Assignment 4+Smoothing

(SNLP tutorial 4)

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Overview

- Task, Metrics
- Differential Privacy
- Homework

Assignment 4

- Exercise 1: Huffman encoding
- Exercise 2: Conditional entropy of DNA
- Bonus: Huffman encoding adaptations

OOV words

Corpus

- Train set:
  🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆
- Test set:
  🍫 🍏 🍟 🍌 🍏 🍆 🍆 🍌 🍒 🍟 🍏 🍏

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Accumulate counts

- 🍏 6 🍌 5 🍆 3 🍒 2
- 🍏 4 🍌 2 🍟 2 🍆 2 🍒 1 🍫 1

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OOV words

- What about 🍫 and 🍟?
- OOV rate: $2+1/4+2+2+1+1+1=27\%$

- Solutions? character-level, subword units

Additive smoothing (add-α-smoothing)

Unigrams

- Add zero counts to frequency table

🍏 6 🍌 5 🍆 3 🍒 2 🍟 0 🍫 0

- Increase all counts by $\alpha = 1$

🍏 6+1 🍌 5+1 🍆 3+1 🍒 2+1 🍟 0+1 🍫 0+1

- Divide by $N = 22$

🍏 0.32 🍌 0.27 🍆 0.18 🍒 0.13 🍟 0.05 🍫 0.05

Perplexity

- Relative frequencies on test corpus:

🍏 0.33 🍌 0.17 🍟 0.17 🍆 0.17 🍒 0.08 🍫 0.08

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- $\text{PP} = 2^{(0.33 \cdot 0.32 + 0.27 \cdot 0.17 + 0.18 \cdot 0.17 + 0.13 \cdot 0.17 + 2 \cdot (0.05 \cdot 0.08))} = 1.4$

Additive smoothing: Bigrams

Recall the additive smoothing formula for unigrams:

$$p_{smoothed}(w_i) = \frac{C(w_i) + \alpha}{N + \alpha|V|} \tag{1}$$

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- What is N? What is V?

Remember from Assignment 2 that:

$$p(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})} \tag{2}$$

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- Smoothe the bigram count: $C(w_{i-1}, w_i) \rightarrow C(w_{i-1}, w_i) + \alpha$
- Normalization: $p_{smoothed}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{?}$

Additive smoothing: Bigrams

Corpus

🍏 🍏 🍏 🍆 🍏 🍌 🍌 🍒 🍏 🍆 🍌 🍌 🍒 🍌 🍏 🍆

Bigrams: 🍏🍏, 🍏🍏, 🍏🍆, 🍆🍏, . . . , 🍏🍆, 🍆🍏 ← circular bigram!

Bigrams: AA, AA, AE, EA, . . . , AE, EA

Additive smoothing: Bigrams: bigram counts

- Collect bigram counts & condtional probabilities for history A

Bigram	$C(w_i, w_{i-1})$	$C(w_{i-1})$	$\frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$
AE	3	6	$1/2$
AA	2	6	$1/3$
AB	1	6	$1/6$

Additive smoothing: Bigrams: add alpha

- We encounter an unknown bigram *AF*

Bigram	$C_\alpha(w_{i-1}, w_i)$	$C(w_{i-1})$	$\frac{C_\alpha(w_{i-1},w_i)}{C(w_{i-1})}$
AE	3+1	6	4/6
AA	2+1	6	3/6
AB	1+1	6	2/6
$\rightarrow$ AF	0+1	6	1/6

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- Not a probabilitiy distribution!
- Solution: We need to adjust the divisor a tiny bit. But how tiny?

Additive smoothing: Bigrams: normalization

- add $\alpha \cdot 4$ to history count!
- Pretend that we have seen the history $|V| = 4$ times more.

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Bigram	$C_\alpha(w_{i-1}) + \alpha\|V\|$	$\frac{C_\alpha(w_{i-1},w_i)}{C(w_{i-1})+\alpha\|V\|}$
AE	$6 + 4$	$4/10$
AA	$6 + 4$	$3/10$
AB	$6 + 4$	$2/10$
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AA	$6+5$	$3/11$
AB	$6+5$	$2/11$
$\rightarrow$ AF	$6+5$	$1/11$
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- $C(A)$ is constant, unsmoothed count
- Probabilities sum up to 1: $4/11 + 3/11 + 2/11 + 1/11 + 1/11 = 1$

Additive smoothing: Bigrams: general case

- General formula for smoothed bigram Probabilities:

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- For n-grams of length n:

$$p(w_i|w_{i-1} : w_{i-n+1}) = \frac{C(w_{i-n+1} : w_i) + \alpha}{C(w_{i-n+1} : w_{i-1}) + \alpha|V_{(w_{i-n+1}:w_{i-1},\bullet)}|} \tag{5}$$

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- $V_{(A,\bullet)}$ is then $AA, AB, AC, AD, AE, AF \Rightarrow |V_{(A,\bullet)}| = 6 = |V|$

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- $V_{(A,\bullet)}$ is then $AA, AB, AC, AD, AE, AF \Rightarrow |V_{(A,\bullet)}| = 6 = |V|$
- We find that the formula we found is identical to the one on the lecture slides!

$$p(w_i|w_{i-1} : w_{i-n+1}) = \frac{C(w_{i-n+1} : w_i) + \alpha}{C(w_{i-n+1} : w_{i-1}) + \alpha|V|} \tag{7}$$

Cross-Validation

TODO

Estimating LOO Parameters

Count Trees

- remove infrequent nodes

TODO

Privacy

TODO differential privacy

Resources

1. UdS SNLP Class, WSD: https://teaching.lsv.uni-saarland.de/snlp/
2. Classical Statistical WSD: https://www.aclweb.org/anthology/P91-1034.pdf
3. n-gram count trees: http://ssli.ee.washington.edu/WS07/notes/ngrams.pdf