Homework 6 (10 points max)

1. (2 points) "Solve" the system using the least squares method:

$$\begin{pmatrix} 2 & 7 & -3 \\ 1 & -1 & 3 \\ 2 & 1 & -1 \\ 2 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 0 \\ -9 \end{pmatrix}$$

2. (2 points) Diagonalize the following symmetric matrices in an orthonormal basis (i.e., obtain the decomposition $A = CDC^T$, where C is an orthogonal matrix and D is diagonal).

   (a)

$$\begin{pmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

   (b)

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

3. (2 points) Given the matrix:

$$A = \begin{pmatrix} 13 & 14 & 4 \\ 14 & 24 & 18 \\ 4 & 18 & 29 \end{pmatrix}$$

   Find any symmetric matrix B such that $B^2 = A$.

4. (2 points) Find the singular value decomposition of the following matrices:

   (a)

$$\begin{pmatrix} 5 & 1 & 8 \\ 7 & 5 & 4 \end{pmatrix}$$

   (b)

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \end{pmatrix}$$

5. (2 points) Let V be a Euclidean space and $P : V \to V$ be the projection operator onto U along W, where $U, W \subseteq V$. Show that $\ker P^* = (\operatorname{Im} P)^\perp$ and $\ker P = (\operatorname{Im} P^*)^\perp$. Deduce from this that the adjoint operator $P^* : V \to V$ will be the projection onto $W^\perp$ along $U^\perp$.