Homework 8 (10 points max)

1. (1 point) Let ξ be a random variable such that $P(\xi = -1) = \frac{1}{4}$, $P(\xi = 2) = \frac{1}{4}$, and for any points $a, b \in [0, 1]$ with $a < b$, we have $P(\xi \in [a, b]) = \frac{b-a}{2}$. Draw the plot of the distribution function $F_\xi(x) = P(\xi \leq x)$. Find the expected value and variance of ξ.

2. (1 point) In a lottery, 1% of the tickets win \$200, 0.01% of tickets win \$1000, and the remaining tickets do not win anything. Find the average winnings in this lottery (per ticket), i.e., the arithmetic mean of the winnings of all tickets. Think about how to formulate this in terms of random variables and expected value.

3. (2 points) There are m objects arranged in a row, k objects are randomly selected, $k < m$. The random variable X equals the number of such objects i that i is selected, but all its neighbors are not. Find the expected value of X.

4. (2 points) In an equilateral triangle ABC of area 1, we select a point M. Find the expected value of the area of ABM.

5. (2 points) Find the expected value and variance of the variable ξ with density $p(x) = \frac{1}{2\alpha} e^{\frac{-|x-a|}{\alpha}}$.

6. Let $P_\xi(x) = P(\xi = x)$ and $F_\xi(x) = P(\xi \leq x)$.
   (a) (1 point) Show that for $a > 0$ and $-\infty < b < \infty$, $P_{a\xi+b}(x) = P_\xi(\frac{x-b}{a})$ and $F_{a\xi+b}(x) = F_\xi(\frac{x-b}{a})$
   (b) (1 point) Find the distribution function $F_{\xi^2}(y)$ (express it through F_ξ and P_ξ)