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Assignment 3 - Discrete Mathematics
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Assignment 3

- Section 1.7

  - 2) Prove: $\neg\exists x\{0 < x < 1000,\ x = \sqrt[3]{c},\ x = a^3 + b^3$ where a, b, and $c \in \mathbb{N}\}$

List of perfect cubes	Possible values for a^3 and b^3
1	none
8	$1 + 1 = 2 < 8$
27	$8 + 8 = 16 < 27$
64	$27 + 27 = 54 < 64$
125	$64 + 64 = 128 > 125,\ 64 + 27 = 91 < 125$
216	$125 + 125 = 250 > 216,\ 125 + 64 = 189 < 216$
343	$216 + 216 = 432 > 343,\ 216 + 125 = 341 < 343$
512	$343 + 343 = 686 > 512,\ 343 + 216 = 559 > 512,\ 216 + 216 = 432 < 512$
729	$512 + 512 = 1024 > 729,\ 512 + 343 = 855 > 729,\ 343 + 343 = 686 < 729$

    $\therefore \neg\exists x\{0 < x < 1000,\ x = \sqrt[3]{c},\ x = a^3 + b^3$ where a, b, and $c \in \mathbb{N}\}$
    Through a very exaustive means.

  - 6) Prove: x, a, and $b \in \mathbb{N}$; $\exists x\{x = a + b,\ a < x,\ b < x\}$
    1+2=3
    $\therefore x$, a, and $b \in \mathbb{N}$; $\exists x\{x = a + b,\ a < x,\ b < x\}$
    This proof is constructive.

  - 10) Prove: Product of two of the the numbers a, b, and c is non-negative.
    Suppose a is positive.
    Suppose b is positive. $a * b =$ a non-negative.
    If b is negative,
    suppose c is positive. $a * c =$ a non-negative.
    If c is negative, $b * c =$ a non-negative.
    If a is negative,
    suppose b is negative. $a * b =$ a non-negative.
    If b is positive,
    suppoce c is negative. $a * c =$ a non-negative.
    If c is positive, $b * c =$ a non-negative.
    If a, b, or c = 0, zero times anything is a non-negative.

    This method of proof is non-constructive.

- 20) prove: $x^2 + \frac{1}{x^2} \geq 2$
  $x^2 + \frac{1}{x^2} - 2 \geq 0$
  $(x - \frac{1}{x})^2 \geq 0$
  If $x - 1/x = 0$
  $(x - \frac{1}{x})^2 = 0$
  If $x - 1/x < 0$
  $(x - \frac{1}{x})^2 > 0$ since any negative real number squared is positive.
  If $x - 1/x > 0$
  $(x - \frac{1}{x})^2 > 0$ since any positive real number squared can never equal to zero or be non-negative.

- 26) $a^2 = b$

if the least signficant digit of a	least significant digit of b
0	0
1 or 9	1
2 or 8	4
3 or 7	9
4 or 6	6
5	5

- Review Exercises
  - 4)
    * a) Two propositions are logically equivalent when they have the same truth values when their propositional variables have the same truth values.
    * b) You can show compund propositions are equivalent by truth tables, deductive resoning, and contradiction.
    * c)
      (1) $\neg p \vee (r \rightarrow \neg q)$ Given
      (2) $\neg p \vee \neg r \vee \neg q$ Material Implication(1)
      (3) $\neg p \vee \neg q \vee \neg r$ Associativity(2)

p	q	r	$\neg p \vee (r \rightarrow \neg q)$	$\neg p \vee \neg q \vee \neg r$
T	T	T	F	F
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	T	T	T	T

  - 6)
    $\forall P(x)$

$\exists P(x)$
$\neg\forall P(x)$
$\neg\exists P(x)$

- 14) A constructive proof will show you explicitly the existance of a case where what ever you are trying to proof is true, while a non-contructive proof will show that a case where what you are trying to prove exists through hypothetical situations.
  Constructive proof:
  prove there exists a where $b + a = b$
  $3 + 0 = 3$
  Nonconstructive proof:
  prove there exists integer a where $a/2 = b$ and b is integer
  suppose a is even.
  $a = 2c$ where c is an integer.
  $2c/2 = c = b$

- Supplementary Exercises
  - 4)
    * a)
      If I will drive to work, then it is raining today.
      If I will not drive to work, then it is not raining today.
      If it is not raining today, then I will not drive to work.
    * b)
      If $x \geq 0$, then $|x| = x$
      If $x < 0$, then $|x| \neq x$
      If $|x| \neq x$, then $x < 0$
    * c)
      If $n^2 > 0$, then $n > 3$
      If $n^2 \leq 0$, then $n \leq 3$
      If $n \leq 3$, then $n^2 \leq 0$
  - 6)
    Inverse of Inverse: $p \to q$
    Inverse of Converse: $\neg q \to \neg p$
    Inverse of Contrapositive: $q \to p$
  - 14)
    * a) $\exists x P(x)$
    * b) $\neg\forall x P(x)$
    * c) $\forall y Q(y)$
    * d) $\forall x \forall y P(x) Q(y)$
    * e) $\exists y \neg Q(y)$
  - 18) $\forall y \neg\exists x G(x, y)\{x > 3\}$

- 30) If it is given that P(x) and P(y) is true for values x and y, then we can simplify what is given by saying that P(x) is true for x and we can also say that P(y) is true for y.
- 34) given: n, $n \in \mathbb{Z}^+$
  prove: $m^2 \leq n < (m+1)^2$
  if $n = a^2$ where $a \in \mathbb{Z}^+$
  $m^2 \leq a^2 < (m+1)^2$
  $m = a$
  If n is not a perfect square, then m^2 would be the closest smaller perfect square, and since there are no perfect squares right next to each other one the integer line, there will always space for an n in between perfect squares(m^2 and $(m+1)^2$).

• Section 2.1

- 2)
  - a) $\{x \in \mathbb{N} | \ x = 3n, \ n \in \mathbb{N}, \ n \geq 4\}$
  - b) $\{x \in \mathbb{Z} | \ -3 \leq x \leq 3\}$
  - c) $\{l \in Alphabet | \ l$ is between 'l' and 'q' in the sequence $Alphabet\}$
- 4)
  - $B \subset A$
  - $C \subset A$
  - $C \subset D$
- 8)
  - a) T
  - b) T
  - c) F
  - d) T
  - e) T
  - f) T
  - g) T
- 16) $B = \{A, x\}$, $A = \{x\}$
- 18)
  - a) 0
  - b) 1
  - c) 2
  - d) 3
- 20) Yes, because if they have the same power set, they have the same elements.

- 22)
  - a) no
  - b) no
  - c) No!
  - d) yes
- 30)
  Suppose $A = \{1\}$ and $B = \{2\}$
  Ax$B = \{(1, 2)\}$
  Bx$A = \{(2, 1)\}$
  $A \neq B$

- Section 2.2
  - 2)
    - a) $A \cap B$
    - b) $A - B$
    - c) $A \cup B$
    - d) $A^c \cup B^c$
  - 4)
    - a) $\{a, b, c, d, e, f, g, h\}$
    - b) $\{a, b, c, d, e\}$
    - c) $\emptyset$
    - d) $\{f, g, h\}$
  - 12)

(1)$A \cup (A \cap B)$	
(2)$(A \cap B) \in A$	Definition of Intersection (1)
(3)$A \cup x = A$, where $x \in A$	Definition of Union
(4)$A \cup (A \cap B) = A$	(1),(2),(3)

  - 16)
    - a)
      $\forall x \in (A \cap B)$
      $x \in A$ and $x \in B$
      and since $x \in A$
      $(A \cap B) \subseteq A$
    - b)
      $\forall x \in A$
      $x \in (A \cup B)$
      $\therefore A \subseteq (A \cup B)$
    - c)
      $\forall x \in A$
      if $x \in B$
      then $x \notin (A - B)$

if $x \notin B$
then $x \in (A - B)$
$\therefore A - B \subseteq A$

* d)
  (1)$A \cap (B - A) = A \cap (B \cap A^c)$
  (2)$A \cap A^c \cap B$ Association (1)
  (3)$\emptyset \cap B$ Complement(2)
  (4)$\emptyset$ Domination(3)

* e)
  (1)$A \cup (B - A)$
  (2)$A \cup (B \cap A^c)$
  (3)$(A \cup B) \cap (A \cup A^c)$ Distribution(2)
  (4)$(A \cup B) \cap \emptyset$ Complemet(3)
  (5)$A \cup B$ Identity(4)

- 18)
  * a)$\forall x \in (A \cup B)$
    $x \in (A \cup B \cup C)$
    $\forall y \in (A \cup B \cup C)$
    $y \in (A \cup B)$ or $y \notin (A \cup B)$
    $\therefore (A \cup B) \subseteq (A \cup B \cup C)$
  * b)
  * c)
  * d)
  * e)
- 30)
- 36)