Conditional Random Fields

(SNLP tutorial)

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Overview

- Sequence Labelling / Entity Recognition
  - Rule-based
  - HMM
  - Bayesian Network
  - Log-linear 1st Order Sequential Model
  - Linear Chain CRF / CRF
- Model comparison
- Code
- Homework

Sequence Labelling / Entity Recognition

- `My name is V. Zouhar, I live in Saarbrücken and my matriculation number is 1234.`

Sequence Labelling / Entity Recognition

- `My name is V. Zouhar, I live in Saarbrücken and my matriculation number is 1234.`
- `My name is [V. Zouhar:person], I live in [Saarbrücken:loc] and my matriculation number is [1234:mat-num].`

Sequence Labelling / Entity Recognition

- `My name is V. Zouhar, I live in Saarbrücken and my matriculation number is 1234.`
- `My name is [V. Zouhar:person], I live in [Saarbrücken:loc] and my matriculation number is [1234:mat-num].`
- NER as Sequence labeling:
  X: sequence of words
  Y: labels {`mat-num, person, location, none`}

Rule-based

- Regex substitute:
  `matriculation (number)? (is)? (\d+)` $\rightarrow$ `[\3:mat-num]`

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- No automated learning

HMM

- Hidden states: {`mat-num`, `person`, `location`, `none`}

HMM

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- Better hidden states: {`mat-num`, `START+person`, `INTERNAL+person`, `END+person`, `location`, `none`, `...`}

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- Transitions: MLE from annotated data

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- Better hidden states: {`mat-num`, `START+person`, `INTERNAL+person`, `END+person`, `location`, `none`, `...`}
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- Emission probabilities: MLE from annotated data (+ smoothing)

HMM: Estimation

- Hidden states: $\pi_1, \pi_2, ..., \pi_N$

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- Emission probability: $p(x_i|\pi_i)$
- $p(x_1, x_2, ..., x_N, \pi_1, \pi_2, ..., \pi_N) = \prod_i p(\pi_i|\pi_{i-1}) \cdot p(x_i|\pi_i)$

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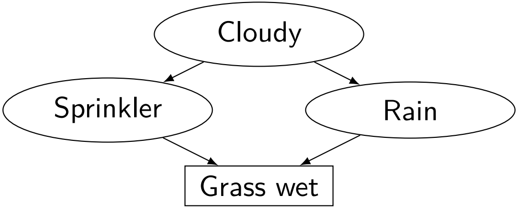
- $p(x_1, x_2, ..., x_N, \pi_1, \pi_2, ..., \pi_N) = \prod_i p(\pi_i|\pi_{i-1}) \cdot p(x_i|\pi_i)$
- Decision rule: $\underset{\pi_1,\pi_2,...,\pi_N}{\arg\max} \left[\prod_i p(\pi_i|\pi_{i-1}) \cdot p(x_i|\pi_i) \right]$

Bayesian Network

- Directed acyclic graph (DAG), $(x \to y) \in E$: y dependent on x

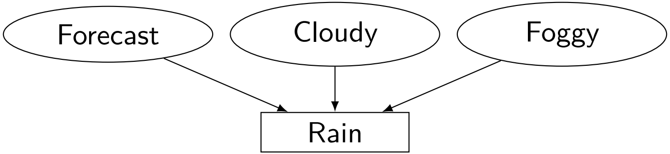
Local Markov Property

Node is conditionally independent of its nondescendants given its parents.
$p(\text{Sprinkler}|\text{Cloudy}, \text{Rain}) = p(\text{Sprinkler}|\text{Cloudy})$

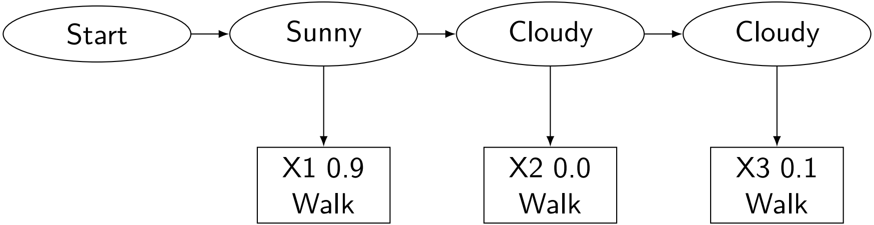


Naïve Bayes

- Assume absolute independence except for the one observed variable
- $p(\pi_j = \text{Yes}|x) = p(\pi_j|x) = \frac{p(x|\pi_j)p(\pi_j)}{p(x)} \propto p(x|\pi_j)p(\pi_j) \approx p(\pi_j)\prod_i p(x_i|\pi_j)$



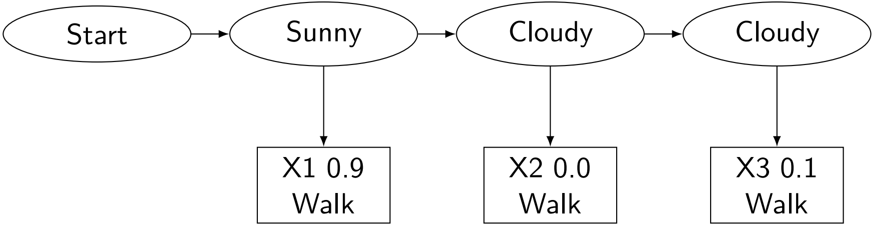
HMM



Sketch of HMM structure

observed variable *Walk duration*, latent variable: $Weather \in \{Sunny, Cloudy\}$

HMM



Sketch of HMM structure
observed variable *Walk duration*, latent variable: $Weather \in \{Sunny, Cloudy\}$

$$p(\pi|x) = \prod_i p(\pi) \cdot p(x_i|\pi_i) \text{ (Naïve Bayes)}$$

$$\Rightarrow$$

$$p(\pi_1, \pi_2, ..., \pi_N|x) = \prod_i p(\pi_i|\pi_{i-1}) \cdot p(x_i|\pi_i) \text{ (HMM)}$$

Logistic Regression

$$p(y|x) = \frac{\exp(\Phi(y,x))}{\sum_{y'} \exp(\Phi(y',x))}$$

$$\arg\max_y \frac{\exp(\Phi(y,x))}{\sum_{y'} \exp(\Phi(y',x))} = \arg\max_y \exp(\Phi(y,x))$$

Log-linear 1st Order Sequential Model

- Sequence of hidden states: y, {`mat-num`, `person`, `location`, `none`}

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- $p(y|x) = \frac{1}{Z(x)} \cdot \prod_j \{a(y_{j-1}, y_j) o(y_j, x_j)\}$
- $argmax\ p(y|x) \ldots$

Log-linear 1st Order Sequential Model

Viterbi:

$$argmax\ p(y|x) = argmax\ \log p(y|x) = argmax\ F(y,x) - \log \sum_{y'} \exp F(y',x)$$

$$= argmax\ F(y,x)$$

$$\alpha_t(y_j) = \max_i \exp \Big(\log \alpha_{t-1}(y_i) + a(y_j, y_i) + o(y_j, x_t) \Big)$$

$$\alpha'_t(y_j) = argmax_i\ \alpha_{t-1}(y_i) + \exp\left(a(y_j, y_i) + o(y_j, x_t)\right)$$

$O(|Y|^2 \cdot T)$

Log-linear 1st Order Sequential Model

Forward:

$$\log fw_t(y_j) = \log \sum_i \exp\left(\log fw_{t-1}(y_i) + a(y_j, y_i) + o(y_j, x_t)\right)$$

$$Z(X) = \sum_i \exp\left(\log fw_{|T|-1}(y_i) + a(y_j, y_i) + o(y_j, x_t)\right)$$

$$\rightarrow$$

$$p(y|x) = \frac{\alpha_{|T|}(y_{:-1})}{Z(x)}$$

$O(|Y|^2 \cdot T)$

Log-linear 1st Order Sequential Model

- Replace $o(y_j, x_t)$ with $\theta_1 h_1(y_j, x_t) + \theta_2 h_2(y_j, x_t) + \ldots$

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- Why not just $\sum_{\text{feature } f} \theta_i f_i(y_i, y_j, x_t)$?

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- Same with $a(y_j, y_i) = \theta'_1 g_1(y_j, y_i) + \theta'_2 g_2(y_j, y_i) + \dots$
- Why not just $\sum_{\text{feature } f} \theta_i f_i(y_i, y_j, x_t)$?
- Why not allow $\sum_{\text{feature } f} \theta_i f_i(y_i, y_j, x, t)$?

Model overview

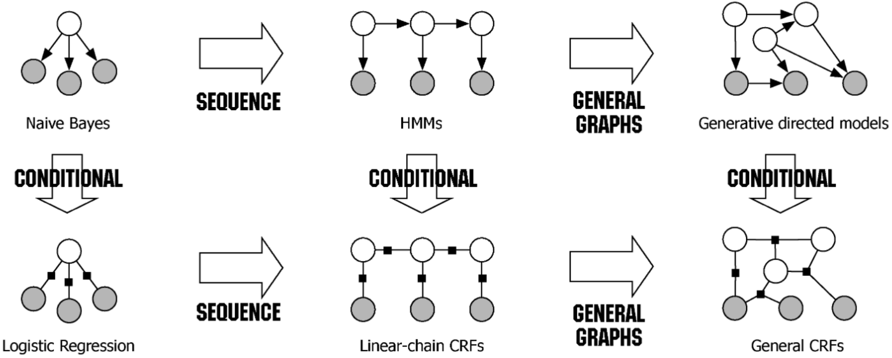

Figure 1: CRF in relation to other models; Source [2]

HMM $\rightarrow$ Linear Chain CRF

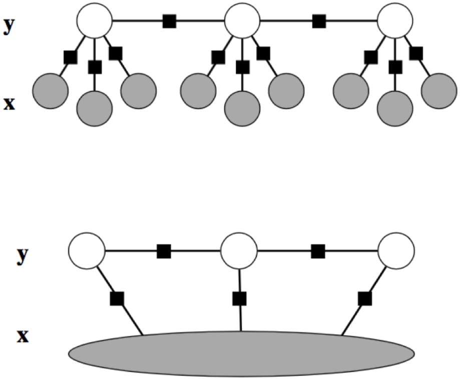

Figure 2: HMM vs. Linear Chain CRF; Source [12]

Model overview

- Multinomial logistic regression:
  $p(y_j|x) = \frac{exp(Z_j \cdot x)}{\sum_i exp(Z_i \cdot x)}$
- Multiclass naïve Bayes:
  $p(y_j|x) = \frac{p(x|y_j)p(y_j)}{p(x)} \propto p(x|y_j)p(y_j) \approx p(y_j) \prod_i p(x_i|y_j)$

Linear Chain CRF

- Sequence of hidden states: y, `{mat-num, person, location, none}`
- Observed sequence of variables: x (words)
- $p(y|x) \propto \prod_t \exp\left\{\sum_{\text{feature } f} \theta_i f_i(y_{t-1}, y_t, x, t)\right\}$
- $p(y|x) = \frac{1}{Z(x)} \prod_t \exp\left\{\sum_{\text{feature } f_i} \theta_i f_i(y_{t-1}, y_t, x, t)\right\}$
- Features: $f_i(y_{t-1}, y_t, x, t) \geq 0$
- Parameters: θ

Linear Chain CRF - Features

$$f_i(y_{t-1}, y_t, x, t) = \begin{cases} 1 & \text{if } \mathrm{cond}_f(y_{t-1}, y_t, x, t) \\ 0 & \text{else} \end{cases}$$

Linear Chain CRF - Features

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$$f_1(y_{t-1}, y_t, x, t) = \begin{cases} 1 & \text{if } x_{t-2} \text{ is capitalized} \\ 0 & \text{else} \end{cases}$$

$$f_a(y_{t-1}, y_t, x, t) = \begin{cases} 1 & \text{if } y_{t-1} = \texttt{number} \wedge y_t = \texttt{none} \\ 0 & \text{else} \end{cases}$$

$$\theta_a = a(\text{number}, \texttt{none})$$

$$f_o(y_{t-1}, y_t, x, t) = \begin{cases} 1 & \text{if } y_t = \texttt{number} \wedge x_t = \texttt{<num>} \\ 0 & \text{else} \end{cases}$$

$$\theta_o = o(\text{number}, \texttt{<num>})$$

Linear Chain CRF - Features

$f_w(y_{t-1}, y_t, x, t) = x_t$ word length

$f_s(y_{t-1}, y_t, x, t) = x_t$ number of non-alphabetic characters

CRF - Operations

Inference:

$$argmax_y \; p(y|x, \theta)$$

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$$p(y|x,\theta)$$

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Decoding:

$$p(y|x, \theta)$$

Training:

$$argmax_\theta \; p(y_D|x_D, \theta)$$

Linear Chain CRF - Estimating θ

Gradient descent (ascent):

$$\frac{\partial \log p(y|x,\theta)}{\partial \theta_i} = \sum_{t=1}^{T} f_i(y_{t-1}, y_t, x, t) - \sum_{y'} \sum_{t=1}^{T} f_i(y'_{t-1}, y'_t, x, t) \cdot p(y'|x)$$

$$\theta_f \leftarrow \theta_f + \epsilon \left[\sum_{t=1}^{T} F(y_{t-1}, y_t, x, t) - \sum_{y'} \sum_{t=1}^{T} F(y'_{t-1}, y'_t, x, t) \cdot p(y'|x,\theta) \right]$$

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Limited-memory BFGS (quasi-Newton method)

Linear Chain CRF - Regularization

Objective function:

$\mathcal{L} = \sum_s \log p(y^{(s)}|x^{(s)}, \theta)$

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$\mathcal{L}_{+lasso} = \sum_s \log p(y^{(s)}|x^{(s)}, \theta) - \lambda_1 \sum_i |\theta_i|$

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Ridge:

$\mathcal{L}_{+ridge} = \sum_s \log p(y^{(s)}|x^{(s)}, \theta) - \frac{\lambda_2}{2} \sum_i \theta_i^2$

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Ridge:

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Elastic net:

$\mathcal{L}_{+elastic} = \sum_s \log p(y^{(s)}|x^{(s)}\theta) - \frac{\lambda_2}{2} \sum_i \theta_i^2 - \lambda_1 \sum_i |\theta_i|$

General CRF

- Factorization to maximal clicques.
- Allow access to a whole clicque

Clique

$G = (V, E) \quad C \subseteq V : \forall x, y \in C : (x, y) \in E$

Maximal Clique

$C \subseteq C' \Rightarrow C = C'$

CRF

$p(Y|X) = \frac{1}{Z(X)} \prod_{C \in Y} \Psi_C(X_C)$
$\Psi_C(Y, X) \sum_i \theta_i f_i(Y_{i-1}, Y_i, X, i) \geq 0$

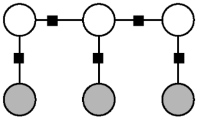

Figure 3: Linear Chain CRF [2]

Code

```
from sklearn_crfsuite import CRF

X_train = [
    [word2features(s, i) for i in range(len(s))]
    for s in train_sents]
y_train = [
    [label for token, postag, label in s]
    for s in train_sents]

crf = sklearn_crfsuite.CRF(
    algorithm='lbfgs',
    c1=0.1, c2=0.1,
    max_iterations=100,
)
crf.fit(X_train, y_train)
```

Notes

Feature selection:

1. Start with all features.
2. a. If there exists a feature removing which worsens the performance by $< t$, remove it. Repeat 2.
3. b. If not, exit.

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Properties

- Hard to setup & train
- Fast inference

Homework

TBD

Resources

1. Overview: https://www.analyticsvidhya.com/blog/2018/08/nlp-guide-conditional-random-fields-text-classification
2. Very detailed: http://homepages.inf.ed.ac.uk/csutton/publications/crftut-fnt.pdf
3. NER using CRF: https://medium.com/data-science-in-your-pocket/named-entity-recognition-ner-using-conditional-random-fields-in-nlp-3660df22e95c
4. Forward-backward for CRF: https://www.cs.cornell.edu/courses/cs5740/2016sp/resources/collins_fb.pdf
5. Academic-level introduction to CRF: https://www.youtube.com/watch?v=7L0MKKfqe98
6. Generalized CRF: https://people.cs.umass.edu/~wallach/technical_reports/wallach04conditional.pdf
7. Accessible introduction: http://pages.cs.wisc.edu/~jerryzhu/cs769/CRF.pdf
8. Python code: https://sklearn-crfsuite.readthedocs.io/en/latest/tutorial.html#let-s-use-conll-2002-data-to-build-a-ner-system

Resources

9. Fast Linear Chain CRFs (C): http://www.chokkan.org/software/crfsuite/
10. Fast Linear Chain CRFs (C++): https://taku910.github.io/crfpp/
11. Bayesian Networks: https://www.ics.uci.edu/~rickl/courses/cs-171/0-ihler-2016-fq/Lectures/Ihler-final/09b-BayesNet.pdf
12. Naïve Bayes to HMM to CRF: http://cnyah.com/2017/08/26/from-naive-bayes-to-linear-chain-CRF/