How do humans catch balls?

Boris Belousov[1]

Jan Peters[2] Gerhard Neumann[2] Walter Kellermann[1]

[1]Friedrich-Alexander-Universität Erlangen-Nürnberg

[2]Technische Universität Darmstadt

Intermediate MSc presentation, 03.11.2015

Outline

About me

2009-2013 BSc in radio engineering and cybernetics, Moscow

- Internship at Intel, multimedia digital signal processor
- Part-time job at Netcracker, solutions for telecom providers

2013-present MSc in communications and multimedia engineering

- Signal processing + machine learning
- Stochastic optimal control, reinforcement learning ...

Computation vs heuristics

The computational view

"...he behaves as if he had solved a set of differential equations in predicting the trajectory of the ball ... At some subconscious level, something functionally equivalent to the mathematical calculations is going on" Dawkins, 1989

The heuristic view

"Fix your gaze on the ball, start running, and adjust your running speed so that the angle of gaze remains constant. A player who relies on the gaze heuristic can ignore all causal variables necessary to compute the trajectory of the ball – the initial distance, velocity, angle, air resistance, speed and direction of wind, and spin, among others." Gigerenzer, 2009

Optic acceleration cancellation (Chapman, 1968)

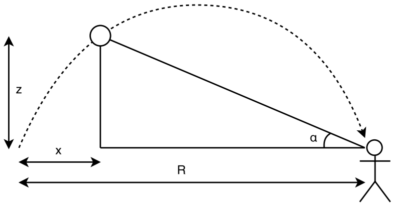


Figure 1: The tangent of the elevation angle increases uniformly with time for a catcher standing still at the ball landing point.

From the equations of motion:

$$z = (V \sin \beta)t - gt^2/2$$
$$x = (V \cos \beta)t$$

one can obtain the range:

$$R = (2V^2 \sin \beta \cos \beta)/g$$

After modest algebraic manipulations we get:

$$\boxed{\tan \alpha = gt/(2V \cos \beta) = (const)t}$$

Constant bearing angle

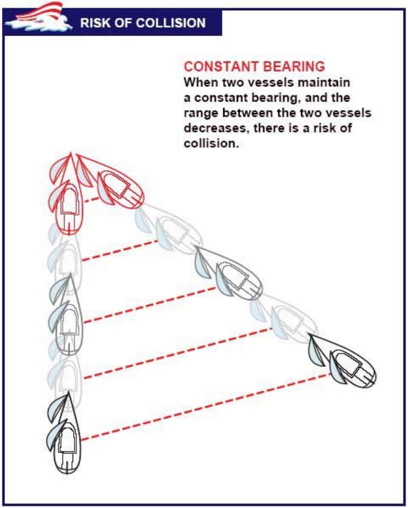


Figure 2: Courtesy of www.boatcourse.com

Outline

Model of the ball

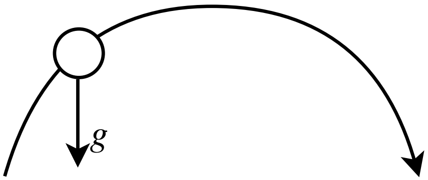


Figure 3: A simple kinematic model of the ball.

$$\mathbf{x}_b = \begin{bmatrix} x_b & y_b & z_b & \dot{x}_b & \dot{y}_b & \dot{z}_b \end{bmatrix}$$

$$\ddot{x}_b = 0$$
$$\ddot{y}_b = 0$$
$$\ddot{z}_b = -g$$

Model of the catcher

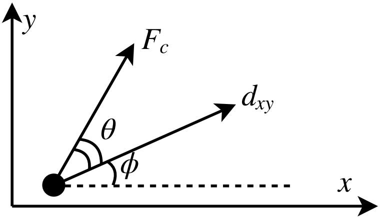


Figure 4: Catcher is a 2D point mass with a 3D gaze vector attached to it.

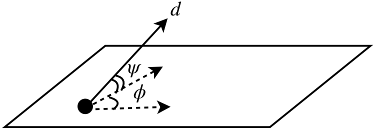


Figure 5: The gaze direction d is defined by the angles ϕ and ψ.

$$\mathbf{x}_c = \begin{bmatrix} x_c & y_c & \dot{x}_c & \dot{y}_c & \phi & \psi \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} F & \theta & \omega_\phi & \omega_\psi \end{bmatrix}$$

$$\ddot{x}_c = F\cos(\phi+\theta) - \mu\dot{x}_c$$

$$\ddot{y}_c = F\sin(\phi+\theta) - \mu\dot{y}_c$$

$$\dot{\phi} = \omega_\phi$$

$$\dot{\psi} = \omega_\psi$$

Forward running is faster than backward running

Model the speed/gaze direction dependence as a constraint on F:

$$F(t) \leq F_1 + F_2 \cos\theta(t)$$

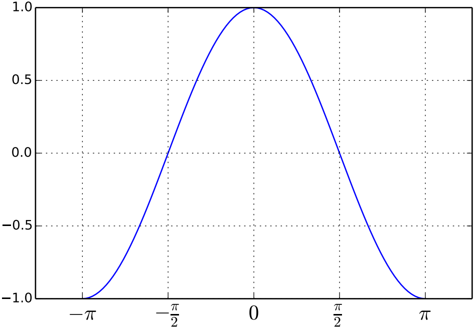


Figure 6: Cosine interpolation of the force.

Outline

Continuous time optimal control

$$\begin{aligned}
\underset{x(\cdot),\,u(\cdot)}{\text{minimize}} \quad & \int_0^T L(x(t),u(t))dt + E(x(T)) \\
\text{subject to} \quad & \\
x(0) - x_0 &= 0, && \text{(fixed initial value)} \\
\dot{x}(t) - f(x(t),u(t)) &= 0, \quad t \in [0,T], && \text{(continuous model)} \\
h(x(t),u(t)) &\leq 0, \quad t \in [0,T], && \text{(path constraints)}
\end{aligned}$$

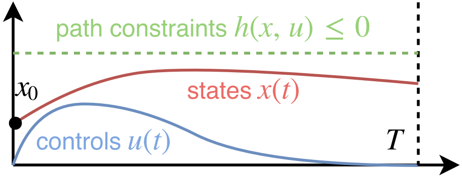


Figure 7: An example trajectory with controls and constraints.

How to solve it?

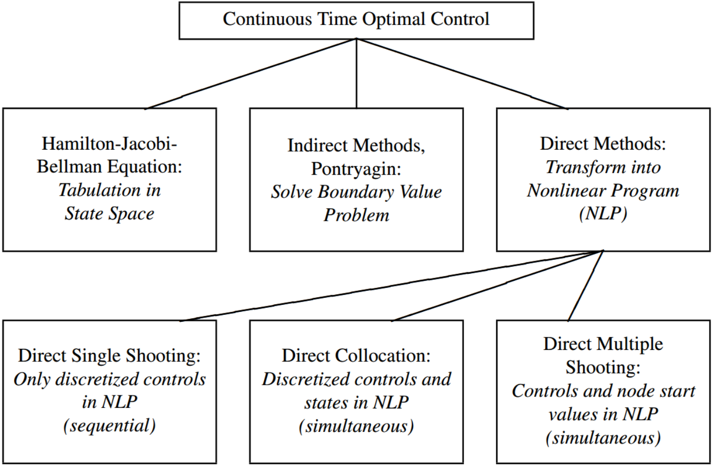
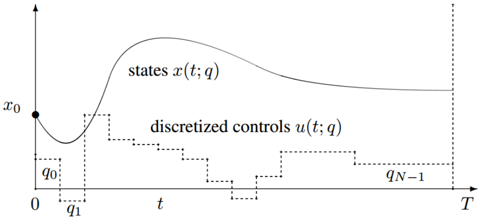
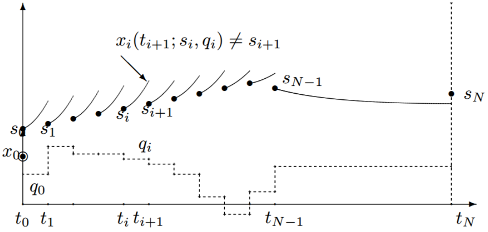


Figure 8: The optimal control family tree.

Single vs multiple shooting





CasADi - symbolic framework for automatic differentiation

The main purpose of CasADi is to be a low-level tool for quick, yet highly efficient implementation of algorithms for nonlinear numerical optimization.

Features

- Solvers
- Integrators
- Symbolic computations
- Automatic differentiation

Symbolic differentiation $\neq$ automatic differentiation

Couple the model of the catcher and the ball

How to incorporate the desire to look at the ball?

Outline

Observation model

Discrete-time dynamics

$$
\begin{aligned}
x_t &= f(x_{t-1}, u_{t-1}) + \epsilon_t, \qquad && \epsilon_t \sim \mathcal{N}(0, Q) \\
z_t &= h(x_t) + \delta_t, && \delta_t \sim \mathcal{N}(0, R(x_t))
\end{aligned}
$$

Observation model

Discrete-time dynamics

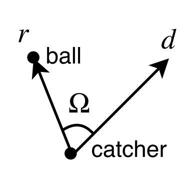
$$
\begin{aligned}
x_t &= f(x_{t-1}, u_{t-1}) + \epsilon_t, & \epsilon_t \sim \mathcal{N}(0, Q) \\
z_t &= h(x_t) + \delta_t, & \delta_t \sim \mathcal{N}(0, R(x_t))
\end{aligned}
$$

Observation model

$$h(x_t) = \begin{bmatrix} x_b & y_b & z_b & x_c & y_c & \phi & \psi \end{bmatrix}$$

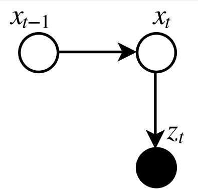
$$\sigma_b^2 = \sigma_{\max}^2(1 - \cos\Omega) + \sigma_{\min}^2$$

$$R(x_t) = diag\{\sigma_b^2, \sigma_b^2, \sigma_b^2, 0, 0, 0, 0\}$$



Belief space planning, framework

Extended Kalman filter : $(\mu_{t-1}, \Sigma_{t-1}, u_{t-1}, z_t) \to (\mu_t, \Sigma_t)$



$$\begin{aligned} \bar{\mu}_t &= f(\mu_{t-1}, u_{t-1}) \\ \bar{\Sigma}_t &= A_{t-1}\Sigma_{t-1}A_{t-1}^T + Q_t \\ K_t &= \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1} \end{aligned}$$

$$\boxed{\begin{aligned} &\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t)) \\ &\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{aligned}}$$

Definition

A `belief` is a distribution of the state x_t given all past observations and controls $b(x_t) = p(x_t \mid z_{1:t}, u_{0:t})$.

Gaussian belief:

$$b_t = (\mu_t, \Sigma_t)$$

Belief space planning, application

Assume maximum likelihood observations, i.e. $z_t = h(\bar{\mu}_t)$. The dynamics of the belief $b_t = (\mu_t, \Sigma_t)$ is then given by:

$$\begin{aligned} \mu_t &= f(\mu_{t-1}, u_{t-1}) \\ \Sigma_t &= (I - K_t C_t)\bar{\Sigma}_t \end{aligned}$$

The cost function is:

$$\mathcal{C} = c_T(\mu_T, \Sigma_T) + \sum_{t=0}^{T-1} c_t(\mu_t, \Sigma_t, u_t)$$

where

$$\begin{aligned} c_T(\mu_T, \Sigma_T) &= \mu_T^T P_T \mu_T + \mathrm{tr}\,(M_T \Sigma_T) \\ c_t(\mu_t, \Sigma_t, u_t) &= u_t^T N_t u_t + \mathrm{tr}\,(M_t \Sigma_t) \end{aligned}$$

Covariance-free trajectory optimization

Single shooting	Multiple shooting
$\begin{aligned} \min_{u_{0:T-1}} \quad & \mathcal{C}(\mu_0, \Sigma_0, u_{0:T-1}) \\ \text{s.t.} \quad & \tilde{f}(\mu_0, u_{0:t-1}) \in \mathcal{X}_{\text{feasible}} \\ & u_t \in \mathcal{U}_{\text{feasible}} \end{aligned}$	$\begin{aligned} \min_{\substack{u_{0:T-1} \\ \mu_{0:T}}} \quad & \mathcal{C}(\mu_{0:T}, \Sigma_0, u_{0:T-1}) \\ \text{s.t.} \quad & \mu_{t+1} = f(\mu_t, u_t) \\ & \mu_t \in \mathcal{X}_{\text{feasible}} \\ & u_t \in \mathcal{U}_{\text{feasible}} \end{aligned}$

Outline

Demo

Show the demo

Successful catch

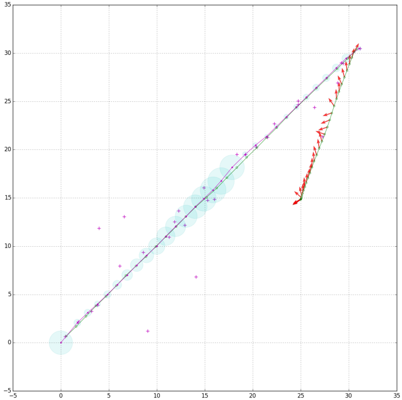


Figure 9: An example of a successful catch.

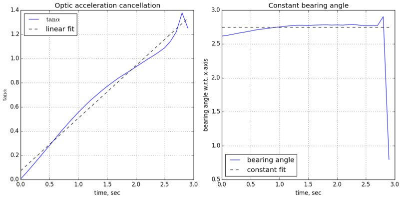


Figure 10: OAC and CBA hold.

Unsuccessful catch

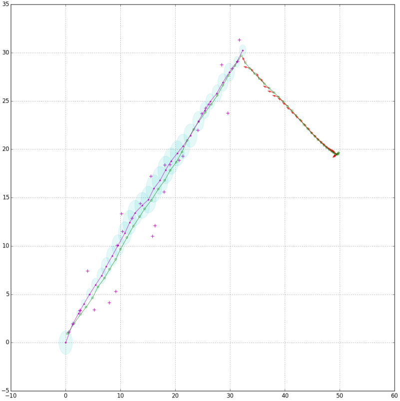


Figure 11: An example of a miss.

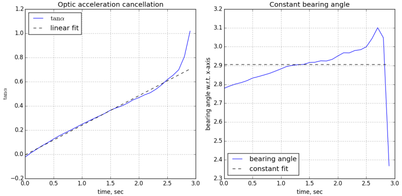


Figure 12: OAC still holds, while CBA does not.

Conclusion and future work

Main results:

- an optimal control based model was developed
- that agrees with heuristics
- and generates reasonable behaviors even when heuristics are not applicable (the ball is outside of the field of view)

To be done:

- run more tests
- check more heuristics