Simplified Simulator for Correlation-based MIMO Radio Channel Model

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***Abstract*— The principal goal of this paper is to provide a simplified and efficient MIMO radio channel simulator based on the most popular correlation-based MIMO radio channel model and the SoS approach which is traditionally used for generation Rayleigh fading waveforms with temporal correlation. First an improved simulation model for the generation of multiple Rayleigh fading waveforms is proposed, which is both simple and efficient with rapid convergence speed. Next a simplified spatial correlation model of outdoor BS is introduced in closed-form expression. This approach has negligible difference compared to the reference correlation values. In the end, capacity of the simulated MIMO radio channel is evaluated with respect to different AOA. The presented simulator is suitable for the theoretical analyses of MIMO radio systems and dynamic MIMO channel simulation.**

***Keywords*-sum-of sinusoids (SoS), Rayleigh fading, Spatial Correlation, power azimuth spectrum.**

I. Introduction

Simulation of mobile radio channels is commonly used in the laboratory for it allows less expensive and more reproducible system tests and evaluations in contrast to field trials. The primary goal of any channel simulation is to reproduce the statistical properties of the real world channel as faithfully as possible with computational simplicity. This paper focuses on space correlated MIMO Rayleigh fading channels simulation based on sum-of-sinusoids (SoS) method and spatial correlation model for MIMO radio systems.

The simulation for MIMO channel requires generation of multiple independent Rayleigh fading waveforms [1]. A number of different methods have been proposed for the simulation of Rayleigh fading channels. These simulators are based on the sum-of-sinusoids [2]-[5], or white noise filtering which filter white noise in frequency domain with inverse discrete Fourier transforms (IDFT) [6] or in time domain with autoregressive (AR) filter [7]or FIR filter [1]. The white noise filtering methods are suitable for the generation of independent Rayleigh fading waveforms, however, a set of independent white Gaussian processes must be generated first, and extra effort is needed for the design and implementation of the digital shaping filters with small bandwidth and special impulse response. As a consequence, SoS method is more popular because of their simplicity, even though they have limitations in their statistical stationary and ergodicity [3]. This paper uses an improved SoS based method to generate multiple independent Rayleigh fading waveforms, i.e., a set of independent complex-valued Gaussian random process conforming to the following criteria: 1) the real and imaginary components of each complex process are zero-mean independent Gaussian processes with identical power spectral density or temporal autocorrelation. Consequently, the envelope is Rayleigh distributed and the phase is uniformly distributed; and 2) the cross correlation between any pair of complex processes should be zero [5].

Early theoretical and simulation studies on MIMO radio systems assumed either fully correlated or fully independent channels [8], while a partially correlated radio channel should be expected in practice for spatial correlation which generally has an adverse effect on capacity or system performance. Simulating realistic correlated channels is thus essential to predict the performance of real MIMO systems.

There are two main approaches to model the spatial correlation, which can be distinguished between physical and nonphysical method. The first one generates the MIMO channel matrix based on a geometrical description of the propagation environment, which are commonly used to predict the performance of MIMO communication systems in realistic propagation environments. On the other hand, the spatial correlation across MIMO channels for nonphysical method is reproduced by the statistical characteristics of the MIMO channels obtained from the measured data. In general, the nonphysical method are easy to simulate and suitable for the theoretical analyses of the correlated MIMO channels. Among the models based on nonphysical approach, the most popular correlation-based model due to the so-called Kronecker model which has been in widespread use for the theoretical analysis of MIMO systems [9].

In this paper, a simplified simulator for frequency-nonselective correlation-based MIMO radio channels is proposed. The simulation process comprises the generation of independent Rayleigh fading waveforms based on SoS and space correlation transformation based on Kronecker model. We remedy the major drawback of SoS based method, which some researchers thought it is difficulty to create multiple uncorrelated fading processes with low complexity [1]. Furthermore, we use a simplified approach to derive the channel spatial correlation in closed form instead of sum of

Bessel functions which do not show a direct dependence of the spatial correlation on the channel and array parameters [11].

This paper is organized as follows. In Section II, the basic framework of MIMO radio channel simulation is introduced. Then, in Section III, the generation of multiple independent Rayleigh fading waveforms is presented. In Section IV, a new closed-form expression of the spatial correlation matrices of MIMO radio channels is proposed. Section V presents the simulation process and capacity results with respect to different AOA. Finally, a summary and conclusions are given in Section VI.

II. DESCRIPTION OF CLASSICAL CORRELATED-BASED MIMO CHANNEL MODEL

We consider a MIMO system with a transmit array of M_T antennas and a receive array of M_R antennas. Correlated channels imply that the elements of the channel matrix are correlated and may be model as

$$vec(H) = R_{MIMO}^{1/2} vec(H_\omega) \tag{1}$$

where $vec(\bullet)$ stacks matrix into a vector columnwise, H_ω is a Rayleigh independent identically distributed spatially white MIMO channel matrix of size $M_R \times M_T$, and R_{MIMO} is a $M_R M_T \times M_R M_T$ covariance matrix defined as

$$R_{MIMO} = \varepsilon\{vec(H)vec(H)^H\} \tag{2}$$

where $\varepsilon\{\bullet\}$ is expectation operator, $(\bullet)^H$ denotes conjugate transpose, and R is a positive semidefinte Hermitian matrix. If R_{MIMO} is full rank (i.e., $R = I_{M_T M_R}$ is identity matrix), then in such a case $H = H_\omega$, which is widely used for early analyses of MIMO systems [8]. Note that the elements of H_ω should satisfy the requirements of independent complex Gaussian processes.

For MIMO channels, modeling efforts have concentrated on the description of the relationship between angle of arrival (AOA) and angle of departure (AOD). Although the aforementioned model is capable of capturing any correlation effects between the elements of H, a simpler and more popular model called Kronecker model [9] assumes that angular power spectrum at the receiver is independent of the transmit direction, and vice versa. This considerably simplifies the mathematical description, and the impulse response matrix (of a frequency nonselective channel) is given by

$$H = R_r^{1/2} H_\omega R_t^{1/2} \tag{3}$$

where R_r and R_t are the spatial correlation matrices at the receiver and transmitter, respectively. This model has been in widespread use for the theoretical analysis of MIMO systems.

From Equations (1)-(3), we can show that R_{MIMO}, R_r and R_t are related by

$$R_{MIMO} = R_t^T \otimes R_r \tag{4}$$

where $\otimes$ is Kronecker product and $(\bullet)^T$ denotes transpose. In this model, the coefficients of R_r, and R_t are characterized by certain AS and AOA/DOD at each link end which will be described in detail in Section IV.

III. GENERATION OF MULTIPLE INDEPENDENT RAYLEIGH FADING WAVEFORMS

For the simulation of correlated-based MIMO channel model, the first step is generation of multiple independent Rayleigh fading waveforms, i.e., the matrix of H_ω in (1). For SoS approaches, the realizable simulation model can be obtained from Clarke's model [2] by using only a finite number of sinusoids N. The underlying process is given by

$$\tilde{c}(t) = \tilde{c}_1(t) + \mathrm{j}\tilde{c}_2(t) \tag{5a}$$

$$\tilde{c}_i(t) = \sum_{n=1}^{N} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \quad i = 1,2 \tag{5b}$$

where $c_{i,n}$, $f_{i,n}$, $\theta_{i,n}$ are also called Doppler coefficients, discrete Doppler frequencies, and Doppler phases, respectively, and the tilde "~" is to emphasize that $\tilde{c}(t)$ is a reduced realization of the reference model $c(t)$.

To remove the shortcoming of Pop's model [3], Zheng and Xiao proposed an improved simulation model [4]. Based on Zheng's model, we introduce an efficient multiple independent Rayleigh waveforms generator which is given as follows.

Assume that P independent complex envelopes are desired, $k = 0,1,...,P-1$, the k^{th} complex faded envelope is defined as $\tilde{c}_k(t) = \tilde{c}_{k1}(t) + \mathrm{j}\tilde{c}_{k2}(t)$, where

$$\tilde{c}_{k,i}(t) = \sum_{n=1}^{N} c_{k,i,n} \cos(2\pi f_{k,i,n} t + \theta_{k,i,n}) \quad i = 1,2 \tag{6a}$$

$$c_{k,i,n} = \sigma_0 \sqrt{\frac{2}{N}} \tag{6b}$$

$$f_{k,1,n} = f_d \cos\left(\frac{2\pi n - \pi + \phi}{4N} + \frac{2\pi k}{4PN}\right) \tag{6c}$$

$$f_{k,2,n} = f_d \sin\left(\frac{2\pi n - \pi + \phi}{4N} + \frac{2\pi k}{4PN}\right) \tag{6d}$$

As a result, an efficient simulator for generation of multiple independent Rayleigh fading waveforms is achieved, which we also refer to as modified Zheng's model.

Since the second-order statistics of the channel characterize the basic structure of stochastic wireless fading channels, we use correlation function to evaluate the properties of the reference model and simulation model. The purpose of Rayleigh fading simulation is to design simulation models with statistical properties converging to the desired properties of Clarke's reference model. The details of reference model may refer to [2] and is omitted in this paper.

The statistical properties of the model are given by

$$\tilde{R}_{c_{k,1}c_{k,1}}(\Delta t) = \tilde{R}_{c_{k,2}c_{k,2}}(\Delta t) = \sigma_0^2 J_0(2\pi f_d \Delta t) \tag{7a}$$

$$\tilde{R}_{c_{k,1}c_{k,2}}(\Delta t) = \tilde{R}_{c_{k,2}c_{k,1}}(\Delta t) = 0 \tag{7b}$$

$$\tilde{R}_{c_k c_k}(\Delta t) = 2\sigma_0^2 J_0(2\pi f_d \Delta t) \tag{7c}$$

$$\tilde{R}_{c_k c_l}(\Delta t) = 0 \tag{7b}$$

For N=8, P=8, $f_d = 50Hz$, and $\sigma_0^2 = 0.5$, the performance evaluation of the multiple independent Rayleigh waveforms simulator defined by (6) is carried out, and two waveform results from the simulation of modified Zheng's model are shown in Figure 1-2. As can be seen from Figure 1, the numerical results show that the autocorrelations of the in-phase component for each waveform, the cross correlations between in-phase component (the real) of the first waveform and the real or imaginary part of the second waveform match the desired ones very well. Figure 2 indicates that the cross correlation between two derived complex-valued random processes is zero, and the autocorrelations of the complex envelope for those two waveforms exactly match the reference model. As a consequence, the complex-valued random processes satisfy the conditions and the Rayleigh fading spectral requirements as independent Rayleigh simulator. Moreover, the statistical properties of the modified model converge to the desired properties even when average over 20 simulation trials.

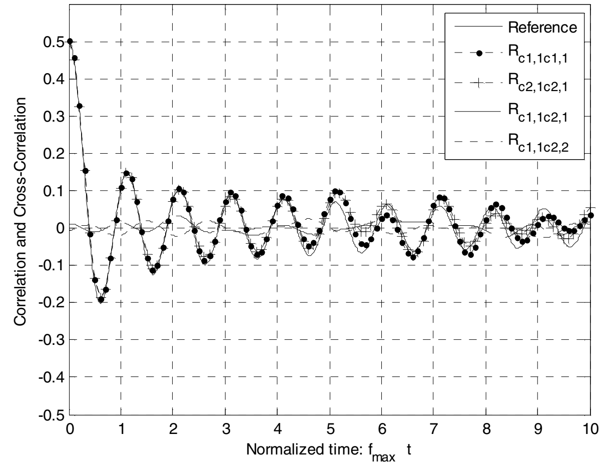

Figure 1. Autocorrelations of in-phase component of the first waveform and the second waveform, cross-correlations between in-phase component of the first waveform and in-phase or quatrature component of the second waveform.

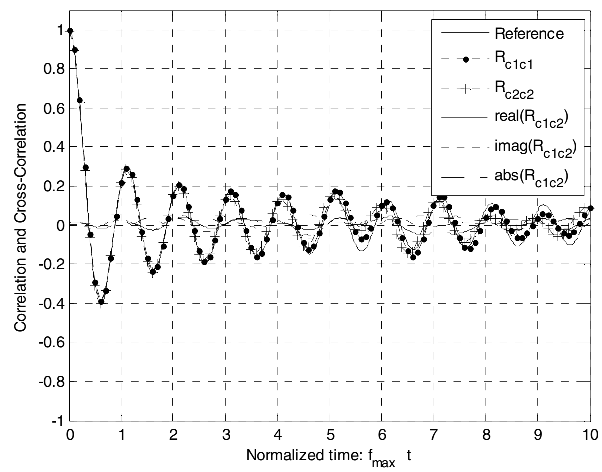

Figure 2. Complex envelope autocorrelations of the the first waveform and the second waveform, real part, imag part and absolute value of complex envelope cross-correlation between the first waveform and the second waveform

IV. NEW SPACE CORRELATION TRANSFORMATION SCHEME

The transformation to obtain the desired spatial correlation between the Gaussian components in H_ω of the channel coefficients is a linear memoryless operation which is performed by equation (1). This Section focuses on the derivation of the spatial correlation of R_{MIMO}, which equals the Kronecker product of the transmit and receive correlation matrices. The Kronecker method uses a reduced set of channel parameters [i.e., angle spread (AS), mean angle of arrival (AOA)/departure (AOD), and the associated power azimuth spectrum], which is both simple and suitable for the theoretical analysis of the correlated MIMO channels, for the parameters are easily obtained from measured data.

Since the power angular spectra on the both links of the MIMO systems are independent for Kronecker approach and the same method is used to calculate each correlation matrix, we use R to characterize the transmit or the receive correlation matrix and M to denote the number of antennas at transmitter or receiver. Let d/λ stands for the normalized distance between elements, where d is the element spacing and λ the wavelength, and $D = 2\pi d/\lambda$. The (m, n) entry of the matrix R for uniform linear array is defined as [11].

$$[R(\phi_0, \sigma_\phi)]_{m,n} = \int_{-\pi}^{\pi} e^{jD(m-n)\sin(\phi_0-\phi)} P_\phi(\phi) d\phi \tag{8}$$

where $P_\phi(\phi)$ is normalized power azimuth spectrum, ϕ is the AOA offset with respect to the mean AOA of ϕ_0 .

Measurements indicate that the power azimuth spectrum is approximately truncated Laplacian distribution at the base station in urban and rural environments with a small AS (i.e., less than 10^o) [12]. Consequently, the sinusoidal function in equation (8) can be approximately represented as

$$\sin(\phi_0 - \phi) \approx \sin\phi_0 - \phi\cos\phi_0 \qquad (9)$$

Substituting (9) into (8), we get

$$[R(\phi_0, \sigma_\phi)]_{m,n} \approx e^{jD(m-n)\sin(\phi_0)} \int_{-\pi}^{\pi} e^{-jD(m-n)\cos(\phi_0)\phi} P_\phi(\phi) d\phi \qquad (10)$$

Equation (9) comprises the product of a complex exponential term times an integral term. The complex exponential term can be given by

$$e^{jD(m-n)\sin(\phi_0)} = e^{jDm\sin(\phi_0)} e^{-jDn\sin(\phi_0)} \qquad (11)$$

where the multiplicative factors at the right-hand side of (11) are the entries of the steering vector of ULA given by

$$a(\phi_0) = [1, e^{-jD\sin(\phi_0)}, ..., e^{-jD(M-1)\sin(\phi_0)}]^T \qquad (12)$$

The integral term is the characteristic function of normalized power azimuth spectrum, which is the truncated Fourier transform evaluated at $\omega = D(m-n)\cos\phi_0$ and represented as

$$[B(\phi_0, \sigma_\phi)]_{m,n} = \int_{-\pi}^{\pi} e^{-jD(m-n)\cos(\phi_0)\phi} P_\phi(\phi) d\phi \qquad (13)$$

Using the definition in (12) and (13), we derive the spatial correlation matrix with complex entries (10) given by

$$R(\phi_0, \sigma_\phi) \approx [a(\phi_0) a(\phi_0)^H] \odot B(\phi_0, \sigma_\phi) \qquad (14)$$

where $\odot$ denotes the Hadamard product and $a(\phi_0)$ is the array response for the mean azimuth AOA (ϕ_0) [10].

For Truncated Laplacian distribution we have

$$P_\phi(\phi) = \begin{cases} \dfrac{\beta}{\sqrt{2}\sigma_\phi} e^{-\left|\sqrt{2}\phi/\sigma_\phi\right|}, & \phi \in [-\pi, \pi) \\ 0, & \text{otherwise} \end{cases} \qquad (15)$$

where $\beta = 1/(1 - e^{-\sqrt{2}\pi/\sigma_\phi})$ is the normalized factor. Substituting (15) into (10), we get

$$[R(\phi_0, \sigma_\phi)]_{m,n} \approx e^{jD(m-n)\sin(\phi_0)} \frac{\beta - E_r}{1 + \dfrac{\sigma_\phi^2}{2}\omega^2} \qquad (16)$$

Where

$$E_r = \beta e^{\sqrt{2}/\sigma_\phi} (\cos\omega\pi - \frac{\sqrt{2}}{2}\omega\sigma_\phi \sin\omega\pi). \qquad (17)$$

TABLE I. REFERNCE AND COMPUTED CORRELATION COEFFICIENT VALUES FOR TEST CASE OF [12]

spacing	AS(degree)	AOA(degree)	Complex		
			Reference	Computed	Error
.5 λ	5	20	0.4743+0.8448i	0.4609+ 0.8511i	1.53
	2	50	-0.7367+0.6725i	-0.7400+0.6689i	0.488
4 λ	5	20	-0.2144+0.2408i	-0.2163 + 0.2360i	1.61
	2	50	0.8025+0.3158i	0.7936 + 0.3386i	2.83
10 λ	5	20	-0.0617+i0.034	-0.0614 + 0.0337i	0.597
	2	50	-0.2762-i0.419	-0.2676 - 0.4242i	2.01

When $P_\phi(\phi)$ is truncated Laplacian distribution, the integration of $P_\phi(\phi)$ truncated over $[-\pi, \pi)$ in (10) is approximately equivalent to the integration over $(-\infty, \infty)$, and (13) can be rewritten as

$$[B(\phi_0, \sigma_\phi)]_{m,n} = \int_{-\infty}^{\infty} e^{-jD(m-n)\cos(\phi_0)\phi} \frac{\beta}{\sqrt{2}\sigma_\phi} e^{-\left|\sqrt{2}\phi/\sigma_\phi\right|} d\phi \qquad (18)$$

Consequently, the correlation between element n and m of (10) is given by

$$[R(\phi_0, \sigma_\phi)]_{m,n} \approx e^{jD(m-n)\sin(\phi_0)} \frac{\beta}{1 + \dfrac{\sigma_\phi^2}{2}\omega^2} \qquad (19)$$

Using this method, a closed-form solution for the spatial-correlation function is obtained and the troublesome integrations of sum of Bessel functions in [11] are avoided. The correctness of computed values is confirmed when looking at Table 1, which compare computed correlation values to the ones presented in [12] for validation purposes. They agree within a small margin of 3%. This method is therefore suitable for dynamic simulation where we just change the direction of ϕ_0 with considerably low computational complexity. A similar result was given in [10], and we verify the validity with respect to the reference values. Moreover, we use different AOA to evaluate the capacity of MIMO radio channel.

V. GENERATION OF MIMO CHANNEL COEFFICIENTS AND PERFORMANCE EVALUATION

A. *Generation of Simulated Correlated Channel Coefficients*

Consider outdoor environment in the uplink with linear array

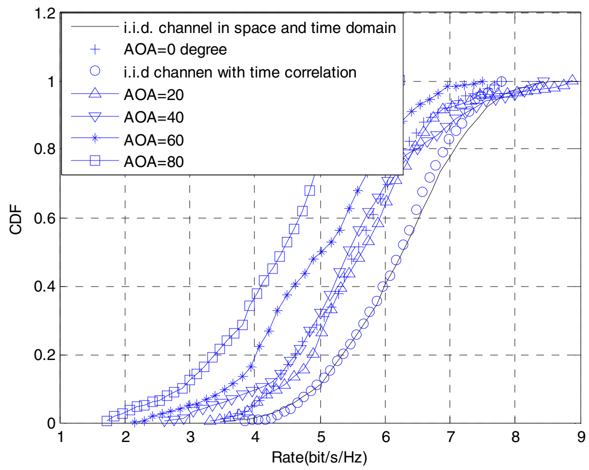

Figure 3. CDF of information for simulated MIMO channel with different AOA

configuration of $M \times N$ between receiver and transmitter, Correlated channel coefficients h_{mn} are generated based on equation (1) such that

$$vec(H) = Cvec(H_{\omega}) \tag{20}$$

where

$$vec(H) = [h_{11}, h_{21}, ..., h_{M1}, h_{M2}, ..., h_{MN}]^T,$$

$$vec(H_{\omega}) = [a_1, a_2, ..., a_{MN}]^T,$$

and the symmetrical mapping matrix C results from the standard Cholesky factorization of the matrix $R_{MIMO} = CC^T$ provided that R_{MIMO} is nonsingular. Subsequently, the generation of the simulated MIMO channel matrix can be deduced from vector $vec(H)$.

B. *Performance of simulator*

Consider outdoor environment in the uplink with linear array configuration of 4×2 at receiver and transmitter respectively, which is a common configuration for future MIMO communications. We use the Case II in SCM link level parameters [12], where BS has 4λ-spacing antenna array with 2 degree RMS angle spread of truncated Laplacian distribution over $[-\pi, \pi)$, and MS has 0.5λ-spacing antenna array with 360 degree uniform PAS. Consider the different AOA with 0, 20, 40, 60, and 80 degree at BS, the cumulative distribution function (CDF) of the information rate of a flat fading MIMO channel is shown Figure 3. The SNR is 10dB and the channel is unknown to the transmitter.

Figure 3 shows the capacity decreases with increasing AOA, i.e., the spatial correlation at BS. We note that the temporal correlation has little impact on the channel capacity. Consequently, the location of antenna array is one of the key issues in MIMO wireless work planning and optimum.

VI. Conclusion

In this paper, an efficient frequency nonselective MIMO channel simulator is proposed, which has both time and space selectivity. The temporal correlation is modeled based on the well known SoS approach to generate multiple independent Rayleigh fading waveforms with considerable simplicity, and the spatial correlation is calculated based on the popular Kronecker model for it is suitable for theoretical analyses. The Kronecker model calculates the spatial correlation by the Kronecker product of transmit correlation matrix and receive matrix, assuming the independence of power azimuth spectrum between transmitter and receiver. We use the simplified method to derive the correlation matrix in closed-form expression avoiding the troublesome integrations in terms of sum of Bessel functions. The presented approach in this paper is suitable for theoretical analyses and also useful for the dynamic MIMO channel simulation.

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