

## Logic and Existential Commitment

### Introduction

The idea that logic is distinguished from other disciplines by its complete lack of existential commitment has a long history of support, and is both familiar and widely accepted today. Kant, for example, claims that the ontological argument cannot be correct because you can't get existential assertions out of logic (and logic plus definitions). Frege and Russell ultimately abandon Logicism because they take mathematics to involve existential commitment and they grant that logic does not. In a recent article, a well-known philosopher claims that, "the idea that logic is free from existential commitment is so ingrained in the contemporary understanding of logic, that it is hard to see how it could—relevantly and soberly—be challenged."<sup>1</sup>

The main job of this paper is to make clear how the idea that logic is free from existential commitment can be relevantly and soberly challenged. I begin by presenting an approach to the conceptual underpinnings of the modern definition of logical consequence in first-order predicate logic which makes logic rely on substantive facts about the world. I will present this account in some detail in order to convince you that it is not perverse or unmotivated. In fact, the account is a modernized version of Tarski's account of logical consequence, is extensionally equivalent with the model-theoretic account, and truncated versions of it appear in formal logic textbooks as a means of motivating the model-theoretic account. I shall then sketch a rationale for adopting this approach to first-order consequence by considering the alternative approach based on denying first-order logic any existential commitments. Divorcing the foundations of logic from ontology is plagued with substantial epistemological difficulties, and at present the advantages of doing this are unclear. I conclude by highlighting why this should increase the comfort level for thinking that ordinary first-order logic depends on substantive facts about the world.

### The Possible Meaning Account of Logical Consequence in First-Order Logic

The conclusion of an argument is a logical consequence of its premises (or, equivalently the argument is valid) if and only if it is impossible for the conclusion to be false while all the

premises are true. To understand logical consequence we must understand how it is possible for sentences to have truth-values other than the ones they actually have. If the conclusion of an invalid argument is *Bill Clinton is a human* we must think that this sentence could (logically) be false. How could *Bill Clinton is a human* be false? Since truth depends both on the use of words and the way the world is, there are two ways of understanding possible truth and falsity

We can say that *Bill married Hillary* could be false and *Bill married Maureen* true if the words *Bill*, *Hillary*, *Maureen*, and *married* are used ordinarily but the world were different (imagine that Bill met Ronald Reagan's daughter at a party while Bill was at Yale, fell in love, and married her.) Alternatively, *Bill married Hillary* could be false and *Bill married Maureen* true if the world is as it is but the words *Bill*, *Hillary*, *Maureen*, and *married* were used differently (imagine that *Bill* and *Hillary* refer to 1, *Maureen* to 2, and *married* to the less than relation.)

This second way of changing a sentence's truth value leads to what might be termed the possible meaning (PM) account of logical consequence: the conclusion of an argument is a logical consequence of its premises iff there is no possible use or meaning of its constituent non-logical elements under which the premises are true and the conclusion is false.<sup>2</sup> To establish the invalidity of an argument whose conclusion is *Bill Clinton is a human* we are not required to search out possible worlds where this is false (arguably such a world cannot be found), but merely appeal to, say, the fact that *Bill Clinton* could be used to refer to the number six, and *is a human* could be used to mean *is a prime number*. This is significant because it is easier to say what uses are possible for terms than it is to say what worlds are possible.

Possible uses for terms are not all that strange. The connection between a word and its ordinary use, is, after all, a matter of convention: *married* didn't have to mean married. And many words have more than one use that is possible relative to the conventions of English. For example, indexicals like *you* and *here* change their referent with the occasion of use; there are many people named *Bill*; *married* can mean *was married to* or *performed the marriage of*, and restrictions on the domains of quantifiers (like *Everybody*) are determined contextually from one

use to another. Think of the different ways *everybody* can be used in the sentence *everybody is looking for you*.

If there were no constraints on possible use (and the meanings of logical terms could change), then every argument would be invalid. Hence, the need to develop some constraints. Consider how we discover the truth values of sentences in which words are used normally. To know the truth value of, say, *Bill married Hillary*, we must know (1) that *Bill married Hillary* is true iff the individual named by *Bill* stands in the relation named by *married* to the individual named by *Hillary*, and (2) what these individuals and relations are.

Knowledge of type (1) is structural knowledge, i.e. we process it as a relational sentence where the proper nouns refer to individuals and ‘married’ to a relation between individuals. To obtain knowledge of type (2), we appeal to the conventions of English and to context to supply the individuals and the relation. The conventions of English narrow the possible relations to just a few (look up *marry* in the dictionary); we also know that *Bill* is used to refer to males and *Hillary* to females. Context then filters out all but one pair of individuals (or so we hope.)

So, the syntactic classification of an expression fixes a broad range of possible uses for it due to the fact that syntactic elements have more or less well-defined semantic roles. A name like *Bill* is broadly used to refer to objects, and the relational predicate *married* is broadly used to refer to a relation between objects. This range is narrowed by the specific conventions of English and dictionary definitions. Formal logic takes the syntactic classification of an expression to be basic in characterizing its possible uses. In other words: the range of possible uses for an expression will be fixed solely by its syntactic classification; as such, these uses will go far beyond what is possible according to the conventions of English; e.g., *Bill* could refer to 1 and *married* to the *less-than* relation.

Moving from expressions to sentences, formal logic also takes structural constraints as basic in characterizing possible uses for sentences. We imagine that a sentence has a structure, as this is determined by the logical constants that appear, if any, and the pattern of the remaining

non-logical elements. The structure of a sentence determines how its unstructured parts (or elements) may be used in relation to one another and how the truth or falsity of the sentence depends upon such a coordinated use of elements. A possible use will be any coordinated use of the elements of a sentence consistent with its structure. For example, taking ‘and’ to be the logical constant in *Bill is a human and Bill married Hillary* determines that it is only true on possible uses for *Bill is a human* and *Bill married Hillary* according to which they both are true. A possible use for a sentence will be any coordinated use of its non-logical elements consistent with its logical structure. Hence, there is no possible use for *Bill is a human and Bill married Hillary* which treats ‘and’ as an *or*, because such a use ignores the sentence’s logical structure and allows uses for the sentence according to which it is true when, say, *Bill married Hillary* is false. This is inconsistent with the fact that *Bill is a human and Bill married Hillary* has a conjunctive form.

In terms of such a picture, an argument is valid if there is no possible use of the non-logical elements in the sentences of which it is comprised for which, in accordance with their structure, the premises are true and the conclusion is false. In order to ascertain whether a given argument is *PM*-valid, analyze the constituent sentences down to their elements and then ask whether there is any use of these elements according to which the premises are true and the conclusion false. If so, then the argument is invalid, the use constituting a counterexample in which the premises are true and the conclusion false. If not, the argument is valid. Obviously, this makes validity relative to the structures we are prepared to see in sentences. Hence the need to develop a catalog of the types of structures we’re prepared to see in English sentences (e.g., truth-functional, subject-predicate, identity, descriptive, and quantificational structures.). In first-order logic, the logical structure of a sentence is its composition out of the classical logical constants and the pattern of its non-logical elements. Sentences from a standard first-order language represent the logical structures (or logical forms) of corresponding ordinary language sentences. The analog of the notion of a possible use of elementary English expressions is an interpretation of a formal language.

In practice, the determination of *PM*-validity boils down to the semantic procedure found in most elementary logic textbooks. For example, consider argument *A*.

*George W. Bush is a Democrat or Al Gore is a Republican.*  
*Al Gore is a Republican*  
*So, George W. Bush is not a Democrat*

*A* is invalid. How can we establish the possibility of true premises and a false conclusion? Well, we first analyze the constituent sentences into elements whose use determines (via definite semantic rules) the truth or falsity of the sentences. Then we identify a possible use of these elements which renders the premises true and the conclusion false.

Taking *or* and *not* to be the only logical constants in the argument, the non-logical elements are: *George W. Bush*, *Al Gore*, *is a Democrat*, and *is a Republican*. Since the argument is invalid, there should be possible uses of the proper nouns and predicate phrases which render the premises true and the conclusion false. On the proposed reading of “possible”, such uses are given merely by associating any object with each proper noun and any property with each predicate phrase. Let the proper nouns retain their meaning, and make *is a Democrat* mean *is a Texan* and *is a Republican* mean *is a Tennessean*.

Note that the *PM* account of logical consequence recognizes possible uses for quantifiers in fixing the extension of logical consequence.<sup>3</sup> For example, the quantifiers in *there are at least two individuals* ( $\exists x \exists y (x \neq y)$ ) can be used to range over the natural numbers or they can be used to range over, say, the offspring of Bill and Hillary Clinton.<sup>4</sup> The fact that the quantifiers are logical constants suggests that the part of their meaning that is fixed on all possible uses of them is the portion of the domain they pick out (e.g., the existential quantifier must refer to at least one member of the domain.) The non-logical element in the meaning of a quantifier is the size of the relevant domain, and therefore it is this that can be varied in order to make a quantification true or false. So, to think that *there are at least two individuals* could logically be false, we are not committed to thinking that there might have been just one thing in the world, rather we are

committed to there being a use for the quantifiers according to which the sentence is, in fact, false (e.g., they could range over, say the offspring of Bill and Hillary Clinton).

I have gone into some detail in presenting the *PM* characterization of logical consequence in first-order logic with the hope that you will appreciate its explanatory power. The *PM* account reduces the validity of an argument to the fact that it preserves truth on all interpretations of its non-logical elements in every (actual but possibly restricted) domain of quantification. The modal notion in the concept of logical consequence gets reduced to two familiar notions: truth and a generalization over possible uses for the non-logical elements of a first-order language. Who could not want this explanation of logical consequence to succeed? The *PM* account is extensionally identical with the standard model-theoretic definition. In fact, the *PM* account can be found in the introduction to formal logic textbooks, where it is used as a means of motivating the later more abstract model-theoretic definition. On my view, the standard model-theoretic account of validity is best viewed as a natural development of the *PM* approach to logical consequence.

By not making what is logically possible turn on the mysterious modal notion “ways the world could be,” the *PM* account explains our knowledge of what is valid by linking our judgments of what is logically possible to our judgments about worldly states of affairs and about possible uses for sentences. I know that *Bill Clinton is a human* could be false, because I know that *Bill Clinton* could be used to refer to six and *is a human* could designate *is a prime number* and I know that, in fact, six is not a prime.

### **A Criticism of the PM Account of Logical Consequence**

So, the epistemic accessibility of *PM*-validity is partly based on the fact that its determination requires existential commitments. To determine whether or not a sentence could logically be false, its logical form directs us to the relevant part(s) of reality that need to be investigated in the search for a falsifying possible use of its non-logical elements. But then our judgments of what is logically possible (and, consequently, our judgments of what is valid) turn

on our perceptions of substantive facts about the make-up of the world. Let's look at some further examples.

The denial of *everything is created by God or nothing is created by God* is not a contradiction because there is a possible use for *is created by God* according to which it is true of something and not of something else (e.g., let it mean, *invented by Thomas A. Edison*). However, the fact that this sentence could logically be false not only turns on the semantic functioning of *everything*, *nothing*, and *or*, but also on the metaphysical fact that there exists at least two things. To be somewhat melodramatic, if Parmenides is correct and there is only "The One," then there is no possible use of *is created by God* according to which it is true of one thing and not another. In such a case the denial of this sentence would be inconsistent, and consequently a *PM*-logical truth. So, the determination that the denial of this sentence is logically possible requires a commitment to the existence of more than one thing.

Here is a different example that makes the same point. Consider the following argument (B)

*Bill Clinton is President and a Democrat*  
*Hillary is not President*  
*So, Al Gore is not President or he is a Democrat.*

Obviously, *B* is invalid: we are in no way justified believing the conclusion on the basis of the truth of the premises. Taking *and*, *not*, and *or* to be the only logical elements, *B* has an invalid argument form: there exists an interpretation for *President*, *Democrat*, *Bill Clinton*, *Hillary*, and *Al Gore* according to which the premises and the denial of the conclusion are, in fact, true (e.g., let *Democrat* mean *saxophone player* and *President* mean *male*).

Such an interpretation of the non-logical elements is not possible if the world contained just two objects, or if all the world's objects fell into two indistinguishable types. If such states of affairs obtained, *B* would have a valid argument form. Hence, the determination that *B* is *PM*-invalid relies on knowing that there are more than two individuals and that the world is not that homogeneous.

Let's look at another example of the way metaphysics penetrates logic on the PM-account. Consider argument *C*.

*Nothing is taller than itself.*

*If  $x$  is taller than  $y$  and  $y$  is taller than  $z$ , then  $x$  is taller than  $z$ .*

*Therefore, there is a tallest object (i.e., an object which is at least as tall as everything else.)*

The premises of *C* tell us is that the *taller than* relation is irreflexive and transitive, and the conclusion says that there is something which is a minimal element of *taller than*. A minimal element is something that is at least as tall as everything else (a relation may have more than one minimal element). The premises are true and (let us suppose) the conclusion is also true. Argument *C* is invalid iff there exists an interpretation of *taller than* which makes it refer to a relation *R* that is irreflexive, transitive, and does not have a minimal element (e.g., in order for *taller than* not to have a minimal element it would have to be the case that each thing is shorter than something or other.) Again, we must look to the world in order to get a counterexample to the argument. Since the extension of an irreflexive, transitive relation that does not have a minimal element must be denumerably infinite, the invalidity of argument *C* depends on the existence of an infinite collection of first-order particulars. If there is not an existent infinity, then *C* is valid.

These examples illustrate that fixing the extension of *PM*-validity in first-order logic requires a prior investigation into the cardinality of the universe of first-order particulars and into the actual extensions of properties and relations. However, this does not force us to regard formal logic as empirical and logical judgments as a posteriori. All that we need to do is to appeal to set-theoretic reality as described by a standard theory like Zermelo-Fraenkel set theory to secure, *a priori*, the correct and necessary extension of validity in first-order logic.

For example, by the axiom of infinity, there exist things (pure sets)  $\alpha$ ,  $\beta$ , and  $\delta$ . The pair-set axiom tells us that for any  $x$  and  $y$ , there is a set that has  $x$  and  $y$  as elements. So, there exists the sets:  $\{\alpha\}$ , and  $\{\alpha, \delta\}$ . We can appeal to these facts to show the invalidity of *B*: let *Democrat*



designate  $\{\alpha\}$  and *President* pick out  $\{\alpha, \delta\}$ , and make *Bill Clinton*, *Hillary*, and *Al Gore* refer to  $\alpha, \beta$ , and  $\delta$  respectively. Argument *C* is invalid because there exists an infinite number of pure sets, and this totality is necessarily structured by a relation that does not have a minimal element (e.g., *is larger than*).

Since set-theoretic truths and truths about possible meanings are knowable *a priori*, the question of the *PM*-validity of a first-order argument is in principle decidable *a priori*.<sup>5</sup> Also, pure sets necessarily exist and their structural features are essential. So, we are guaranteed a sufficient range of possible uses for the non-logical elements of a first-order language to secure standard first-order logic. To the extent that set-theoretic truths are topic neutral, we preserve the topic neutrality of classical logic.

But even if it is granted that the truths of logic are, in a sense, still *a priori* and necessary (and even topic neutral), the fact is that the *PM* approach makes logic substantial. What cannot be denied is that we fix the extension of *PM* validity by looking to the world. But, as Wittgenstein says in the *Tractatus*, “Whenever a question can be decided by logic at all it must be possible to decide it without further ado. (And if we get into a position where we have to look at the world for an answer to such a problem, that shows that we are on a completely wrong track.)”<sup>6</sup>

It is not uncommon to think that the significance of the logical consequence relation lies precisely in the fact that its determination is antecedent of metaphysical inquiry. It is said that “logic is about more than just what is; it is also about what might be” and “...we value the logical consequence relation because it tells us something about the limits on how things *can be*.”<sup>7</sup> And Etchemendy remarks that “if the assessment of [validity] rests on substantive facts about the size of the universe, then ...it would have been wrong had the facts in question been otherwise. This is damaging to the claim to have captured the ordinary concept of [validity].”<sup>8</sup>

The import of these remarks can be understood in terms of the idea that we use logic to extend our knowledge about the world. We rely on logic to help formulate our scientific and philosophical theories, and to determine their consequences. Hence, logic is an important tool in deciding what to believe about the world. In order for logic to do this, knowledge of what is logically possible must be prior to knowledge of actual states of affairs, and, therefore logic is prior to *all* extra-linguistic fields of study. To paraphrase Russell, this is the lordliness of logic which the logician should preserve.<sup>9</sup> But on the *PM* approach, this is not preserved: logic does not take an absolute epistemic priority over all other knowledge. Rather, there is an interaction between logic and knowledge of set theory. Logic is of little help in deciding between standard and non-standard set theory when its very use relies on standard set theory.

Let me put the criticism of the *PM* account in a more concrete form. Take finitism to be the position that the non-existence of an actual infinity is necessary in a metaphysical sense. Can the finitist rationally hold that argument *C* is invalid? The logic-is-lordly camp answers yes because the source of the intuition that *C* is invalid is not some conviction about the number of individuals in the world. Argument *C* would still be invalid even if the universe were finite. But on the *PM* account, the issue of whether or not *C* is valid is not independent of the issue of whether or not there is an existent infinity. The *PM* theorist must think that it is irrational for the finitist to accept the invalidity of argument *C*. Against this, the logic-is-lordly camp think that the finitist's denial of the (metaphysical) possibility of an existent infinity is substantive (non-logical), and therefore is irrelevant to the determination of *C*'s invalidity.

The criticism here of the *PM* account is based on a rejection of any explanation of logical consequence that supervenes logical possibility on the extensional. A supervenience account of logical consequence requires a choice of an ontology by means of which we represent logically possible situations and fix the extension of validity. But if logic is supposed to fix the limits on the ways reality could be, then a decision about whether or not there is an existent infinity cannot be prior to a decision about what is logically possible. The question is: how we are to understand

a *logically possible situation* in a way that does not make this appeal to the possibility of an existent infinity carry existential commitment? As understood here, the logic-is-lordly camp thinks that an argument is valid only because there is no way the world could be which would make the premise(s) true and the conclusion false. How are we to understand the modality in *there is no way the world could be*?

### Logical Primitivism

One way of proceeding is as follows. It is wrong to think that the extension of validity is *a priori* because knowledge about the existence of sets is *a priori*. Rather, judgments about what is valid and invalid are *a priori* because they are prior to knowledge about extra-linguistic existents (e.g., pure sets, bicycles, Bill and Hillary, etc.) But then we must ground the invalidity of an argument like *C* on an intuition of a logically possible state of affairs in which the universe is infinitely large. In terms of possible worlds: *C* has a counterexample because, in part, *is taller than* has an infinite extension in a logically possible world. This logically possible world is basic: it does not supervene on actual states of affairs or on what is metaphysically possible. After all, the lordliness of logic demands that we do not make logic hostage to the resolution of issues about what is the case or about what might—in a metaphysical sense—be the case.

According to this way of thinking, a conceptual analysis of validity must appeal to possible worlds in order to preserve the lordliness of logic. We can reflect this by amending the *PM* characterization of validity in first-order logic so that it includes an appeal to possible worlds: an argument is valid because there is no possible world in which the conclusion is false while all the premises are true on any possible use of the argument's non-logical elements.

The extension of validity is now determined on the basis of primitive (logical) intuitions that (1) an existent infinity is possible and (2) it is possible that it be heterogeneous enough to secure the possible uses for predicates and individual constants required to get first-order logic, i.e., arbitrary objects and sub-collections of this totality can serve as the extensions of names and predicates of a first order language. In order for these modal intuitions to do the philosophical

work required of them, they must be primitive and not reducible to judgments about the actual or metaphysically (and epistemological) possible features of the world.

So I believe that in order for the modal assumptions necessary to ensure the proper yield of validity to be non-substantial, a basic, primitive notion of logical possibility must be countenanced in defining validity. This is arguable. For example, one might think that it is inappropriate for purposes of logic to individuate possible uses for expressions **given** the actual state of the world. Suppose it is a fact about the relational predicate *is taller than* that, given the type of expression it is, it is capable of having an infinite extension. If the actual world is finite, this fact about the possible use of *is taller than* will not be captured. To capture this semantic possibility we must appeal to semantically possible worlds. To say that there is an infinite semantically possible world is just to say that nothing about the semantic functioning of language disallows infinite quantifier domains, or rules out a possible use of a predicate with an infinite extension. We can think that this claim carries no commitments about whether our world is infinitely large or whether a world with infinite cardinality is really (metaphysically) possible. Here logical possibility gets reduced to semantic possibility and the determination of what is logically possible requires no existential commitments.

I reject this because I do not believe that the appeal to possible uses of the non-logical elements in a language can be divorced from an appeal to substantive facts (modal or otherwise) about the language-independent world. I am inclined to think that possible uses for linguistic items are limited by the possibilities for real change. The possibility of using *is taller than* to pick out an existent infinity is parasitic on the possibility of an existent infinity. On my view, facts about the possible semantic values of expressions are not independent of ontological commitments. The possible uses of expressions are assignment of objects and sets of objects to the language's singular terms and predicates. That these meaning assignments are successful depends on the existence of these objects and sets of objects. So, the very notion of a possible use or meaning of a term relies on substantive claims about the world.<sup>10</sup> There is nothing about

the semantic functioning of *Bill Clinton* qua individual constant that rules out it designating an impossible thing like a circular square. But *Bill Clinton* cannot be used to refer to a circular square or to the greatest prime number because such things cannot exist.<sup>11</sup>

So in my view, it is inconsistent for the finitist to maintain that there is necessarily a finite number of objects in the world and hold that argument *C* is invalid because *is taller than* could be used to pick out a relation with an infinite extension. Again, whether, *is taller than* could have had an infinite extension is not independent of whether an infinite number of objects could exist. Since she thinks that such a totality is impossible, the finitist cannot countenance a possible use for *taller than* which designates such a totality.

It is one thing to say that first-order logic must dispense with the idea of providing existential commitments for our judgments of validity, but it is entirely something else to accept modal intuitions about an infinite totality and its heterogeneity as a brute datum not susceptible of further explanation or justification, and intelligible without reference to anything beyond themselves. This would place first-order logic within the sphere of empirical psychology, and satisfy nobody genuinely concerned with the credentials of logic.

So, logical primitivists owe us an account of how we could come to understand what is logically possible, as we in fact do, independently of our mathematics and science. For example, do we follow Kant and attribute these basic modal intuitions to the a priori structure of the understanding. Well, this makes logic psychologistic; not a popular view since Frege's devastating criticism. Russell holds that the logically possible truth of the axiom of infinity is more epistemically transparent than its actual truth. In fact, he admits that he does not know whether or not there is an existent infinity. But then the truth conditions for its possible truth must be *distinct* from the truth conditions for its actual truth. In order to account for this distinction must the possible existence of an infinite totality be unpacked (*a la* David Lewis) in terms of the existence of an infinite totality of possible (non-actual) objects? If so, then we can say that the latter's existence is more epistemically transparent than the existence of an infinite

totality of actual objects. Surely a Platonic insight into a Meinongian realm of logically possible objects forces existential commitments on us that we can do without.

Perhaps a phenomenological account of the relevant intuitions would be useful. I cannot get into this here other than to say it is unclear to me what the specifics of such an account would look like. For example, it is problematic to claim that the mere apprehension of the proposition expressed by a token formulation of the axiom of infinity gives rise to the intuition that it could logically be true. The possibility of the axiom of infinity is not so self-evident.

At the very least, it seems a little far-fetched for a person to claim a “primitive”, pre-theoretic status for the notion of logical possibility after learning about logical possibility in high-powered courses in mathematical logic. My students do not arrive to logic class with full-bodied intuitions about logical possibility. Indeed, the vast majority of my intelligent, symbolic logic students find the possibility of a completed infinity counterintuitive. Compare this with the more refined intuitions of mathematicians. Offhand, I think that a good case can be made for thinking that intuitions about the possibility of an existent infinity are informed by one’s mathematics. At any rate, to echo a point made by Stewart Shapiro,<sup>12</sup> without an account of how we come by these allegedly basic logical intuitions it is empty to use the word “primitive.” To the best of my knowledge, such an account has yet to appear.

Furthermore, it is far from obvious to me that an irreducible *logical* intuition about the possibility of an existent infinity is epistemically safer than one about the actual existence of such a totality. Suppose that the intuition that the axiom of infinity could logically be true is basic and non-substantial. Couldn’t the intuition still be erroneous? Profound and perceptive thinkers from the Greeks to the Intuitionists have questioned the coherence of the notion of an actual infinity. Many others think that truth of the logical possibility of the axiom of infinity is not self-evident, and that the required evidence is less than conclusive. Hartry Field, for example, points to the fact that the inability to derive a contradiction from fairly rich mathematical theories in which the axiom of infinity or (its equivalent) is a theorem (e.g., Zermelo-Fraenkel or Von Neumann-

Godel-Bernays) counts as evidence for the possible truth of the axiom of infinity.<sup>13</sup> But this is inductive evidence about what is derivable in a theory, and the claim that the axiom of infinity is logically possible has the epistemic status of a hypothesis.

Given that there is no proof that the concept of an existent infinity is contradiction free, the logical primitivist must countenance the possibility that the concept, in fact, harbors a contradiction in which case arguments like *C* are valid after all. There is no guarantee that *C* is invalid.<sup>14</sup> The epistemic transparency of the possibility of an existent infinity is not obviously entailed by the fact that it is non-substantial. If we grant—as it is becoming increasingly common to grant—that calling the intuition of a possible infinity “logical” does not *ipso facto* make it epistemically privileged *vis-à-vis* intuitions about the existence of an infinite totality, then one will be comfortable with the guarantee of *C*’s invalidity offered by the *PM* account and, perhaps, wonder what all the fuss is about with thinking that existential commitments are required to ensure the proper yield of validity in first-order logic.

## Conclusion

To demarcate what is valid from what is invalid in first-order logic, we must appeal in the logical metatheory either to the actual existence of an infinite totality and the extensions of properties in this totality or appeal to the mere possibility of such a totality and the merely possible extensions of properties. The *PM* account is aligned with the school of thought, which denies that the logical notions of possibility and necessity are irreducible and basic. The existential commitments of logic are relegated to the set-theoretic metatheory where we, for example, appeal to the number of sets in the world in order to represent logically possible situations. Prominent members of this school include Quine, Resnik, Maddy, and Shapiro. The major challenge facing these philosophers and the *PM* account is answering Benacerraf’s questions about how it is possible for humans to know anything about sets, and about why we are confident that our assertions about them are true. Obviously, this challenge has implications in

the epistemology of logic, if, like the PM account, we anchor our determination of what is logically true and valid in a set-theoretic ontology.

The opposite school of thought, which I have called logical primitivism, denies logic any existential commitments. The modal terminology in the metatheory of first-order logic is taken at face value, not reduced to something non-modal. Of course, as with the other school of thought, the details of one's reasons for subscribing to logical primitivism vary from one person to another. If we follow, for example, the early Wittgenstein, Russell, and more recently Etchemendy, then we are logical primitivists because we think that logic is prior to investigations of the world. On the other hand, the logical primitivism of Field, Hellman, and Chihara is motivated by the enterprise of seeing how far we can go without asserting the existence of abstract objects like sets.

Borrowing from Shapiro,<sup>15</sup> we may say that the PM account grounds logic in ontology, while logical primitivism bases the logical enterprise exclusively in an ideology. What we have to consider, then, is the tradeoff between a vast ontology and an increased ideology. I don't believe that we can accurately assess the balance of the tradeoff one way or the other until we get answers to Benacerraf's question as applied to the relevant primitive modal intuitions: how is it possible for humans to know about primitive logical possibility? What confidence can we have on our assertions about the ways the world could logically be?

I have been gesturing at one rationale for thinking that logic is based on a choice of an ontology: the implausibility of thinking that there is a pre-theoretic grasp of the modal notions in logic that happen to exactly match the hard won set-theoretic results in model theory. Since I believe that sets are indispensable to mathematics and science, I believe that the *PM* approach doesn't create any problems in the epistemology of logic that we do not already have in the epistemology of science and mathematics. By invoking an awareness of basic modal facts peculiar to logic, logical primitivism burdens logicians with having to explain this species of awareness. Perhaps its an indication of how heavy this burden is that a logical primitivist like



Russell, in *Introduction to Mathematical Philosophy*, remarks, “Logic, I should maintain, must no more admit a unicorn than zoology can; for logic is concerned with the real world just as truly as zoology, though with its more abstract and general features.”<sup>16</sup> If we understand set-theoretic facts as facts about the abstract and general features of the world, then the *PM* account makes validity turn on these features of the world and on modal facts about language.

I suspect that the current research related to uncovering the cash value of basing logic in an ideology will, ironically, increase the comfort level for thinking that first-order logic requires existential commitments, and, perhaps, pave the way for a more universal acceptance of second-order logic as logic.

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<sup>1</sup> Bob Hale “Is Platonism Epistemologically Bankrupt?” *The Philosophical Review* Vol. 102, No. 2 (1994): 299-325, 318.

<sup>2</sup> While this approach to logical consequence has certain affinities to what is referred to in the literature as interpretational semantics, popularized in J. Etchemendy, The Concept of Logical Consequence, (Stanford: CSLI Publications, 1999), Ch. 4, my source for the basic ideas behind this account is S. Lehmann “Slightly Non-Standard Logic” Logique Et Analyse 92 (1980), pp. 379-392.

<sup>3</sup> This feature provides the basis for thinking that PM-validity is not co-extensive with the classical account of first-order validity according to which it is not logically possible that, say, ' $\exists x(x=x)$ ' be false. Classical semantics rules out the empty domain, and on the analysis of *it is logically possible that p* in terms of *there exists a model for p*, this makes ' $\exists x(x=x)$ ' a logical truth. But on the PM approach, the possible uses for variables are to range as widely as possible. This sentence could (logically) be false when the quantifier is used to range over the empty set. Clearly, the legitimacy of this use is independent of the issue of whether the universe could (in a metaphysical sense) have been empty.

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<sup>4</sup> This feature of the PM theory doesn't seem to be universally acknowledged. For example, Read argues against the PM approach as follows. If we allow substitution on 'taller than', we can't explain why 'Iain is taller than Bill and Bill is taller than Mary' entails 'Iain is taller than Mary'. But if 'taller than' is held constant, then from  $\forall x \exists y \sim (x \text{ is taller than } y)$  we can conclude that  $\exists x \exists y (x \neq y \ \& \ x \text{ is taller than } y)$ . Either way, the PM approach generates counterintuitive results. But this is overly harsh, because the PM approach can account for the invalidity of the last argument by appealing to the fact that the quantifiers could be used to range over a domain consisting of, say, two equal sized twins. Again, unlike the Tarskian approach, the PM account allows restrictions on the world's domain in order to change the truth-values of sentences in fixing the extension of validity and logical truth. See Stephen Read "Formal and Material Consequence" Journal of Philosophical Logic 23 (1994), pp. 247-265. Read may reply that his dilemma is aimed at the Tarskian approach to validity (which he does discuss.) However Read's remarks (ibid.) on extensions of the Tarskian approach and on pg. 255 on completeness of interpretational semantics suggest he is also thinking of the PM approach.

<sup>5</sup> This argument is due to McGee. See, V. McGee, "Two Problems with Tarski's Theory of Consequence", Proceedings of the Aristotelian Society 92 (1992), pp. 273-292. For any model of a set of wffs from a standard first-order language, there exists an isomorphic model with a domain of pure sets. If  $M$  is an interpretation with domain  $D$ , and  $D'$  is a set that has the same cardinality as  $D$ , then one can define a model  $M'$  with domain  $D'$  such that  $M$  is isomorphic with  $M'$ . That for any set  $S$ , there is a pure set  $S'$  with the same cardinality as  $S$  is more or less implied by the definition of "cardinal number" in standard ZFC set theory.  $\text{Card}(S)$ , for any set  $S$ , is the least ordinal  $\alpha$  such that  $S$  and  $\alpha$  are equinumerous (have the same cardinality). It is a consequence of the axiom of choice (plus the usual construction of ordinals) that there exists at least one such  $\alpha$  for each  $S$ . By construction all ordinals (hence all cardinals) are pure.

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<sup>6</sup> L. Wittgenstein, Tractatus. trans. by D.F. Pears and B.F. McGuinness (London: Routledge and Kegan Paul, 1961) 5.551.

<sup>7</sup> W. Hanson, “The Concept of Logical Consequence”, The Philosophical Review 106, (1997), pp.365-409, 374.

<sup>8</sup> J. Etchemendy, op. cit., p. 109.

<sup>9</sup> B. Russell, Introduction to Mathematical Philosophy (IMP). (New York: Macmillan Co., 1919). Russell remarks that, “there is a certain lordliness which the logician should preserve: he must not condescend to derive arguments from the things he sees about him.” P. 192.

<sup>10</sup> More precisely, letting  $\alpha$  be a first-order particular or a set of first-order particulars, there is a possible use for a term  $t$  according to which it designates  $\alpha$  only if

- (i.) nothing about the semantic functioning of  $t$  rules out it referring to  $\alpha$ , and
- (ii.)  $\alpha$  could exist (if  $\alpha$  exists, then (ii.) is satisfied).

Only if both (i.) and (ii.) are satisfied can  $t$  be used to pick out  $\alpha$ . (i.) is a semantic claim that is independent of metaphysical commitments and is non-substantive. There is a problem with fleshing out (i.) as a subjunctive conditional—if  $\alpha$  could exist, then  $\alpha$  could be the semantic value of  $t$ . According to some popular theories, the truth-values of such subjunctive conditionals are dependent on the structure of metaphysical possibility. But the account here needs to understand them as making claims that are independent of the metaphysical modal facts.

<sup>11</sup> The examples of semantic facts used to illustrate Etchemendy’s notion of non-substantiality are facts about the functioning of non-logical and logical terms, and are what ground specific instances of (i.).

<sup>12</sup> S. Shapiro. “Modality and Ontology” Mind 102 (1993), pp.455-481, 475.

<sup>13</sup> H. Field. “Is Mathematical Knowledge Just Logical Knowledge” The Philosophical Review 93 (1984), pp. 509-552, 520-521.

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<sup>14</sup> I do not believe that there is anything obviously incoherent in the possibility of an existent infinity. But I agree with Resnik that statements of the possibility of infinite collections are hypotheses, and, therefore, could turn out false. See, M. Resnik, “How Nominalist is Hartry Field’s Nominalism?” Philosophical Studies 47 (1985), pp. 163-181, 173.

<sup>15</sup> Shapiro, op. cit.

<sup>16</sup> IMP p. 169.