Mathematical Statistics. Homework 1 (10 points max)

1. (4 points) Many "named" distributions are special cases of the more distributions. **For each of the following distributions derive the form of the pdf, verify it is a pdf (0.2 points), and calculate the mean (0.4 points) and variance (0.4 points)**.

   (a) (1 point) If $X \sim \exp(\beta)$, then $Y = X^{1/\gamma}$ has the Weibull(γ, β) distribution, where $\gamma > 0$ is a constant

   (b) (1 point) If $X \sim$ gamma(a, b), then $Y = (1/X)$ has the Inverted Gamma IG(a, b) distribution.

   (c) (1 point) If $X \sim$ gamma$(3/2, \beta)$, then $Y = (X/\beta)^{1/2}$ has the Maxwell distribution.

   (d) (1 point) If $X \sim \exp(1)$, then $Y = \alpha - \gamma \log X$ has the Gumbel(α, γ) distribution, where $-\infty < \alpha < \infty$ and $\gamma > 0$.

2. (5 points) Derive the Stirling's Formula from CLT. For that

   (a) (1 point) Prove that

   $$P\left(\frac{\bar{X}_n - 1}{1/\sqrt{n}} \leq x\right) \to P(Z \leq x),$$

   where Z is a standard normal random varaible and $X_i \sim \exp(1)$ for $i = 1, 2, \ldots$.

   (b) (4 points) Differentiate both sides of the approximation in part (a). *HINT:* $\sum\limits_{i=1}^{n} X_i = gamma(n, 1)$

   (c) Substitute $x = 0$.

3. (1 point) What is the probability that the larger of two continuous iid random variables will exceed the population median m (Or it can be formulated like $P(\max(X_1, X_2) > m)$)? Generalize result to samples of size n.