

## Homework 2 (10 points max)

1. (2 points) Find the determinants of the following matrices:

$$(a) \begin{pmatrix} 1 & 2 & 3 \\ 5 & 1 & 4 \\ 3 & 2 & 5 \end{pmatrix} \quad \text{and} \quad (b) \begin{pmatrix} 5 & 1 & 7 & 3 \\ 1 & 0 & 2 & 0 \\ -2 & 2 & 5 & 4 \\ 3 & 0 & 4 & 0 \end{pmatrix}$$

2. (1 point) Calculate the characteristic polynomial of the matrix

$$\begin{pmatrix} -1 & 5 & 4 \\ 3 & -2 & 0 \\ -1 & 3 & 6 \end{pmatrix}$$

3. (2 points) Find the determinants of the following matrices:

$$(a) \begin{pmatrix} -t & & & & a_1 \\ a_2 & -t & & & \\ & \ddots & \ddots & & \\ & & a_{n-1} & -t & \\ & & & a_n & -t \end{pmatrix} \quad \text{and} \quad (b) \begin{pmatrix} \lambda & 1 & \cdots & 1 & \lambda \\ 1 & \lambda & \cdots & \lambda & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & \lambda & 1 \\ 1 & 1 & \cdots & 1 & \lambda \end{pmatrix}$$

4. (2 points) Let  $X = (X_1 | \dots | X_n) \in M_n(\mathbb{R})$  and  $\lambda_1, \dots, \lambda_n \in \mathbb{R}$ . Find  $\det(\lambda_1 X_1 X_1^T + \dots + \lambda_n X_n X_n^T)$ .
5. (2 points) Let  $A \in M_n(\mathbb{R})$  be an arbitrary matrix. Construct a matrix  $B \in M_n(\mathbb{R})$  by shifting all columns of matrix  $A$  two positions to the right in a cycle and adding the result to  $A$ . Express the determinant of  $B$  in terms of the determinant of  $A$ .
6. (1 point) Let  $A, B \in M_n(\mathbb{R})$  with  $A$  being invertible. Show that the characteristic polynomials of the matrices  $AB$  and  $BA$  coincide.