# Information Retrieval + Q&A (SNLP Tutorial 12)

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- Precision |retrieved∩relevant| |retrieved|

#### How to cheat so that...

- precision is high?
- recall is high?

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## $\{Precision, Recall\}@k : Retrieve k documents (top k scoring)$

- Recall@ $k \frac{|\text{retrieved}@k \cap \text{relevant}|}{|\text{relevant}|}$
- Precision@ $k \frac{|\text{retrieved@}k \cap \text{relevant}|}{|\text{retrieved@}k|} = \frac{|\text{retrieved@}k \cap \text{relevant}|}{k}$

- Average precision:  $AveP(q) = \frac{\sum_{k} P@k \times rel(k)}{|relevant|}$
- $rel(k) = \begin{cases} 1 & k\text{-th document relevant} \\ 0 & otherwise \end{cases}$

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- Mean average precision  $MAP(Q) = \frac{\sum_{q \in Q} AveP(q)}{|Q|}$
- Q can be a "test set"

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- Mean average precision  $MAP(Q) = \frac{\sum_{q \in Q} AveP(q)}{|Q|}$
- F-score  $2 \cdot \frac{P \cdot R}{P+R}$

- Taking the rank into consideration: Mean Reciprocal Rank
- $MRR(Q) = \frac{1}{|Q|} \sum_{q \in Q} \frac{1}{\operatorname{rank}_q}$ rank<sub>q</sub> = position of the first relevant document

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document	rank	relevant
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b	1	
С		
d		+
е	2	+
f	3	

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• 
$$Q = \{\text{example}\}, MRR(Q) = \frac{1}{\text{rank}_{example}} = \frac{1}{2}$$

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Always depends on the task.

Query: Goethe, devil

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  - A (Wolfgang Goethe, Mephistopheles devil)
  - C (unrelated context)

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- How to rank them?
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- Can these inferences be made automatically?

Solution: vector with counts of words:

```
(<the>, <a>, <dog>, , fent>, ...)
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- Issue: how do we know which words are useful?

# Term Frequency - Inverse Document Frequency

#### TF-IDF

$$tf(term, doc) = \frac{count_{doc}(term)}{|doc|}$$
 
$$df(term) = \frac{|\{doc|term \in doc, doc \in D\}|}{|D|}$$
 
$$idf'(term) = \frac{|D|}{df(term)}, idf(term) = \log_2\left(\frac{|D|}{df(term)}\right)$$
 
$$tf - idf(term, doc) = tf(term, doc) \times idf(term)$$

# Term Frequency - Inverse Document Frequency

#### TF-IDF

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$$\textit{idf}'(\textit{term}) = \frac{|\textit{D}|}{\textit{df}(\textit{term})}, \textit{idf}(\textit{term}) = \log_2\left(\frac{|\textit{D}|}{\textit{df}(\textit{term})}\right)$$
 
$$\textit{tf} - \textit{idf}(\textit{term}, \textit{doc}) = \textit{tf}(\textit{term}, \textit{doc}) \times \textit{idf}(\textit{term})$$

#### Augmented TF

$$tf'(term, doc) = 0.5 + 0.5 \cdot \frac{count_{doc}(term)}{max_{term'}\{count_{doc}(term')\}}$$

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- Solution: vector of tf-idf
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- Issue: demon Mephistopheles are equally separate concepts as devil lasagne
- Issue: independent terms assumption

#### Document Retrieval - Probabilistic Retrieval

- Goal: Find P[R|d,q]
- Ranking: Proportional to relevance odds

$$O(R|d,q) = \frac{P[R|d,q]}{P[\bar{R}|d,q]}$$

 Different probabilistic models calculate these probabilities differently e.g. Binary Independence model, Poisson model, BM25

For Poisson, 
$$P[d|\lambda] = \prod_{t \in V} \frac{e^{-\lambda_t \cdot \lambda_t^{d_t}}}{d_t!}$$

# Document Retrieval - Statistical Language Model

- Pretend the query was generated by a LM based on the document
- Ranking: Proportional to query likelihood
- $argmax_d \ p(d|q) = argmax_d \ \frac{p(q|d) \cdot p(d)}{p(q)} = argmax_d \ p(q|d) \cdot p(d)$   $\approx argmax_d \ p_{LM}(q|d) \cdot p(d)$   $p(d) \approx \frac{1}{|D|} \text{ or } p(d) \text{ is query independent}$  $\approx argmax_d \ p_{LM}(q|d)$
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- LMs can be smoothed, as you remember e.g. Interpolation, Dirichlet smoothing
- Jelinek-Mercer smoothing:  $p(q_i|d,D) = \lambda \cdot p(q_i|d) + (1-\lambda) \cdot p(q_i|D)$ High  $\lambda$ : documents with all query words (conjunctive) Low  $\lambda$ : suitable for long queries (disjunctive)

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- Issue: Without word embeddings, no word relatedness Query: Goethe, devil
  - A: Wolfgang's idea of the demon Mephistopheles who makes a bet with God
- Can we model word co-occurence for a topic?

# Solution 4 (Latent Semantic Analysis)

- Assumption: Documents are composed of *k* latent topics.
- Solution: Perform dimensionality reduction → eigenvalues, singular value decomposition
- $A_{i,j} = \#$ occurences of term  $t_i$  in document  $d_j$

	$d_1$	$d_2$	$d_3$	$d_4$
Wolfgang	1	1	0	0
Mephistopheles	1	0	0	0
Faust	0	1	0	0
Goethe	0	1	0	1
devil	0	1	1	0
demon	1	0	0	1
lasagne	0	0	1	0
German	0	1	0	1

	$d_1$	$d_2$	d <sub>3</sub>	$d_4$
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lasagne	0	0	1	0
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	$c_1$	<i>c</i> <sub>2</sub>	<i>c</i> <sub>3</sub>
Wolfgang	1	0	0
Mephistopheles	0	1	0
Faust	1	0	0
Goethe	1	0	0
devil	0	1	0
demon	0	1	0
lasagne	0	0	1
German	1	0	0

3 latent concepts:

{Goethe (Wolfgang, Faust, German), devil (Mephistopheles, demon), lasagne}

$$d_1 = 1 \times c_1 + 1 \times c_2$$

• Given: *A*, *k* 

- Given: A, k
- $A' = argmin_{A'rankk}||A A'||$

- Given: A, k
- $A' = argmin_{A'rankk} ||A A'||$  Distance e.g. Frobenius  $(\sqrt{\sum_{i,j} a_{i,j}})$

# Singular Value Decomposition

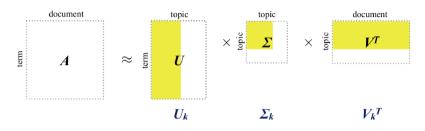


Figure 1: SVD for LSA

- U = eigenvectors of  $A^T A$  (# intersection of documents  $d_i$  and  $d_j$ )
- V = eigenvectors of  $AA^T$  (# documents in which both terms  $t_i$  and  $t_j$  occur)
- $S = \text{roots of corresponding eigenvalues of } A^T A$
- $A = USV^T$

# Eigen{vector,value}

Nonzero  $v \in \mathbb{R}^n, \lambda \in \mathbb{R}$ 

### Eigenvector

$$Av = \lambda v$$
  $Av = \lambda Iv$   $(A - \lambda I)v = 0$   $ker(A - \lambda I)$ 

"Directions (v) which A only scales."

#### Eigenvalue

$$Av = \lambda v$$

"The stretch  $(\lambda)$  of eigenvector v by A."

### **SVD**

#### Proof sketch

$$A = USV^T, A^T = VSU^T, S$$
 diagonal  $U^TU = VV^T = I$  orthogonal  $AA^TU = US^2 \rightarrow U$  eigenvectors of  $AA^T, S$  root of eigenvalues  $(\forall i: AA^TU_{i,*} = U_{i,*} \cdot S_{i,i}^2)$   $A^TAV = VS^2 \rightarrow V$  eigenvectors of  $A^TA, S$  root of eigenvalues  $(\forall i: A^TAV_{i,*} = V_{i,*} \cdot S_{i,i}^2)$ 

### LSA

- Order eigenvalues by descending values  $(S_{i,i} > S_{i+1,i+1} \ge 0)$  (proof next slide)
- Take top-k eigenvectors + values (or all above threshold)

### **LSA**

- Order eigenvalues by descending values  $(S_{i,i} > S_{i+1,i+1} \ge 0)$  (proof next slide)
- Take top-k eigenvectors + values (or all above threshold)
- Term  $\rightarrow$  latent representation:  $U_k S_k$
- Document  $\rightarrow$  latent representation:  $(S_k V_k^T)^T = V_k S_k^T = V_k S_k$

# Properties of S

### Descending

$$U' = U$$
 +swapped  $i, j$  column,  $S' = S$  +swapped  $i, j$  values,  ${V'}^T = V^T$  +swapped  $i, j$  row  $U' = U \times C(i, j), S' = S \times C(i, j), {V'}^T = V^T \times R(i, j)$   $U'S' = (US)$  with swapped  $i, j$  columns,  $U'S' = (US) \times C(i, j)$   $U'S'V'^T = (US) \times C(i, j) \times V^T \times R(i, j) = (US) \times C(i, j) \times C(i, j) \times V^T = USV^T$ 

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#### Non-negative

$$A^T A$$
 is positive semidefinite  $\Rightarrow S_{i,i} \ge 0$   
 $\forall x \ne \overrightarrow{0} : x^T A^T A x = (Ax)^T (Ax) = ||Ax|| \ge 0$ 

## LSA Concepts

- $U_k S_k$  maps terms to latent "concepts"  $(m \to k)$
- $V_k S_k$  maps documents to "concepts"  $(n \to k)$

	$d_1$	$d_2$	d <sub>3</sub>	$d_4$
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• Choose k=2

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- Choose k=2
- Representation of Goethe: fourth row of  $U_k$   $(m \times k \to 1 \times 2)$  scaled by  $S_k$ :  $[0.13, -0.13]^T$

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- Representation of devil: fifth row of  $U_k$   $(m \times k \to 1 \times 2)$  scaled by  $S_k$ :  $[0.58, -0.01]^T$

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- Representation of  $d_1$ : first column of  $V_k^T$   $(k \times n \to 2 \times 1)$  scaled first by  $S_k$ :  $r_d = [0.3, 0.02]^T$

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- Map query to our topic space:  $q \rightarrow U_k^t \cdot q = q' = [0.355, -0.07]^T$

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- Map query to our topic space:  $q o U_k^t \cdot q = q' = [0.355, -0.07]^T$
- Query-document match: dot product, cosine similarity:  $\frac{r_q \cdot r_d}{|r_q| \cdot |r_d|} = \frac{0.01205}{0.10879} \approx 0.11$

# LSA Graphics

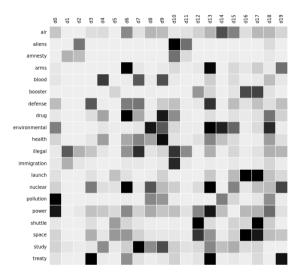


Figure 2: Term-document matrix, no ordering, k = 5; Source [6]

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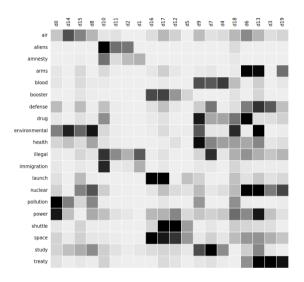


Figure 3: Term-document matrix, group documents, k = 5; Source [6]

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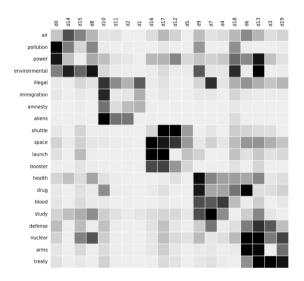


Figure 4: Term-document matrix, group documents+terms, k = 5; Source [6]

#### LSA Code

```
from sklearn.decomposition import TruncatedSVD
from sklearn.feature extraction.text import TfidfVectorizer
vectorizer = TfidfVectorizer(stop_words='english',
    max_features= 1000,
    \max df = 0.5,
    smooth_idf=True)
X = vectorizer.fit_transform(documents)
svd model = TruncatedSVD(n components=20)
svd model.fit(X)
                     Compression: m \times n \rightarrow m \times k + n \times k + k \times k
```

#### Considerations

#### Pros:

- Easy to implement
- Explainable terms
- Quite fast runtime
- Handles synonymy of words

#### Cons:

- Only surface dependencies
- Determination of k
- SVD difficult to update

### **Dense Vectors**

TODO

#### Resources

- Python code: https://medium.com/acing-ai/what-is-latent-semantic-analysis-lsa-4d3e2d18417a
- $\textbf{@} \ \, \mathsf{Comprehensive} \ \, \mathsf{tutorial} \ \, \mathsf{for} \ \, \mathsf{LSA+SVD} \colon \mathsf{https:}//\mathsf{www.engr.uvic.ca}/\sim \mathsf{seng474/svd.pdf}$
- SVD example: http://web.mit.edu/be.400/www/SVD/Singular\_Value\_Decomposition.htm
- Computation: https://en.wikipedia.org/wiki/Singular\_value\_decomposition#Calculating\_the\_SVD
- $\verb| Omputation: https://www.cs.utexas.edu/users/inderjit/public\_papers/HLA\_SVD.pdf| \\$
- $\textbf{ 0} \ \ \ Visualization: \ https://topicmodels.west.uni-koblenz.de/ckling/tmt/svd\_ap.html \\$
- $\hbox{ $\emptyset$ Computation: $https://en.wikipedia.org/wiki/Jacobi\_eigenvalue\_algorithm} \\$
- Option code: https://www.analyticsvidhya.com/blog/2018/10/stepwise-guide-topic-modeling-latent-semantic-analysis/
- ① Jelinek-Mercer: http://ctp.di.fct.unl.pt/~jmag/ir/slides/a05%20Language%20models.pdf
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