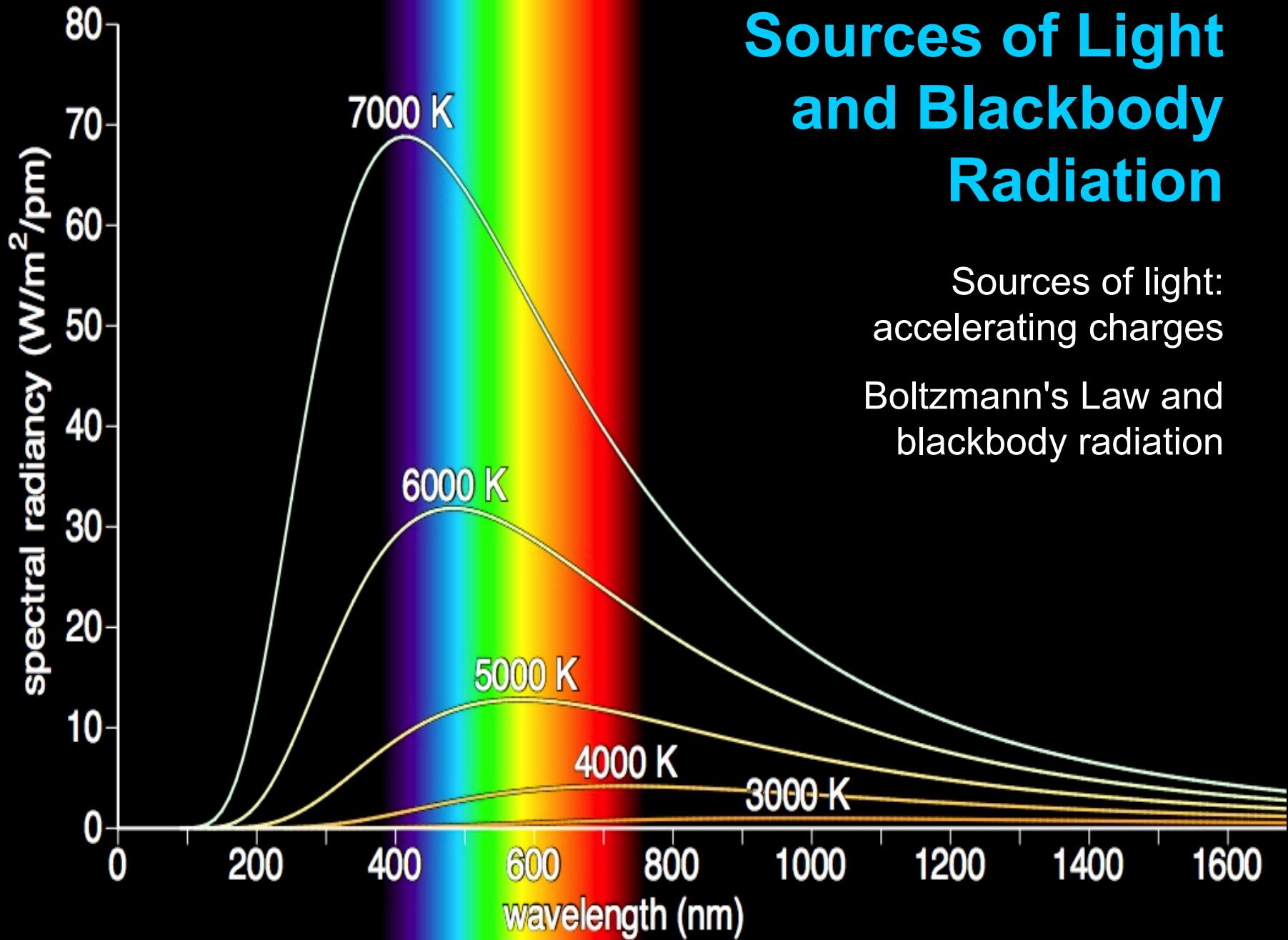


Sources of Light and Blackbody Radiation

Sources of light:
accelerating charges

Boltzmann's Law and
blackbody radiation



Blackbody Radiation

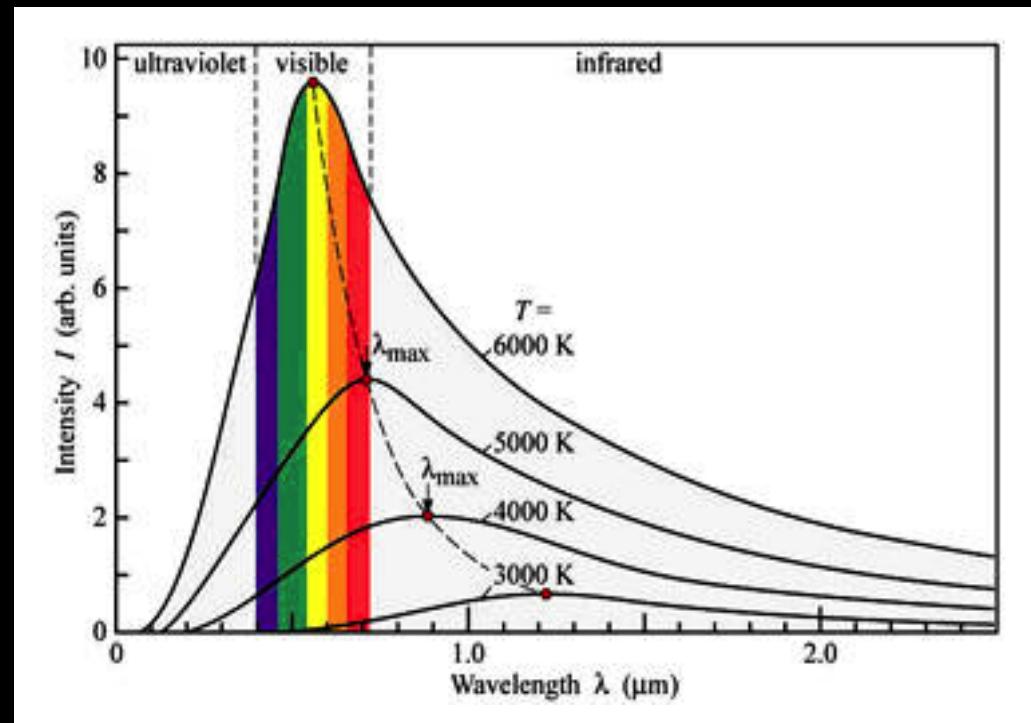
Example of a blackbody:
almost everything!

Blackbody radiation is emitted from a hot body. It's anything but black!

The name comes from the assumption that the body absorbs at every frequency and hence would look black at low temperature.

Blackbodies emit spectra that depend only on their temperature.

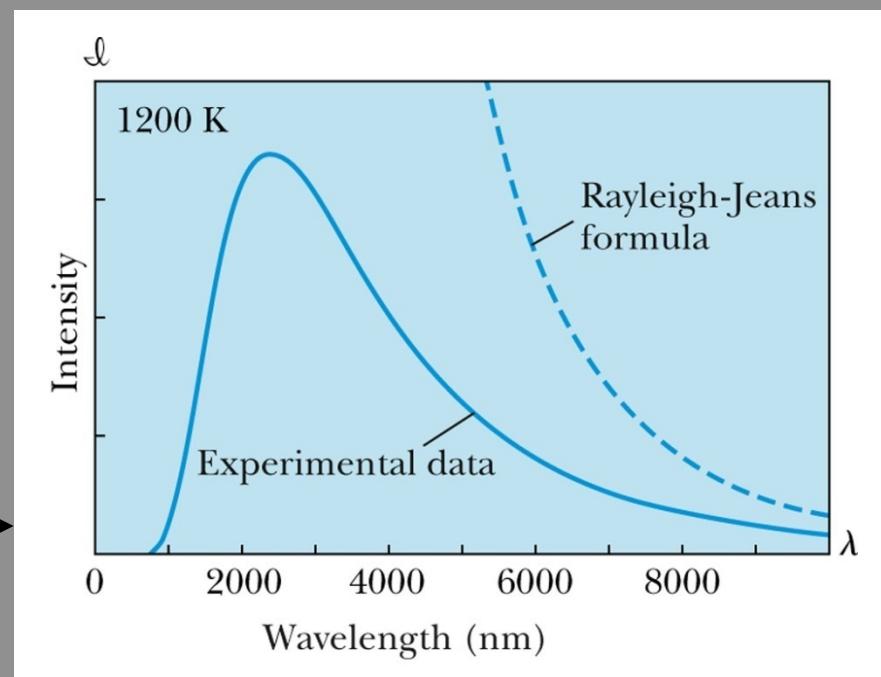
19th-century physicists recognized that such sources were fundamentally important and so merited thought.



Explaining blackbody radiation was one of the great challenges of the 19th century.

In a speech to the Royal Institution in 1900, Lord Kelvin described two “dark clouds on the horizon” of physics:

One was the “aether.”
The other was the failure of classical physics to explain blackbody radiation.



This huge discrepancy was known as the “ultraviolet catastrophe.”

Where does light come from?

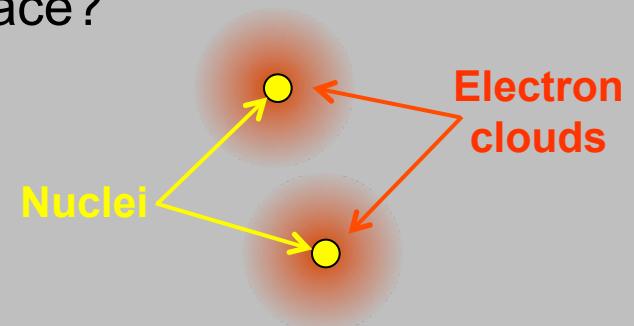
We've seen that Maxwell's Equations (i.e., the wave equation) describe the **propagation** of light.

But where does light come from in the first place?

Light is emitted by matter. This happens via the matter's **polarization**:

$$\mathcal{P}^I(t) = \sum_i q_i \overset{\mathbf{r}}{x}_{q_i}(t) = N q \overset{\mathbf{r}}{x}_q(t)$$

↑
if all the charges and
their motions are the
same

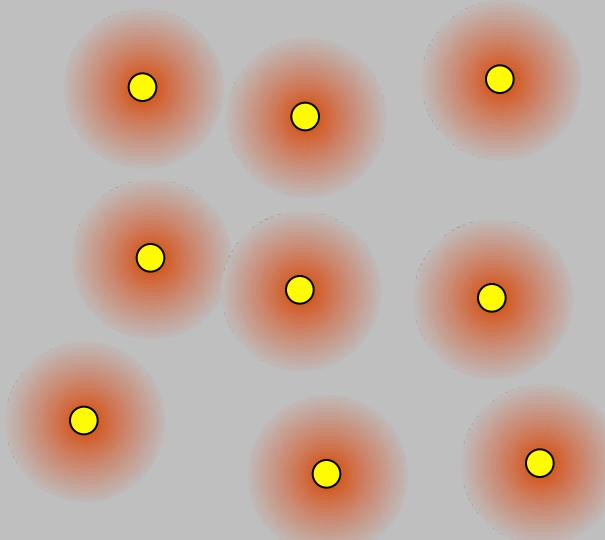


Note that matter's polarization is analogous to the polarization of light. Indeed, it will yield emission of light with the same polarization direction.

where N is the number density of charged particles, q_i is the charge of each particle, and $\overset{\mathbf{r}}{x}_{q_i}(t)$ is the position of the charge, q_i .

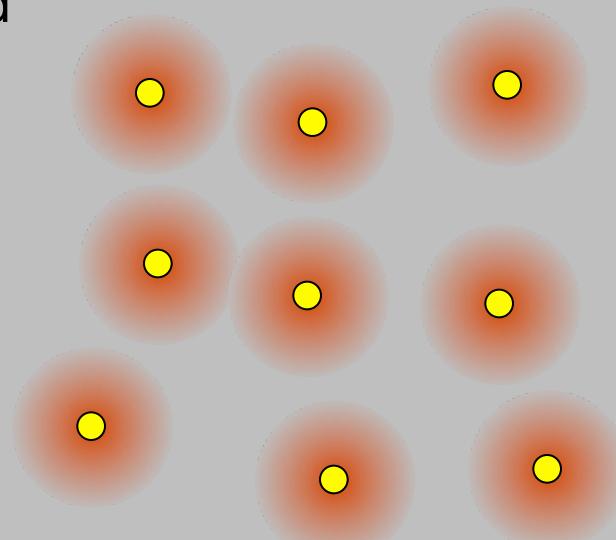
Polarized and Unpolarized Media

Slightly polarized medium (lots of cancellations, but not zero)



Unpolarized media

Highly polarized medium



On the right, the displacements of the charges are correlated, so it is polarized at any given time (and its polarization is oscillating).

It will emit light at its oscillation frequency.

Maxwell's Equations in a Medium

The induced polarization, $\overset{\text{I}}{\mathcal{P}}$, contains the effect of the medium and is included in Maxwell's Equations:

$$\overset{\text{r}}{\nabla} \cdot \overset{\text{r}}{\mathcal{E}} = 0$$

$$\overset{\text{r}}{\nabla} \times \overset{\text{r}}{\mathcal{E}} = -\frac{\partial \overset{\text{I}}{\mathcal{B}}}{\partial t}$$

$$\overset{\text{r}}{\nabla} \cdot \overset{\text{r}}{\mathcal{B}} = 0$$

$$\overset{\text{r}}{\nabla} \times \overset{\text{r}}{\mathcal{B}} = \mu_0 \epsilon_0 \frac{\partial \overset{\text{r}}{\mathcal{E}}}{\partial t} + \mu_0 \frac{\partial \overset{\text{I}}{\mathcal{P}}}{\partial t}$$

This extra term also adds to the wave equation, which is known as the **inhomogeneous wave equation**:

$$\overset{\text{r}}{\nabla}^2 \overset{\text{r}}{\mathcal{E}} - \frac{1}{c^2} \frac{\partial^2 \overset{\text{r}}{\mathcal{E}}}{\partial t^2} = \mu_0 \frac{\partial^2 \overset{\text{I}}{\mathcal{P}}}{\partial t^2} = \mu_0 Nq \frac{\partial^2 \overset{\text{r}}{x}_q}{\partial t^2}$$

The polarization is the **source term** and tells us what light will be emitted.

Note that $\frac{\partial^2 \overset{\text{r}}{x}_q}{\partial t^2}$ is just the charges' acceleration!

So light is emitted by accelerating charges!

Sources of Light

Accelerating charges emit light.

Linearly accelerating charge:



Synchrotron radiation—
light emitted by charged
particles deflected by a
magnetic field:



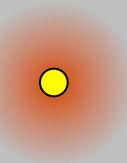
Bremsstrahlung (Braking radiation)—
light emitted when charged particles
collide with other charged particles:



But most light in the universe is emitted by atomic and molecular vibrations.

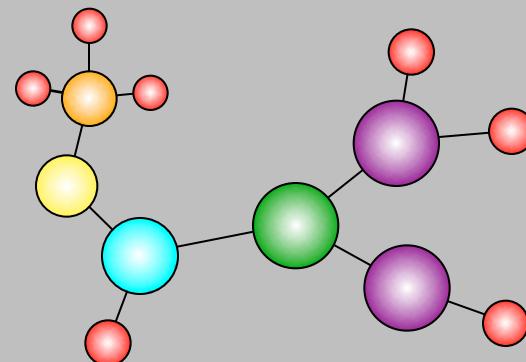
Electrons vibrate in their motion around nuclei.

High frequency: $\sim 10^{14} - 10^{17}$ cycles per second.



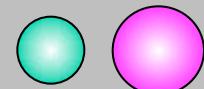
Nuclei in molecules vibrate with respect to each other.

Intermediate frequency:
 $\sim 10^{11} - 10^{13}$ cycles per second.



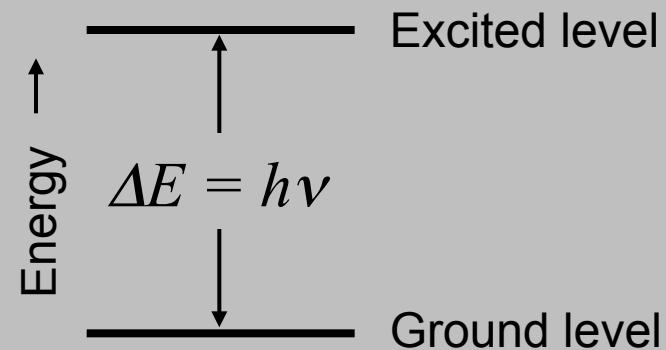
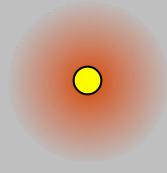
Nuclei in molecules rotate.

Low frequency: $\sim 10^9 - 10^{10}$ cycles per second.



Atomic and molecular vibrations correspond to excited energy levels in quantum mechanics.

Energy levels are everything in quantum mechanics.



The atom is vibrating at frequency, ν .

The atom is at least partially in an excited state.

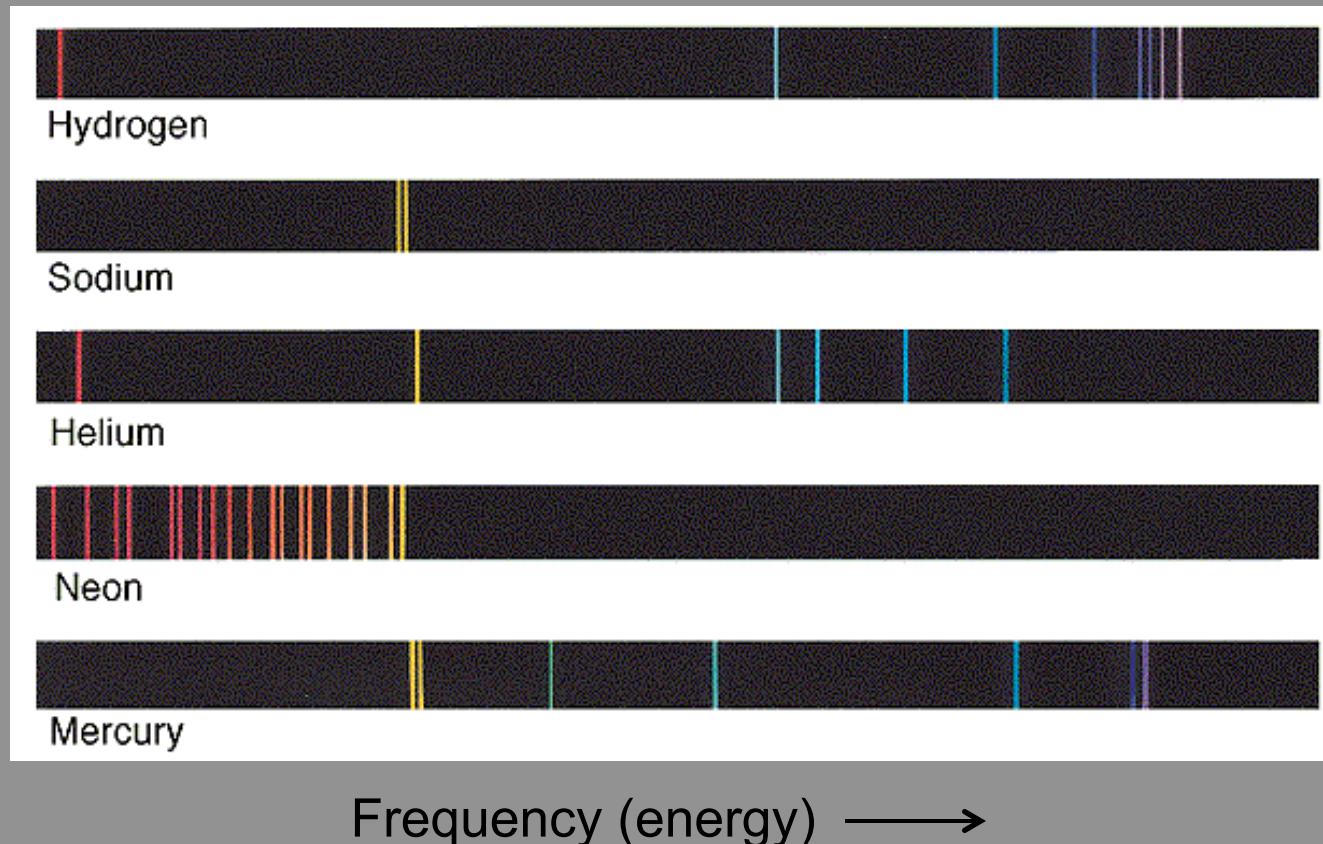
Excited atoms emit photons spontaneously.

When an atom in an excited state falls to a lower energy level, it emits a photon of light.



Molecules typically remain excited for no longer than a few nanoseconds. This is often also called **fluorescence** or, when it takes longer (say, ms), **phosphorescence**.

Different atoms emit light at different widely separated frequencies.



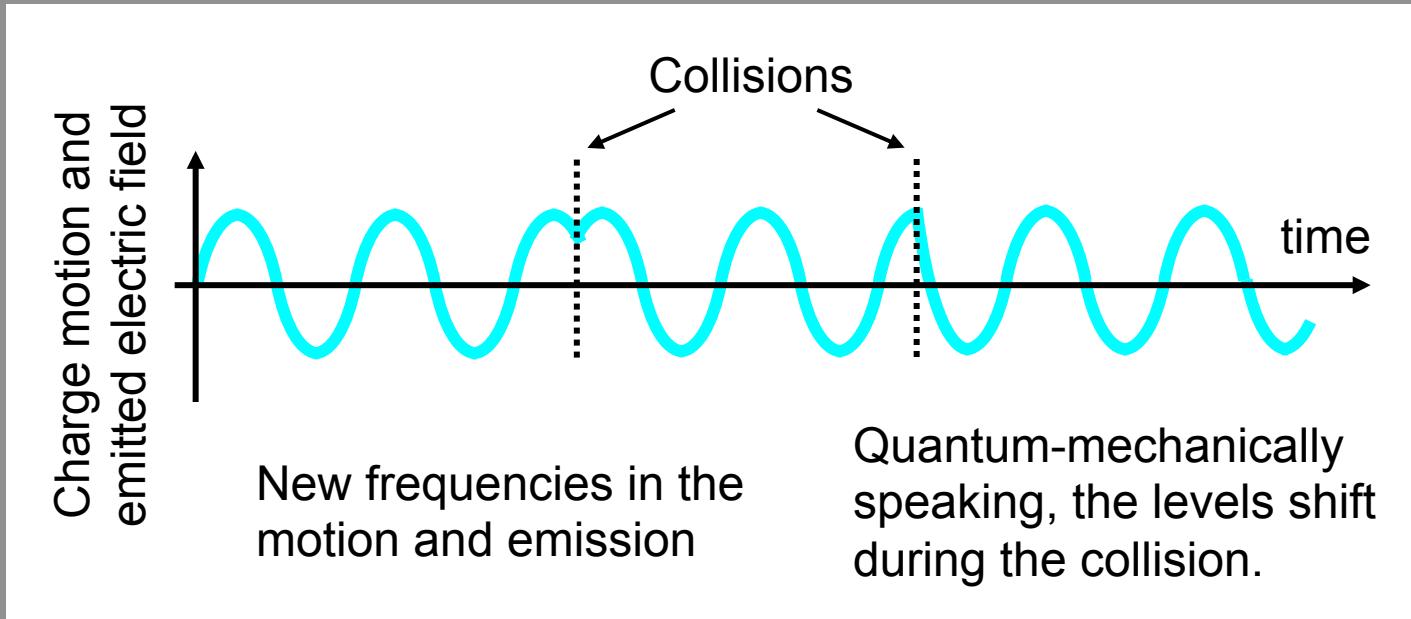
Each colored **emission line** corresponds to a difference between two energy levels.

These are **emission spectra** from gases of hot atoms.

Atoms lack nuclear vibrations and rotations and so have relatively simple energy level systems (and hence simple spectra).

Collisions broaden the frequency range of the charge motion and light emission.

A collision abruptly changes the phase of the motion and the light emission. So atomic and molecular emissions usually have broader spectra.

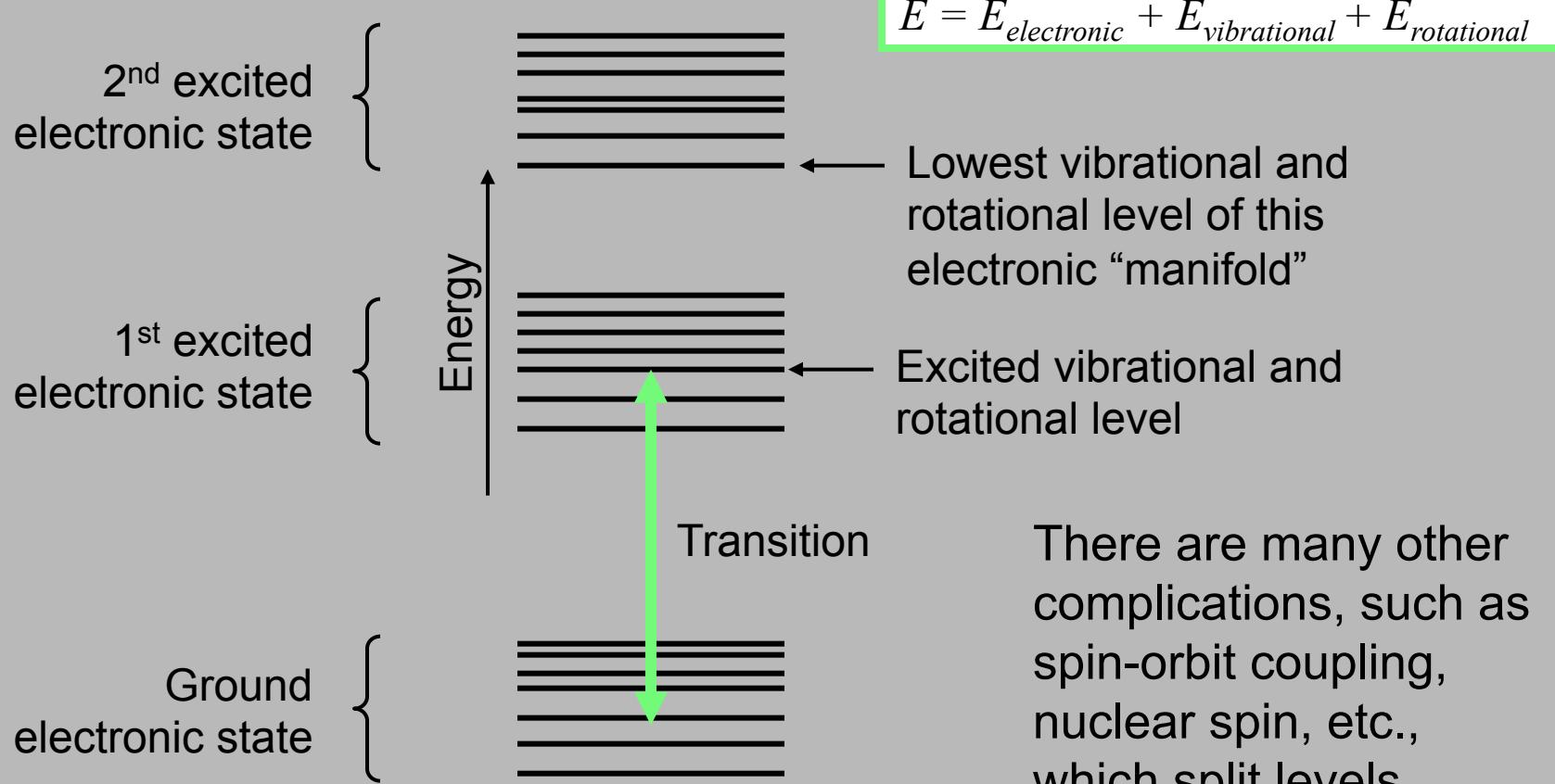


Gases at atmospheric pressure have emission widths of $\sim 10^9$ Hz.

Solids and liquids emit much broader ranges of frequencies ($\sim 10^{13}$ Hz!).

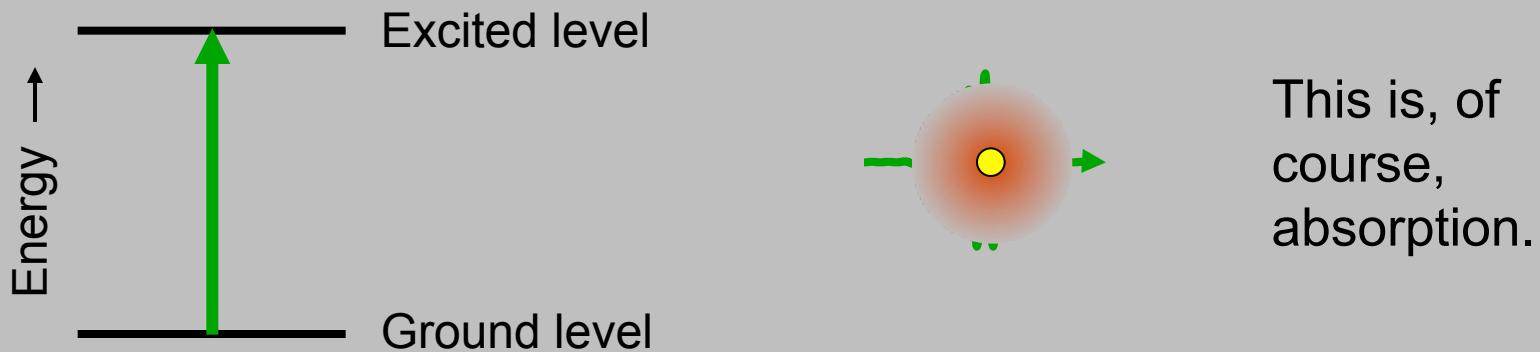
Molecules have many energy levels.

A typical molecule's energy levels:

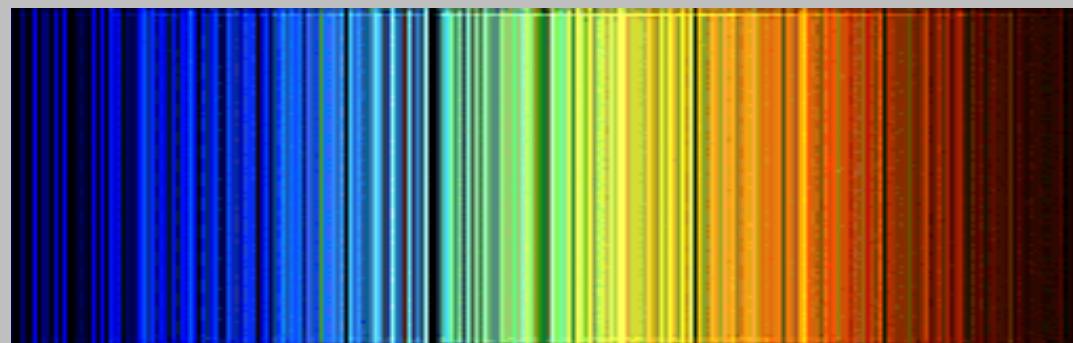


As a result, molecules generally have very complex spectra.

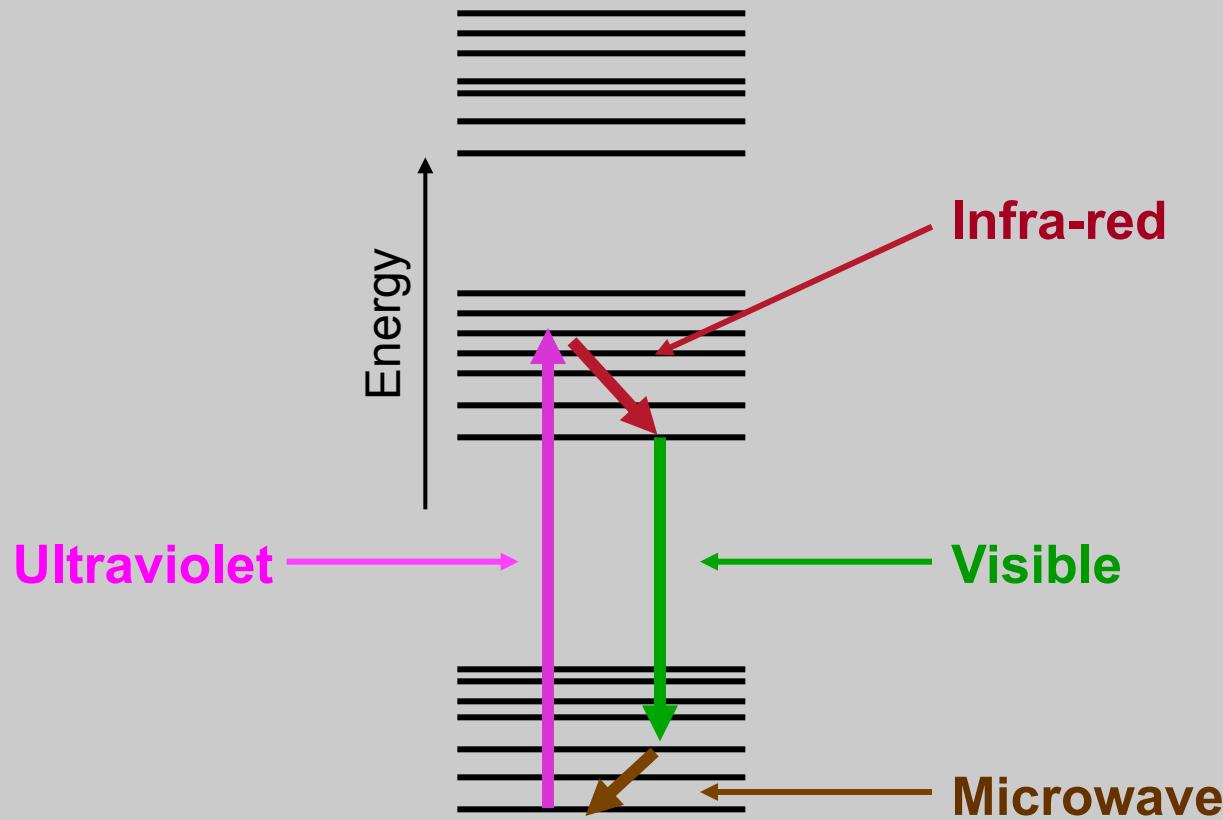
Atoms and molecules can also absorb photons, making a transition from a lower level to a more excited one.



Absorption lines in an otherwise continuous light spectrum due to a cold atomic gas in front of a hot source.



Decay from an excited state can occur in many steps.



The light that's eventually re-emitted after absorption often occurs at other colors.

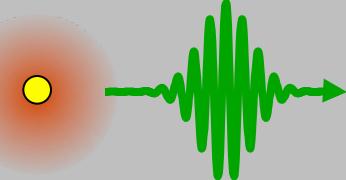
Einstein showed that another process,
stimulated emission, can also occur.

Spontaneous
emission

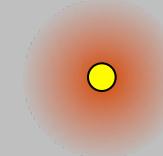
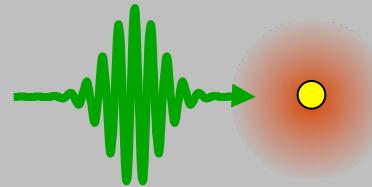
Before



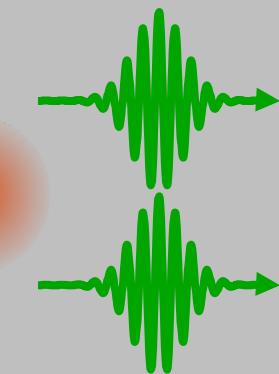
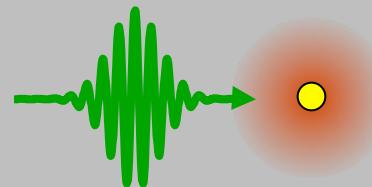
After



Absorption



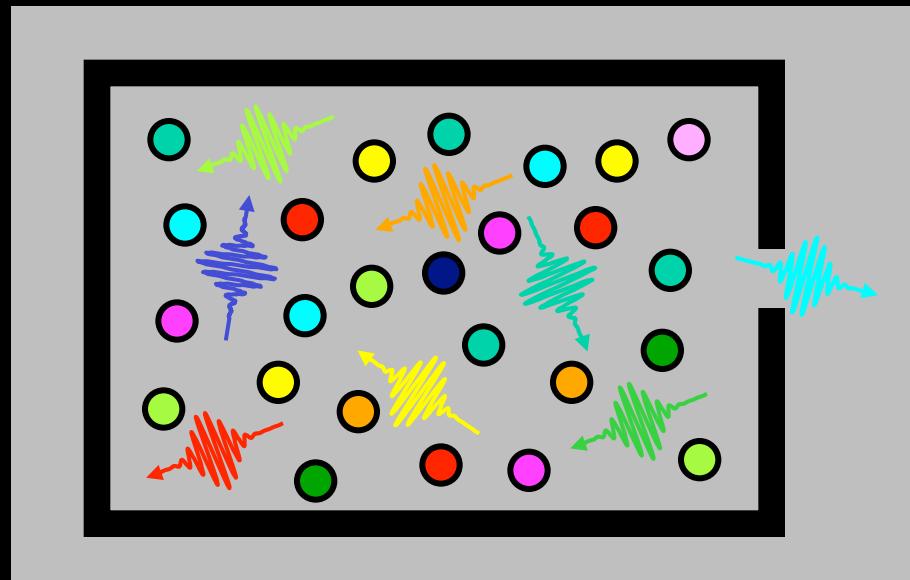
**Stimulated
emission**



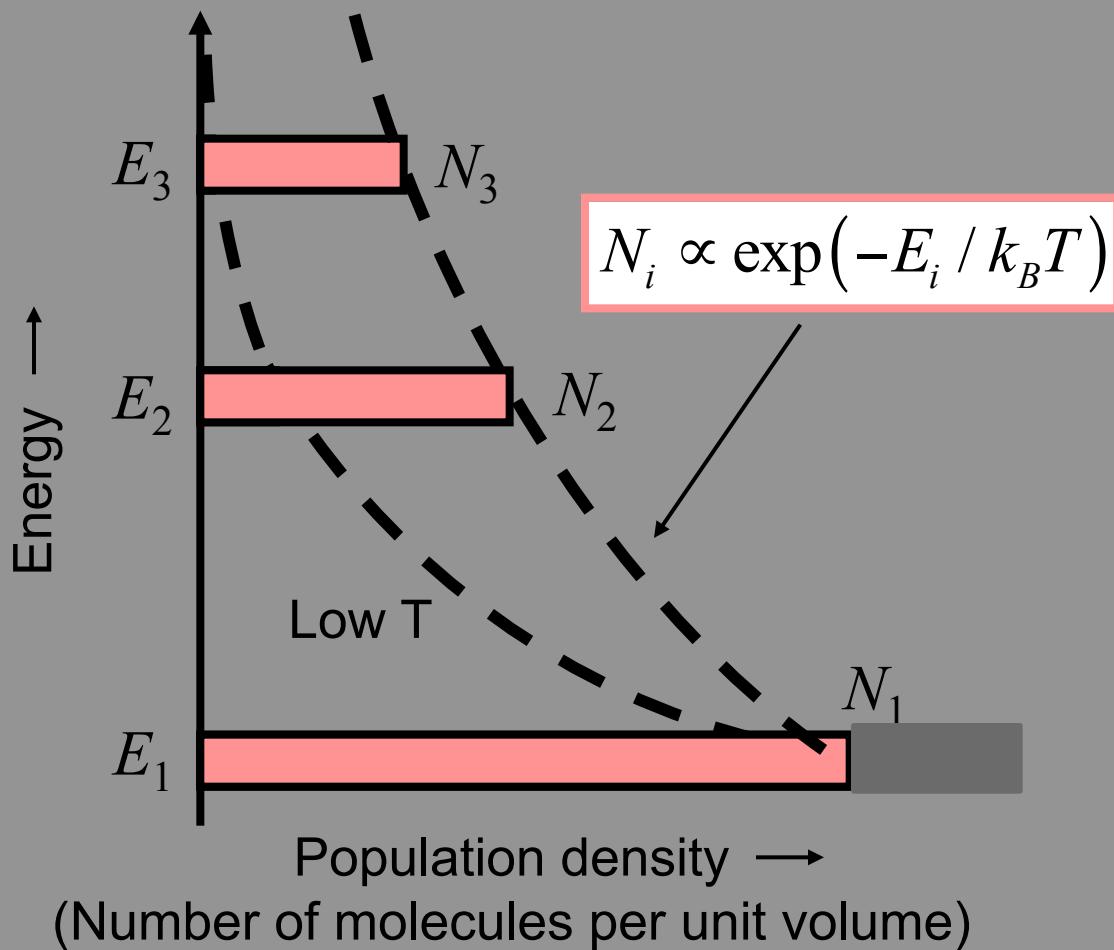
Blackbody Radiation

Blackbody radiation results from a combination of **spontaneous emission, absorption, and stimulated emission** occurring in a medium at a given temperature.

It assumes that the box is filled with many different molecules that together have transitions (absorptions) at every wavelength.



In what energy levels do molecules reside? Boltzmann Population Factors

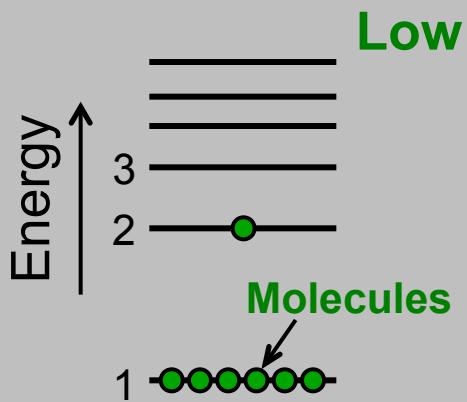


N_i is the number density (also known as the population density) of molecules in state i (i.e., the number of molecules per cm^3).

T is the temperature, and k_B is Boltzmann's constant = $1.3806503 \times 10^{-23} \text{ J}/\text{K}$

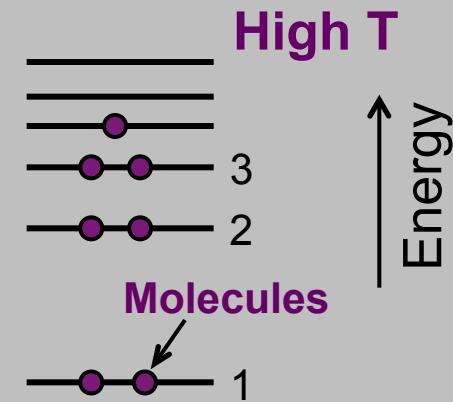
The Maxwell-Boltzmann Distribution

In the absence of collisions, (low T) molecules tend to remain in the lowest energy state available.



Collisions can knock a molecule into a higher-energy state. The higher the temperature, the more this happens.

$$\frac{N_2}{N_1} = \frac{\exp[-E_2 / k_B T]}{\exp[-E_1 / k_B T]} = \exp[-(E_2 - E_1) / k_B T]$$



The ratio of the population densities of two states is:

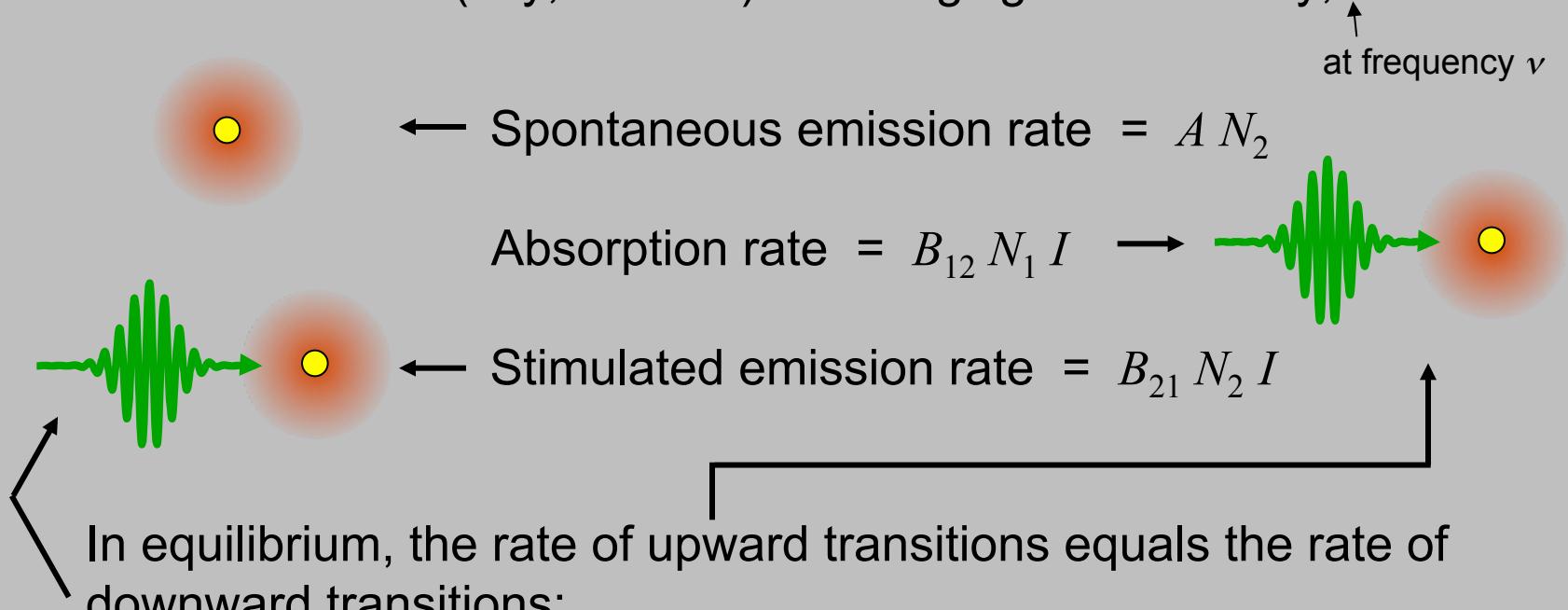
$$N_2 / N_1 = \exp(-\Delta E / k_B T), \quad \text{where } \Delta E = E_2 - E_1 = h\nu$$

frequency of
a photon for
an $E_2 - E_1$
transition

As a result, higher-energy states are always less populated than the ground state, and absorption is stronger than stimulated emission.

Einstein A and B Coefficients

In 1916, Einstein considered the various transition rates between molecular states (say, 1 and 2) involving light of intensity, I :



$$A N_2 + B_{21} N_2 I = \text{Down} = \text{Up} = B_{12} N_1 I$$

Dividing by $N_1 (A + B_{21}I)$ yields N_2/N_1 :

$$N_2 / N_1 = (B_{12} I) / (A + B_{21} I) = \exp(-h\nu / k_B T)$$

Recalling the Maxwell-Boltzmann distribution for N_2 / N_1

Blackbody Radiation

Now solve for the intensity in: $(B_{12} I) / (A + B_{21} I) = \exp(-h\nu/k_B T)$

Multiply by $(A + B_{21} I) \exp(h\nu/k_B T)$: $B_{12} I \exp(h\nu/k_B T) = A + B_{21} I$

Solve for I :

$$I = A / [B_{12} \exp(h\nu/k_B T) - B_{21}]$$

Dividing numerator and denominator by B_{21} :

$$I = (A/B_{21}) / [(B_{12}/B_{21}) \exp(h\nu/k_B T) - 1]$$

Now, when $T \rightarrow \infty$, I should also. As $T \rightarrow \infty$, $\exp(h\nu/k_B T) \rightarrow 1$.

So:

$$(B_{12}/B_{21}) - 1 = 0$$

And:

$$B_{12} = B_{21} \equiv B \quad \leftarrow \text{Coeff up = coeff down!}$$

And:

$$I = (A/B) / [\exp(h\nu/k_B T) - 1]$$

Blackbody Radiation

$$I = (A/B) / [\exp(h\nu/k_B T) - 1]$$

We can eliminate A/B based on other information and, writing in terms of the intensity **per unit frequency**, I_ν :

$$I_\nu = \frac{8\pi h\nu^3 / c^2}{\exp(h\nu / k_B T) - 1}$$

This is the total intensity per unit frequency (that is, in a range from ν to $\nu + \delta\nu$) emitted by an arbitrary blackbody.

We considered only two levels, but our approach was general and so applies to any two levels and hence to the entire spectrum.

Notice that it's independent of the size of the object, but, to obtain the emitted power, multiply by the surface area of the blackbody.

Writing the Blackbody Spectrum vs. Wavelength

Total intensity → $I = \int I_\nu(\nu) d\nu = \int \frac{8\pi h\nu^3 / c^2}{\exp(h\nu / k_B T) - 1} d\nu$

Units of $I_\nu(\nu)$: intensity per unit frequency

Change variables from ν to λ :

$$I = \int \frac{8\pi h (c/\lambda)^3 / c^2}{\exp(h(c/\lambda) / k_B T) - 1} \left| \frac{d\nu}{d\lambda} \right| d\lambda$$
$$\frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$I = \int \frac{8\pi h c^2 / \lambda^5}{\exp(hc / \lambda k_B T) - 1} d\lambda = \int I_\lambda(\lambda) d\lambda$$

$$I_\lambda(\lambda) = \frac{8\pi h c^2 / \lambda^5}{\exp(hc / \lambda k_B T) - 1}$$

Units of $I_\lambda(\lambda)$: intensity per unit wavelength

Total Emitted Blackbody Intensity: the Stefan-Boltzmann Law

Performing either of the integrals of blackbody emitted light over all frequencies or wavelengths yields a very simple result for the total emitted blackbody intensity at its surface:

$$I_{total} = \sigma T^4$$

where:

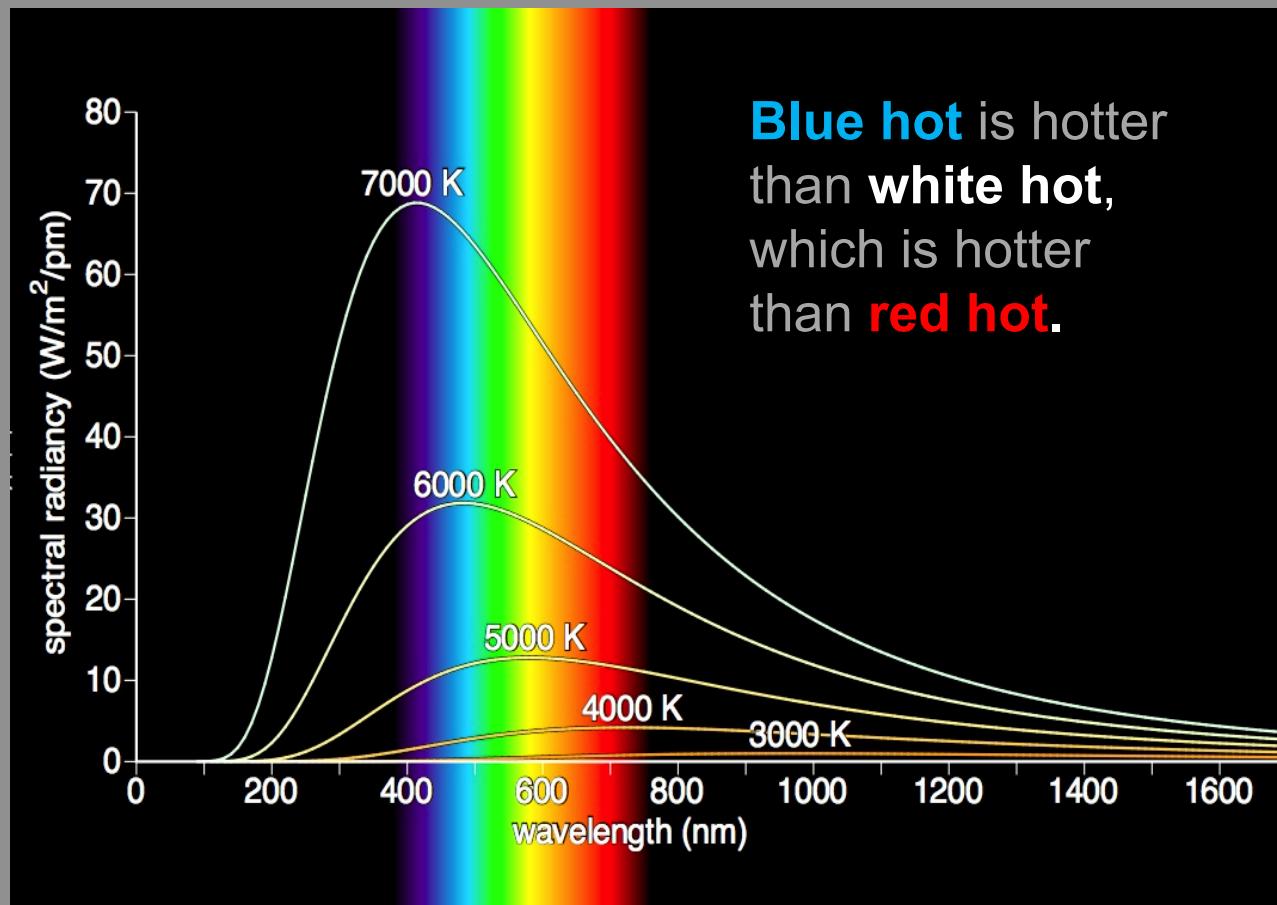
$$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.670373 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

σ is called the Stefan-Boltzmann constant (not to be confused with Boltzmann's constant, k_B).

Multiply by the blackbody surface area to find the total emitted power.

Blackbody Emission Spectrum

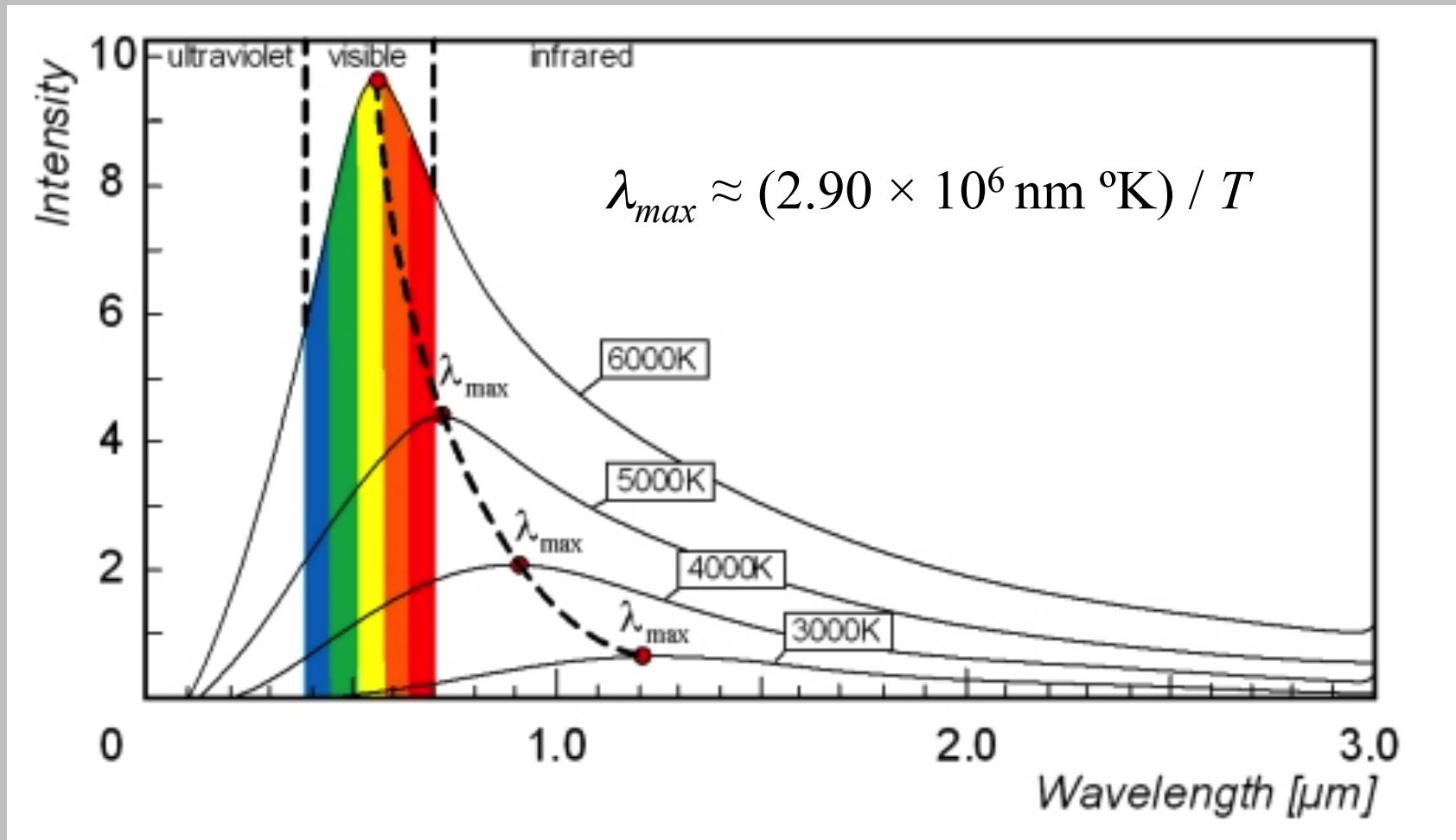
The higher the temperature, the more the emission at all wavelengths and the shorter the peak wavelength.



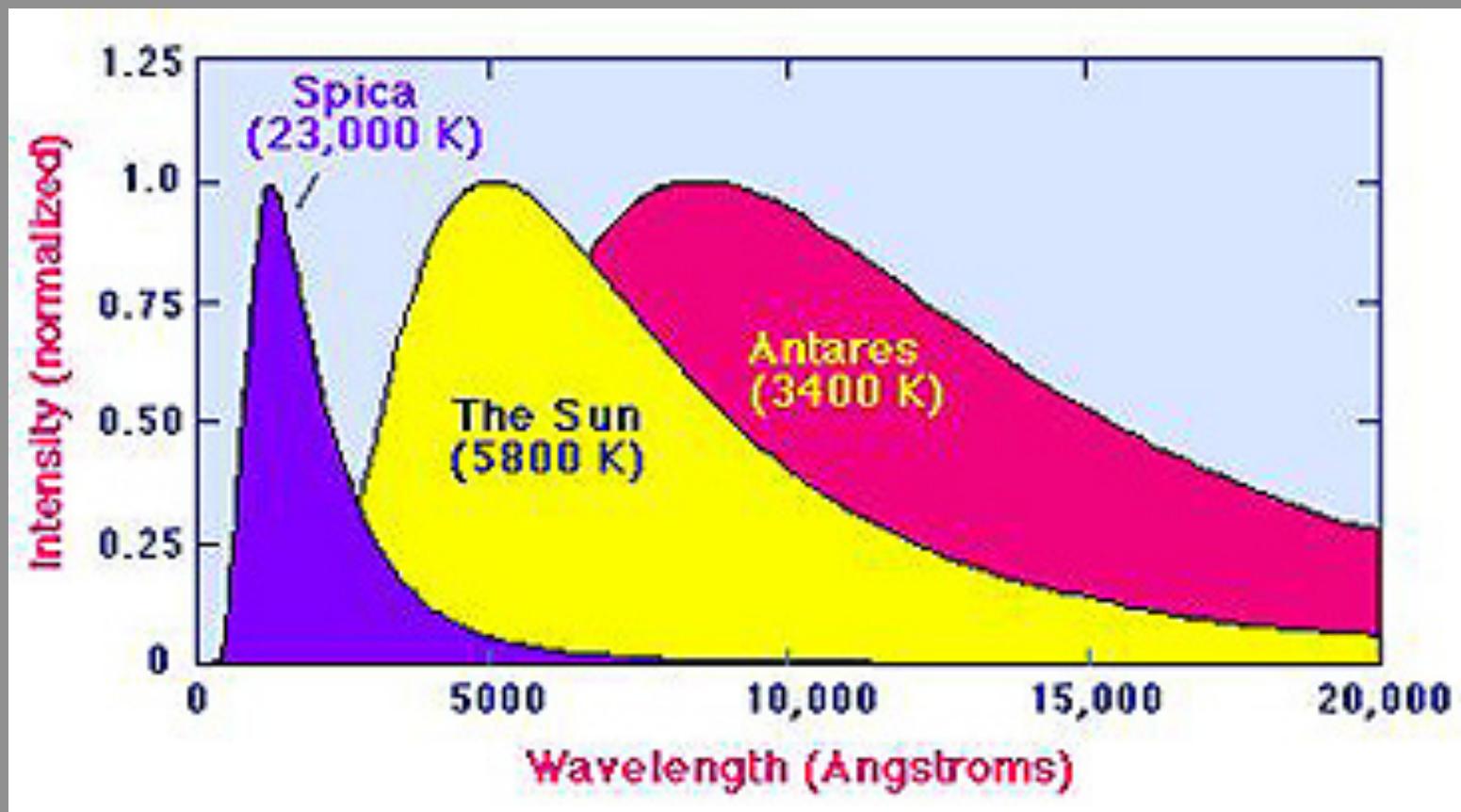
Blue hot is hotter than **white hot**, which is hotter than **red hot**.

The sun's surface is $\sim 5800^\circ$ K, so its blackbody spectrum peaks at ~ 500 nm—in the green. However, blackbody spectra are broad, so it contains red, yellow, and blue, too, and so looks white.

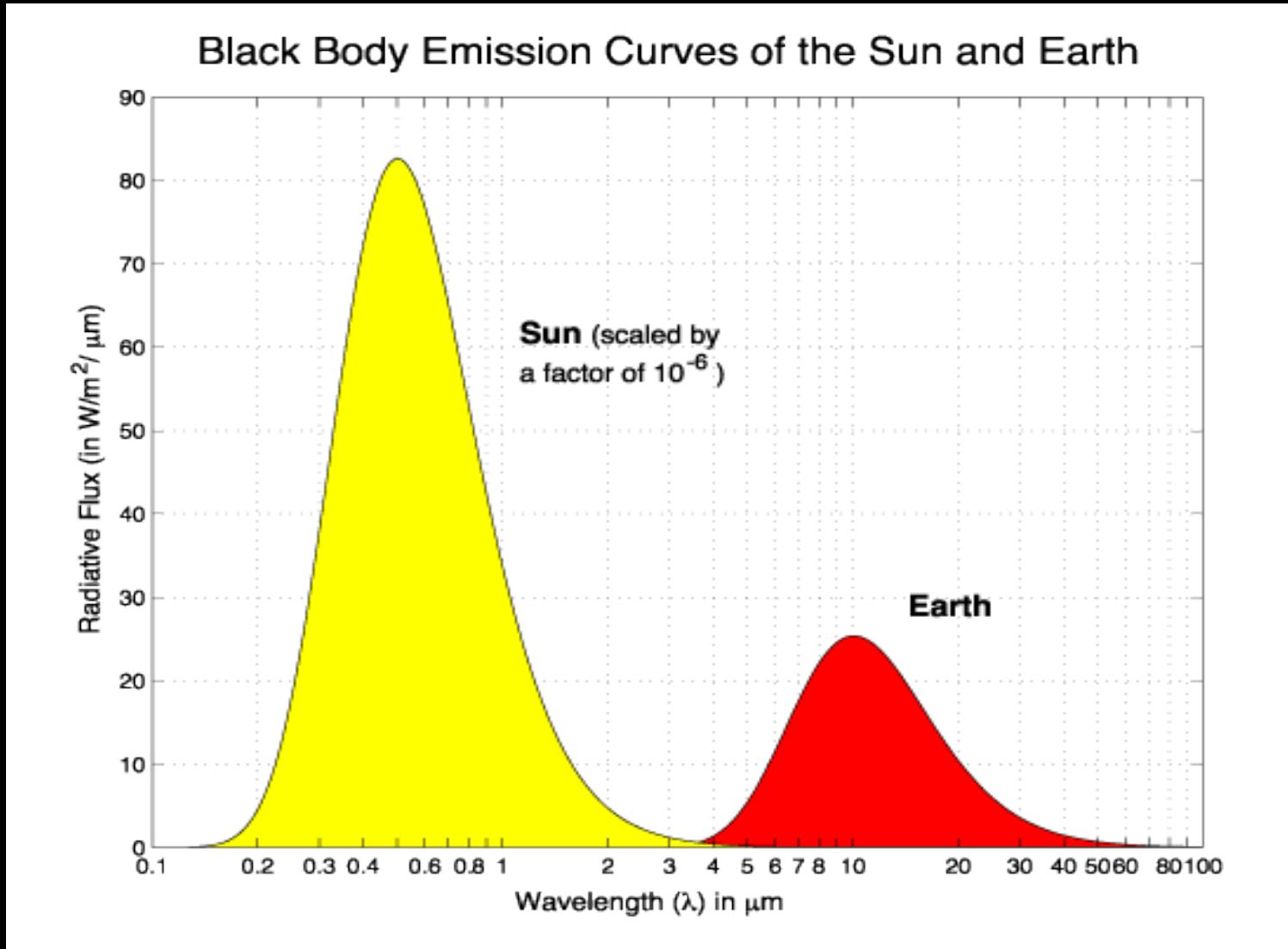
Wien's Law: The blackbody peak wavelength scales as 1/Temperature.



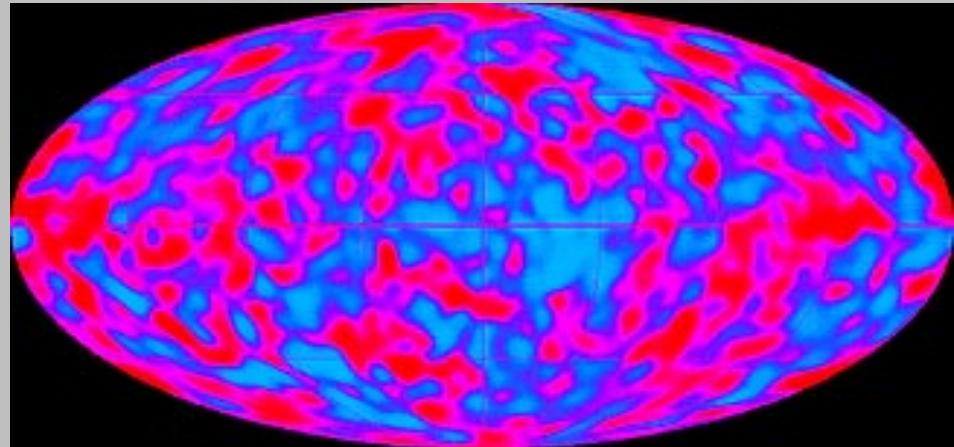
We can tell how hot a star is by its emission spectrum.



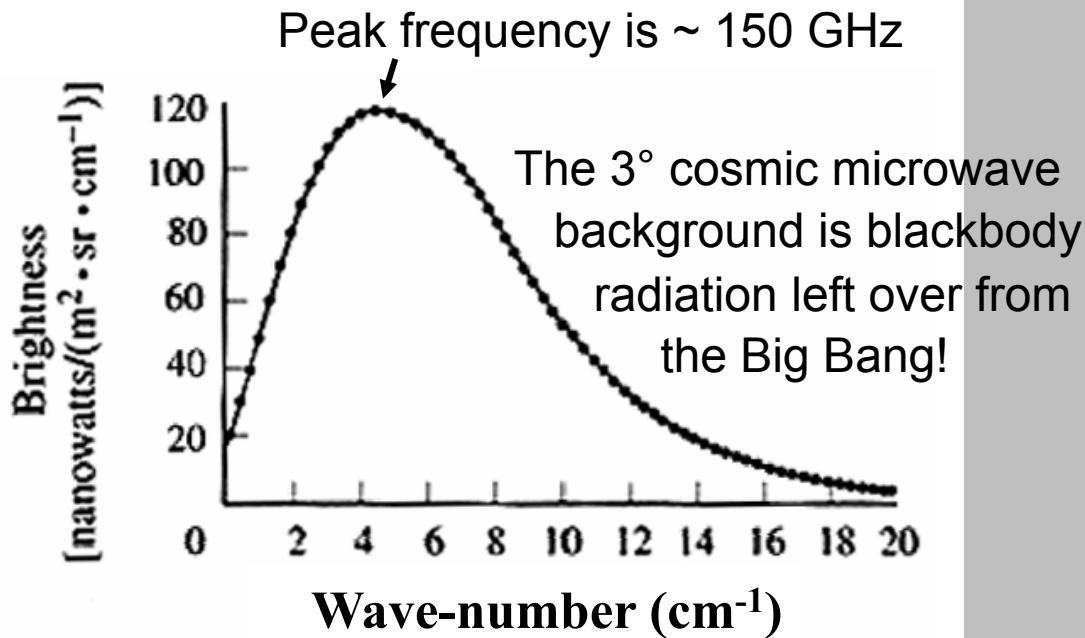
The earth is a blackbody, too.



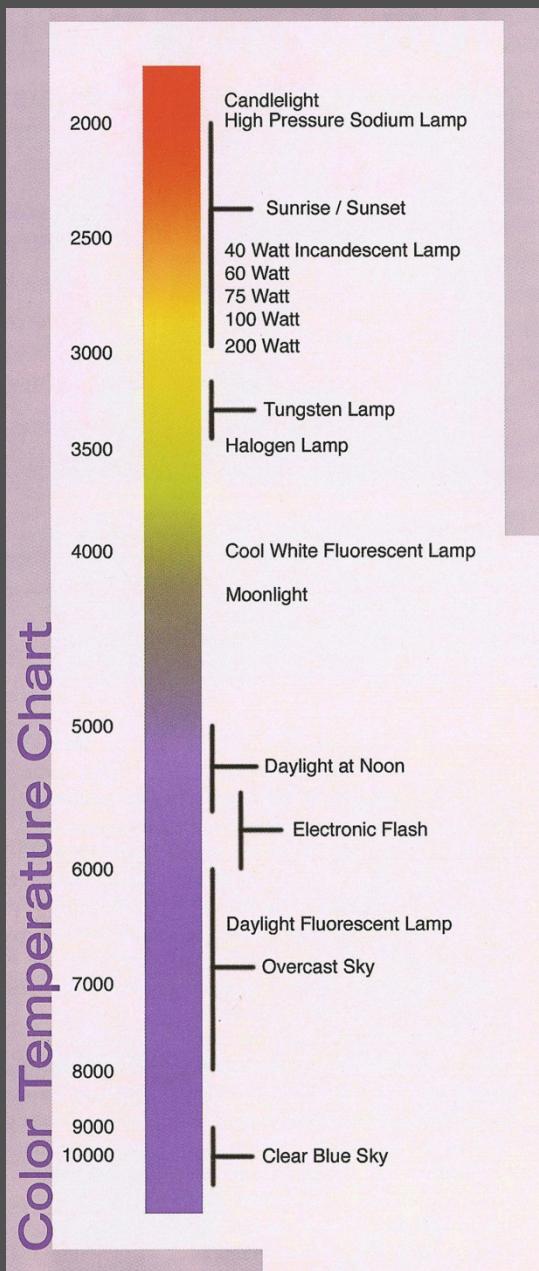
Cosmic Microwave Background



Microwave background vs. angle. Note the variations.



Interestingly, blackbody radiation retains a blackbody spectrum despite the expansion the universe. It does get colder, however.



Color Temperature

Blackbodies are so pervasive that a light spectrum is often characterized in terms of its temperature even if it's not exactly a blackbody.

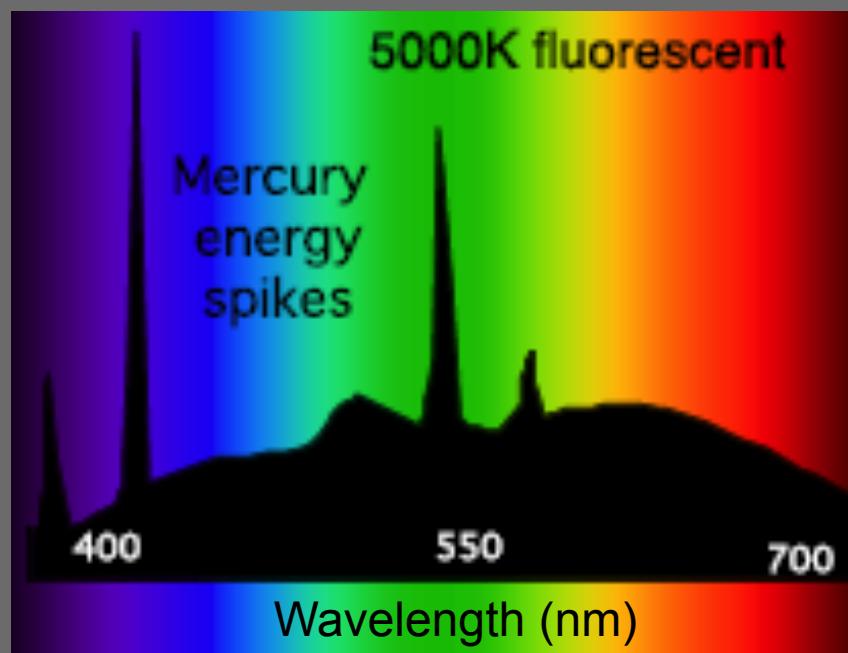
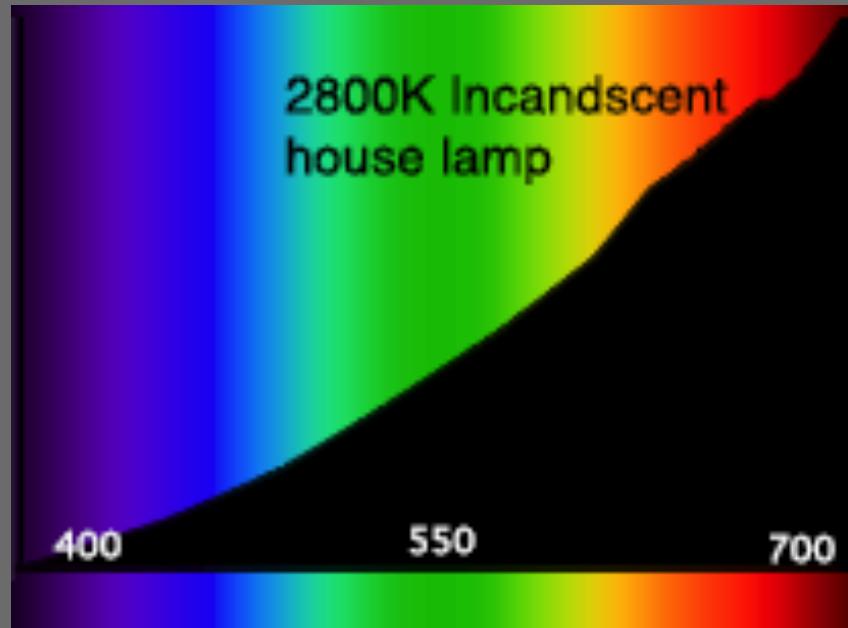
Keep in mind that blackbody spectra are broad, so they usually look white with a tint of the peak color (wavelength).

Light Bulb Spectra

Ordinary light bulbs have a color temperature of $\sim 3000^\circ \text{ K}$, so much of their light is in the IR and is wasted. They appear yellow.

Fluorescent light bulbs have a blackbody component, but additional (non-equilibrium) narrow-spectrum emission lines.

Fluorescent light bulbs aren't really blackbodies, but their spectra are often fit to a blackbody curve.

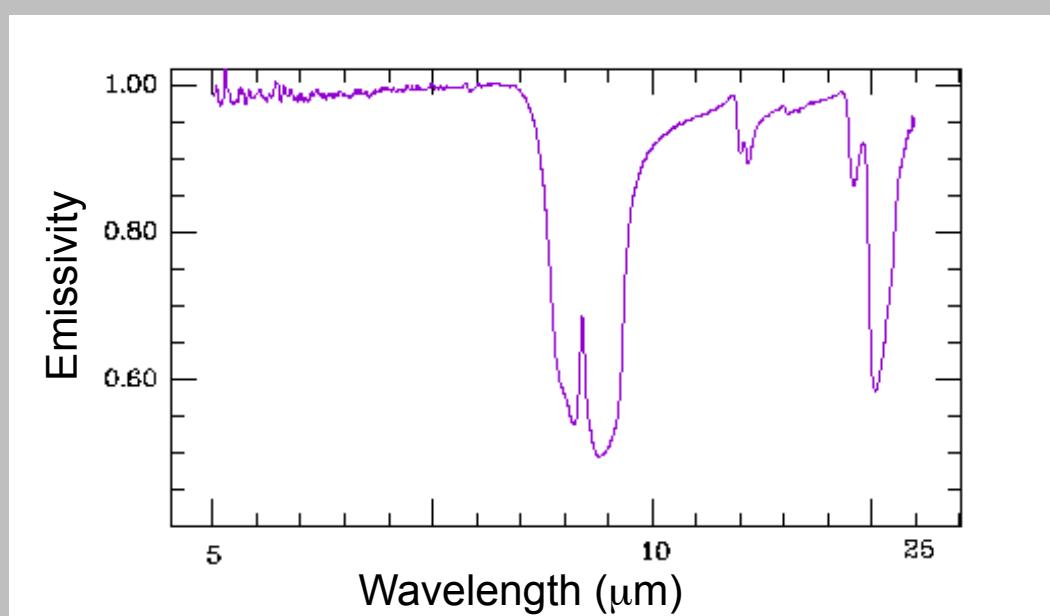
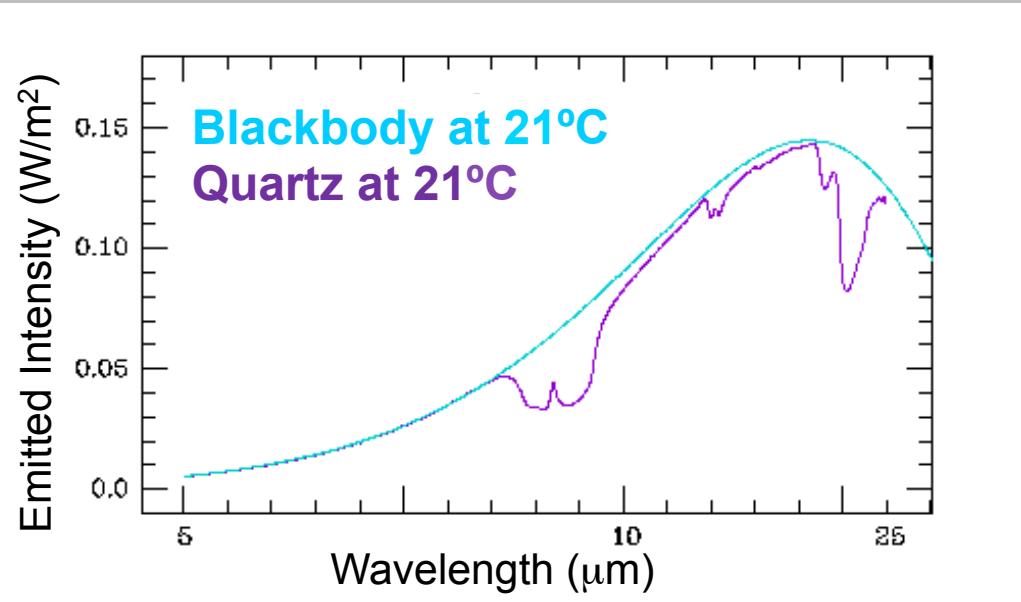


Emissivity

Our analysis assumed a perfectly black object. However, the object may not have transitions at all wavelengths, and it may not be “optically thick.”

The **emissivity** takes this into account. It is the ratio of the actual emitted spectrum and the theoretical blackbody spectrum.

It depends on the medium and its density.



$$E = \frac{hc}{\lambda}$$

Particles: photon energy

Wave-like properties

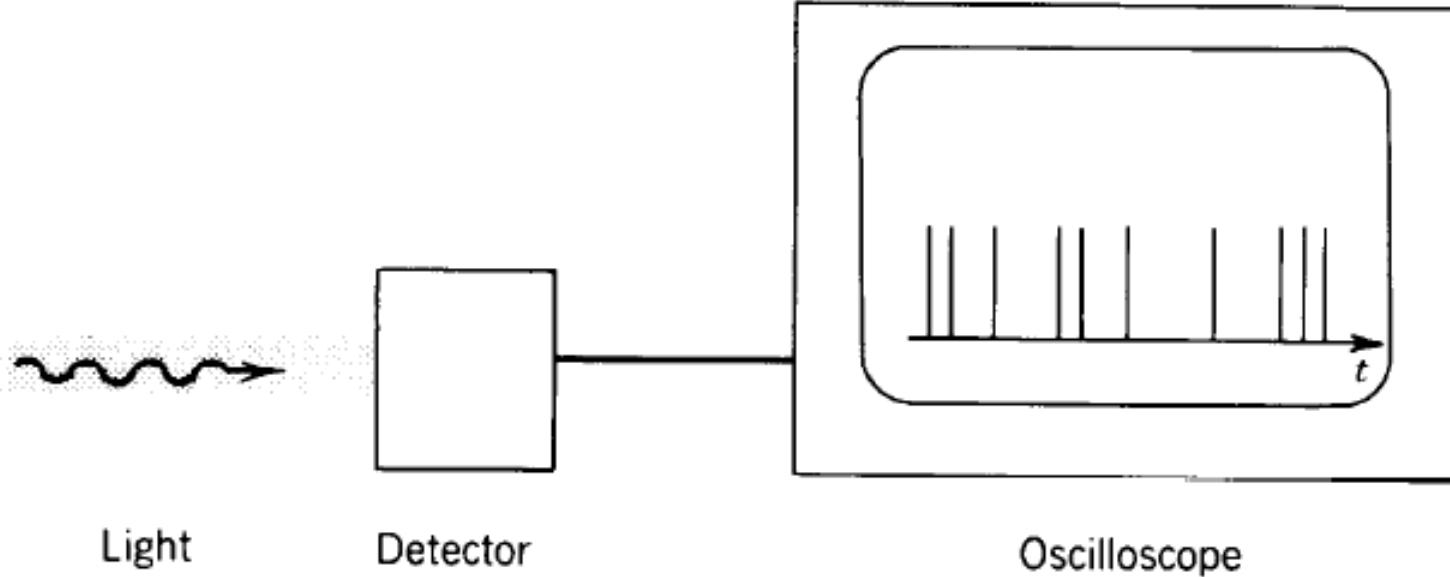


Figure 11.2-1 Photon registrations at random localized instants of time.

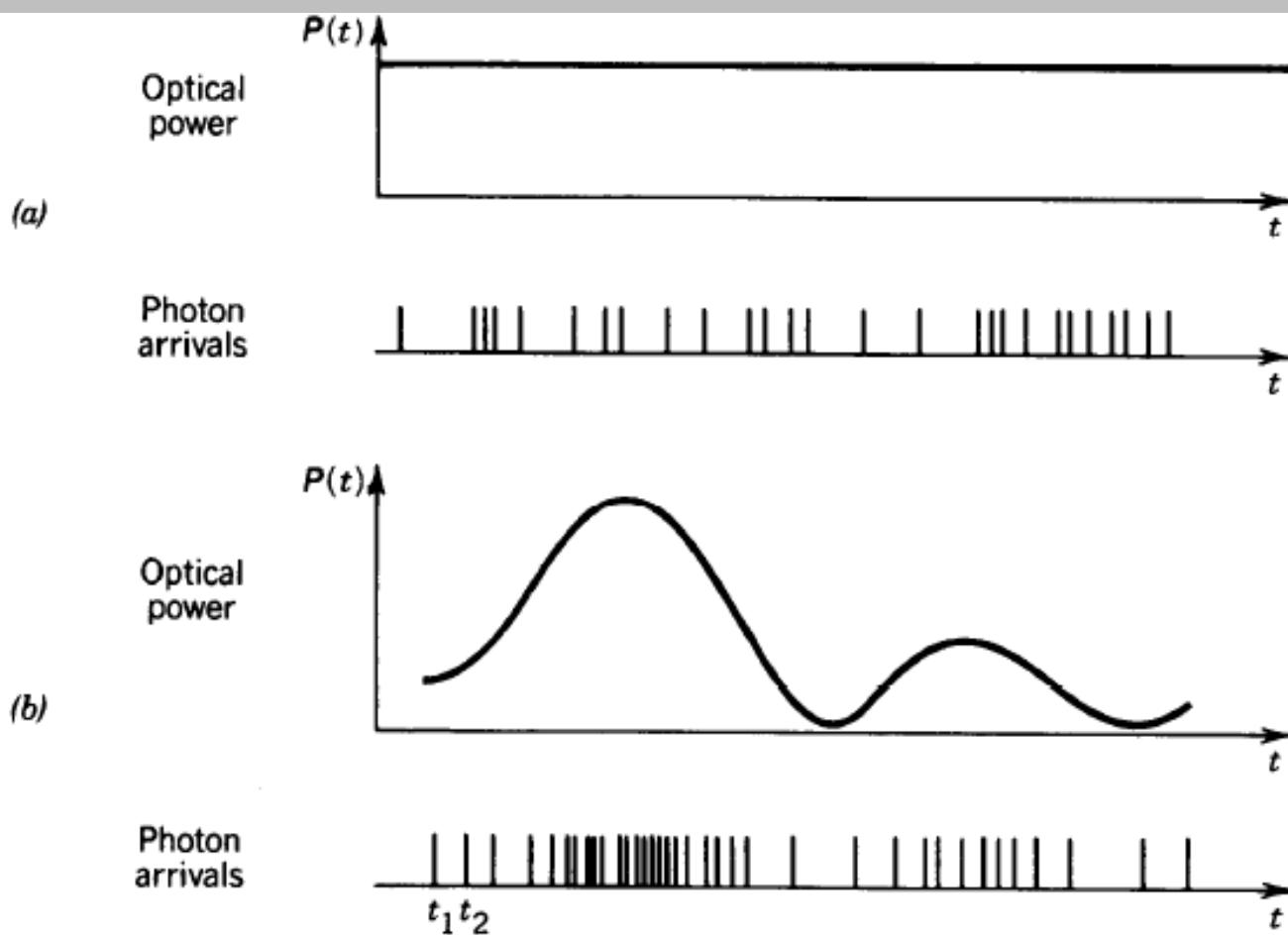


Figure 11.2-3 (a) Constant optical power and the corresponding random photon arrival times.
 (b) Time-varying optical power and the corresponding random photon arrival times.

$$\lambda = 1.24 \times 10^{-6} / E$$



Wavelength

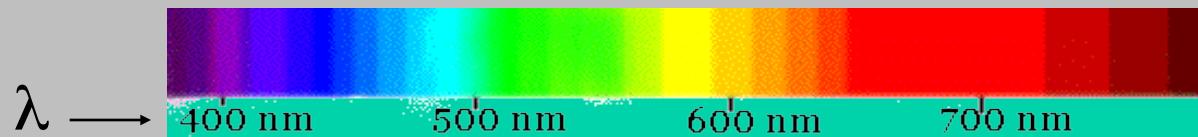
**Associated
with colours**



Energy

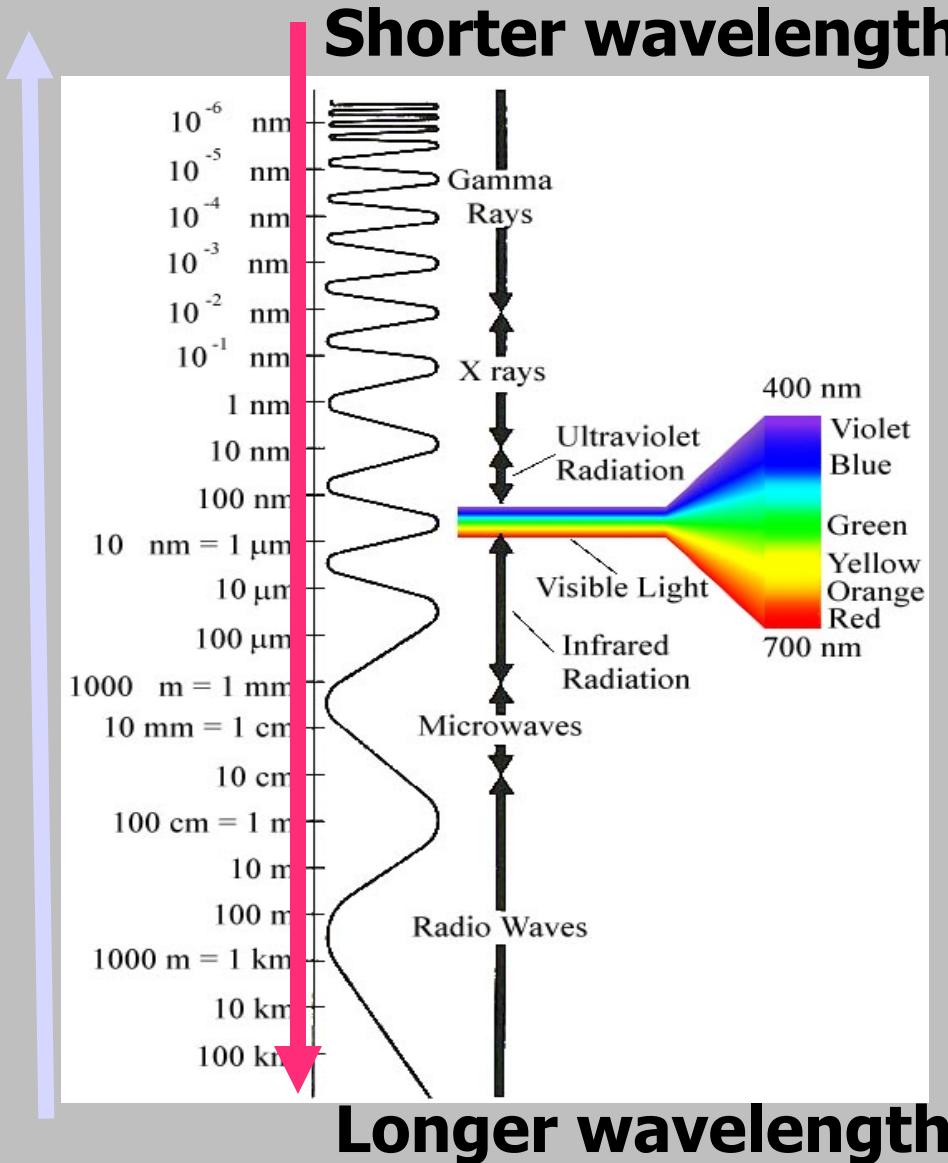
**Each colour has energy
associated with it**

calculate the photon energy of violet, blue, green, orange and red lights.



Electromagnetic Spectrum

Larger Photon Energy (eV)



V ~ 3.17eV

B ~ 2.73eV

G ~ 2.52eV

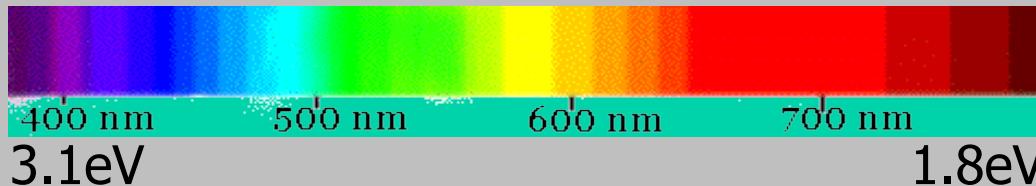
Y ~ 2.15eV

O ~ 2.08eV

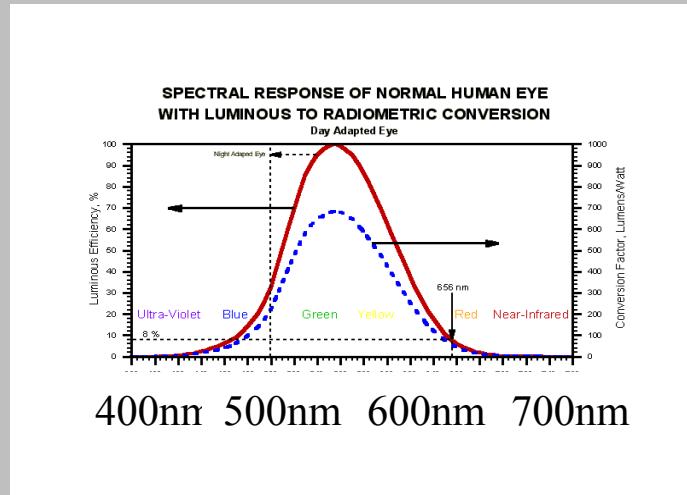
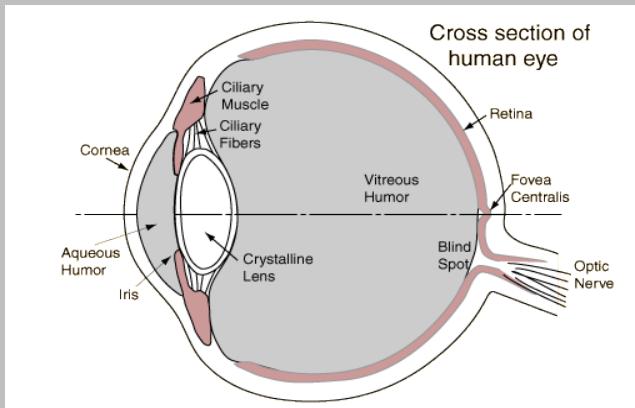
R ~ 1.62eV

Visible Lights

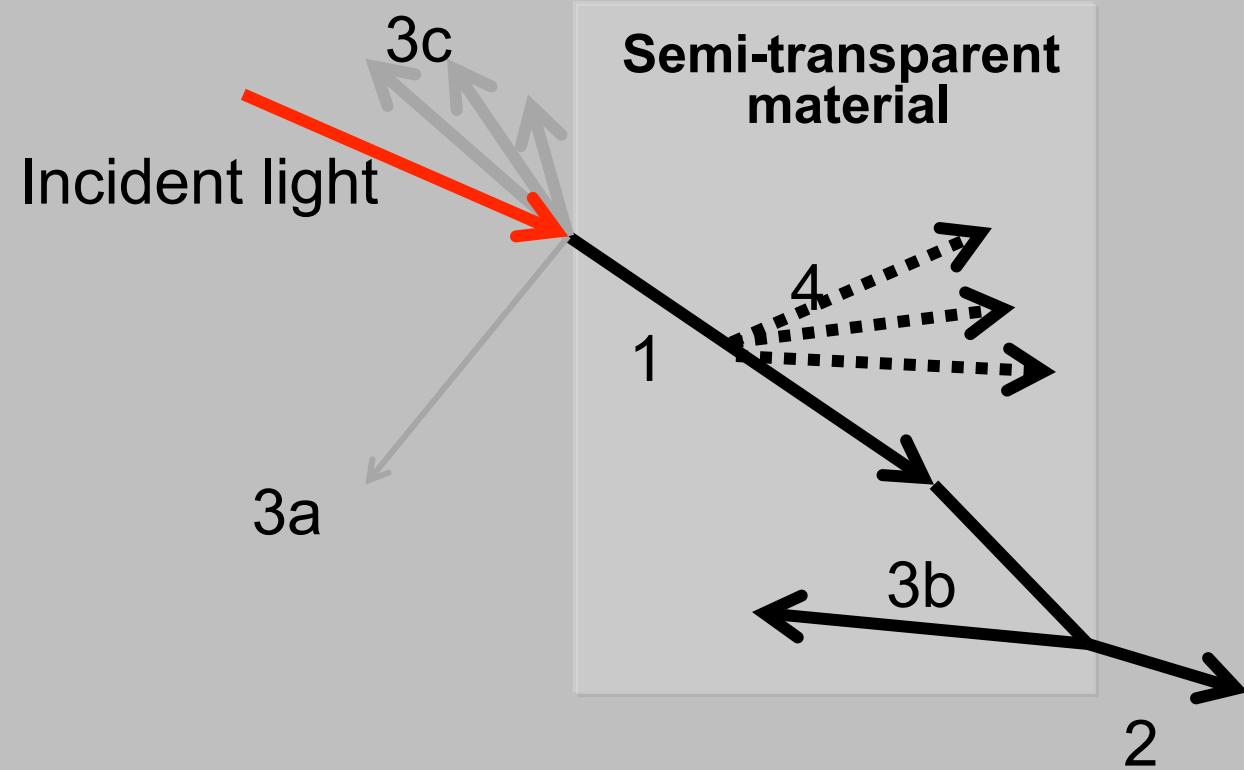
- Lights of wavelength detected by human eyes ~ 450nm to 650nm is called visible light:



- Human eyes can detect lights with different colours
- Each colour is detected with different efficiency.

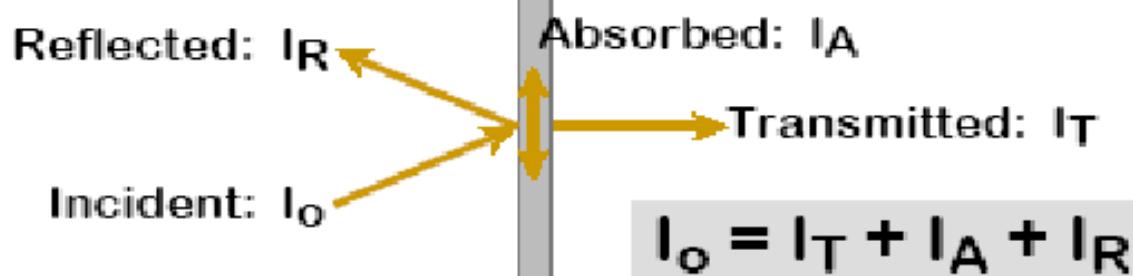


Interaction Between Light and Bulk Material



- 1 - Refraction**
 - 2- Transmission**
 - 3a – Specular reflection**
 - 3b – Total internal reflection**
 - 3c – Diffused reflection**
 - 4 – Scattering**
- There is also dispersion –where different colours bend differently*

Light interaction with solids



Optical Classification

| | |
|---------------------|--|
| Transparent | γ = lattice parameter; unit cell x-shear strain (6.2) |
| Transluscent | Δ = state change in a parameter |
| | ϵ = eng. strain; shear strain (6.3) |
| | κ = dielectric constant (18-16) |
| | ϵ_r = dielectric constant (6.6) |
| | ϵ_T = true strain (6.6) |
| | η = viscosity (12.7) |

Appearance of insulator, metal and semiconductor

- Appearance in term of colour depends on the interaction between the light with the electronics configuration of the material.
- Normally,
 - High resistivity material: insulator → transparent
 - High conductivity material: metals → metallic luster and opaque
 - Semiconductors → coloured, opaque or transparent, colour depending on the band gap of the material
- For semiconductors the energy band diagram can explain the appearance of the material in terms of lustre and colouration

Why is Silicon Black and Shiny?



Need to know, the energy gap of Si

$$E_{\text{gap}} = 1.2 \text{ eV}$$

Need to know visible light photon energy

$$E_{\text{vis}} \sim 1.8 - 3.1 \text{ eV}$$

E_{vis} is larger than Silicon E_{gap}

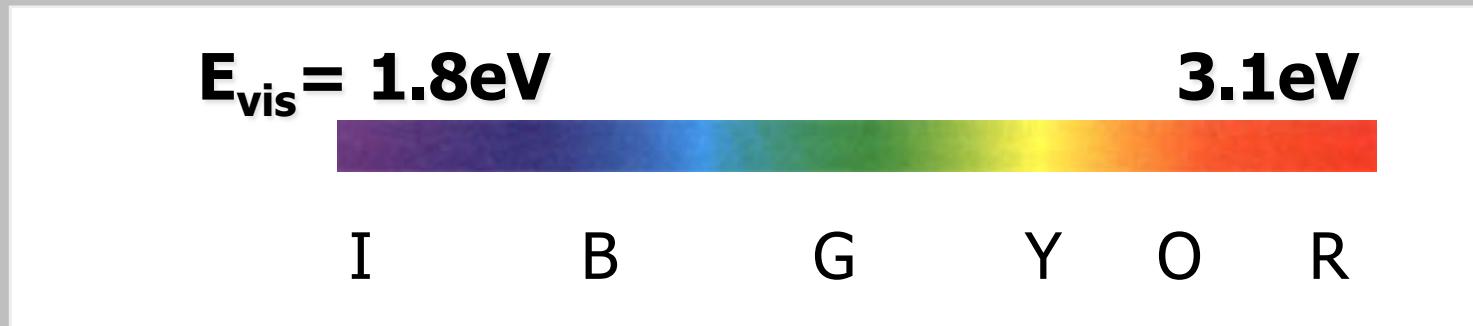
All visible light will be absorbed

Silicon appears black

Why is Si shiny?

A lot of photons absorption occurs in silicon, there are significant amount of electrons on the conduction band. These electrons are delocalized which induce the lustre and shines.

Colours of Semiconductors



- If Photon Energy, $E_{vis} > E_{gap} \rightarrow$ Photons will be absorbed
- If Photon Energy, $E_{vis} < E_{gap} \rightarrow$ Photons will transmitted
- If Photon Energy is in the range of E_{gap} ;
 - Those with higher energy than E_{gap} will be absorbed.
 - We see the colour of the light being transmitted
- If all colours are transmitted = White

Why do you think glass is transparent?

- Glass is insulator (huge band gap)
- The electrons find it hard to jump across a big energy gap ($E_{gap} \gg 5\text{eV}$)
- $E_{gap} \gg E$ visible spectrum $\sim 2.7 - 1.6\text{eV}$
- All colored photon are transmitted, no absorption hence light transmit – transparent.
- Defined transmission and absorption by Lambert's law:
 - $I = I_o \exp(-\alpha l)$
 - I = incident beam
 - I_o = transmitted beam
 - α = total linear absorption coefficient (m^{-1})
 - α = takes into account the loss of intensity from both scattering centers and absorption centers.
 - α = approaching zero for pure insulator.

Thermal Light

Light emitted from atoms, molecules, and solids, under conditions of thermal equilibrium and in the absence of other external energy sources, is known as thermal light.

Luminescent Light

An applied external source of energy may cause an atomic or molecular system to undergo transitions to higher energy levels.

In the course of decaying to a lower energy, the system may subsequently emit optical radiation.

Such “nonthermal” radiators are generally called luminescent radiators and the radiation process is called luminescence.

Semiconductor

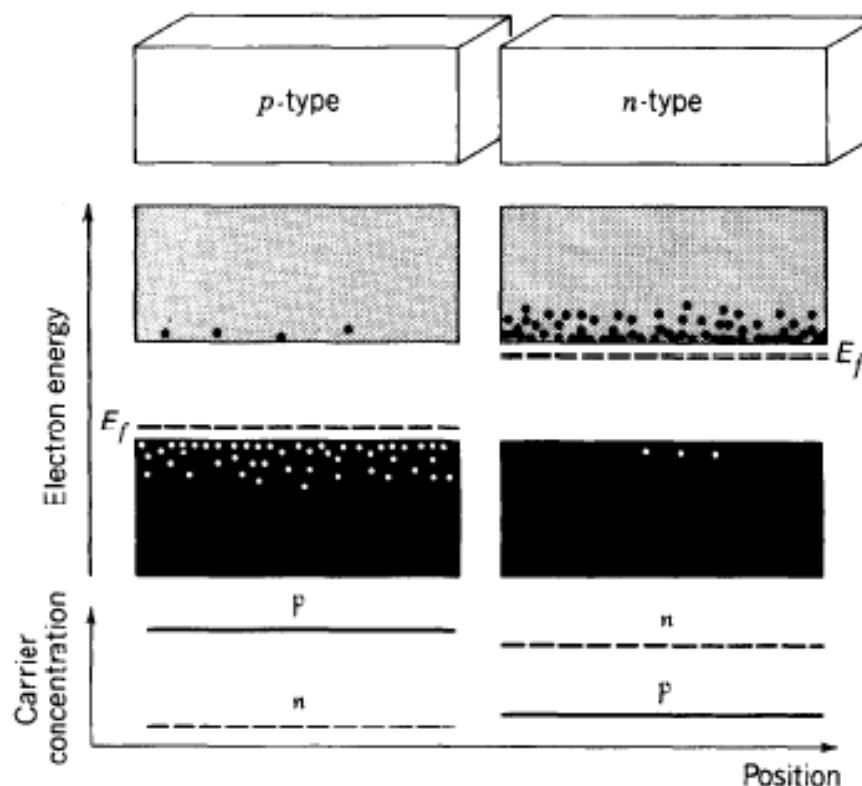


Figure 15.1-15 Energy levels and carrier concentrations of a *p*-type and an *n*-type semiconductor before contact.

Semiconductor

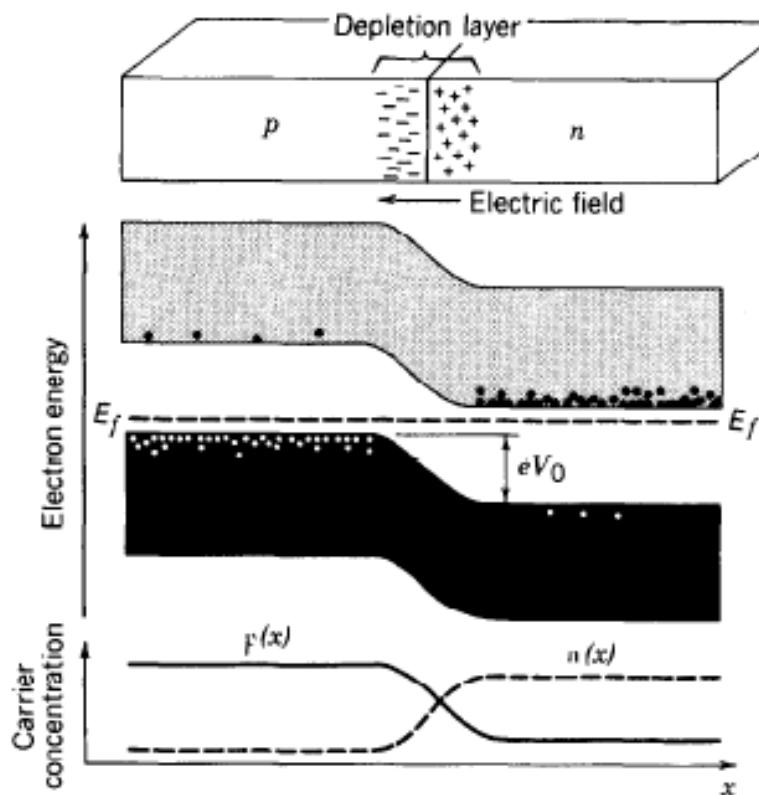


Figure 15.1-16 A $p-n$ junction in thermal equilibrium at $T > 0$ K. The depletion-layer, energy-band diagram, and concentrations (on a logarithmic scale) of mobile electrons $n(x)$ and holes $p(x)$ are shown as functions of position x . The built-in potential difference V_0 corresponds to an energy eV_0 , where e is the magnitude of the electron charge.

A forward-biased p-n junction

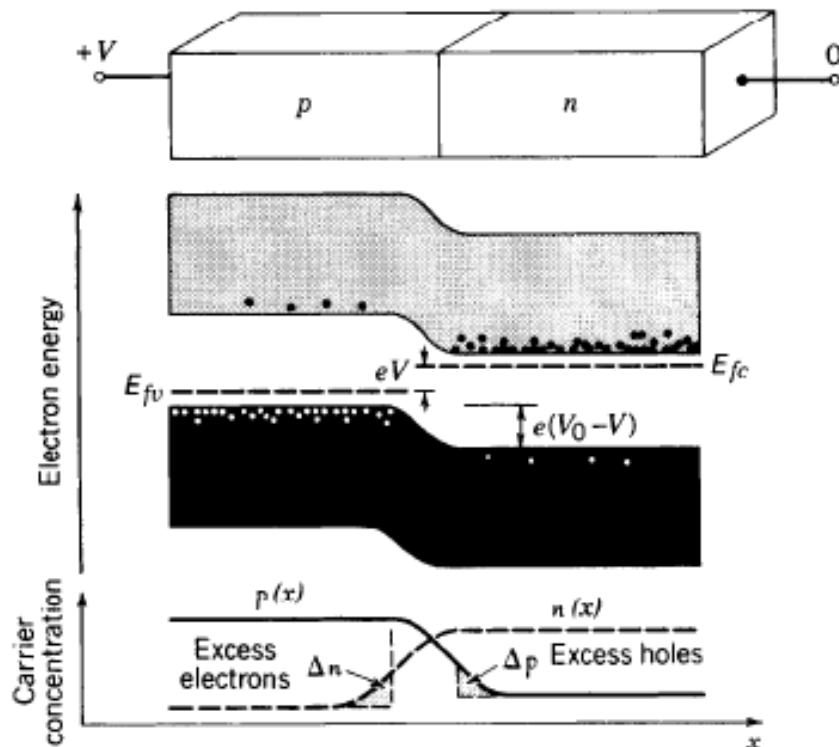
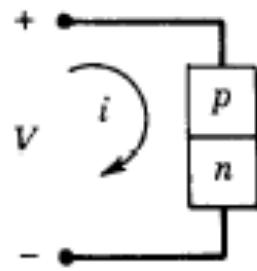
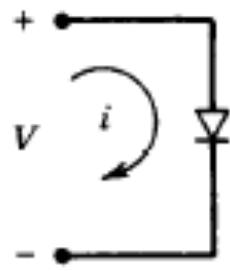


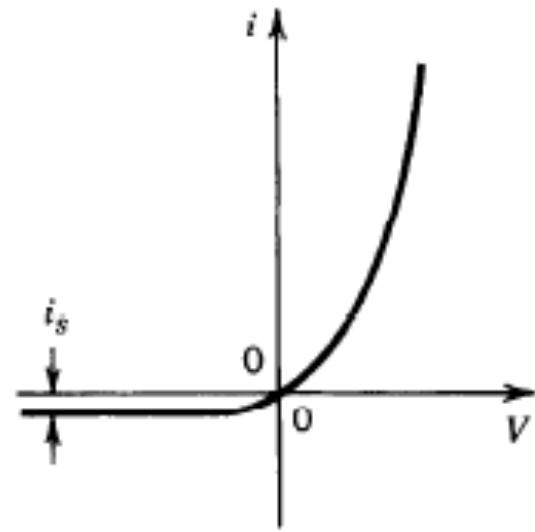
Figure 15.1-17 Energy-band diagram and carrier concentrations in a forward-biased *p-n* junction.



(a)



(b)



(c)

Figure 15.1-18 (a) Voltage and current in a $p-n$ junction. (b) Circuit representation of the $p-n$ junction diode. (c) Current–voltage characteristic of the ideal $p-n$ junction diode.

$$i = i_s \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right],$$

(15.1-24)
Ideal Diode
Characteristic

A forward-biased semiconductor

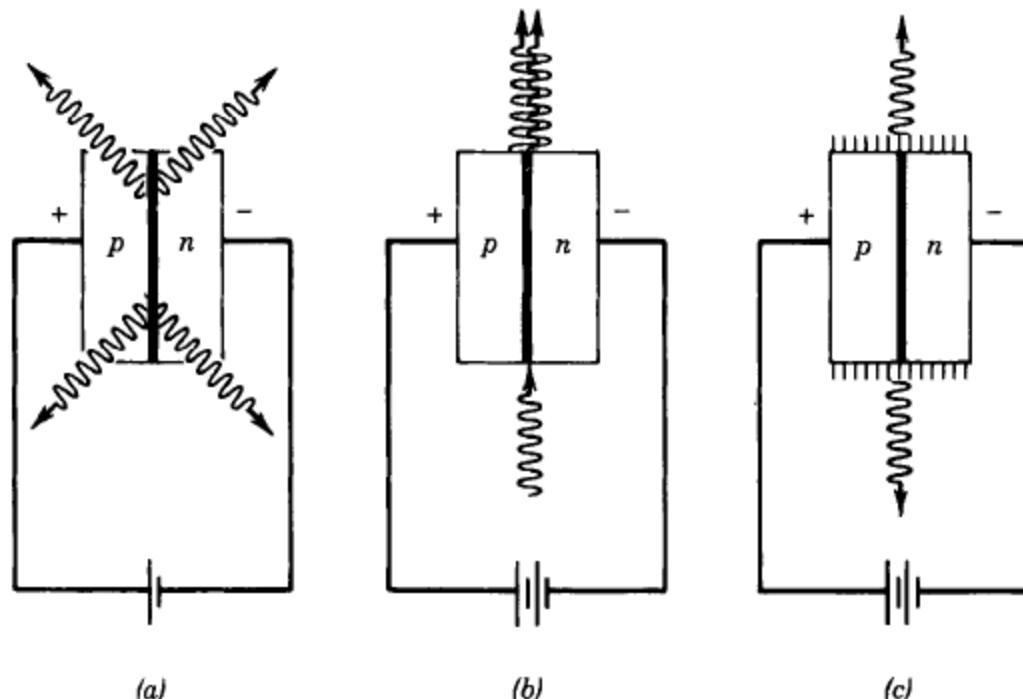


Figure 16.0-1 A forward-biased semiconductor $p-n$ junction diode operated as (a) an LED, (b) a semiconductor optical amplifier, and (c) a semiconductor injection laser.