

# Assignment 10 + Conditional Random Fields

## (SNLP Tutorial 11)

Vilém Zouhar, Awantee Deshpande, Julius Steuer

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# Organisation

- Check that you have the finalised versions of the tutorial slides ([github.com/zouharvi/uds-snlp-tutorial](https://github.com/zouharvi/uds-snlp-tutorial))
- Check if you are eligible for the exam, and register accordingly.
- Project will be released on Friday, expected deadline at the end of August (tentatively 20th Aug), will be specified in the project instructions.
- Next week's tutorial discussion: Open Q&A
- Send a list of questions to me (Teams or Private Piazza Post)
- Discussion of sample exam

# Assignment 10

- Exercise 1: Lesk's Algorithm
- Exercise 2: Expectation Maximisation
- Exercise 3: Yarowsky Algorithm

# Overview

- Sequence Labelling / Entity Recognition
  - ▶ Rule-based
  - ▶ HMM
  - ▶ Bayesian Network
  - ▶ Log-linear 1st Order Sequential Model
  - ▶ Linear Chain CRF / CRF
- Model comparison
- Implementations

# Sequence Labelling / Entity Recognition

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- My name is [Joachim:PERSON], I live in [Saarbrücken:LOC], and my matriculation number is [1234:MATNUM].

# Sequence Labelling / Entity Recognition

- My name is Joachim, I live in Saarbrücken, and my matriculation number is 1234.
- My name is [Joachim:PERSON], I live in [Saarbrücken:LOC], and my matriculation number is [1234:MATNUM].
- NER as Sequence labelling:
  - X: sequence of words
  - Y: labels {MATNUM, PERSON, LOCATION, NONE}

# Rule-based

- Regex substitute:  
`matriculation (number)? (is)? (\d+) → [\3:mat-num]`



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- No automated learning

# Generative vs. Discriminative Models

- What's the difference?

# Generative vs. Discriminative Models

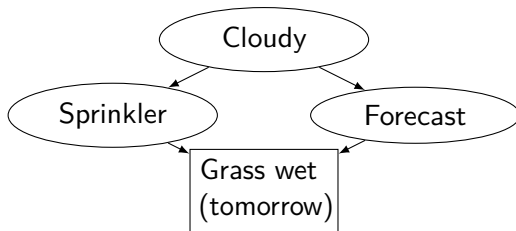
- What's the difference?
- Generative: Model actual distribution of data, learn joint probability and predict conditional probability using Bayes Theorem i.e. predict  $P(Y|X)$  using  $P(X|Y)$  and  $P(Y)$   
e.g. Naive Bayes, HMMs
- Discriminative: Model decision boundary between classes, learn conditional probability directly, estimate parameters for  $P(Y|X)$  directly from data  
e.g. MaxEnt Classifier, CRFs

# Bayesian Network

- Directed acyclic graph (DAG),  $(x \rightarrow y) \in E : y$  dependent on  $x$

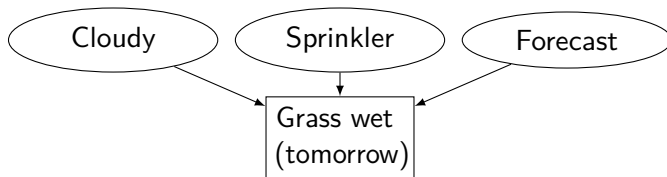
## Local Markov Property

- Node is conditionally independent of its nondescendants given its parents.  
 $p(\text{Sprinkler} | \text{Cloudy}, \text{Rain}) = p(\text{Sprinkler} | \text{Cloudy})$
- How does this benefit us?

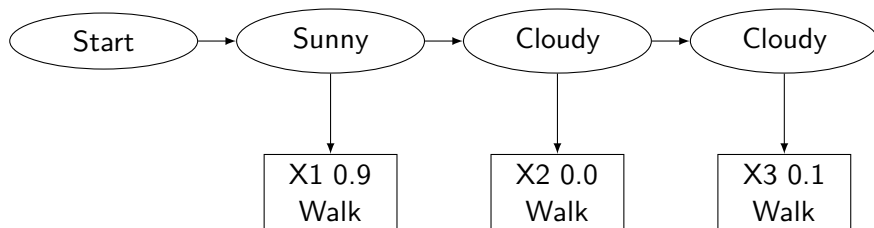


# Naïve Bayes

- Assume absolute independence except for the one observed variable
- $p(y = \text{Yes}|x) = p(y_j|x) = \frac{p(x|y_j)p(y_j)}{p(x)} \propto p(x|y_j)p(y_j) \approx p(y_j) \prod_i p(x_i|y_j)$



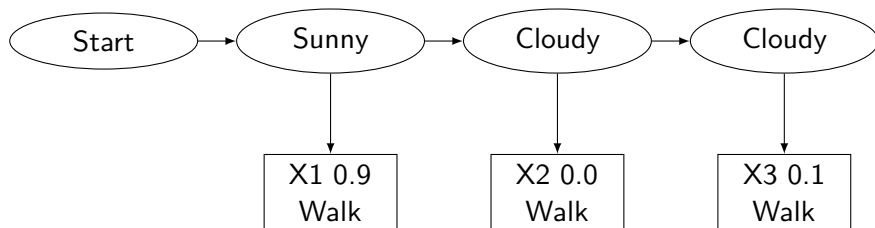
# HMM



Sketch of HMM structure

observed variable *Walk duration*, latent variable:  $Weather \in \{Sunny, Cloudy\}$

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$$p(y|x) = p(y) \cdot \prod_i o(x_i, y) \text{ (Naïve Bayes)}$$

$\Rightarrow$

$$p(Y|X) = \prod_i a(y_{i-1}, y_i) \cdot o(y_i, x_i) \text{ (HMM)}$$



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- Emission probabilities: MLE from annotated data (+ smoothing)
- $p(Y|X) = \prod_i a(y_{i-1}, y_i) \cdot o(y_i, x_i)$

## Questions

- What are the drawbacks of HMMs?

# Logistic Regression

$$p(y|x) = \frac{\exp(\Phi(y,x))}{\sum_{y'} \exp(\Phi(y',x))}$$

$$\arg \max_y \frac{\exp(\Phi(y,x))}{\sum_{y'} \exp(\Phi(y',x))} = \arg \max_y \exp(\Phi(y,x))$$



# Model overview

- Multinomial logistic regression:

$$p(y_j|x) = \frac{\exp(Z_j \cdot x)}{\sum_i \exp(Z_i \cdot x)}$$

- Multiclass naïve Bayes:

$$p(y_j|x) = \frac{p(x|y_j)p(y_j)}{p(x)} \propto p(x|y_j)p(y_j) \approx p(y_j) \prod_i p(x_i|y_j)$$

# Log-linear 1st Order Sequential Model

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- $p(y|x) = \frac{1}{Z(x)} \cdot \exp \{ \sum_i \log a(y_{i-1}, y_i) + \log o(y_i, x_i) \}$

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# Log-linear 1st Order Sequential Model

Viterbi:

$$\operatorname{argmax}_y p(y|x) = \operatorname{argmax}_y \log p(y|x) = \operatorname{argmax}_y F(y, x) - \log \sum_{y'} \exp F(y', x)$$

$$= \operatorname{argmax}_y F(y, x)$$

$$\alpha_t(y_j) = \max_i \exp \left( \log \alpha_{t-1}(y_i) + a(y_j, y_i) + o(y_j, x_t) \right)$$

$$\alpha'_t(y_j) = \operatorname{argmax}_i \alpha_{t-1}(y_i) + \exp (a(y_j, y_i) + o(y_j, x_t))$$

$$O(|Y|^2 \cdot T)$$



# Log-linear 1st Order Sequential Model

Forward:

$$\log fw_t(y_j) = \log \sum_i \exp \left( \log fw_{t-1}(y_i) + a(y_j, y_i) + o(y_j, x_t) \right)$$

$$Z(X) = \sum_i \exp \left( \log fw_{|T|-1}(y_i) + a(y_j, y_i) + o(y_j, x_t) \right)$$

$\rightarrow$

$$p(y|x) = \frac{\alpha_{|T|}(y_{:-1})}{Z(x)}$$

$$O(|Y|^2 \cdot T)$$

# Log-linear 1st Order Sequential Model

- Replace  $o(y_j, x_t)$  with  $\lambda_1 h_1(y_j, x_t) + \lambda_2 h_2(y_j, x_t) + \dots$

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- Replace  $o(y_j, x_t)$  with  $\lambda_1 h_1(y_j, x_t) + \lambda_2 h_2(y_j, x_t) + \dots$
- Same with  $a(y_j, y_i) = \lambda'_1 g_1(y_j, y_i) + \lambda'_2 g_2(y_j, y_i) + \dots$

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- Same with  $a(y_j, y_i) = \lambda'_1 g_1(y_j, y_i) + \lambda'_2 g_2(y_j, y_i) + \dots$
- Why not just  $\sum_{\text{feature } f} \lambda_i f_i(y_i, y_j, x_t)$  ?

# Model overview

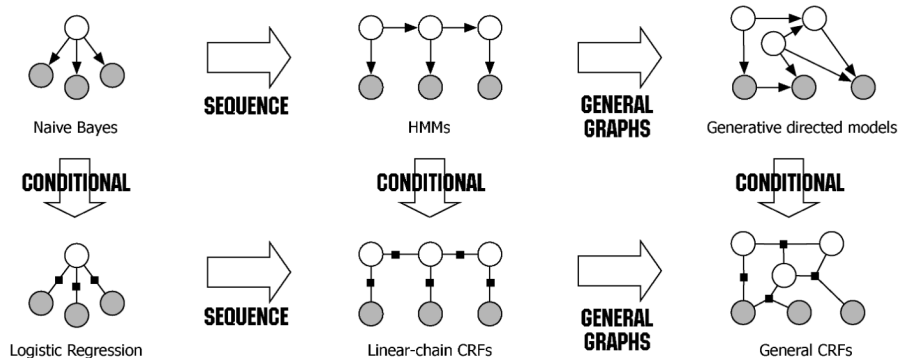


Figure 1: CRF in relation to other models; Source [2]

## HMM $\rightarrow$ Linear CRF

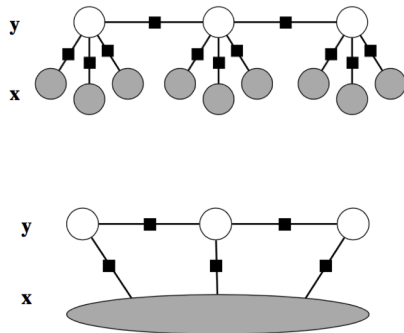


Figure 2: HMM vs. Linear Chain CRF; Source [12]

### Question

- What is the difference between HMM and CRF?

# Conditional Random Fields

- Factorization to maximal cliques.
- Allow access to a whole clique

## Clique

$$G = (V, E) \quad C \subseteq V : \forall x, y \in C : (x, y) \in E$$

## CRF

$$p(y|x) = \frac{1}{Z(x)} \prod_{c \in C} \psi_c(x_c)$$

## Maximal Clique

$$C \subseteq C' \Rightarrow C = C'$$

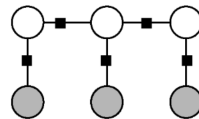


Figure 3: Linear CRF [2]

# Linear CRF

- Sequence of hidden states:  $y$ , {MATNUM, PERSON, LOCATION, NONE}
- Observed sequence of variables:  $x$  (words)
- $p(y|x) \propto \prod_i \exp \{ \sum_j \lambda_j f_j(y_{i-1}, y_i, x, i) \}$
- $p(y|x) = \frac{1}{Z(x)} \prod_i \exp \{ \sum_j \lambda_j f_j(y_{i-1}, y_i, x, i) \}$
- Features:  $f_j(y_{i-1}, y_i, x, i)$
- Parameters:  $\lambda$
- Clique template:  $\{ \Psi_i(y_{i-1}, y_i, x, i) | \forall i \in \{1 \dots n\} \}$



## Linear CRF - Binary Features

$$f_j(y_{i-1}, y_i, x, i) = \begin{cases} 1 & \text{if } \text{cond}_f(y_{i-1}, y_i, x, i) \\ 0 & \text{else} \end{cases}$$

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$$f_1(y_{i-1}, y_i, x, i) = \begin{cases} 1 & \text{if } x_{i-2} \text{ is capitalized} \\ 0 & \text{else} \end{cases}$$

$$f_a(y_{i-1}, y_i, x, i) = \begin{cases} 1 & \text{if } y_{i-1} = \text{number} \wedge y_t = \text{none} \\ 0 & \text{else} \end{cases}$$

$$\lambda_a = a(\text{number}, \text{none})$$

$$f_o(y_{i-1}, y_i, x, i) = \begin{cases} 1 & \text{if } y_i = \text{number} \wedge x_i = \langle \text{num} \rangle \\ 0 & \text{else} \end{cases}$$

$$\lambda_o = o(\text{number}, \langle \text{num} \rangle)$$

## Linear Chain CRF - Non-binary Features

$$f_w(y_{i-1}, y_i, x, i) = |x_i| \text{ word length}$$

$$f_s(y_{i-1}, y_i, x, i) = |c| \text{ number of non-alphabetic characters}$$

# Linear Chain CRF - Non-binary Features

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## Questions

- How do we interpret the values of  $\lambda_j$  for the features  $f_j$ ? ( $\lambda_j > 0$ ,  $\lambda_j = 0$ ,  $\lambda_j < 0$ ?)
- How are  $\lambda$ s estimated?
- How many such features can we create?

# CRF - Operations

Training:

$$\operatorname{argmax}_{\lambda} p(y_D|x_D, \lambda)$$

Interpretation: Given label sequences and inputs, find parameters of the CRF  $M$  that maximise  $p(y|x, \lambda)$ .

Done using gradient methods, Forward-Backward algorithm etc.

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Done using gradient methods, Forward-Backward algorithm etc.

Inference (Viterbi):

$$\operatorname{argmax}_y p(y|x, \lambda)$$

Interpretation: Given input  $x$  and CRF  $M$ , find optimal  $y$ .

Decoding (forward):

$$\max_y p(y|x, \lambda)$$

## Linear Chain CRF - Estimating $\lambda$

Gradient descent (ascent):

$$\frac{\partial \log p(y|x, \lambda)}{\partial \lambda_i} = \sum_{t=1}^T f_i(y_{t-1}, y_t, x, t) - \sum_{y'} \sum_{t=1}^T f_i(y'_{t-1}, y'_t, x, t) \cdot p(y'|x)$$

$$\lambda_f \leftarrow \lambda_f + \epsilon \left[ \sum_{t=1}^T F(y_{t-1}, y_t, x, t) - \sum_{y'} \sum_{t=1}^T F(y'_{t-1}, y'_t, x, t) \cdot p(y'|x, \lambda) \right]$$

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Limited-memory BFGS (quasi-Newton method)



# Feature selection:

## Alternative 1

- 1 Start with all features.
- 2
  - a. If there exists a feature removing which worsens the performance by  $< t$ , remove it. Repeat 2.
  - b. If not, exit.

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## Properties

- Hard to setup & train
- Fast inference

# Linear Chain CRF - Regularization

Objective function:

$$\mathcal{L} = \sum_s \log p(y^{(s)} | x^{(s)}, \lambda)$$

LASSO:

$$\mathcal{L}_{+lasso} = \sum_s \log p(y^{(s)} | x^{(s)}, \lambda) - \lambda_1 \sum_i |\lambda_i|$$

Ridge:

$$\mathcal{L}_{+ridge} = \sum_s \log p(y^{(s)} | x^{(s)}, \lambda) - \frac{\lambda_2}{2} \sum_i \lambda_i^2$$

Elastic net:

$$\mathcal{L}_{+elastic} = \sum_s \log p(y^{(s)} | x^{(s)}, \lambda) - \frac{\lambda_2}{2} \sum_i \lambda_i^2 - \lambda_1 \sum_i |\lambda_i|$$

# Code

```
from sklearn_crfsuite import CRF
X_train = [
    [word2features(s, i) for i in range(len(s))]
    for s in train_sents]
y_train = [
    [label for token, postag, label in s]
    for s in train_sents]
crf = sklearn_crfsuite.CRF(
    algorithm='lbfgs',
    c1=0.1, c2=0.1,
    max_iterations=100,
)
crf.fit(X_train, y_train)
```

- Fast Linear Chain CRFs (C): <http://www.chokkan.org/software/crfsuite/>
- Fast Linear Chain CRFs (C++): <https://taku910.github.io/crfpp/>

# Resources

- ❶ Hidden Markov Model: <https://web.stanford.edu/~jurafsky/slp3/A.pdf>
- ❷ Bayesian Networks: <https://www.ics.uci.edu/~rickl/courses/cs-171/0-ihler-2016-fq/Lectures/Ihler-final/09b-BayesNet.pdf>
- ❸ Overview: <https://www.analyticsvidhya.com/blog/2018/08/nlp-guide-conditional-random-fields-text-classification>
- ❹ Very detailed: <http://homepages.inf.ed.ac.uk/csutton/publications/crftut-fnt.pdf>
- ❺ Academic-level introduction to CRF: <https://www.youtube.com/watch?v=7L0MKKfqe98>
- ❻ Generalized CRF:  
[https://people.cs.umass.edu/~wallach/technical\\_reports/wallach04conditional.pdf](https://people.cs.umass.edu/~wallach/technical_reports/wallach04conditional.pdf)
- ❼ Accessible introduction: <http://pages.cs.wisc.edu/~jerryzhu/cs769/CRF.pdf>
- ❽ Forward-backward for CRF:  
[https://www.cs.cornell.edu/courses/cs5740/2016sp/resources/collins\\_fb.pdf](https://www.cs.cornell.edu/courses/cs5740/2016sp/resources/collins_fb.pdf)

# Resources

- ⑨ NER using CRF: <https://medium.com/data-science-in-your-pocket/named-entity-recognition-ner-using-conditional-random-fields-in-nlp-3660df22e95c>
- ⑩ Python code: <https://sklearn-crfsuite.readthedocs.io/en/latest/tutorial.html#let-s-use-conll-2002-data-to-build-a-ner-system>
- ⑪ Naïve Bayes, HMM, CRF:  
<http://cnyah.com/2017/08/26/from-naive-bayes-to-linear-chain-CRF/>
- ⑫ Highly Informative Naïve Bayes, HMM, MaxEnt, CRF:  
[https://ls11-www.cs.tu-dortmund.de/\\_media/techreports/tr07-13.pdf](https://ls11-www.cs.tu-dortmund.de/_media/techreports/tr07-13.pdf)