Homework 7 (10 points max)

1. (1 point) Vasily decided to impress Vasilisa and claims that if he rolls two dice, he will definitely get a six. Vasilisa, on the other hand, decided to give in to Vasily and will ignore all his rolls if the dices show the same numbers. Find the probability that Vasily will impress Vasilisa under these conditions.

2. (1 point) In a beautiful city "X", the population has grown to 10 people. On this occasion, an amusement park was opened, where you can jump on a trampoline. It turns out that the trampoline always breaks if there are 5 people on it, in case of 4 people it breaks with a probability of 0.8, in case of 3 people with a probability of 0.6, in case of 2 people with a probability of 0.4, in case of 1 person with a probability of 0.2, and the manufacturer guarantees that their reliable and quality trampolines do not break on their own. Despite the festive event, the inhabitants of the city "X" are very busy, each of them can come to the park with a probability of 0.5. Find out what is the probability that the trampoline business will fail on the first day.

3. (2 points) There are three balls in the bag: two black and one white. We perform the following actions: take out the ball, record its color and put back a ball of another color. Such actions are performed three times. Considering such an experiment as random, describe the probability space (Ω, P), that is, the space of elementary outcomes Ω and the probability measure $P : 2^{\Omega} \to [0, 1]$ on it. What is the probability of getting a white ball on the third step?

4. (2 points) Given the set $X = \{1, 2, 3, 4\}$. We will choose a graph G on vertices from X with equal probability. Find the following probabilities

   (a) The probability that the graph is connected

   (b) The probability that the graph is a tree

   (c) The probability that the graph is a tree, given that it is connected

   (d) The probability that the graph is connected, given that it is not a tree

5. (2 points) Lucky Arkady came to the casino with two coins. Making a bet he wins with probability p and loses with probability q. For winning or losing, he gains or respectively loses one coin. Arkady decided to go big and play until he completely loses his money. Find the probability that he will make at least 5 bets.

6. (2 points) A disc with radius $r < \frac{1}{4}$ is thrown uniformly into a square with side 2 in such a way that it falls entirely into the square. Describe the probability space and find the probability that the thrown disc will cross at least one diagonal of the square.