Problems (10 points max)

1. (2 points) Let ξ and η be independent random variables with exponential distributions, i.e.,

$$p_\xi(x) = p_\eta(x) = \lambda e^{-\lambda x}\theta(x)$$

where

$$\theta(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

Find the following densities: $p_{\xi,\xi+\eta}(x,y)$ and $p_{\xi|\xi+\eta=z}(x)$.

Important: For the joint density, indicate the region where it equals zero.

2. (2 points) Let ξ and η be two independent normal distributions with parameters (a_1, σ_1^2) and (a_2, σ_2^2). Find the density distribution of the vector $(\xi + \eta, \xi - \eta)$. Will the coordinates of this vector be independent?

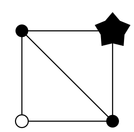
3. (2 points) Let ξ and η be independent random variables with distributions $\mathcal{N}(0,1)$. Find the density of the variable $\chi = \xi^2 + \eta^2$.

4. (2 points) Random variables X and Y are independent. X has a Laplace distribution with density $\frac{1}{2}e^{-|x|}$, while Y is uniformly distributed in the interval $[1,2]$.

   (a) Find the distribution density of the random variable $-2Y$

   (b) Find the distribution density of the random variable $X - 2Y$

5. (2 points) Consider the following maze:



A person leaves the point marked with $\circ$, choosing a direction with equal probability and randomly and independently at each step. As a random variable ξ, consider the number of steps needed to first reach the point marked with star. Find the expected value of ξ.