Assignment 2 + Entropy, KL Divergence
(SNLP Tutorial 3)

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Assignment 2

- Exercise 1: Perplexity Calculation
- Exercise 2: Formulating n-gram models
- Exercise 3: Perplexity Calculation for n-grams
- Bonus: Alternative metric to perplexity

Overview of Formulas

Concepts and formulations.

- Information Content
- Entropy
- Joint entropy
- Conditional entropy
- Mutual Information
- Cross-entropy
- KL-Divergence

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- $D(p||q) = -\sum_{x \in X} p(x) \cdot \log \frac{p(x)}{q(x)}$

How do they relate to each other?

- Chain Rule:
$$H(X, Y) = H(X) + H(Y|X)$$
$$H(X_1...X_n) = H(X_1) + H(X_2 \mid X_1) + ... + H(X_n \mid X_1, ...X_{n-1})$$

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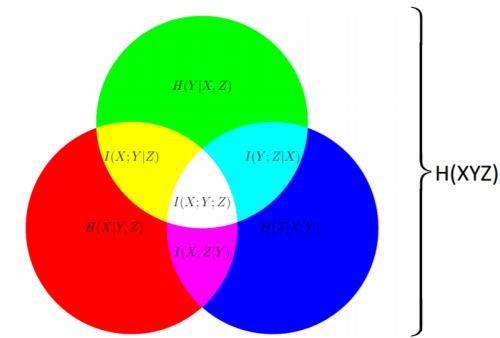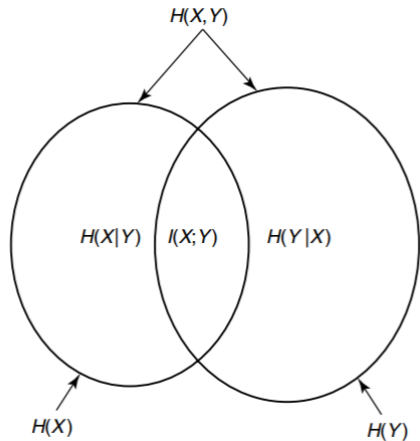
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- Apply to 3 variables

$$I(X; Y \mid Z) = I((X; Y)|Z) = H(X \mid Z) - H(X \mid Y, Z)$$

How do they relate to each other?

Examples

1. Entropy calculation

X \ Y	0	1
0	1/2	1/5
1	1/3	0

Find

- $H(X), H(Y)$
- $H(X, Y)$
- $H(X \mid Y), H(Y \mid X)$
- $I(X; Y)$
- $H(Y) - H(Y \mid X)$

Examples

2. Entropy of functions

What is the (in)equality relationship between H(X) and H(Y) when

- $y = f(x)$ # general case
- $y = 2^x$
- $y = sin(x)$

KL-divergence

TODO, examples, exercises, questions

Resources

1. TODO