Oil Prices and the Macroeconomy: Online Appendix

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1 Introduction

This appendix has two purposes. First, we revisit the basic model and discuss a few additional issues not touched upon in the chapter itself. Second, we introduce several extensions of the basic model. The first extension is a version of the basic model that includes capital accumulation. We then consider two variants of the model with capital: one with the production function of Kim and Loungani (1992) and another with the production function of Finn (2000). After that, we consider an extension of the basic model that adds household demand for oil.

For each extension, we work through many of the equations and discuss additional issues related to the modeling or calibration that we feel are worthwhile pointing out. Interested readers are recommended to consult the references for additional details about the models. Some impulse response functions are presented at the back of the appendix. The Matlab code is posted online for each variant.

2 Revisiting the basic model

Our handbook chapter contains a very simple DSGE model with oil. The basic model includes firm demand for oil, an exogenous supply of oil, and endogenous oil prices. The equations in the text are based on a decentralized model with a representative agent and a representative firm. The representative agent consumes a final good and earns income by providing labor to the private sector and from dividends it receives from the firm. The representative firm operates under perfect competition and produces the final good using labor and imported oil. Each period the economy in question trades some of its output of the final good for oil. The model assumes that trade balances each period. Readers interested in the exact equations can find them in the chapter text. Here, we expand upon the discussion found in the handbook chapter by considering two issues that modelers will likely run into at some point in time.

2.1 A potential numerical issue

In most cases, it is necessary to numerically solve the DSGE model and that requires calibrating it. Here we briefly discuss a potential numerical issue that can arise when doing so.

Consider a standard CES function of the form

$$y_t = \left[(1-\alpha_o)(n_t)^{\frac{\sigma-1}{\sigma}} + \alpha_o (o_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

Numerically solving for the weight, given by α_o, can be difficult when the elasticity of substitution or the cost-share are small as the weight's value will start approaching 0.

To see this explicitly, one can re-write the firm's first-order condition for oil as

$$\alpha_o = p_t^o \left(\frac{o_t}{y_t} \right)^{\frac{1}{\sigma}}. \tag{2}$$

One can usually normalize p^o to be 1, in which case it's immediately clear how the weight is influenced by the calibration of the cost-share and the elasticity of substitution. In the chapter, we set the GDP-share of oil to 0.02, which implies a cost-share of 0.0196. The elasticity of substitution was set to 0.25. This gives a value of 0.00000015. The problem becomes worse as the elasticity or the cost-share become smaller, and this can often create headaches for choosing starting values and getting a solution from the solver.

For this particular model, it is not really an issue as the first-order condition lets one solve for the weight directly so long as one is OK with normalizing the price to 1 and the modeler can calibrate the cost-share of oil (with respect to gross output). In more complicated models, though, it can sometimes be difficult to do this. In those cases, there are several ways of skirting the issue.

In the handbook chapter, we worked with a CES function of the form

$$y_t = \left[(1-\alpha_o) \left(z_t^n n_t \right)^{\frac{\sigma-1}{\sigma}} + \alpha_o \left(z^o o_t \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

We relied on a particularly convenient normalization that set the steady state value of z^n to 1 and z^o to the inverse of o. In that case, the weight α_o is exactly equal to the cost-share of oil. This avoids the numerical issue involved in solving for the steady state and, in fact, one can solve the steady state for this particular model by hand. We will make use of this approach in many of the variants that we introduce later in the appendix.

Another approach involves exponentiating the CES weights in a particular way. An example of this is approach can be found in Bodenstein et al. (2011). We refer the interested reader to that paper for specifics. The use of the production function in Finn (2000) also avoids this issue, since it is Cobb-Douglas in nature.

2.2 A digression on calculating GDP

The introduction of oil leads to differences between GDP and gross output. It also introduces a relative price that varies over time in response to shocks in the model. When the elasticity of substitution is different from 1, this variation in the relative price means that cost-shares and expenditure-shares will also vary over time. All of this can make it a bit confusing when trying to keep track of GDP.

For example, in the basic model the expenditure approach would suggest calculating real GDP as c_t. On the other hand, the value-added approach would suggest using the equation $y_t - p_t^o o_t$. This equation, though, builds in the relative price of oil, which will vary over time. One way to deal with this is to consider a fixed-weight measure by holding relative prices fixed at their initial value, i.e. $y_t - o_t$ (making use of our normalization of p^o to 1). Using this approach, one can also derive a GDP deflator using the ratio of nominal and real GDP. Note, however, that this fixed-price measure will not equal consumption, since the low elasticity of substitution means the cost-share of oil changes over time. Since our assumption is that the economy "imports" oil, if the cost-share goes up then the representative agent gets to consume less because more of its income is spent on oil.

The code contains several different equations for those interested in this:

$$gdp_t^{ex} = c_t, \tag{4}$$

$$gdp_t^{va} = y_t - o_t, \tag{5}$$

$$p_t^g = \frac{y_t - p_t^o o_t}{y_t - o_t}, \tag{6}$$

where the superscripts ex and va represent the expenditure approach and the value-added approach, respectively, and p_t^g is the GDP deflator.

3 Adding capital to the basic model

Capital accumulation is a common feature in RBC models and adding capital to the basic model is fairly straight forward. In this section, we consider an extension of the production function used in the basic model where value-added is a Cobb-Douglas function of labor and capital. Gross output is still produced using a CES production function that combines oil and value-added. This collapses to the basic model when the labor-share of value-added is set to 1.

There are other possibilities available to the modeler when it comes to the production function. Later sections in this appendix introduce the production function of Kim and Loungani (1992), where capital and energy are nested in their own CES function, and the production function of Finn (2000), which incorporates capital utilization.

3.1 The model

The representative agent chooses consumption, investment, next period's capital stock and labor to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) - \chi n_t^{1+\eta}/(1+\eta)], \tag{7}$$

subject to the budget constraint

$$c_t + i_t = w_t n_t + r_t^k k_{t-1} + d_t, \tag{8}$$

and the capital accumulation equation

$$i_t = k_t - (1-\delta) k_{t-1}. \tag{9}$$

In terms of parameters, β is the discount factor, δ is the depreciation rate of capital, η is the inverse Frisch-elasticity of labor and χ is the weight on the dis-utility from working. The variable c_t is consumption, i_t investment in capital, k_{t-1} the current stock of capital (a state variable), n_t is hours worked, and r_t^k is the return to capital. Please note that in our notation $t-1$ is used to denote variables that were chosen at time $t-1$. For capital, this implies that k_{t-1} is a state variable at time t and k_t is chosen at that period.[3]

After some simplification of the equations, the first-order conditions can be written as

$$1/c_t = \lambda_t, \tag{10}$$

$$w_t \lambda_t = \chi n_t^{\eta}, \tag{11}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k + (1-\delta) \right], \tag{12}$$

where λ_t as the multiplier on the budget constraint.

Gross output is produced using a CES production where the inputs are value-added and oil. Value-added is itself a Cobb-Douglas aggregate of labor and capital. The specific form we use is

$$y_t = \{(1-\alpha_o) \left[(z_t^n n_t)^{\theta} k_{t-1}^{1-\theta} \right]^{\frac{\sigma-1}{\sigma}} + \alpha_o (z_t^o o_t)^{\frac{\sigma-1}{\sigma}} \}^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where θ is the labor-share of value-added. Please note that allow z^o to vary over time, which introduces an "oil-efficiency" shock into the model.

[3] An alternative notation is to use k_{t+1} and k_t.

The first order conditions for this problem are given by

$$p_t^o = \alpha_o \left(\frac{y_t}{o_t}\right)^{\frac{1}{\sigma}} z_t^{o\frac{\sigma-1}{\sigma}}, \tag{14}$$

$$w_t n_t = \theta(1-\alpha_o) y_t^{\frac{1}{\sigma}} \left[(z_t^n n_t)^\theta k_{t-1}^{1-\theta}\right]^{\frac{\sigma-1}{\sigma}}, \tag{15}$$

$$r_t^k k_{t-1} = (1-\theta)(1-\alpha_o) y_t^{\frac{1}{\sigma}} \left[(z_t^n n_t)^\theta k_{t-1}^{1-\theta}\right]^{\frac{\sigma-1}{\sigma}} \tag{16}$$

where we have made use of the fact that $y_t^{\frac{1}{\sigma}} = \{(1-\alpha_o)\left[(z_t^n n_t)^\theta k_{t-1}^{1-\theta}\right]^{\frac{\sigma-1}{\sigma}} + \alpha_o (o_t z_t^o)^{\frac{\sigma-1}{\sigma}}\}^{\frac{\sigma}{\sigma-1}-1}$.

3.2 Calibration and steady state

We use a procedure similar to the one in the handbook chapter to calibrate and solve for the steady state. We continue to calibrate to a quarterly frequency. The parameter β is set to 0.99, η to 1, δ to 0.025 and θ to 0.64. None of those parameters are unique to models with oil. Given the calibration of β and δ, one immediately gets the value of r^k from the household's first order condition for capital. The value of σ is set to 0.25, as in the basic model.

We normalize n, y and p^o to 1 in the steady state. We calibrate the value of o using the same GDP-share data we used in the basic model. This gives a value of 0.0196 for the cost-share of oil and for o. From the firm's first order condition for oil, you can see that setting $z^o = 1/o$ ensures that α_o is equal to the cost-share of oil. From the production function one can then show that $k^{\frac{\theta-1}{\theta}}$ is the value of z^n consistent with our normalization. From the firm's first order condition for capital, one can then solve for k. With this in hand, we get the value of z^n and then the firm's first order condition for labor gives the value of w. One gets a value for i from the law of motion equation for capital, and the steady state value of consumption is then given by the resource constraint, i.e. $c = y - p^o o - i$.

There are, of course, other ways to calibrate the steady state than the procedure used here. For example, one could match the capital-GDP ratio or the ratio of investment spending to GDP. This usually necessitates allowing either δ or β to be free parameters. Likewise, one could pick an energy-capital ratio (as done in Kim and Loungani (1992)) or some other energy-related quantity. It really depends upon the preference of the modeler, the questions being asked and the data that is available.

3.3 Impulse response functions

We show impulse response functions for oil prices and GDP in Figure (1). We normalize the size of the three shocks so that they all increase the price of oil by 10 percent. This implies that we are looking at a negative oil supply shock, a positive productivity shock and a negative oil-efficiency shock.

3.4 An alternative way to write the production function

It is possible to write down the firm's problem directly in terms of the value-added input. First, define value-added as

$$v_t = \left(z_t^n n_t\right)^{\theta} k_{t-1}^{1-\theta}. \tag{17}$$

Next, re-write the production function as

$$y_t = \left[\left(1-\alpha_o\right) v_t^{\frac{\sigma-1}{\sigma}} + \alpha_o \left(o_t z_t^o\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{18}$$

We do not derive the equations here, but one can solve the firm's profit maximization problem if desired.

It's also useful to consider a cost-minimization problem where the firm chooses labor and capital inputs to minimize the cost of producing a set amount of value-added. That is,

$$\min_{n_t, k_{t-1}} p_t^v v_t = w_t n_t + r_t^k k_{t-1}, \tag{19}$$

subject to

$$v_t = \left(z_t^n n_t\right)^{\theta} k_{t-1}^{1-\theta}, \tag{20}$$

where p_t^v is the price of a unit of value-added (denominated in terms of the final good). The first-order conditions are

$$w_t = \theta \frac{v_t}{n_t}(-\Phi_t), \tag{21}$$

$$r_t^k = (1-\theta)\frac{v_t}{k_{t-1}}(-\Phi_t), \tag{22}$$

where Φ_t is the multiplier on the constraint and $-\Phi_t = p_t^v$. The first-order conditions can be used to derive an explicit equation for the price of value-added,

$$p_t^v = \left(\frac{w_t}{z_t^n \theta}\right)^{\theta} \left(\frac{r_t^k}{1-\theta}\right)^{1-\theta}. \tag{23}$$

This can be done by dividing the two first-order conditions, solving for n_t as a function of everything else, substituting this into the equation for value-added, using the first-order condition for capital to substitute out the k_{t-1} term and then solving for p_t^v.

4 The production function of Kim and Loungani (1992)

We now consider a model which uses the production function of Kim and Loungani (1992), where gross output is produced using a Cobb-Douglas function of labor and the service flow from a capital good. The latter is produced using a CES production function that combines capital and energy. We use the word energy here on purpose, as the calibration in Kim and Loungani (1992) is based on the use of oil and other fossil fuels (coal and natural gas). To be consistent with this, we define energy use as e_t rather than o_t. This distinction turns out to be of great importance, though, only in the calibration of some of the steady state shares.

Before moving onto the details, it is important to point out that we do not strictly follow the model, notation or calibration found in the original work. Some notable differences include:

- Kim and Loungani (1992) assumed that energy prices followed an exogenous stochastic process, whereas we assume that the supply of energy is exogenous.
- The original uses a series for real energy use and the capital stock to calibrate the energy-capital ratio. We match a GDP-share for energy expenditures.[4]
- The original uses a price series for energy to pin down the parameters of the stochastic process for energy prices.
- The original work considers an annual calibration. We follow suit in the code.
- We work with a different utility function.
- We introduce an energy-efficiency shock into the model.
- We continue to use the notation found in the handbook chapter.

4.1 The model

The utility maximization problem for the household side is equivalent to the one in the basic model extended to include capital accumulation. To conserve space, we do not reproduce the household equations here.

Our production function is given by

$$y_t = \left(z_t^n n_t\right)^{\theta} \left[\left(1-\alpha_e\right) k_{t-1}^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e \left(z_t^e e_t\right)^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}(1-\theta)}, \tag{24}$$

[4] For those just learning about these models, a useful exercise would be to head to the original paper for the exact details of how the authors approached the calibration.

where θ is the cost-share of labor in gross output, e_t is energy use, σ_e is the elasticity of substitution between capital and energy, and α_e is the share of energy in the production of the service flow from the capital good. Besides the change in notation, the other difference between this function and the original is we have added in the energy-efficiency shock, z_t^e.

The first order conditions for the firm are given by

$$w_t = \theta \frac{y_t}{n_t}, \tag{25}$$

$$r_t^k = \frac{(1-\theta)(1-\alpha_e) y_t k_{t-1}^{-\frac{1}{\sigma_e}}}{(1-\alpha_e)\, k_{t-1}^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e \left(z_t^e e_t\right)^{\frac{\sigma_e - 1}{\sigma_e}}}, \tag{26}$$

$$p_t^e = \frac{(1-\theta)\alpha_e y_t e_t^{-\frac{1}{\sigma_e}} z_t^{e\,\frac{\sigma_e - 1}{\sigma_e}}}{(1-\alpha_e)\, k_{t-1}^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e \left(z_t^e e_t\right)^{\frac{\sigma_e - 1}{\sigma_e}}}. \tag{27}$$

4.2 Calibration and steady state

Our calibration approach differs somewhat from the original work. We first normalize energy prices, output and labor to 1. We then calibrate the steady state value of e using a GDP-share of total spending on fossil fuels. Following the original, we consider total spending by firms and households on not only oil, but also natural gas and coal. This differs from the basic model in two ways: it broadens the definition of energy and it lumps together firm and household spending. The data source, however, is the exact same one we used to get a measure of spending on petroleum when calibrating the basic model.[5] Based on this data, the average GDP-share over the period of 1986-2015 is 0.054. This gives a cost-share of energy in gross output of 0.051.

As mentioned earlier we consider an annual calibration. The parameter β is set to 0.96, η to 1, δ to 0.10 and θ to 0.64. The value of σ_e is set to 0.59, based on one of the values considered in Kim and Loungani (1992).[6] Given the calibration of β and δ, one immediately gets the value of r^k from the household's first order condition for capital. The value of w can be derived from the firm's first-order condition for labor.

We use a numerical solver to find the steady state values for α_e, k, z^e and z^n. The production function and the firm's first order conditions for capital and energy provide three of the necessary equations. Using the firm's first-order conditions for capital and energy, one can see that setting $z^e = k/e$ ensures that α_e is exactly equal to the share of energy in producing the service flow of the capital good.[7] We impose this constraint as the fourth equation in the code. It can be shown, using the production function, that this implies that $z^n = k^{\frac{\theta-1}{\theta}}$.

[5] The data can be found in an accompanying Excel file.

[6] The equivalent parameter in the original is given by $\nu = -\frac{\sigma_e - 1}{\sigma_e}$.

[7] Divide one of the first order conditions by the other and then solve for α_e to see this more clearly.

Given the solutions for α_e, k, z^e and z^n, one can then solve for the remaining steady state values using the model's equations.

4.3 Impulse response functions

We show impulse response functions for energy prices and GDP in Figure (2). We normalize the size of the three shocks so that they all increase the price of energy by 10 percent. This implies that we are looking at a negative energy supply shock, a positive productivity shock and a negative energy-efficiency shock.

5 The production function of Finn (2000)

Finn (2000) introduces a Cobb-Douglas production function where gross output is produced using labor and capital. In her setup, however, the utilization rate of capital can vary over time, the utilization of capital requires energy, and the depreciation rate of capital is positively related with the utilization rate. Energy use is broadly defined to include electricity, natural gas, coal and oil. As with Kim and Loungani (1992), this is important primarily in regards to steady state shares. Energy prices are exogenous in the original work but we assume it is the supply of energy that is exogenous rather than the price.

5.1 The model

The original work provides a very good description of the model's equations and how to solve the steady state. We provide an overview here but refer the reader to the original for full details.

The production function is given by

$$y_t = \left(z_t^n n_t\right)^{\theta} \left(k_{t-1} u_t\right)^{1-\theta}, \tag{28}$$

where θ is the labor-share of gross output and u_t is the utilization rate of capital. Utilization of capital is linked to energy via the following equation,

$$\frac{e_t}{k_{t-1}} = \frac{\nu_0 u_t^{\nu_1}}{\nu_1}, \tag{29}$$

where e_t is energy demanded by the firms, $\nu_0 > 0$ and $\nu_1 > 1$. Depreciation is positively related with the utilization rate,

$$\delta_t = \frac{\omega_0 u_t^{\omega_1}}{\omega_1}, \tag{30}$$

with $\omega_0 > 0$ and $\omega_1 > 1$.

The firm's problem is to maximize profit,

$$\max \pi_t = y_t - w_t n_t - r_t^k k_{t-1} u_t, \tag{31}$$

choosing labor input, n_t, and effective capital, $k_{t-1}u_t$. The two first order conditions are given by

$$w_t = \theta \frac{y_t}{n_t}, \tag{32}$$

$$r_t^k = (1-\theta)\frac{y_t}{k_{t-1}u_t}. \tag{33}$$

In Finn's setup, the household chooses energy use, utilization, investment and next period's capital stock, in addition to consumption and labor supply. Making use of our utility function and notation, we have the utility maximization problem as

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) - \chi n_t^{1+\eta}/(1+\eta)], \tag{34}$$

subject to the budget constraint

$$c_t + i_t + p_t^e e_t = w_t n_t + r_t^k u_t k_{t-1} + d_t, \tag{35}$$

and the capital accumulation equation

$$i_t = k_t - (1-\delta_t)k_{t-1}. \tag{36}$$

We don't go over the algebra here but it's possible to boil the household's first-order conditions into the following four equations:

$$\frac{1}{c_t} = \lambda_t, \tag{37}$$

$$\chi n_t^{1+\eta} = w_t \lambda_t, \tag{38}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k u_{t+1} + (1-\delta_{t+1}) - \frac{p_{t+1}^e e_{t+1}}{k_t} \right], \tag{39}$$

$$r_t^k = \omega_0 u_t^{\omega_1 - 1} + p_t^e \nu_0 u_t^{\nu_1 - 1}. \tag{40}$$

5.2 Calibration and steady state

Our procedure for calibration and solving the steady state is very similar to that in Finn (2000). We set β to 0.99, δ to 0.025, and θ to 0.64. We normalize y, p, and n to 1, which allows us to solve for w using the firm's first order condition. We calibrate the value of e by matching a GDP-share, except this time we match total spending on all forms of energy. The GDP-share from 1986-2015 is 0.075, which translates into a value of 0.069 for the ratio of e to y. We follow Finn (2000) and set the utilization rate to 0.82. One can then solve for k using the household's first order condition for capital, after substituting out a two terms using the firm's first order condition for capital and the equation for utilization. Given k, one can then solve for r^k using the firm's first order condition for capital and get the value of z^n using the production function. The values for i, c, λ, and χ come immediately thereafter from other steady state equations.

Solving for the values of ω_0, ω_1, ν_0 and ν_1 is trickier and requires a numerical solver. We follow the procedure in Finn (2000) to get the values of those four parameters. Interested readers are referred to the original work for a detailed description of how this is done.

5.3 Impulse response functions

We show impulse response functions for energy prices, GDP and utilization of capital in Figure (3). We normalize the size of the shocks so that they all increase the price of energy by 10 percent. This implies that we are looking at a negative energy supply shock and a positive productivity shock.

6 A model with household demand for oil

We now extend the basic model (without capital) to include household demand for oil. We introduce a very simple model that distinguishes between a non-oil consumption good, which we denote as c_{nt}, and consumption of oil, o_{ct}. In practice, exactly what "oil" refers to will depend upon the question of interest and the country being modeled. If one specifically focuses on oil, then consumption would typically mean consumption of fuels used for transportation and home heating. In the U.S., recently, this would primarily be gasoline and diesel/heating oil. Of course, one can consider broader measures of energy that would include natural gas and/or electricity. In the U.S., for example, both of those forms of energy are important for home heating purposes. In the context of the models introduced here, broadening the definition would only be important for calibrating steady state shares.

It is worthwhile pointing out that more sophisticated models introduce a durable good that is used in conjunction with oil (or energy) to provide a service flow from the durable good. We do not present such a model here but for those interested in learning more about them we recommend Dhawan and Jeske (2008) or Plante and Traum (2012) as useful references.

6.1 The model

The representative agent chooses consumption of oil, consumption of the non-oil good, and labor to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) - \chi n_t^{1+\eta}/(1+\eta)], \tag{41}$$

where β is the discount factor, c_t is aggregate consumption, n_t is hours worked, η is the inverse Frisch-elasticity of labor and χ is the weight on the dis-utility from working.

It is common to a use a CES function for the aggregator, as this allows one to vary the elasticity of substitution between the oil and non-oil consumption good. We consider the following form,

$$c_t = \left[(1-\alpha_{oc})\, c_{nt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_{oc} \left(z_t^{oc} o_{ct} \right)^{\frac{\sigma_c - 1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c - 1}}, \tag{42}$$

where α_{oc} is the consumption-expenditure share of oil, σ_c is the elasticity of substitution between the oil and non-oil consumption good.

Utility is maximized subject to a budget constraint,

$$c_{nt} + p_t^o o_{ct} = w_t n_t + d_t, \tag{43}$$

where p_t^o is the relative price of oil, w_t is the real wage and d_t are dividends from the firm. The household takes prices and wages as given when making its decisions.

Denoting λ_t as the multiplier on the budget constraint, the first-order conditions are

$$(1-\alpha_{oc}) \frac{c_t^{\frac{1}{\sigma_c}}}{c_t} c_{nt}^{-\frac{1}{\sigma_c}} = \lambda_t, \tag{44}$$

$$\alpha_{oc} \frac{c_t^{\frac{1}{\sigma_c}}}{c_t} o_{ct}^{-\frac{1}{\sigma_c}} z_t^{oc\, \frac{\sigma_c - 1}{\sigma_c}} = p_t^o \lambda_t, \tag{45}$$

$$w_t \lambda_t = \chi n_t^{\eta}. \tag{46}$$

The production side of the model is exactly the same as the basic model, although the notation is bit more detailed. We go over some of the equations again just to present the new notation, which makes its way into the computer code.

The firm uses a CES production function of the form

$$y_t = \left[(1-\alpha_{op}) \left(z_t^n n_t \right)^{\frac{\sigma_p - 1}{\sigma_p}} + \alpha_{op} \left(z_t^{op} o_{pt} \right)^{\frac{\sigma_p - 1}{\sigma_p}} \right]^{\frac{\sigma_p}{\sigma_p - 1}}, \tag{47}$$

where o_{pt} is oil used by the firm, z_t^n is a labor-augmenting productivity shock and z_t^{op} is an oil-efficiency shock. The term α_{op} controls the cost-share of oil in gross output, y_t. The σ_p term is the elasticity of substitution between oil and labor (value-added).

The first-order conditions for the firm are given by

$$w_t = (1-\alpha_{op}) \left(\frac{y_t}{n_t} \right)^{\frac{1}{\sigma_p}} z_t^{n \frac{\sigma_p - 1}{\sigma_p}}, \tag{48}$$

$$p_t^o = \alpha_{op} \left(\frac{y_t}{o_{pt}} \right)^{\frac{1}{\sigma_p}} z_t^{op \frac{\sigma_p - 1}{\sigma_p}}. \tag{49}$$

The market clearing condition for the oil market is now given by

$$o_{pt} + o_{ct} = o_t^s. \tag{50}$$

One can derive a resource constraint for the economy using the household's budget constraint and the firm's first order conditions. The resource constraint for the economy is

$$c_{nt} + p_t^o o_{ct} = y_t - p_t^o o_{pt}. \tag{51}$$

6.2 Calibration and steady state

Relative to the baseline model, there are a few additional parameters and starting values. Notwithstanding this, the procedure we use is quite similar to the one we used to calibrate the baseline model.

The parameters β, η and χ follow the calibration used for the baseline model, i.e. β is 0.99, η is 1 and the value of χ is determined from the household's first order condition for labor. The parameters σ_c and σ_p are directly connected with the price-elasticity of demand for oil. While the model as written out allows the elasticities to differ between households and firms, it is usually the case that the two are set equal to each other. So, in practice, there is just one parameter that needs to be set. As discussed in the handbook chapter, a generally accepted finding is that short-run price elasticities for oil are low, i.e. well below unity. For illustrative purposes, we consider a value of 0.25 and follow suit with the literature by setting both σ_p and σ_c equal to that value.

We continue to normalize y, n, z^n and p^o to 1. Using the steady-state production function, one can then show that $z^o = 1/o$. This normalization will ensure that α_{op} equal the cost-share of oil. We calibrate the value of o using the same GDP-share data we used in the basic model. This gives a value of 0.0196 for the cost-share of oil and for o. The value for α_{op} can be solved for using the firm's first order condition for oil. The firm's first order condition for labor gives $w = 1 - \alpha_{op}$.

The calibration of α_{oc} is done using the expenditure data on household spending already introduced in the handbook chapter. For the U.S., data on nominal spending on petroleum products is available from the U.S. Energy Information Administration (EIA) from 1970 to 2015.[8] This covers spending by both firms and households. Another set of data from the Bureau of Economic Analysis (BEA) provides annual nominal spending by households on motor gasoline, heating oil and other fuels.[9] This can be subtracted off from the EIA data to produce an estimate of nominal spending by firms on petroleum products and another for households.

Using this data, we found that over the Great Moderation period (1986-2015) the average (nominal) GDP-share for total petroleum spending was 0.04. This was evenly split between households and firms. Defining g^c as the GDP-share of household spending on oil, we have $o_c = g_c\,(y - o_p)$, where we have imposed our normalization that $p^o = 1$. With the value of o_c in hand, one can then solve for c_n using the resource constraint.

The modeler can face some of the same numerical issues solving for the value of α_{oc} as can occur with α_{op} due to the small expenditure-share of oil and the low elasticity of substitution. Again, this can be avoided by assuming a particular normalization for z^{oc}, which ends up being $z^{oc} = \frac{c_n}{o_c}$. This normalization will ensure that the CES weight is equal to the expenditure share of oil. In either case, the value for α_{oc} can be solved for by combining the household's first order condition for oil and the non-oil good. The value for c comes from its definition, while λ and χ are solved from the remaining equations.

6.3 Calculating GDP

The addition of household demand for oil means we need to revisit the equation used to calculate GDP. The expenditure approach would suggest working with some variant of the equation $c_{nt} + p_t^o o_{ct}$. In the code, we use a fixed-price measure based on this,

$$gdp_t^{ex} = c_{nt} + o_{ct}. \tag{52}$$

[8] See Table ET1 of the U.S. Energy Information Administration's 2015 State Energy Data report. Please note the data used here includes spending on natural gas liquids.

[9] See specifically "Motor vehicle fuels, lubricants and fluids" and "Fuel oil and other fuels."

6.4 Impulse response functions

We show impulse response functions for oil prices and GDP Figure (4). We normalize the size of the shocks so that they all increase the price of energy by 10 percent. This implies that we are looking at a negative energy supply shock, a positive productivity shock and negative energy efficiency shocks (i.e. it takes more oil to do what was once done).

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7 Figures

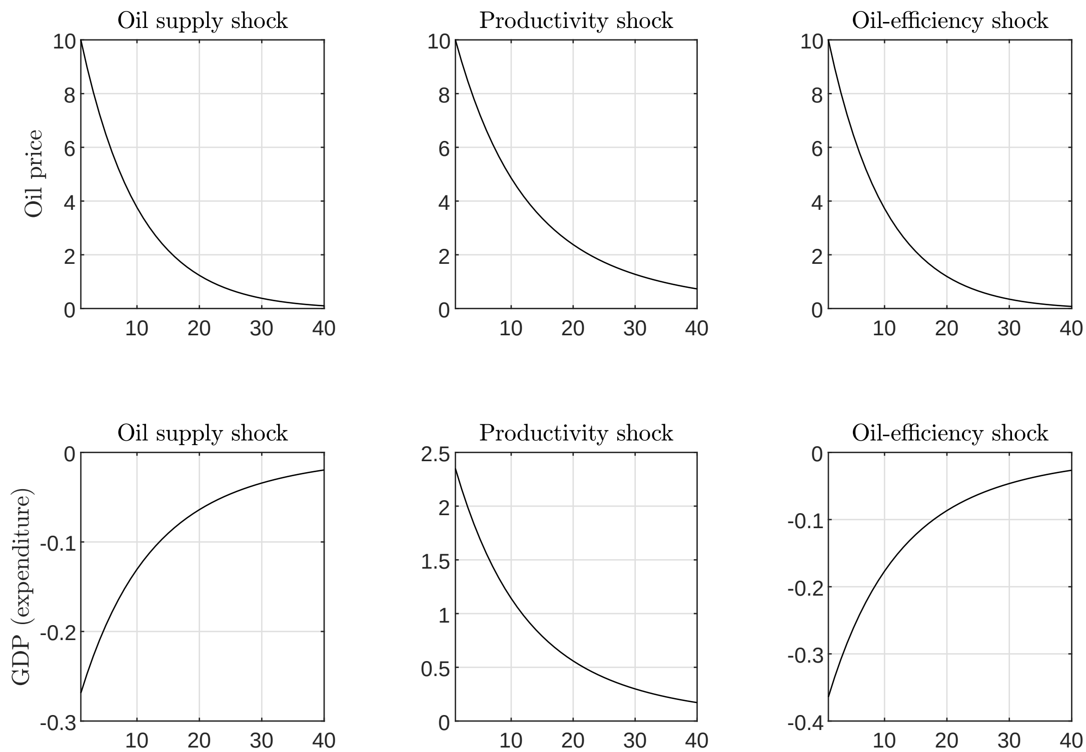

Figure 1: Impulse response functions for oil prices and GDP in the model with capital.

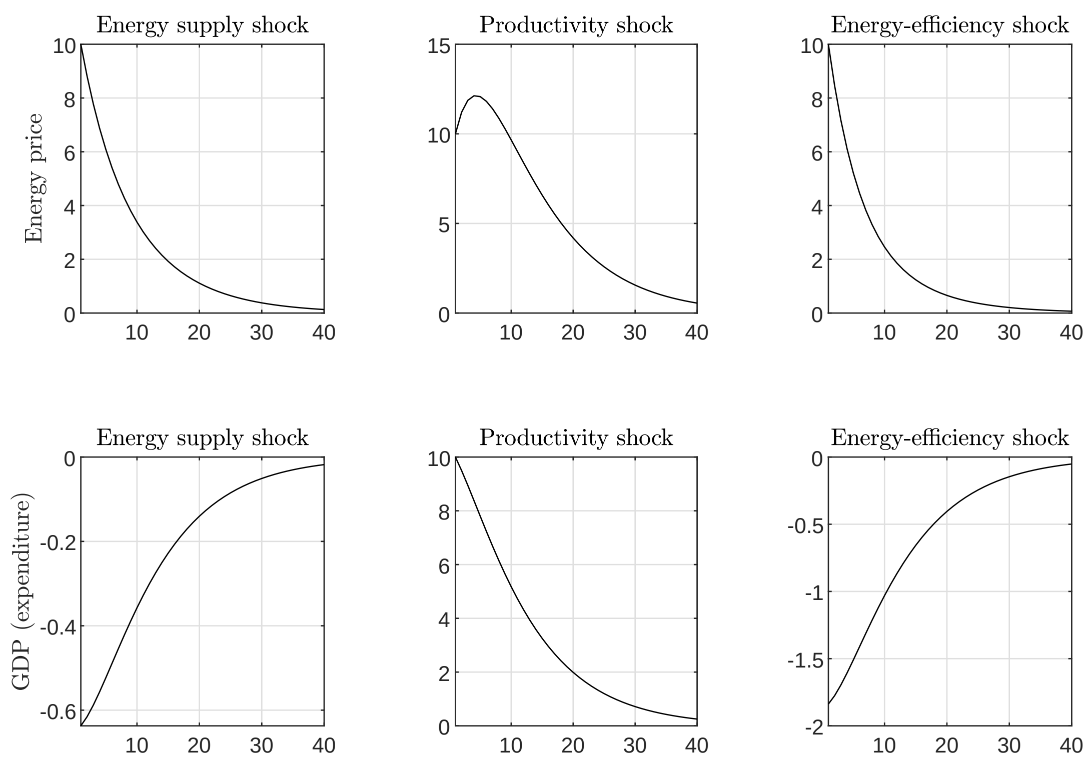

Figure 2: Impulse response functions for oil prices and GDP in the model with capital. This uses the Kim and Loungani production function.

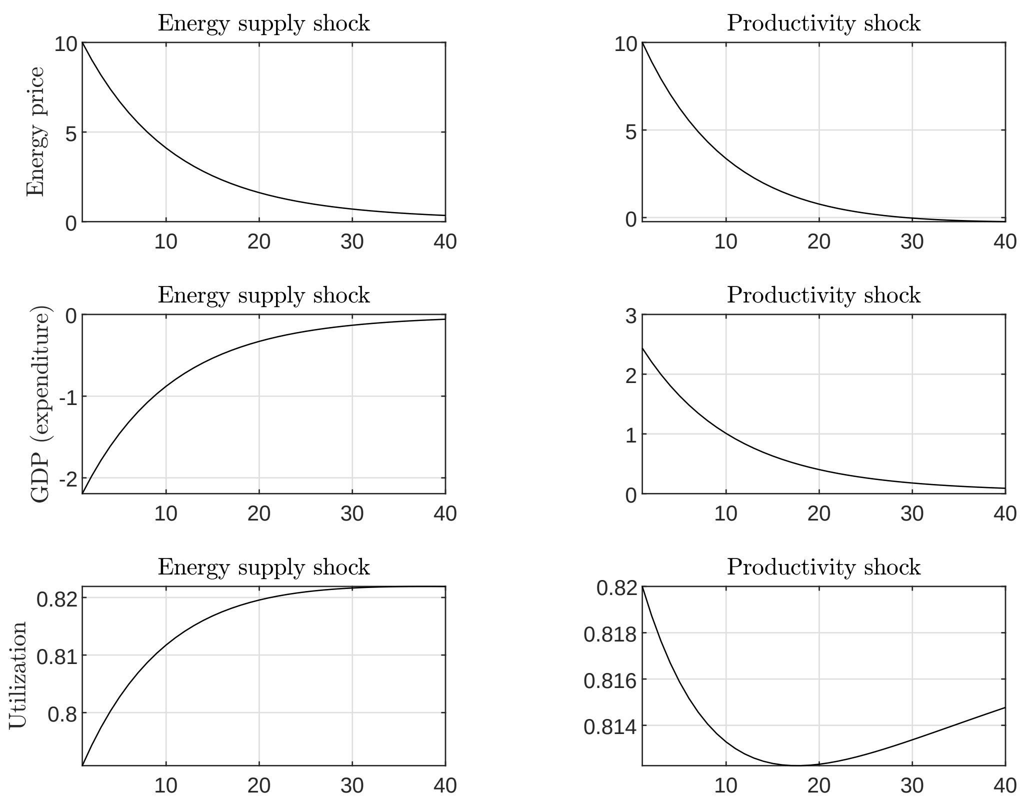

Figure 3: Impulse response functions for oil prices and GDP in the model with capital. This uses the Finn production function.

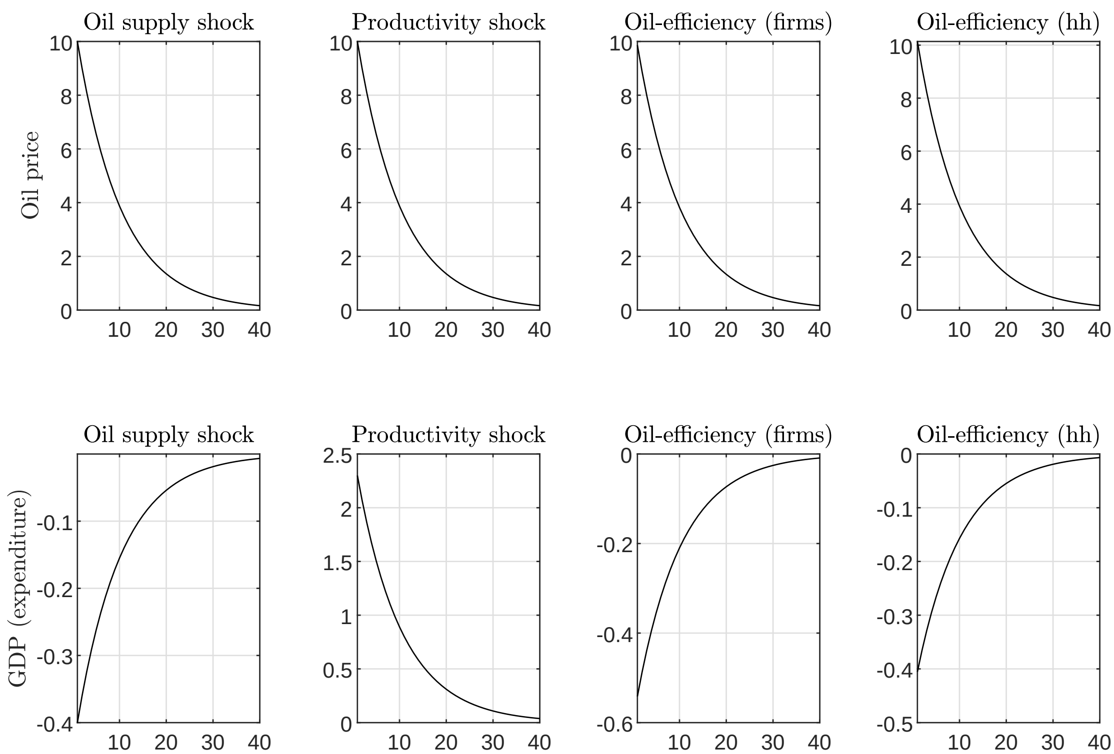

Figure 4: Impulse response functions for oil prices and GDP in the model with household demand.