A Generalized Blind Channel Estimation Algorithm for OFDM Systems with Cyclic Prefix

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Abstract—A subspace-based blind channel estimation algorithm with cyclic prefix is proposed in this paper. A systematic approach is used to construct a new signal matrix in the proposed algorithm. Compared with conventional blind channel estimation algorithm, the proposed algorithm has lower computational complexity and higher probability of full row rank for the corresponding signal matrix. In addition, fewer OFDM symbols can be used to satisfy the necessary condition for achieving a full-row-rank signal matrix. Simulation results show that the proposed algorithm outperforms conventional methods in mean-squared error and bit error rate under static channel. Even with a smaller number of received OFDM symbols, the proposed algorithm can perform well.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1] is a popular transmission technique in wireless communication systems due to its advantages of high spectrum efficiency and high transmitted data rate. In an OFDM system, it is robust to frequency selective fading because a wideband spectrum can be divided into many narrowband subchannels by subcarriers. The fading effects on each subcarrier can be regarded as frequency non-selective. In addition, a guard interval is usually added to OFDM symbols. If the guard interval is larger than the maximum channel delay, OFDM signal can avoid inter-symbol interference (ISI).

To obtain better performance in OFDM systems, coherent demodulation and accurate estimation of the channel impulse response (CIR) are required. Therefore, channel estimation is mandatory and important in OFDM systems. The channel estimation can be categorized into two kinds by training signals existing or not. Training-based channel estimation algorithms [2], [3] need extra pilot tones or pilot symbols. On the other side, blind channel estimation algorithms need not any additional training signal. The blind channel estimation algorithms can also be divided into non-subspace-based [4] or subspace-based [5], [6]. In this paper, we only focus on subspace-based blind channel estimation methods. Recently,

Su [6] introduced a new system parameter called repetition index to subspace-based method. We denote this method as "Su method". While the repetition index is chosen as large as possible, the required OFDM symbols may be reduced for achieving the same performance of conventional method. However, the Su method will suffer from high computational complexity and low probability of full row rank for the signal matrix. In this paper, a subspace-based blind channel estimation algorithm which will overcome these drawbacks is proposed. In addition, when the repetition index is equal to one, our proposed algorithm is a generalized version of the subspace-based blind channel estimation algorithm presented by Roy [5], which is denoted as "Roy method".

The rest of this paper is organized as follows. Section II introduces the OFDM signal model. In Section III, a generalized subspace-based blind channel estimation algorithm is proposed. Simulation results of the proposed algorithm and conclusions are given in Section IV and Section V, respectively.

Notations: $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ represent the operators of transpose, complex conjugate, and transpose-conjugate, respectively. If the vector $\mathbf{u} = [u_0, u_1, ..., u_{N-1}]^T$, the notation $\mathbf{u} \mid_{\alpha}^{\beta}$ means the $(\alpha - \beta + 1) \times 1$ vector $[u_{\beta}, u_{\beta+1}, ..., u_{\alpha}]^T$ for $0 \le \beta \le \alpha \le N-1$.

II. SIGNAL MODEL

A. CP-Based OFDM Signal Model

In this subsection, an OFDM signal model with multiple antennas is considered. The system model of an OFDM system with cyclic prefix for two receive antennas is shown in Fig. 1. The k^{th} transmitted OFDM data symbol in frequency domain can be expressed by

$$\mathbf{d}_{N}(k) = \left[d_{0}(k), d_{1}(k), \cdots, d_{N-1}(k) \right]^{T},$$
 (1)

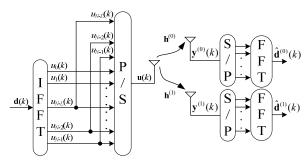


Figure 1. OFDM system model.

where $d_i(k)$ is selected from a complex modulated signal constellation with unit energy, i = 0, 1, ..., N-1 is the index of subcarrier with total N subcarriers. Then the transmitted data symbol passes through inverse discrete Fourier transform (IDFT) or inverse fast Fourier transform (IFFT) operator to obtain time-domain OFDM symbol $\mathbf{u}_N(k) = [u_0(k), u_1(k), ...,$ $u_{N-1}(k)$ ^T= $\mathbf{F}_N \cdot \mathbf{d}_N(\mathbf{k})$. The matrix \mathbf{F}_N denotes the IDFT matrix. The last L time-domain signals of $\mathbf{u}_N(k)$ are denoted as $\mathbf{u}_{CP}(k)$ = $[u_{N-L}(k), u_{N-L+1}(k), \dots, u_{N-1}(k)]^T$, and they will be duplicated and appended to the head of $\mathbf{u}_N(k)$ to be cyclic prefix (CP). L represents the length of cyclic prefix. Accordingly, the k^{th} transmitted OFDM symbol in time domain with cyclic prefix is parallel-to-serial converted and can be represented by a vector form

$$\mathbf{u}(k) = \left[\mathbf{u}_{CP}^{T}(k) \ \mathbf{u}_{N}^{T}(k)\right]^{T}$$
 (2)

with dimension $(N+L)\times 1$.

If there are M antennas at the receiver, the CIR with maximum channel length L at the m^{th} antenna is defined by

$$\mathbf{h}^{(m)} = [h_0^{(m)}, h_1^{(m)}, \dots, h_L^{(m)}]^T.$$
 (3)

Synchronization between the transmitter and the receiver is assumed to be performed perfectly before channel estimation. Consequently, the carrier frequency offset and timing offset are not considered in the following signal model equations. Now, we define a Toeplitz matrix with dimension $O\times(O+L)$

$$H_{Q}^{(m)} = \begin{bmatrix} h_{L}^{(m)} & h_{L-1}^{(m)} & \dots & h_{0}^{(m)} & 0 & \dots & 0 \\ 0 & h_{L}^{(m)} & h_{L-1}^{(m)} & \dots & h_{0}^{(m)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \dots & \ddots & 0 \\ 0 & \dots & 0 & h_{L}^{(m)} & h_{L-1}^{(m)} & \dots & h_{0}^{(m)} \end{bmatrix}, (4)$$

where Q is an arbitrary value. The received time-domain OFDM symbol corresponding to k^{th} transmitted signal at the mth antenna can be represented by

$$\mathbf{y}^{(m)}(k) = \mathbf{H}^{(m)}\mathbf{u}(k) + \mathbf{n}^{(m)}(k), \qquad (5)$$

where $\mathbf{H}^{(m)} = H_{N}^{(m)}$ is a Toeplitz matrix defined in Eq. (4) with Q=N and $\mathbf{n}^{(m)}(k)$ is an AWGN noise vector with dimension $N\times 1$. The k^{th} received time-domain symbol $\mathbf{y}_{ISI}^{(m)}(k)$ with dimension $L \times 1$ can be written as

$$\mathbf{y}_{ISI}^{(m)}(k) = \mathbf{H}_{LT}^{(m)} \mathbf{u}_{CP}(k) + \mathbf{H}_{UT}^{(m)} \mathbf{u}_{CP}(k-1) + \mathbf{n}_{ISI}^{(m)}(k), (6)$$

where

$$\mathbf{H}_{LT}^{(m)} = \begin{bmatrix} h_0^{(m)} & \mathbf{0} \\ \vdots & \ddots & \\ h_{L-1}^{(m)} & \cdots & h_0^{(m)} \end{bmatrix} \quad \text{and} \quad \mathbf{H}_{UT}^{(m)} = \begin{bmatrix} h_L^{(m)} & \cdots & h_1^{(m)} \\ & \ddots & \vdots \\ \mathbf{0} & & h_L^{(m)} \end{bmatrix}$$

are both $L \times L$ matrices as well as $\mathbf{n}_{ISI}^{(m)}(k)$ is the AWGN noise vector with dimension $L \times 1$.

PROPOSED ALGORITHM Ш

Channel Equations of the Proposed Algorithm

Our proposed algorithm is presented in this subsection and only ISI-free symbols will be considered here. A column vector $\overline{\mathbf{y}}_{i}^{(m)}(k)$, $0 \le j \le P-1$, is defined by

$$\overline{\mathbf{y}}_{i}^{(m)}(k) = \left[\mathbf{y}^{(m)}(k) \mid_{N-1-i}^{L-j}\right],\tag{7}$$

where the size of $\overline{\mathbf{y}}_{i}^{(m)}(k)$ is $(N-L)\times 1$. In the same way, another

 $N\times 1$ column vector $\overline{\mathbf{u}}_{i}(k)$, $0 \le j \le P-1$, can be defined by

$$\overline{\mathbf{u}}_{j}(k) = \left[\mathbf{u}(k) \mid_{N+L-1-j}^{L-j}\right]. \tag{8}$$

$$\overline{\mathbf{u}}_{j}(k) = [\mathbf{u}(k)|_{N+L-1-j}]. \tag{8}$$
Eq. (7) can also be written as
$$\overline{\mathbf{y}}_{j}^{(m)}(k) = \overline{\mathbf{H}}_{P}^{(m)} \overline{\mathbf{u}}_{j}(k), \tag{9}$$

where $\overline{\mathbf{H}}_{P}^{(m)} = H_{N-I}^{(m)}$ is a channel matrix with dimension (N-L)×N. Let P columns of $\bar{\mathbf{y}}_{i}^{(m)}(k)$ be combined to form an (N- $L)\times P$ matrix

$$\overline{\mathbf{Y}}_{P}^{(m)}(k) = [\overline{\mathbf{y}}_{0}^{(m)}(k) \ \overline{\mathbf{y}}_{1}^{(m)}(k) \cdots \overline{\mathbf{y}}_{P-1}^{(m)}(k)] = \mathbf{H}_{P}^{(m)} \overline{\mathbf{U}}_{P}(k) , (10)$$

where $\overline{\mathbf{U}}_{P}(k) = [\mathbf{u}_{0}(k) \mathbf{u}_{1}(k) \cdots \mathbf{u}_{P-1}(k)]$ is a signal matrix with dimension $N \times P$. $\overline{\mathbf{Y}}_{P}^{(m)}(k)$ including P columns can be obtained when only one OFDM symbol is received. We can observe the permutation approach of the proposed signal matrix $\overline{\mathbf{U}}_{P}$ is different from that of the signal matrix of Su method. If there are I OFDM symbols received, Eq. (10) also can be rewritten by

$$\overline{\mathbf{Y}}_{P,I}^{(m)} = \overline{\mathbf{H}}_{P}^{(m)} \overline{\mathbf{U}}_{P,I}, \tag{11}$$

$$\overline{\mathbf{Y}}_{P,I}^{(m)} = \left[\overline{\mathbf{Y}}_{P}^{(m)}(0) \ \overline{\mathbf{Y}}_{P}^{(m)}(1) \ \cdots \ \overline{\mathbf{Y}}_{P}^{(m)}(I-1) \right]$$

$$\overline{\mathbf{U}}_{P,I} = \left[\overline{\mathbf{U}}_P(0) \ \overline{\mathbf{U}}_P(1) \ \cdots \ \overline{\mathbf{U}}_P(I-1) \right].$$

If a diversity combining rule is applied to these M antennas, the combined signal can be expressed by

$$\overline{\mathbf{Y}}_{P,I} = \overline{\mathbf{H}}_P \overline{\mathbf{U}}_{P,I} \,, \tag{12}$$

where

$$\overline{\mathbf{Y}}_{P,I} = \left[\overline{\mathbf{Y}}_{P,I}^{(0)^T}, \overline{\mathbf{Y}}_{P,I}^{(1)^T}, \cdots, \overline{\mathbf{Y}}_{P,I}^{(M-1)^T}\right]^T$$

$$\overline{\mathbf{H}}_P = \left[\overline{\mathbf{H}}_P^{(0)^T}, \overline{\mathbf{H}}_P^{(1)^T}, \cdots, \overline{\mathbf{H}}_P^{(M-1)^T}\right]^T.$$

For example, if N=8, L=2, P=3, I=1, and M=2 are considered and then $\mathbf{u}(k)$ and $\mathbf{y}^{(m)}(k)$ can be written as $\mathbf{u}(k)$ = $[u_6, u_7, u_0, ..., u_7]^T$ and $\mathbf{y}^{(m)}(k) = [y_0^{(m)}, ..., y_7^{(m)}]^T$, respectively. Therefore, Eq. (12) becomes

$$\begin{bmatrix} y_2^{(0)} & y_1^{(0)} & y_0^{(0)} \\ y_3^{(0)} & y_2^{(0)} & y_1^{(0)} \\ y_5^{(0)} & y_3^{(0)} & y_2^{(0)} \\ y_6^{(0)} & y_5^{(0)} & y_4^{(0)} \\ y_2^{(1)} & y_1^{(1)} & y_0^{(1)} \\ y_3^{(1)} & y_2^{(1)} & y_1^{(1)} \\ y_3^{(1)} & y_2^{(1)} & y_1^{(1)} \\ y_5^{(1)} & y_4^{(1)} & y_3^{(1)} & y_2^{(1)} \\ y_5^{(1)} & y_4^{(1)} & y_3^{(1)} & y_4^{(1)} \\ y_5^{(1)} & y_4^{(1)} & y_3^{(1)} & y_4^{(1)} \\ y_5^{(1)} & y_4^{(1)} & y_3^{(1)} & y_4^{(1)} \\ y_5^{(1)} & y_5^{(1)} & y_4^{(1)} & y_3^{(1)} \\ y_5^{(1)} & y_5^{(1)} & y_4^{(1)} & y_3^{(1)} \\ y_5^{(1)} & y_5^{(1)} & y_5^{(1)} & y_4^{(1)} \\ y_5^{(1)} & y_5^{(1)} & y_5^{(1)} & y_4^{(1)} \\ y_5^{(1)} & y_5^{(1)} & y_5^{(1)} & y_5^{(1)} \\ y_5^{(1)} & y_$$

Since our signal model is a generalized version of Roy method, the channel matrix $\overline{\mathbf{H}}_P$ equals that of Roy method. Therefore, the channel matrix $\overline{\mathbf{H}}_P$ is also full column rank [5].

We can get the annihilators of $\overline{\mathbf{Y}}_{P,I}$ by taking SVD of $\overline{\mathbf{Y}}_{P,I}$. Assume $\overline{\mathbf{W}}_{n}(i)$ (i=0, 1, ..., M[N-L]-N-1) are annihilators, and then the following equation will hold

$$\overline{\mathbf{W}}_{n}(i)^{H} \overline{\mathbf{Y}}_{PI} = \mathbf{0}^{T}. \tag{13}$$

On the other hand, the annihilators of $\overline{\mathbf{Y}}_{P,I}$ will also be the annihilators of $\overline{\mathbf{H}}_P$ only if the signal matrix $\overline{\mathbf{U}}_{P,I}$ is full row rank. Assume $\overline{\mathbf{W}}_n(i)$ is the annihilator of $\overline{\mathbf{H}}_P$, we can get

$$\overline{\mathbf{W}}_{n}(i)^{H} \overline{\mathbf{H}}_{P} = \mathbf{0}^{T}. \tag{14}$$

Before the following discussions, $\overline{\mathbf{h}}^{(m)} = \left[h_L^{(m)}, h_{L-1}^{(m)}, \cdots, h_0^{(m)}\right]^T$ and $\overline{\mathbf{h}} = \left[\mathbf{h}^{(0)}, \mathbf{h}^{(1)}, \cdots, \mathbf{h}^{(M-1)}\right]^T$ are defined, and then Eq. (14) can be rewritten as

$$\overline{\mathbf{W}}_{n}(i)^{T}\overline{\mathbf{H}}_{P} = \overline{\mathbf{h}}^{T}W_{i}. \tag{15}$$

Suppose the vector $\overline{\mathbf{W}}_n(i)$ is $[w_{i,0}, w_{i,1}, \cdots, w_{i,M(N-L)-1}]^T$, and thus the elements of the $M(L+1) \times N$ matrix W_i will come from $\overline{\mathbf{W}}_n(i)$. Now, if we define another matrix \mathbf{W} with dimension $M(L+1) \times N [M(N-L)-N]$

$$\mathbf{W} = \left[W_0, W_1, \dots, W_{M(N-L)-N-1} \right]^*, \tag{16}$$

 $\overline{\mathbf{h}}^T\mathbf{W} = \mathbf{0}$ can be obtained. If the noise is present, we can get the estimated annihilator $\hat{\overline{\mathbf{W}}}_n(i)$ by taking SVD on $\overline{\mathbf{V}}_{P,J}$. These estimated annihilators $\hat{\overline{\mathbf{W}}}_n(i)$ are from the M(N-L)-N singular vectors associated with the M(N-L)-N smallest singular values. Thereupon, the CIR can be estimated by

$$\hat{\overline{\mathbf{h}}} = \underset{\|\overline{\mathbf{h}} = 1\|}{\arg\min} \sum_{i=0}^{M(N-L)-N-1} \left\| \hat{\overline{\mathbf{W}}}_{n}(i)^{H} \overline{\mathbf{H}}_{P} \right\|^{2}, \quad (17)$$

where

$$\left\| \widehat{\overline{\mathbf{W}}}_{n}(i)^{H} \overline{\mathbf{H}}_{p} \right\|^{2} = \widehat{\overline{\mathbf{W}}}_{n}(i)^{H} \overline{\mathbf{H}}_{p} \overline{\mathbf{H}}_{p}^{H} \widehat{\overline{\mathbf{W}}}_{n}(i) = \overline{\mathbf{h}}^{T} (\widehat{W}_{i})^{*} \widehat{W}_{i}^{T} (\overline{\mathbf{h}})^{*}.$$

If we define $\tilde{\mathbf{h}} = (\overline{\mathbf{h}})^*$, the channel estimator is rewritten by

$$\hat{\widetilde{\mathbf{h}}} = \arg\min_{\|\widetilde{\mathbf{h}} = 1\|} (\widetilde{\mathbf{h}}^H \hat{W} \hat{W}^H \widetilde{\mathbf{h}}). \tag{18}$$

B. The Range of Repetition Index P

In the proposed algorithm, the repetition index P can not be chosen too large. In our observation, the value of P in our signal must satisfy the condition

$$P \le L + 1. \tag{18}$$

Therefore, the maximum value of repetition index P is L+1. In other words, P value can be enlarged by selecting longer length of cyclic prefix.

C. Necessary Condition for the Proposed Algorithm

The proposed algorithm will work only if $\overline{\mathbf{U}}_{p,j}$ is full row rank. Therefore, we have to choose the number of column not smaller than the number of row. Because the dimension of $\overline{\mathbf{U}}_{p,j}$ is $N \times PI$, the following equations should hold

$$N \le PI \tag{19}$$

or

$$P \ge N/I \ . \tag{20}$$

Then the minimum value of P is $P = \lceil N/I \rceil$. On the other side, the necessary condition for Su method [6] is

$$P \ge (2N-1)/(I-2)$$
. (21)

Therefore, we can declare that the proposed algorithm requires fewer OFDM symbols (I) to satisfy the necessary condition under the same N and P.

D. Discussions

The computational complexity of the proposed algorithm is based on SVD of the matrix $\overline{\mathbf{Y}}_{P,I}$ whose dimension is $M(N-L)\times PI$, and thus the complexity is proportional to $O((M^*(N-L))^3)$. Because the complexity of Su method is proportional to $O((2N+P+L-1)^3)$ [6], the complexity will increase with large P and L. Consequently, we can declare that if M is not larger than two, the complexity of the proposed algorithm will be lower than that of Su method.

In addition to the low complexity, high probability of full row rank is also provided by the proposed algorithm. That is, our proposed algorithm can work properly with BPSK modulation constellation and smaller number of received OFDM symbols. It will be shown in the simulation results in the next Section.

E. Equalization and Phase Ambiguity Problem

Although we obtain the estimated channel coefficients by the proposed algorithm, there is still a phase ambiguity between the true channel and the estimated channel. In this paper, one additional pilot symbol is used to solve the phase ambiguity problem. After solving this problem, zero-forcing (ZF) method and equal gain combining (EGC) method are both performed to demodulate the received data symbols.

IV. SIMULATION RESULTS

In order to validate the performance of the proposed algorithm, Monte Carlo simulations are carried out in this Section. Perfect timing and frequency synchronization are assumed to be performed before channel estimation and the following system parameters and specifications are considered:

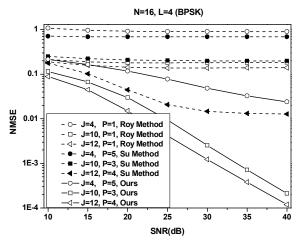


Figure 2. Normalized MSE comparison with BPSK constellation.

1) BPSK is used for the OFDM data symbols; 2) the number of subcarriers N is 16; 3) the length of the guard interval is L=4; 4) two receive antennas are applied; 5) static channel environment is considered and the channel coefficients are defined as follows [5]

$$\mathbf{h}^{(1)} = [0.3825 + 0.0010j, 0.5117 + 0.2478j, -0.3621 + 0.3320j, -0.4106 + 0.3428j, 0.0087 + 0.0546j]^T$$

$$\mathbf{h}^{(2)} = [-0.2328 + 0.1332j, -0.3780 - 0.3794j, -0.0320 - 0.4532j, \\ 0.5081 - 0.0125j, 0.4195 + 0.0220j]^T$$

The normalized mean-squared-error (NMSE) performance is shown in Fig. 2. The NMSE is defined by

$$NMSE = \left(1/\|\mathbf{h}\|\right) \left(1/N_r\right)_{q=1}^{N_r} \|c\hat{\mathbf{h}}_q - \mathbf{h}\|^2\right], \qquad (22)$$

where N_r denotes the number of computer runs, the index qrepresents the q^{th} Monte Carlo run, and the parameter c means the phase ambiguity which can be solved by one extra pilot symbol. Moreover, $\hat{\mathbf{h}}_{a}$ is the q^{th} estimated channel with a phase ambiguity, and h is the true channel. Since the received data symbols of Roy method will suffer from ISI, we only focus on NMSE performance of Roy method. For Su method and Roy method, the NMSE's are almost constant at any signal-to-noise (SNR) when the number of received OFDM symbols (I) is 4, or 10. While I is increasing up to 12, the NMSE of Su method becomes an decreasing function of SNR. However, the NMSE of Roy method is still constant at any SNR. The NMSE's of our proposed algorithm are all decreasing functions of SNR with the number of received OFDM symbols larger than 3 and they are always smaller than that of Su method and Roy method. It is obvious that the proposed algorithm is better than Su method and Roy method in NMSE performance.

In Fig. 3, bit error rate (BER) performance comparison is depicted. Similarly to the NMSE performance, the BER performance of Su method is poor when I is 4, or 10. The major reason is the necessary condition for the Su method is not satisfied. However, our proposed algorithm can work well both in these cases. When I is 12, the proposed algorithm and

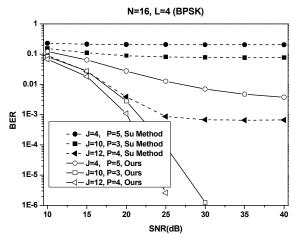


Figure 3. BER performance comparison with BPSK constellation.

Su method both meet the necessary condition, but the Su method still does not work well because the signal matrix of the Su method has low probability of full row rank. On the other hand, the proposed algorithm works very well in these cases, and then we can declare that the proposed algorithm has high probability of full row rank compared with the Su method. The BER performance of our proposed algorithm is always better than that of Su method in any SNR with BPSK modulation.

V. CONCLUSIONS

A generalized subspace-based blind channel estimation algorithm for OFDM systems is proposed in this paper. The necessary condition for the proposed algorithm and the range of the repetition index are also derived. With the aid of repetition index, the proposed algorithm can perform well even with a smaller number of received OFDM symbols. From simulation results, the proposed algorithm not only outperforms the Su method and Roy method under static channel, but also is contained with lower computational complexity and higher probability of full row rank. Finally, we believe that our proposed algorithm is also better than Su method and Roy method under time-varying channels.

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