Differential Drive Kinematics

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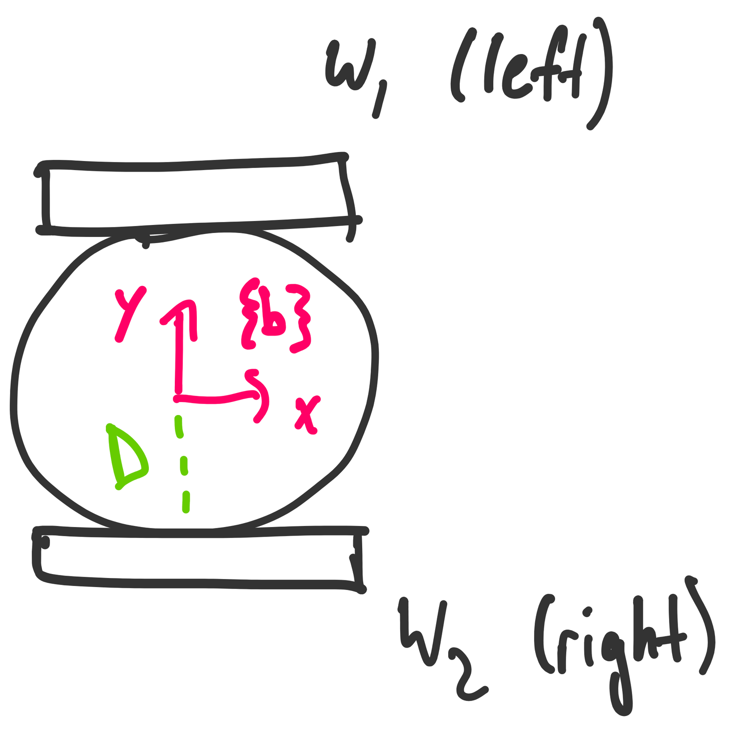

$L = 0$ b/c 2 wheels

Body twist to wheel motion

$$V_1 = A_{1b} \; V_b$$

W_1: $$\begin{bmatrix} \dot\theta \\ r\dot\phi_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ L & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot\phi_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -D/r & 1/r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

W_2: $$\begin{bmatrix} \dot\theta \\ r\dot\phi_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ L & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot\phi_2 \\ 0 \end{bmatrix} = \begin{bmatrix} D/r & 1/r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

Inverse kinematics

$$\begin{bmatrix} \dot\phi_1 \\ \dot\phi_2 \end{bmatrix} = \overbrace{\frac{1}{r} \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix}}^{H} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} \qquad \dot\phi = H V_b$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} \qquad 0 = Q V_b$$

$$\dot\phi_1 = -\frac{D}{r}\dot\theta + \frac{v_x}{r} \qquad \text{(eqn 1.1)}$$

$$\dot\phi_2 = \frac{D}{r}\dot\theta + \frac{v_x}{r} \qquad \text{(eqn 1.2)}$$

Forward Kinematics

$u = \dot\phi = \phi$ b/c $\Delta t = 1$

$$V_b = H^+ u = \frac{r}{2} \begin{bmatrix} -1/D & 1/D \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

$$= \frac{r}{2} \begin{bmatrix} \frac{\phi_2 - \phi_1}{D} \\ \phi_1 + \phi_2 \\ 0 \end{bmatrix} \qquad \text{(eqn 2.1)}$$

Integrate V_b to get $T_{bb'} = T(\Delta\theta_b, \Delta x_b, \Delta y_b)$ (eqn 2.2)

$\Delta q_b = (\Delta\theta_b, \Delta x_b, \Delta y_b)$ (eqn 2.3)

$\Delta q = A(\theta, 0, 0)\Delta q_b$ (eqn 2.4)

$q' = q + \Delta q$ (eqn 2.5)

$$\begin{matrix} \dot q = A(\theta,0,0) V_b \\ \parallel \qquad\qquad\qquad \parallel \\ \Delta q = A(\theta,0,0)\Delta q \end{matrix}$$

b/c $\Delta t = 1$

$$A(\theta,0,0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\Delta q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \Delta\theta_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix}$$

$$= \begin{bmatrix} \Delta\theta_b \\ \Delta x_b \cos\theta - \Delta y_b \sin\theta \\ \Delta x_b \sin\theta + \Delta y_b \cos\theta \end{bmatrix}$$