A Novel Method for Estimating the Power Azimuth Spectrum of the Wireless Channel

Ji Li, Xiao-Zhou Li, Dong-Ming Zhou, and Er-Yang Zhang

Abstract—**A simple and novel method for estimating the power azimuth spectrum (PAS) is presented in this letter. Analysis shows that the expected power received by the directional antenna can be expressed as the integral of the PAS multiplied by the antenna pattern. Therefore, the PAS can be estimated by conducting azimuth-scanning power measurements with directional antenna and solving the integral equation. The influence of the antenna directivity on estimation is also analyzed.**

Index Terms—**Deconvolution, directional antenna, Fredholm integral equations of the first kind, power azimuth spectrum (PAS).**

I. Introduction

POWER azimuth spectrum (PAS) is an important statistical properties of the wireless channel, since it influences the Doppler power spectrum [1] and the spatial correlation of the wireless multiinput–multioutput (MIMO) channel [2], directly. Therefore, it is essential to obtain and model the realistic PAS for channel simulation and performance analysis of the wireless system. Some measurement campaigns have been carried out and valuable results have been published in [3]–[5]. By employing expensive measurement systems and complex algorithms of array processing, the channel parameters were estimated jointly in those works. But, in some cases, such as modeling the frequency flat channel using Jake's model [1] and Clarke model [6], analyzing the capacity of the wireless MIMO channel [2], the PAS is more relevant. In this letter, we present a novel and simple method for estimating the PAS by conducting power measurements with directional antenna.

II. Power Azimuth Spectrum

The PAS describes the spatial distribution of the expected power related to the azimuth. Consider the non line-of-sight frequency flat channel with one transmitter and one receiver and there is no relative motion between them, i.e., the static scenario. Under the classical plane wave assumption, where the electromagnetic field in space is a superposition of plane waves [1], the received signal can be expressed as

$$y(t) = \sum_{l=0}^{L-1} a_l \sqrt{G(\theta_l - \psi)} \cdot s(t) + n(t) \tag{1}$$

where $y(t)$, $s(t)$, and $n(t)$ denote the received signal, the transmitted signal and the additive noise, respectively. L is the number of sub-rays. $G(\theta)$ is the antenna pattern of the receiver and $\theta = 0^\circ$ corresponds to the main lobe direction, ψ is the main lobe direction reference to the line-of-sight. Random variable a_l is the complex amplitude of the lth sub-ray and relative to the corresponding azimuth θ_l. It is assumed that $a_0, a_1, \ldots, a_{L-1}$ are independent [3]. According the definitions above, the PAS can be written as [3]

$$P_A(\theta_l) = c \cdot E\left[|a_l|^2 \middle| \theta_l\right] \cdot f_A(\theta_l) \tag{2}$$

where $f_A(\theta)$ is the probability density function (pdf) of the azimuth, c is the normalization coefficient and will be further ignored for simplification. Equation (2) means that the PAS is proportional to the product of the expected power conditioned on the azimuth and the pdf of the corresponding azimuth. This is physically intuitive.

III. Measurement Equation

Let the transmitted signal be the unmodulated carrier wave with frequency ω_0. Then the expectation of the received power conditioned on the main lobe direction ψ can be written

$$\begin{aligned} E\left[|y(t)|^2 \middle| \psi\right] &= E\left[\left|\sum_{l=0}^{L-1} a_l \sqrt{G(\theta_l - \psi)} \cdot e^{j\omega_b l} + n(t)\right|^2\right] \\ &= E\left[\left|\sum_{l=0}^{L-1} a_l \sqrt{G(\theta_l - \psi)}\right|^2\right] + E\left[|n(t)|^2\right] \\ &= \sum_{l=0}^{L-1} E\left[|a_l|^2\right] \cdot G(\theta_l - \psi) + N_0 \\ &= \sum_{l=0}^{L-1} E\left[|a_l|^2 \middle| \theta_l\right] P(\theta = \theta_l) \\ &\quad \cdot G(\theta_l - \psi) + N_0 \end{aligned} \tag{3}$$

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where N_0 is the power of the noise, it can be easily measured by shutting down the transmitter and reasonably assumed to be irrelative to ψ. The term $P(\theta = \theta_1)$ denotes the probability of variable θ when it equals to the specific value θ_l. When $L \to \infty$, (3) could be expressed in integral form

$$\begin{aligned} Z(\psi) &\triangleq E\left[|y(t)|^2 \middle| \psi\right] - N_0 \\ &= \int_\theta E\left[|a|^2 \middle| \theta\right] G(\theta - \psi) \cdot f_A(\theta) d\theta \\ &= \int_\theta P_A(\theta) \cdot G(\theta - \psi) d\theta. \end{aligned} \tag{4}$$

This is a Fredholm integral equation of the first kind. When $G(\theta)$ is evenly symmetric, the integral will be a convolution. Thus, the estimation of the PAS can be achieved by deconvolution.

IV. Estimating the PAS

Let us consider the power measurement campaigns with directional antenna scanning uniformly over $[0, 2\pi)$ in azimuth. We can get a observation set $\{Z(\psi_n)\}$ where $\psi_n (n = 0 \dots N-1)$ is the main lobe direction in the nth measurement campaign. By discretizing the PAS and the antenna pattern, (4) can be expressed as

$$Z(\psi_n) = \sum_{n'=0}^{N-1} P_A(\psi_{n'}) G(\psi_n - \psi_{n'}) \cdot \Delta\psi, \quad 0 \le n \le N-1 \tag{5}$$

where $\Delta\psi = 2\pi/N$ is the azimuth step of scanning and $G(\theta)$ is assumed to be evenly symmetric. Since $P_A(\theta)$ and $G(\theta)$ can be treated as periodic functions with period 2π, (5) is a circular convolution [7]. Substituting ψ_n with the subscript n and ignoring the constant coefficient $\Delta\psi$, we can rewrite (5) as

$$Z(n) = P_A(n) \circledcirc G(n), \quad 0 \le n \le N-1 \tag{6}$$

where $\circledcirc$ denotes the N-point circular convolution. Therefore, the deconvolution can be realized by applying discrete Fourier transform (DFT).

Let $z(k)$, $p_A(k)$ and $g(k)$ be the N-point DFT of the $Z(n)$, $P_A(n)$ and $G(n)$, respectively. Thus, we get

$$z(k) = P_A(k) g(k), \quad 0 \le k \le N-1 \tag{7}$$

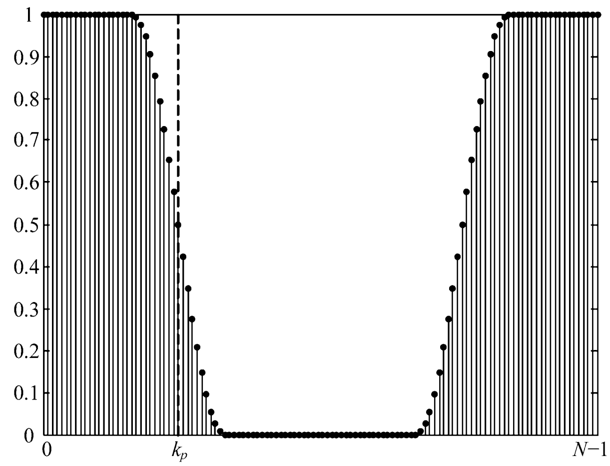

Fig. 1. Frequency spectrum of the raised cosine rolloff filter.

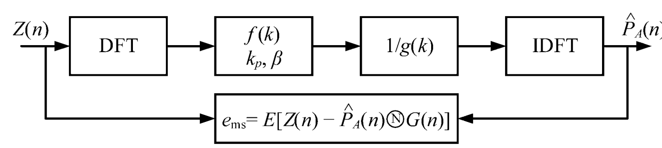

Fig. 2. Signal flow graph for estimating the PAS.

and

$$\hat{P}_A(n) = \mathrm{IDFT}[P_A(k)] = \mathrm{IDFT}\left[\frac{z(k)}{g(k)}\right] \tag{8}$$

where $\mathrm{IDFT}[\cdot]$ means inverse DFT operation.

In fact, deconvolution is an ill-conditioned problem [8]. We will keep the concept of "frequency spectrum" in $z(k)$, $p_A(k)$ and $g(k)$ in order to describe the ill-condition intuitively, although the $Z(n)$, $P_A(n)$ and $G(n)$ are not time defined sequences. Usually, $g(k)$ is lowpass so that $|1/g(k)|$ will tend to infinity in the high frequency area. It means that even tiny disturbances in the observation would lead to enormous changes in the estimated values.

Mostly, in the realistic multipath environments, the DFTs of the PASs have lowpass characteristic, so we can prefilter the $z(k)$ using a low-pass filter to overcome the ill-condition. The raised cosine rolloff filter is employed due to the simplicity and the suppressed tail after the IDFT. The N-point discrete raised cosine rolloff filter is expressed as shown in (9) at the bottom of the next page where $\beta \in [0, 1]$ is the rolloff factor, $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote the ceil and floor functions, respectively. The waveform of the $f(k)$ is shown in Fig. 1.

Fig. 2 shows the signal flow graph for estimating the PAS. The minimum mean square error (mmse) estimation of the PAS can be achieved by simultaneously varying k_p and β within their respective range until the mean square error e_{ms} is minimum.

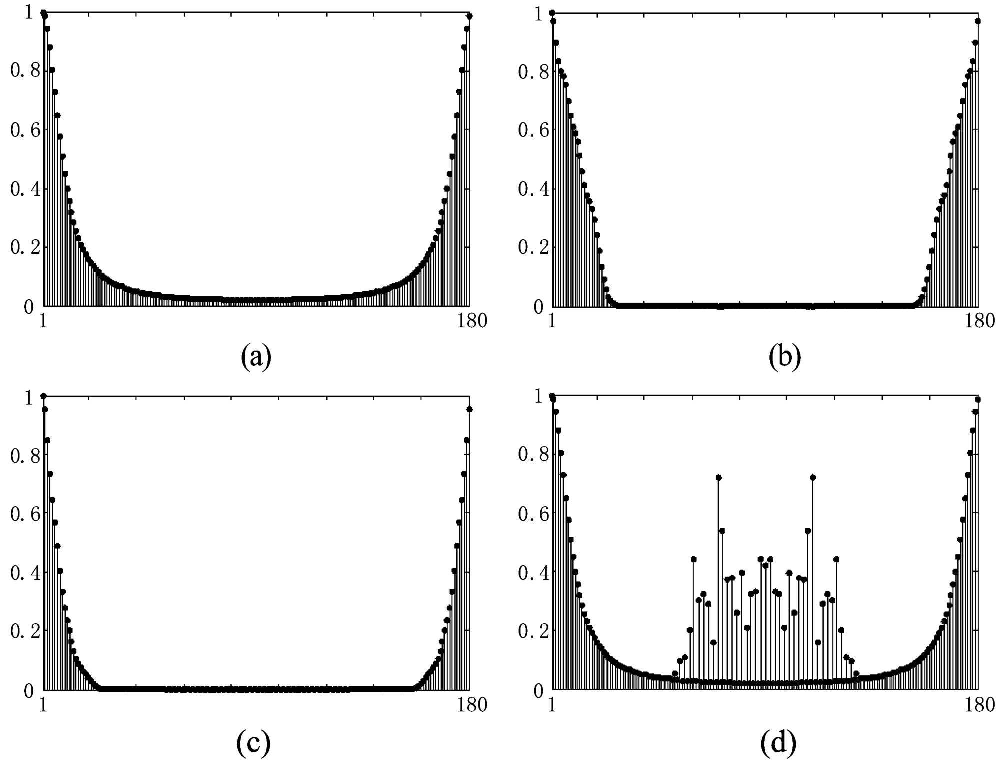

Fig. 3. Frequency spectrum of the sequences. (a) $|p_A(k)|$. (b) $|g(k)|$. (c) $|z(k)|$. (d) $|z(k)/g(k)|$.

V. Influence of the Antenna Directivity on Estimation

The antenna directivity is the key factor that influences the performance of the estimation. As shown in (7), the high frequency components of the $p_A(k)$ are suppressed by $g(k)$ during the measurement campaigns and can not be recovered exactly by an inverse operation in (8) due to the ill-condition. Fig. 3 illustrate the waveforms of the $z(k)$, $p_A(k)$, $g(k)$ and $z(k)/g(k)$ as an example. It is shown in Fig. 3(d) that there are irregular variations in the high frequency area of the $z(k)/g(k)$ due to the ill-condition.

It is obvious that the resolution of estimating the PAS is decided by the antenna directivity essentially. An antenna with strong directivity can keep more information of the PAS during the measurement campaigns, so that an accurate estimation can be obtained. The mmse estimations of a Laplacian PAS achieved by using three antennas with different directivity are shown in Fig. 4, comparing with the theoretical curve. The corresponding patterns are also illustrated. The theoretical PAS is Laplacian distributed with std 5° and the observation sequence $Z(n)$ is calculated using (5). As expected, the estimation corresponding to the antenna with the strongest directivity gives the best match for the theoretical curve.

VI. Conclusion

In this letter, a novel and simple method for estimating the PAS of the wireless channel is presented. The measurement equation is a Fredholm integral equation of the first kind when conducting power measurement campaigns with the directional antenna. The PAS can be estimated by deconvolution as the antenna pattern is known. The mmse estimation can be achieved by choosing the proper parameters of the raised cosine rolloff filter which is employed to overcome the ill-condition of the deconvolution. The directivity of the antenna influences the performance of the estimation essentially and the stronger directivity will lead to a more accurate result.

$$f(k) = \begin{cases} 1, & 0 \le k \le k_p - \lceil \beta k_p \rceil \frac{1}{2} + \frac{1}{2}\cos\left\{ \frac{\pi}{2\lceil \beta k_p \rceil} \left[k - (k_p - \lceil \beta k_p \rceil) \right] \right\} k_p - \lceil \beta k_p \rceil \le k \le k_p + \lceil \beta k_p \rceil \\ 0 & k_p \lceil \beta k_p \rceil < k \le \left\lfloor \frac{N-1}{2} \right\rfloor \\ f(N-1-k) & \left\lfloor \frac{N-1}{2} \right\rfloor < k \le N-1 \end{cases} \tag{9}$$

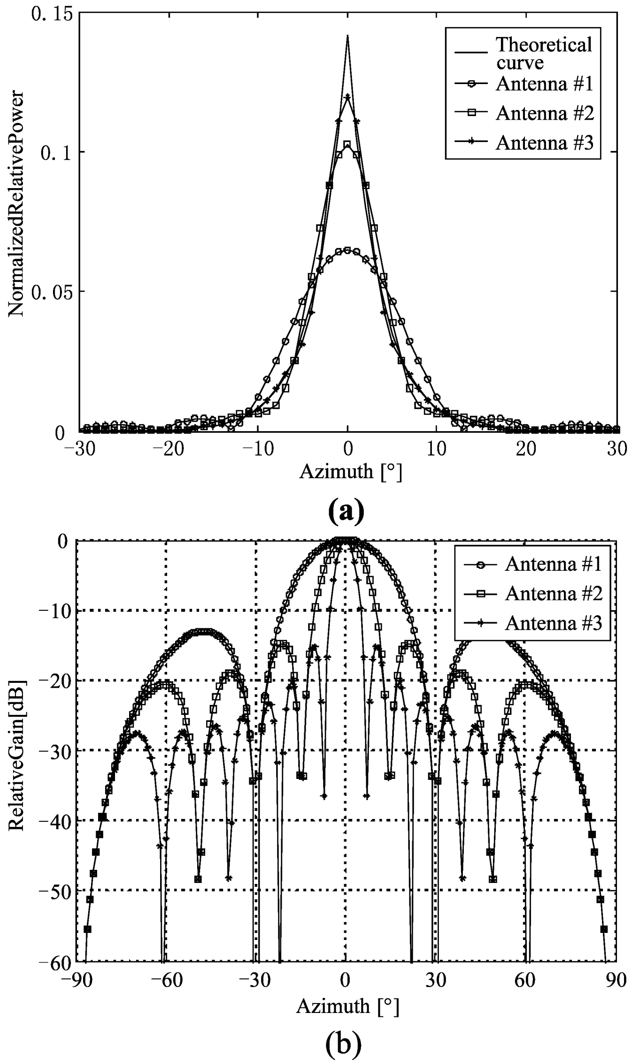

Fig. 4. (a) Estimations of the PAS achieved by using different antennas. (b) Corresponding patterns of the directional antennas.

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