# Written Assignment 1
# Due October 9 at 5:00 PM

This assignment asks you to prepare written answers to questions on regular languages and finite automata. Each of the questions has a short answer. You may discuss this assignment with other students and work on the problems together, however, your write-up must be your own individual work. Remember that written assignments are to be turned in either at the start of lecture, electronically, or in the CS143 homework box outside Gates 411 by 5:00 PM on the due date.

1. Give a DFA for each of the following languages over the alphabet $\Sigma = \{a, b\}$.

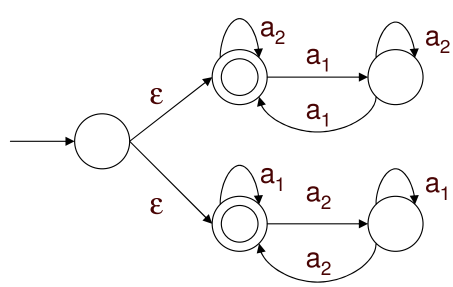
   - $L_1$: *All strings that contain at least two a's*
   - $L_2$: *All strings that contain at least one b*
   - $L_3$: *All strings that contain at least two a's and at least one b*
   - $L_4$: *All strings that contain at most one a or no b's*

   **Aside**: This example illustrates that regular languages are closed under intersection and complementation. Note that $L_3 = L_1 \cap L_2$ and $L_4 = \Sigma^* - L_3$, where $\Sigma^*$ represents the language containing all strings over the alphabet $\Sigma$.

2. Let $\Sigma_m = \{a_1, \ldots, a_m\}$ be an alphabet containing $m$ elements, for some integer $m \geq 1$. Let $L_m$ be the following language that includes all strings in which at least one of the characters occurs an even number of times, i.e.

   *All strings in which* $a_i$ *occurs an even number of times for some* $i$*, where* $1 \leq i \leq m$

   The following figure shows an NFA for the language $L_2$.




   Construct a DFA for the language $L_2$. Also construct an NFA for the language $L_3$.

   **Aside:** Non-deterministic finite automata (NFAs) are no more powerful than DFAs in terms of the languages that they can describe. However, NFAs can be exponentially more succinct than DFAs, as this problem demonstrates. For the language $L_m$, there exists an NFA of size at most $2m+1$ while any DFA must have size at least $2^m$. Note that the DFA for the language $L_3$ is not as easy to construct as the NFA for the language $L_3$.

3. Write regular expressions for the following languages over the alphabet $\Sigma = \{0, 1\}$:

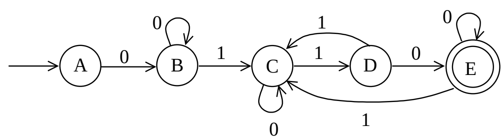
   (a) All strings that contain at least one 0 and at least one 1 and that also end with at least two 1s.

   (b) All strings that do not begin with 01.

   (c) All strings that contain an odd number of 1s.

   (d) All strings that contain exactly two 1s and at least one 0.

4. Give a one-sentence description and a regular expression for the language over the alphabet $\Sigma = \{0, 1\}$ described by the following deterministic finite automaton (DFA):




5. For each of the following lexical specifications, give a regular expression describing the language of possible outputs. Assume that all inputs are strings of 0s and 1s only.

   (a) Specification 1:

```
0       { print ("c"); }
00      { print ("a"); }
1       { print ("b"); }
```

   (b) Specification 2:

```
(0+)(1+) { print("a"); }
0         { print("b"); }
1         { print("c"); }
```