

# Assignment 5,6 + Smoothing 2

## (SNLP Tutorial 6)

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[github.com/zouharvi/uds-snlp-tutorial](https://github.com/zouharvi/uds-snlp-tutorial)

- Contributions welcome
- “Cheating” allowed

# Assignment 5

- Exercise 1: OOV Words
- Exercise 2: Additive smoothing
- Exercise 3: Perplexity, infinite smoothing, interpolation
- Bonus: Other language models

# Cross-validation

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- If two models have same average performance ( $k$ -fold cross-validation), are they the same?
- Other usage of  $k$ -fold cross-validation (or split in general)?

# Smoothing Techniques - Basics

- To keep a language model from assigning 0 or  $\sim 0$  probabilities to \_\_\_\_\_
- Generally we can smooth any arbitrary \_\_\_\_\_
- Different ways to do this...

# Floor Discounting

$$P(w|h) = \frac{N(w, h) + \epsilon}{N(h) + \epsilon \cdot V}$$

Variants: Laplace smoothing, Lidstone smoothing, add- $\alpha$  smoothing. . .

## Questions

- What is  $N(h)$  for unigram  $N(w)$ ?
- What is  $N(h)$  for n-gram  $N(w, h)$ ?
- What is  $N(h)$  for zero-gram?

# Good-Turing

Data: 🍏 🍏 🍏 🍆 🍌 🍌 🍒 🍆 🍌 🍌 🍒 🍆 🍇 🌿

# Good-Turing

Data:          

- $N_4 = \{\text{banana}\}$
- $N_3 = \{\text{apple}, \text{eggplant}\}$
- $N_2 = \{\text{cherry}\}$
- $N_1 = \{\text{grapes}, \text{branch}\}$
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$$p_r = \frac{(r+1)N_{r+1}}{N_r} \cdot \frac{1}{N}$$

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$$p_r = \frac{(r+1)N_{r+1}}{N_r} \cdot \frac{1}{N}$$

- Nominator: expected total number of occurrences of words that occur  $r+1$  times
- Denominator-left: previous bucket size
- Fraction-left: expected number of occurrences of a single word from that bucket
- Denominator-right: divide by total occurrences

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- Two items have same original frequency. What will be their new probability?



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# Good-Turing - Questions

- Two items have same original frequency. What will be their new probability?
- Let  $k$  be the maximum occurrence of a word. What's the issue?
- A similar issue related to the one above?
- Do the probabilities sum up to 1?
- How to make it work for anything above unigrams?

# Linear Intepolation/Jelinek-Mercer Smoothing

Train: 🍏 🍒 🍌 🍌

Train bigrams: (🍏, 🍒) (🍒, 🍌) (🍌, 🍌)

Test: 🍒 🍏 🍒

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Train bigrams: (🍏, 🍒) (🍒, 🍌) (🍌, 🍌)

Test: 🍒 🍏 🍒

$$P(w|h) = \lambda P(w|h) + (1 - \lambda)P(w)$$

Can be generalised to higher order n-grams.

## Questions

- What condition must be fulfilled for higher n-grams?
- How is  $\lambda_i$  determined?
- Can you smooth the above probabilities?

# Backing-Off models

- What other way can we use the lower-order n-gram distributions?
- Is a lot of context always a good thing?
- Idea behind back-off models: Use information from a lower order n-gram distribution.

$$P(w|h) = \begin{cases} \frac{N(w,h)}{N(h)} + \alpha(h)\beta(w|h) & \text{for } N(w,h) > 0 \\ \alpha(h)\beta(w|h) & \text{otherwise} \end{cases}$$

- How come the coefficient is a function of the history and not a fixed constant?

# Absolute Discounting

## Corpus

- Train 🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆
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## Distribution

- Vocabulary counts

🍏 6     🍌 5     🍆 3     🍒 2     🍷 0     🍷 0

- Decrease all non-zero counts by some parameter  $d = 0.75$

🍏  $6 - 0.75$      🍌  $5 - 0.75$      🍆  $3 - 0.75$      🍒  $2 - 0.75$      🍷 0     🍷 0

- Divide by  $N = 16$

🍏 0.33     🍌 0.26     🍆 0.14     🍒 0.11     🍷 0     🍷 0






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$$\text{Sum} = 0.33 + 0.26 + 0.14 + 0.11 = 0.84 \neq 1.$$

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Sum =  $0.33+0.26+0.14+0.11 = 0.84 \neq 1$ .

Idea: Utilise this probability mass for zero counts.

# Absolute Discounting

$$P(w|h) = \frac{c(w, h) - d}{c(h)}$$

Adjust the probability mass  $1 - \sum_h \frac{c(w, h) - d}{c(h)}$

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e.g. For bigrams,

$$P_{abs}(w_i|w_{i-1}) = \frac{\max\{N(w_{i-1}, w_i) - d, 0\}}{\sum_{w'} N(w_{i-1}, w')} + \lambda(w_{i-1})P_{abs}(w_i)$$

$$P_{abs}(w_i) = \frac{\max\{N(w_i) - d, 0\}}{\sum_{w'} N(w')} + \lambda(.)P_{unif}(w_i)$$

$$\text{where } \lambda(w_{i-1}) = \frac{d}{\sum_{w'} N(w_{i-1}, w')} \cdot N_{1+}(w_{i-1}, \bullet)$$

$$\lambda(.) = \frac{d}{\sum_{w'} N(w')} \cdot N_{1+}$$

# Absolute Discounting - Questions

- How does the discounting parameter  $d$  affect perplexity?
- What values can  $d$  take? Why?
- What if we set  $d$  to  $\infty$ ?

# Kneser-Ney Smoothing

Idea: Can we use the lower order distributions in a better way?

I WENT TO THE GROCERY \_\_\_\_\_ .

Options:

$W_1$ : STORE

$W_2$ : YORK

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Options:

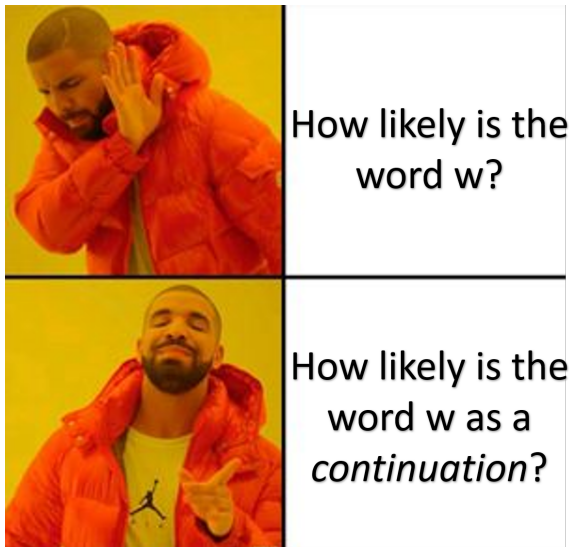
$W_1$ : STORE

$W_2$ : YORK

Use the fact that YORK generally appears as context or *continuation* of the word NEW.



# Kneser-Ney Smoothing



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$$P_{\text{continuation}}(w) \propto |\{w' : C(w', w) > 0\}|$$

Normalize by all bigram types. :  $|\{(w_i, w_j) : C(w_i, w_j) > 0\}|$

$$P_{KN}(w_i | w_{i-n+1:i-1}) = \frac{\max\{C_{KN}(w_{i-n+1:i-1}, w_i) - d, 0\}}{\sum_{w'} C_{KN}(w_{i-n+1:i-1} w')} + \lambda(w_{i-1}) P_{KN}(w_i)$$

$$\text{where } C_{KN}(\bullet) = \begin{cases} \text{count}(\bullet) & \text{for highest order} \\ \text{continuation\_count}(\bullet) & \text{for lower orders} \end{cases} \quad (1)$$

Will be covered in detail in the next tutorial...

# Pruning

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- Idea: Use some information-theory based approach to determine the nature of the probabilities, and then prune the lower orders. Known as *Stolcke Pruning*.

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## Questions

- Does pruning assign 0 probability to the pruned n-grams?
- Can we prune an entire branch/subtree? What does this mean?
- What is a good pruning strategy?

# Assignment 6

- Exercise 1: MAP and MLE estimates
- Exercise 2: Good Turing Smoothing
- Exercise 3: Cross-Validation

# Resources

- ① UdS SNLP Class: <https://teaching.lsv.uni-saarland.de/snlp/>
- ② n-gram models: <https://web.stanford.edu/~jurafsky/slp3/3.pdf>
- ③ Entropy pruning: <https://arxiv.org/pdf/cs/0006025.pdf>
- ④ Twitter emojis
- ⑤ Smoothing overview: [http://mlwiki.org/index.php/Smoothing\\_for\\_Language\\_Models](http://mlwiki.org/index.php/Smoothing_for_Language_Models)