Inverted Pendulum Control

using Linear Quadratic Regulator

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Outline

1. Setup of Inverted Pendulum kit
2. Linear model and LQR
3. Calculation of feedback matrix

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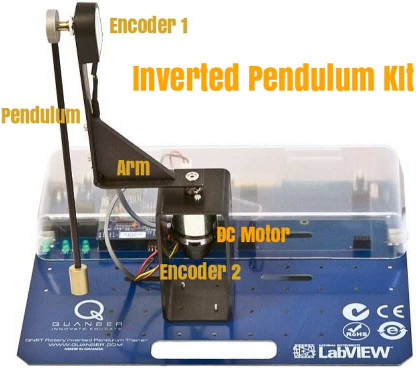
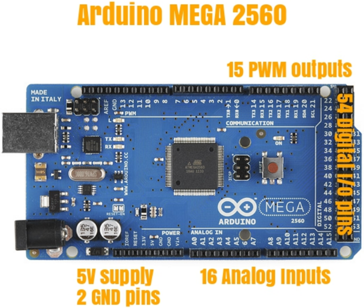
Setup of inverted pendulum

Inverted Pendulum Kit and Arduino





Circuit Diagram

Connections between encoder, decoder, Arduino and motor driver

Fetching Decoder Values

- Each decoder sends 8 bits of data at a time
- Each decoder has 2 select pins for sending
- We just need the last two bytes of data from the decoder
- Then we merge the two bytes to get a single 16 bit data
- Then we convert the data into an integer value
- Then we scale it by $\frac{360}{2000} = 0.18$ to give value in degrees.

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Plant Equation

- The arm rotates about the vertical axis and its angle is denoted by θ
- The pendulum rotates about its pivot angle is called α
- In the linearzed model the states are $x_1 = \theta$, $x_2 = \alpha$, $x_3 = \dot{\theta}$, $x_4 = \dot{\alpha}$.

$$x = [x_1, x_2, x_3, x_4]^T$$

- The linear state-space representation of the Inverted Pendulum is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

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Linear Quadratic Regulator

- Given plant model

$$\dot{x}(t) = Ax(t) + Bu(t)$$

  linear quadratic regulator problem is to find a control input u that minimizes the cost function J

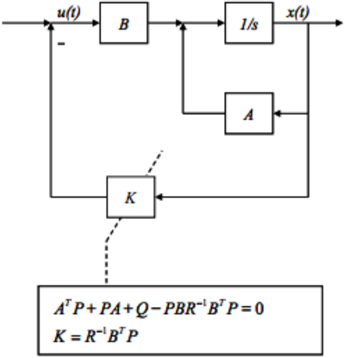
$$J = \int_0^\infty x^T(t)Qx(t) + u^T(t)Ru(t)dt$$

- Q is positive semidefinite matrix and R is an positive definite symmetric matrix

Outline

Calculating K matrix

Mathematical Approach



Linear Quadratic Regulator

Calculating K matrix

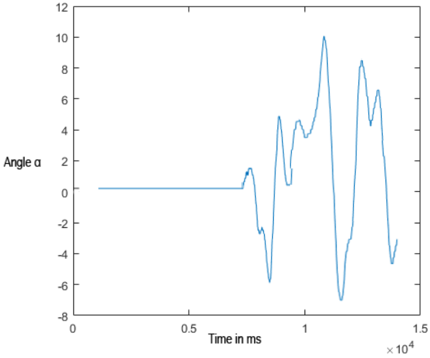
Using Matlab

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 123.979 & -1.577 & 0 \\ 0 & 111.623 & -0.725 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 56.389 \\ 25.930 \end{bmatrix}$$

Computing K matrix in Matlab:

```
Q = [90,0,0,0; 0,5,0,0; 0,0,2,0; 0,0,0,1;]
R = 1
K = lqr(A,B,Q,R)
% output:
K = [ -9.487   98.198   -4.151   13.021]
```

Results



Challenges faced

- Tuning the weights in a Q matrix is a tedious job, it takes a lot of trial and error to get a desirable output
- Same values of matrix Q and R fail to stabilize two different system though the physical specifications of different kit are almost same.

Thank You