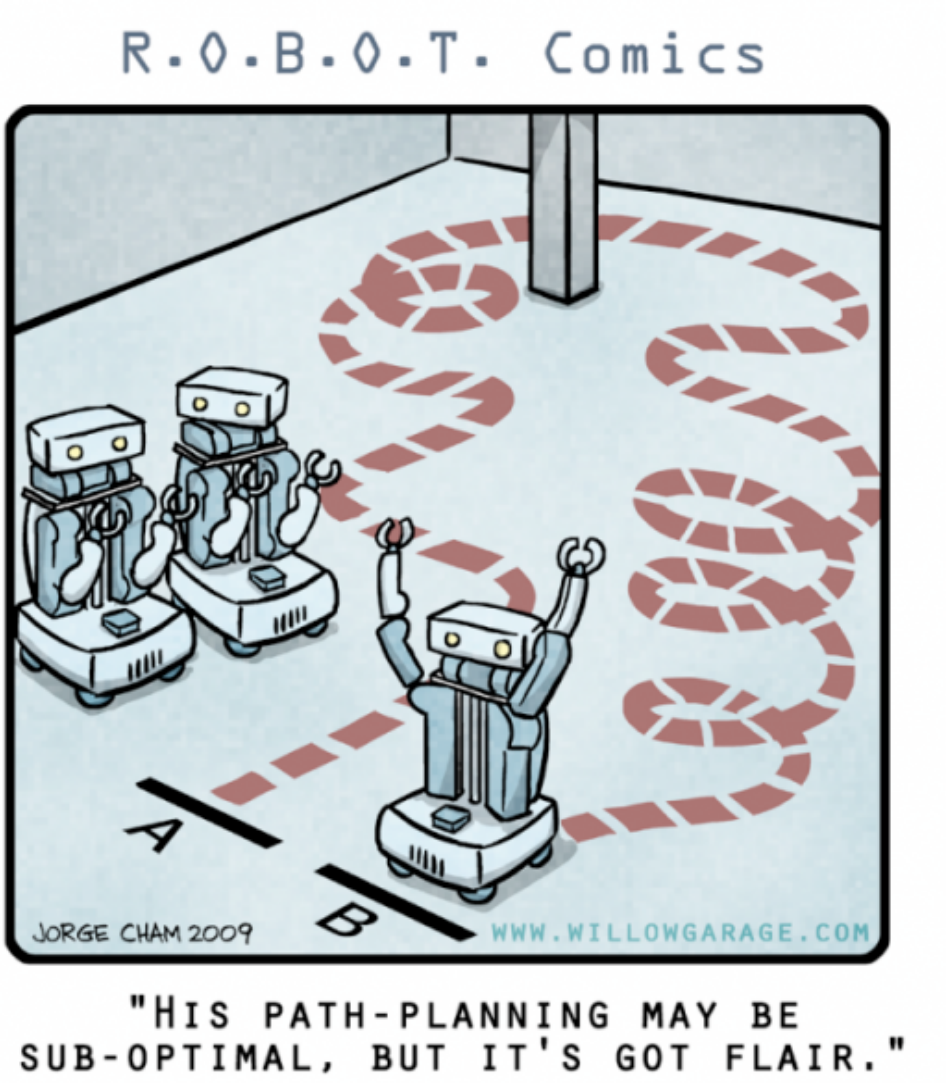


Path planning

the best way to get from A to B

EECS 367
Intro. to Autonomous Robotics

ME/EECS 567 ROB 510
Robot Modeling and Control

Fall 2019

autorob.org

Feature | Geek Life | Profiles

Manuela Veloso: RoboCup's Champion

This roboticist has transformed robot soccer into a global phenomenon

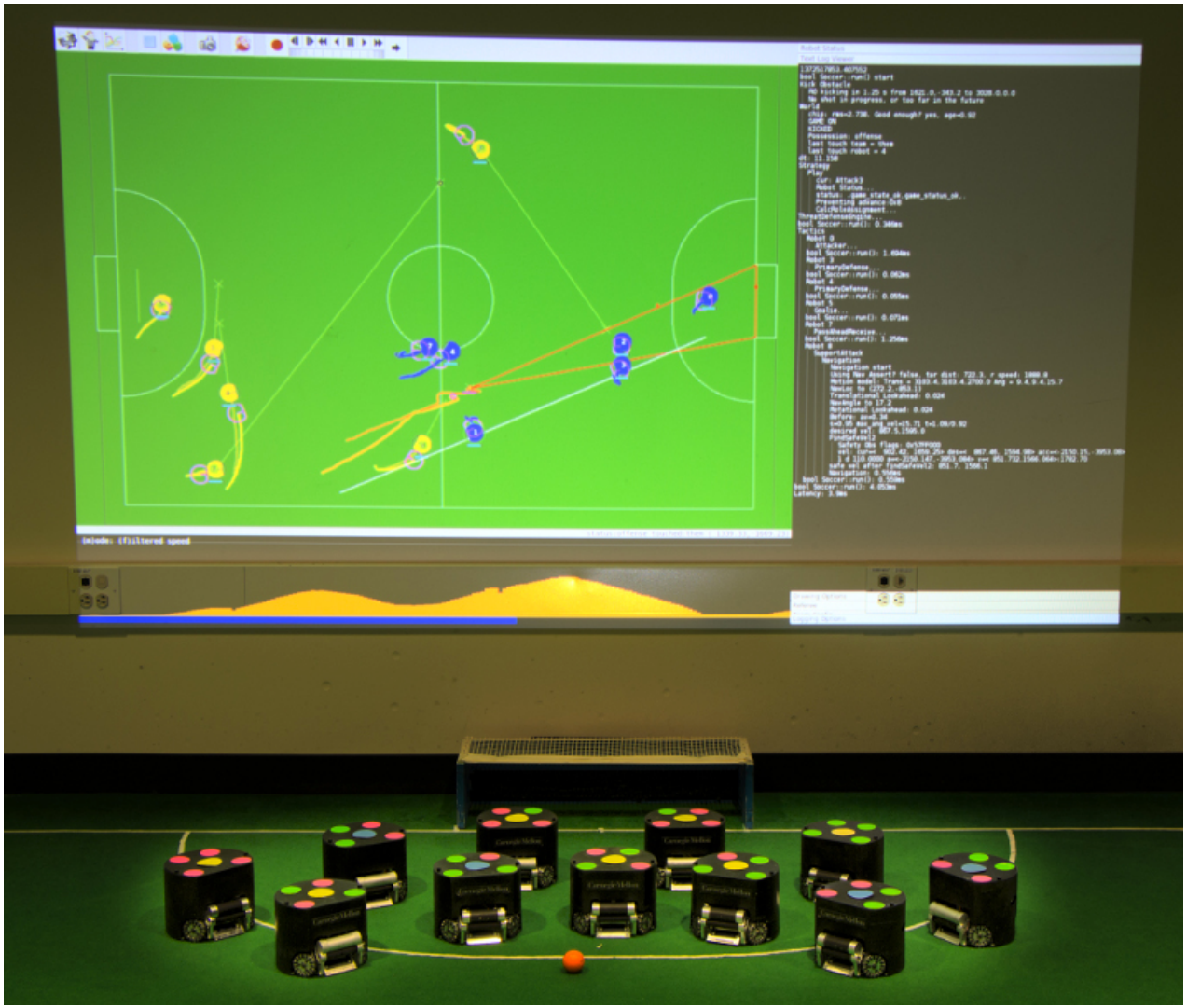
Posted 28 Feb 2015 | 17:00 GMT

By **PRACHI PATEL**

Michigan Robotics 367/510/567 - autorob.org

CMDragons RoboCup Small 2006

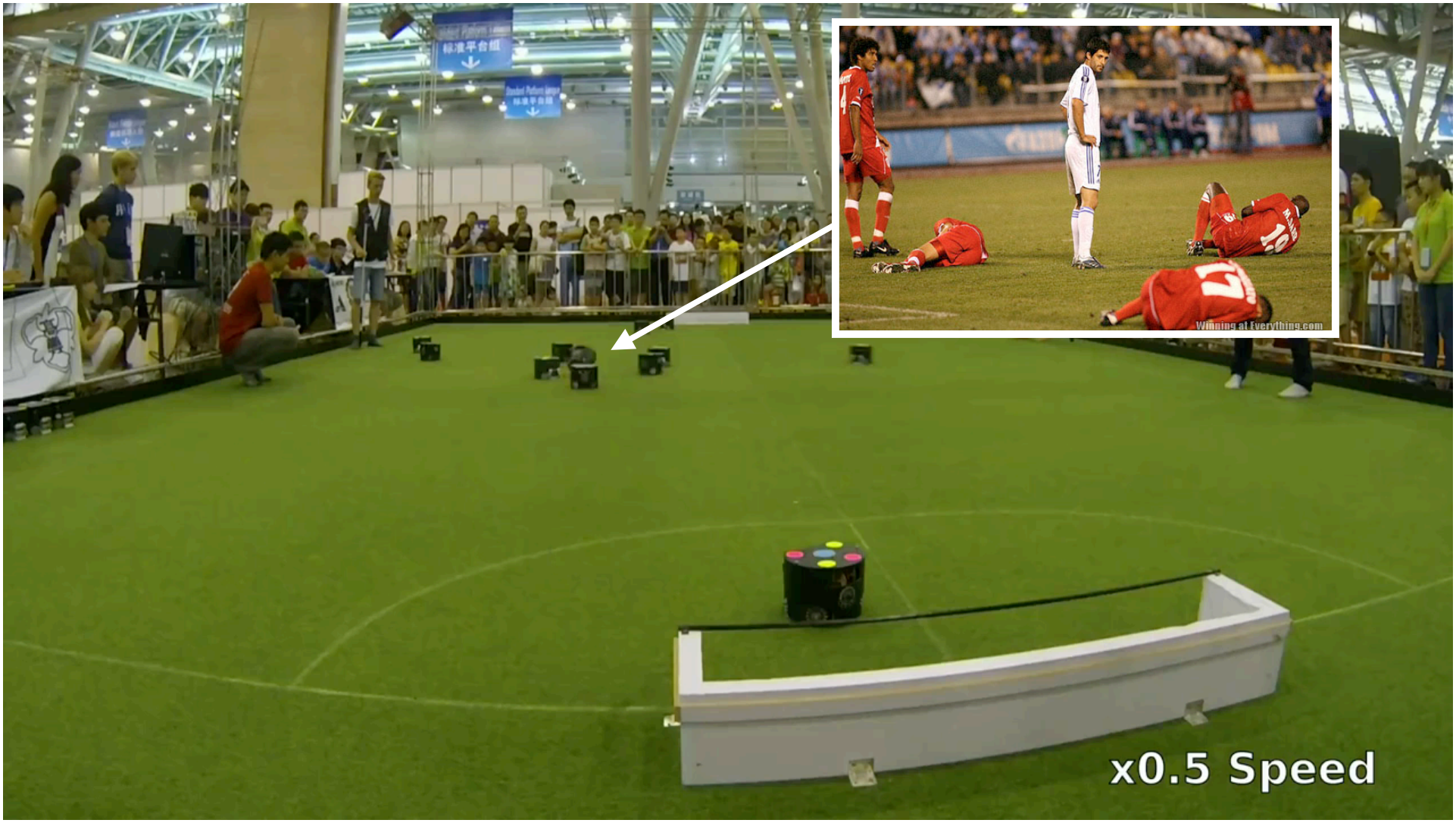
https://youtu.be/-Y4H3Sox_4I



CMDragons - http://www.cs.cmu.edu/~robosoccer/small/

CMDragons 2015 Pass-ahead Goal

CMDragons 2015 slow-motion multi-pass goal



CMDragons 2015 slow-motion multi-pass goal

http://www.cs.cmu.edu/~coral/projects/cobot/

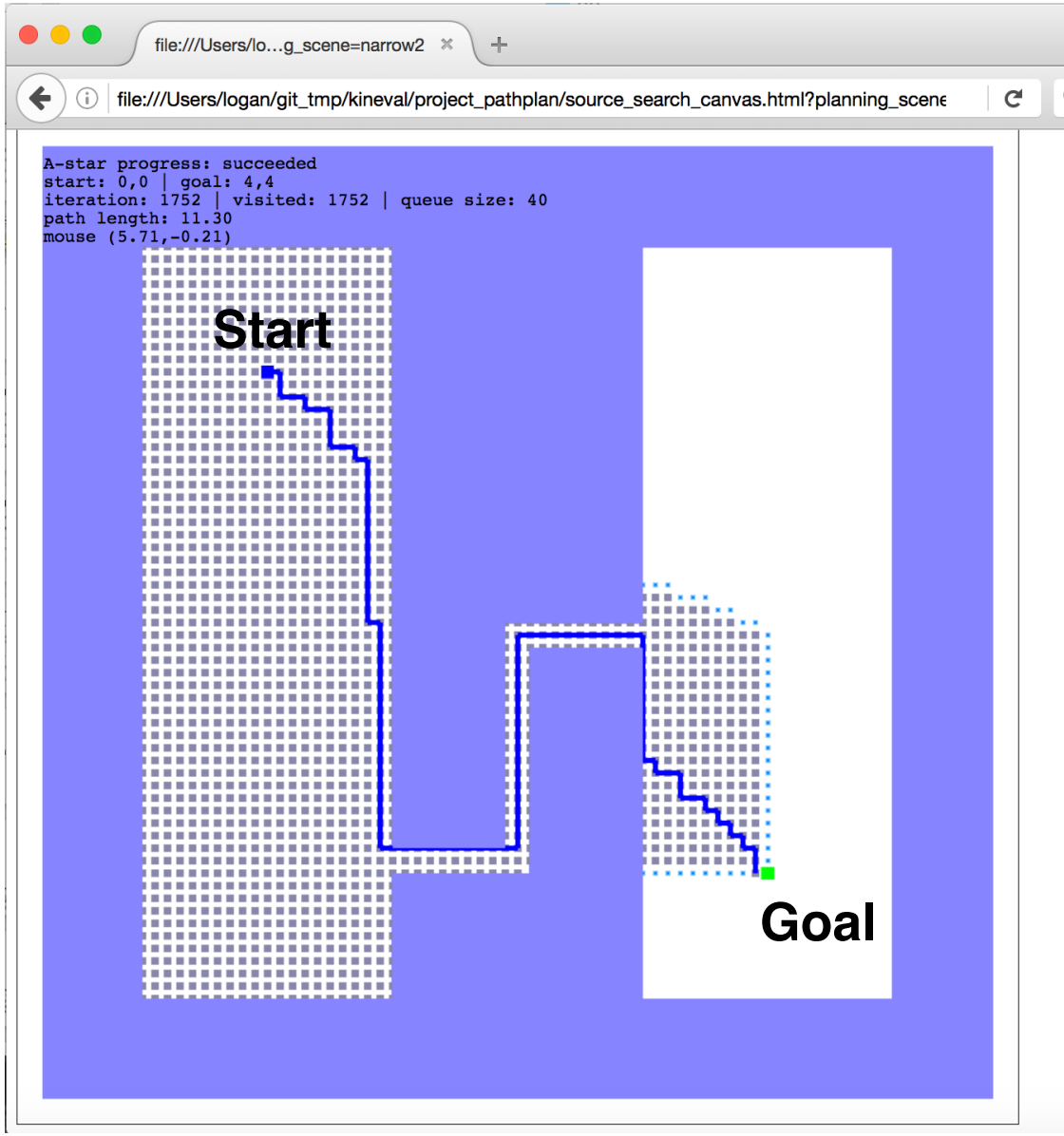
http://www.cs.cmu.edu/~coral/projects/cobot/

http://www.cs.cmu.edu/~coral/projects/cobot/

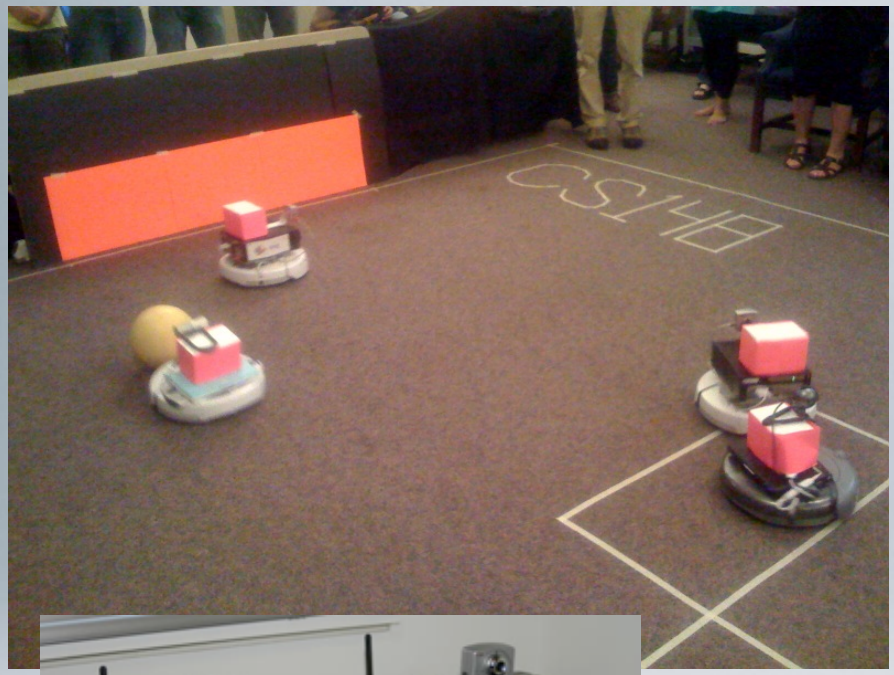
CoBot greets me in 2013

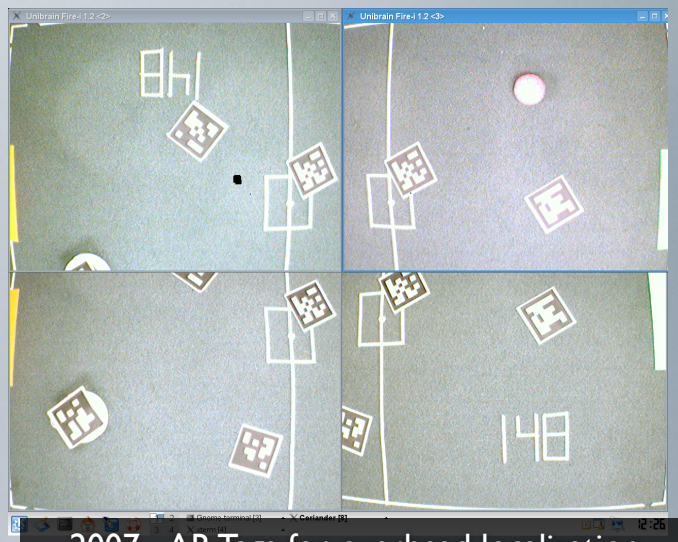
Project 1:
2D Path Planning

- A-star algorithm for search in a given 2D world
- Implement in JavaScript/ HTML5
- Heap data structure for priority queue
- Grad: DFS, BFS, Greedy
- Submit through your git repository

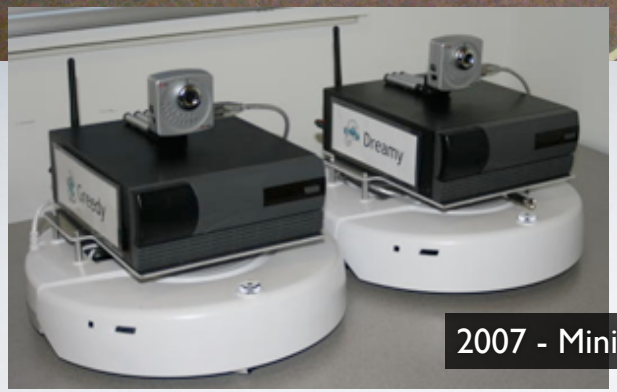


2007-10: SOCCER WITH IROBOT CREATE





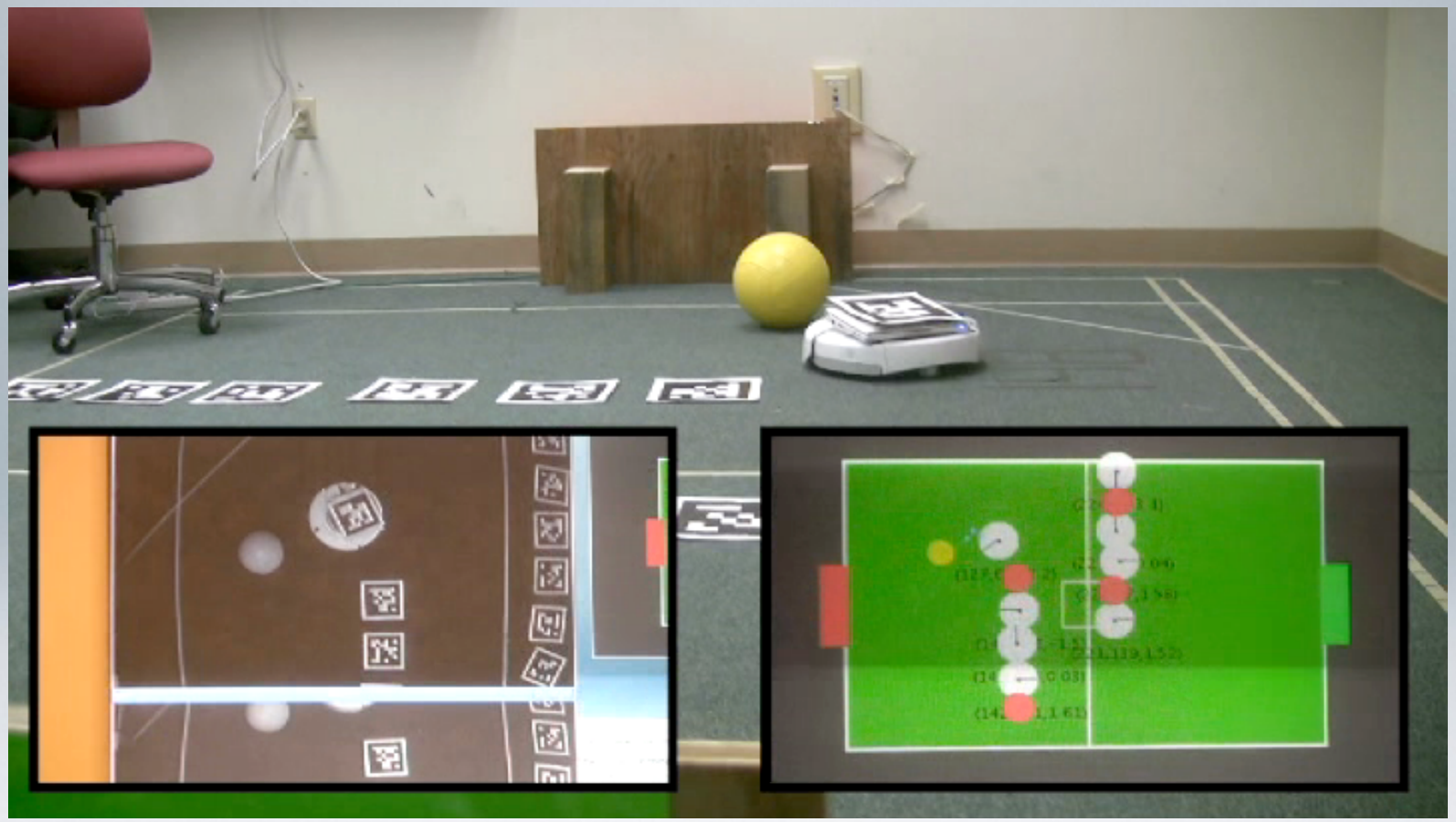
2007 - AR Tags for overhead localization

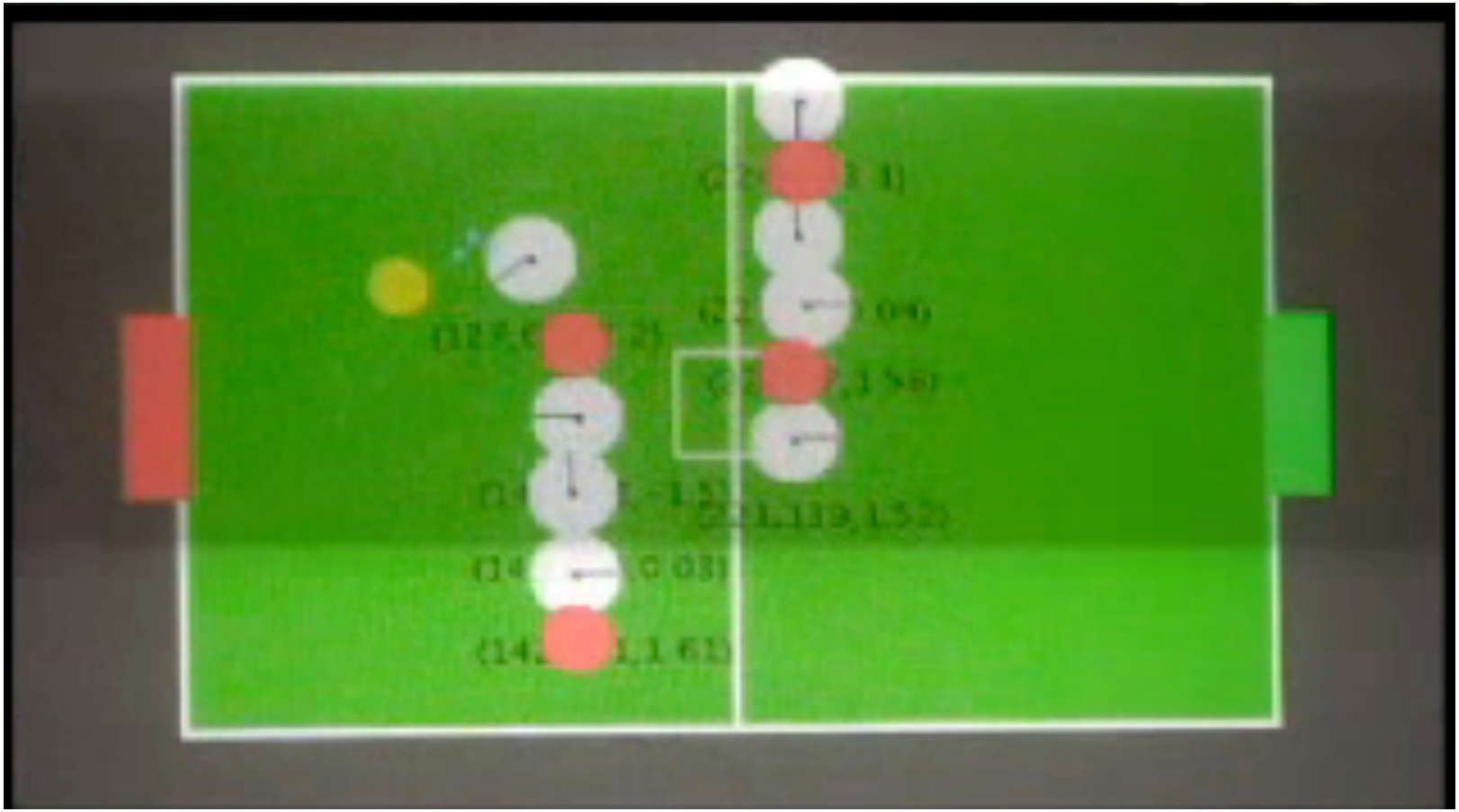


2007 - Mini ITX

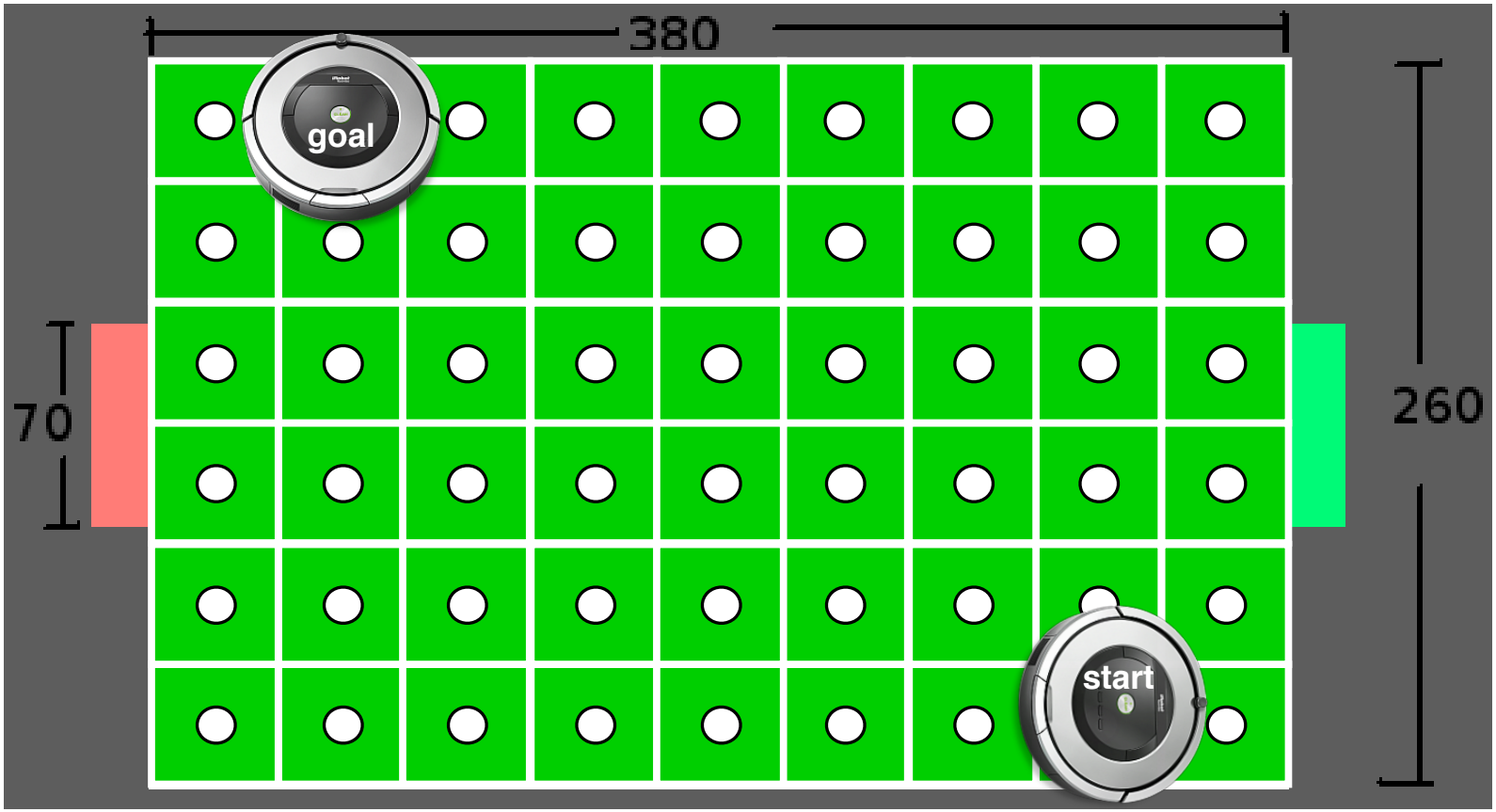
2009 - Asus EEE

http://www.youtube.com/watch?v=88zR6IC7S0g

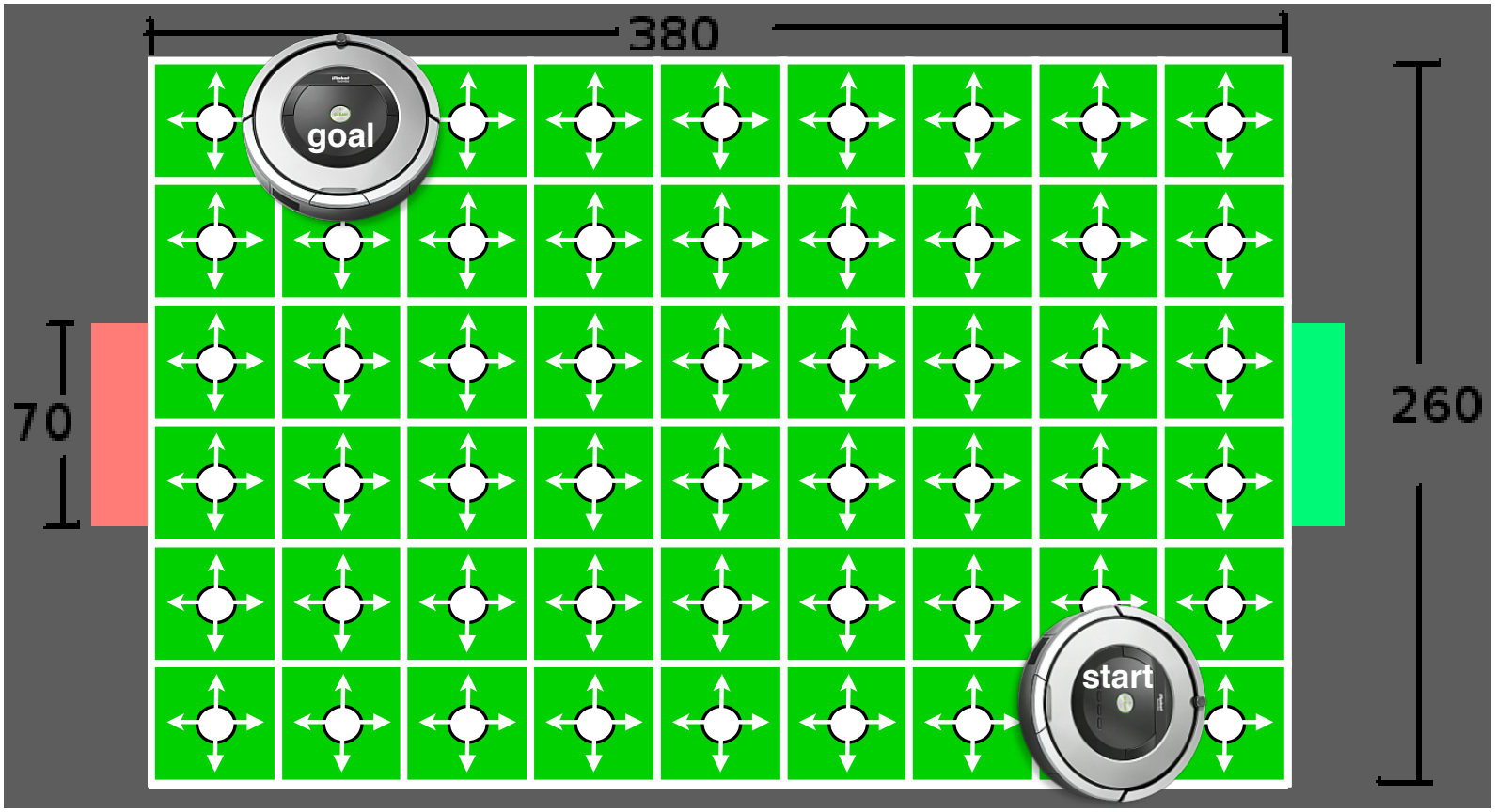




Consider all possible poses as uniformly distributed graph vertices



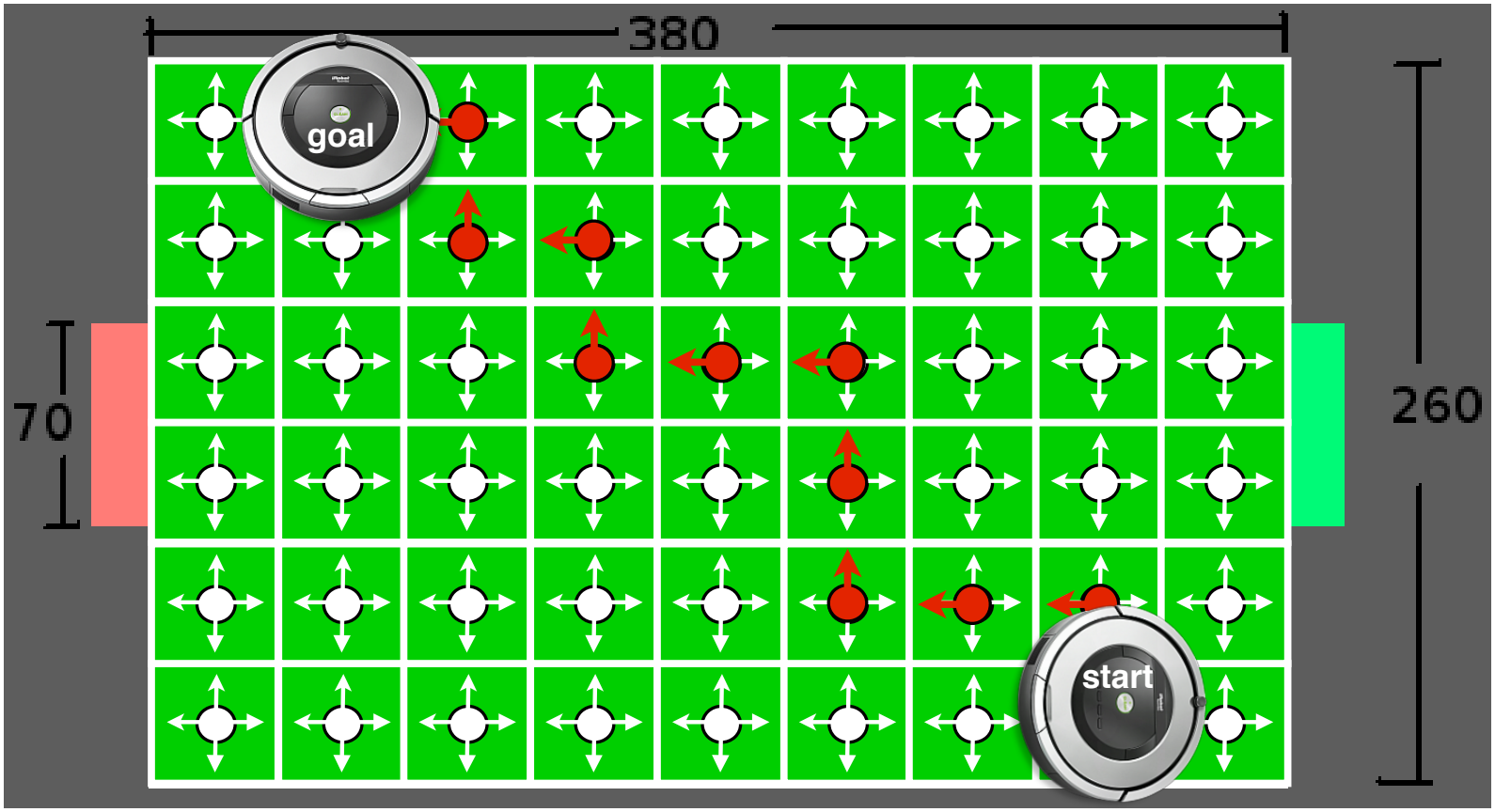
Consider all possible poses as uniformly distributed graph vertices
Edges connect adjacent (traversable) poses, weighted by distance



Consider all possible poses as uniformly distributed graph vertices

Edges connect adjacent (traversable) poses, weighted by distance

How to find a valid path in this graph?

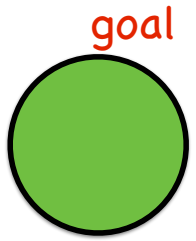


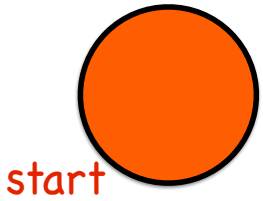
Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- **Graph Search (fixed graph)**
  - **Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first**
- Sampling-based Search (build graph):
  - Probabilistic Road Maps, Rapidly-exploring Random Trees
- Optimization (local search):
  - Gradient descent, potential fields, Wavefront

Consider a simple search graph

Consider each possible robot pose as
node in a graph





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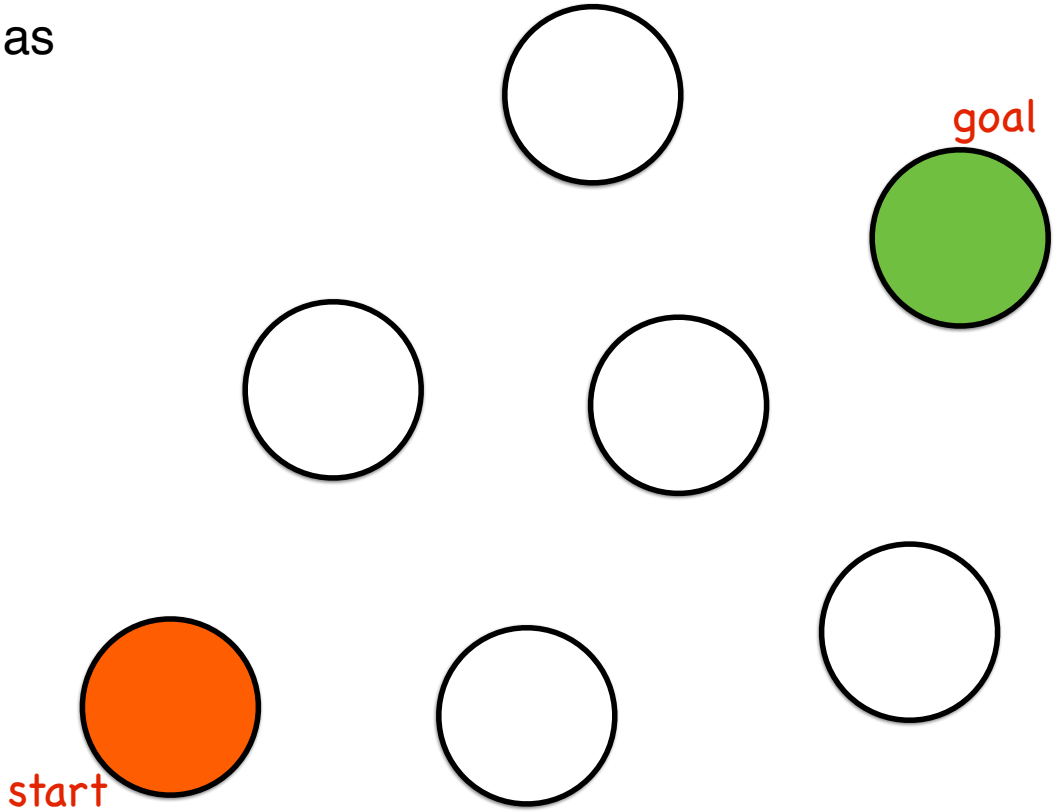
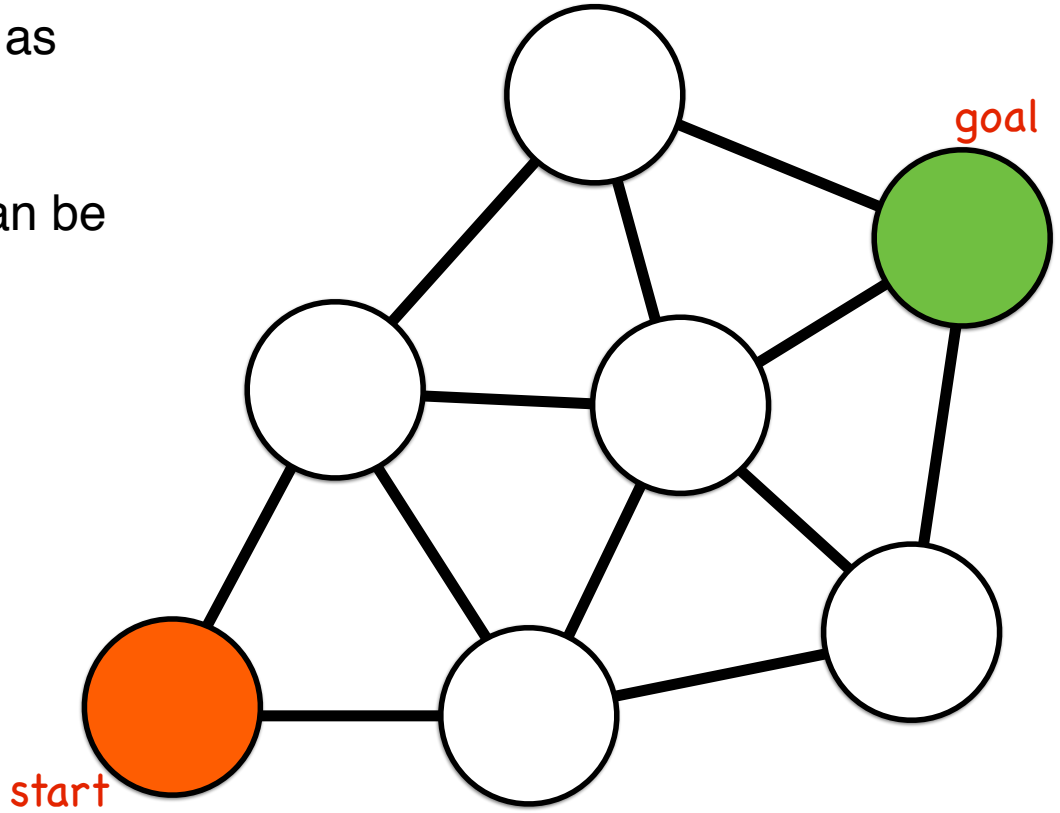


Consider a simple search graph

Consider each possible robot pose as node in a graph

Graph edges connect poses that can be reliably moved between

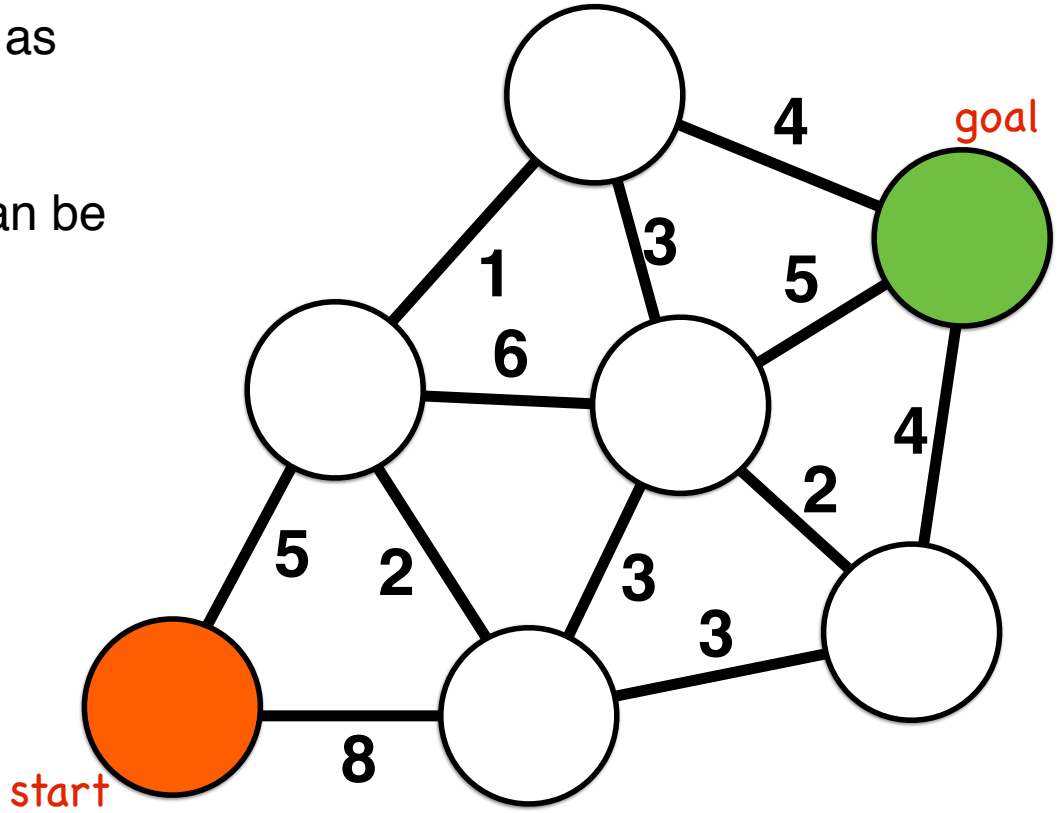


Consider a simple search graph

Consider each possible robot pose as node in a graph

Graph edges connect poses that can be reliably moved between

Edges have a cost for traversal



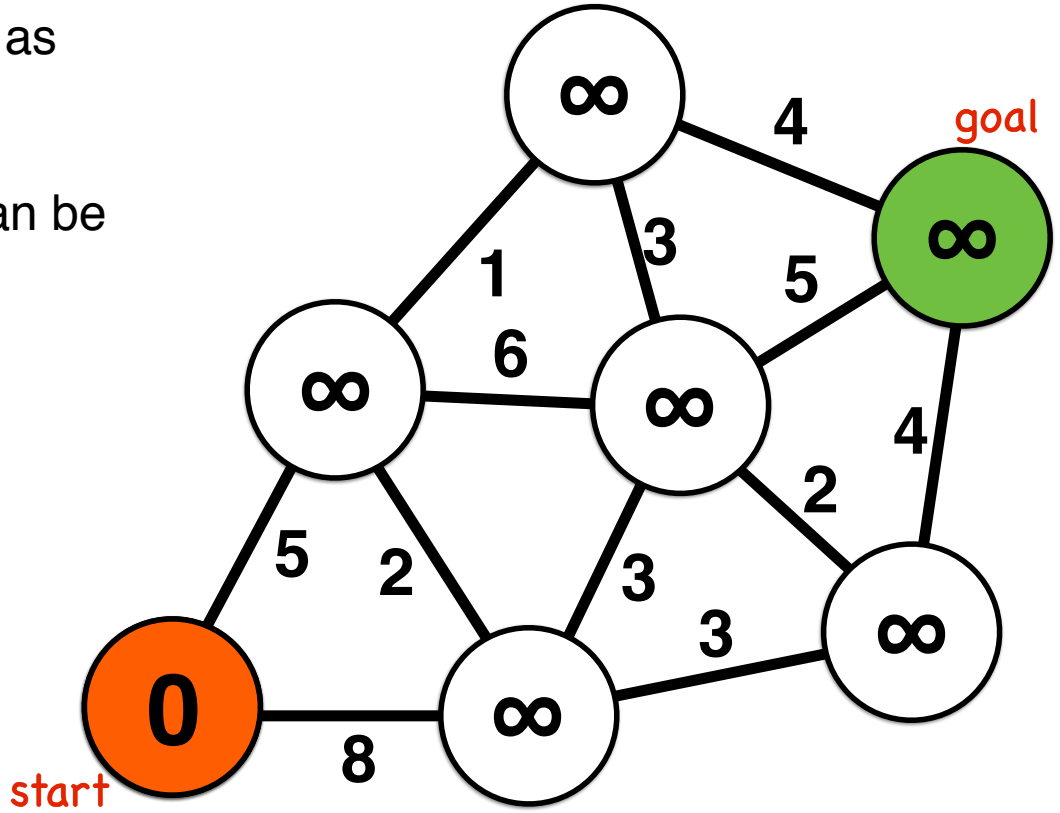
Consider a simple search graph

Consider each possible robot pose as node in a graph

Graph edges connect poses that can be reliably moved between

Edges have a cost for traversal

Each node maintains the **distance** traveled from start as a scalar cost



Consider a simple search graph

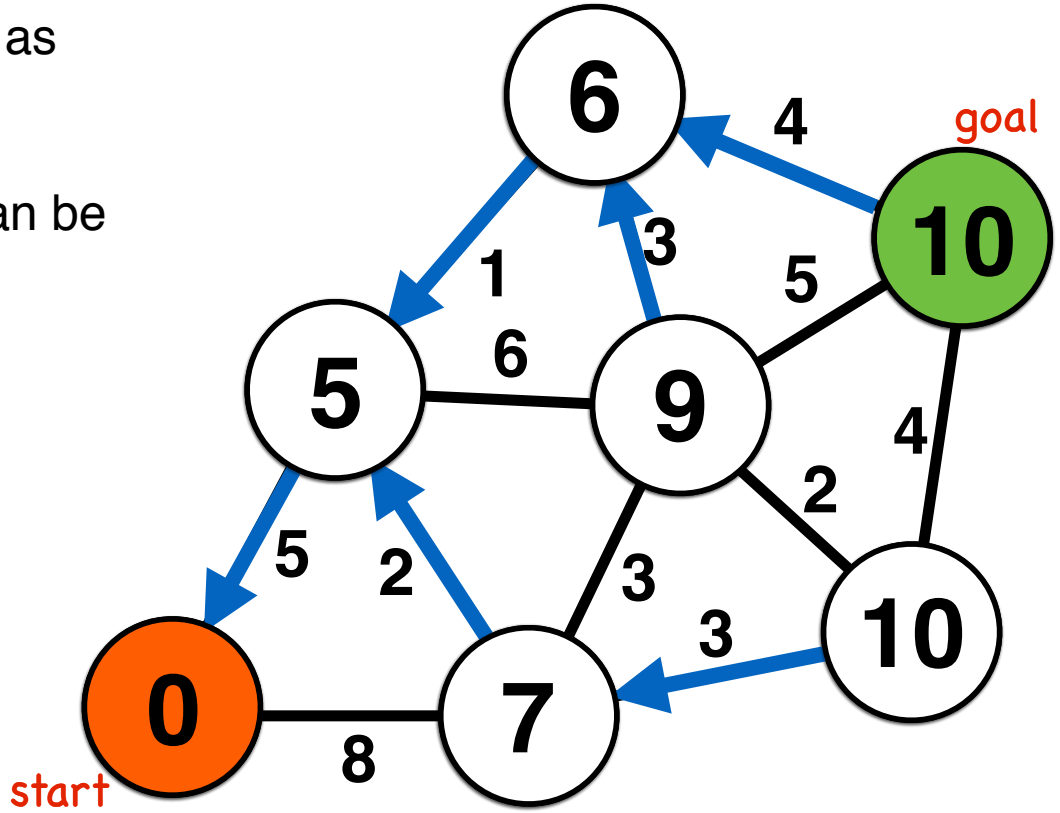
Consider each possible robot pose as node in a graph

Graph edges connect poses that can be reliably moved between

Edges have a cost for traversal

Each node maintains the **distance** traveled from start as a scalar cost

Each node has a **parent** node that specifies its route to the start node

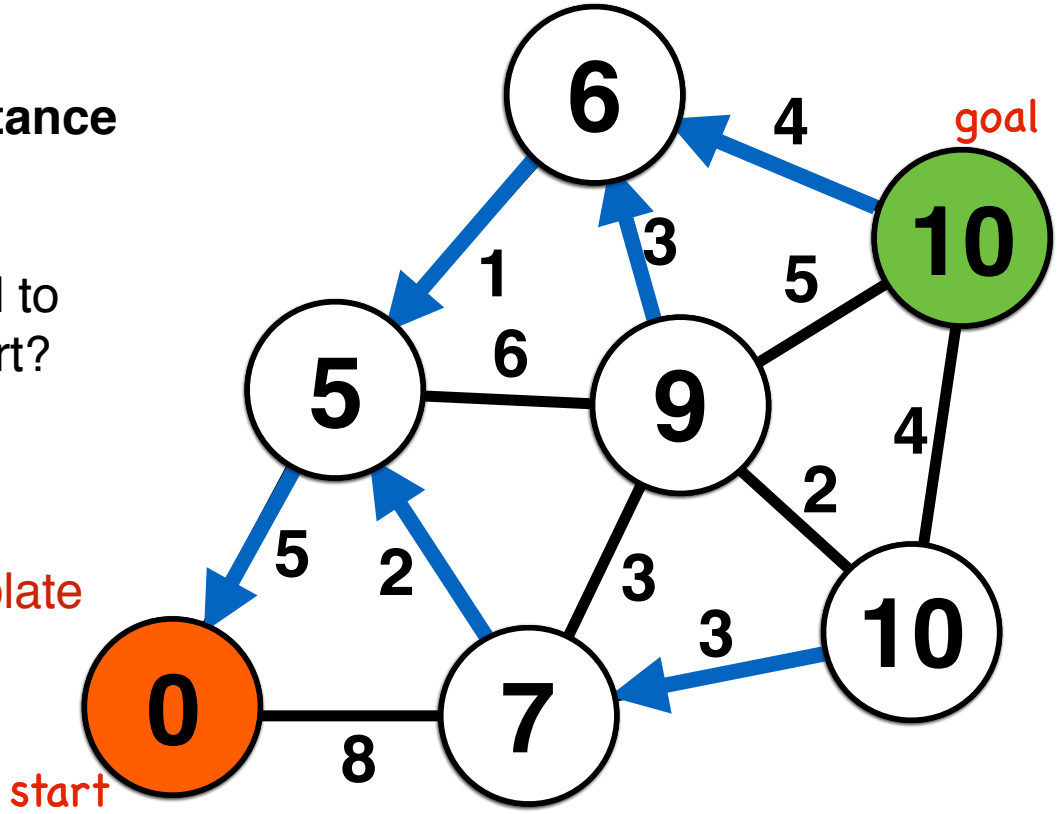


Path Planning as Graph Search

Which route is best to optimize **distance** traveled from start?

Which **parent** node should be used to specify route between goal and start?

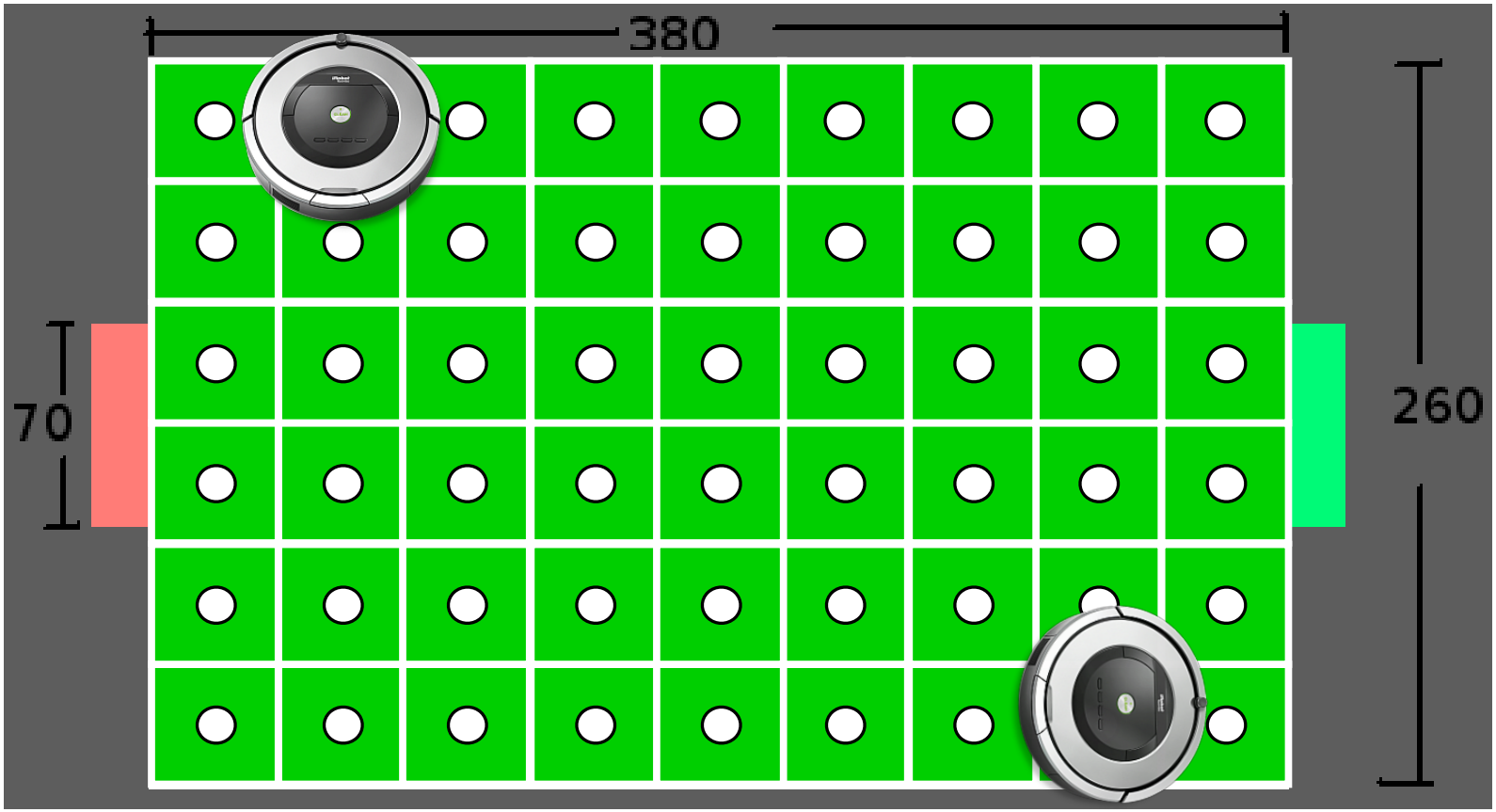
We will use a single algorithm template for our graph search computation



Depth-first search

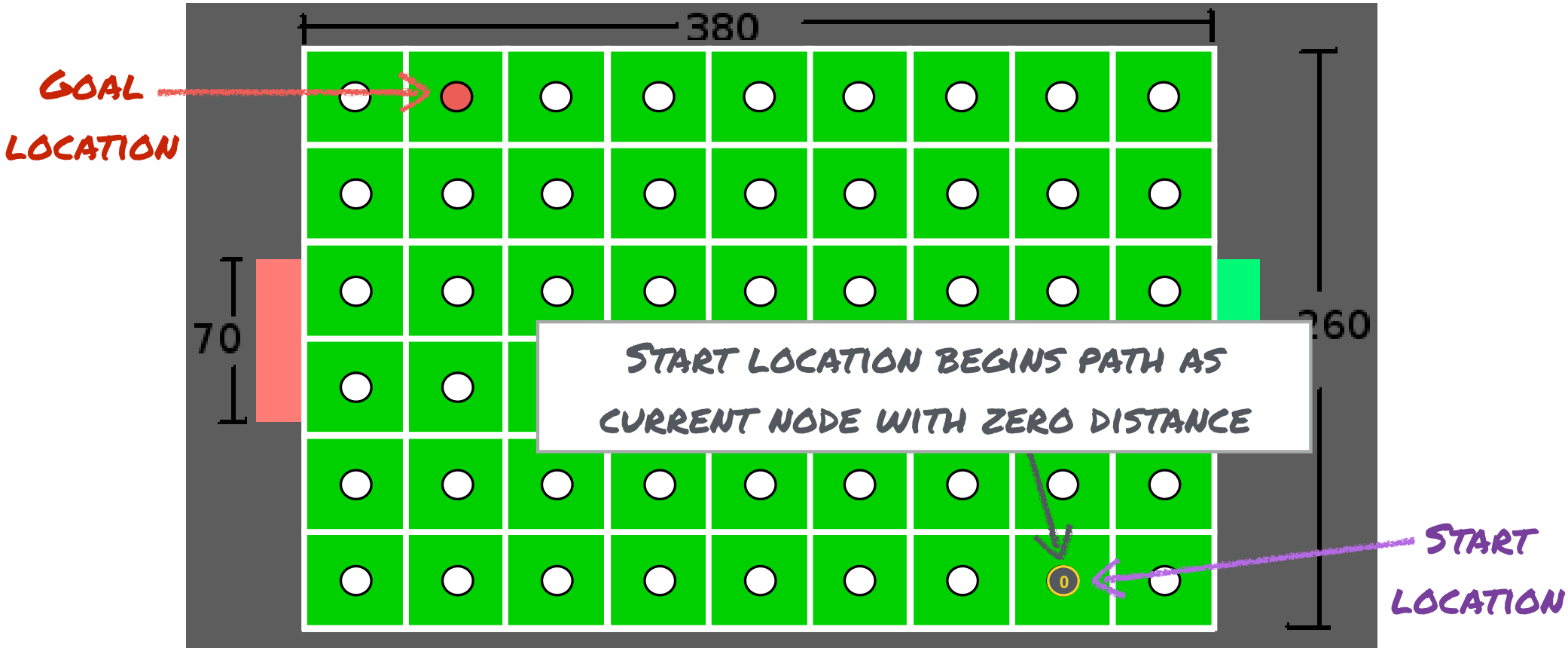
intuition and walkthrough

Depth-first search



Depth-first search

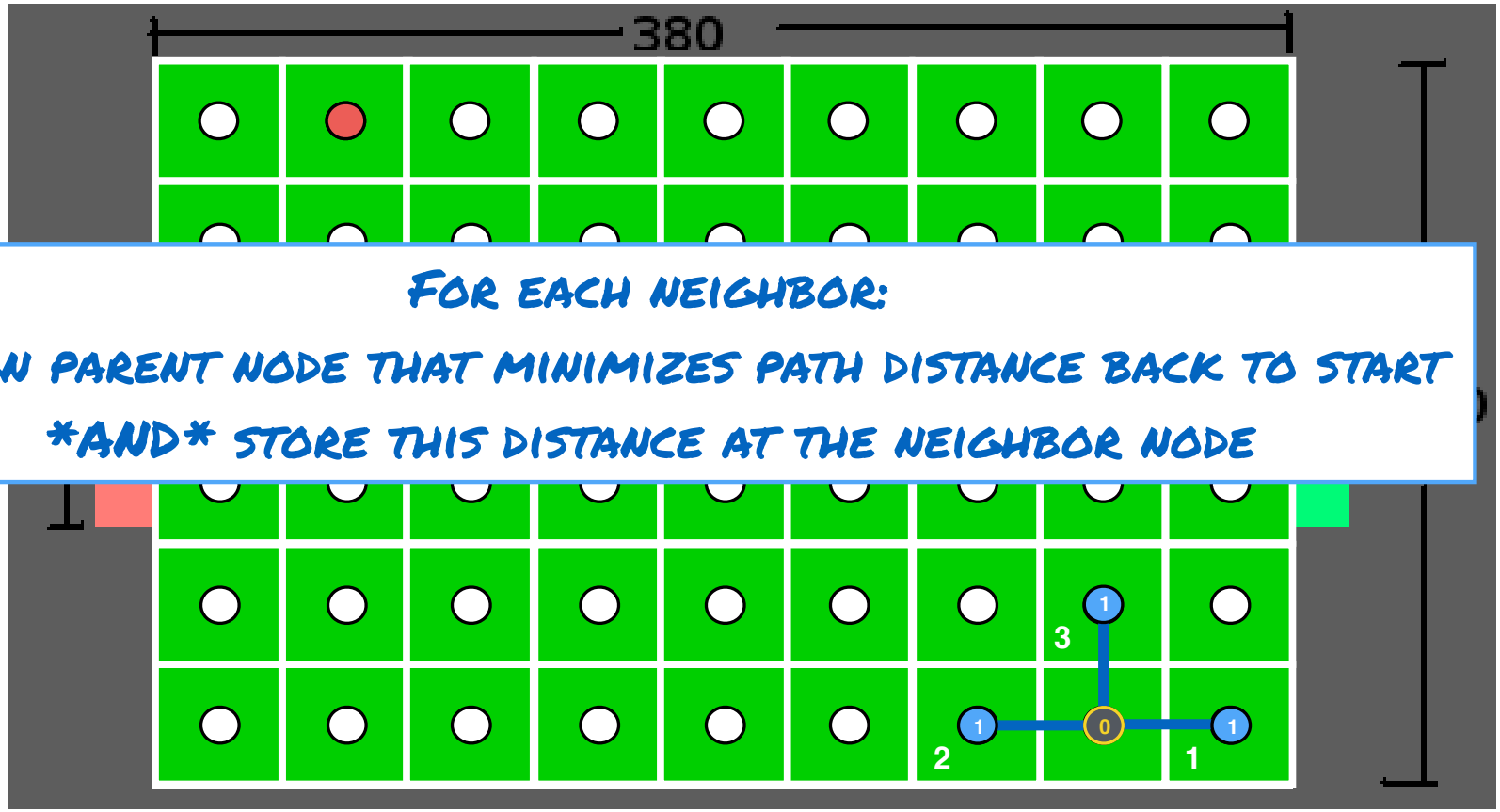
Depth-first search



Depth-first search

Depth-first search

Depth-first search

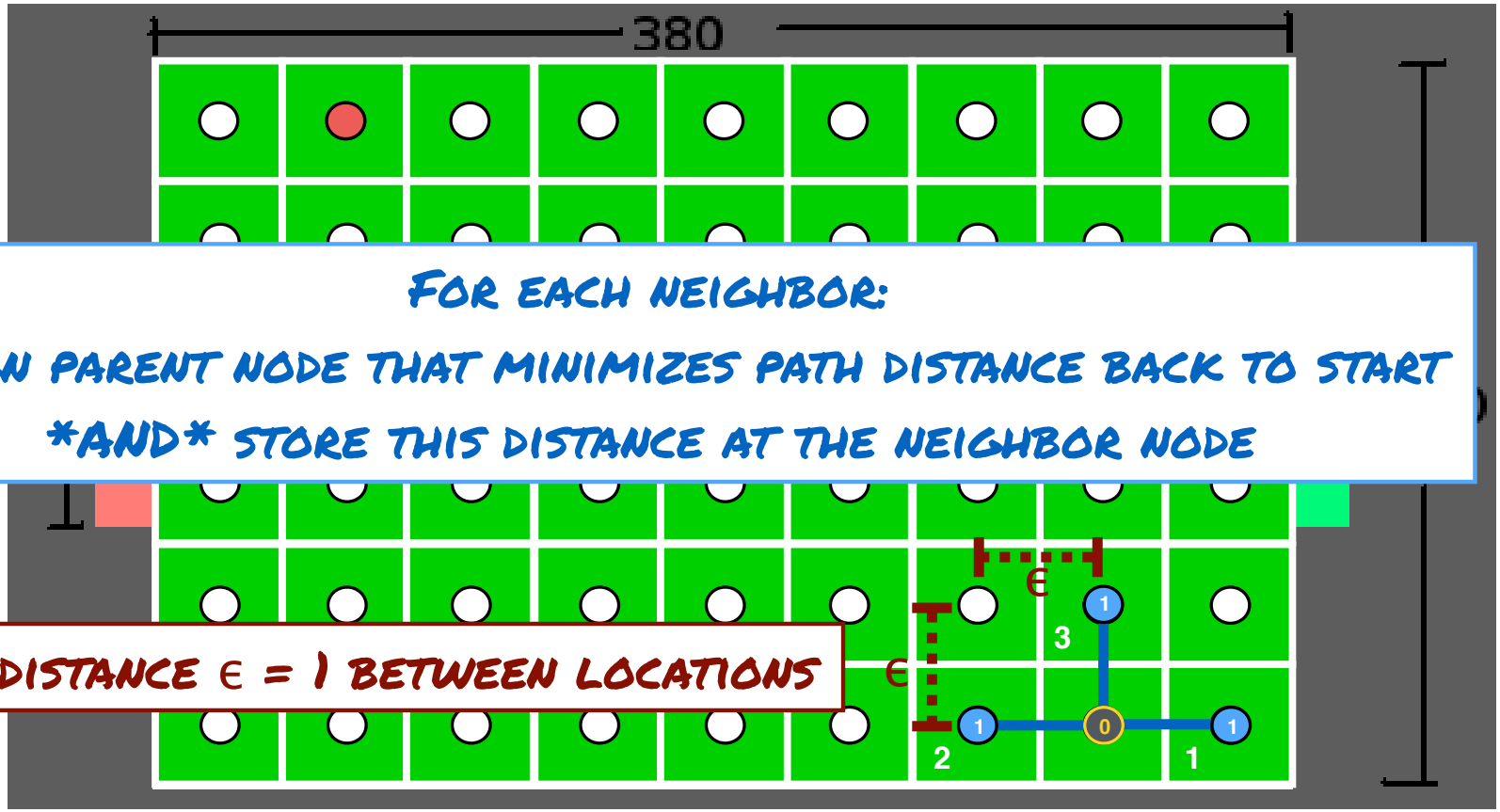


For each neighbor:

assign parent node that minimizes path distance back to start

and store this distance at the neighbor node

Depth-first search



For each neighbor:

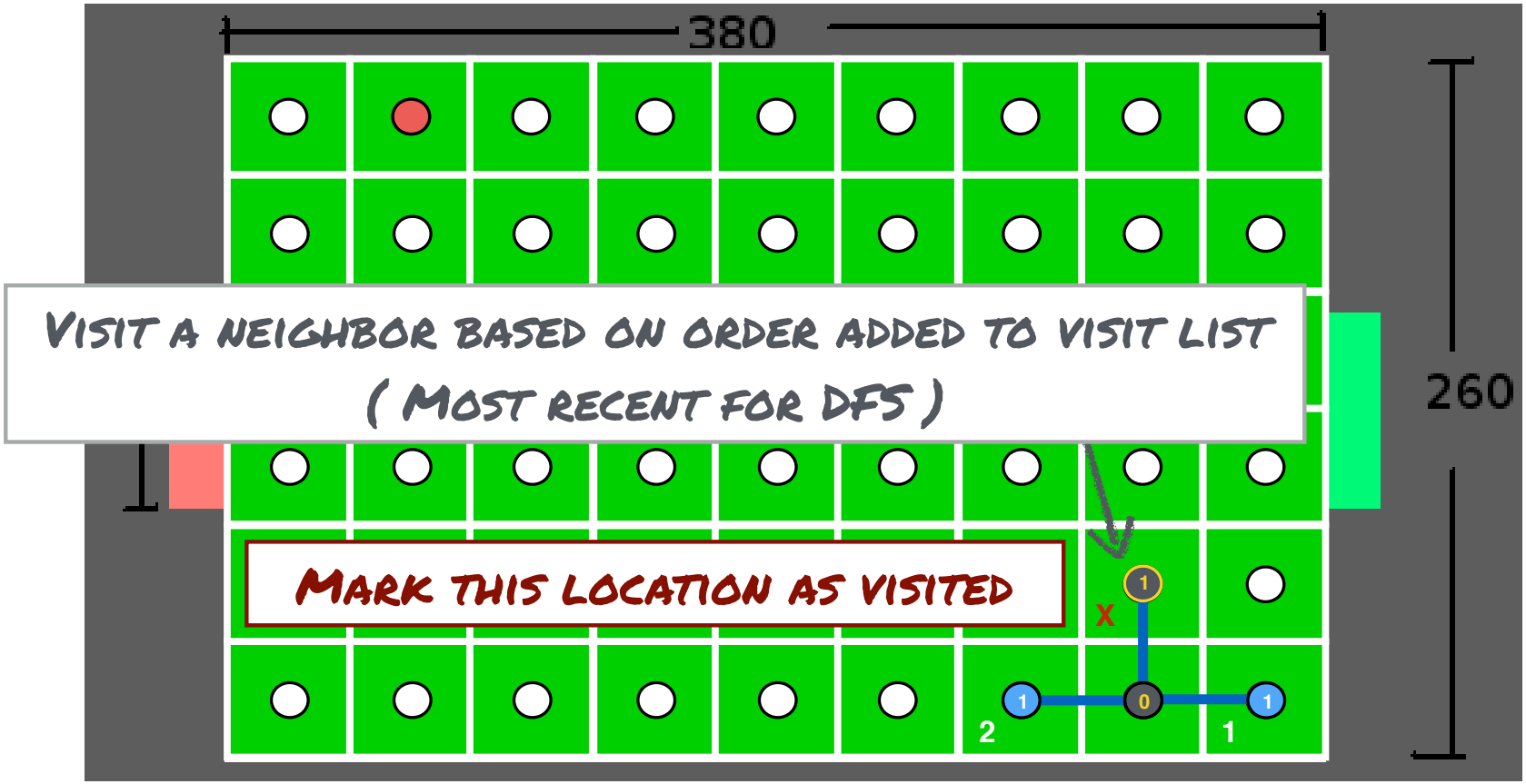
assign parent node that minimizes path distance back to start

and store this distance at the neighbor node

assume distance ϵ = 1 between locations

Depth-first search

Depth-first search



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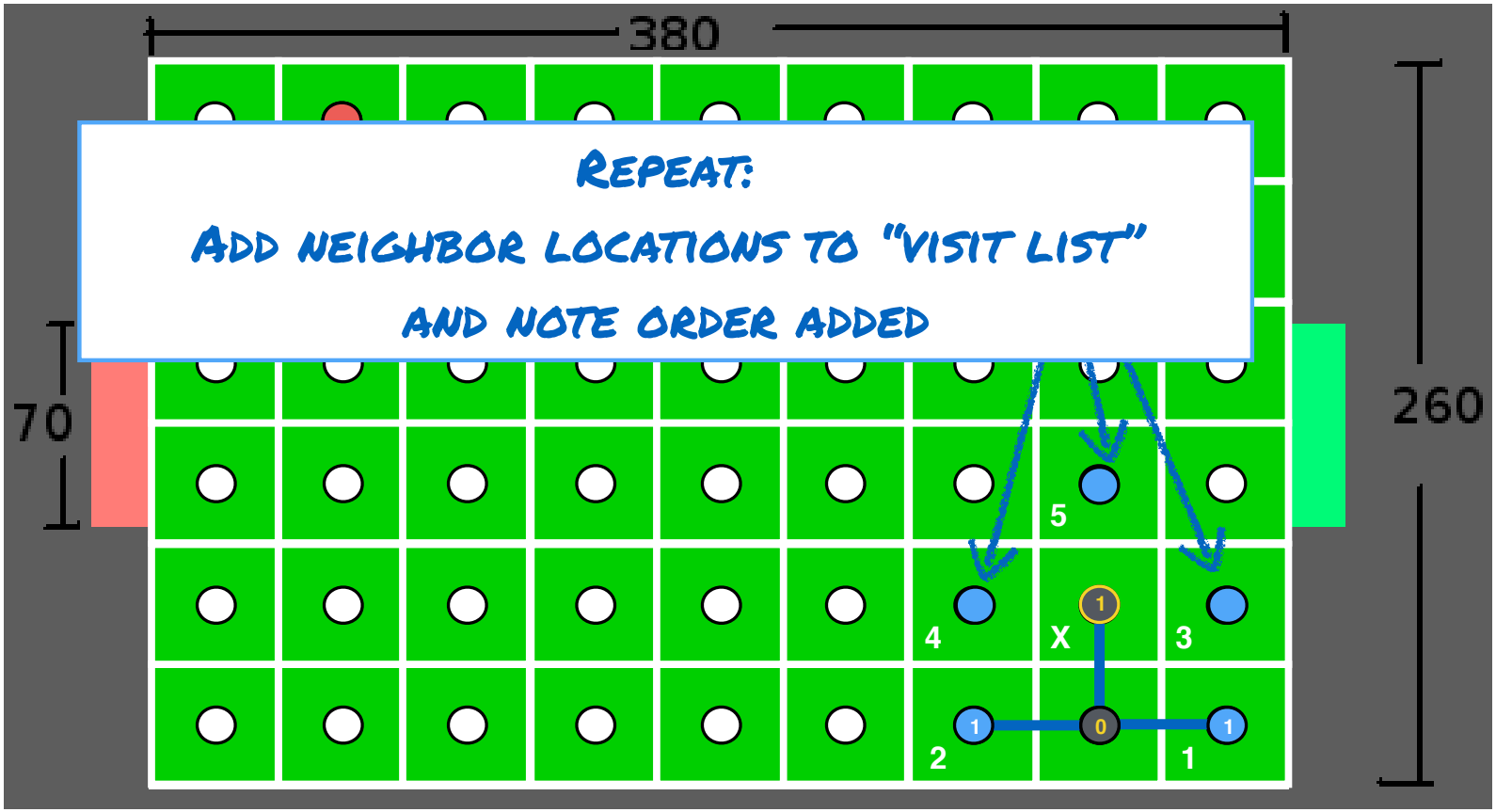

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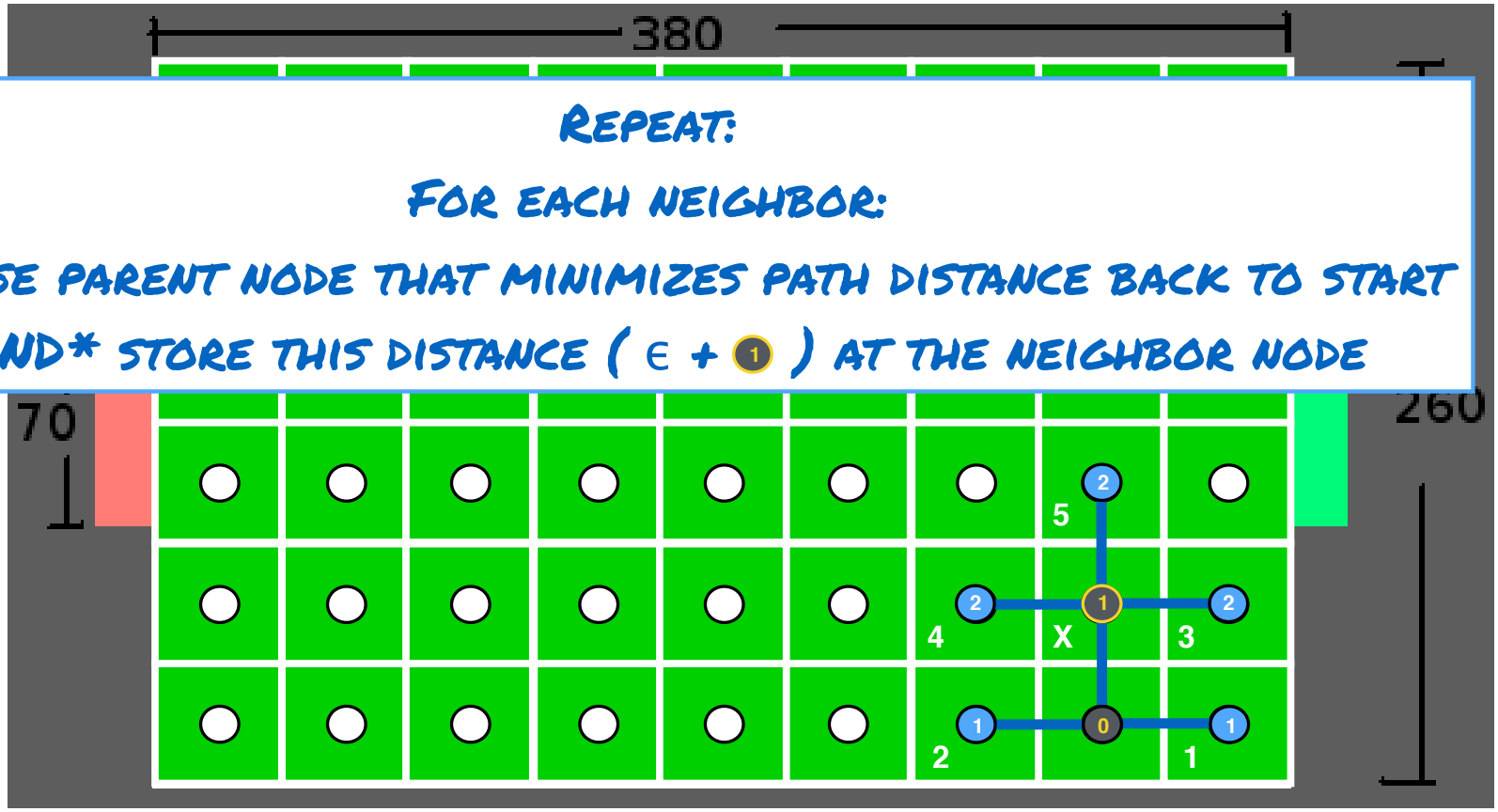

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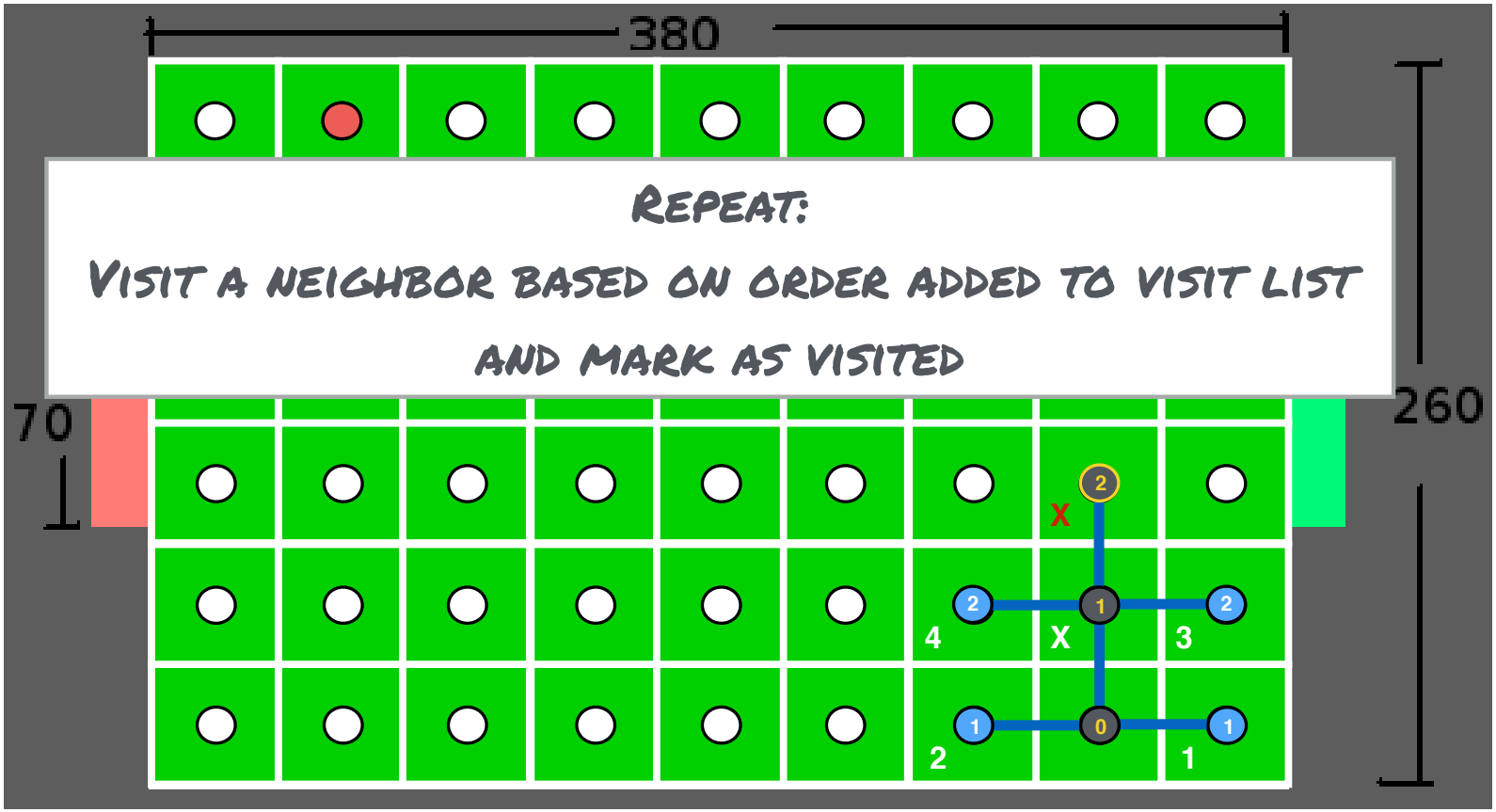

Depth-first search

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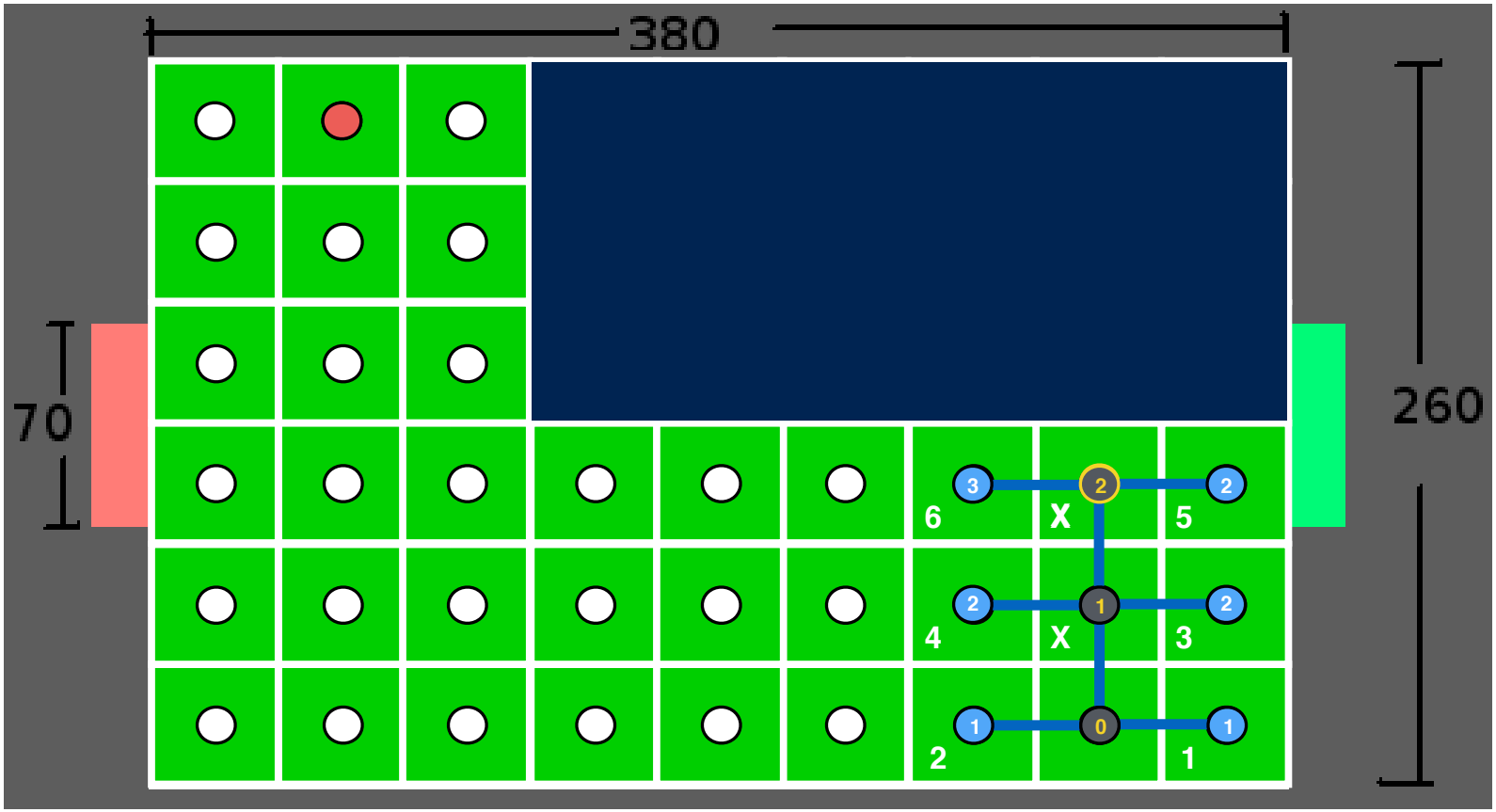

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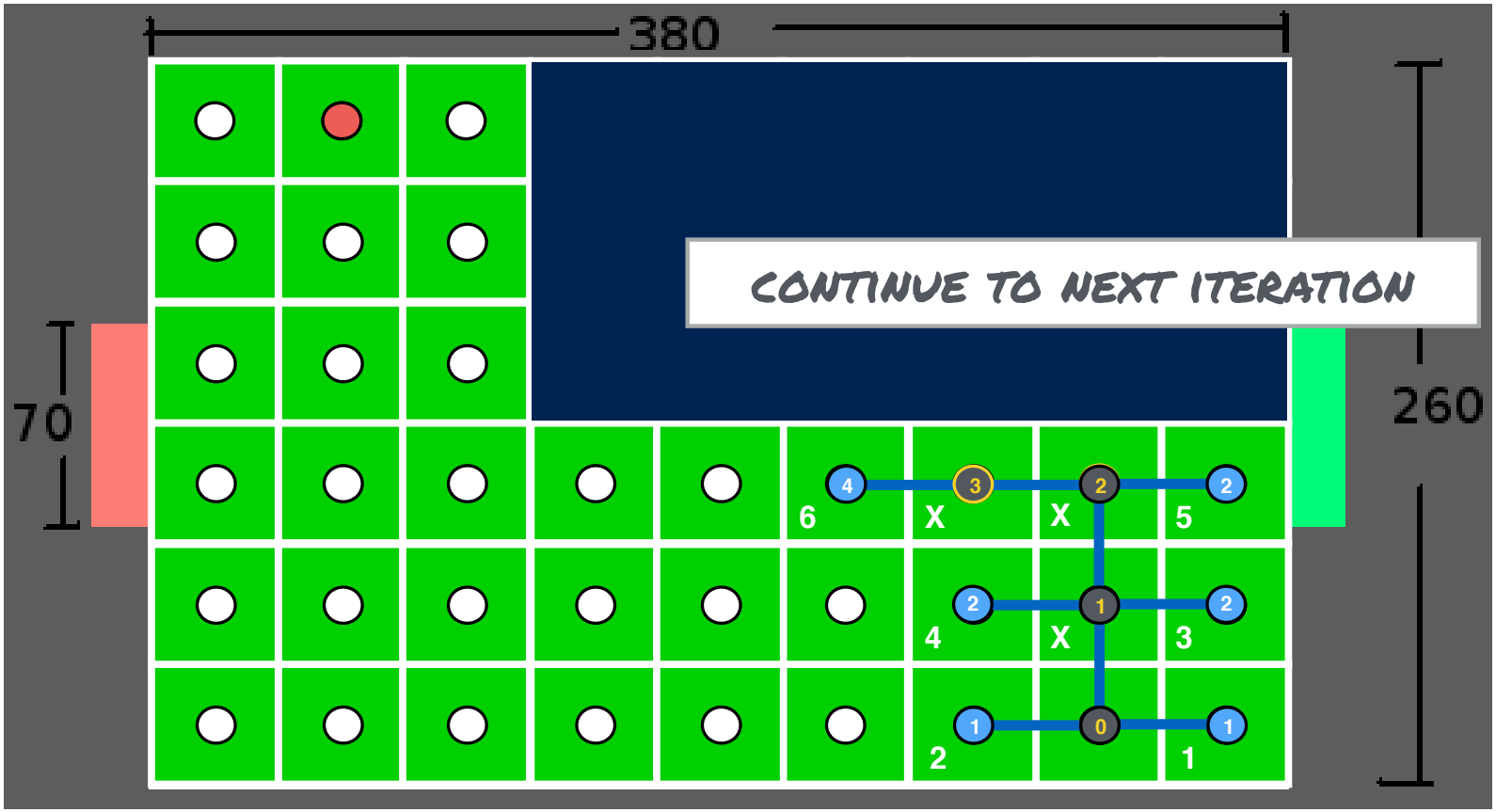

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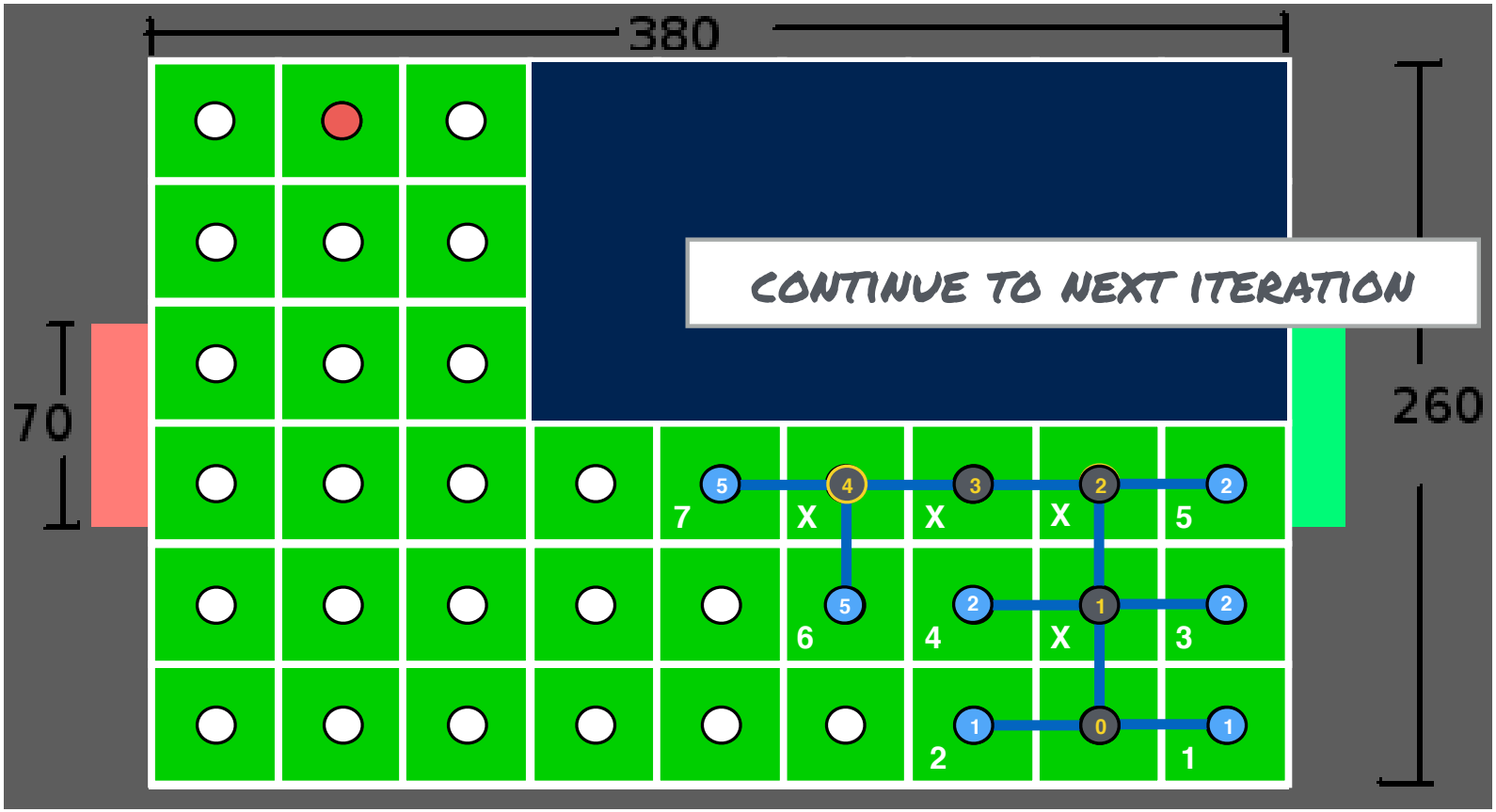

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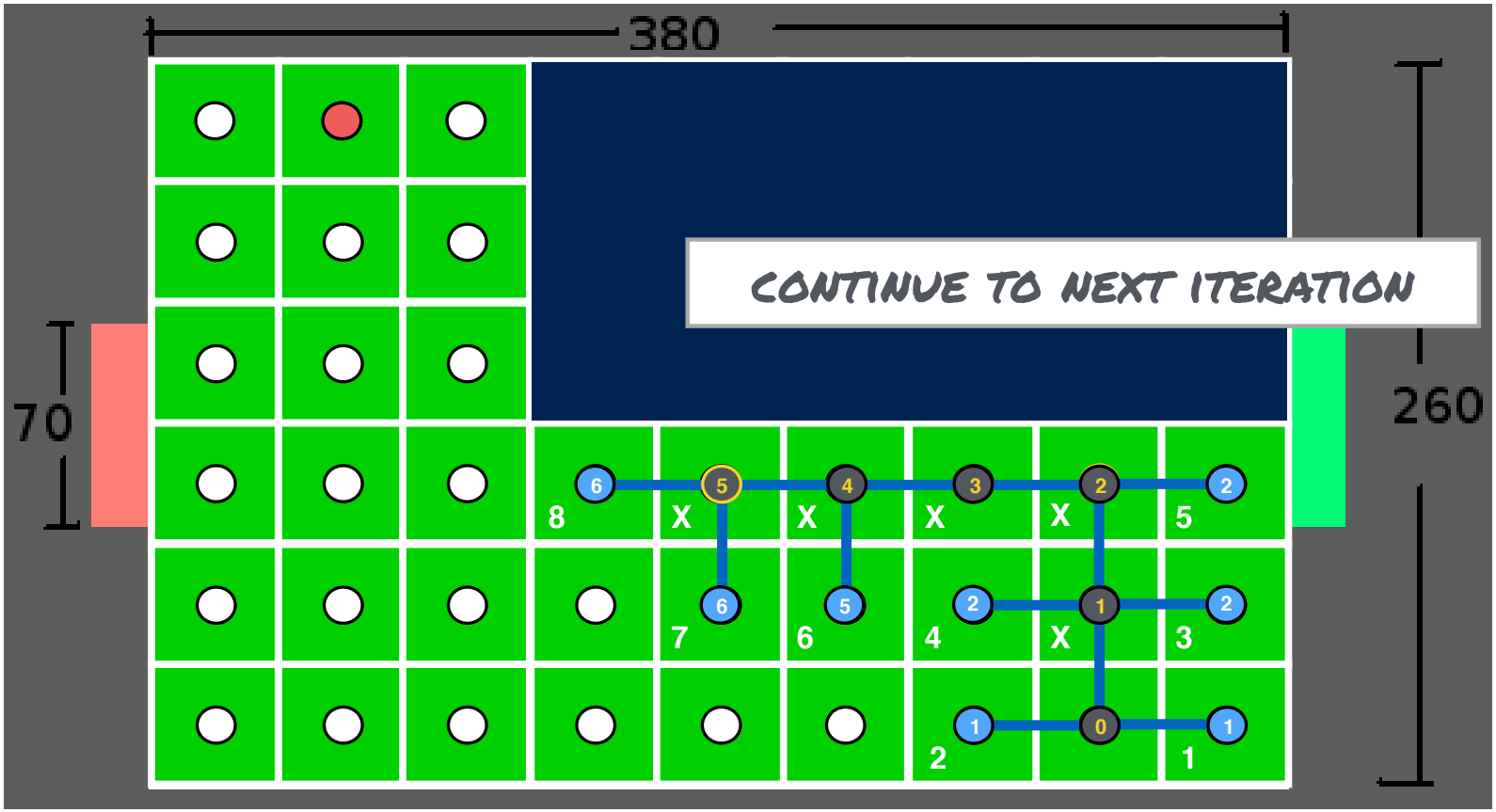

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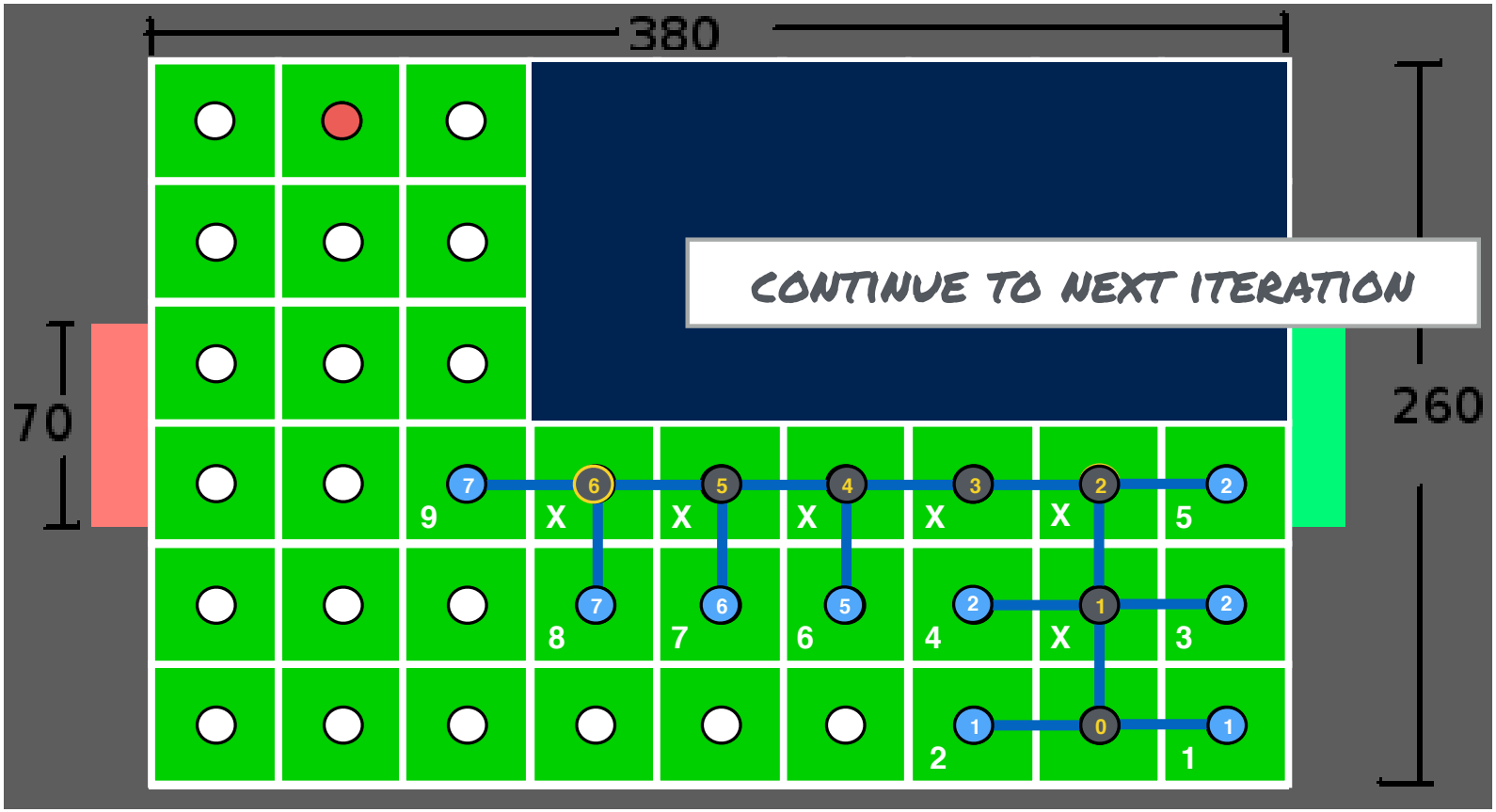

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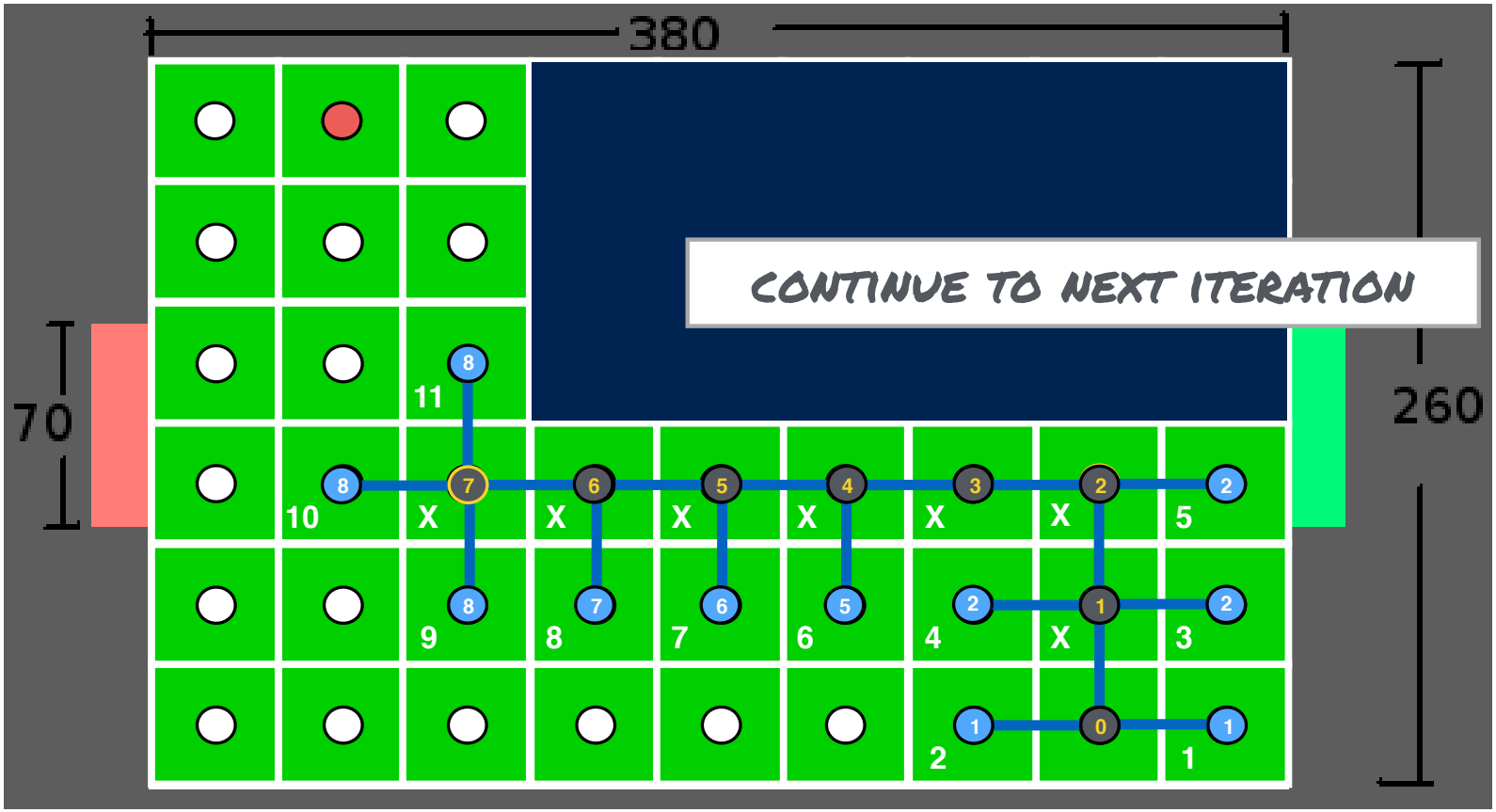

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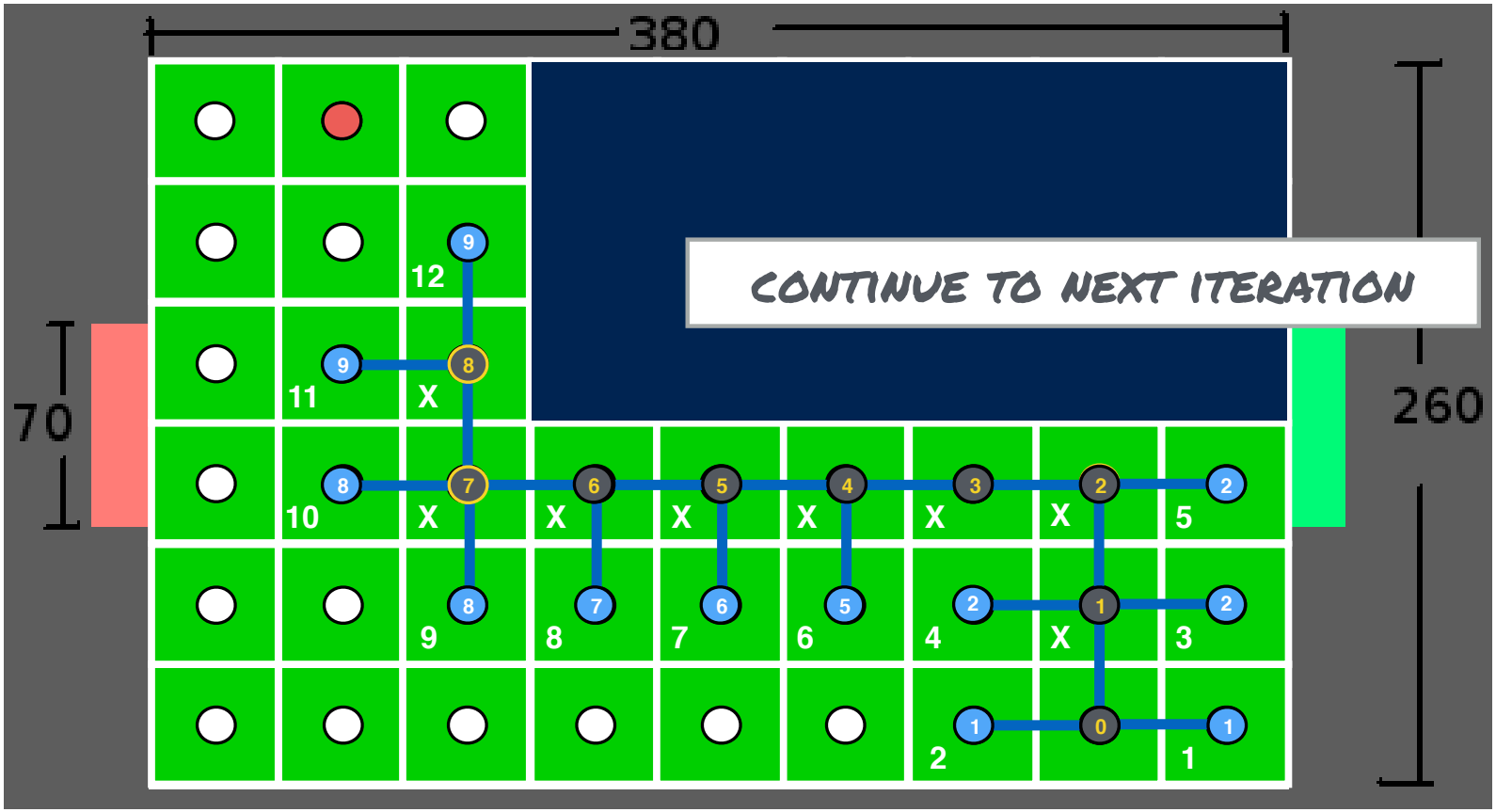

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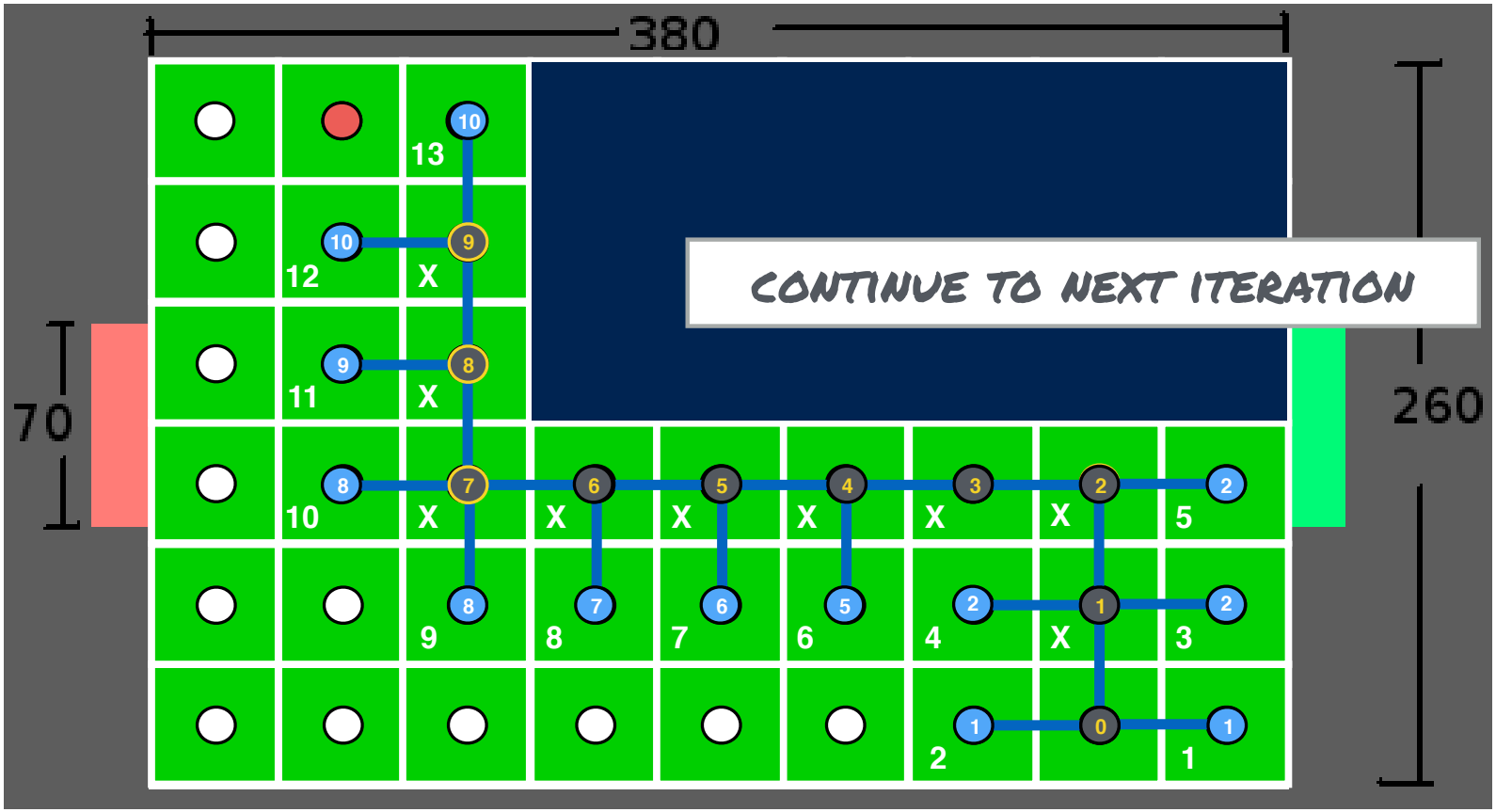

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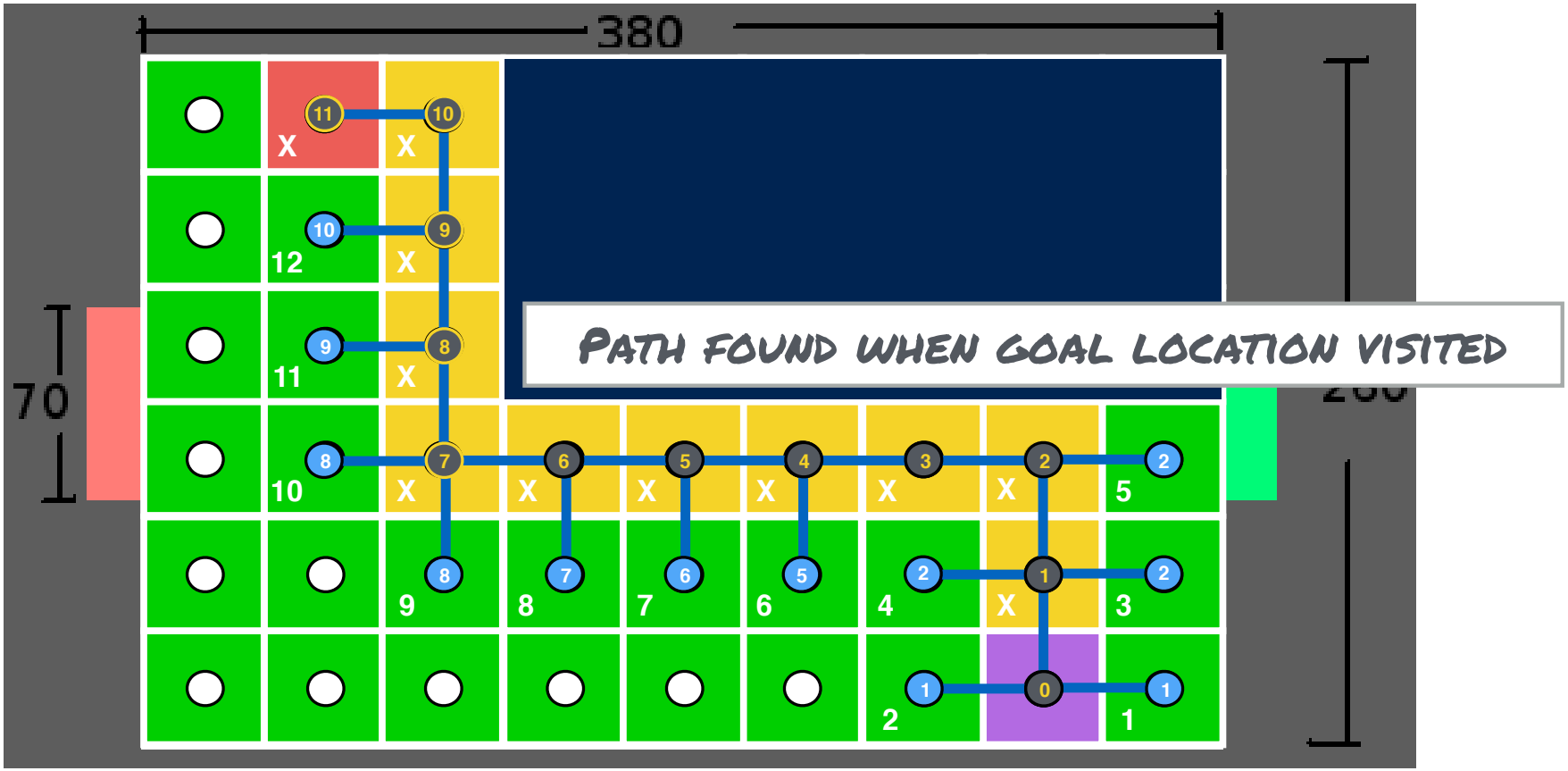
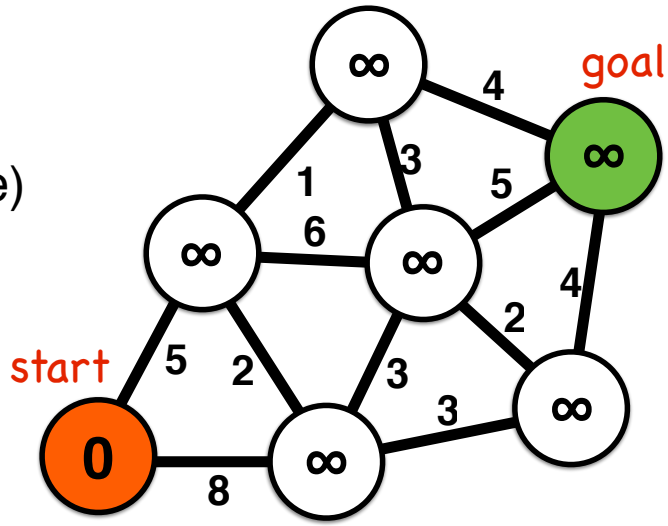


Let’s turn this idea into code

Search algorithm template

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distStraightLine(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distStraightLine(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



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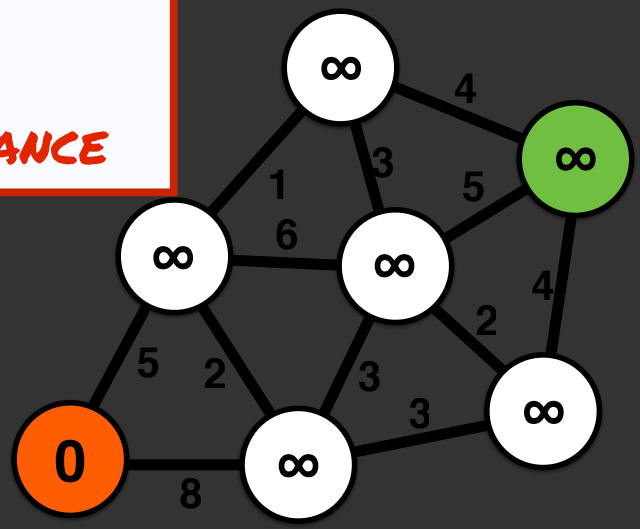
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_list ← start_node

INITIALIZATION

- EACH NODE HAS A DISTANCE AND A PARENT
  - DISTANCE: DISTANCE ALONG ROUTE FROM START
  - PARENT: ROUTING FROM NODE TO START
- VISIT A CHOSEN START NODE FIRST
- ALL OTHER NODES ARE UNVISITED AND HAVE HIGH DISTANCE

end if
end for loop
end while loop
output ← parent, distance



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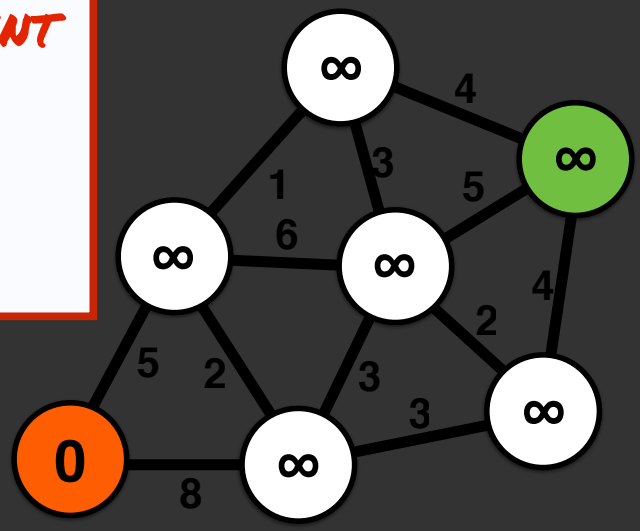
while visit_list != empty && current_node != goal
cur_node ← highestPriority(visit_list)
$visited_{cur\_node}$ ← true

end for loop
end while loop
output ← parent, distance

MAIN LOOP

- VISITS EVERY NODE TO COMPUTE ITS DISTANCE AND PARENT
- AT EACH ITERATION:
  - SELECT THE NODE TO VISIT BASED ON ITS PRIORITY
  - REMOVE CURRENT NODE FROM VISIT_LIST



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                parent_nbr ← current_node
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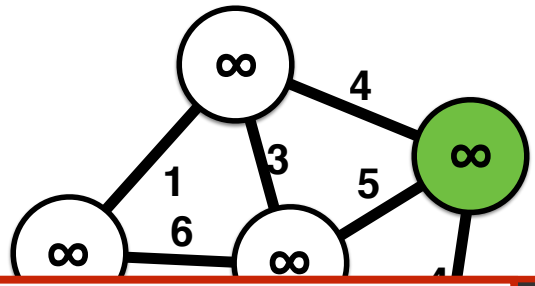
            end if
```



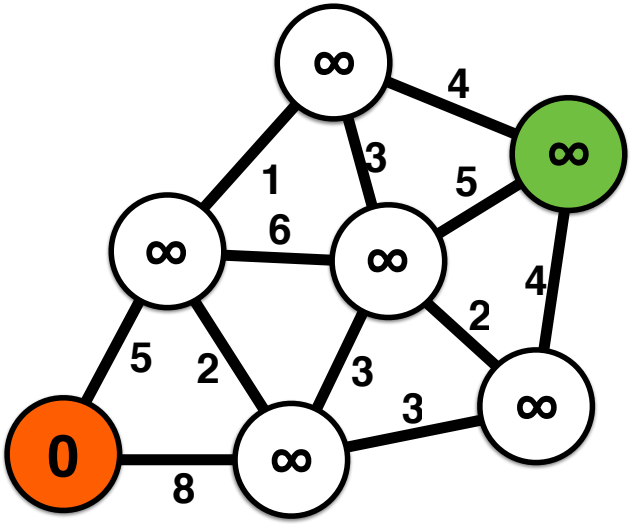
FOR EACH ITERATION ON A SINGLE NODE

- ADD ALL UNVISITED NEIGHBORS OF THE NODE TO THE VISIT LIST
- ADD THE NODE AS A PARENT TO A NEIGHBOR, IF IT CREATES A SHORTER ROUTE

Search algorithm template

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance
```

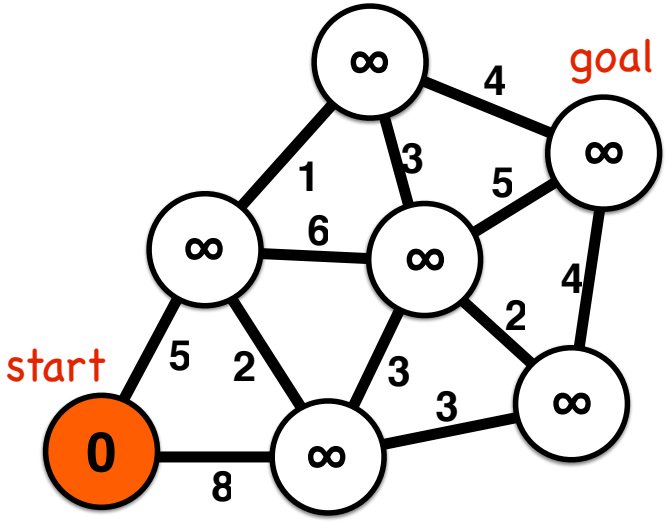
OUTPUT THE RESULTING ROUTES AND DISTANCES



Depth-first search

Search algorithm template

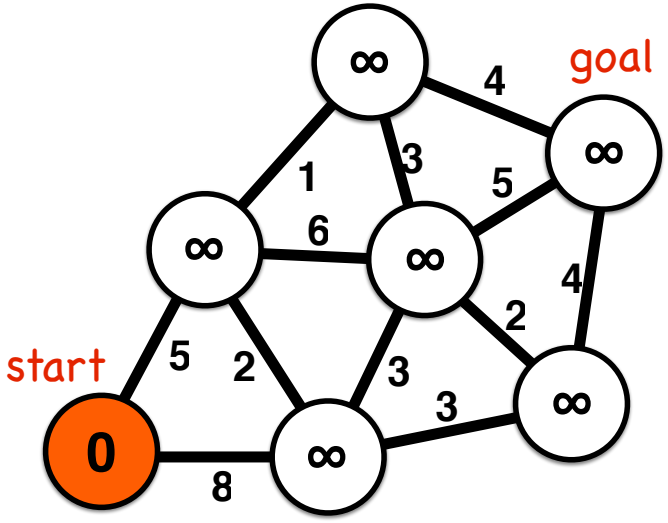
all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← **highestPriority**(visit_list)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



Depth-first search

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_stack ← start_node
    while **visit_stack** != empty && current_node != goal
        cur_node ← **pop**(**visit_stack**)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            push(nbr to **visit_stack**)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance

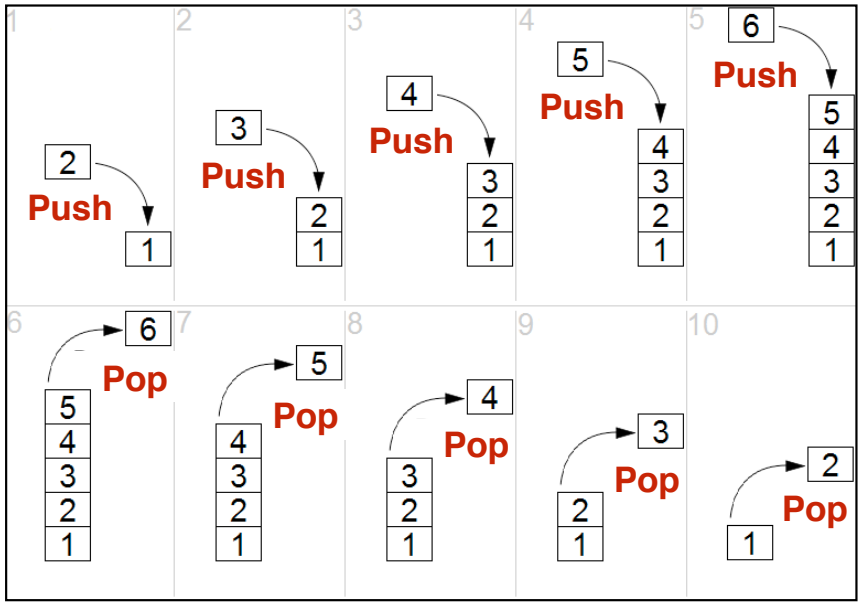
PRIORITY:
MOST RECENT



Stack data structure

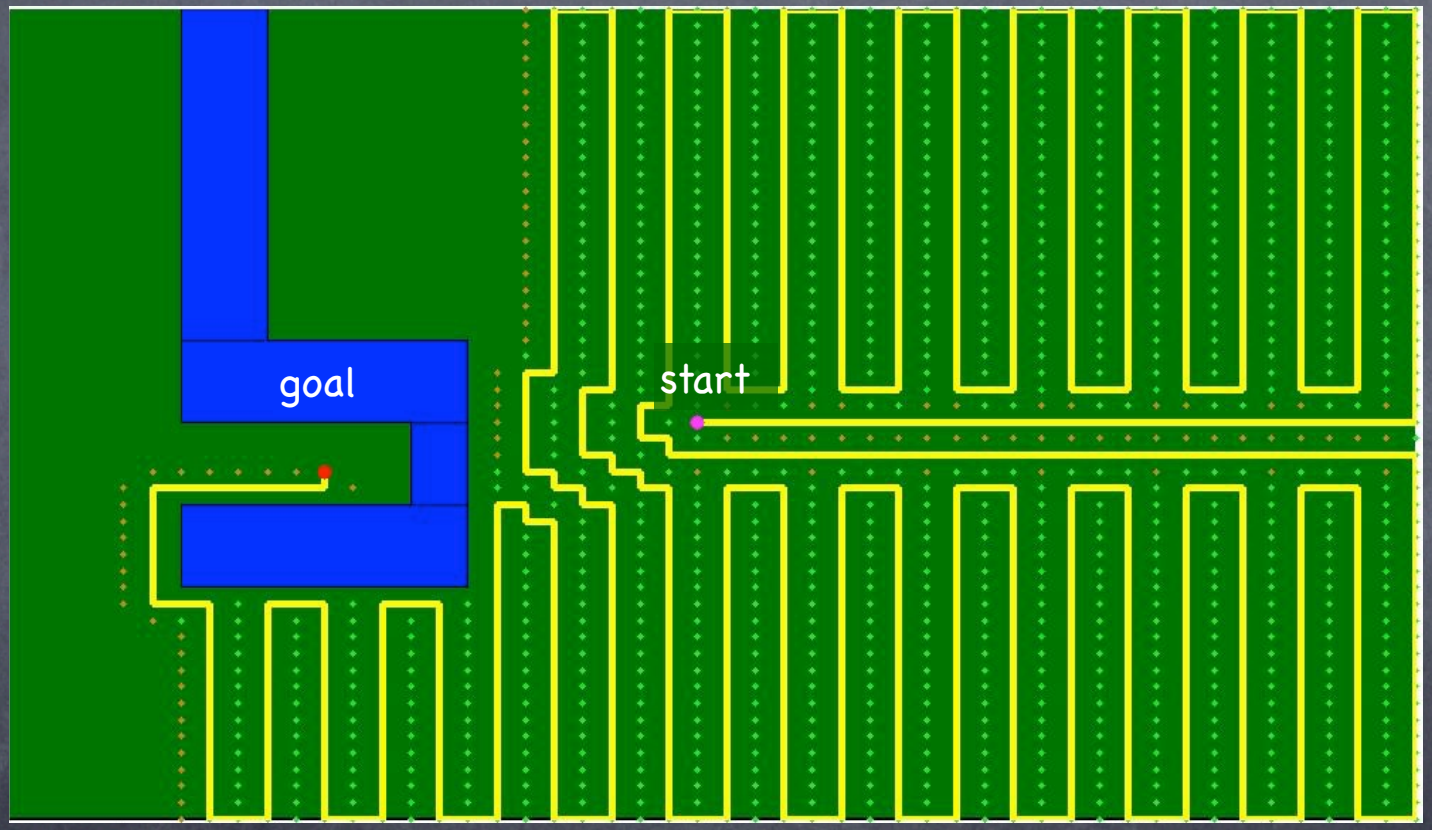
A stack is a “last in, first out” (or LIFO) structure, with two operations:
push: to add an element to the top of the stack
pop: to remove and element from the top of the stack

Stack example for reversing the order of six elements



matlab example: DFS

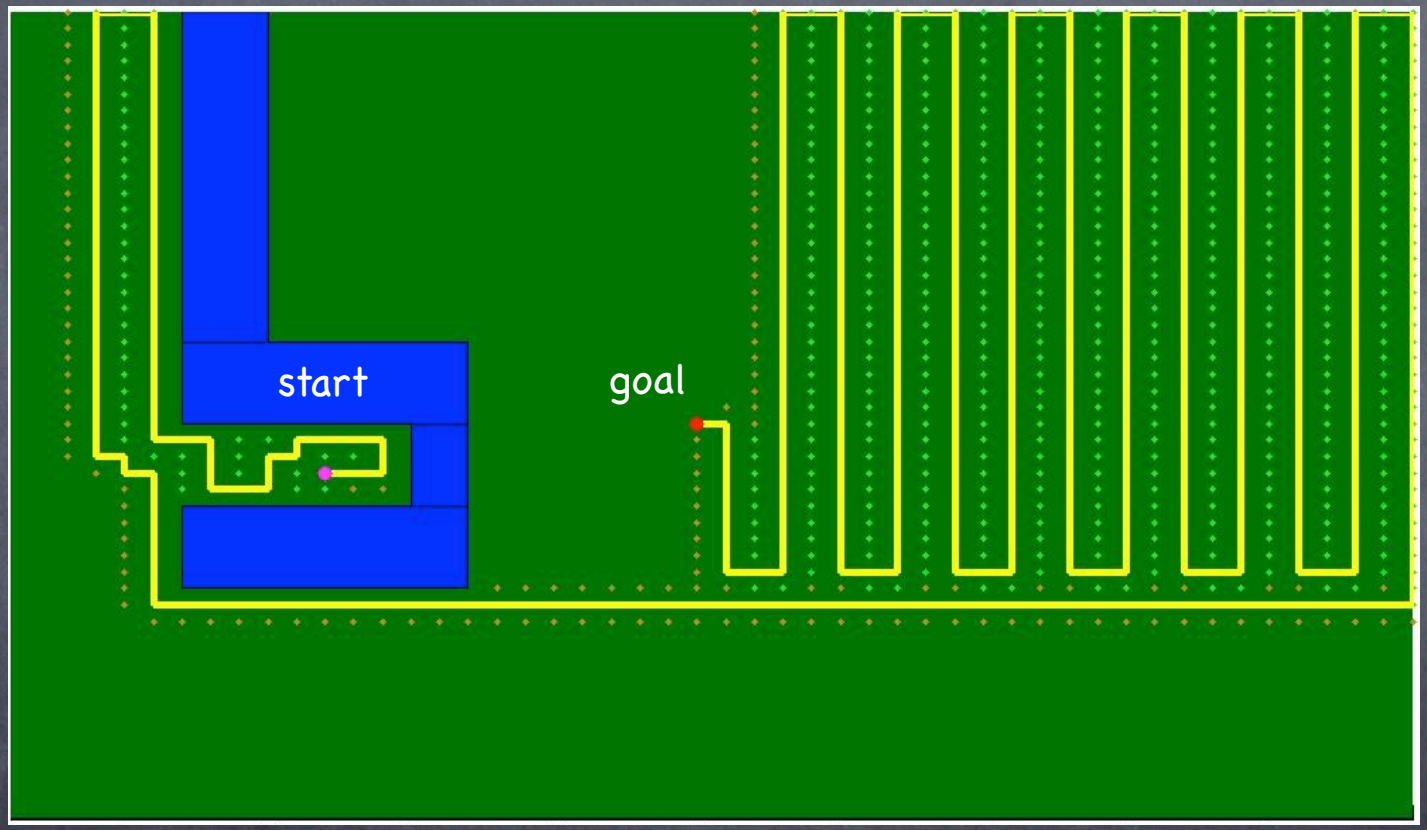
pathplan.m



visited locations in light green
4-connected grid, right visited first

matlab example: DFS

pathplan.m

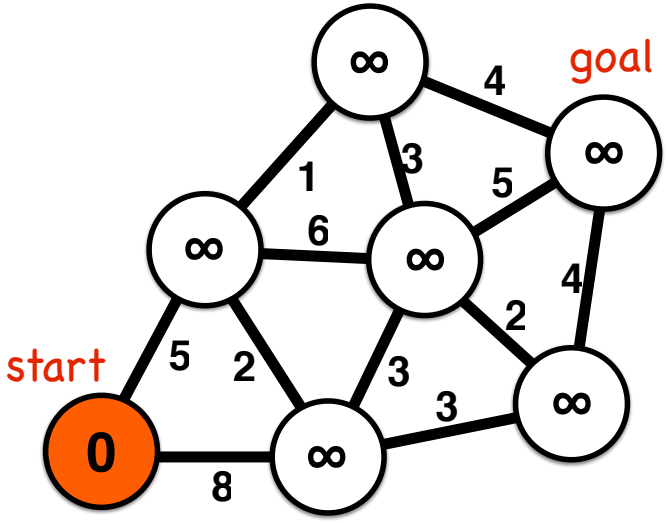


visited locations in light green
4-connected grid, right visited first

Breadth-first search

Search algorithm template

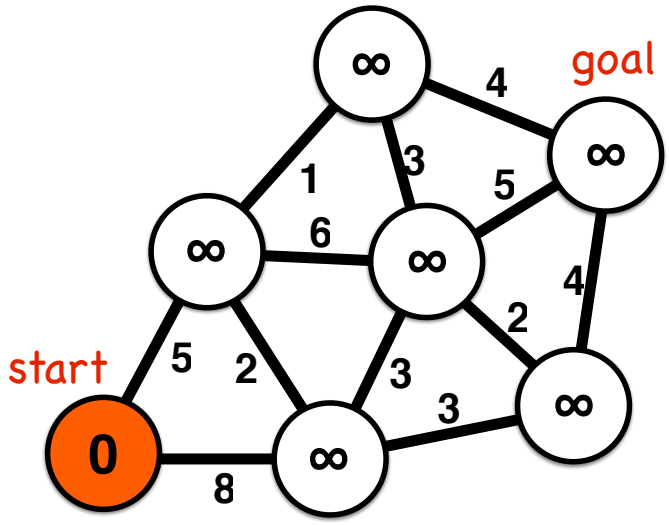
all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



Breadth-first search

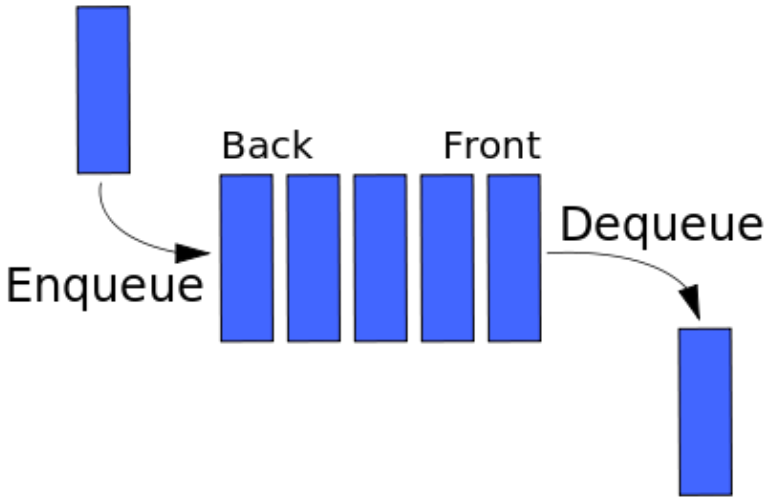
all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while **visit_queue** != empty && current_node != goal
        cur_node ← **dequeue**(**visit_queue**)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to **visit_queue**)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance

PRIORITY:
LEAST RECENT



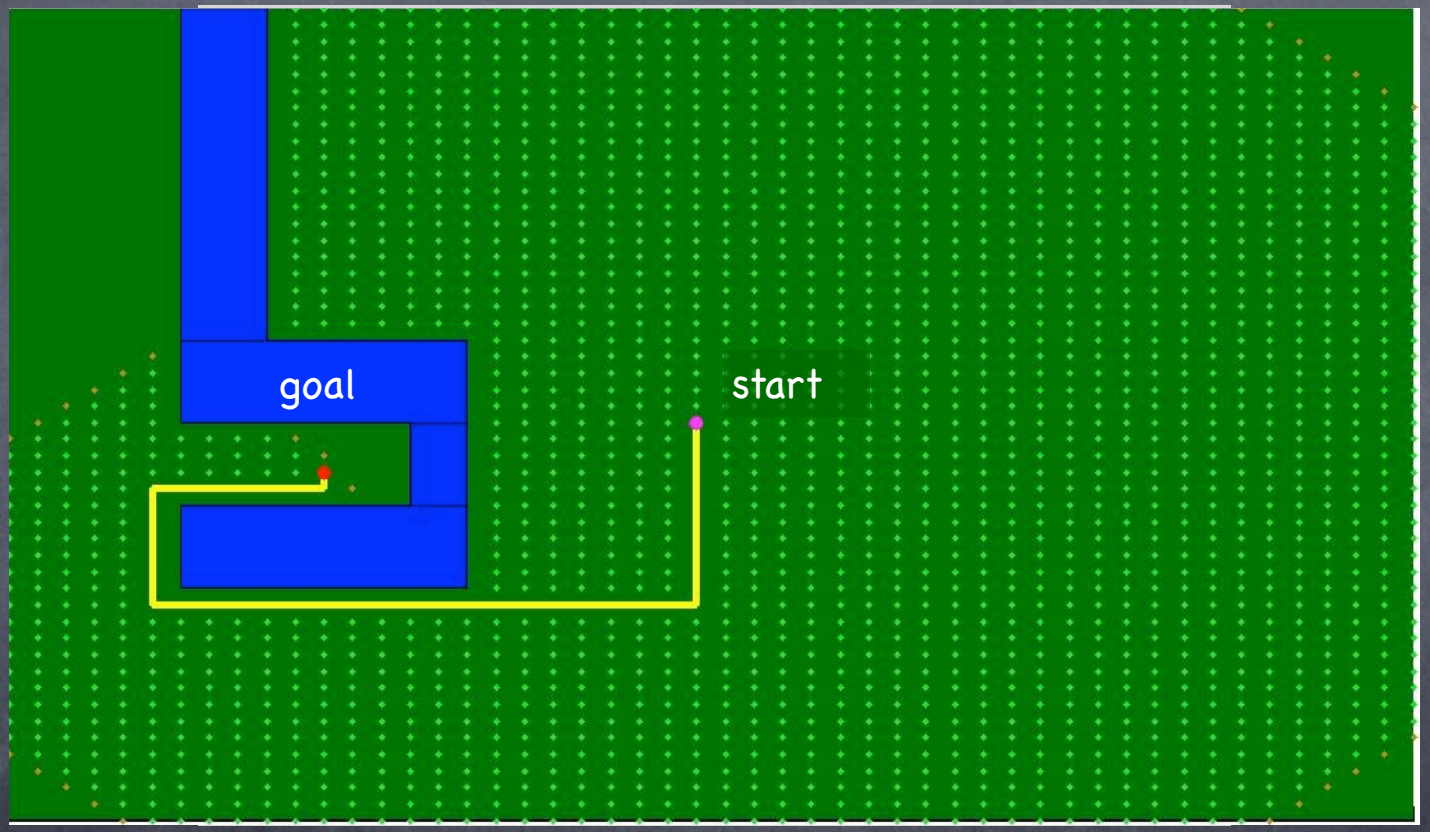
Queue data structure

A queue is a “first in, first out” (or FIFO) structure, with two operations
enqueue: to add an element to the back of the stack
dequeue: to remove an element from the front of the stack



matlab example: BFS

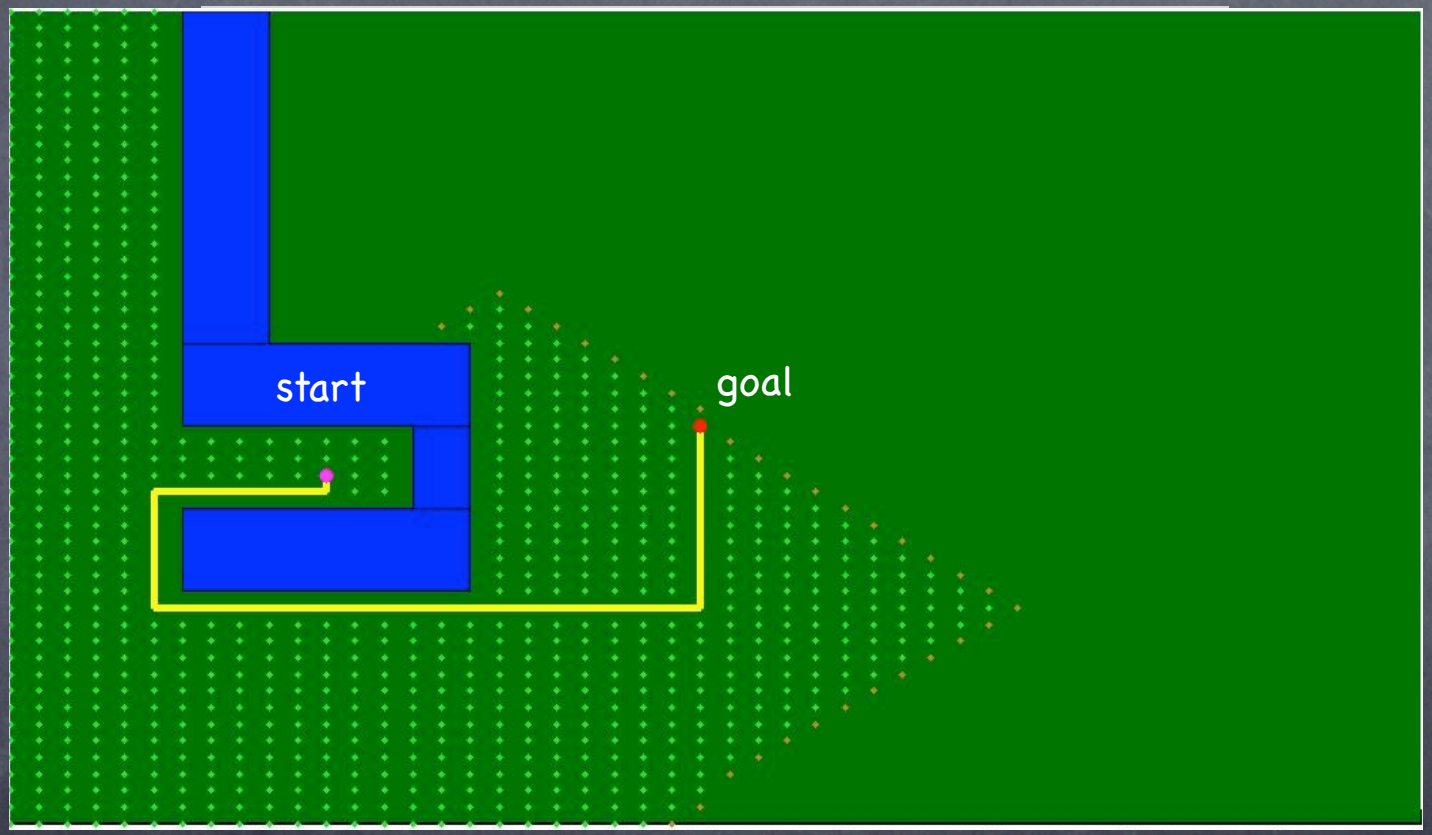
pathplan.m



visited locations in light green
4-connected grid, right visited first

matlab example: BFS

pathplan.m

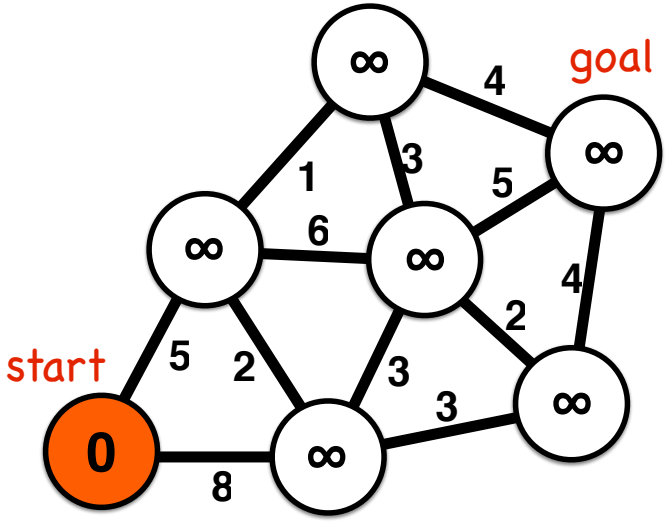


visited locations in light green
4-connected grid, right visited first

Dijkstra shortest path

Search algorithm template

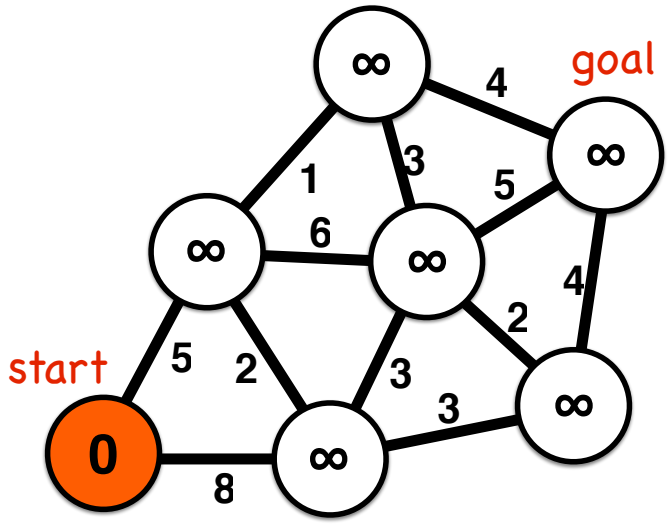
all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← **highestPriority**(visit_list)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while **visit_queue** != empty ~~**&& current_node != goal**~~
        cur_node ← **min_distance**(**visit_queue**)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to **visit_queue**)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance

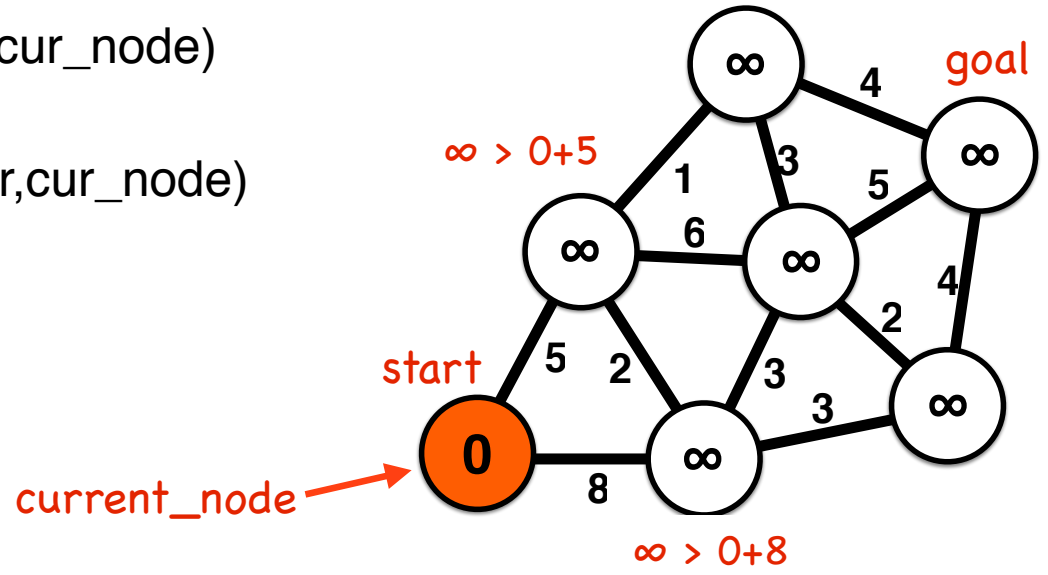
PRIORITY:
MINIMUM ROUTE DISTANCE
FROM START



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance

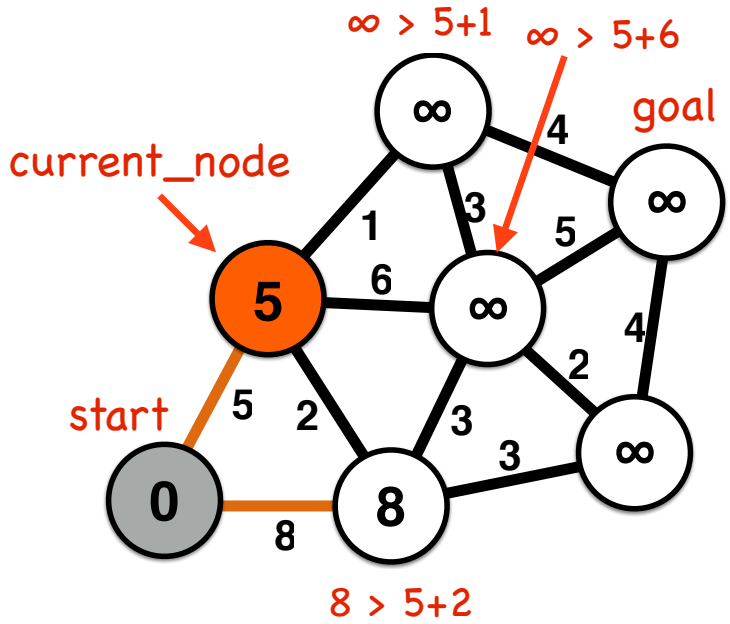
Dijkstra walkthrough



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance

Dijkstra walkthrough



Dijkstra shortest path algorithm

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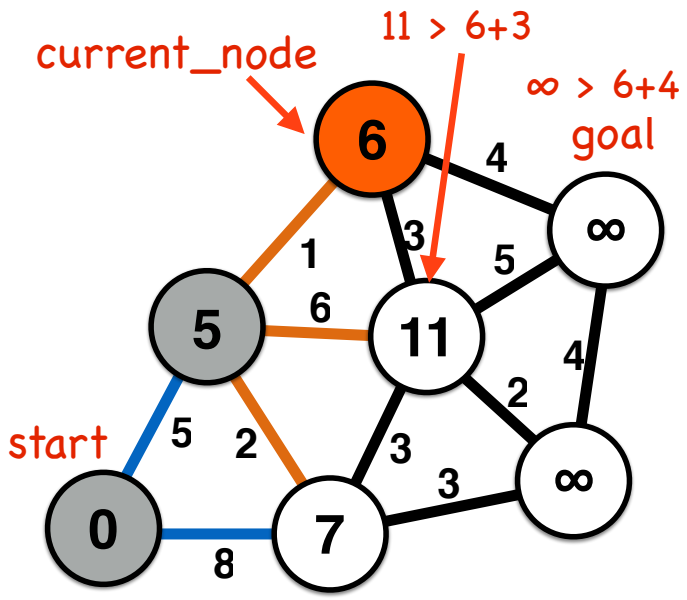
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance

Dijkstra walkthrough



Dijkstra shortest path algorithm

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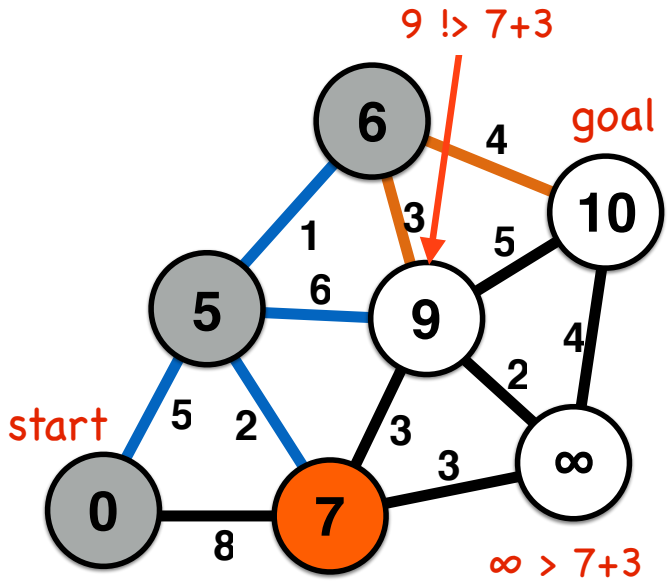
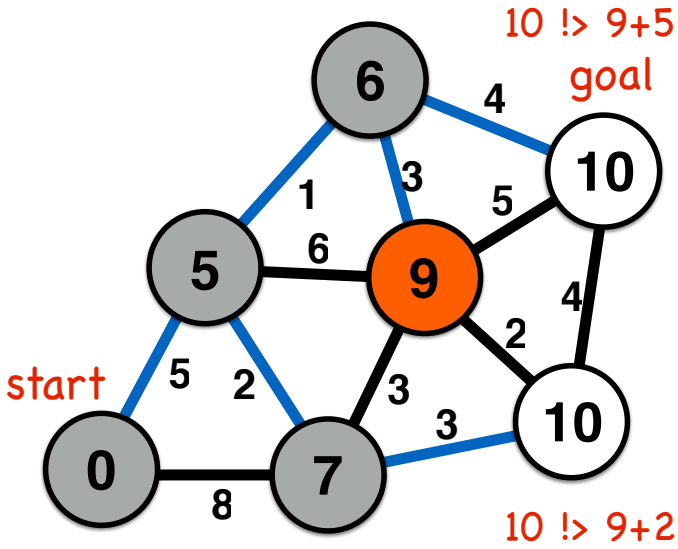
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



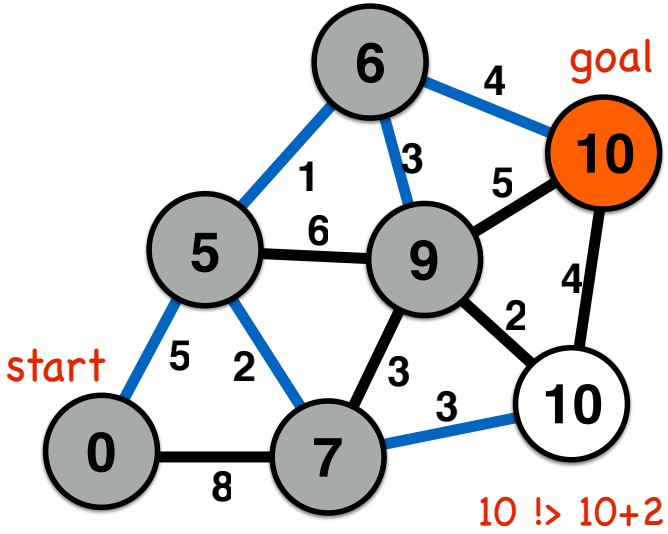
Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



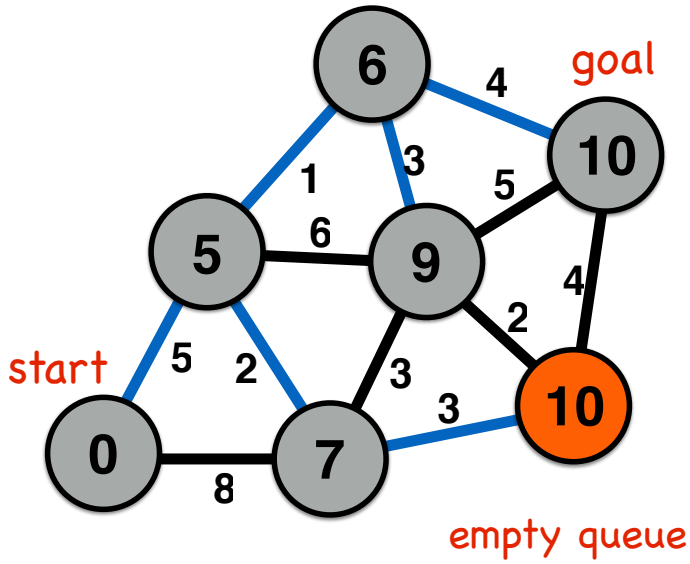
Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



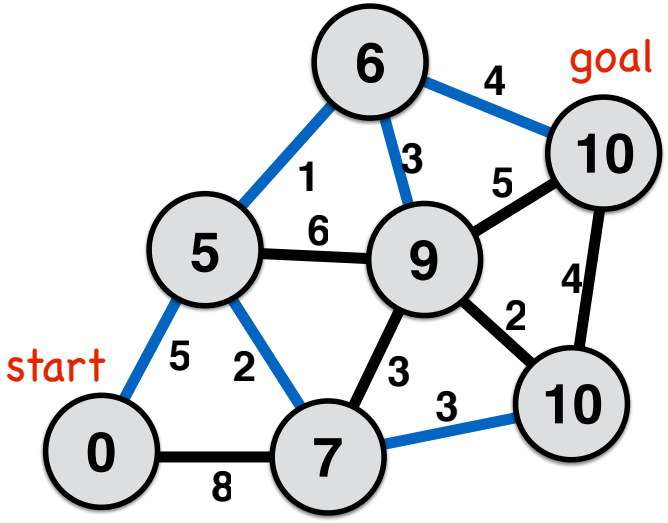
Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



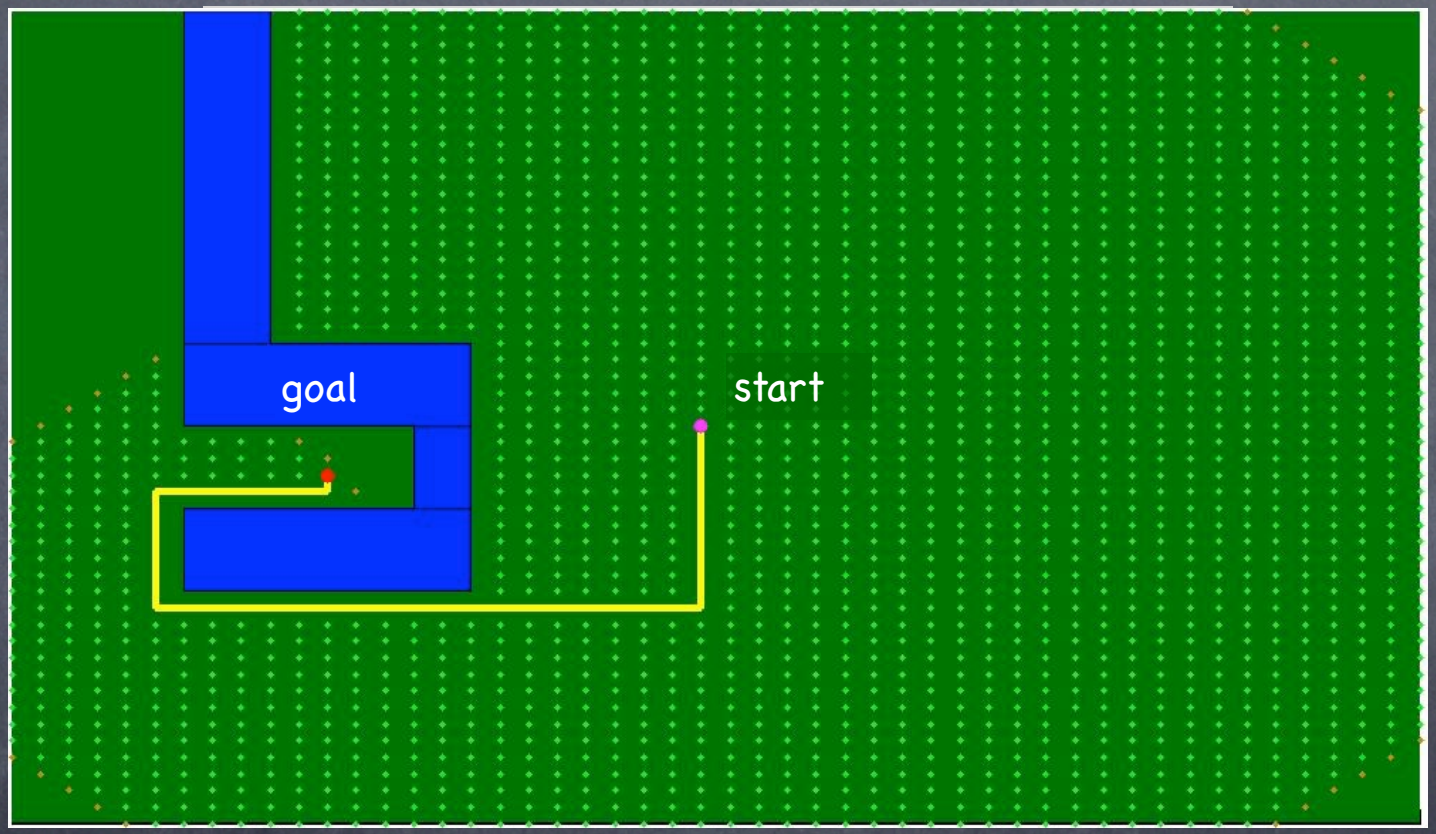
Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



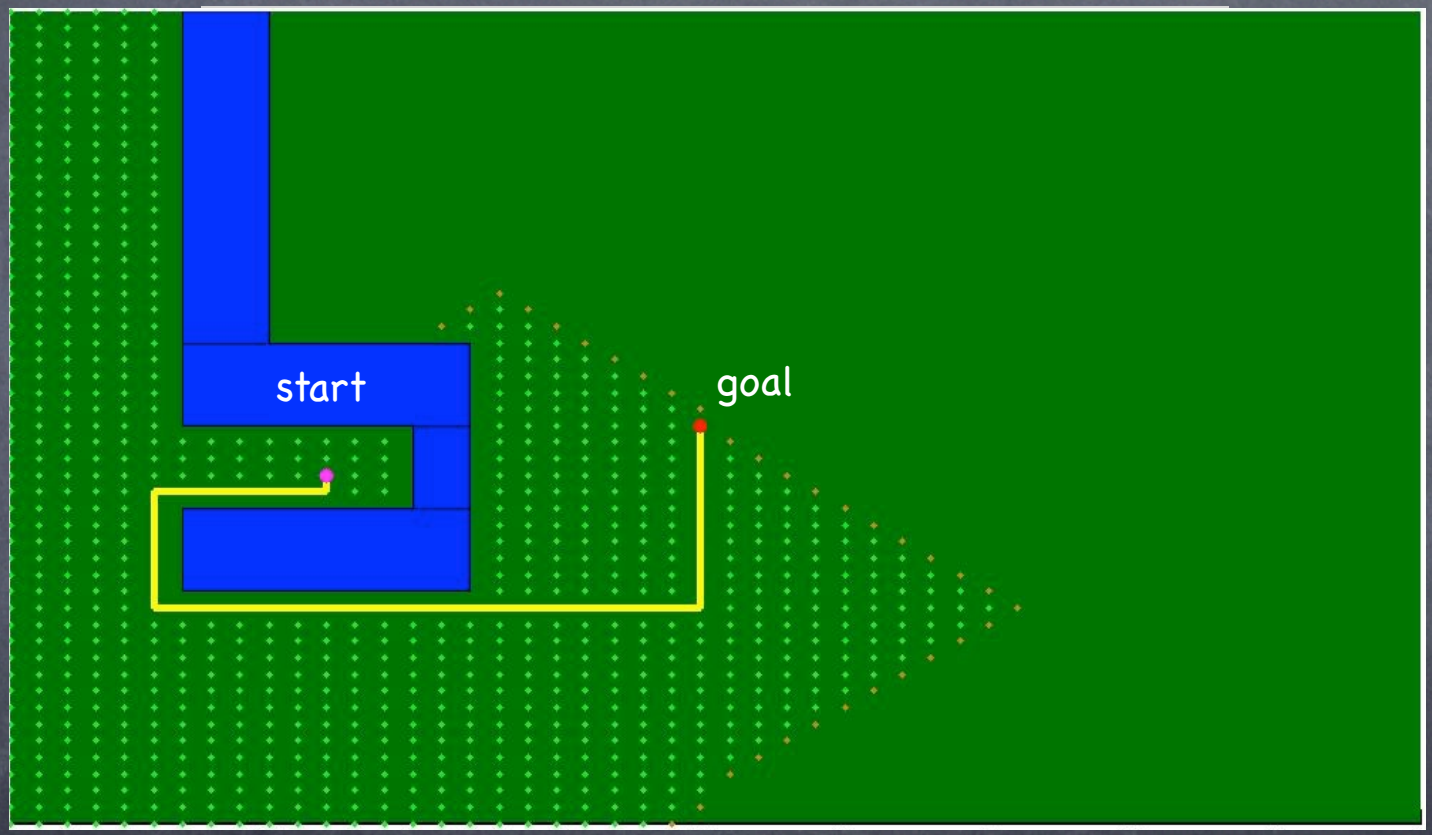
matlab example: Dijkstra

pathplan.m



matlab example: Dijkstra

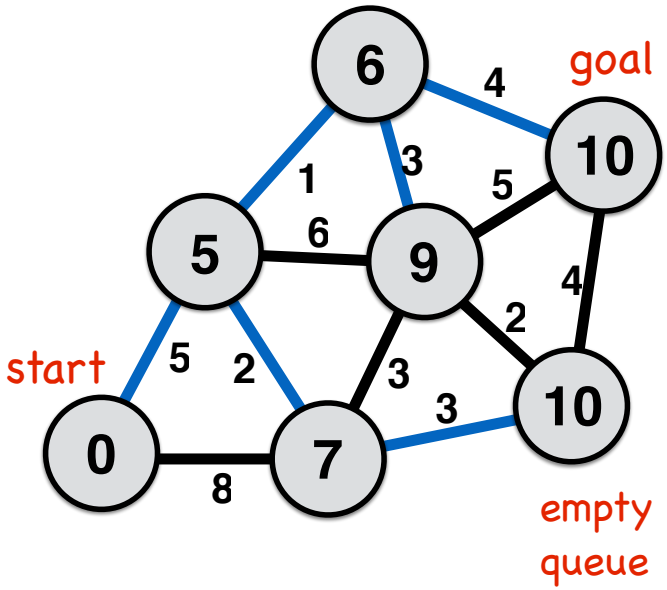
pathplan.m



A-star shortest path

Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance



A-star shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while (visit_queue != empty) && current_node != goal
        cur_node ← **dequeue(visit_queue, f_score)**
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
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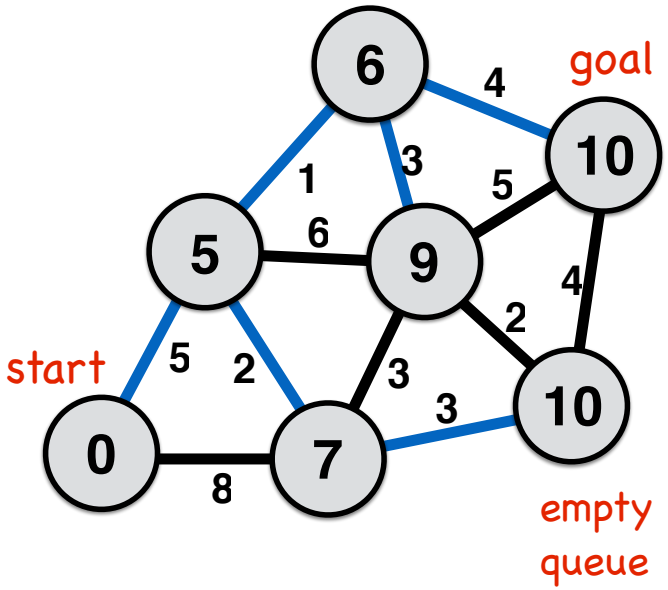
                f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$
            end if
        end for loop
    end while loop
output ← parent, distance



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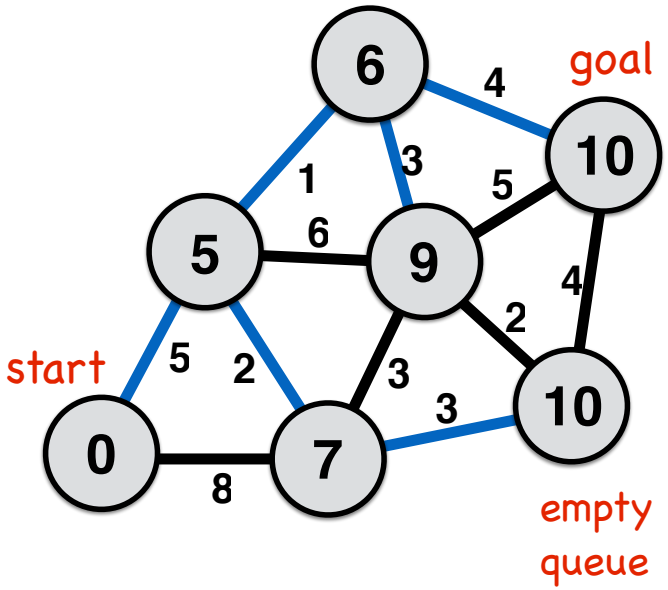
                f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$
            end if
        end for loop
    end while loop
output ← parent, distance

priority queue wrt. f_score
(implement min binary heap)

g_score: distance along current path back to start

h_score: best possible distance to goal



A-star shortest path algorithm

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while (visit_queue != empty) && current_node != goal
cur_node ← **dequeue(visit_queue, f_score)**

priority queue wrt. f_score
(implement min binary heap)

$visited_{cur\_node}$ ← true
for each nbr in not_visited(adjacent(cur_node))
enqueue(nbr to visit_queue)
if $dist_{nbr} > dist_{cur\_node}$ + distance(nbr,cur_node)
$parent_{nbr}$ ← current_node
$dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

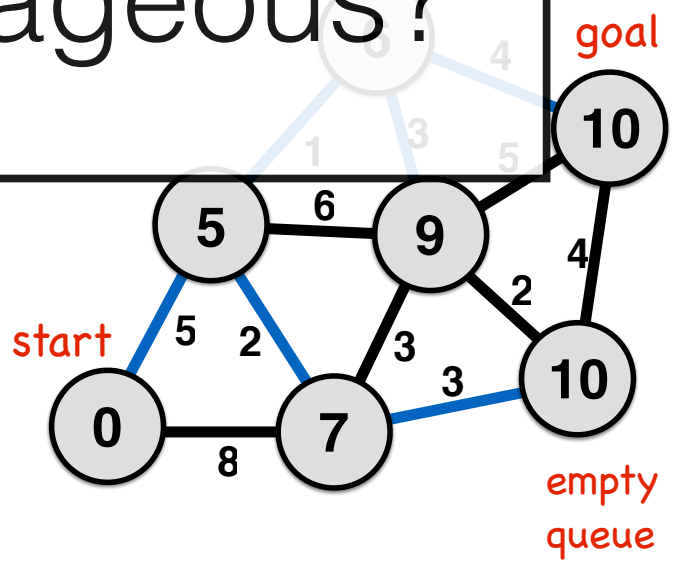
f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$

end if
end for loop
end while loop
output ← parent, distance

g_score: distance along current path back to start

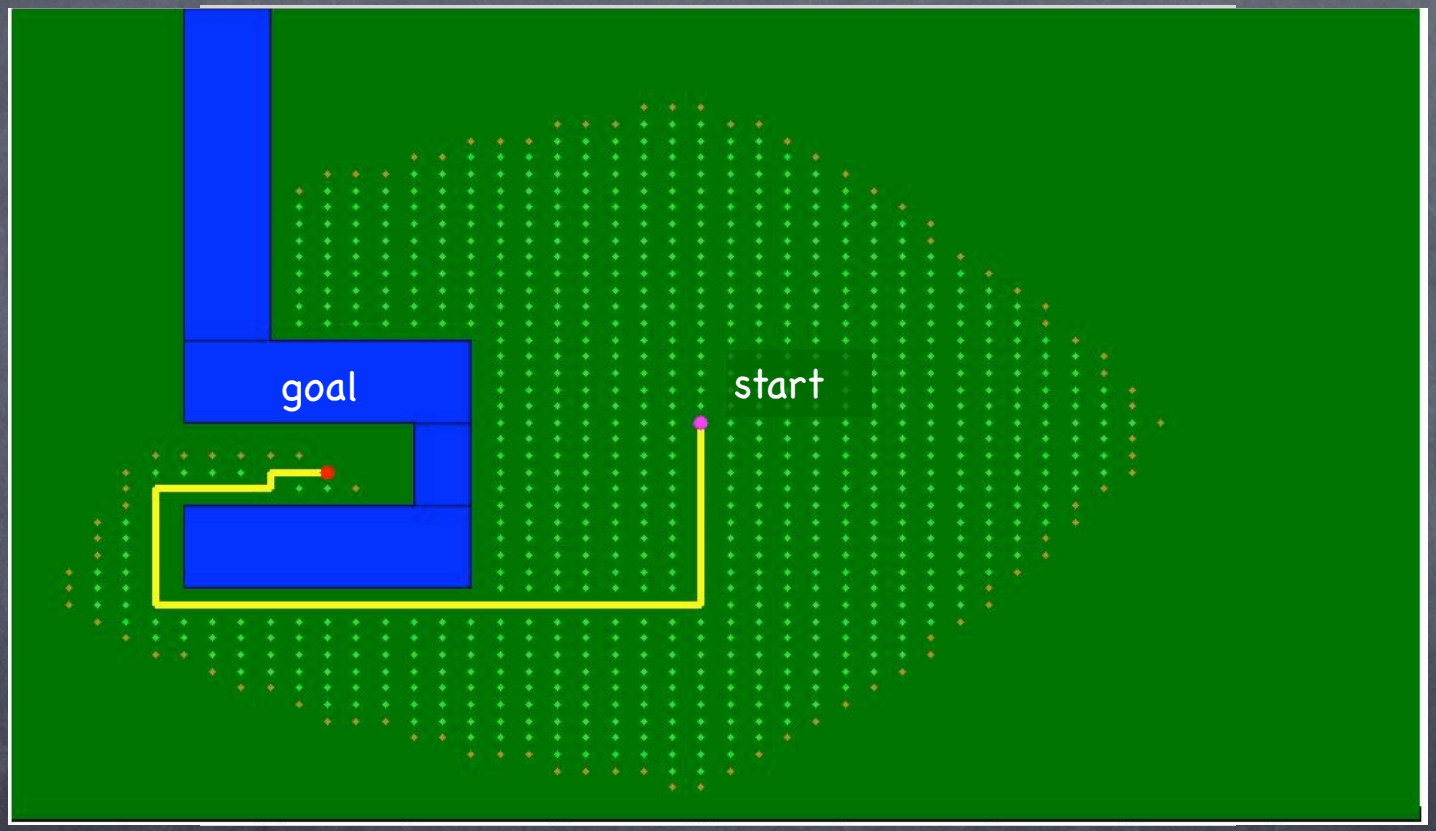
h_score: best possible distance to goal

Why is A-star advantageous?



matlab example: A*

pathplan.m



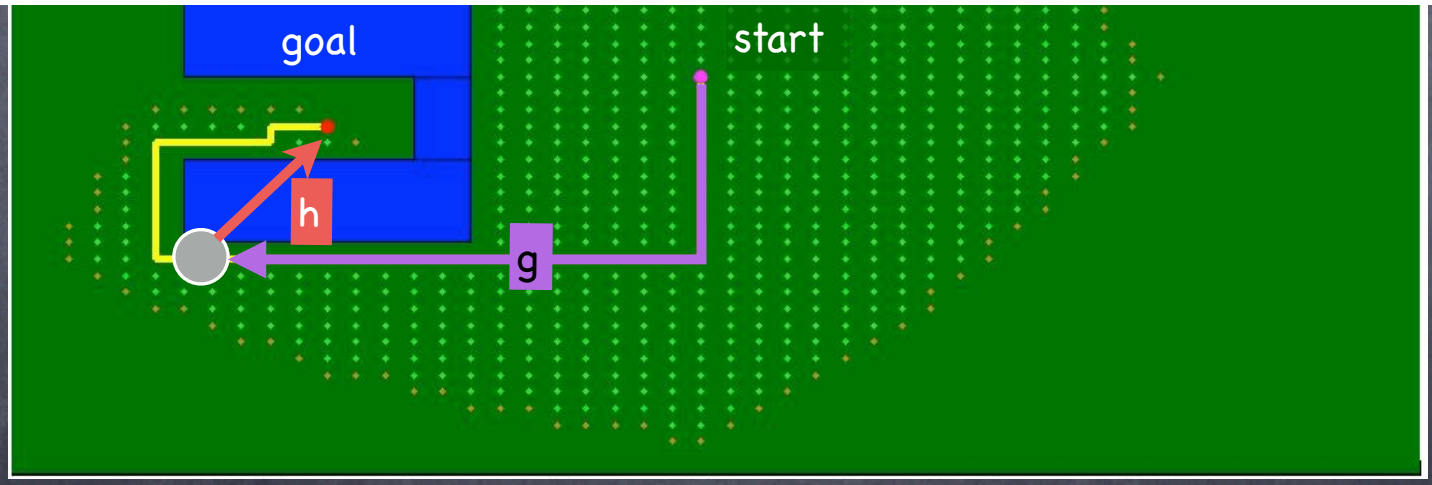
The straight line h_score is an admissible and consistent heuristic.

A heuristic function is **admissible** -> never overestimates the cost of reaching the goal.

Thus, h_score(x) <= the lowest possible cost from current location to the goal.

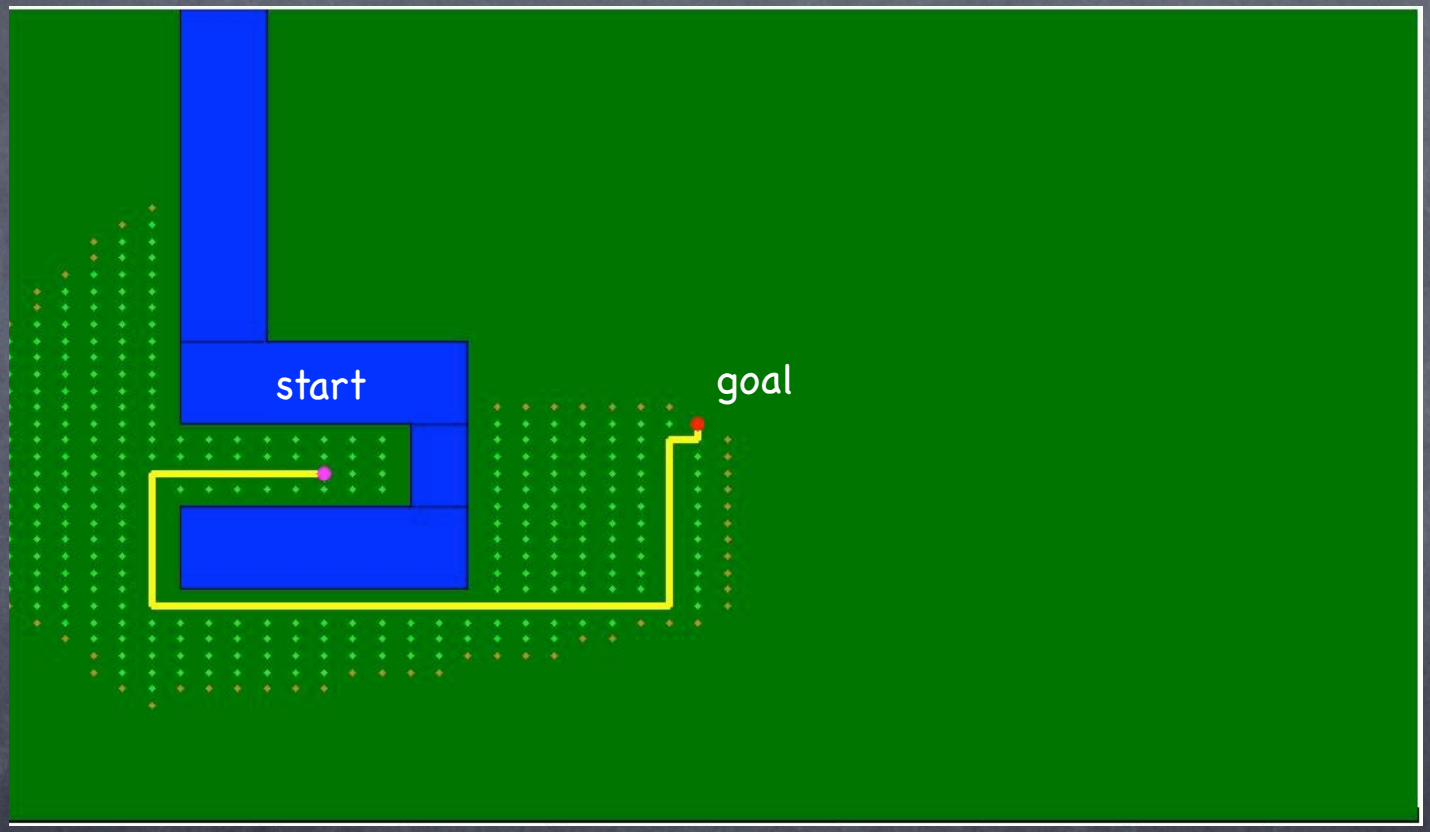
A heuristic function is **consistent** -> obeys the triangle inequality

Thus, the value of the h_score(x) <= cost(x,action,x') + h_score(x')

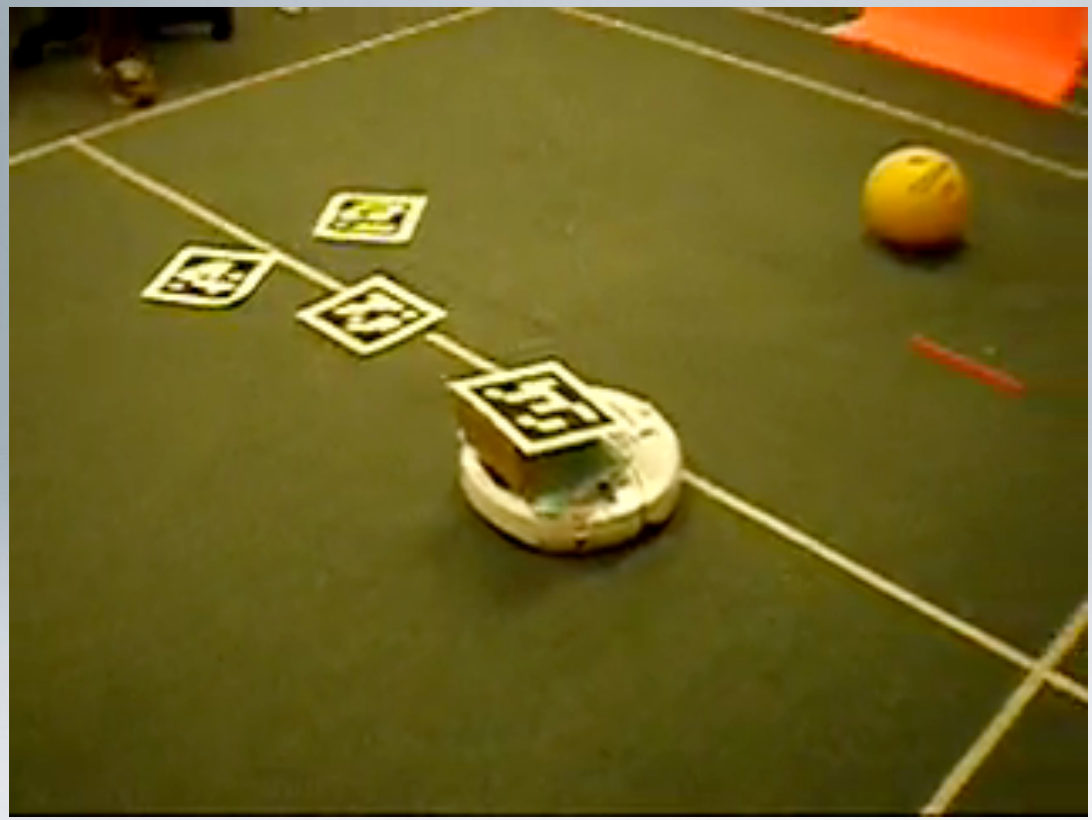


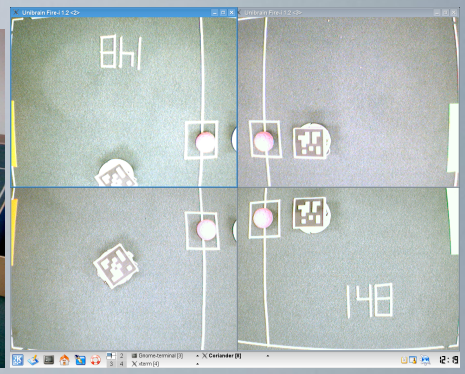
matlab example: A*

http://www.cs.brown.edu/courses/cs148/pub/pathplan.m



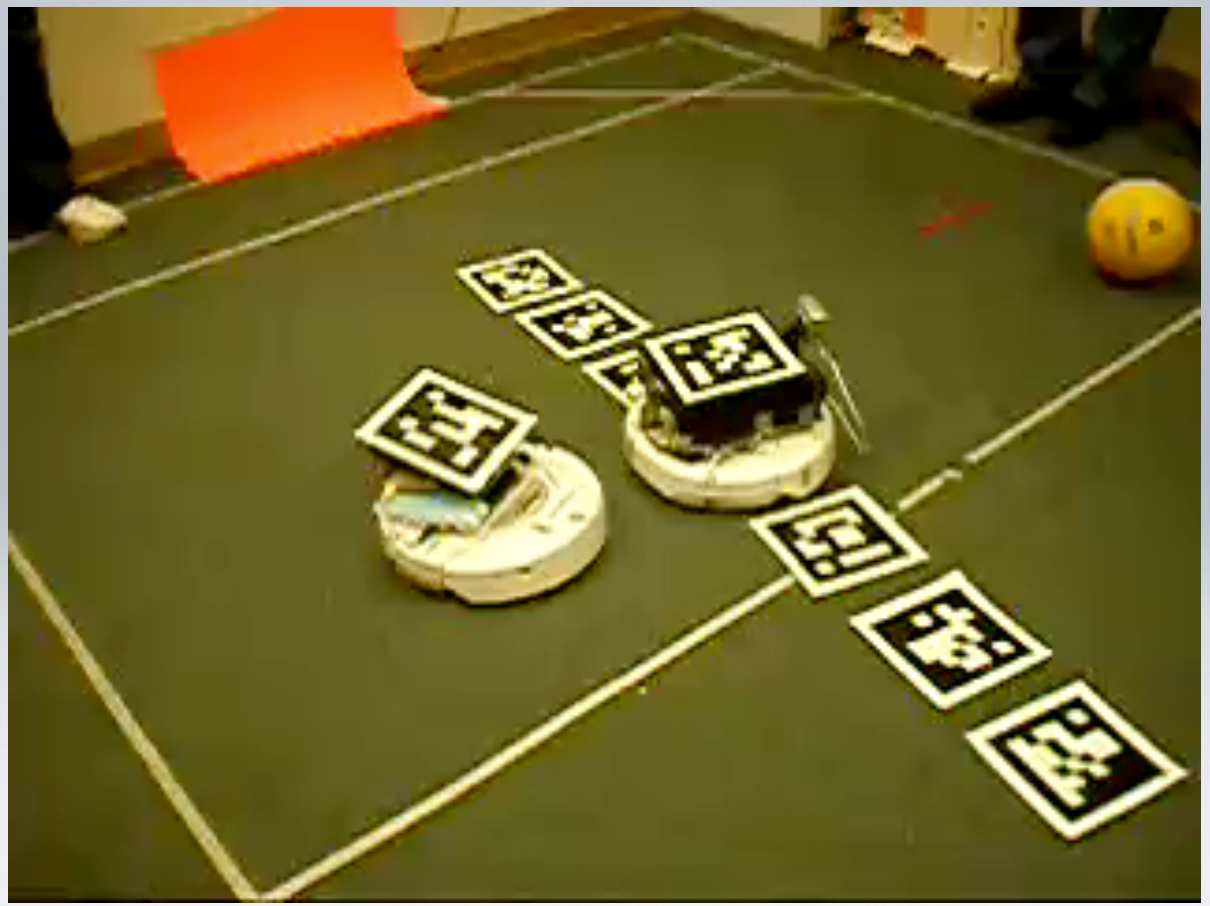
A* PATH PLANNING





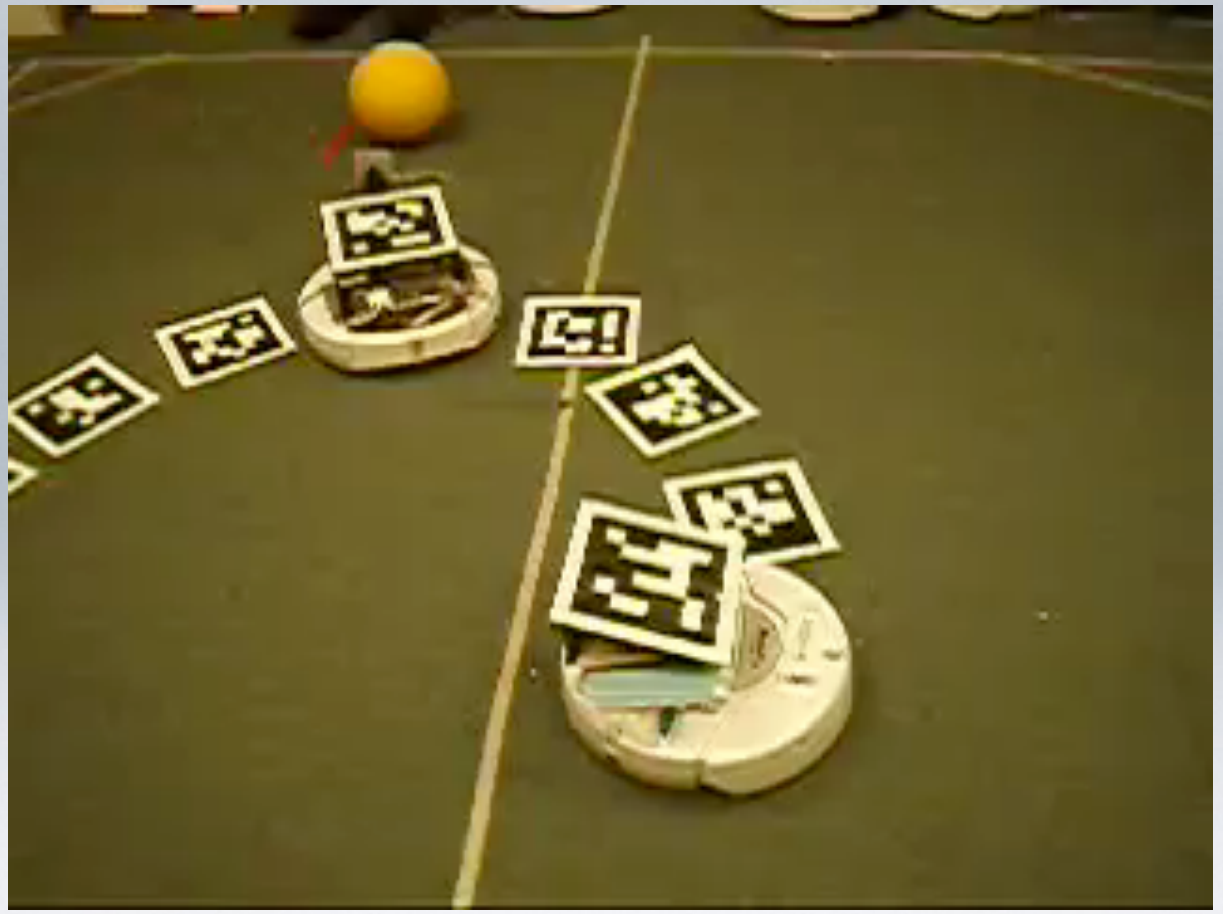
Lisa Miller, http://www.youtube.com/watch?v=2Z2RyeofsZg

NAVIGATING AROUND A WALL



Lisa Miller

AVOIDING DEAD-ENDS



Lisa Miller

http://www.youtube.com/watch?v=k6Kj4VjTKc8

Suppose f_score = g_score

Greedy best-first search

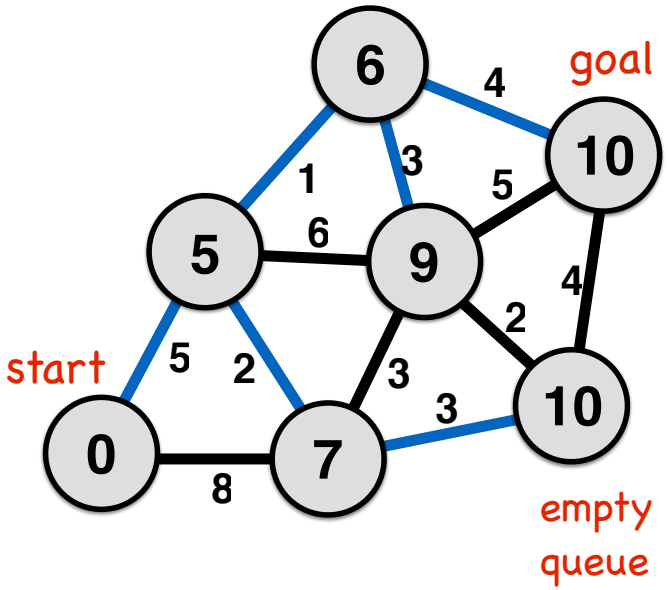
A-star shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while (visit_queue != empty) && current_node != goal
        cur_node ← dequeue(visit_queue, f_score)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
                f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$
            end if
        end for loop
    end while loop
output ← parent, distance

priority queue wrt. f_score
(implement min binary heap)

g_score: distance along current path back to start

h_score: best possible distance to goal



Greedy best-first search

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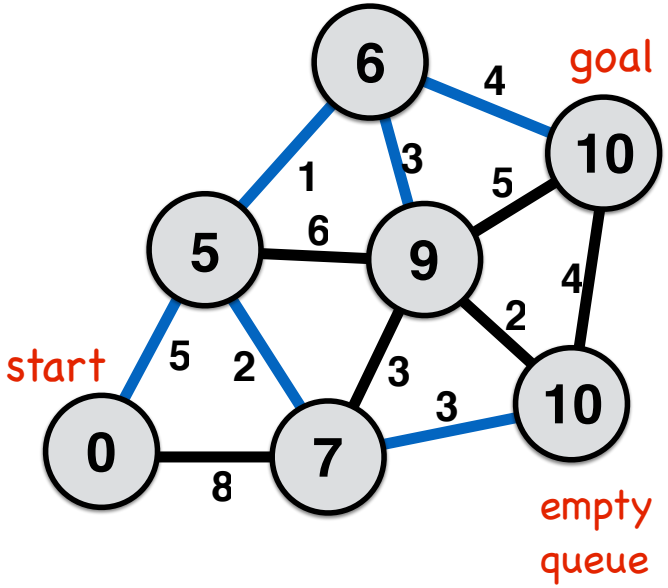
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while (visit_queue != empty) && current_node != goal
        cur_node ← dequeue(visit_queue, f_score)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue(nbr to visit_queue)
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
                f_score ← ~~$distance_{nbr}$~~ + $line\_distance_{nbr,goal}$
            end if
        end for loop
    end while loop
output ← parent, distance

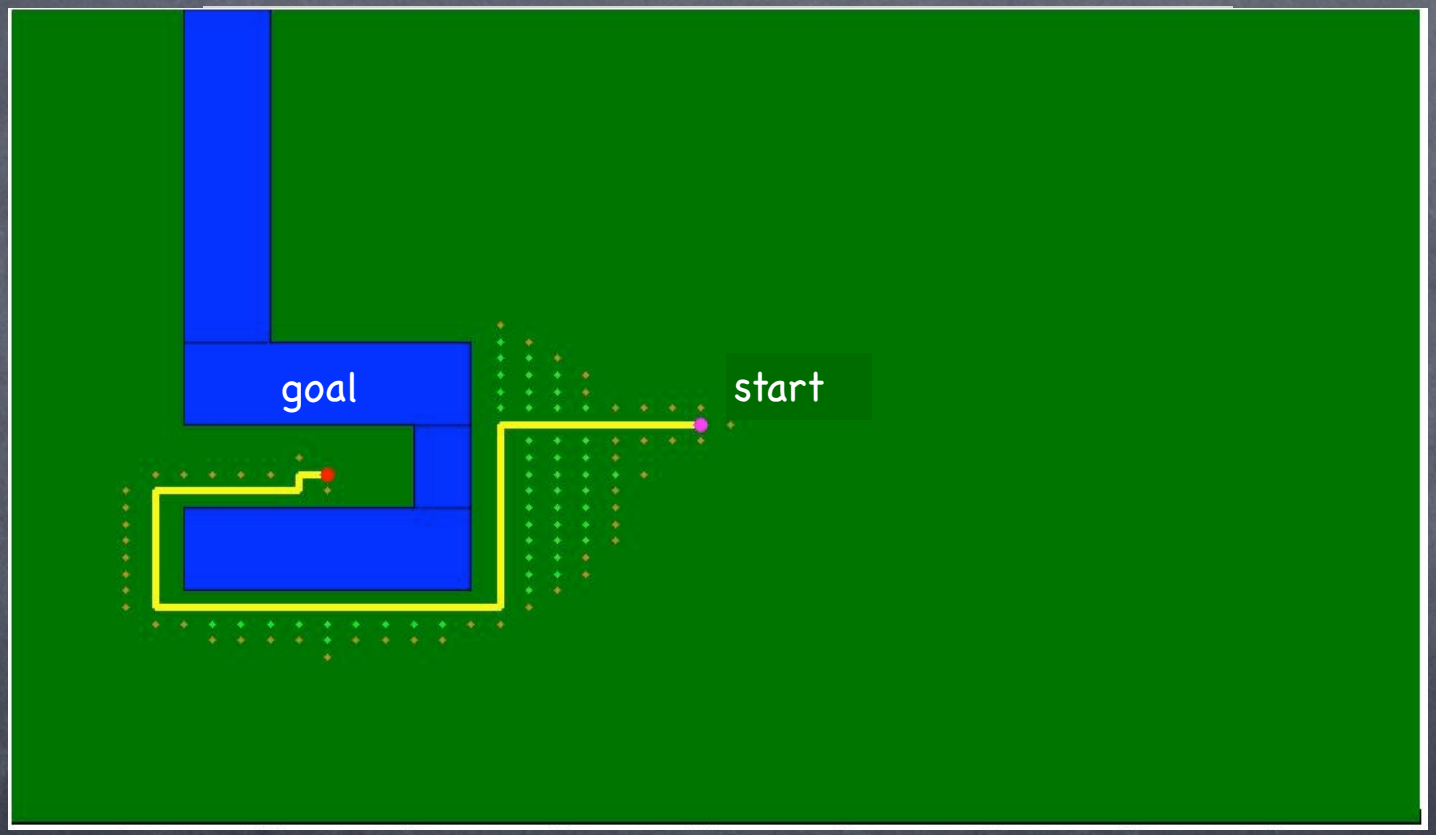
priority queue wrt. f_score
(implement min binary heap)

h_score:
best possible
distance to goal



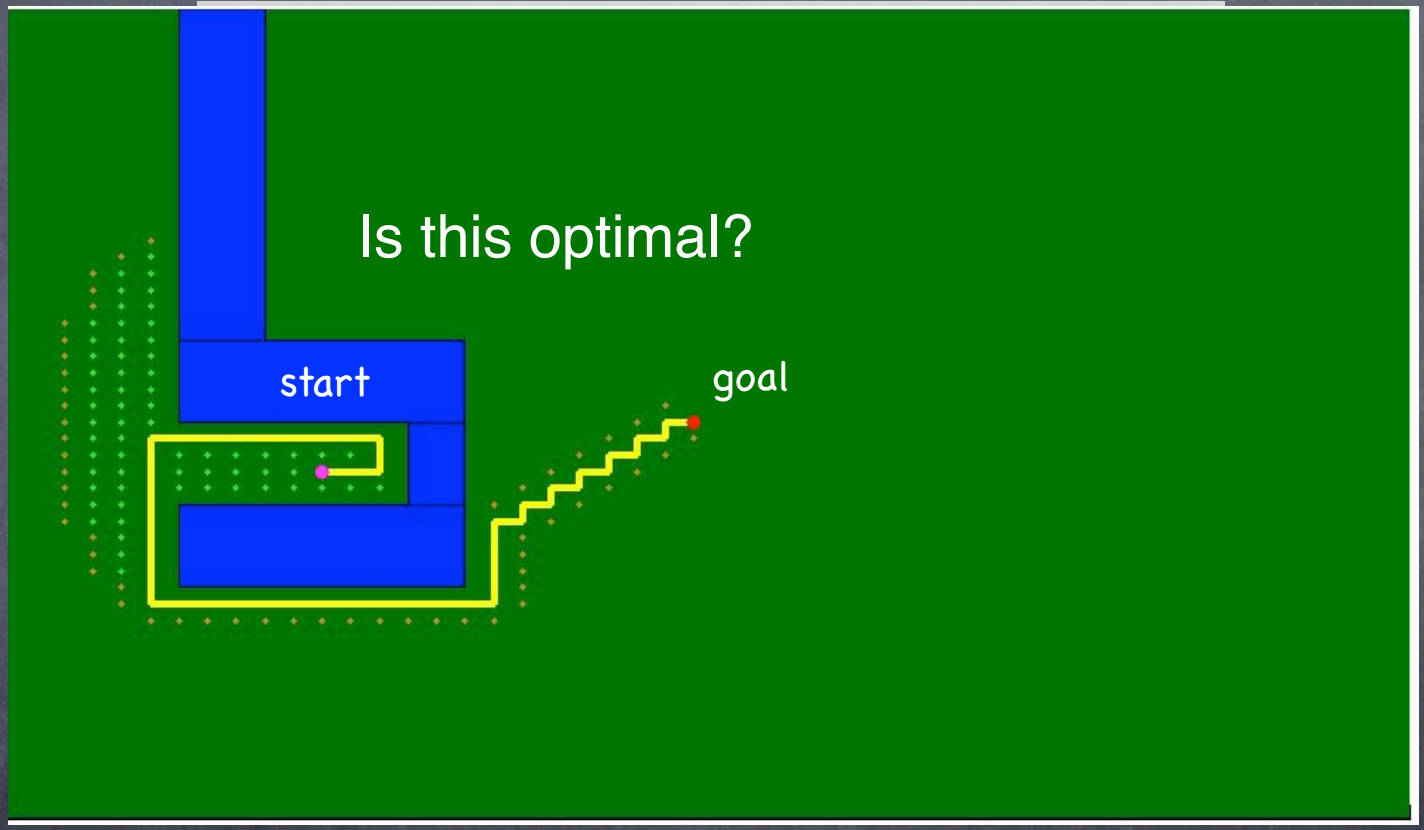
matlab example: gr.best-first

pathplan.m



matlab example: gr.best-first

pathplan.m



Heaps and Priority Queues

A-star shortest path algorithm

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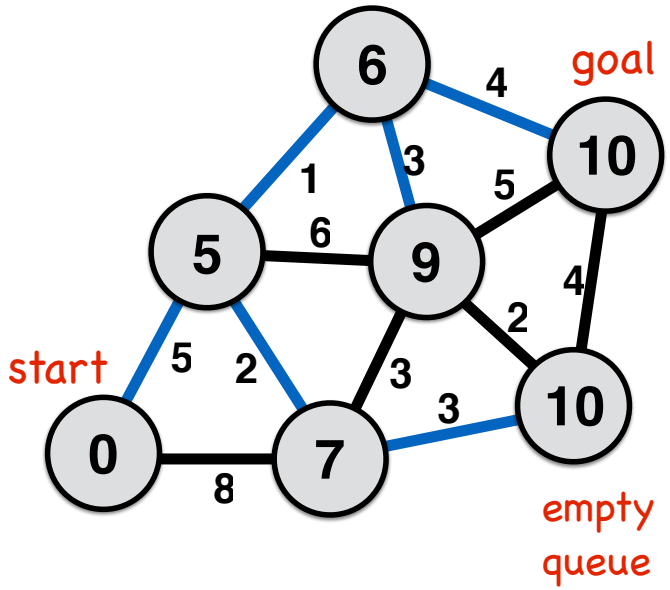
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
    while (visit_queue != empty) && current_node != goal
        dequeue: cur_node ← f_score(visit_queue)
        $visited_{cur\_node}$ ← true
        for each nbr in not_visited(adjacent(cur_node))
            enqueue: nbr to visit_queue
            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
                $parent_{nbr}$ ← current_node
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
                f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$
            end if
        end for loop
    end while loop
output ← parent, distance

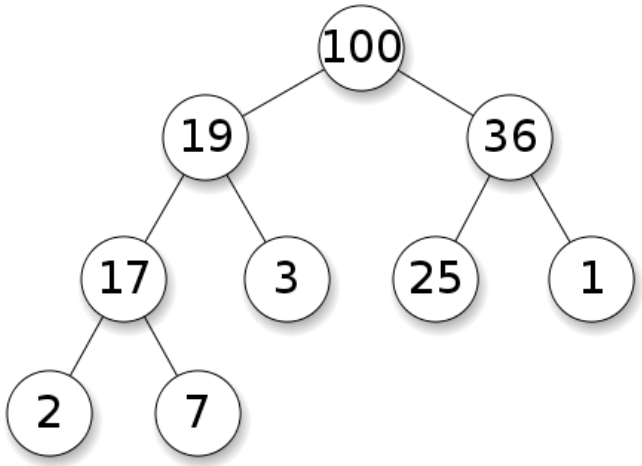
MIN BINARY HEAP FOR PRIORITY QUEUE



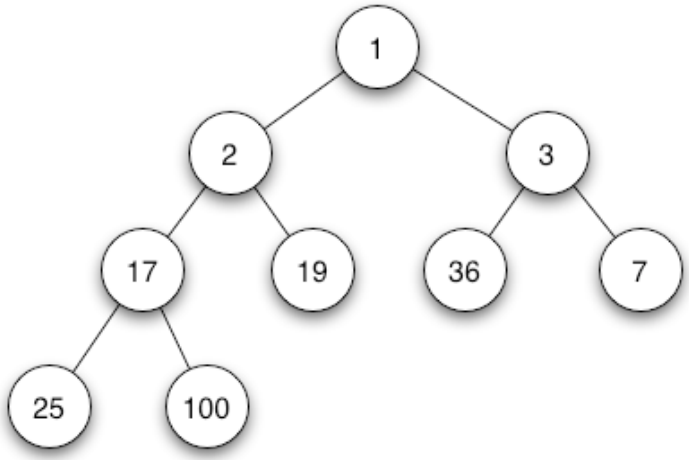
Binary Heaps

A heap is a tree-based data structure satisfying the heap property:
every element is greater (or less) than its children

Binary heaps allow nodes to have up to 2 children

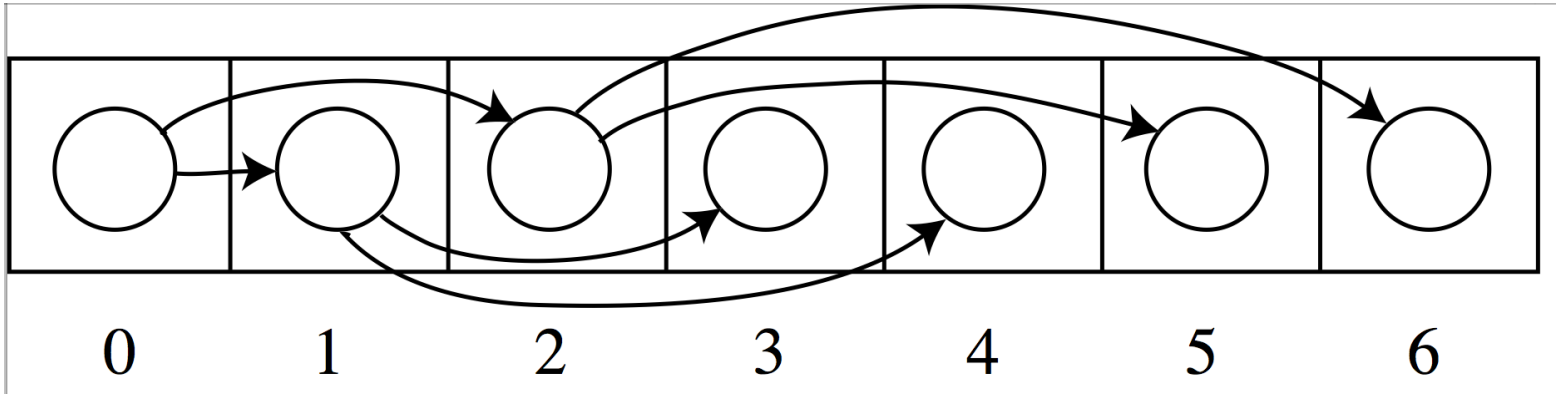


max heap



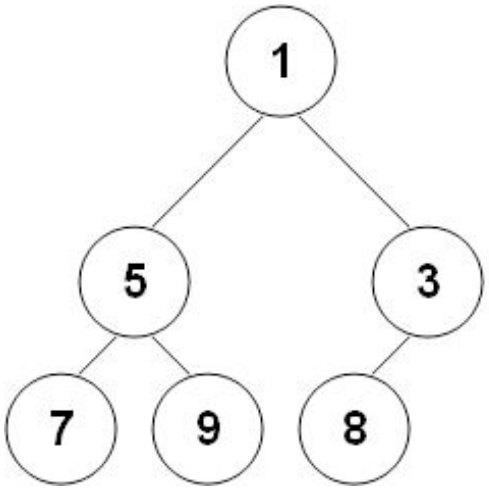
min heap

Heaps as arrays



- Heap element at array location *i* has
  - children at array locations *2i+1* and *2i+2*
  - parent at *floor((i-1)/2)*

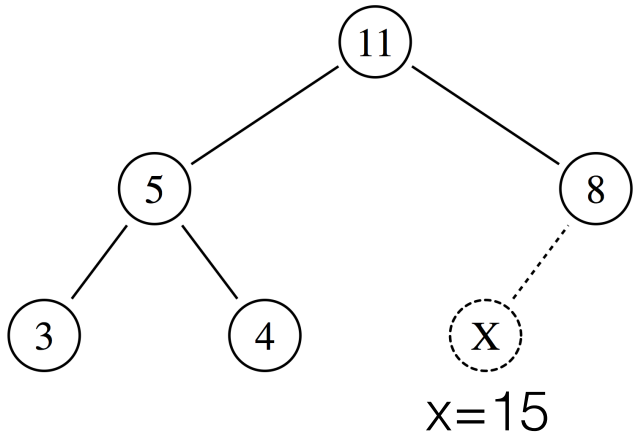
Heap array example



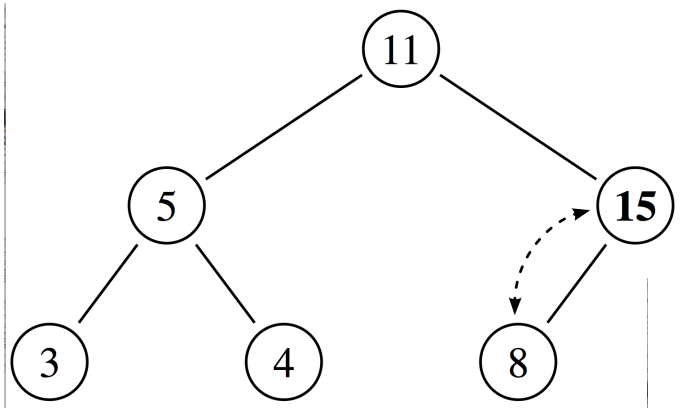
Node	1	5	3	7	9	8
Index	0	1	2	3	4	5

Heap operations: Insert

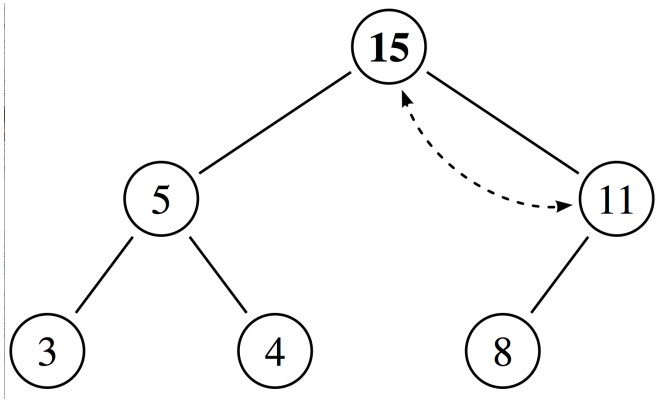
1) add new element to end of tree



2) swap with parent



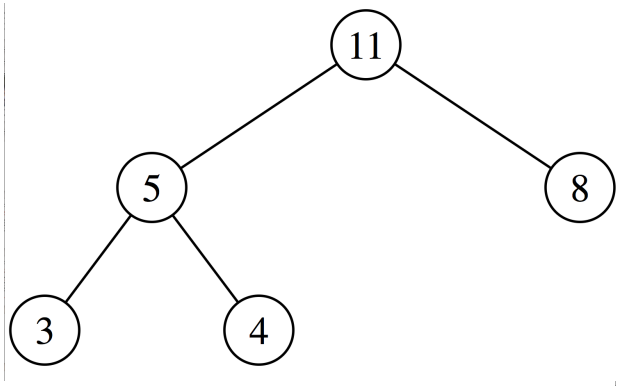
3) until heaped, do (2)



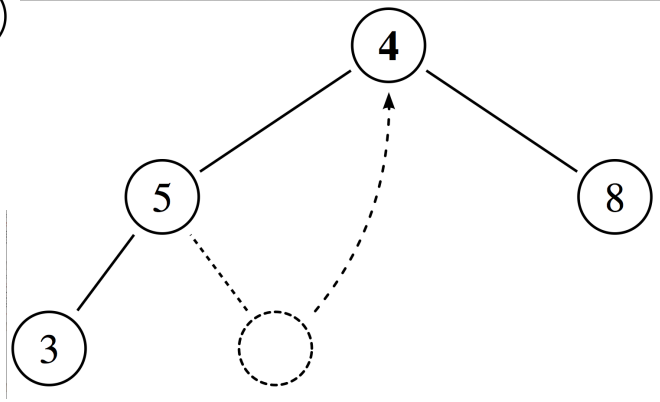
For priority queue, previously non-queued locations will be inserted with f_score priority

Heap operations: Extract

1) extract root element

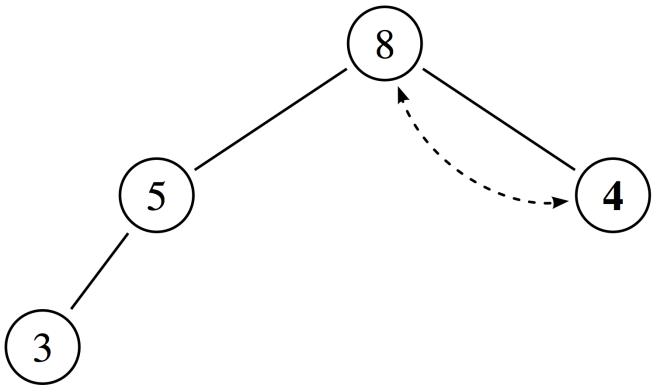


2) put last element at root



3) swap with higher priority child

4) until heaped, do (3)

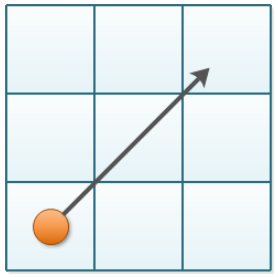


For priority queue, the root of the heap will be the next node to visit

Considerations

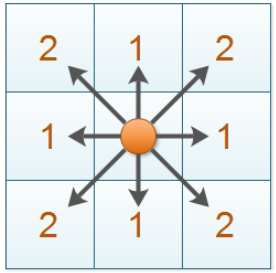
- How many operations are needed for heap insertion and extraction?
- How much better is a min heap than an array wrt. # of operations?
- Can there be duplicate heap elements for the same robot pose?
- How should we measure distance on a uniform grid?
- Is a choice of distance measure both metric and admissible?

Euclidean Distance



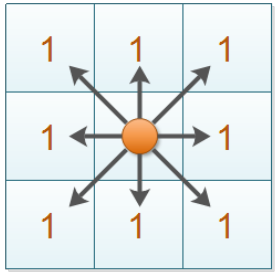
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Manhattan Distance



$$|x_1 - x_2| + |y_1 - y_2|$$

Chebyshev Distance

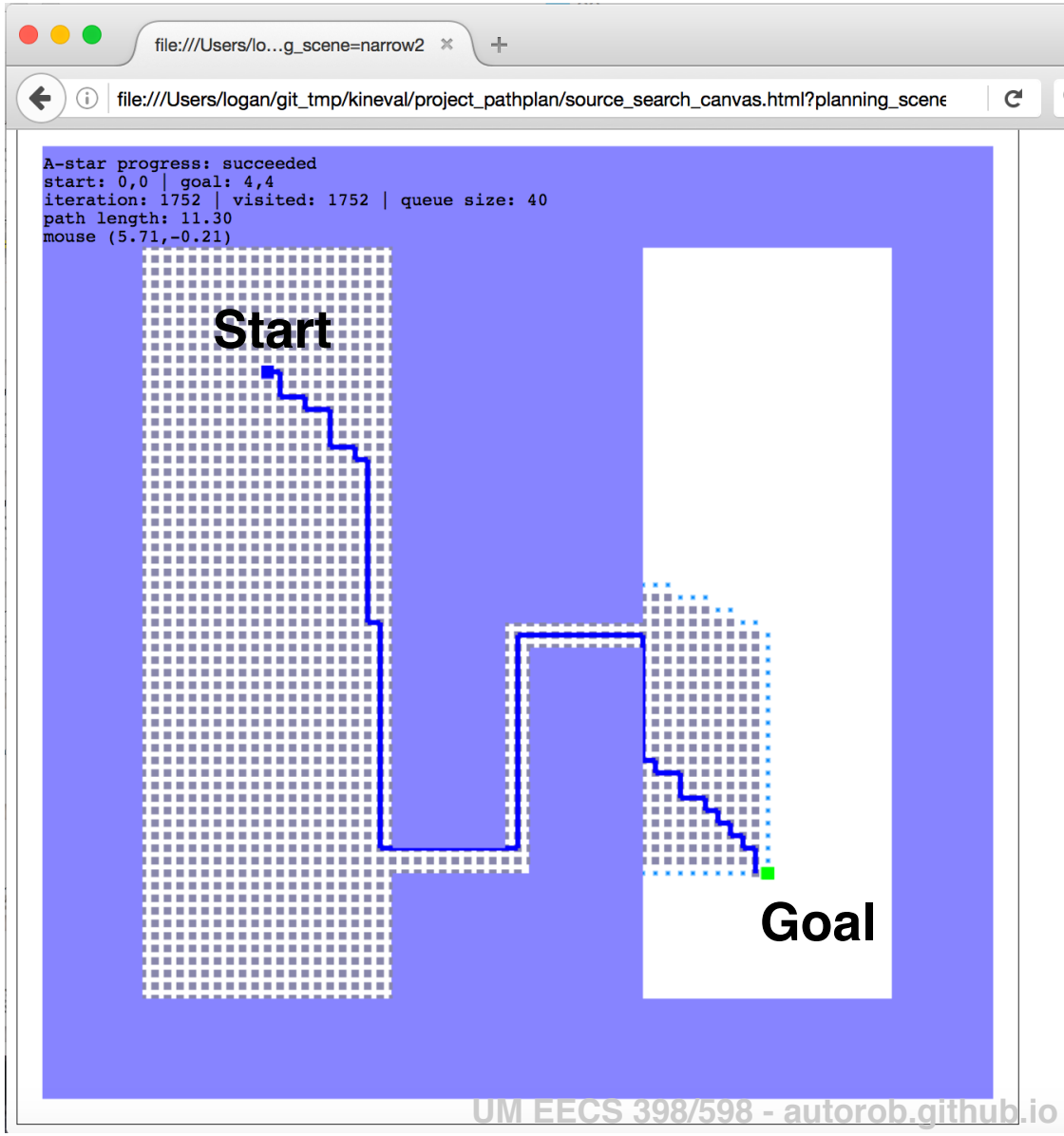


$$\max(|x_1 - x_2|, |y_1 - y_2|)$$

https://lyfat.wordpress.com/2012/05/22/euclidean-vs-chebyshev-vs-manhattan-distance/

Project 1:
2D Path Planning

- A-star algorithm for search in a given 2D world
- Heap data structure for priority queue
- Implement in JavaScript/ HTML5 (next lecture)
- Grad: DFS, BFS, Greedy
- Submit through your git repository



```
<html>
<title>How do we implement this planner?</title>

<body>
<h1>Next lecture:</h1>
<p>JavaScript/HTML5 and git Tutorial</p>

<a href="http://autorob.github.io">
EECS 367 Introduction to Autonomous Robotics <br>
ROB 510 ME/EECS 567 Robot Kinematics and Dynamics
</a>

</body>
</html>
```

How do we implement this planner?

next_lecture.html

Next lecture:

JavaScript/HTML5 and git Tutorial

EECS 367 Introduction to Autonomous Robotics
ROB 510 ME/EECS 567 Robot Kinematics and Dynamics