Assignment 4,5 + Smoothing (SNLP tutorial 4)

Vilém Zouhar, Awantee Deshpande, Julius Steuer

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Overview

- Task, Metrics
- Differential Privacy
- Homework

Entropy

- Amount of information / compressed size in bits
- $H(p) = E[-\log(p(V))] = -\sum p(v)\log(p(v))$
- For binomial distribution highest in the middle
- For uniform distribution: log(W)
- Entropy is always non-negative
- \bullet H((W,W))=H(W)+H(W) when statistically independent p(w1,w2)=p(w1)p(w2)
- Conditional entropy: $H(X|Y) = -\sum p(x,y) \log p(x|y)$

Kullback-Leibler Divergence

- $D(p||q) = \sum p_i \log p_i/q_i$
- Not symmetric
- Non-negative
- How many extra bits if we use bad encoding
- Cross-entropy: $-\sum p_i \log q_i$

Code

- Mapping of word to a finite string of a *D*-nary alphabet
- Prefix code
- $\sum D^{-l_i} \leq 1$
- true for prefix codes
- for every length distribution satisfying this, there exists a prefix code
- Expected length: $\sum l_i p(w_i)$
- Optimal length: $-\log_D p(w_i)$

Correlation Function

• $p_d(w1, w2)/(p(w1)p(w2))$

OOV words

Corpus



























OOV words

Corpus

- Train set:
- Test set:

Accumulate counts



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OOV words

- What about and 2?
- OOV rate: 2 + 1/4 + 2 + 2 + 1 + 1 + 1 = 27%

Additive smoothing (add- α -smoothing)

Unigrams

Add zero counts to frequency table















ullet Increase all counts by lpha=1



















• Divide by N = 22



















Perplexity

• Relative frequencies on test corpus:















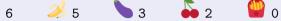


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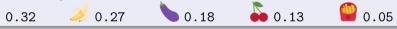
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Perplexity

• Relative frequencies on test corpus:

















 $PP = 2^{(0.33 \cdot 0.32 + 0.27 \cdot 0.17 + 0.18 \cdot 0.17 + 0.13 \cdot 0.17 + 2 \cdot (0.05 \cdot 0.08))} = 1.4$

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• What is N? What is V?

Remember from Assignment 2 that:

$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$
(2)

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- Smoothe the bigram count: $C(w_{i-1}, w_i) \rightarrow C(w_{i-1}, w_i) + \alpha$
- Normalization: $p_{smoothed}(w_i|w_{i-1}) = \frac{C(w_{i-1},w_i) + \alpha}{?}$



Additive smoothing: Bigrams: bigram counts

• Collect bigram counts & condtional probabilities for history A

Bigram	$C(w_i, w_{i-1})$	$C(w_{i-1})$	$\frac{C(w_{i-1},w_i)}{C(w_{i-1})}$
AE	3	6	1/2
AA	2	6	1/3
AB	1	6	1/6

Additive smoothing: Bigrams: add alpha

• We encounter an unknown bigram AF

Bigram	$C_{\alpha}(w_{i-1},w_i)$	$C_{\alpha}(w_{i-1})$	$\frac{C_{\alpha}(w_{i-1},w_i)}{C_{\alpha}(w_{i-1})}$
AE	3+1	6 + 1	4/7
AA	2 + 1	6 + 1	3/7
AB	1 + 1	6 + 1	2/7
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- Solution: We need to adjust the divisor a tiny bit. But how tiny?

- add α 3 to history count!
- Pretend that we have seen the history |V| = 3 times more.

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• Now the probabilities sum up to 1: 4/10 + 3/10 + 2/10 + 1/10 = 1

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AA	7 + 4	3/11
AB	7 + 4	2/11
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- $C_{\alpha}(A)$ is constant
- Probabilities sum up to 1: 4/11 + 3/11 + 2/11 + 1/11 + 1/11 = 1

• General formula for smoothed bigram Probabilities:

$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha|V|}$$
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• For n-grams of length *n*:

$$p(w_i|w_{i-1}:w_{i-n+1}) = \frac{C(w_{i-n+1}:w_i) + \alpha}{C(w_{i-n+1}:w_{i-1}) + \alpha |V_{(w_{i-n+1}:w_{i-1},\bullet)}|}$$
(5)

Kneser-Ney Smoothing

TODO

• absolute discounting

Cross-Validation

TODO

Estimating LOO Parameters

TODO ??

Laplace Smoothing

add epsilon

TODO

Linear Discounting

• linear interpolation

Good-Turing Discounting

TODO

Count Trees

• remove infrequent nodes

TODO

Privacy

TODO differential privacy

Resources

- UdS SNLP Class, WSD: https://teaching.lsv.uni-saarland.de/snlp/
- Classical Statistical WSD: https://www.aclweb.org/anthology/P91-1034.pdf
- on-gram count trees: http://ssli.ee.washington.edu/WS07/notes/ngrams.pdf