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MODALITIES AND QUANTIFICATION

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1. The problems of modal logic. The purpose of this article is to give a survey of some results I have found in investigations concerning logical modalities. The results refer: (1) to semantical systems, i.e., symbolic language systems for which semantical rules of interpretation are laid down; (2) to corresponding calculi, i.e., syntactical systems with primitive sentences and a rule of inference; (3) to relations between a semantical system and the corresponding calculus.

The semantical systems to be dealt with are the following: propositional logic (PL), functional logic (FL), and the corresponding modal systems, viz. modal propositional logic (MPL) and modal functional logic (MFL). MPL is built out of PL by the addition of the symbol 'N' for logical necessity; likewise MFL out of FL. In terms of Lewis's symbol '$\Diamond$' for logical possibility, 'Np' means the same as '$\sim\Diamond\sim p$'. All other logical modalities can, of course, be defined on the basis of 'N'; e.g., impossibility by 'N$\sim p$', possibility by '$\sim$N$\sim p$', contingency by '$\sim$Np.$\sim$N$\sim p$', etc.

The calculi corresponding to these semantical systems are the following: the propositional calculus (PC), the functional calculus (FC), and the modal calculi (MPC and MFC) again constructed by the addition of 'N'.

Lewis's systems of strict implication[1] are forms of MPC. So far, no forms of MFC have been constructed, and the construction of such a system is our chief aim. The corresponding semantical systems MPL and MFL are constructed chiefly for the purpose of enabling us to show that the modal calculi MPC and MFC are adequate, i.e., that every sentence provable in them is L-true (analytic). With the help of a normal form, it can further be shown that for MPC the inverse holds also; MPC is complete in the sense that every sentence which is L-true in MPL is provable in MPC. The reduction to the normal form constitutes a decision method for MPC and MPL. For MFC likewise a method of reduction to a normal form will be given. This reduction removes all occurrences of 'N' of higher order, i.e., such that the scope of one 'N' contains another 'N'. A decision method for MFC is of course not possible; however, the reduction makes it possible to apply to MFC the known decision methods for special cases in FC.

The semantical systems FL and MFL contain an infinite number of individual constants. Therefore the representation of these systems requires a very strong metalanguage, dealing with classes of classes of sentences. Consequently, the semantical concepts defined, e.g., L-truth, are indefinite (non-effective) to a high degree. The chief reasons for constructing corresponding calculi are here, as usually in the case of logical calculi, the following two: (1) avoidance of any reference to the meanings of the signs and sentences, (2) use of basic concepts which are effective. The second purpose is here, as generally in the case of calculi without transfinite rules, fulfilled in the following sense. Al-

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[1] C. I. Lewis and C. H. Langford, ***Symbolic logic***, 1932; the systems are developed from those in Lewis's earlier book (1918).