

KL Divergence

(SNLP tutorial 3)

Vilém Zouhar, Awantee Deshpande, Julius Steuer

4th, 6th May 2021

Organisational Issues

- Teammates
- Assignment submissions
 - ▶ Naming your assignment folder: Name1_id1_Name1_id2.zip
 - ▶ Your Notebooks and files should be directly inside the main folder (no unnecessary nesting)
 - ▶ Do not submit the following files:
 - ▶ ★ `--pycache__`
 - ▶ ★ `.ipynb_checkpoints`
 - ▶ ★ `data/*`
 - ▶ ★ Any other pdf or information file accompanying the assignment
 - ▶ Only submit: Notebook + Python files. Otherwise points can be deducted.

Part 1: Discussion of Assignment 1

- Exercise 1: Instructions for setup
- Exercise 2: Mandelbrot distribution + Stick breaking
- Exercise 3: Zipf's Law at word level
- Bonus: Zipf's Law at character level

Part 2: Overview of current topics

- Basics of Probability Theory
- Perplexity
- Maximum Likelihood Estimation
- Smoothing

Probability Theory for Language Models

Predict

$P(w_1, w_2 \dots w_N)$ which can be decomposed as $\prod P(w_i | h_i)$

Bonus question

Compare for uniform, unigram, bigram, trigram... ngram models.

- Where do we assume statistical independence?
- What is this kind of assumption called?

Probability Theory for Language Models

Entropy as Expectation value

$$E[f(V)] = \sum_{w_i \in V} p(w_i) f(w_i)$$

Entropy is a property of any distribution, e.g. that of a unigram language model.

$$H = E[-\log(p(V))] = - \sum_{w_i \in V} p(w_i) \log(p(w_i))$$

What does this mean? What are we capturing by the entropy of the LM distribution?

Probability Theory for Language Models

Entropy as Expectation value

$$E[f(V)] = \sum_{w_i \in V} p(w_i) f(w_i)$$

Entropy is a property of any distribution, e.g. that of a unigram language model.

$$H = E[-\log(p(V))] = - \sum_{w_i \in V} p(w_i) \log(p(w_i))$$

What does this mean? What are we capturing by the entropy of the LM distribution?

Consider a bigram model where

$$E[-\log P(w|in')] = 10.42 \text{ \{e.g. ('in', 'fact'), ('in', 'that'), ('in', 'my')\}}$$

$$E[-\log P(w|the')] = 15.11 \text{ \{e.g. ('the', 'day'), ('the', 'most'), ('the', 'end')\}}$$

What do the expectation values indicate here?

Bonus Questions

- 1 What is the entropy of a fair die $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$?

Bonus Questions

- 1 What is the entropy of a fair die $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$?
- 2 What is the entropy of a loaded die $q = (\frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12})$?

Bonus Questions

- 1 What is the entropy of a fair die $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$?
- 2 What is the entropy of a loaded die $q = (\frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12})$?
- 3 What is the cross-entropy of the same distribution? $H(p, p)$

Bonus Questions

- 1 What is the entropy of a fair die $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$?
- 2 What is the entropy of a loaded die $q = (\frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12})$?
- 3 What is the cross-entropy of the same distribution? $H(p, p)$
- 4 What is the cross-entropy of the loaded die q if we assume a wrongly loaded die $k = (\frac{1}{6}, \frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12}, \frac{1}{6})$?

Bonus Questions

- 1 What is the entropy of a fair die $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$?
- 2 What is the entropy of a loaded die $q = (\frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12})$?
- 3 What is the cross-entropy of the same distribution? $H(p, p)$
- 4 What is the cross-entropy of the loaded die q if we assume a wrongly loaded die $k = (\frac{1}{6}, \frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12}, \frac{1}{6})$?
- 5 What would happen if we assumed the correct distribution?

Bonus Questions

- 1 What is the entropy of a fair die $p = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$?
- 2 What is the entropy of a loaded die $q = (\frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12})$?
- 3 What is the cross-entropy of the same distribution? $H(p, p)$
- 4 What is the cross-entropy of the loaded die q if we assume a wrongly loaded die $k = (\frac{1}{6}, \frac{1}{12}, \frac{1}{6}, \frac{1}{6}, \frac{3}{12}, \frac{1}{6})$?
- 5 What would happen if we assumed the correct distribution?
- 6 What does the difference tell us?

Perplexity

Formulae

$$PP = 2^{-\frac{1}{n} \sum_1^n \log p(w_i | w_{:i})}$$

$$PP = 2^{-\sum_{w,h} f(w,h) \log_2 P(w | w_{:i})}$$

Note that $w_{:i}$ is all the previous (left) history up to i (exclusive).

How do these two formulae relate to each other?

Maximum Likelihood Estimation

- A way to estimate language model (distribution) parameters
- Trying to maximize probability of the training data
- NOTE: Separate the text itself from the language model
- LMs exist independent of the text and MLE only maximizes their performance on the text

LM Smoothing

Bonus Questions

- ① What happens if an unknown token is encountered and the LM assigns it 0 probability?

LM Smoothing

Bonus Questions

- ① What happens if an unknown token is encountered and the LM assigns it 0 probability?
- ② What are some quick solutions to this issue?

LM Smoothing

Bonus Questions

- ① What happens if an unknown token is encountered and the LM assigns it 0 probability?
- ② What are some quick solutions to this issue?
- ③ How are LMs useful in downstream tasks?

LM Smoothing

Bonus Questions

- ① What happens if an unknown token is encountered and the LM assigns it 0 probability?
- ② What are some quick solutions to this issue?
- ③ How are LMs useful in downstream tasks?

LM Smoothing

Bonus Questions

- ① What happens if an unknown token is encountered and the LM assigns it 0 probability?
- ② What are some quick solutions to this issue?
- ③ How are LMs useful in downstream tasks?

Different smoothing methods will be covered in the further chapters.

Homework

- Exercise 1: Perplexity calculation by hand
- Exercise 2: Plotting n-gram distributions
- Exercise 3: MLE language models, Perplexity calculation
- Bonus: Custom alternative to perplexity

Bonus Question

- *If the most frequent unigram is X , will the most frequent bigram begin with X ?
(R.M.V., 2021)*

Resources

- ① Why is Perplexity used over Entropy?

<https://stats.stackexchange.com/questions/285798/perplexity-and-cross-entropy-for-n-gram-models>

- ② On Redundancy in Natural Languages

<http://www-math.ucdenver.edu/~wcherowi/courses/m5410/m5410lc1.html>