CHANNEL ESTIMATION AND TRACKING IN SPATIALLY CORRELATED MIMO OFDM SYSTEMS

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ABSTRACT

This paper addresses the problem of channel estimation for MIMO OFDM systems where the channels may be spatially correlated. Starting from OFDM transmission model with independent MIMO channels, we derive a new state-space model that accounts for spatial correlation at transmitter and receiver antenna arrays. Kalman filter is then applied to estimate and track the time-varying channels. Correlation parameters are also estimated from the received data.

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems have received great attention in recent years since they allow improving the spectral efficiency ($bits/s/Hz$) and link quality of the system tremendously. The capacity gains depend on the available channel information at either the receiver or transmitter, on the signal to noise ratio (SNR), and on the spatial correlation [4, 9].

In realistic scenarios, both uncorrelated and correlated channels are encountered. One way of modeling a correlated MIMO channel is based on defining two correlation matrices, one at the transmitter and the other at the receiver antenna array. A correlation matrix for the MIMO channels may be constructed. It is then multiplied by a channel matrix with independent and identically distributed (*i.i.d.*) entries in order to get a correlated MIMO channel matrix [3, 5, 9].

Accurate channel estimation plays an important role in achieving the high capacity of MIMO systems. The problem of correlated channel coefficients in a Single Input Single Output system has been also considered in [10] where the time-varying channel was modeled as a multichannel autoregressive process of order p. The state transition matrix has been estimated from the received data by using higher order statistics.

In this paper we derive a state-space model for correlated channels starting from a model for uncorrelated channels. The correlation matrix is estimated from the received data and is a part of the state variable. The rest of the paper is organized as follows. In the next section we describe how correlated channels can be modeled based on uncorrelated channels. In section 3 we describe the state-space model in the case of correlated channels and in section 4 we present how we can estimate the correlation from the received data. Simulation results are presented in section 5.

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2. CORRELATED MIMO CHANNELS

An important MIMO channel model based on measurements is described in [5]. This model, referred to as I-METRA, has been used as simulation tool during the standardization process of 3G systems. Assuming a m transmitter, n receiver narrowband scenario, the channel matrix $\mathbf{H}$ will be of size $n \times m$. The input-output relationship may then be written as:

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{w}_k, \quad (1)$$

where $\mathbf{r}$ is the received signal, $\mathbf{x}$ are the transmitted symbols, $\mathbf{w}$ is additive Gaussian noise, k is the time index.

Following the I-METRA model, we define a set of two correlation matrices at the transmitter and receiver antenna arrays, namely $\mathbf{R}_{Tx}$ and $\mathbf{R}_{Rx}$ with respective dimensions $m \times m$ and $n \times n$. The correlated channel matrix may then be expressed as:

$$\mathbf{H}^{(c)} = \mathbf{R}_{Rx}^{1/2} \mathbf{H}^{(i)} (\mathbf{R}_{Tx}^{T})^{1/2},$$

where $\mathbf{H}^{(c)}$ is the correlated MIMO channel matrix and $\mathbf{H}^{(i)}$ is the MIMO channel matrix with *i.i.d.* Rayleigh distributed entries and matrix square roots may be obtained, e.g. via Cholesky decomposition.

Modeling spatial correlation at the transmitter and receiver antennas can be done independently from one another. The justification for this approach relies on the fact that only immediate surroundings of the antenna array cause the correlation between array elements and have no impact on correlations observed between the elements of the array at the other end of the link [2, 5]. Hence, the $mn \times mn$ $\mathbf{R}_{MIMO}$ spatial correlation matrix of the MIMO radio channel can be expressed as the Kronecker product of spatial correlation matrices at the transmitter and the receiver:

$$\mathbf{R}_{MIMO} = \mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx}. \quad (2)$$

A correlated channel matrix $\mathbf{H}^{(c)}$ may be generated using the $n \times m$ matrix $\mathbf{H}^{(i)}$ and a given MIMO correlation matrix $\mathbf{R}_{MIMO}$ as follows. The Cholesky decomposition of the positive definite MIMO correlation matrix is:

$$\mathbf{R}_{MIMO} = \mathbf{C}\mathbf{C}^{H}. \quad (3)$$

Using the *vec* operator, the channel matrix elements can be stacked into a column vector as: $\mathbf{h} = vec(\mathbf{H})$, where $\mathbf{h}$ has dimension $mn \times 1$. The correlated channel matrix in column vector form is obtained by:

$$vec\left(\mathbf{H}_k^{(c)}\right) = \mathbf{C}\ vec\left(\mathbf{H}_k^{(i)}\right),$$

which is equivalent to:

$$\mathbf{h}_k^c = \mathbf{C}\mathbf{h}_k^i, \tag{4}$$

where k is the time index. Using a 2×2 system as an example (Figure 1), the channel vectors from (4) contain the elements:

$$\mathbf{h}_k^c = [h_{11,k}^c \ h_{21,k}^c \ h_{12,k}^c \ h_{22,k}^c]^T. \tag{5}$$

In case of frequency selective time-varying model with L-tap channels in each MIMO branch, we have a relationship similar to (4):

$$\mathbf{h}_k^c = \tilde{\mathbf{C}}\mathbf{h}_k^i, \tag{6}$$

where $\tilde{\mathbf{C}} = \mathbf{C} \otimes \mathbf{I}_L$ is of size $mnL \times mnL$, $\mathbf{I}_L$ being the identity matrix of size $L \times L$. The underlying assumption, is that each delayed path component $(1, \dots, L)$ exhibits similar spatial correlation defined by $\mathbf{C}$. Each $h_{rs,k}^c$ in (5) has now been replaced by a $L \times 1$ vector containing the path components corresponding to the rs-th MIMO branch, $(r, s = \{1, 2\})$.

Just to recap, spatially correlated MIMO channels were generated from MIMO channel matrices with i.i.d. elements [3, 5, 9] by applying a given correlation structure. In case of frequency selective channels the channel taps are stacked into a vector and the correlation is spanned using the Kronecker product in order to match the dimensionality.

3. STATE-VARIABLE MODEL

In wireless mobile communication environments characterized by time selectivity, estimation and tracking of channel parameters is crucial for reliable tranmission. In the case of linear FIR channels, we can model the transmission using linear state-space model. Kalman filter (KF) may then be applied in order to estimate the state vector comprised of channel coefficients. We consider the m-input n-output frequency selective MIMO OFDM scenario (Fig. 1 illustrates simple 2×2 system). The state-space model is the following:

$$\begin{aligned} \mathbf{h}_{k+1}^i &= \mathbf{A}_d\mathbf{h}_k^i + \mathbf{v}_k & (7)\\ \mathbf{r}_k &= \tilde{\mathbf{X}}_k\mathbf{h}_k^i + \mathbf{w}_k. & (8) \end{aligned}$$

The state vector $\mathbf{h} = [\mathbf{h}_{11}, \mathbf{h}_{21}, \cdots, \mathbf{h}_{mn}]^T$ where $\mathbf{h}_{rs} = [h_{rs,1}, \dots, h_{rs,L}]^T$, and $rs = 11, \dots, mn$ is of size $mnL \times 1$. The observation $\mathbf{r}_k$ is $nN \times 1$ vector, where N is the number of subcarriers. Typically $N \gg L$. The circulant matrix containing the transmitted OFDM modulated symbols $\tilde{\mathbf{X}}$ is of size $nN \times mnL$. It admits a left inverse of dimension $mnL \times nN$ which will be denoted by $\tilde{\mathbf{X}}^+$, i.e. $\tilde{\mathbf{X}}^+\tilde{\mathbf{X}} = \mathbf{I}$. The state transition matrix $\mathbf{A}_d$ is a $mnL \times mnL$ diagonal matrix. State noise $\mathbf{v}$ and measurement noise $\mathbf{w}$ with covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ are assumed to be zero mean circular white Gaussian, mutually uncorrelated and also uncorrelated with the state vector.

Combining equations (6) and (7) leads to the following dynamic model for the *spatially correlated* channel:

$$\mathbf{h}_{k+1}^c = \mathbf{A}_d\mathbf{h}_k^c + \tilde{\mathbf{C}}\mathbf{v}_k, \tag{9}$$

under the condition $\mathbf{A}_d = a\mathbf{I}$, $|a| < 1$.

As in [6] we consider that the first-order AR model approximation is enough to capture the channel dynamics over time. There may also exist one-lag spatial correlation between the taps. Consequently, the state transition matrix is not necessarily diagonal, and we will denote it by $\mathbf{A}$. Combining equations (6) and (7) leads to the following dynamic model for the *spatio-temporal correlated* channel:

$$\mathbf{h}_{k+1}^c = \mathbf{A}_c\mathbf{h}_k^c + \tilde{\mathbf{C}}\mathbf{v}_k, \tag{10}$$

where $\mathbf{A}_c = \tilde{\mathbf{C}}\mathbf{A}\tilde{\mathbf{C}}^{-1}$. The measurement equation remains:

$$\mathbf{r}_k = \tilde{\mathbf{X}}_k\mathbf{h}_k^c + \mathbf{w}_k. \tag{11}$$

Using the state-space model defined by the set of equations (10) and (11), KF can be applied to estimate the state. The update of the covariance matrix of the prediction error requires the channel correlation matrix and the state noise covariance:

$$\mathbf{P}_{(k|k-1)} = \mathbf{A}_c\mathbf{P}_{(k-1|k-1)}\mathbf{A}_c^H + \tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H. \tag{12}$$

4. CORRELATED CHANNEL ESTIMATION

As noted previously, the correlation matrix $\tilde{\mathbf{C}}$ is needed in KF equations. In fact, we do not need to know explicitly the correlation matrix, but the structure $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$. In the following, two methods for estimating this quantity will be presented.

4.1. A priori computation

The first method presented here to estimate $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ is based on adaptively updating a quantity based on the received data, on the transmitted symbols and on the state transition matrix. The technique is based on investigating the following quantity:

$$\begin{aligned} \mathbf{B}_1^k \ = \ & E\Big[\Big(\tilde{\mathbf{X}}_k^+\mathbf{r}_k - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{r}_{k-1}\Big) \\ & \Big(\tilde{\mathbf{X}}_k^+\mathbf{r}_k - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{r}_{k-1}\Big)^H\Big]. \end{aligned} \tag{13}$$

We will first consider the term $\mathbf{S}_k = \tilde{\mathbf{X}}_k^+\mathbf{r}_k - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{r}_{k-1}$. Using the measurement equation (11), $\mathbf{S}_k$ can be further written as:

$$\begin{aligned} \mathbf{S}_k &= \tilde{\mathbf{X}}_k^+\mathbf{r}_k - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{r}_{k-1} \qquad (14)\\ &= \mathbf{h}_k^c + \tilde{\mathbf{X}}_k^+\mathbf{w}_k - \mathbf{A}_c\mathbf{h}_{k-1}^c - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{w}_{k-1}. \end{aligned}$$

According to (10) we also have: $\mathbf{h}_k^c = \mathbf{A}_c\mathbf{h}_{k-1}^c + \tilde{\mathbf{C}}\mathbf{v}_{k-1}$, thus the last form of the previous equation can be further written as $\mathbf{S}_k = \tilde{\mathbf{C}}\mathbf{v}_{k-1} + \tilde{\mathbf{X}}_k^+\mathbf{w}_k - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{w}_{k-1}$. Going back to (13), using the latter form of $\mathbf{S}_k$ and taking the expectation will lead to:

$$\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H = \mathbf{B}_1^k - \tilde{\mathbf{X}}_k^+\mathbf{R}\left(\tilde{\mathbf{X}}_k^+\right)^H - \mathbf{A}_c\tilde{\mathbf{X}}_{k-1}^+\mathbf{R}\left(\tilde{\mathbf{X}}_{k-1}^+\right)^H\mathbf{A}_c^H, \tag{15}$$

where $\mathbf{B}_1^k$ is estimated as follows:

$$\begin{aligned} \hat{\mathbf{B}}_1^k \ = \tfrac{1}{k-3}\textstyle\sum_{n=3}^k \ & \Big[\Big(\tilde{\mathbf{X}}_n^+\mathbf{r}_n - \mathbf{A}_c\tilde{\mathbf{X}}_{n-1}^+\mathbf{r}_{n-1}\Big) \\ & \Big(\tilde{\mathbf{X}}_n^+\mathbf{r}_n - \mathbf{A}_c\tilde{\mathbf{X}}_{n-1}^+\mathbf{r}_{n-1}\Big)^H\Big]. \end{aligned} \tag{16}$$

In a mobile system the spatial correlation matrix $\tilde{\mathbf{C}}$ may be also time varying due to moving scatterers or the mobile terminal. The recursive update of $\mathbf{C}\mathbf{Q}\tilde{\mathbf{C}}^H$ in (15) allows also the tracking of the correlation matrix.

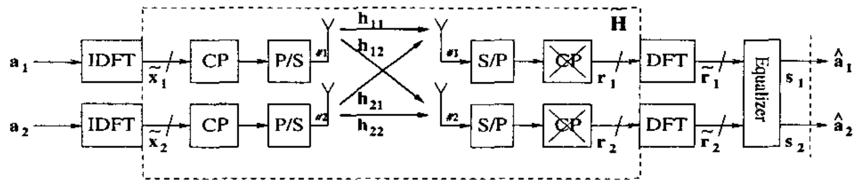

Fig. 1. 2×2 MIMO OFDM transmission.

4.2. Covariance matching

The second method for estimating $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ is based on matching the covariance of the theoretical and estimated residual processes [7]. Let us form the following residual process:

$$\mathbf{q}_k = \mathbf{h}_{k+1}^c - \mathbf{A}_c\hat{\mathbf{h}}_{(k|k)}^c \tag{17}$$

which can be further written as:

$$\mathbf{q}_k \;\; = \;\; \mathbf{A}_c[\mathbf{h}_k^c - \hat{\mathbf{h}}_{(k|k)}^c] + \tilde{\mathbf{C}}\mathbf{w}_k. \tag{18}$$

The theoretical covariance of the residual process is given by:

$$\mathbf{S} = \mathbf{A}_c\mathbf{P}_{(k|k)}\mathbf{A}_c^H + \tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H. \tag{19}$$

The mean $\bar{\mathbf{q}}$ of the residual process $\mathbf{q}$ can be computed recursively.

$$\bar{\mathbf{q}}(k) = \frac{k-1}{k}\bar{\mathbf{q}}(k-1) + \frac{1}{k}\mathbf{q}(k). \tag{20}$$

It follows that a recursive estimate of the residual process covariance matrix can be computed as:

$$\hat{\mathbf{S}}_k = \frac{k-1}{k}\hat{\mathbf{S}}(k-1) + \frac{1}{k}\left[\mathbf{q}_k - \bar{\mathbf{q}}_k\right]\left[\mathbf{q}_k - \bar{\mathbf{q}}_k\right]^H. \tag{21}$$

Matching $\mathbf{S}$ with $\hat{\mathbf{S}}_k$ we obtain:

$$\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}_k^H = \hat{\mathbf{S}}_k - \frac{1}{k}\mathbf{A}_c\mathbf{P}_{(k|k)}\mathbf{A}_c^H. \tag{22}$$

Discussion - the major difference between the two methods is that the second one has to work inside the KF algorithm since it depends on quantities generated by the KF, i.e. $\mathbf{P}_{k|k}$. The first method does not depend on parameters KF and this might be an advantage. The computation of $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ may be done prior to actually running KF, provided that the needed quantities ($\mathbf{r}$, $\tilde{\mathbf{X}}$, $\mathbf{R}$ and $\mathbf{A}_c$) are available.

The KF parameter acquisition stage is using known training symbols before actual data is transmitted. After this stage, the algorithm may switch to decision directed mode, where received symbols are equalized using MMSE techniques as presented in [8].

5. SIMULATIONS

We will first illustrate the performance of the proposed method using a simple 2×2 MIMO spatially correlated system without OFDM transmission. An example where we know all the quantities involved in the model is needed in order to verify the convergence of estimated quantities to the true ones. We consider a narrowband (L=1) system where the matrices $\mathbf{A}_c$, $\tilde{\mathbf{C}}$, $\mathbf{Q}$, $\mathbf{R}$ are known. The size of the state is 4×1 and the observation is of dimension 32×1. The Tx and Rx correlation matrices are the ones presented in [1] corresponding to the Nokia scenario. The state matrix is close to identity ($a = 0.99$) and the correlation matrix $\tilde{\mathbf{C}}$ is randomly generated. We estimate the MIMO correlation structure via the product $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ of dimension 4×4 according to (15). The convergence of the elements of the first two rows of the matrix to the true values is shown in Figure 2.

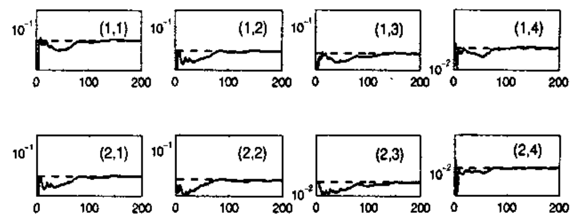

Fig. 2. Tracking of the elements of the first two rows of the matrix $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$. Dash line represents true values, continuous line represents estimated values.

In order to investigate the convergence of the estimated matrix to the true one, we use the following error criterion:

$$\mathcal{E}_{\mathbf{M}}(k) = \frac{||\mathbf{M}(k) - \hat{\mathbf{M}}(k)||_F}{||\mathbf{M}(k)||_F}, \tag{23}$$

where $||\cdot||_F$ denotes the Frobenius norm, $\hat{\mathbf{M}}$ is an estimate of the true matrix $\mathbf{M}$. This criterion is applied to $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ which is computed with (15) and (22). Using equation (23) for $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$, the error $\mathcal{E}_{\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H}$ converges as in Fig. 3. Both results have been averaged over 30 realizations.

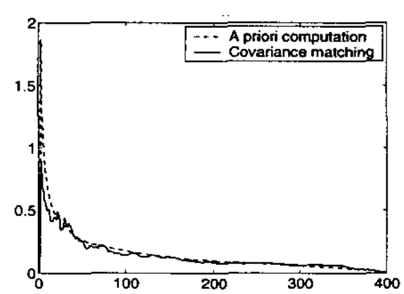

Fig. 3. Error in $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ estimation using Frobenius norm criterion of (23). The result is averaged over 30 realizations.

In the next part of the simulation results section we consider a MIMO-OFDM spatially correlated scenario with the same settings as above, i.e. 2×2 system, $L = 1$. The state matrix is

4×4 and the same dimension for the correlation matrix. BPSK modulation is used. The carrier frequency is $f_0 = 2.4$ GHz and the number of subcarriers is $N = 32$. The receiver speed is 100 km/h and the channel is generated using a realistic channel generator for mobile communications. Hence, the statistic $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ has to be estimated. In the simulations only the parameter acquisition stage is considered. Hence, we transmit the same OFDM symbol. The number of subcarriers must be such that the dimension of the observation vector is larger than dimension of the state. The dimension of the state is mnL, in our simulation $2 \times 2 \times 1 = 4$. In MIMO-OFDM this is obviously the case since the observation vector is of dimension nN, where N is the number of subcarriers and typically $nN \gg mnL$.

The Tx and Rx correlation matrices are adopted from the Nokia scenario in [1]. Since the process noise covariance matrix is not known, a convergence of the estimated parameters to the true ones cannot be demonstrated. However, the estimated parameters do converge to steady state values. The Mean Square Error (MSE) of channel estimation using estimated model parameters is depicted in Fig. 4.

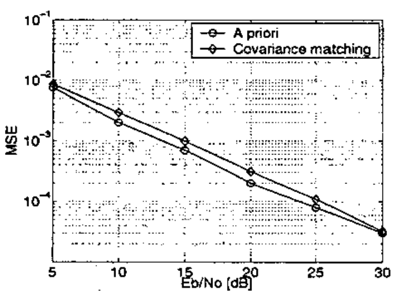

Fig. 4. MSE for channel estimates using estimated state-space model parameters computed with (15) and (22).

The system performance is characterized by bit error rate (BER). We have investigated raw BER using channel state information (CSI) by the receiver and estimated channel with estimated model parameters, Fig. 5.

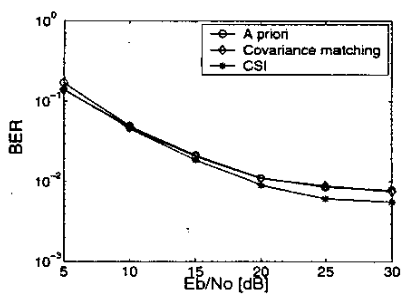

Fig. 5. BER of the receiver when channel estimation is performed using true and estimated state-space parameters.

The track of the real part of the channel taps is depicted in Figure 6 where KF is using the estimated correlation as shown in (15). Dash lines represents true values, continuous line represents estimated values. Note that high fidelity tracking performance is obtained due to good estimates of $\tilde{\mathbf{C}}\mathbf{Q}\tilde{\mathbf{C}}^H$ that lead to close to optimal performance of KF.

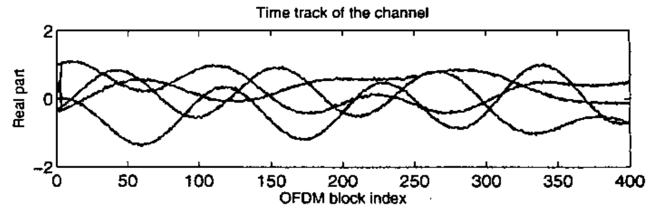

Fig. 6. Time track of real part of the channel using estimated model parameters.

6. CONCLUSION

In this paper we derived a recursive estimator for tracking spatially correlated MIMO channels. Estimates of the MIMO channel and correlation structure may be obtained at the arrival of each observation vector. The reliable performance of the proposed method has been demonstrated in both simple MIMO system and MIMO-OFDM type of scenarios.

7. REFERENCES

[1] "IST-1999-11729 METRA Project", available at http://www.ist-metra.org/

[2] D. Chizhik, J. Ling, P. W. Wolniansky, R. A. Valenzuela, N. Costa, K. Huber, "Multiple-Input-Multiple-Output Measurements and Modeling in Manhattan", IEEE J. on Selected Areas in Comm., vol. 21, no. 3, 2002.

[3] D. Gesbert, and J. Akhtar, "Breaking the barriers of Shannon's capacity: An overview of MIMO wireless systems", Telektronikk Telenor J., to appear, 2002.

[4] A. Goldsmith, S. A. Jafar, N. Jindal, S. Vishwanath, "Capacity Limits of MIMO Channels", IEEE J. on Selected Areas in Comm., vol. 21, no. 5, pp. 684-702, 2003.

[5] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, F. Frederiksen, "A Stochastic MIMO Radio Channel Model With Experimental Validation", IEEE J. on Selected Areas in Comm., 20(6):1211-1225, 2002.

[6] C. Komninakis, C. Fragouli, A. H. Sayed, and R.D. Wesel, "Multi-input multi-output fading channel tracking and equalization using Kalman estimation" Signal Processing, IEEE Transactions on, vol.50, no.5, pp. 1065-1076, 2002.

[7] K. A. Myers, B. D. Tapley "Adaptive Sequential Estimation with Unknown Noise Statistics", IEEE Transactions on Automatic Control, vol. AC-21 ,No.4, August 1976, pp. 520-523.

[8] T. Roman, M. Enescu, V. Koivunen, "Time-domain method for tracking dispersive channels in MIMO OFDM systems", ICASSP, Vol. 4, pp. 393 - 396, 2003.

[9] D-S. Shiu, G. J. Foschini, M. J. Gans, J. M. Kahn, "Fading Correlation and Its Effects on the Capacity of Multielement Antenna Systems", IEEE Trans. on Comm., 40(3):502-513, 2000.

[10] M.K. Tsatsanis, G.B. Giannakis, G. Zhou, "Estimation and Equalization of fading channels with random coefficients", Signal Processing, 53, pp.: 211-229, 1996.