Assignment 4,5 + Smoothing 1

(SNLP Tutorial 5)

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Assignment 4

- Exercise 1: Huffman encoding
- Exercise 2: Conditional entropy of DNA
- Bonus: Huffman encoding adaptations

OOV words

Corpus

- Train set:
  🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆
- Test set:
  🍫 🍏 🍟 🍌 🍏 🍆 🍆 🍌 🍒 🍟 🍏 🍏

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Accumulate counts

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- 🍏 4 🍌 2 🍟 2 🍆 2 🍒 1 🍫 1

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- What about 🍫 and 🍟?

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OOV words

- How do we even know this will be an issue?

Subword Units

Solution to OOV words: go lower

- Characters: $V = \{a, b, c, \ldots, \_\}$

Questions

Subword Units

Solution to OOV words: go lower

- Characters: $V = \{a, b, c, \ldots, \_\}$
- Syllables: $V = \{bo, ve, r, how, \ldots, \_\}$

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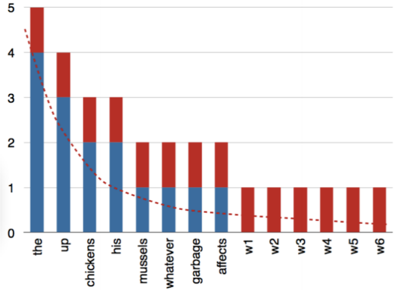
Questions

- Can we still get an unknown "word"?
- How do we define perplexity for subword language models?

Smoothing

- Words present in vocabulary, but have ~0 probabilities
- Words present in vocabulary, but have unseen context

Solution: Assign probability mass from frequent events to infrequent events (Smoothing/Discounting)



- Will cover different smoothing methods over the next few tutorials

Additive smoothing (add-α-smoothing)

Distribution

- Add zero counts to frequency table

🍏 6 🍌 5 🍆 3 🍒 2 🍟 0 🍫 0

- Increase all counts by $\alpha = 1$

🍏 6+1 🍌 5+1 🍆 3+1 🍒 2+1 🍟 0+1 🍫 0+1

- Divide by $N = 22$

🍏 0.32 🍌 0.27 🍆 0.18 🍒 0.13 🍟 0.05 🍫 0.05

Perplexity

- Relative frequencies on test corpus:

🍏 0.33 🍌 0.17 🍟 0.17 🍆 0.17 🍒 0.08 🍫 0.08

- Recall perplexity formula:

$$PP = \exp - \sum_{w,h} f(w,h) \cdot \log_2 p(w|h) \tag{1}$$

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Perplexity

- Relative frequencies on test corpus:

🍏 0.33 🍌 0.17 🍟 0.17 🍆 0.17 🍒 0.08 🍫 0.08

- PP: $2^{-(0.33\cdot(-1.64)+0.17\cdot(-1.89)+0.17\cdot(-4.32)+0.17\cdot(-2.47)+0.08\cdot(-2.94)+0.08\cdot(-4.32))} = 2^{(-2.6)} \approx 6$
- What would be PP with unsmoothed model?

Additive smoothing: Bigrams

Recall the additive smoothing formula for unigrams:

$$C^*(w_i) = C(w_i) + \alpha \tag{2}$$

$$N^* = \sum_{w_i \in V} C^*(w_i) = N + \alpha|V| \tag{3}$$

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$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})} \tag{5}$$

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Smoothen the bigram count: $C(w_{i-1}, w_i) \rightarrow C(w_{i-1}, w_i) + \alpha$

Additive smoothing: Bigrams

Corpus

🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆

Bigrams: 🍏🍏, 🍏🍏, 🍏🍆, 🍆🍏, . . . , 🍏🍆, 🍆🍏 ← circular bigram!

Bigrams: AA, AA, AE, EA, . . . , AE, EA

Additive smoothing: Bigrams: bigram counts

- Collect bigram counts & condtional probabilities for history A

Bigram	$C(A, w_i)$	$C(A)$	$\frac{C(A,w_i)}{CA)}$
AE	3	6	$1/2$
AA	2	6	$1/3$
AB	1	6	$1/6$

Additive smoothing: Bigrams: add alpha

- We encounter an unknown bigram *AF*

Bigram	$C(A, w_i)$	$C(A)$	$\frac{C_\alpha(A,w_i)}{C(A)}$
AE	3+1	6	4/6
AA	2+1	6	3/6
AB	1+1	6	2/6
$\rightarrow$ AF	0+1	6	1/6

Additive smoothing: Bigrams: add alpha

- We encounter an unknown bigram AF

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AE	3+1	6	4/6
AA	2+1	6	3/6
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$\rightarrow$ AF	0+1	6	1/6

- Not a probabilitiy distribution!

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AA	2+1	6	3/6
AB	1+1	6	2/6
$\rightarrow$ AF	0+1	6	1/6

- Not a probabilitiy distribution!
- Solution: We need to adjust the divisor a tiny bit. But how tiny?

Additive smoothing: Bigrams: normalization

- Add $\alpha \cdot 4$ to history count
- Pretend that we have seen the history $|V| = 4$ times more.

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Bigram	$C(A) + \alpha\|V\|$	$\frac{C_\alpha(A,w_i)}{C(A)+\alpha\|V\|}$
AE	$6 + 4$	$4/10$
AA	$6 + 4$	$3/10$
AB	$6 + 4$	$2/10$
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$\rightarrow$ AF	$6+4$	$1/10$

- Now the probabilities sum up to 1: $4/10 + 3/10 + 2/10 + 1/10 = 1$

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- What is $|V|$ now?

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Bigram	$C(A) + \alpha\|V\|$	$\frac{C_\alpha(A,w_i)}{C(A)+\alpha\|V\|}$
AE	$6 + 5$	$4/11$
AA	$6 + 5$	$3/11$
AB	$6 + 5$	$2/11$
$\rightarrow$ AF	$6 + 5$	$1/11$
$\rightarrow$ AD	$6 + 5$	$1/11$

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$\rightarrow$ AF	$6 + 5$	$1/11$
$\rightarrow$ AD	$6 + 5$	$1/11$

- $C(A)$ is constant, unsmoothed count
- Probabilities sum up to 1: $4/11 + 3/11 + 2/11 + 1/11 + 1/11 = 1$

Additive smoothing: Bigrams: general case

- General formula for smoothed bigram Probabilities:

$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha|V|} \tag{6}$$

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$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha|V_{(w_{i-1},\bullet)}|} \tag{7}$$

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- For n-grams of length n:

$$p(w_i|w_{i-1} : w_{i-n+1}) = \frac{C(w_{i-n+1} : w_i) + \alpha}{C(w_{i-n+1} : w_{i-1}) + \alpha|V_{(w_{i-n+1}:w_{i-1},\bullet)}|} \tag{8}$$

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- We already know the shared (train + test) vocabulary V

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- $V_{(A,\bullet)}$ is then $AA, AB, AC, AD, AE, AF \Rightarrow |V_{(A,\bullet)}| = 6 = |V|$

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- $V_{(A,\bullet)}$ is then $AA, AB, AC, AD, AE, AF \Rightarrow |V_{(A,\bullet)}| = 6 = |V|$
- We find that the formula we found is identical to the one on the lecture slides!

$$p(w_i|w_{i-1} : w_{i-n+1}) = \frac{C(w_{i-n+1} : w_i) + \alpha}{C(w_{i-n+1} : w_{i-1}) + \alpha|V|} \tag{10}$$

Backing-off

MARY HAD A LITTLE LAMB

- Consider the bigram (LITTLE MARY)
- Consider the trigram (HAD A LAMB)

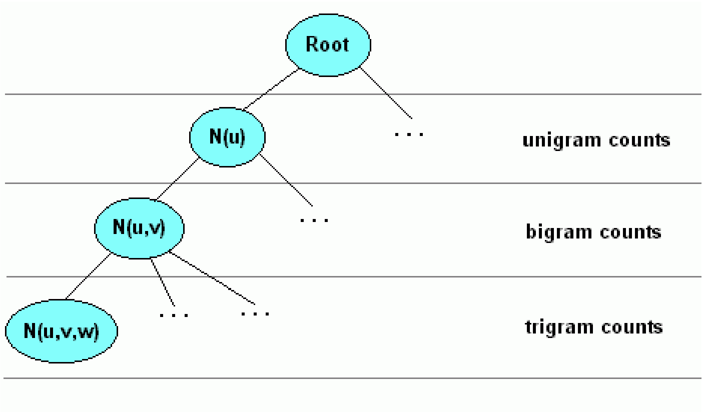
For a trigram $p(w_3|w_2, w_1)$, use probability of bigram $P(w_3|w_2)$, else back-off to unigram probability $P(w_3)$.

$$0.5 \cdot p(w_3|w_2, w_1) + 0.25 \cdot p(w_3|w_2) + 0.25 \cdot p(w_3)$$

$$0.5 \cdot p(\text{lamb}|\text{a}, \text{had}) + 0.25 \cdot p(\text{lamb}|\text{a}) + 0.25 \cdot p(\text{lamb})$$

Will be covered in more detail in further tutorials.

Count Trees



Root
N(u)
. . .
unigram counts
N(u,v)
. . .
bigram counts
N(u,v,w)
. . .
. . .
trigram counts

Assignment 5

- Exercise 1: OOV Words
- Exercise 2: Additive smoothing
- Exercise 3: Perplexity, infinite smoothing, interpolation
- Bonus: Other language models

Resources

1. UdS SNLP Class: https://teaching.lsv.uni-saarland.de/snlp/
2. Additive smoothing: https://en.wikipedia.org/wiki/Additive_smoothing
3. n-gram count trees: http://ssli.ee.washington.edu/WS07/notes/ngrams.pdf
4. n-gram models: https://web.stanford.edu/~jurafsky/slp3/3.pdf
5. Count-trees figure: https://www.w3.org/TR/ngram-spec/