

Assignment 3

• Section 1.7

- 2) Prove: $\neg \exists x \{0 < x < 1000, x = \sqrt[3]{c}, x = a^3 + b^3 \text{ where } a, b, \text{ and } c \in \mathbb{N}\}$

List of perfect cubes	Possible values for a^3 and b^3
1	none
8	$1 + 1 = 2 < 8$
27	$8 + 8 = 16 < 27$
64	$27 + 27 = 54 < 64$
125	$64 + 64 = 128 > 125$, $64 + 27 = 91 < 125$
216	$125 + 125 = 250 > 216$, $125 + 64 = 189 < 216$
343	$216 + 216 = 432 > 343$, $216 + 125 = 341 < 343$
512	$343 + 343 = 686 > 512$, $343 + 216 = 559 > 512$, $216 + 216 = 432 < 512$
729	$512 + 512 = 1024 > 729$, $512 + 343 = 855 > 729$, $343 + 343 = 686 < 729$

$\therefore \neg \exists x \{0 < x < 1000, x = \sqrt[3]{c}, x = a^3 + b^3 \text{ where } a, b, \text{ and } c \in \mathbb{N}\}$

Through a very exhaustive means.

- 6) Prove: $x, a, \text{ and } b \in \mathbb{N}; \exists x \{x = a + b, a < x, b < x\}$

$$1+2=3$$

$\therefore x, a, \text{ and } b \in \mathbb{N}; \exists x \{x = a + b, a < x, b < x\}$

This proof is constructive.

- 10) Prove: Product of two of the the numbers a, b, and c is non-negative.

Suppose a is positive.

Suppose b is positive. $a * b =$ a non-negative.

If b is negative,

suppose c is positive. $a * c =$ a non-negative.

If c is negative, $b * c =$ a non-negative.

If a is negative,

suppose b is negative. $a * b =$ a non-negative.

If b is positive,

suppose c is negative. $a * c =$ a non-negative.

If c is positive, $b * c =$ a non-negative.

If a, b, or c = 0, zero times anything is a non-negative.

This method of proof is non-constructive.

- 20) prove: $x^2 + \frac{1}{x^2} \geq 2$
 $x^2 + \frac{1}{x^2} - 2 \geq 0$
 $(x - \frac{1}{x})^2 \geq 0$
If $x - 1/x = 0$
 $(x - \frac{1}{x})^2 = 0$
If $x - 1/x < 0$
 $(x - \frac{1}{x})^2 > 0$ since any negative real number squared is positive.
If $x - 1/x > 0$
 $(x - \frac{1}{x})^2 > 0$ since any positive real number squared can never equal to zero or be non-negative.

– 26) $a^2 = b$

if the least significant digit of a	least significant digit of b
0	0
1 or 9	1
2 or 8	4
3 or 7	9
4 or 6	6
5	5

• Review Exercises

- 4)
- * a) Two propositions are logically equivalent when they have the same truth values when their propositional variables have the same truth values.
 - * b) You can show compound propositions are equivalent by truth tables, deductive reasoning, and contradiction.
 - * c)

(1) $\neg p \vee (r \rightarrow \neg q)$	Given
(2) $\neg p \vee \neg r \vee \neg q$	Material Implication(1)
(3) $\neg p \vee \neg q \vee \neg r$	Associativity(2)

p	q	r	$\neg p \vee (r \rightarrow \neg q)$	$\neg p \vee \neg q \vee \neg r$
T	T	T	F	F
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

- 6)
 $\forall P(x)$

$\exists P(x)$
 $\neg \forall P(x)$
 $\neg \exists P(x)$

- 14) A constructive proof will show you explicitly the existence of a case where what ever you are trying to prove is true, while a non-constructive proof will show that a case where what you are trying to prove exists through hypothetical situations.

Constructive proof:

prove there exists a where $b + a = b$

$$3 + 0 = 3$$

Nonconstructive proof:

prove there exists integer a where $a/2 = b$ and b is integer

suppose a is even.

$a = 2c$ where c is an integer.

$$2c/2 = c = b$$

- Supplementary Exercises

- 4)

- * a)

If I will drive to work, then it is raining today.

If I will not drive to work, then it is not raining today.

If it is not raining today, then I will not drive to work.

- * b)

If $x \geq 0$, then $|x| = x$

If $x < 0$, then $|x| \neq x$

If $|x| \neq x$, then $x < 0$

- * c)

If $n^2 > 0$, then $n > 3$

If $n^2 \leq 0$, then $n \leq 3$

If $n \leq 3$, then $n^2 \leq 0$

- 6)

Inverse of Inverse:

$$p \rightarrow q$$

Inverse of Converse:

$$\neg q \rightarrow \neg p$$

Inverse of Contrapositive:

$$q \rightarrow p$$

- 14)

- * a) $\exists x P(x)$

- * b) $\neg \forall x P(x)$

- * c) $\forall y Q(y)$

- * d) $\forall x \forall y P(x) Q(y)$

- * e) $\exists y \neg Q(y)$

- 18) $\forall y \neg \exists x G(x, y) \{x > 3\}$

- 30) If it is given that $P(x)$ and $P(y)$ is true for values x and y , then we can simplify what is given by saying that $P(x)$ is true for x and we can also say that $P(y)$ is true for y .
- 34) given: $n, n \in \mathbb{Z}^+$
 prove: $m^2 \leq n < (m+1)^2$
 if $n = a^2$ where $a \in \mathbb{Z}^+$
 $m^2 \leq a^2 < (m+1)^2$
 $m = a$
 If n is not a perfect square, then m^2 would be the closest smaller perfect square, and since there are no perfect squares right next to each other on the integer line, there will always be space for an n in between perfect squares (m^2 and $(m+1)^2$).

- Section 2.1

- 2)
 - * a) $\{x \in \mathbb{N} \mid x = 3n, n \in \mathbb{N}, n \geq 4\}$
 - * b) $\{x \in \mathbb{Z} \mid -3 \leq x \leq 3\}$
 - * c) $\{l \in Alphabet \mid l \text{ is between 'l' and 'q' in the sequence } Alphabet\}$
- 4)
 - * $B \subset A$
 - * $C \subset A$
 - * $C \subset D$
- 8)
 - * a) T
 - * b) T
 - * c) F
 - * d) T
 - * e) T
 - * f) T
 - * g) T
- 16) $B = \{A, x\}, A = \{x\}$
- 18)
 - * a) 0
 - * b) 1
 - * c) 2
 - * d) 3
- 20) Yes, because if they have the same power set, they have the same elements.

– 22)

* a) no

* b) no

* c) No!

* d) yes

– 30)

Suppose $A = \{1\}$ and $B = \{2\}$

$A \times B = \{(1, 2)\}$

$B \times A = \{(2, 1)\}$

$A \neq B$

• Section 2.2

– 2)

– 4)

– 12)

– 16)

– 18)

– 30)

– 36)