Channel Estimation Based on Extended Kalman Filtering for MIMO-OFDM Systems

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Abstract

MIMO-OFDM systems are considered as the solution scheme for future wideband wireless communication systems. And channel state information play a crucial role in data detection module. But Kalman filtering (KF) is a linear processing method which is not fit for processing nonlinear problem. While wireless channels have some nonlinear characteristics. Owing to the above reasons, this paper describes a channel estimation method based on extend Kalman filtering (EKF) for MIMO-OFDM systems. The proposed method based on EKF can exploit pilot symbols and an extended Kalman filter to implement channel estimation without the aid of any prior knowledge of channel statistics. In comparison with the channel estimation methods based on the least square (LS) and the least mean square (LMS) algorithms, the method base on EKF has better performances theoretically. Computer simulations also demonstrate the channel estimation method based on EKF outperforms the channel estimation methods based on LS and the LMS. Hence, the channel estimation method based on EKF can offer a dramatic system performance improvement at the cost of modest computational complexity.

Keywords: MIMO-OFDM, Channel estimation, EKF, LMS

1. Introduction

Since the realization of high data wireless access is demanded by many applications, more bandwidth needs to be allocated to achieve high data rate transmission in traditional systems. But increasing the bandwidth is often undesirable due to system complexity or spectral limitations. Therefore, multiple-input multiple-out (MIMO) systems [1] with multiple transmit and receive antennas can theoretically be employed to achieve over narrowband channels an improvement of spectral efficiency proportional to the number of transmit antennas. And space-time coding technique [2-3] is often applied to MIMO systems to mitigate channel fading without sacrificing bandwidth and becoming attractive in broadband wireless systems. Moreover, orthogonal frequency division multiplexing (OFDM) is an effective technique for mitigating the effects of delay spread in a frequency selective fading channel. By using OFDM modulation with cyclic prefix, frequency selective sub channels; thereby space-time processing technique can be effectively applied to improve the system performance. A system that combines MIMO system, space-time coding and OFDM can provide spectral efficiency and higher data transmission over a fading channel.

In MIMO-OFDM systems [4], channel estimation is important to the data detection process. And there are many channel estimation methods proposed for MIMO-OFDM systems. Several pilot symbols assisted channel estimation schemes have been introduced in [5-6] for providing the receivers with the channel state information for large diversity and multiplexing gains. And those channel estimation methods, which are based on the adaptive filters [7] including LMS, KF and EKF, are presented respectively in [8-10]. Since some characteristics of wireless channels are not always linear, some channel estimation methods are employed to track or estimate the nonlinear characteristics of wireless channels. In order to improve the accuracy of channel state information, this paper proposes a channel estimation method base on extended Kalman filtering for MIMO-OFDM systems. Compared with [5-6], this method proposed in this paper presupposes no prior knowledge of channel and noise statistics; compared with [8], this proposed method exploits an extended Kalman filter instead of a LMS filter for channel tracking and outperforms LMS method; compared with [9], this proposed method employs EKF signal processing method without the assumption that the receiver have prior knowledge of channel statistics such as AR model channel; compared with [10], this proposed method has a simpler preprocessing process and obtains channel information more easily. Therefore, the proposed channel estimation method based on EKF has practical value for the implementation of MIMO-OFDM systems.

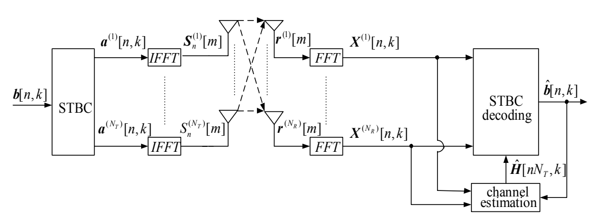

Figure 1. MIMO-OFDM system

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2. System model

As Figure 1 shows, we use N_T transmit antennas, N_R receive antennas, n OFDM symbols and K subcarriers in a MIMO-OFDM system. The transmitted symbol vector is $\boldsymbol{a}[n,k]=\left[\boldsymbol{a}^{(1)}[n,k] \quad ... \quad \boldsymbol{a}^{(N_T)}[n,k]\right]^T$, $n\in Z$, $k=0......K-1$, where $\boldsymbol{a}^{(i)}[n,k]$ denotes the symbol transmitted at the symbol time n, subcarrier k, and antenna i. The nth OFDM symbol $\boldsymbol{S}_n[m]$ can be obtained by performing an inverse fast discrete Fourier transform (IFFT) to the $\boldsymbol{a}[n,k]$ and inserting a cyclic prefix (CP) of length L_{CP},

$$\boldsymbol{S}_n[m]=\begin{cases}\dfrac{1}{\sqrt{KN_T}}\displaystyle\sum_{k=0}^{K-1}\boldsymbol{a}[n,k]e^{j2\pi mk/K}, & m=-L_{CP},...,K-1\\ 0, & \text{else}\end{cases} \tag{1}$$

Thus the duration of each OFDM symbol is $N=K+L_{cp}$. The overall baseband transmitted signal is $\boldsymbol{S}[m]=\sum_{n=-\infty}^{+\infty}\boldsymbol{S}_n[m-nN]$.

The signal from each receiver is formed by the parameter matrix $\boldsymbol{H}[m,l]$ of the fading MIMO $N_R\times N_T$ channel [5], the transmitted signal $\boldsymbol{S}[m]$, and the noise $\boldsymbol{\eta}[m]$. $\boldsymbol{\eta}[m]$ is stationary white Gauss noise which distribution is expressed by $\boldsymbol{N}(0,\boldsymbol{\sigma}_\eta^2)$. Thus the received signal can be expressed as follows,

$$\boldsymbol{S}_n[m]=\begin{cases}\dfrac{1}{\sqrt{KN_T}}\displaystyle\sum_{k=0}^{K-1}\boldsymbol{a}[n,k]e^{j2\pi mk/K}, & m=-L_{CP},...,K-1\\ 0, & \text{else}\end{cases} \tag{2}$$

In this paper, we assume the channel of this MIMO-OFDM system is wide-sense stationary uncorrelated scattering (WSSUS). The receiver signal $\boldsymbol{r}[m]$ is demodulated by removing cyclic prefix and performing fast Fourier transform (FFT).

$$\boldsymbol{X}[n,k]=\frac{1}{\sqrt{K}}\sum_{m=0}^{K-1}\boldsymbol{r}[nN+m]e^{-j2km/K} \tag{3}$$

If $Nf_{Doppler}<<1$ and $\boldsymbol{H}[m,l]=\boldsymbol{h}_l[n]$ (Here $n=m$) varies negligibly within one OFDM symbol, the input/output relation can be expressed as below,

$$\boldsymbol{X}[n,k]=\hat{\boldsymbol{H}}[n,k]\boldsymbol{a}[n,k]+\hat{\boldsymbol{\eta}}[n,k] \tag{4}$$

Here $\boldsymbol{X}[n,k]$, $\hat{\boldsymbol{H}}[n,k]$ and $\hat{\boldsymbol{\eta}}[n,k]$ are all $N_R\times N_T$ matrixes, and $\boldsymbol{a}[n,k]$ is $N_T\times N_T$ matrix. The channel matrix $\hat{\boldsymbol{H}}[n,k]$ can be obtained by the channel estimation method based on EKF which is described as follows.

3. Channel estimation based on EKF

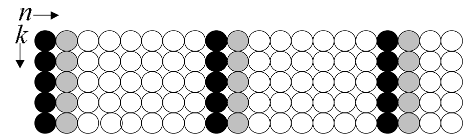

Figure 2. Pilot structure in each antenna

3.1. Pilot structure

Different methods and principles of pilot design are fit for different channel estimation methods. In order to process signals simply in this paper, the pilot sequences in different subcarries but in the same OFDM symbol are same in one antenna, and these pilot sequences from different antennas are orthogonal each other. $\boldsymbol{P}$ is the pilot matrix which consists of the pilot sequences from each antenna. $\boldsymbol{P}=[\boldsymbol{P}_1...\boldsymbol{P}_{N_T}]$, here $\boldsymbol{P}_n$ is a pilot sequence of an antenna and a cyclic sequence whose period is N_T, i.e. , $\boldsymbol{P}_{n+N_T}=\boldsymbol{P}_n$.

$$\boldsymbol{P}\boldsymbol{P}^H=\boldsymbol{P}^H\boldsymbol{P}=N_T\boldsymbol{I} \tag{5}$$

For a 2×2 MIMO-OFDM system, the pilot sequences are inserted as Figure 2 shows, i.e. , $\boldsymbol{P}_1=[1,1]^T$, $\boldsymbol{P}_2=[1,-1]^T$.

3.2. EKF Filtering

In order to using extended Kalman filtering to track or estimate channel instantly, the channel estimation proposed in this paper utilizes the least sequences (LS) algorithm [7]. Thus we can attain the channel transfer functions in frequency domain very easily. Considering a MIMO-OFDM system with N_T antennas, the relation between transmit and receive signals can be expressed as

$$\boldsymbol{X}[n,k]=\boldsymbol{P}[n,k]\boldsymbol{H}[n,k]+\boldsymbol{\eta}[n,k] \tag{6}$$

Where $\boldsymbol{P}[n,k]$ denotes the pilot symbols which is another form of $\boldsymbol{a}[n,k]$ as equation (4) shows. Apply the LS channel estimation method, we can conclude equation (7).

$$\tilde{\boldsymbol{H}}[n,k]=\boldsymbol{P}^{-1}[n,k]\boldsymbol{X}[n,k] \tag{7}$$

Thus the channel state information $\tilde{\boldsymbol{h}}_l[n]$ can be achieved by applying an inverse fast Fourier transform (IFFT) to the transfer function $\tilde{\boldsymbol{H}}[n,k]$. And we can conclude (8).

$$\tilde{\boldsymbol{h}}_l[n]=\boldsymbol{h}_l[n]+\boldsymbol{z}_l[n] \tag{8}$$

Here $\boldsymbol{z}_l[n]$ is a zero-mean, complex Gaussian vector whose distribution is $\boldsymbol{N}(0,\boldsymbol{\sigma}_z^2)$.

The Kalman filtering method exploits the state-space model in time/delay domain as (9) shows.

$$\boldsymbol{h}_l[n+1]=\boldsymbol{F}\boldsymbol{h}_l[n]+\boldsymbol{\omega}_l[n] \tag{9}$$

Here $\boldsymbol{F}$ is the $M\times M$ *matrix, $M=N_T\times N_R$,* and the $\boldsymbol{\omega}_l[n]$ is the

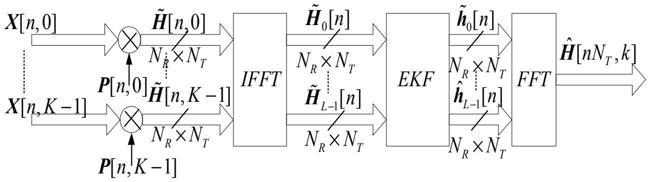

Figure 3. Principle of channel estimation

M×1 innovations noise vector. If $\boldsymbol{F}, \boldsymbol{\sigma}_z^2$, $\boldsymbol{\sigma}_\omega^2$ are known in advance, we can obtain the estimated channel state parameter $\hat{\boldsymbol{h}}_l[n]$ by exploiting Kalman filtering from (8) and (9). But we can not know the key parameters in advance, so we must utilize extended Kalman filtering to estimate channel state parameters.

Considering that wireless channels have some nonlinear characteristics, the channel estimation based on EKF is in essence a linear method which can do with nonlinear problems by the exploiting approximate linear method. So we construct the augmented state space equation (10) and the measurement equation (11) respectively.

$$\boldsymbol{x}[n+1] = \boldsymbol{G}(\boldsymbol{x}[n]) + \boldsymbol{\omega}[n] \tag{10}$$

$$\hat{\boldsymbol{h}}[n] = \boldsymbol{Q}\boldsymbol{x}[n] + \boldsymbol{z}[n] \tag{11}$$

Here $\boldsymbol{x}[n] = \begin{bmatrix} \boldsymbol{h}_0[n] \\ . \\ . \\ . \\ \boldsymbol{h}_{L-1}[n] \\ vec\{\boldsymbol{F}\} \end{bmatrix}$ and $\boldsymbol{G}(\boldsymbol{x}[n]) = \begin{bmatrix} \boldsymbol{F}\boldsymbol{h}_0[n] \\ . \\ . \\ . \\ \boldsymbol{F}\boldsymbol{h}_{L-1}[n] \\ vec\{\boldsymbol{F}\} \end{bmatrix}$.

And we define the *LM×1 measurement* vectors $\hat{\boldsymbol{h}}[n] \triangleq \left[\tilde{\boldsymbol{h}}_0^T[n] \ \ ... \ \ \tilde{\boldsymbol{h}}_{L-1}^T[n]\right]^T$, $\boldsymbol{z}[n] \triangleq \left[\boldsymbol{z}_0^T[n] \ \ ... \ \ \boldsymbol{z}_{L-1}^T[n]\right]$ and *LM×(L+M)M* matrix $\boldsymbol{Q} \triangleq [\boldsymbol{I}_{LM} \ \ \boldsymbol{0}_{LM \times M^2}]$ .Thus we can conclude the equations (12~16) [7] as follows.

$$\hat{\boldsymbol{x}}[n+1\,|\,n] = \boldsymbol{G}(\hat{\boldsymbol{x}}[n\,|\,n]) = \begin{bmatrix} \hat{\boldsymbol{F}}[n\,|\,n]\hat{\boldsymbol{h}}_0[n\,|\,n] \\ . \\ . \\ . \\ \hat{\boldsymbol{F}}[n\,|\,n]\hat{\boldsymbol{h}}_{L-1}[n\,|\,n] \\ vec\{\hat{\boldsymbol{F}}[n\,|\,n]\} \end{bmatrix} \tag{12}$$

Here the equation (12) updates the time parameter.

$$\hat{\boldsymbol{x}}[n\,|\,n] = \hat{\boldsymbol{x}}[n\,|\,n-1] + \boldsymbol{K}[n](\tilde{\boldsymbol{h}}[n] - \tilde{\boldsymbol{h}}[n\,|\,n-1]) \tag{13}$$

Here the equation (13) updates the measurement equation.

$$\boldsymbol{K}[n] = \boldsymbol{\Omega}[n\,|\,n-1]\boldsymbol{Q}^H(\boldsymbol{Q}\boldsymbol{\Omega}[n\,|\,n-1]\boldsymbol{Q}^H + \boldsymbol{\sigma}_z^2\boldsymbol{I}_{LM})^{-1} \tag{14}$$

$$\boldsymbol{\Omega}[n\,|\,n] = (\boldsymbol{I}_{(L+M)M} - \boldsymbol{K}[n]\boldsymbol{Q})\boldsymbol{\Omega}[n\,|\,n-1] \tag{15}$$

$$\boldsymbol{\Omega}[n+1\,|\,n] = \boldsymbol{U}[n]\boldsymbol{\Omega}[n\,|\,n]\boldsymbol{U}^H[n] + \boldsymbol{\sigma}_\omega^2 \begin{bmatrix} \boldsymbol{I}_{LM} & \boldsymbol{0}_{LM \times M^2} \\ \boldsymbol{0}_{M^2 \times LM} & \boldsymbol{0}_{M^2 \times M^2} \end{bmatrix} \tag{16}$$

Where $\boldsymbol{U}[n]$ is defined as (17),

$$\boldsymbol{U}[n] = \left.\frac{\boldsymbol{G}(\boldsymbol{x})}{\boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}[n|n]} = \begin{bmatrix} \boldsymbol{F}[n|n] & \boldsymbol{0}_{M \times M} & . \ . \ . & \boldsymbol{0}_{M \times M} & \boldsymbol{h}_0^T[n|n] \otimes \boldsymbol{I}_M \\ \boldsymbol{0}_{M \times M} & \boldsymbol{F}[n|n] & . \ . \ . & \boldsymbol{0}_{M \times M} & \boldsymbol{h}_0^T[n|n] \otimes \boldsymbol{I}_M \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \boldsymbol{0}_{M \times M} & \boldsymbol{0}_{M \times M} & . \ . \ . & \boldsymbol{F}[n|n] & \boldsymbol{h}_0^T[n|n] \otimes \boldsymbol{I}_M \\ \boldsymbol{0}_{M^2 \times M} & \boldsymbol{0}_{M^2 \times M} & . \ . \ . & \boldsymbol{0}_{M^2 \times M} & \boldsymbol{I}_{M^2} \end{bmatrix} \tag{17}$$

Here $\otimes$ denotes the Kronecker product.

Because the measurement and time update equations are a recursive process for estimating the channel state parameters in time/delay domain, we must initialize the vectors as $\hat{\boldsymbol{x}}[1\,|\,0] = \left[\hat{\boldsymbol{h}}_0[0] \ \ ... \ \ \hat{\boldsymbol{h}}_{L-1}[0] \ \ vec\{\boldsymbol{I}_M\}\right]^T$ and $\boldsymbol{\Omega}[1\,|\,0] = \boldsymbol{I}_{(L+M)M}$.

As the equations (14) and (16) show, the innovations noise variance and the measurement noise variance are necessary for EKF filtering. But the innovations noise variance can not be estimated directly due to $\omega_l[n]$ not being observed. So we construct equation (18) to estimate $\boldsymbol{\omega}_l[n]$ [11-12].

$$\hat{\boldsymbol{\omega}}_l[n] = \hat{\boldsymbol{h}}_l[n\,|\,n] - \hat{\boldsymbol{F}}[n\,|\,n]\hat{\boldsymbol{h}}_l[n-1\,|\,n-1] \tag{18}$$

Herein, since $\hat{\boldsymbol{h}}_l[n\,|\,n]$ and $\hat{\boldsymbol{F}}[n\,|\,n]$ can be obtained by the equations mentioned above, we can conclude the innovations noise variance $\hat{\boldsymbol{\sigma}}_\omega^2[n]$ which can be expressed as $\hat{\boldsymbol{\sigma}}_\omega^2[n] = \dfrac{1}{LnM}\sum_{l=0}^{L-1}\sum_{m=1}^{n} \|\,\hat{\boldsymbol{\omega}}_l[m]\,\|^2$.

The measurement noise variance also can not be observed but estimated. And we attain it as below.

$$\hat{\boldsymbol{z}}_l[n] = \hat{\boldsymbol{h}}_l[n] - \hat{\boldsymbol{h}}_l[n\,|\,n-1] \tag{19}$$

Here $\hat{\boldsymbol{h}}_l[n\,|\,n-1] = \hat{\boldsymbol{F}}[n-1\,|\,n-1]\hat{\boldsymbol{h}}[n-1\,|\,n-1]$. So the measurement noise variance can be expressed as $\hat{\boldsymbol{\sigma}}_z^2[n] = \dfrac{1}{LnM}\sum_{l=0}^{L-1}\sum_{m=1}^{n} \|\,\hat{\boldsymbol{z}}_l[m]\,\|^2$.

3.3. Postprocessing Process

Because $\hat{\boldsymbol{h}}_l[n]$ can be drawn, we can obtain the channel transfer functions in time/frequency domain by FFT as

$$\hat{\boldsymbol{H}}[n,k] = \frac{1}{N_T}\sum_{l=0}^{L-1}\hat{\boldsymbol{h}}_l[n]e^{-j2kl/K} \tag{20}$$

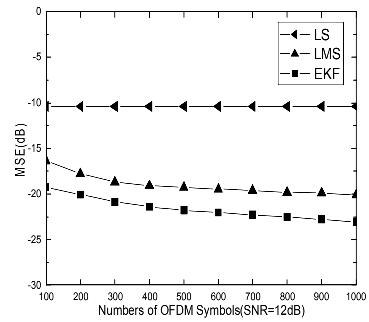

Figure 4. Convergence behavior

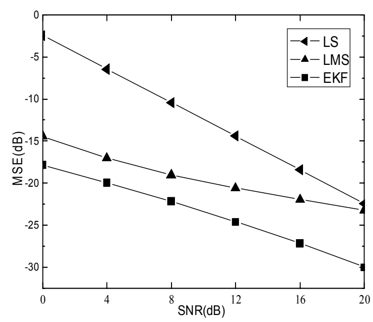

Figure 5. MSE versus SNR

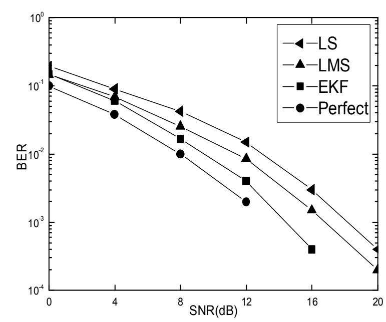

Figure 6. BER versus SNR

4. Computer simulations

The simulation system applies a 2×2 MIMO-OFDM system. The carrier frequency is 4G Hz, the bandwidth is 6 MHz, the subcarrier number is 64, the length of CP is 16, and the modulation scheme is QPSK. The space-time coding scheme is Alamouti's STBC with 1/2 rate and the decoding scheme is Maximum likelihood algorithm. The simulation channel is an ITU Jakes model.

Because the LS channel estimation method is a part of the LMS or the EKF channel estimation method, its computational complexity is lowest. The computational complexity of the LMS method is lower than the EKF method. Though the EKF method is more complex than the LMS method,the EKF method has better performance than the LMS one. Hence, the choice of different methods is a trade-off between good performance and low computational complexity.

As Figure 4 shows, the EKF method has better convergence than LMS method. From Figure 5 and Figure 6, the MSE and BER performances of the EKF method are better than those of the LMS and the LS methods at 600 OFDM symbols. Though it has highest computational complexity, the EKF method is still used to estimate channel due to its highest performance.

5. Conclusions

Considering the wireless channels probably have some nonlinear characteristics, this paper describes a channel estimation method based on EKF for MIMO-OFDM systems. This method exploits pilot symbols and the EKF algorithm to estimate channel without any prior statistical knowledge of channel or noise. Simulation results prove that the proposed EKF method has moderate computational complexity and higher performance than the LMS and the LS methods. Considering different characteristics between LMS algorithm and EKF algorithm, we can select a reasonable channel estimation method for MIMO-OFDM systems according to the nonlinear or linear characteristics of channels. Therefore, the proposed method has some academic and engineering value for future research of MIMO-OFDM systems.

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