Example: Pendulum

Equation of motion
(with rotational inertia *I*)

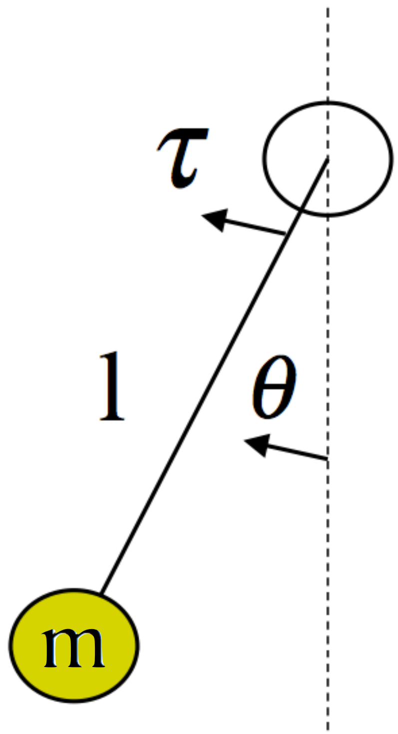
$$I\ddot{\theta} = -mgl\sin(\theta) + \tau$$

with Parallel Axis Theorem ($I=ml^2$)

$$\ddot{\theta} = -\frac{g}{l}\sin(\theta) + \frac{\tau}{ml^2}$$

Numerical integration over time

$$\boxed{\begin{aligned}\theta_{t+\Delta t} &= \theta_t + \dot{\theta}_t \Delta t \\ \dot{\theta}_{t+\Delta t} &= \dot{\theta}_t + \ddot{\theta}_t \Delta t\end{aligned}}$$



Motor produces torque
(angular force)

Angle expresses pendulum
range of motion

Pendulum of length *l* with
point mass *m*

Example: Pendulum

Equation of motion
(with rotational inertia I)

$$I\ddot{\theta} = -mgl\sin(\theta) + \tau$$

with Parallel Axis Theorem ($I = ml^2$)

What is this?

Numerical integration over time

$$\theta_{t+\Delta\theta} = \theta_t + \dot{\theta}_t \Delta\theta$$

$$\dot{\theta}_{t+\Delta\theta} = \dot{\theta}_t + \ddot{\theta}_t \Delta\theta$$

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ular force)

resses pendulum
e of motion

Hint

Pendulum of length l with
point mass m

Second-order state

Reminder:

- State in Newtonian physics has both position (θ) and velocity ($\dot{\theta}$)

$$\boxed{\begin{aligned}\theta_{t+\Delta t} &= \theta_t + \dot{\theta}_t \Delta t \\ \dot{\theta}_{t+\Delta t} &= \dot{\theta}_t + \ddot{\theta}_t \Delta t\end{aligned}}$$