An Improved Kalman Filter Algorithm for UWB Channel Estimation

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***Abstract*—This paper deals with channel estimation for ultra-wideband (UWB) wireless communication systems in dense multipath environments. An improved kalman filter algorithm is proposed for estimating and tracking the channel impulse response (CIR) of time-varying channels. This algorithm includes two steps: frequency-domain pre-filter and kalman filter. By introducing frequency-domain pre-filter to simplify kalman measurement equation before traditional kalman filter, the kalman measurement update equations which include the solutions of measurement update, kalman gain and error covariance matrix will be represented as more concise format. As a result, the kalman iterative equations can be simplified so that the computational complexity is reduced. Simulations and comparisons are implemented to illustrate the promising performance of the proposed algorithm.**

Keywords - UWB; channel estimation; frequency-domain pre-filter; kalman filter

I. [1]INTRODUCTION

As a promising candidate for future short-range high-speed wireless communication, ultra-wide bandwidth (UWB) technology has attracted growing interests for the last decade. [1]-[4] provided an overview on this signaling format and in depth discussions on its potential to countering multiple-access interference and multipath propagation. Among the existing receiver structures for UWB systems, adaptive equalization and diversity combining (RAKE) techniques have been extensively applied in improving the receiver performance [5]-[7], and the knowledge of fading coefficients and delays for different channel paths are required to realize these techniques, in other words, accurate channel estimation is important.

There has been some work on channel estimation: In [8], maximum-likelihood (ML) algorithm was developed in the presence of multiple-access interference, both data aided and non-data aided estimation were discussed to find the delay and fading coefficients of the multipath components. Based on the least square rule, the joint channel estimation and synchronization method is developed in [9], nevertheless, the two-dimensional searching process and the high sampling rate would bring a burdensome complexity. [10] proposed a blind channel estimation algorithm for UWB systems that employ pulse-position modulation, which exploited the first-order cyclostationarity in the received signal and had low complexity, not surprisingly, the receiver performance wasn't satisfactory. A subspace approach to blind estimation of UWB channels was derived for multi-user receivers in [11], however, the increment of computational complexity didn't bring a significant improvement on system performance.

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Besides, the most important problem of all the algorithms mentioned above is that they all modeled the channel parameters as quasi-static, in other words, the CIR was assumed to be time-invariant in the duration needed for channel estimation, which was unrealistic in the real short-range high-speed wireless environment.

In this paper, we model the channel parameters as time-varying and develop an improved kalman filter algorithm based on [12] to estimate the CIR. By introducing the frequency-domain pre-filter to simplify measurement equation before traditional kalman filter, the kalman measurement update equations which include the solutions of measurement update, kalman gain and error covariance matrix will be represented as more concise format. As a result, the kalman iterative equations can be simplified so that the computational complexity is reduced. Perfect synchronization is assumed to be accomplished at the receiver.

The rest of this paper is organized as follows: Section II introduces the UWB system and channel modeling. Section III proposes the improved kalman filter algorithm. Simulations and related performance analysis are provided in section IV. At the end of this paper, summarizing remark is given.

Notation: $\lceil \cdot \rceil$ denotes top operation, $\otimes$ is circle convolution. Boldface types $\mathbf{a}$ and $\mathbf{A}$ represent a vector or matrix. $(\ )^T$ and $(\ )^H$ denotes transpose and conjugate transpose of a vector or matrix, respectively. $(\ )^{-1}$ stands for the inverse operator. $\mathbf{F}$ represents the fast Fourier matrix and $\mathbf{F}^{-1}$ is the inverse fast Fourier matrix.

II. UWB SYSTEM AND CHANNEL MODELING

A time hopping (TH) UWB system with pulse amplitude modulation (PAM) is considered in this paper. The transmitted signal is expressed as

$$s(t) = \sum_{k=-\infty}^{+\infty} a_k g_T(t - kT_b), \tag{1}$$

where a_k denotes the k-th information symbol taking value ± 1 with equal probability, T_b is symbol duration, g_T represents the transmitted symbol waveform with the normalized energy and is expressed as

$$g_T(t) = \sum_{j=0}^{N_f-1} g(t - jT_f - c_j T_c), \tag{2}$$

where $g(t)$ represents the elementary pulse (referred to as monocycle) with duration T_p, N_f denotes the number of frames per symbol, T_f is the frame period which satisfies $T_b = N_f T_f$, T_c stands for the chip period and $\{c_j\}_{j=0}^{N_f-1}$ is the time-hopping sequence whose elements are integer values randomly chosen in the range $0 \le c_j \le N_h - 1$.

A typical model of multipath fading channels in UWB wireless communication systems can be expressed as

$$h(t) = \sum_{l=0}^{L-1} \gamma_l \delta(t - \tau_l), \tag{3}$$

where $\{\gamma_l\}_{l=0}^{L-1}$ and $\{\tau_l\}_{l=0}^{L-1}$ denote channel fading coefficients and delays along different paths, respectively. L is the total number of channel taps. The channel fading coefficients and path delays are independent of each other. To separate the multipath spreading effect from the first delay τ_0, the *l-th* relative path delay is defined as: $\tau_{l,0} = \tau_l - \tau_0$, with $\tau_{0,0} = 0$, $\tau_{l,0} < \tau_{l+1,0}$, $\forall l$. In low-duty-cycle UWB systems, the frame duration satisfies $T_f \ge \tau_{L-1,0} + T_p + (N_h - 1)T_c$ to avoid ISI and IFI.

Assuming perfect synchronization at the receiver end and the k th observation with symbol-long can be represented as

$$r_k(t) = a_k \sum_{l=0}^{L-1} \gamma_l g_T(t - \tau_l) + z_k(t), \quad t \in [0, T_b), \tag{4}$$

where $z_k(t)$ denotes the zero-mean additive white Gaussian noise (AWGN). To estimate $h(t)$, we transmit a special training sequence, i.e., $a_k \equiv 1$. (4) can be rewritten as

$$r_k(t) = \sum_{l=0}^{L-1} \gamma_l g_T(t - \tau_l) + z_k(t), \quad t \in [0, T_b), \tag{5}$$

Let $T_s = T_p / 2$ be the Nyquist sampling interval, $N = \lceil T_b / T_s \rceil$ is the number of samples within one symbol. Then, the discrete form of $r_k(t)$ can be expressed as

$$r_k(n) = g_T(n) \otimes h(n) + z_k(n), \quad n \in [0, N-1], \tag{6}$$

where $g_T(n)$ and $z_k(n)$ are the discrete form of $g_T(t)$ and $z_k(t)$, respectively. $h(n)$ denotes the equivalent discrete impulse response of $h(t)$. Combining N samples $r_k(n)$ into a column vector $\mathbf{r}_k = [r_k(0) \cdots\cdots r_k(N-1)]^T$, the matrix form of (6) can be expressed as

$$\mathbf{r}_k = \mathbf{X}\mathbf{h} + \mathbf{z}_k, \tag{7}$$

where $\mathbf{X}$, $\mathbf{h}$ and $\mathbf{z}_k$ are defined as

$$\mathbf{X} = \begin{bmatrix} g_T(0) & g_T(N\text{-}1) & \cdots & g_T(1) \\ g_T(1) & g_T(0) & \cdots & g_T(2) \\ \cdots & \cdots & \cdots & \cdots \\ g_T(N\text{-}1) & g_T(N\text{-}2) & \cdots & g_T(0) \end{bmatrix},$$

$$\mathbf{h} = [h(0) \cdots\cdots h(N\text{-}1)]^T,$$

$$\mathbf{z}_k = [z_k(0) \cdots\cdots z_k(N\text{-}1)]^T,$$

and the covariance matrix of $\mathbf{z}_k$ is $\mathbf{C}_{\mathbf{z}_k}$.

In practical UWB systems, the approximation of CIR by constants during channel estimation has proven to be unsatisfactory, especially when the estimation needs a relative long time to finish. So the more realistic model for UWB channel is that the CIR is considered as random process slowly changing from one symbol to another. Thus, by describing the CIR as first-order auto regressive (AR) model, the relationship of CIR between the k th symbol duration and the k-1 th symbol duration can be expressed as [12]:

$$\mathbf{h}_k = \mathbf{h}_{k-1} + \mathbf{w}_{k-1}, \tag{8}$$

where $\mathbf{w}_{k-1} = [w_{k-1}(0) \cdots\cdots w_{k-1}(N-1)]^T$ is known as WGN, stands for process noise, and is independent of the CIR. The covariance matrix of $\mathbf{w}_{k-1}$ is $\mathbf{C}_{\mathbf{w}_{k-1}}$ which reflects the changes between $\mathbf{h}_k$ and $\mathbf{h}_{k-1}$. As a result, (7) can be rewritten as

$$\mathbf{r}_k = \mathbf{X}\mathbf{h}_k + \mathbf{z}_k. \tag{9}$$

Taking (8) as *state equation* and (9) as *measurement equation*, the kalman filter is introduced in [12] to estimate the CIR. Based on [12], we added an frequency-domain pre-filter before traditional kalman filter to estimate the CIR with lower computational complexity and higher performance.

III. IMPROVED KALMAN FILTER ALGORITHM FOR CHANNNEL ESTIMATION

Firstly, the traditional kalman iterative equations for UWB channel estimation introduced by [12] are listed below.

The one-step prediction equations:

$$\hat{\mathbf{h}}_{k|k-1} = \hat{\mathbf{h}}_{k-1}, \tag{10}$$

$$\mathbf{M}_{k|k-1} = \mathbf{M}_{k-1} + \mathbf{C}_{\mathbf{w}_{k-1}}, \tag{11}$$

The measurements update equations:

$$\hat{\mathbf{h}}_k = \hat{\mathbf{h}}_{k-1} + \mathbf{K}_k\left(\mathbf{r}_k - \mathbf{X}\hat{\mathbf{h}}_{k|k-1}\right), \tag{12}$$

$$\mathbf{K}_k = \mathbf{M}_{k|k-1}\mathbf{X}^H\left(\mathbf{X}\mathbf{M}_{k|k-1}\mathbf{X}^H + \mathbf{C}_{\mathbf{z}_k}\right)^{-1}, \tag{13}$$

$$\mathbf{M}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{X})\mathbf{M}_{k|k-1}. \tag{14}$$

where subscript k means in the k th symbol duration and $k\,|\,k-1$ denotes the one-step prediction from k-1 th symbol to k th symbol. $\mathbf{h}$ is the state vector and $\hat{\mathbf{h}}$ is its prediction. $\mathbf{M}$ stands for the $N\times N$ error covariance matrix of $\hat{\mathbf{h}}$, and $\mathbf{K}$ represents the $N\times N$ kalman gain matrix. Once the initial values $\hat{\mathbf{h}}_0$, $\mathbf{M}_0$ and $\mathbf{C}_{\mathbf{w}_k}$ are given according to [13], this iterative process continues until k is equal to the length of training sequence and the $\hat{\mathbf{h}}_k$ from last iteration is the final estimation of CIR. More details about kalman filter can refer to [13].

Now let's focus on the measurements update equations (12)-(14), the matrix $\mathbf{X}$ appears 5 times in the solutions of $\hat{\mathbf{h}}_k$, $\mathbf{K}_k$ and $\mathbf{M}_k$ for each iteration, which means that if we are going to carry out L iterations, then we have to calculate the $N\times N$ matrix multiplication for $5L$ times. Obviously, this will bring a burdensome complexity during the whole process of iterations, yet no works have been done on how to improve it by eliminating the matrix $\mathbf{X}$ in (12)-(14), and that's the goal of our efforts in this paper.

As is discussed in [13], the computational complexity of kalman measurements update equations (12)-(14) depends on measurement equation (9). It means, if we could eliminate the matrix $\mathbf{X}$ in (9) and directly describe the received samples as the sum of the CIR and noise, then, equations (12)-(14) will be represented as more concise format without $\mathbf{X}$. Consequently, the computational complexity is reduced and the efficiency of kalman algorithm is improved.

To simplify the measurement equation (9), the frequency-domain pre-filter is proposed by us.

According to (6), the discrete Fourier transform (DFT) of $r_k(n)$ can be computed as

$$R_k(m) = G(m)H_k(m) + Z_k(m), \quad m \in [0, N-1], \tag{15}$$

where $R_k(m)$ is the DFT coefficients of $r_k(n)$. $G(m)$, $H_k(m)$ and $Z_k(m)$ denote the DFT coefficients of $g_T(n)$, $h_k(n)$ and $z_k(n)$, respectively. Combining N samples $R_k(m)$ into a column vector $\mathbf{R}_k = [R_k(0) \cdots\cdots R_k(N\text{-}1)]^T$, the matrix form of (15) can then be expressed as

$$\mathbf{R}_k = \mathbf{G}\mathbf{H}_k + \mathbf{Z}_k, \tag{16}$$

where $\mathbf{G}$, $\mathbf{H}_k$, and $\mathbf{Z}_k$ are defined as

$$\mathbf{G} = \begin{bmatrix} G(0) & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & G(N-1) \end{bmatrix},$$

$$\mathbf{H}_k = [H_k(0) \cdots\cdots H_k(N-1)]^T,$$

$$\mathbf{Z}_k = [Z_k(0) \cdots\cdots Z_k(N-1)]^T.$$

The entries along the principal diagonal of $\mathbf{G}$ are nonzero by choosing proper transmitted waveform, the inverse matrix $\mathbf{G}^{-1}$ of $\mathbf{G}$ is then easily computed due to its diagonal structure. The two ends of (16) are left-multiplied by $\mathbf{G}^{-1}$ respectively, we have

$$\mathbf{Y}_k = \mathbf{G}^{-1}\mathbf{R}_k = \mathbf{H}_k + \mathbf{V}_k, \tag{17}$$

where $\mathbf{V}_k = \mathbf{G}^{-1}\mathbf{Z}_k$.

By doing this, the measurement equation has been successfully described as the sum of CIR and noise in frequency-domain. To get the time-domain expression of this equation, the inverse discrete Fourier transform (IDFT) of $\mathbf{Y}_k$ is done as

$$\mathbf{y}_k = \mathbf{F}^{-1}\mathbf{Y}_k = \mathbf{h}_k + \mathbf{v}_k, \tag{18}$$

where $\mathbf{v}_k = \mathbf{F}^{-1}\mathbf{V}_k = \mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k$.

Because of the foundational restriction of kalman filter, we need to check whether $\mathbf{v}_k$ is a WGN vector for different k. In fact, we have

$$E(\mathbf{v}_k) = E(\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k) = \mathbf{F}^{-1}\mathbf{G}^{-1}E(\mathbf{Z}_k) = \mathbf{0}_{N\times 1},$$

$$\begin{aligned} E(\mathbf{v}_k\mathbf{v}_{k+1}^H) &= E\left[(\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k)(\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_{k+1})^H\right] \\ &= E\left[\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k\mathbf{Z}_{k+1}^H(\mathbf{G}^{-1})^H(\mathbf{F}^{-1})^H\right] \\ &= \mathbf{F}^{-1}\mathbf{G}^{-1}E(\mathbf{Z}_k\mathbf{Z}_{k+1}^H)(\mathbf{G}^{-1})^H(\mathbf{F}^{-1})^H \\ &= \mathbf{0}_{N\times N} \end{aligned}.$$

So $\mathbf{v}_k$ for different k is proved to be a WGN vector with covariance matrix

$$\begin{aligned} \mathbf{C}_{\mathbf{v}_k} &= E(\mathbf{v}_k\mathbf{v}_k^H) = E\left[(\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k)(\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k)^H\right] \\ &= E\left[\mathbf{F}^{-1}\mathbf{G}^{-1}\mathbf{Z}_k\mathbf{Z}_{k+1}^H(\mathbf{G}^{-1})^H(\mathbf{F}^{-1})^H\right] \\ &= \mathbf{F}^{-1}\mathbf{G}^{-1}E(\mathbf{Z}_k\mathbf{Z}_{k+1}^H)(\mathbf{G}^{-1})^H(\mathbf{F}^{-1})^H \\ &= (N_0/2)\mathbf{F}^{-1}\mathbf{G}^{-1}(\mathbf{F}^{-1}\mathbf{G}^{-1})^H \end{aligned}, \quad (19)$$

where $N_0/2$ is the power spectral density of $z_k(t)$. Therefore, (18) can be regarded as our new measurement equation of kalman filter.

One point must be made that the white noise $\mathbf{Z}_k$ is band-limited in (17) by $\mathbf{G}^{-1}$ and therefore has the same bandwidth as the transmitted waveform, which shows the potential to improve the receiver SNR.

Now we look back on the derivation of the new measurement equation, the inverse matrix $\mathbf{G}^{-1}$ plays an important role in simplifying the original measurement equation (9) by eliminating the matrix $\mathbf{G}$ in frequency-domain. This procedure is called frequency-domain pre-filter and the diagonal matrix $\mathbf{G}^{-1}$ is nominated as pre-filter matrix. Taking (18) as *new measurement equation* and (8) still *state equation*, the new kalman measurements update equations after frequency-domain pre-filter are listed as follows:

The new measurements update equations:

$$\hat{\mathbf{h}}_k = \hat{\mathbf{h}}_{k-1} + \mathbf{K}_k(\mathbf{y}_k - \hat{\mathbf{h}}_{k|k-1}), \quad (20)$$

$$\mathbf{K}_k = \mathbf{M}_{k|k-1}(\mathbf{M}_{k|k-1} + \mathbf{C}_{\mathbf{v}_k})^{-1}, \quad (21)$$

$$\mathbf{M}_k = (\mathbf{I} - \mathbf{K}_k)\mathbf{M}_{k|k-1}. \quad (22)$$

Compared with traditional kalman measurement update equations (12)-(14), the new measurement update equations (20)-(22) which include the iterative solutions $\hat{\mathbf{h}}_k$, $\mathbf{K}_k$ and $\mathbf{M}_k$ have been successfully represented as more concise format after frequency-domain pre-filtering. Of course, the process of frequency-domain pre-filter can also bring some additional computation which includes DFT, left-multiplication of $\mathbf{G}^{-1}$ and IDFT. Since DFT and IDFT can be replaced by FFT and IFFT, and $\mathbf{G}^{-1}$ is an diagonal matrix, the additional computation brought by frequency-domain pre-filter is far less than what it reduced. So, the introduction of frequency-domain pre-filter dramatically reduces the computational complexity and improves the efficiency of kalman algorithm.

In summary, the proposed algorithm can be carried out as follows:

step1. Give the length of training sequence denoted by M, initialize $\hat{\mathbf{h}}_0$, $\mathbf{M}_0$ and $\mathbf{C}_{w_k}$ ($0 \le k \le M-1$), let $k=1$.

step2. Sample the new received k th symbol and get $\mathbf{r}_k$.

step3. Perform the frequency-domain pre-filtering to $\mathbf{r}_k$ according to (15)-(18) and get $\mathbf{y}_k$.

step4. Take $\mathbf{y}_k$ into new kalman iterative equations (10)-(11), (20)-(22) and get $\hat{\mathbf{h}}_k$, $\mathbf{M}_k$.

step5. When $k < M$, let $k = k+1$, return to step 2; else, go to Step 6.

step6. $\hat{\mathbf{h}}_M$ is the final estimation for CIR of our algorithm.

IV. SIMULATION AND PERFORMANCE ANALYSIS

In this section, primary simulations and comparisons are presented to validate the proposed algorithm. We employ the elementary pulse $g(t)$ as the second derivative of the Gaussian function, i.e.

$$g(t) = [1 - 4\pi(t/t_m)^2]\exp[-2\pi(t/t_m)^2].$$

which is normalized to have unit energy and with the shaping parameter $t_m = 0.2877ns$ and duration $T_p = 1ns$. Each information-bearing symbol consists of $N_f = 12$ frames, each with duration $T_f = 50ns$ .TH codes of period N_f are employed, which are generated from a uniform distribution over $[0, N_h - 1]$ with $N_h = 10$. $T_c = 2ns$ is chosen as the chip duration.

Firstly, comparisons are made between our proposed algorithm and traditional kalman filter algorithm when giving the length of training sequence $M = 100$. As shown in Fig. 1, the MSE curve of the improved approach is about 3dB lower than the MSE curve of traditional kalman filter algorithm. The improved BER performance by employing our channel estimation scheme is also illustrated in Fig. 2. With SNR increases, the BER of our algorithm decreases more quickly compared with the BER of the traditional one. The comparisons suggest that the improved kalman filter algorithm is more beneficial to the overall system performance than traditional kalman filter algorithm under the same conditions.

Secondly, the proposed algorithm is tested for various M values. As shown in Fig. 3, the BER versus SNR for four M values: 20, 50, 100 and 200 is plotted. We observe that the BER curves decrease monotonically with increasing SNR, as well as increasing M values. It can be obviously concluded that the better performance can be achieved by choosing a larger M .

V. CONCLUSIONS

In this paper, an improved kalman filter algorithm is proposed for UWB channel estimation. By introducing the frequency-domain pre-filter to simplify the measurement equation before traditional kalman filter, our algorithm proved to be less computation complexity and better performance. It can be obviously concluded that the proposed algorithm provides an effective and desirable strategy for UWB channel estimation over slow time-varying multipath fading channels.

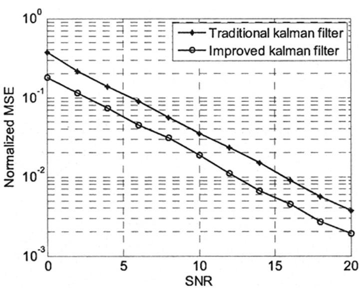

Figure 1. Normalized MSE versus SNR.

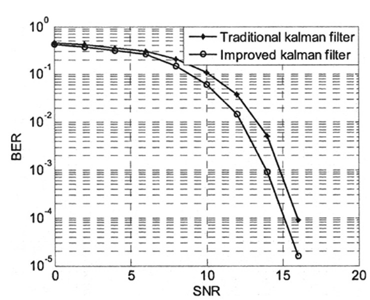

Figure 2. BER versus SNR.

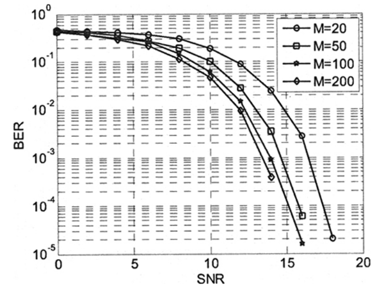

Figure 3. BER versus SNR for the proposed algorithm with $M = 20, 50, 100, 200$.

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