# Assignment 1 + Language Properties (SNLP tutorial 2)

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## Organisational Issues

- Teammates
- Assignment submissions
- Naming your assignment folder: Name1\_id1\_Name1\_id2.zip
- Your Notebooks and files should be directly inside the main folder (no unnecessary nesting)
- Do not submit the following files:
- .ipynb\_checkpoints
- ★ data/\*
- Any other pdf or information file accompanying the assignment
- Only submit: Notebook + Python files. Otherwise points can be deducted.

## Part 1: Discussion of Assignment 1

- Exercise 1: Instructions for setup
- Exercise 2: Mandelbrot distribution + Stick breaking
- Exercise 3: Zipf's Law at word level
- Bonus: Zipf's Law at character level

# Part 2: Overview of current topics

- Basics of Probability Theory
- Perplexity
- Maximum Likelihood Estimation
- Smoothing

# Probability Theory for Language Models

#### Predict

 $P(w_1, w_2...w_N)$  which can be decomposed as  $\prod P(w_i|h_i)$ 

#### Bonus question

Compare for uniform, unigram, bigram, trigram... ngram models.

- Where do we assume statistical independence?
- What is this kind of assumption called?

# Probability Theory for Language Models

#### Entropy as Expectation value

$$E[f(V)] = \sum_{w_i \in V} p(w_i) f(w_i)$$

Entropy is a property of any distribution, e.g. that of a unigram language model.

$$H = E[-\log(p(V))] = -\sum_{w_i \in V} p(w_i)\log(p(w_i))$$

What does this mean? What are we capturing by the entropy of the LM distribution?

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Consider a bigram model where

$$E[-\log P(w|'in')] = 10.42 \text{ (e.g. ('in', 'fact'), ('in', 'that'), ('in', 'my'))}$$

$$E[-\log P(w|'the')] = 15.11 \text{ (e.g. ('the', 'day'), ('the', 'most'), ('the', 'end')}$$

What do the expectation values indicate here?

**1** What is the entropy of a fair die  $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$ ?

- What is the entropy of a fair die p = (<sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>)?
  What is the entropy of a loaded die q = (<sup>1</sup>/<sub>12</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>1</sup>/<sub>6</sub>, <sup>3</sup>/<sub>12</sub>)?

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- What would happen if we assumed the correct distribution?
- What does the difference tell us?

# Perplexity

#### Formulae

$$PP = 2^{-\frac{1}{n} \sum_{1}^{n} \log p(w_{i}|w_{i})}$$

$$PP = 2^{-\sum_{w,h} f(w,h) \log_{2} P(w|h)}$$

How do these two formulae relate to each other?

#### Maximum Likelihood Estimation

- A way to estimate language model (distribution) parameters
- Trying to maximize probability of the training data
- NOTE: Separate the text itself from the language model
- LMs exist independent of the text and MLE only maximizes their performance on the text

#### **Bonus Questions**

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Different smoothing methods will be covered in the further chapters.

#### Homework

- Exercise 1: Perplexity calculation by hand
- Exercise 2: Plotting n-gram distributions
- Exercise 3: MLE language models, Perplexity calculation
- Bonus: Custom alternative to perplexity

• If the most frequent unigram is X, will the most frequent bigram begin with X? (R.M.V., 2021)

#### Resources

- Why is Perplexity used over Entropy? https://stats.stackexchange.com/questions/285798/perplexity-and-cross-entropy-for-n-gram-models
- On Redundancy in Natural Languages http://www-math.ucdenver.edu/~wcherowi/courses/m5410/m5410lc1.html