# A GREEK PARALLEL TO BOETHIUS' <u>DE HYPOTHETICIS</u> SYLLOGISMIS

BY

#### SUSANNE BOBZIEN

Pagination may vary from published paper. Please quote from published paper only.

ABSTRACT: In this paper I present the text, a translation, and a commentary of a long anonymous scholium to Aristotle's <u>Analytics</u> which is a Greek parallel to Boethius' <u>De Hypotheticis Syllogismis</u>, but has so far not been recognized as such. The scholium discusses hypothetical syllogisms of the types <u>modus</u> <u>ponens</u> and <u>modus tollens</u> and hypothetical syllogisms constructed from three conditionals ('wholly hypothetical syllogisms'). It is Peripatetic, and not Stoic, in its theoretical approach as well as its terminology. There are several elements of early Peripatetic hypothetical syllogistic preserved in it, and there is a large number of close parallels to Boethius' <u>De Hypotheticis Syllogismis</u> which we find in no other source. It is very likely that there was a Greek source from which both the scholium and large parts of Boethius' <u>De Hypotheticis Syllogismis</u> are ultimately derived.

Boethius' logical treatise <u>De Hypotheticis Syllogismis</u> (<u>HS</u>) is the only complete work on hypothetical syllogisms that has survived from antiquity. With regard to most of the text we do not know what sources he drew from.<sup>1</sup>) There is evidence that several other ancient writers composed works on hypothetical syllogisms;<sup>2</sup>) however, none of them has come down to us. We have a number of brief compendia on hypothetical syllogistic from late antiquity, e.g. in Philoponus,<sup>3</sup>) which belong to a tradition different from Boethius' treatise. The earlier Byzantine texts on hypothetical syllogisms depend on this tradition;<sup>4</sup>) but in the 13th century we find a

<sup>&</sup>lt;sup>1</sup> The exception is <u>HS</u> 2.1.1-6 (Obertello), where Cicero, or a Latin source drawing from Cicero, is a likely source.

<sup>&</sup>lt;sup>2</sup> In Latin, Marius Victorinus (cf. Cassiodorus Senator, <u>Instit</u>. II 3.13); in Greek perhaps Ammonius (cf. [Amm.] APr. 67.32).

<sup>&</sup>lt;sup>3</sup> Philop. <u>APr.</u> 244.1-246.14, [Ammonius] <u>APr.</u> 67.32-69.28; cf. <u>Scholia in Amm. APr.</u> (CAG IV.6) X1,1-XII.3; Alcin. <u>Didasc.</u> 158.5-159.30 (Whittaker), and my <u>Wholly Hypothetical Syllogisms</u>, Phronesis 42 (2000), 87-137, sections 2 and 10.

<sup>&</sup>lt;sup>4</sup> See K. Ierodiakonou, <u>The Hypothetical Syllogisms in the Greek and Latin Medieval Traditions</u>, Cahiers de L'institut du Moyen-age Grec et Latin 66 (1996), 96-116.

translation by Holobolus of Boethius' <u>De Hypotheticis Syllogismis</u> into Greek.<sup>5</sup>) Parallels between Boethius and some Arabic texts suggest that there has been a Greek source common to Boethius and part of the Arabic tradition.<sup>6</sup>) But no such Greek text has - as yet - been identified.

In this paper I discuss a longish anonymous scholium to Aristotle's <u>Analytics</u> which is a Greek parallel to Boethius' <u>De Hypotheticis Syllogismis</u>. The scholium is available in print only in Theodor Waitz's edition of Aristotle's <u>Organon</u> (Leipzig 1844). It is Codex Laur. 72.5, ff 210-12, appended at the end of a manuscript of the <u>Prior</u> and <u>Posterior Analytics</u>. Dieter Harlfinger has dated this part of the codex to the second half of the 10th century; this gives us a <u>terminus ante quem</u>. The scholium has, I believe, so far not been recognized as a parallel to Boethius, nor has it been discussed in the literature on hypothetical syllogisms. I am also not aware of any translation. The scholium is important for the history of hypothetical syllogistic, because it is the only extant Greek text that provides a close parallel to the particular theory Boethius presents in Latin. We can assume that the scholium was composed no later than the 10th century (see above). But it preserves elements of a theory that was most probably developed before the 6th century. There are a number of idiosyncrasies in the terminology, a fact that sets the text apart from all other Greek sources on hypothetical syllogistic, and thus adds to its interest.

In the following I present the text of the scholium, a translation, and a commentary, including some general remarks about the theory the scholium preserves.

# Aristotle, Organon, ed. Waitz, vol. I, pp. 9-10, Anon. Scholium on Hypothetical Syllogisms<sup>9</sup>)

<sup>&</sup>lt;sup>5</sup> D. Z. Nikitas, <u>Eine byzantinische Übersetzung von Boethius' "De hypotheticis syllogismis"</u> (Göttingen 1982).

<sup>&</sup>lt;sup>6</sup> M. Maroth, Die hypothetischen Syllogismen, Acta Antiqua 27 (1979), 407-36.

<sup>&</sup>lt;sup>7</sup> D. Harlfinger, in <u>Aristoteles Graecus</u>, vol. 1, Paul Moraux (ed.) (Berlin 1976), Nachträge, discusses Laur. 72.5, on pp. 475-80.

<sup>&</sup>lt;sup>8</sup> Except that Prantl, <u>Geschichte der Logik im Abendlande</u>, vol. I (Leipzig 1855), quotes part of the scholium in footnote 167 (p.656).

<sup>&</sup>lt;sup>9</sup> I am extremely grateful to Sten Ebbesen, who very kindly collated Waitz' edition of the scholium with the manuscript for me. He points out to me that "Waitz' transcription of the text ... is very faithful to the manuscript. ... however ... the ms uses iota adscriptum, and ... repeatedly uses letters with a line above them both for cardinal and ordinal numbers." In order not to forestall possible interpretations of the text, I have reinstated letters with a line above them for the occurrences of A, B,  $\Gamma$  in the representation of the syllogistic modi in Waitz' text. However, I believe that Waitz was correct in using A, B,  $\Gamma$  (i.e. 'term letters') instead of 'the first', 'the second, 'the third' (i.e. place holders for propositions), and have translated the text accordingly (see also the commentary).

(1) ἐν τοῖς ὑποθετιχοῖς συλλογισμοῖς πوῶτοί εἰσιν οἱ ἐχ δύο ὄφων συνημμένων ἢ διαλελυμένων εἶτα οἱ ἐκ τριῶν. $^{10}$ ) (2) δυσι $^*$  συνημμένοις ὅροις γίνονται συλλογισμοί τέσσαρες διὰ τῆς ἐπὶ τὸν πρότερον ἐπανόδου καὶ τέσσαρες διὰ τῆς ἐπὶ τὸν ἔσχατον. λαμβάνονται γὰρ ἢ καταφατικῶς οί δύο ὅροι ἢ ἀποφατικῶς ὁ δεύτερος ἢ ὁ πρότερος ἀποφατικῶς ἢ ἀμφότεροι ἀποφατικῶς. καὶ ἡ μὲν έπὶ τὸν\*\* ποౖῶτον ἐπάνοδος κατὰ θέσιν γίνεται, ἢ δ'ἐπὶ τὸν δεύτεοον κατὰ ἀναίοεσιν. οἷον εἰ τὸ ā, τὸ  $\beta$ . ἐπάνοδος· τὸ  $\bar{a}$  δὲ. Συμπέρασμα· τὸ  $\beta$  ἄρα. εἶτα εἰ τὸ  $\bar{a}$ , οὐ τὸ  $\bar{b}$ · τὸ  $\bar{a}$  δὲ, οὐ τὸ  $\beta$  ἄρα.  $πάλιν εἰ οὐ τὸ <math>\bar{a}$ , τὸ  $\bar{β}$ · οὐ τὸ  $\bar{a}$  δὲ, τὸ  $\bar{β}$  ἄρα. τέταρτον εἰ οὐ τὸ  $\bar{a}$ , οὐδὲ τὸ  $\bar{β}$ · οὐ τὸ  $\bar{a}$  δὲ, οὐδὲ τὸ  $\bar{β}$ ἄρα. ἐπὶ τὸν δεύτερον ἐπάνοδος· εἰ τὸ ā, τὸ β· οὐ τὸ β δὲ, οὐδὲ τὸ ā ἄρα. εἶτα εἰ τὸ ā, οὐ τὸ β· τὸ eta δὲ, οὐ τὸ  $ar{a}$  ἄφα. ἔτι εἰ οὐ τὸ  $ar{a}$ , τὸ eta· οὐ τὸ eta δὲ, τὸ  $ar{a}$  ἄφα. τέταφτον εἰ οὐ τὸ  $ar{a}$ , οὐ τὸ eta· τὸ etaδὲ, καὶ τὸ ā ἄρα. (3) ἑξῆς τοῖς ἐκ συνημμένων ὅρων οἱ ἐκ διαλελυμένων εἰσὶ συλλογισμοί, ὧν ἡ ἐπάνοδος ἐφ' ὁπότερον ἂν τύχη, οἷον εἰ ἐξ ἀνάγκης τὸ ā ἢ τὸ β, μὴ ἔστι δὲ τὸ β, τὸ ā ἄρα· ἢ μὴ ἔστι δὲ τὸ ā, τὸ β ἄρα. (4) ἐχ δὲ τριῶν ὅρων συνημμένων ὀκτὼ μὲν συλλογισμοὶ διὰ τῆς ἐπὶ τὸν  $\pi \varrho \tilde{\omega}$ τον ἐπανόδου ὀκτ $\dot{\omega}$  δὲ διὰ τῆς ἐπὶ τὸν $^{***}$  ὕστε $\varrho$ ον· συντί $\vartheta$ ενται γὰ $\varrho$  οἱ τ $\varrho$ εῖς ὅ $\varrho$ οι ἢ καταφατικῶς ἢ αποφατικῶς ὁ τوίτος ἢ καὶ ὁ τوίτος καὶ ὁ δεύτερος ἢ μόνος ὁ δεύτερος. Πάλιν τε αὖ ἢ ἀποφατικῶς οἱ τρεῖς ἢ καταφατικῶς ὁ τρίτος ἢ καὶ ὁ τρίτος καὶ ὁ δεύτερος ἢ μόνος ὁ δεύτερος. ὥστε γίνεσθαι τὴν σύνθεσιν ὀκταχῆ, (5) οἷον ὡς ἐπὶ ἑνὸς τρόπου $^{\sharp \cdot}$  εἰ τὸ ā, τὸ  $\beta \cdot$  εἰ τὸ  $\beta$ , τὸ γ̄. εἰ τὸ ā, ἄρα τὸ γ̄. τὸ σχῆμα πρῶτον. ὁ γὰρ συνάγων ὅρος μέσος ἐστὶ δὶς λαμβανόμενος, λήγων μὲν ἐν τῷ προτέρῳ συνημμένῳ ἡγούμενος δὲ ἐν τῷ δευτέρῳ. διὸ καὶ ἀναπόδεικτοι οί συλλογισμοὶ οὐ δεόμενοι ἀναλύσεως πρὸς ἀπόδειξιν. Καὶ οἱ ἄλλοι δὲ τρόποι τοῦ πρώτου σχήματος όμοίως ἔχουσι συντιθέμενοι καθάπες εἴςηται. (6) Δεύτεςον δὲ σχῆμα ἐν ῷ ὁ συνάγων ὅςος τὴν αὐτὴν ἔχει σχέσιν πρὸς ἑκάτερον τῶν συναγομένων καθὸ ἡγεῖται ἐν ἑκατέρω συνημμένω, πλὴν ἐν μὲν τῷ ἐτέρῳ καταφατικῶς, ἐν δὲ τῷ ἐτέρῳ /p.10/ ἀποφατικῶς. διὸ καὶ ἀναλυόμενοι εἰς τὸ πρώτον αποδείχνυνται οἷον ἐπὶ ἑνὸς τρόπου ἐκ καταφατικῶν τῶν συναγομένων ὅρων συνεστῶτος $^{\#}$  εἰ τὸ  $\bar{a}$ , τὸ  $\bar{\beta}$  εἰ οὐ τὸ  $\bar{a}$ , τὸ  $\bar{\gamma}$  εἰ οὐ τὸ  $\bar{\beta}$ , ἄρα τὸ  $\bar{\gamma}$ . (7) ἀναλύεται δὲ εἰς τὸ πρῶτον οὕτως. εἴτε καταφατικοὺς ἔχει τοὺς συναγομένους εἴτε ἀποφατικοὺς εἴτε ἀνομοίους μετὰ τοῦ συνάγοντος ἐναλλὰξ τιθεμένου. 11) λαμβάνεται ἐν τῆ ἀναλύσει τὸ ἀντικείμενον τῷ προτέρφ συνημμένω καὶ ἐπάγεται τὸ δεύτερον, ἵνα ὁ ἀντικείμενος τῷ λήγοντι ἐν τῷ πρώτω συνημμένω  $\langle \sigma$ υνάγηται πρὸς τὸν λήγοντα ἐν τῷ δευτέρ $\phi$  συνημμέν $\phi \rangle^{\#\#}$  οἵον ἐπὶ τοῦ προκειμένου τρόπου  $\left[\tilde{\eta}\right]^+$ εἰ οὐ τὸ  $\beta$ , οὐ τὸ  $\bar{a}\cdot$  εἰ οὐ τὸ  $\bar{a}$ , τὸ  $\bar{\gamma}\cdot$  εἰ οὐ τὸ  $\beta$ , ἄρα τὸ  $\bar{\gamma}$ , ὅπερ ἔδει δεῖξαι. (8) Τρίτον σχῆμα ἐν  $\ddot{\omega}$ ό συνάγων ὅوος τὴν μὲν αὐτὴν ἔχει σχέσιν πρὸς ἑκάτερα τῶν συναγομένων, λήγων δὲ ἐν ἑκατέρφ τῷ συνημμένῳ καταφατικῶς τε καὶ ἀποφατικῶς, οἶον ῶς ἐπὶ ἑνὸς τρόπου πάλιν εκ καταφατικῶν τῶν συναγομένων ὅρων συνεστῶτος: εἰ τὸ  $\bar{a}$ , τὸ  $\beta$ : εἰ τὸ  $\bar{\gamma}$ , οὐ τὸ  $\bar{\beta}$ : εἰ τὸ  $\bar{a}$ , ἄρα οὐ τὸ  $\bar{\gamma}$ : (9) ή δὲ ἀνάλυσις αὐτοῦ γίνεται εἰς τὸ πρῶτον σχῆμα οὕτως. τὸ ἀντικείμενον τῷ δευτέρφ συνημμένφ λαμβάνεται μετὰ τὸ πρότερον συνημμένον καὶ συνάγεται τὸ ἡγούμενον τοῦ προτέρου συνημμένου πρὸς τὸ αντιχείμενον τῷ ἡγουμένω ἐν τῷ δευτέρω, οἵον ὡς ἐπὶ τοῦ προχειμένου τρόπου· εἰ τὸ ā, τὸ

<sup>&</sup>lt;sup>10</sup> I have changed Waitz' punctuation.

<sup>&</sup>lt;sup>11</sup> I have changed Waitz' punctuation.

β; εἰ τὸ β, οὐ τὸ ϙ̄ εἰ τὸ ᾱ, ἄρα οὐ τὸ ϙ̄ ὅπερ ἔδει δεῖξαι. 12) (10) ἐπὶ ὕλης ὡς ἐν τῷ πρώτῳ σχήματι εἰ ἡμέρα ἐστίν, φῶς ἐστίν εἰ φῶς ἐστίν, ὁρᾶται τὰ ὁρατά εἰ ἡμέρα ἐστίν, ὁρᾶται ἄρα τὰ ὁρατά. δευτέρου σχήματος καὶ πρώτου τρόπου εἰ ἡμέρα ἐστίν, φῶς ἐστίν εἰ οὐχ ἡμέρα ἐστίν, ὁ ἤλιος ὑπὸ γῆν εἰ μὴ φῶς ἐστίν, ὁ ἤλιος ὑπὸ γῆν εἰ μὴ φῶς ἐστίν, ὁ ἤλιος ὑπὸ γῆν εἰ μὴ φῶς ἐστίν, ὁ ἤλιος ὑπὸ γῆν ἐστίν ἡμέρα εἰ οὐχ ἡμέρα ἐστίν, ὁ ἤλιος ὑπὸ γῆν εἰ μὴ φῶς ἄρα ἐστίν, ὁ ἤλιος ὑπὸ γῆν ἐστίν τρίτου σχήματος καὶ πρώτου τρόπου εἰ ἡμέρα ἐστί, φῶς ἐστίν εί ἀθεώρητα τὰ ὁρατά, οὐκ ἔστι φῶς εἰ ἡμέρα ἄρα ἐστίν, οὐκ ἀθεώρητα τὰ ὁρατά. (11) Εἰσίν οἱ τρόποι ὀκτὰν τοῦ δευτέρου σχήματος καὶ τοῦ τρίτου ὀκτώ· δύο ἐκ καταφατικῶν συναγόμενοι δύο ἐξ ἀποφατικῶν τέσσαρες ἐξ ἀνομοίων μετὰ ὁμοίου ἢ ἀνομοίου τιθεμένου τοῦ συνάγοντος.

\*δύο Waitz,  $\beta$  ms;  $^{13}$ ) \*\* $\tau$ ων ms; \*\*\* $\tau$ ὸ ms;  $^{\#}$ τρίτου ms;  $^{\#}$ συνεστώς ms;  $^{\#\#}$ (συνάγηται πρὸς τὸν λήγοντα ἐν τῷ δευτέρω συνημμένω) addidi;  $^{14}$ )  $^{+}$ [ $\tilde{\eta}$ ] delevi.

### **Translation**

(1) Among the hypothetical syllogisms there are first those <that come to be> from two terms connected or separated; then those <that come to be> from three terms. (2) By means of two connected terms there come to be four syllogisms through the repetition of the first <term> and four through the <repetition> of the last. For either the two terms are taken affirmatively, or the second is taken negatively, or the first is taken negatively, or both are taken negatively. The repetition of the first comes to be by positing, that of the second by taking away, such as 'if A, B'; repetition: 'but A'; conclusion: 'hence B'; then, 'if A, not B; but A; hence not B'; again, 'if not A, B; but not A; hence B'; fourth, 'if not A, not B; but not A; hence not B'. Repetition of the second: 'if A, B; but not B; hence neither A'; then, 'if A, not B; but B; hence not A'; moreover, 'if not A, B; but not B; hence A'; fourth 'If not A, not B; but B; hence A also'. (3) The syllogisms from connected terms are followed by those from separated terms, in which the repetition can be of either <term>, such as if from necessity either A or B, but B is not, A is hence; or but A is not, hence B. (4) From three connected terms <there come to be> eight syllogisms through the repetition of the first term and eight through the repetition of the last. For the terms are compounded, either all three terms affirmatively, or the third negatively, or both the third and the second negatively, or only the second <negatively>. Again, either all three <terms are compounded> negatively, or the third affirmatively, or the third and second <affirmatively>, or only the second <affirmatively>. Hence the compound comes to be eight-fold: (5) Such as in the first mode 'if A, B; if B, C; if A, hence C.' The figure is the first; for the term which conjoins,

<sup>&</sup>lt;sup>12</sup> I have changed Waitz' punctuation.

<sup>&</sup>lt;sup>13</sup> Sten Ebbesen drew my attention to this possibility of reading the ms.

<sup>&</sup>lt;sup>14</sup> Sten Ebbesen kindly assisted me with this emendation.

which is middle, is taken twice, and follows in the first conditional but leads in the second. Wherefore, also, these syllogisms are indemonstrables, since they do not require a reduction for proof. And the other modes of the first figure work in the same way, being compounded as has been said. (6) The second figure is the one in which the term that conjoins has the same position with respect to each of the terms that are conjoined, insofar as it leads in either conditional, except that it is taken affirmatively in one, and negatively /p.10/ in the other. Wherefore, also, they are proved to reduce to the first <figure>, such as in the case of the first mode which is compounded from affirmative conjoined terms: 'if A, B; if not A, C; if not B, hence C.' (7) It is reduced to the first <figure> as follows: the conjoined <terms> are either affirmative or negative or unlike, with the conjoining <term> posited alternately. In the reduction the contraposition of the first conditional is taken and the second <conditional> added, in order that the contradictory of the consequent in the first <conditional> <is conjoined to the consequent in the second>; for example in the case of the above mode it would be 'if not B, not A; if not A, C; if not B, C', quod erat demonstrandum. (8) The third figure is the one in which the term that conjoins has the same position with respect to each of the terms that are conjoined, following in both conditionals, affirmatively in the one, negatively in the other, such as in the first mode again, which is put together from two affirmative terms that are conjoined: if A, B; if C, not B; if A, hence not C. (9) Its reduction to the first figure comes to be in this way: the contraposition of the second conditional is taken after the first conditional, and the antecedent <term> of the first conditional is conjoined to the contradictory of the antecedent <term> in the second; as for example in the case of the above mode: 'if A, B; if B, not C; if A, hence not C', guod erat demonstrandum. (10) With matter, as in the first figure

if it is day, it is light if it is light, the visible things are seen if it is day, the visible things are hence seen

and the first mood of the second figure

if it is day, it is light if it is not day, the sun is above the earth if it is not light, the sun is above the earth

#### reduction:

if it is not light, it is not day if it is not day, the sun is above the earth hence if it is not light, the sun is above the earth

and the first mood of the third figure

if it is day, it is light if the visible things are unseen, it is not light hence if it is day, the visible things are unseen.

(11) There are eight modes of the second figure and eight of the third; two conjoin from affirmative <terms>, two from negative <terms>, four from unlike <terms>, with the conjoining term posited as like or unlike.

## Commentary

Waitz remarks that the position of the scholium after the <u>Prior</u> and <u>Posterior Analytics</u> is inappropriate.<sup>15</sup>) However, it is by no means surprising: At least from the 3rd century AD onwards hypothetical syllogistic had become a standard part of the logic syllabus, despite its not being discussed in Aristotle's logical works. As a result, in the handbooks or compendia hypothetical syllogisms would be discussed briefly after the categorical syllogisms, and in the commentaries usually in the context of <u>APr.</u> I 23 and I 29. It becomes thus understandable why, in a collection of manuscripts of the texts of the <u>Organon</u> we may find a section on hypothetical syllogistic appended to the <u>Analytics</u>.<sup>16</sup>)

- (1) The two types of hypothetical syllogisms that are distinguished here correspond very roughly to the ancient 'mixed hypothetical syllogisms' and 'wholly hypothetical syllogisms'.<sup>17</sup>) In this scholium the two types of syllogisms have been given no names, but are simply described. Boethius, in HS, does the same.
- ὑποθετικοῖς συλλογισμοῖς: this is Peripatetic nomenclature; presumably contrasted with categorical syllogisms (κατηγορικοὶ συλλογισμοί).
- οἱ ἐκ δύο ὅρων συνημμένων ἢ διαλελυμένων: these are the syllogisms that correspond, more or less, to the modi ponens, tollens, ponendo tollens, and tollendo ponens. The total absence in

<sup>15</sup> Cf. Waitz' comments before the scholium: "Simile additamentum in cod. Laur. 72, 5, de quo modo diximus, invenimus adiectum ad finem Anal. post. quod quamquam ab hoc loco plane alienum esse patet (agit enim de syllogismis hypotheticis), tamen valde dignum mihi visum est quod h. l. addam." For information, I also adduce Waitz' comments after the scholium: "neque verum hunc Analyticorum finem esse neque omnino scriptum ab Aristotele non est quod probemus iis, qui vel leviter noverint Aristotelis Analytica, sed ab homine compositum esse non solum artis logicae peritissimo, sed etiam ingenii acumine paestanti et e dicendi genere patere mihi videtur, quod ab scholiastarum ambagibus alienum est, et ex ipsa re, de qua nove clare apte disseruit. Quae ab Aristotelis usu recedant paucis annotare placet. ἐπάνοδος idem est quod apud Aristot. μετάληψις vel μεταλαμβανόμενον, de quo cf. quae diximus ad 41a39, συνάγων ὅρος vocatur μέσος ab Aristotele, συναγόμενοι - τὰ ἄκρα, τὸ συνημμένον - ἡ πρότασις. Propositionem autem εἰ τὸ Α, οὐ τὸ Β converti posse in hanc: εἰ τὸ Β, οὐ τὸ Α, non expresse quidem demonstravit Aristoteles, sed facile hoc colligitur ex iis quae tradit 57b1 vel 53b12."

<sup>&</sup>lt;sup>16</sup> Of course, there may also simply have been some blank parchment left at the end of the <u>Analytics</u> in an ancestor of our manuscript, which was then used by the author of the scholium, without there necessarily being any relation to the preceding or following text - as Sten Ebbesen points out to me.

<sup>&</sup>lt;sup>17</sup> Cf. e.g. Alcin. <u>Didasc</u>. 158.23-4, 159.7-30.

- the scholium of hypothetical syllogisms with a negated conjunction as first premiss indicates that the theory preserved in it is at least in part based on early Peripatetic material.<sup>18</sup>)
- οἱ ἐκ δύο ὅρων συνημμένων: τὸ συνημμένον (i.e. ἀξίωμα) was originally the Stoic term for conditionals, but was soon adopted by Peripatetics and Platonists. However, unusually, here the adjective is used for two terms that are connected by the conjunction 'if': 'if A, B'; possibly short for 'if something is A, it is B'. Boethius HS 2.1.7 provides a parallel: syllogismis quorum propositiones in conexione positae duobus terminis constant.<sup>20</sup>)
- διαλελυμένων: I have not found this expression used anywhere else for the description of disjunctive propositions. Terms used elsewhere for disjunctions and disjunctive relations include διεζευγμένον, διαζευκτικόν, διάζευξις, διάστασις, διαιgετικόν, διαίgεσις; Boethius has propositiones ... per disjunctionem and in disjunctione ... positi (HS 3.10.3).
- εἶτα οἱ ἐκ τριῶν: i.e. ὅρων, see (4). The only parallel description in an ancient source is Boethius' ex tribus terminis at HS 1.6.3. Some of the Aristotle commentators refer to these 'wholly hypothetical syllogisms' as 'through three' (διὰ τρίων) arguments or syllogisms (Alex. APr. 326.8-9, Philop. APr. 243.13-15).
- (2) The eight syllogisms in this passages are term-logical parallels to <u>modus ponens</u> and <u>modus tollens</u>, each given with the four permutations one obtains by taking the terms positively or negatively:

7

<sup>&</sup>lt;sup>18</sup> For the characteristics of early Peripatetic hypothetical syllogistic (as contrasted with later Peripatetic and Stoic theories) see my <u>Pre-Stoic hypothetical syllogistic in Galen</u>, in the Proceedings of the Symposium 'Galen beyond Kühn', ed. V. Nutton, forthcoming 2002.

<sup>&</sup>lt;sup>19</sup> Since the ms uses letters with lines above them for both ordinal and cardinal numbers (see above n. 9), and cardinal numbers were used by the Peripatetics and Platonists for terms, whereas ordinal numbers were used by the Stoics for whole propositions, further evidence is required for deciding between the two possibilities.

<sup>&</sup>lt;sup>20</sup> On this point see also my Wholly Hypothetical Syllogisms.

not A not A A A

In ancient logic, this eightfold classification of the <u>modi ponens</u> and <u>tollens</u> has a parallel only in Boethius <u>HS</u>, cf. 2.1.7 <u>quatuor enim fiunt per praecedentis positionem ... quatuor vero per sequentis negationem</u>. Boethius' order of the eight is the same as that in the scholium. An example for (i) would be: 'If <it is a> human being, then <it is an> animal; but <it is a> human being; therefore <it is an> animal.'

- ἐπάνοδος: the expression is used for denoting the co-assumptions in hypothetical syllogisms (usually  $\pi \rho \delta_5 \lambda \eta \psi_{15}$ ). Such use is to my knowledge not attested elsewhere. I assume that the word has been chosen because of its meaning of 'repetition'. In the entire literature on hypothetical syllogisms, I have only found one text which provides a parallel: this is the Latin translation of Averroes or [Averroes], in particular where he reports Al-Farabi's view on hypothetical syllogisms: ... in syllogismo conditionali ponuntur duae praemissae, quarum una est conditionalis et repetita est categorica. ... et illi sunt in rei veritate syllogismi conditionales, quorum repetitum et coniunctio est ignota ... Abunazar (i.e. Al-Farabi), quod non sit syllogismus conditionalis ille, cuius repetitum sit per se notum et coniunctio per syllogismum ... (Quaes. in Prior. Resol., f. 368. r. A, my italics).<sup>21</sup>) This parallel is noteworthy, since Al-Farabi wrote in the first half of the 10th century, which is close to the time to which the manuscript pages of the scholium have been dated. It also may be taken as corroboration of the claim that there was a Greek source on hypothetical syllogisms on which both Boethius and some Arabic commentaries depend. Boethius uses assumptio, which is Cicero's term for πρόσληψις (e.g. <u>Inv.</u> I 59, <u>Div.</u> II 108).<sup>22</sup>) However, interestingly, at <u>HS</u> 2.2.7 Boethius writes '... prior propositionis pars in assumptione repetitur' ... '23) which shows that the idea that in the co-assumption one part of the hypothetical premiss is repeated was known to him. We can imagine that the use of  $\frac{\partial \vec{r}}{\partial t}$  in its meaning 'repetition' arose from this view of the co-assumption.
- ἀποφατικῶς and καταφατικῶς: Peripatetic terminology; cf. e.g. Arist. <u>APr.</u> II.15 64a13-15;
   Boethius uses <u>affirmative</u> and <u>negative</u> (<u>HS</u> 3.3.2).

8

<sup>&</sup>lt;sup>21</sup> Cf. <u>Prior. Resol.</u> f. 83 v. A; cf. also Averroes (or [Averroes]), <u>Epitome</u>, f. 346 v. A, illa quidem, quae ex duobus imperfectis concludentibus, sunt duarum specierum, quarum una est <u>repetens</u> antecedens per se et concludens consequens per se, ... in secunda autem specie <u>repetitur</u> oppositum consequentis et concluditur oppositum antecedentis ... (f. 346 v. B, my italics).

<sup>&</sup>lt;sup>22</sup> As Boethius uses this term when drawing from Cicero at <u>HS</u> 2.1.1 - 2.1.6, we cannot say with certainty what the Greek was for the parallel passages to our scholium: even in the unlikely case that it had been  $\frac{\partial \hat{n}}{\partial t} = \frac{\partial \hat{n}}{\partial$ 

<sup>&</sup>lt;sup>23</sup> I am grateful to Christopher Martin for having reminded me of this passage.

- κατὰ θέσιν and κατὰ ἀναίφεσιν: Peripatetic terminology, see e.g. Philop. <u>APr</u>. 264-6, [Amm.]
   APr. 68.23-41.
- συμπέρασμα: used equally by Stoics and Peripatetics for 'conclusion', but in later antiquity reclaimed as Peripatetic, see Philop. <u>APr</u>. 242.30-1.
- (3) Only two modes of the disjunctive syllogisms are given:

$$\begin{array}{cccc} \text{(i)} & A \text{ or } B & \text{(ii)} & A \text{ or } B \\ & \underline{\text{not } A} & & \underline{\text{not } B} \\ & B & & A \end{array}$$

It is uncertain whether positive co-assumptions ('repetitions') with negative conclusions were also admitted; and accordingly, whether the permutations with positive and negative terms would produce eight or sixteen modes. One interesting point in this sentence is the fact that we have 'either A or B, but <u>it is</u> not B', since it suggests that we have an abbreviated version of '<u>it is</u> either A or B; but <u>it is</u> not B; hence <u>it is</u> A', etc. This confirms the assumption that the letters  $\bar{a}$ ,  $\beta$ , and  $\bar{\gamma}$  in the scholium are to be replaced by terms rather than propositions, and thus taken as cardinal, rather than ordinal numbers. It also finds a parallel in Boethius (<u>HS</u> 3.10.3 ff).

- ἐξ ἀνάγκης: the use of a phrase in a hypothetical premiss that indicates necessity (presumably necessitas consequentiae) is a mark of early Peripatetic theory, cf. e.g. Arist. APr. I 32 47a28-31, Top. B4 111b19. Boethius uses such phrases frequently, e.g. ex necessitate, necesse est, necessario (HS 2.9.1-3.6.4 passim).<sup>24</sup>)
- (4) From here to the end, the scholium presents the 'wholly hypothetical syllogisms' and their three figures. The eight possible combinations of the premisses (in the first figure) are again obtained by permutations with positive and negative terms:
  - (i) if A, B (ii) if A, B (iii) if A, not B if A, not B (iv) if not B, C if B, C if B, not C if not B, not C if not A, not B if not A, not B if not A, B if not A, B (v) (vi) (vii) (viii)

if not B, C

The only parallel which presents these eight premiss pairings - if in a different order - is Boethius HS 2.9.1-2.11.7.

if B, C

if B, not C

- ἐκ δὲ τρίων ὅρων συνημμένων: viz. in the premisses, with A and B connected in the first, B and C in the second.
- σύνθεσις and συντίθεσθαι: for the compounding (σύνθεσις) of terms see Alex. <u>APr.</u> 300.16, 317.28, 319.8. cf. also Boethius <u>HS</u> 3.1.1, <u>componentur</u>.
- ἐπάνοδος: if one expects syllogisms with the paradigm form

if not B, not C

9

<sup>&</sup>lt;sup>24</sup> Cf. also my Wholly hypothetical syllogisms, 92-3 and 113.

then the claim that the syllogisms come to be by repetition (or co-assumption) of the first or last term will seem distinctly odd. For no such repetition (or co-assumption) occurs in syllogisms of that form. There is however nothing odd when one envisages the underlying forms as

$$\begin{array}{ccc} \text{if A, B} & \text{if A, B} & \text{etc.} \\ \text{if B, C} & \text{if B, C} \\ \underline{A} & \text{not C} \\ \text{hence C} & \text{hence not A} \end{array}$$

- (5) For similar accounts of the first figure cf. Alex. <u>APr.</u> 326.26-9, Philop. <u>APr.</u> 302.16-19.
- συνάγων ὅρος / συναγόμενοι ὅροι: these expressions are used to denote the common (κοινός) term and the extreme (ἄκρα) terms. The terminology is not known from other ancient texts on hypothetical syllogisms, but one can imagine how it came about. συνάγειν was taken in its meaning 'to bring together', 'zusammenführen' (so e.g. at (9)). The middle term is then the term that brings together the extreme terms, and the latter are brought together in the conclusion of an inference (cf. e.g. Alex. APr. 316.6). (Cf. Boethius, HS 1.9.1, namque ut fiat extremorum conclusio, medius terminus intercedit, cuius communitas extrema coniungit.)

-

<sup>25</sup> But why then the formulation 'εἰ τὸ A, ἄρα τὸ C' instead of 'τὸ A δέ, ἄρα τὸ C'? (Cf. Boethius HS 2.9.7 at vero est a, non est igitur c). These formulations - which seem to take the whole conditional sentence as belonging to the conclusion, but only the ἄρα-clause as inferred - would tally with the view of those ancient logicians who held that in a conditional proposition the consequent is asserted on the supposition of the antecedent (cf. Amm. APr. 17.19-24 and 27.7-11 and my Propositional logic in Ammonius in: H. Linneweber-Lammerskitten, G. Mohr (ed.) Logische Analyse und systematische Philosophie, Festschrift for Gerhard Seel (Würzburg 2001) forthcoming). The difference between such a view and the one which interprets the 'antecedent' as co-assumption is logically significant; however, one can easily imagine someone to move from one to the other without realizing this.

<sup>&</sup>lt;sup>26</sup> This meaning of συνάγειν is different (i) from that in passages such as Alex. <u>APr.</u> 45.1 and Philop. <u>APr.</u> 256.16, where, e.g. in a syllogism, the middle term either conjoins or disjoins (διιστάναι, χωρίζειν) the

- μέσος ἐστι: this expression is used solely in the first figure, where the common term has indeed middle position.  $\mu \dot{\epsilon} \sigma \sigma_{\xi}$  is here used descriptively, not as name of the common term.
- έν τῶ προτέρω συνημμένω: here το συνημμένον is used in its meaning 'conditional', as in Stoic and later Peripatetic logic, not as earlier in (1) - (4), meaning 'connection', as conexio in Boethius' HS. The author of the scholium thus uses συνημμένος in two different, if related, ways. (Boethius, too, talks about the two conditionals in the syllogisms from three terms: ex tribus terminis ... quasi ex duabus conditionalibus ... tres quidem termini hi ... duae vero conditionales hoc modo, HS 1.6.3.)
- ήγούμενος and λήγων: this originally Stoic pair of terms for the antecedent and consequent in conditionals is found in Peripatetic and Platonist texts of the second and third century AD (e.g. Alcin. Didasc. 159, Alex. Top. 331.11-12, APr. 327.17-18), but is later usually replaced by the pair  $\dot{\eta}\gamma oi\mu = vos$  and  $\dot{\epsilon}\pi \dot{o}\mu = vos$  (cf. [Amm.] APr. 68.4-8, Philop. APr. 242.29-32 and 243.4-7).
- αναπόδεικτοι: it is unclear whether all sixteen modes of the first figure are indemonstrables, or only the first eight. (In principle, the second eight could be reduced to the first eight by contraposition of the conclusion.)<sup>27</sup>)
- ἀναλύσεως: the reduction is from second and third figure syllogisms to the indemonstrable first figure ones. (Cf. also Alex. APr. 7.24-5, 'reducing imperfect syllogisms to perfect ones is called analyzing', which is the seventh in Alexander's list of eight types of analysis.)

(6) and (8): The eight possible combinations of the premisses of the second figure, again obtained by permutations with positive and negative terms, are:

*(:::*)

*(::*)

*(*:)

(1)	if A, B if not A, C	(11)	if A, B if not A, not C	(111)	if A, not B if not A, not C	(1V)	if A, not B if not A, C
(v)	if not A, B if A, C	(vi)	if not A, not B if A, not C	(vii)	if not A, B if A, not C	(viii)	if not A, not B if A, C .
Those of the third figure are:							
(i)	if A, B if C, not B	(ii)	if A, B if not C, not B	(iii)	if A, not B if C, B	(iv)	if A, not B if not C, B
(v)	if not A, B if C, not B	(vi)	if not A, not B if not C, B	(vii)	if not A, B if not C, not B	(viii)	if not A, not B if C, B.

extremes, the former in affirmative conclusions, the latter in negative ones; (ii) from the standard sense of inferring, as in a deduction, where the conclusion is inferred from the premisses, or in a conditional the consequent from the antecedent.

<sup>&</sup>lt;sup>27</sup> Boethius, at HS 2.9.4-5, seems to hold that the first figure syllogisms need reduction to syllogisms of a different order. Perhaps he wanted 'if A, B; if B, C; A; therefore C' to be reduced to 'if A, B; if B, C; therefore if A, C'.

For the accounts of the second and third figures there are no close parallels, but cf. Alex. <u>APr.</u> 327.15-18, 7-1, and especially Boethius <u>HS</u> 3.1.1-2 (<u>inaequimoda propositio</u>) and 3.4.2. Note that the accounts are of second and third figure <u>syllogisms</u>, i.e. of formally <u>valid</u> arguments. The order of the second and third figure is as in Boethius, and as reported for Theophrastus, but differs from that in Alexander, Alcinous, Philoponus.<sup>28</sup>)

• σχέσις: used here to refer to the position of the term.

(7) and (9): The reduction of a second figure syllogism to the first figure consists basically in taking the contraposition of its first premiss, and leaving the rest as it is; that of a third figure syllogism to the first figure in taking the contraposition of the second premiss, and leaving the rest as it is. The description of the reductions in the scholium is awkward. That of the second figure seems corrupt. I have restored it in parallel with the description of that of the third figure, where the reduction runs thus: 'the contraposition of the second conditional (if B, not C) is taken after the first conditional (if A, B), and the antecedent <term> of the first conditional (A) is conjoined to the contradictory of the antecedent <term> in the second (not C)'.

- $\dot{\epsilon}\nu a\lambda\lambda \dot{a}\xi$ : i.e. once affirmatively, once negatively, see section (6).
- ἀντικείμενον: the term is used both for 'contradictory' and for 'contraposition', although the latter is usually called ἀντιστροφή; contraposition is here of terms, not propositions.<sup>29</sup>)
- $\delta \pi \epsilon \varrho$   $\delta \delta \epsilon i \delta \epsilon i \delta \epsilon i \delta \epsilon i$ : a formula originally used in mathematical treatises.

to be replaced with whole sentences, not with terms. 'If it is day, it is light' is a standard example used by the Stoics and generally in later ancient logic. 'If it is day, the sun is above the earth' is found from the second century AD onwards (e.g. Galen Inst.Log. 9.1-2, Gellius Noct.Att. 16.8.9, Simpl. Ench. 432 Hadot). I have not found 'if it is light, the visible things are seen' and its variations anywhere else yet. The example of the second figure has a false second premiss and a false second conclusion. This is rare in ancient logic. Perhaps someone removed the negations in the consequents in order to obtain a second figure example in the first mode. Section (10) may be a later interpolation, since (11) goes back to discuss the second and third figures only. This assumption would square with the facts (i) that sections (1) to (9) and (11) are concerned with terms, whereas (10) appears to consider whole propositions, and (ii) that in section (10) 'hence'

<sup>&</sup>lt;sup>28</sup> Cf. on this point my Why the order of the figures of the hypothetical syllogisms was changed, CQ 50.1 (2000), 247-51.

<sup>&</sup>lt;sup>29</sup> For contraposition of terms cf. e.g. Arist. <u>Top.</u> 113b15-27, Alex. <u>APr.</u> 326.27-327.2; 327.23-35; 328.24, Galen <u>Inst.Log.</u> 6.4, Philop. <u>APr.</u> 46.17-19 and my <u>Wholly Hypothetical Syllogisms</u>, 23-4 and 31-2.

 $(\ddot{a}\varrho a)$  is positioned in the antecedent of the conclusion, whereas in (4) to (9) it introduces the 'consequent' (see above).

- üλη: this expression is used by Peripatetics and Platonists in the context of logic for concrete examples (as opposed to letters), cf. e.g. [Amm.] <u>APr.</u> 68.33-6.
- (11) This last sentence returns to the second and third figures: in line with the way they were introduced in sections (6) and (8), it is now said that there are eight modes per figure. It seems hence as if sixteen valid modes have been left out (eight from the second and eight from the third figure), namely those one obtains by contraposition of the conclusion from the sixteen ones described. How can this be explained? The first sentence of (4) implied that there should be sixteen syllogisms in each figure. Thus, perhaps, there are eight modes  $(\tau \varrho \acute{o}\pi o\iota)$  per figure, but sixteen syllogisms? This would mean that the modes are fully determined by the types of premiss pairings, and the syllogisms (or valid argument forms) by these together with the types of conclusion.
- μετὰ ὁμοίου ἢ ἀνομοίου τιθεμένου τοῦ συνάγοντος: in line with sections (6), (7), and (8), this should mean 'posited once affirmatively, once negatively'.

#### Conclusion

In the commentary section it should have become increasingly apparent that the anonymous scholium on hypothetical syllogisms in Waitz is Peripatetic, and not Stoic, in its theoretical approach as well as its terminology. There are several elements of early Peripatetic hypothetical syllogistic preserved in it, although section (10) is likely to be witness to a later development of Peripatetic or Platonist hypothetical syllogisms. The most striking feature in the scholium is the large number of close parallels to Boethius' <u>De Hypotheticis Syllogismis</u>. Since it is rather unlikely that the scholium is based on a Latin source, we can assume that there must have been a Greek source from which both the scholium and large parts of Boethius' <u>De Hypotheticis Syllogismis</u> are ultimately derived.

I conclude with a list of the idiosyncrasies of the scholium, which may help to identify its author, or the school to which the author belonged, or to define the time of composition of the scholium more accurately: (i) the use of  $\delta\iota a\lambda\epsilon\lambda\nu\mu\dot{\epsilon}\nu o\varsigma$  for disjunction, (ii) the use of  $\delta\varrho o\varsigma$  συνάγων and  $\delta\varrho o\iota$  συναγομένοι for middle and extreme terms, (iii) the use of  $\dot{\epsilon}\pi\dot{a}\nu o\delta o\varsigma$  for co-assumption, (iv) the use of  $\dot{a}\nu\tau\iota\kappa\dot{\epsilon}\iota\mu\epsilon\nu o\nu$  for contraposition, (v) the example of a conditional 'if it is light, the visible things are seen'.