Example: Pendulum

Equation of motion
(with rotational inertia I)

$$I\ddot{\theta} = -mgl\sin(\theta) + \tau$$

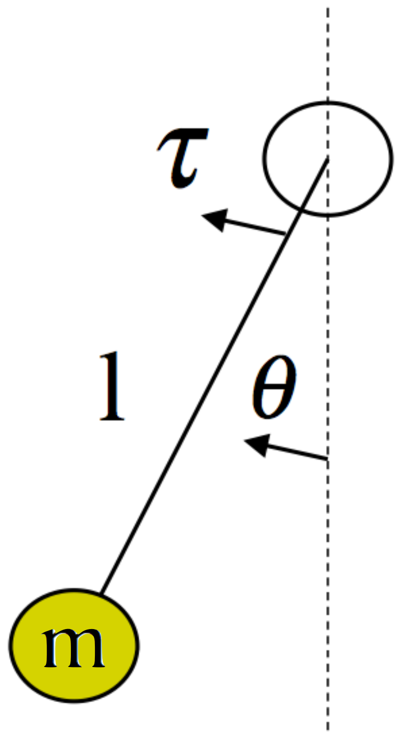
with Parallel Axis Theorem ($I=ml^2$)

$$\ddot{\theta} = -\frac{g}{l}\sin(\theta) + \frac{\tau}{ml^2}$$

Numerical integration over time

$$\theta_{t+\Delta t} = \theta_t + \dot{\theta}_t \Delta t$$

$$\dot{\theta}_{t+\Delta t} = \dot{\theta}_t + \ddot{\theta}_t \Delta t$$



Motor produces torque (angular force)

Angle expresses pendulum range of motion

Pendulum of length *l* with point mass *m*

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What is this?

Numerical integration over time

$$\theta_{t+\Delta\theta} = \theta_t + \dot{\theta}_t \Delta\theta$$

$$\dot{\theta}_{t+\Delta\theta} = \dot{\theta}_t + \ddot{\theta}_t \Delta\theta$$

Hint

Pendulum of length l with
point mass m

Second-order state

Reminder:

- State in Newtonian physics has both position (θ) and velocity ($\dot{\theta}$)

$$\boxed{\begin{aligned} \theta_{t+\Delta t} &= \theta_t + \dot{\theta}_t \Delta t \\ \dot{\theta}_{t+\Delta t} &= \dot{\theta}_t + \ddot{\theta}_t \Delta t \end{aligned}}$$

Velocity Verlet

$$y(t + \Delta t) = y(t) + \dot{y}(t)\Delta t + \frac{1}{2}a(t)\Delta t^2$$

$$\dot{y}(t + \Delta t) = \dot{y}(t) + \frac{a(t) + a(t + \Delta t)}{2}\Delta t$$

ASSUMES THAT ACCELERATION $a(t + \Delta t)$
ONLY DEPENDS ON POSITION $y(t + \Delta t)$