Latest commit 5fd521e 26 days ago

2 contributors

file:///Users/cgenkin/courses/cs148_2014/3bot/mrt_canvas_stencil.html

various 3D worlds for testing robots included by:
home.html?world=worlds/world_local_minima.js

GitHub, Inc. (US) | https://github.com/ohseejay/kineval-stencil | rrt connect

ohseejay / kineval-stencil

Code Issues 0 Pull requests 0 Wiki Pulse Graphs

Stencil code for KinEval (Kinematic Evaluator) for robot control

2 commits 1 branch

Branch: master New pull request New file Upload

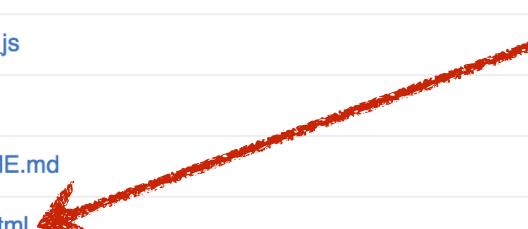
odestcj initial commit

- js
- kineval
- pendularm
- robots
- rrt
- tutorial_js
- worlds
- README.md
- home.html

home.html

```
<script src="worlds/world_basic.js"></script>
...
function my_animate() {
    ...
    // detect robot collisions
    kineval.robotIsCollision();
    ...
    // if requested, perform configuration space
    motion planning to home pose
    kineval.planMotionRRTConnect();
}
```

initial commit 2 months ago
initial commit 2 months ago
initial commit 2 months ago



world file can be alternatively
loaded by a script tag

home.html

```
<script src="worlds/world_basic.js"></script>
```

```
...
```

```
function my_animate() {
```

```
...
```

```
// detect robot collisions
```

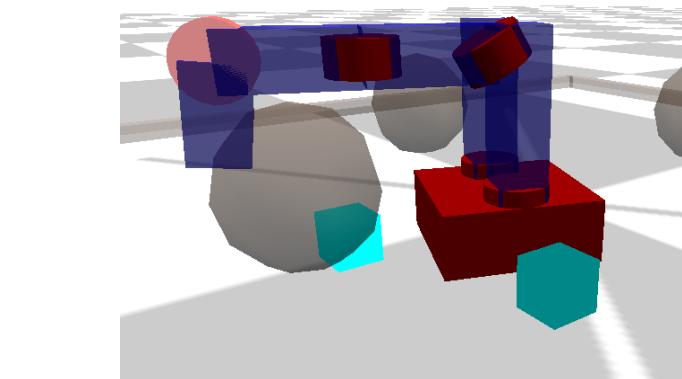
```
kineval.robotIsCollision();
```

```
...
```

```
// if requested, perform configuration space motion planning to home pose
```

```
kineval.planMotionRRTConnect();
```

```
}
```



detect if current
configuration is in collision
(colliding link turns red)

iterate motion planner

Branch: master **kineval-stencil / kineval /**

New file Upload files Find file History

 odestcj initial commit

Latest commit 2a1bd6e on Jan 11

..		
kineval.js	initial commit	2 months ago
kineval_collision.js	initial commit	2 months ago
kineval_controls.js	initial commit	2 months ago
kineval_forward_kinematics.js	initial commit	2 months ago
kineval_inverse_kinematics.js	initial commit	2 months ago
kineval_matrix.js	initial commit	2 months ago
kineval_quaternion.js	initial commit	2 months ago
kineval_robot_init.js	initial commit	2 months ago
kineval_rosbridge.js	initial commit	2 months ago
kineval_rrt_connect.js	initial commit	2 months ago
kineval_servo_control.js	initial commit	2 months ago
kineval_startingpoint.js	initial commit	2 months ago
kineval_threejs.js	initial commit	2 months ago
kineval_userinput.js	initial commit	2 months ago

Update collision detection with your forward kinematics



Implement RRT-Connect planner



kineval_collision.js

```
kineval.poseIsCollision = function robot_collision_test(q) {  
    // perform collision test of robot geometry against planning world  
  
    // test base origin (not extents) against world boundary extents  
    if ((q[0]<robot_boundary[0][0])||(q[0]>robot_boundary[1][0])||  
        (q[2]<robot_boundary[0][2])||(q[2]>robot_boundary[1][2]))  
        return robot.base;  
  
    // traverse robot kinematics to test each body for collision  
    // STENCIL: implement forward kinematics for collision detection  
    return robot_collision_forward_kinematics(q);  
}
```

input: q (robot configuration)
output: false (for no collision)
or name of link in collision



"electric fence"
world boundary
detection is
provided

Uncomment this call; and

Implement this function to traverse forward kinematics for collisions;
Use provided link collision function for AABB test of each link



kineval_rrt_connect.js

```
kineval.robotRRTPlannerInit = function robot_rrt_planner_init() {  
    // set start (q_init) and goal (q_goal) configurations  
}  
  
function robot_rrt_planner_iterate() {  
  
    var i;  
    rrt_alg = 1; // 0: basic rrt (OPTIONAL), 1: rrt_connect (REQUIRED)  
  
    if (rrt_iterate && (Date.now()-cur_time > 10)) {  
        cur_time = Date.now();  
  
        // implement one planning iteration here with calls to rrt_extend, etc.  
  
        // if plan found, store configuration sequence in kineval.motion_plan  
        //   kineval.motion_plan[kineval.motion_plan_traversal_index]  
    }  
}
```

make sure to test
against all provided
worlds!

