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A New Algorithm for MIMO Channel Tracking Based on Kalman Filter

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Abstract—**The estimation of a time-varying fading channel in multiple-input multiple-output (MIMO) wireless systems is a difficult task for the receiver. Its performance can be improved if an appropriate channel estimation filter is used according to the prior knowledge of the fading channel. We propose an algorithm which uses the Kalman filter based on Clarke's model to track the MIMO time-varying fading channel by using superimposed periodic training sequences. To reduce the complexity of the high-dimensional Kalman filter for channel estimation of the paths, we use a low-dimensional Kalman filter for the estimation of each path. An analytical formula for the estimation error due to the temporal variation of the channel coefficients is given and verified by link level simulations based on synthetic and measured impulse responses. Simulation results show this algorithm is effective for the estimation of the time-varying fading channel in MIMO system when the performance of the channel estimation is presented in terms of the mean-square error(MSE).**

Ⅰ. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems with some transmit and receive antennas have attracted much attention as a promising technique for achieving high bit-rate and high capacity transmission in wireless environment [1][2]. Accurate channel state information (CSI) is a prerequisite for the full exploitation of the advantages of MIMO systems. However, the perfect CSI is never known to the receiver in real wireless scenarios. So channel estimation becomes a key topic for the applications of the MIMO system. A popular estimation technique incorporated in an iterative receiver is iterative channel estimation [3][4]. In an iterative channel estimation method, both pilot symbols and soft (or hard) estimates of the data symbols are used to improve the channel quality in a semiblind manner. For time-varying channels, training sequences have to be sent frequently and periodically to keep up with the changing channel. The problem of determining the optimal number of pilot symbols in MIMO system is addressed in [5]. A linear least squares (LLS) approach by constructing optimal training sequences for MIMO communication systems is presented in [6]. More recently, a superimposed training based approach has been explored where a training sequence is added at a low power to the information sequence at the transmitter before modulation and transmission [7]. Some useful power is wasted in superimposed training which could have been allocated to the information sequence, but there is no loss in information rate. Channel estimation for MIMO time-invariant or slowly time-varying channels is considered using superimposed training in [7][8].

An efficient algorithm for tracking the time-varying Rayleigh fading MIMO channels based on ML algorithm is proposed in [9]. At a cost of complexity, both least-squares (RLS) [6][10] and Kalman filter [11][12] algorithms are extensively used for channel estimation in wireless communications. Although in [11]-[14], Kalman filter are applied for tracking MIMO channels, they didn't take into account the prior information of the power spectral density of the channel process.

In this paper, after developing a state-space model in a time-varying fading channel based on Clarke's model [15], we use Kalman filter to estimate and track the MIMO time-varying channel with the help of periodic pilots. The performance of the time-varying fading MIMO channel estimation is analyzed. An analytical formula for the estimation error due to the temporal variation of the channel coefficients is given and verified by link level simulations based on synthetic and measured impulse responses. The time-domain approach shows highly reliable tracking performance and robustness. The matrix inversion lemma is applied in order to reduce the computational complexity. The simulation results show that the proposed algorithm substantially improves the estimation performance in the presence of additive white Gaussian noise(AWGN).

The rest of the paper is organized as follows. In section 2, the time-varying fading MIMO channel is given. Section 3 describes the channel model approach of Clarke's model. In section 4 we introduce Kalman filter channel tracking algorithm based on Clarke's model. Section 5 demonstrates the simulation results using mobile 2-transmitter and 2-receiver MIMO system. Finally, section 6 concludes the paper.

Ⅱ. SYSTEM MODEL

Let us consider an MIMO system with M transmit antennas and N receive antennas. We assume that the fading is to be uncorrelated across antennas where each individual realization of the channel path is independent for all time steps k. The received signal at the jth receive antenna can be written as:

$$r_j(k) = \sum_{i=1}^{M}\sum_{l=0}^{L-1} h_{ij}^l(k)x_i(k-l) + v_j(k) \quad (1)$$

where $h_{ij}^l(k)$ is the lth tap of the impulse response between the ith transmit antenna and the jth receive antenna at time k. Here the order of the impulse response of every channel is assumed to be L. $x_i(k)$ is the transmit signal of transmit antenna i at time k. $v_j(k)$ is the zero mean AWGN of variance σ_n^2 observed at the jth receive antenna at time k.

The time-varying fading channel is modeled as a wide-sense-stationary complex Gaussian process with zero-mean, which makes the marginal distribution of the phase and amplitude at any given time uniform and Rayleigh respectively. The channel can be modeled by a linear time-varying filter having the complex low-pass impulse response. The channel impulse response (CIR) at time k can be represented as:

$$\begin{aligned} h_{ij}(k,l) &= \sum_{l=1}^{L} h_{ij}^l(k) \\ &= \sum_{l=1}^{L} \alpha_l(k)\exp\{j\varphi_l(k)\}\delta(k-\tau_l) \end{aligned} \quad (2)$$

where $\delta(\cdot)$ is the Dirac function, $\varphi_l(k)$ is the phase response of the lth path. $\alpha_l(k)$ and τ_l are the amplitude and time delay respectively, associated with the lth propagation path.

$$S_{h_{ij}^l h_{ij}^l}(f) = \begin{cases} \dfrac{1}{\pi f_D}\dfrac{1}{\sqrt{1-(\frac{f}{f_D})^2}}, & |f| < f_D \\ 0, & otherwise \end{cases} \quad (3)$$

It is a non-rational power spectral density and Doppler spectrum in (3) of the channel process for each channel tap [16]. Where f_D is the maximum Doppler frequency (the Doppler spread of the channel) in Hertz, which leads to a corresponding continuous-time autocorrelation of each channel tap as:

$$R(\tau) = J_0(2\pi f_D \tau) \quad (4)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind. For a frequency selective MIMO channel $R_{ij}^l(\tau) = E\{h_{ij}^l(t)h_{ij}^l(t-\tau)^*\}$. The discretized autocorrelation which takes into account the sampling rate T of the system is $R_{ij}^l(k) = J_0(2\pi f_D T|k|)$ for each integer time lag k. Each channel tap is a zero-mean complex Gaussian random process like $h_{ij}^l(k)$ described above, which is uncorrelated with and independent from any other tap process.

III. ARMA FILTER DESIGN

The dynamics of the channel tap $h_{ij}^l(k)$ can be approximated as an ARMA process of order p. For optimum selection of channel ARMA model parameters from correlation functions, the parameters can be obtained by approximating the square root of the spectrum of (3) as it is translated into discrete frequency [16]. The ARMA filter with frequency response equal to the square root of (3) performs a linear operation. By filtering of the Gaussian variables, the resulting sequence remains Gaussian, with a spectrum $S_{out}(f) = S_{in}(f)|H(f)|^2$, where $|H(f)|^2$ is the squared magnitude response of the filter and equal to (3). The speed of channel variations depends on the Doppler, or equivalently on the relative velocity between the receivers and the transmitters elements. A reasonable assumption, which is conventional in most of the scenarios, is each path with possibly the same Doppler rates. Obviously, this structure can be replicated as many path as the receiver desires.

An IIR filter of order $2K$, synthesized as a cascade of K second-order canonic sections(biquads), is designed to approximate the square root of the spectrum of (3) as it is translated into discrete frequency. We follow an approach proposed by K.Steiglitz [17] and use the method adopted by Christos [16] to search for the optimum real coefficients a_k, b_k, c_k, d_k, $k = 1,\cdots,K$ and the scaling factor A_0, such that the magnitude response of the filter is given by (5).

$$H(z) = A_0 \prod_{k=1}^{K} \frac{1 + a_k z^{-1} + b_k z^{-2}}{1 + c_k z^{-1} + d_k z^{-2}} \quad (5)$$

For $z = e^{jw}$ approaches the desired magnitude response $Y(w)$.

Based on the channel transport function $H(z)$, a state-space model for the fading channel can be built as in [15]. Defining $x[n] := [h(n),\cdots,h(n+p-1)]^T$ to be the channel impulse response coefficients vector at time n, we can get the following p order auto-regressive (AR) process,

$$x[n] = \Phi x[n-1] + Gw[n] \quad (6)$$

which is the state equation with $w[n]$ being the white Gaussian process noise. The matrices Φ and G are defined as:

$$\Phi := \begin{bmatrix} 0_N & I_N & \cdots & 0_N \\ \vdots & & \ddots & \vdots \\ 0_N & & \cdots \; 0_N & I_N \\ -B[1] & -B[2] & \cdots & -B[p] \end{bmatrix}, G := [0,\cdots,0,Q]^T,$$

where I_N and 0_N are $N \times N$ identity matrix and all-zero matrix, respectively. The observation equation of the state-space model is:

$$y(n) = A[n]x[n] + v(n) \tag{7}$$

where $A := [a_1, \cdots, a_p]$. The state-space model of (6) and (7) allows us to use Kalman filter to track the time-varying fading channel tap adaptively.

Ⅳ. MIMO CHANNEL TRACKING BASED ON KALMAN FILTER

We aim to improve the performance of MIMO channel estimation based on the prior information of the power spectral density of the channel process. Equation (6) and (7) derived from the Doppler power spectral density of the channel process can be taken as the state space models of the Kalman filter. During the training period, the transmitted symbols are known to the receiver. The periodic training symbols are sent in order to avoid loosing the tracking, which could occur if most of the MIMO channels happen to fall in a deep fade simultaneously. So the Kalman filter algorithm can be used to track the channel variation.

Additional non-random periodic sequences $\{c_i(k) = c_i(k+mP), (i = 1,2,\cdots,M)\} \; \forall m,n$ are superimposed on information-bearing symbols $\{b_i(k), (i = 1,2,\cdots,M)\}$ on all transmit antennas, so $x_i(k) = b_i(k) + c_i(k), (i = 1,2,\cdots,M)$ are actually transmitted. And P is the period of the $c_i(k)$. The proposed MIMO channel tracking algorithm will be described in detail. In the sequel, we assume that:

(1) Channel: a set of channels $\{h_{ij}^l(k)\}$ is assumed to be quasi-static over Ln symbols.

(2) Receiver: receiver is assumed to be matched to the transmitted pulse.

Thus (1) can be rewritten as:

$$r(k) = x(k) \cdot h(k) + v(k) \tag{8}$$

where $r(k) = [r_1(k)\; r_2(k) \cdots r_N(k)]^T$

$$x(k) = [x_1(k)\mathrm{I}_N \cdots x_M(k)\mathrm{I}_N$$
$$\cdots x_1(k-L+1)\mathrm{I}_N \cdots x_M(k-L+1)\mathrm{I}_N]$$

$$h(k) = [h_{11}^0(k) \cdots h_{1N}^0(k) \cdots h_{M1}^0(k) \cdots h_{MN}^0(k)$$
$$\cdots h_{11}^{L-1}(k) \cdots h_{1N}^{L-1}(k) \cdots h_{M1}^{L-1}(k) \cdots h_{MN}^{L-1}(k)]^T$$
$$v(k) = [v_1(k)\; v_2(k) \cdots v_N(k)]^T$$

Exact modeling of the process h_{ij}^l with an ARMA model is impossible. For implementation of a channel estimator, we focus on a second-order channel model based on (5) for simplicity and its true dynamics of real channels [10], where the parameters are selected according to the normalized Doppler frequency $f_D T$ (assumed to be known). Therefore, the time evolution of the MIMO channel taps can be described as:

$$\vec{h}(k) = D\vec{h}(k-1) + w(k) \tag{9}$$

where $\vec{h}(k) = [h^T(k)\; h^T(k+1)]^T$, D is the state transition matrix and w is the state noise. The updating of each channel tap in (9) is realized according to (5). We can get

$$D = \begin{bmatrix} \mathrm{I}_{MNL} & 0 \\ -d_1 \cdot \mathrm{I}_{MNL} & -c_1 \cdot \mathrm{I}_{MNL} \end{bmatrix}.$$

Here, 0 denotes a matrix of zeros whose dimensions will be obvious from the context. The observation matrix is:

$$C = [A_0(b_1 - d_1)\mathrm{I}_{MNL} \quad A_0(a_1 - c_1)\mathrm{I}_{MNL}] \tag{10}$$

Equations (8), (9) and (10) form the state-space model of the MIMO system. Considering that the model is linear and the noise is Gaussian, Kalman filtering can be applied to estimate the state matrix $\vec{h}$. The proposed channel estimation method is suitable to track even fast fading channels because of the iterative structure of the Kalman filter. For each k, do the Kalman filter update according to

$$\hat{\vec{h}}[k|k-1] = D(k,k-1)\hat{\vec{h}}[k-1|k-1]$$
$$\hat{\vec{h}}[k|k] = D(k,k-1)\hat{\vec{h}}[k-1|k-1] + K(k)v(k)$$
$$P(k|k-1) = D(k,k-1)P(k-1|k-1)D^T(k,k-1)$$
$$+ B(k-1)Q(k-1)B^T(k-1)$$
$$K(k) = P(k|k-1)C^T(k)[C(k)P(k|k-1)C^T(k)$$
$$+ R(k)]^{-1}$$
$$P(k|k) = [I - K(k)C(k)]P(k|k-1)$$

where $R = \sigma^2 \mathrm{I}$ is the covariance matrix of the measurement noise v,which is assumed to be uncorrelated. $P(k|k-1)$ and $P(k|k)$ are priori estimate error covariance and posteriori estimate error covariance respectively. $K(k)$ is the Kalman gain. Q being the covariance matrix of the state noise. In our simulation in section 5, Q is chosen according to [11][12].

The major computational cost of the algorithm lies in the calculation of the matrix inversion in the Kalman gain expression. Because the matrix discussed above is trigonal matrix, the proposed algorithms is achieved significantly lower complexity.

Channel estimate at instance k is:

$$\hat{\hat{h}}(k) = [\mathrm{I}_{MNL}, 0]\hat{\hat{h}}[k|k] \tag{11}$$

Assuming the fixed total power P_t, the perfect CSI can be obtained in the sense that it achieves the minimum mean squared error of the channel estimation. From (11), the MSE of channel estimates $\hat{\hat{h}}(k)$ is given by:

$$MSE_k = \frac{1}{MNL} E\{\left|\hat{\hat{h}}(k) - h(k)\right|^2\} \tag{12}$$

where $E\{x\}$ denotes the ensemble average of x.

V. SIMULATION RESULTS

In this section, we provide computer simulation results to demonstrate the performance of the algorithm discussed above. The simulation experiment is based on the MIMO system with 2 transmit antennas and 2 receive antennas. The extension to the M transmit antennas and N receive antennas case is a straightforward generalization. The transmitted pulse is a raised cosine with roll-off 0.5. The four sub-channels $\{h(1,1), h(1,2), h(2,1), h(2,2)\}$ are assumed to be independent on each other. We consider the tapped-delay-line channel model with the parameters according to TABLE□.

Ideal timing and frequency synchronization are assumed. The performance of the system is measured in terms of the mean square error (MSE) and the symbol error rate (SER) versus signal-to-noise ratio (SNR) of two different channel estimation schemes. The SNR refers to the energy per bit over one-sided noise spectral density with both information and superimposed training sequence counting toward the bit energy.

Figure 1 shows the MSE of the Kalman filter channel estimation versus the received SNR with different $f_D T$ and Doppler frequency. The MSE is the average of the channel estimation for 3000 slots. From Figure 1 we can see that the MSE of the algorithm proposed by this paper is relatively good in high speed environment.

TABLE I. THE PARAMETERS FOR SIMULATIONS

Chip rate	1.28 Mcps
Spreading factor	16
Multipaths	2
f_D	50-200 Hz
Modulation mode	QPSK
Channel	Rayleigh(Clarke)

We also implemented a conventional (time-multiplexed) training-based approach where the center 144 chips of the slot are reserved for training and the remaining bits are information bits. The training sequences at two Tx's are mutually independent i.i.d sequences. Figure 2 shows the MSE of the Kalman filter channel estimator and conventional channel estimator versus the received SNR with different Doppler frequency. In the time-varying channel, Doppler frequency of 200 Hz and 100 Hz, which corresponds to 108 km/h and 54 km/h, is considered in the simulation. The MSE is also the average of the transmission of 3000 slots. The performance of the proposed method is shown in different SNR, where tracking is performed in terms of MSE. There is no loss in information rate when superimposed training is added compared with conventional training.

Furthermore, although the channel estimation errors may be much lower for superimposed training with iterated enhancement, this channel accuracy advantage does not necessarily translate into a large SER advantage because of SNR penalty (power wasted in superimposed training).

Figure 3 shows SER of the Kalman filter channel estimation and conventional channel estimator versus the received SNR in different Doppler. The receiver SER performance is improved significantly by using the proposed algorithm of this paper. The SER performance is relatively better compared with the conventional channel estimation algorithm especially in high mobile speed environment. The SER is again the average of the transmission of 3000 slots. As is evident from these graphs, the algorithm proposed in this paper is able to closely track the fast fading channel impulse response of each path individually and hence results in better MSE and SER performance than conventional algorithms.

VI. CONCLUSION

In this paper, we propose an algorithm which uses the Kalman filter solution for the estimation of a time-varying fading channel in MIMO system. The state space model is developed based on the power spectral density of the channel process. It makes the most of the prior information of the time-varying fading channel estimator. The computational complexity is reduced by applying the matrix inversion lemma. The analytical formula for the estimation error due to the temporal variation of the channel coefficients is given and verified by link level simulations based on synthetic and measured impulse responses. The simulation results show the proposed method in particular at high mobile speed is superior to the conventional MIMO channel estimators.

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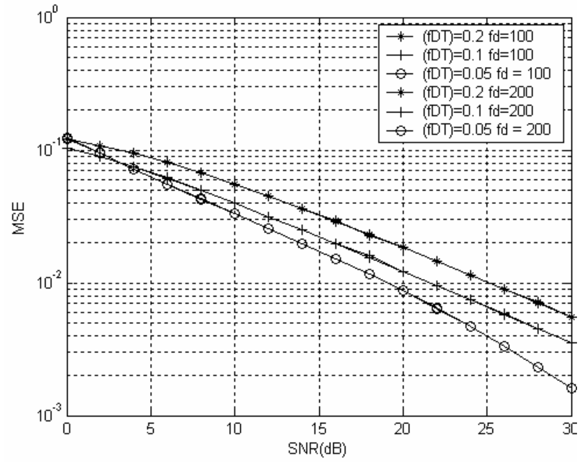

Fig.1. Channel estimate MSE of the Kalman filter estimators in different Doppler

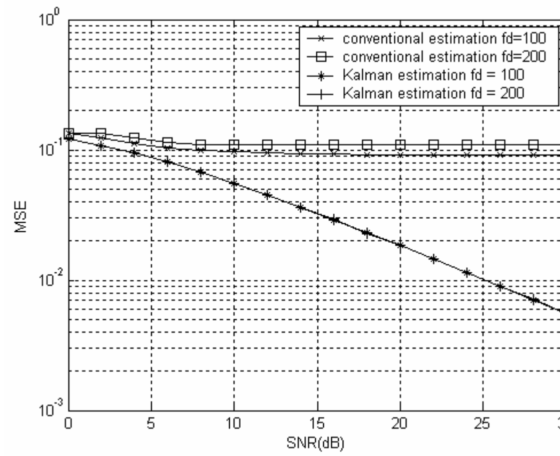

Fig. 2. Channel estimate MSE of the Kalman filter and conventional estimators in different Doppler

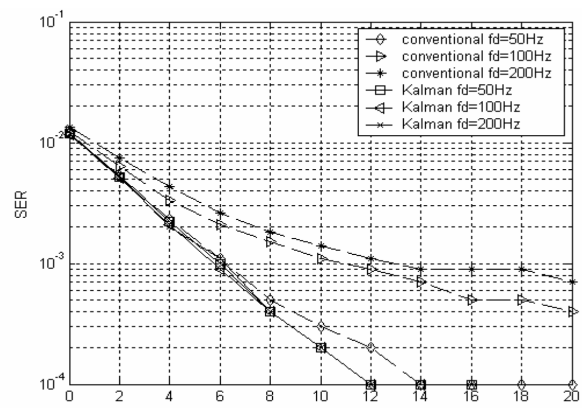

Fig. 3. Symbol error rate of the Kalman filter and conventional estimators in different Doppler