Assignment 4+Smoothing

(SNLP Tutorial 5)

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Assignment 4

- Exercise 1: Huffman encoding
- Exercise 2: Conditional entropy of DNA
- Bonus: Huffman encoding adaptations

OOV words

Corpus

- Train set:
  🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆
- Test set:
  🍫 🍏 🍟 🍌 🍏 🍆 🍆 🍌 🍒 🍟 🍏 🍏

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Accumulate counts

- 🍏 6 🍌 5 🍆 3 🍒 2
- 🍏 4 🍌 2 🍟 2 🍆 2 🍒 1 🍫 1

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OOV words

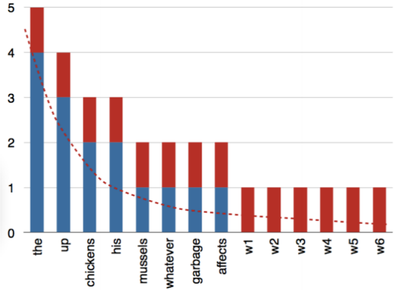
- What about 🍫 and 🍟?
- OOV rate: $2/6 = 33\%$

- Solutions? character-level, subword units

Smoothing

- Words present in vocabulary, but have ~0 probabilities
- Words present in vocabulary, but have unseen context

Solution: Assign probability mass from frequent events to infrequent events (Smoothing/Discounting)



- Will cover different smoothing methods over the next few tutorials

Additive smoothing (add-α-smoothing)

Unigrams

- Add zero counts to frequency table

🍏 6 🍌 5 🍆 3 🍒 2 🍟 0 🍫 0

- Increase all counts by $\alpha = 1$

🍏 6+1 🍌 5+1 🍆 3+1 🍒 2+1 🍟 0+1 🍫 0+1

- Divide by $N = 22$

🍏 0.32 🍌 0.27 🍆 0.18 🍒 0.13 🍟 0.05 🍫 0.05

Perplexity

- Relative frequencies on test corpus:

🍏 0.33 🍌 0.17 🍟 0.17 🍆 0.17 🍒 0.08 🍫 0.08

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- $\text{PP} = 2^{(0.33 \cdot 0.32 + 0.27 \cdot 0.17 + 0.18 \cdot 0.17 + 0.13 \cdot 0.17 + 2 \cdot (0.05 \cdot 0.08))} = 1.4$

Additive smoothing: Bigrams

Recall the additive smoothing formula for unigrams:

$$p_{smoothed}(w_i) = \frac{C(w_i) + \alpha}{N + \alpha|V|} \quad (1)$$

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- What is N? What is V?

Remember from Assignment 2 that:

$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})} \tag{2}$$

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- Smoothen the bigram count: $C(w_{i-1}, w_i) \rightarrow C(w_{i-1}, w_i) + \alpha$
- Normalization: $p_{smoothed}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{?}$

Additive smoothing: Bigrams

Corpus

🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆

Bigrams: 🍏🍏, 🍏🍏, 🍏🍆, 🍆🍏, . . . , 🍏🍆, 🍆🍏 $\leftarrow$ circular bigram!

Bigrams: AA, AA, AE, EA, . . . , AE, EA

Additive smoothing: Bigrams: bigram counts

- Collect bigram counts & condtional probabilities for history A

Bigram	$C(w_i, w_{i-1})$	$C(w_{i-1})$	$\frac{C(w_{i-1},w_i)}{C(w_{i-1})}$
AE	3	6	$1/2$
AA	2	6	$1/3$
AB	1	6	$1/6$

Additive smoothing: Bigrams: add alpha

- We encounter an unknown bigram *AF*

Bigram	$C_\alpha(w_{i-1}, w_i)$	$C(w_{i-1})$	$\frac{C_\alpha(w_{i-1}, w_i)}{C(w_{i-1})}$
AE	$3+1$	6	$4/6$
AA	$2+1$	6	$3/6$
AB	$1+1$	6	$2/6$
$\rightarrow$ AF	$0+1$	6	$1/6$

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- Not a probabilitiy distribution!

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AA	2+1	6	3/6
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$\rightarrow$ AF	0+1	6	1/6

- Not a probabilitiy distribution!
- Solution: We need to adjust the divisor a tiny bit. But how tiny?

Additive smoothing: Bigrams: normalization

- add $\alpha \cdot 4$ to history count!
- Pretend that we have seen the history $|V| = 4$ times more.

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Bigram	$C_\alpha(w_{i-1}) + \alpha\|V\|$	$\frac{C_\alpha(w_{i-1},w_i)}{C(w_{i-1})+\alpha\|V\|}$
AE	$6 + 4$	$4/10$
AA	$6 + 4$	$3/10$
AB	$6 + 4$	$2/10$
$\rightarrow$ AF	$6 + 4$	$1/10$

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- Now the probabilities sum up to 1: $4/10 + 3/10 + 2/10 + 1/10 = 1$

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- We encounter another n-gram AD
- What is $|V|$ now?

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Bigram	$C_\alpha(w_{i-1}) + \alpha\|V\|$	$\frac{C_\alpha(w_{i-1},w_i)}{C(w_{i-1})+\alpha\|V\|}$
AE	$6 + 5$	$4/11$
AA	$6 + 5$	$3/11$
AB	$6 + 5$	$2/11$
$\rightarrow$ AF	$6 + 5$	$1/11$
$\rightarrow$ AD	$6 + 5$	$1/11$

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AB	$6+5$	$2/11$
$\rightarrow$ AF	$6+5$	$1/11$
$\rightarrow$ AD	$6+5$	$1/11$

- $C(A)$ is constant, unsmoothed count
- Probabilities sum up to 1: $4/11 + 3/11 + 2/11 + 1/11 + 1/11 = 1$

Additive smoothing: Bigrams: general case

- General formula for smoothed bigram Probabilities:

$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha|V|} \tag{3}$$

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- $|V|$ = Number of bigram **types** starting with w_{i-1}

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$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha|V_{(w_{i-1},\bullet)}|} \tag{4}$$

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- For n-grams of length n:

$$p(w_i|w_{i-1} : w_{i-n+1}) = \frac{C(w_{i-n+1} : w_i) + \alpha}{C(w_{i-n+1} : w_{i-1}) + \alpha|V_{(w_{i-n+1}:w_{i-1},\bullet)}|} \tag{5}$$

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- We already know the shared (train + test) vocabulary V

Additive smoothing: Bigrams: general case

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- We already know the shared (train + test) vocabulary V
- $V_{(A,\bullet)}$ is then $AA, AB, AC, AD, AE, AF \Rightarrow |V_{(A,\bullet)}| = 6 = |V|$

Additive smoothing: Bigrams: general case

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- We already know the shared (train + test) vocabulary V
- $V_{(A,\bullet)}$ is then $AA, AB, AC, AD, AE, AF \Rightarrow |V_{(A,\bullet)}| = 6 = |V|$
- We find that the formula we found is identical to the one on the lecture slides!

$$p(w_i|w_{i-1}:w_{i-n+1}) = \frac{C(w_{i-n+1}:w_i)+\alpha}{C(w_{i-n+1}:w_{i-1})+\alpha|V|} \tag{7}$$

Backing-off

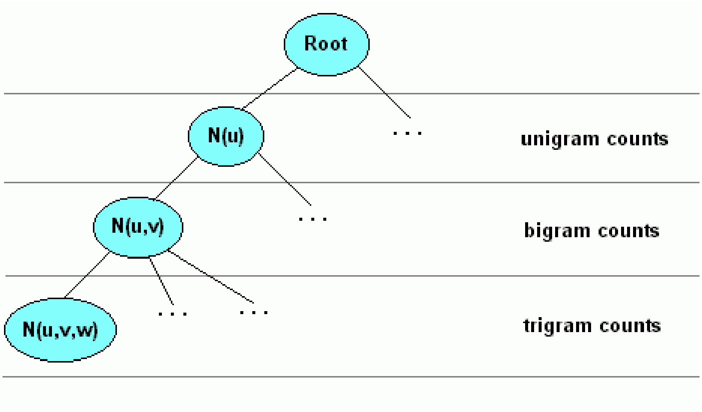
MARY HAD A LITTLE LAMB

- Consider the bigram (LITTLE MARY)
- Consider the trigram (HAD A LAMB)

For a trigram $P(w_3|w_2, w_1)$, use probability of bigram $P(w_3|w_2)$, else back-off to unigram probability $P(w_3)$.

Will be covered in more detail in further tutorials.

Count Trees



Source:https://www.w3.org/TR/ngram-spec/

Assignment 5

- Exercise 1: OOV Words
- Exercise 2: Additive smoothing
- Exercise 3: Perplexity, infinite smoothing, interpolation
- Bonus: Other language models

Resources

1. UdS SNLP Class: https://teaching.lsv.uni-saarland.de/snlp/
2. Additive smoothing: https://en.wikipedia.org/wiki/Additive_smoothing
3. n-gram count trees: http://ssli.ee.washington.edu/WS07/notes/ngrams.pdf
4. n-gram models: https://web.stanford.edu/~jurafsky/slp3/3.pdf