Assignment 2 + Entropy, KL Divergence

(SNLP Tutorial 3)

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Assignment 2

- Exercise 1: Perplexity Calculation
- Exercise 2: Formulating n-gram models
- Exercise 3: Perplexity Calculation for n-grams
- Bonus: Alternative metric to perplexity

Overview of Formulas

Concepts and formulations. (Express also as expectation values.)

- Information Content
- Entropy
- Joint entropy
- Conditional entropy
- Mutual Information
- Cross-entropy
- KL-Divergence

- $I(x) = -\log p(x)$
- $H(X) = -\sum_{x \in X} p(x) \cdot \log p(x)$
- $H(X, Y) = -\sum_{x \in X} \sum_{y \in Y} p(x, y) \cdot \log p(x, y)$
- $H(X, Y) = -\sum_{x \in X} \sum_{y \in Y} p(x, y) \cdot \log p(y \mid x)$
- $I(X; Y) = -\sum_{x,y} p(x, y) \cdot \log \frac{p(x,y)}{p(x) \cdot p(y)}$
- $H(p, q) = -\sum_{x} p(x) \cdot \log q(y)$
- $D(p||q) = -\sum_{x \in X} p(x) \cdot \log \frac{p(x)}{q(x)}$

How do they relate to each other?

- Chain Rule:

$$H(X, Y) = H(X) + H(Y|X)$$

$$H(X_1...X_n) = H(X_1) + H(X_2 \mid X_1) + ... + H(X_n \mid X_1, ..., X_{n-1})$$

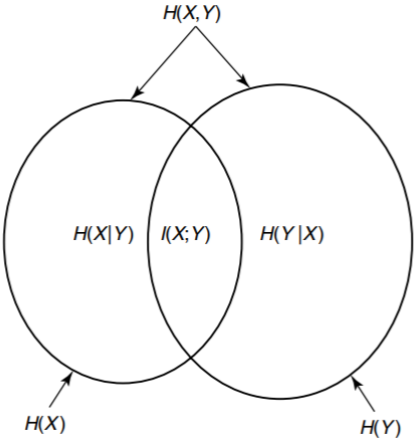
- Mutual Information and Entropy

$$I(X; Y) = H(X) - H(X \mid Y) = H(X) + H(Y) - H(X, Y)$$

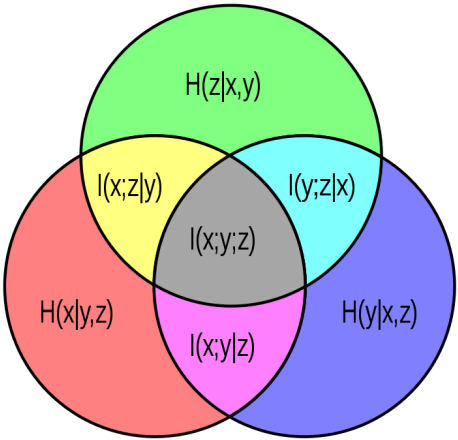
- Apply to 3 variables

$$I(X; Y \mid Z) = I((X; Y)|Z) = H(X \mid Z) - H(X \mid Y, Z)$$

How do they relate to each other?



Source: https://syncedreview.com/2020/11/30/synced-tradition-and-machine-learning-series-part-1-entropy/



Source: https://en.wikipedia.org/wiki/Information_diagram

Examples - Entropy calculation

X \ Y	0	1
0	1/2	1/6
1	1/3	0

Find

- $H(X), H(Y)$
- $H(X, Y)$
- $H(X|Y), H(Y|X)$
- $I(X; Y)$
- $H(Y) - H(Y|X)$

Examples - Entropy of functions

What is the (in)equality relationship between H(X) and H(Y) when

- $y = f(x)$ # general case
- $y = 2^x$
- $y = sin(x)$

KL-divergence

Question: Can we use the chain rule on KL-Divergence?

$$D(p(x,y) \mid\mid q(x,y)) = D(p(x) \mid\mid q(x)) + D(p(y \mid x) \mid\mid q(y \mid x))$$

Applications of KL Divergence:

- Bayesian inference
- Compression techniques
- Variational autoencoders

Resources

1. http://csustan.csustan.edu/~tom/sfi-csss/info-theory/info-lec.pdf
2. https://www.cs.cmu.edu/~odonnell/toolkit13/lecture20.pdf
3. https://syncedreview.com/2020/11/30/synced-tradition-and-machine-learning-series-part-1-entropy/