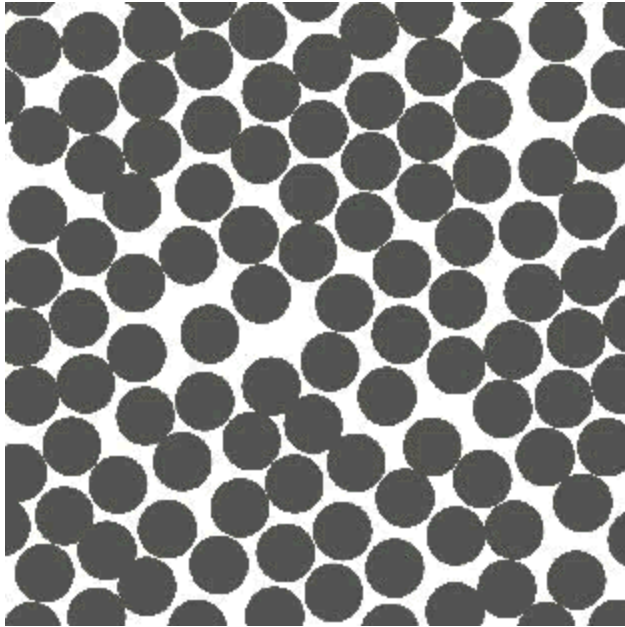


# Image Analysis



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NSF-CMMI



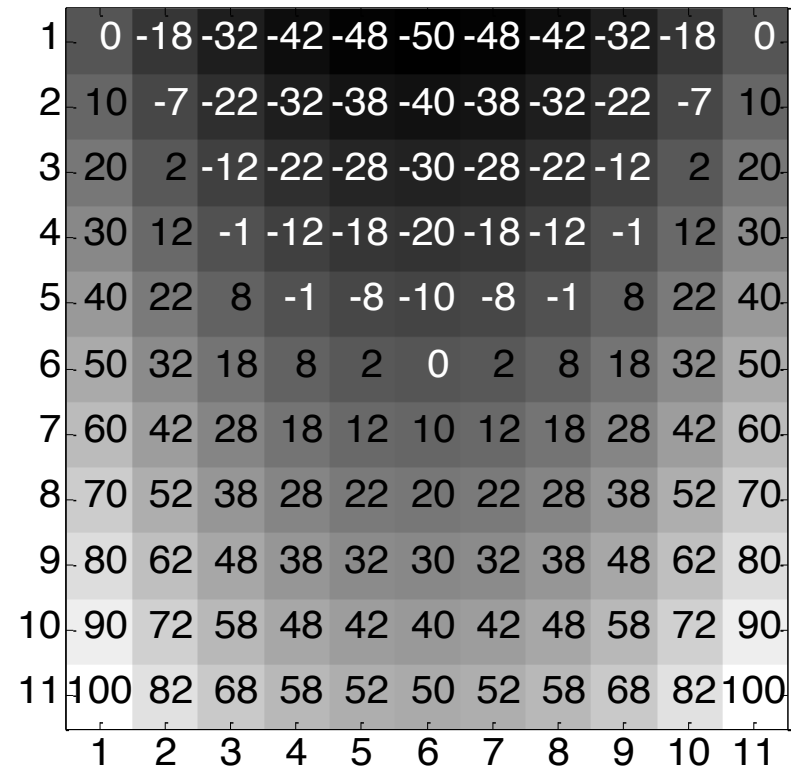
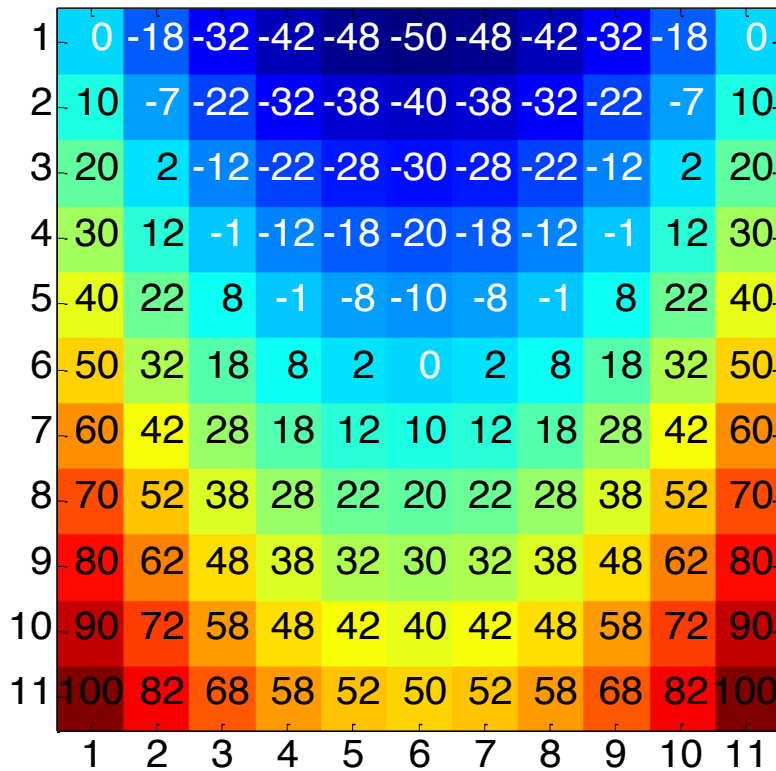
# Introduction

- The brain is the best image analyzer.
- If you *cannot* do it with your brain, you *cannot* do with a computer.
- If you *can* do it with your brain, you can rarely do it with a computer.
- Making you image better will pay off.

# Outline

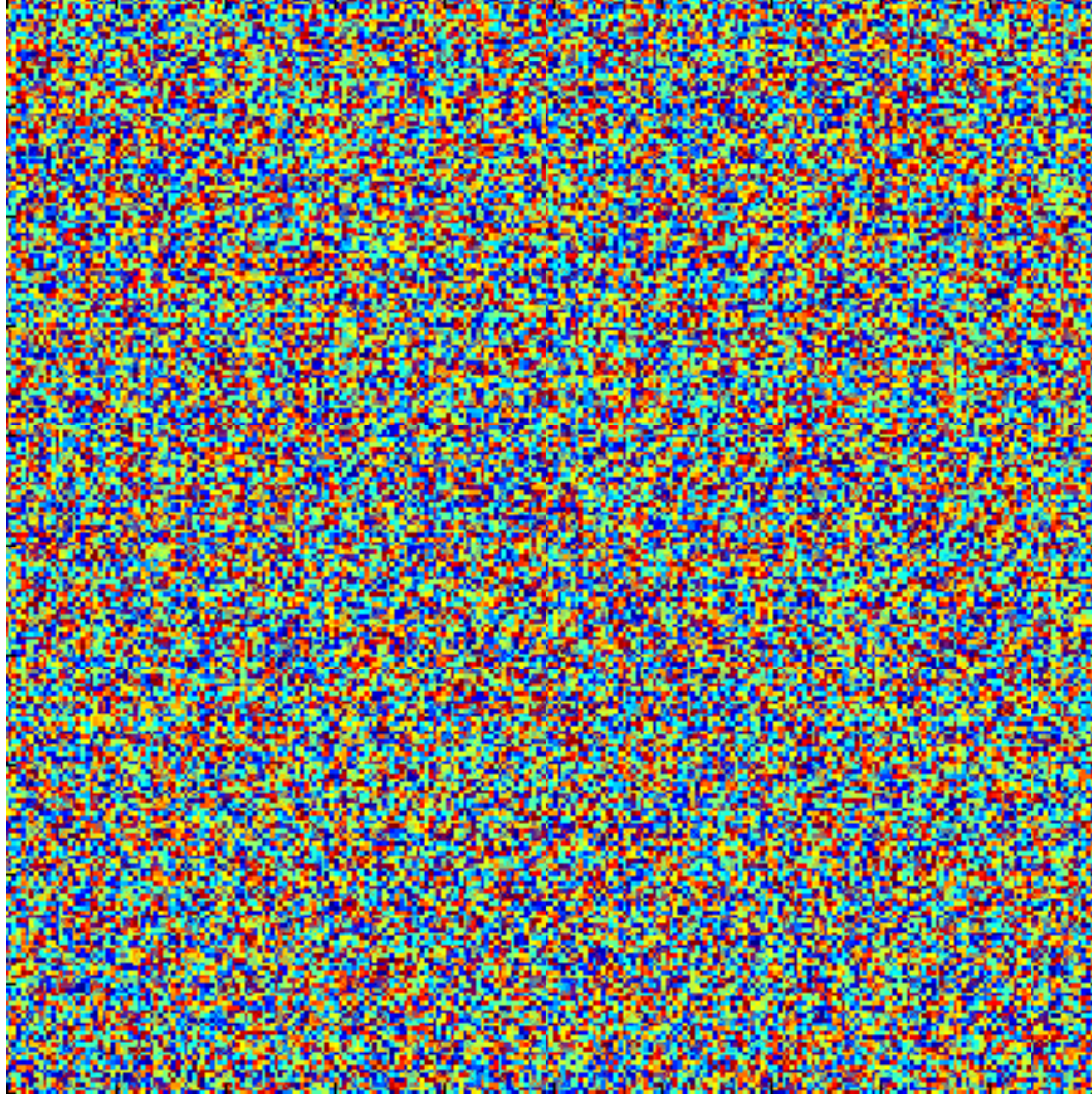
- Image Analysis tools
  - Fast Fourier Transform (FFT)
  - Convolution and Cross-correlation
- Particle Imaging Velocimetry (PIV)
- Particle Tracking
- Oscillatory Image Demodulation (OID)

# Image



$I_{nm}$

# Video Image Data



$$I_{nm}(k\Delta t)$$

# FFT

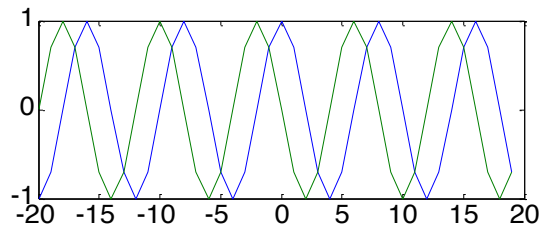
- Fast Fourier Transform

$$F_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f_{kl} e^{-2\pi i(kn/K + lm/L)}$$

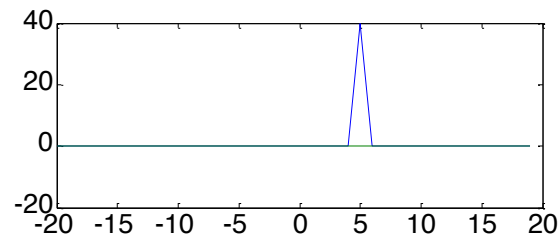
$$f_{kl} = \frac{1}{NM} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} F_{nm} e^{+2\pi i(kn/N + lm/M)}$$

# FFT

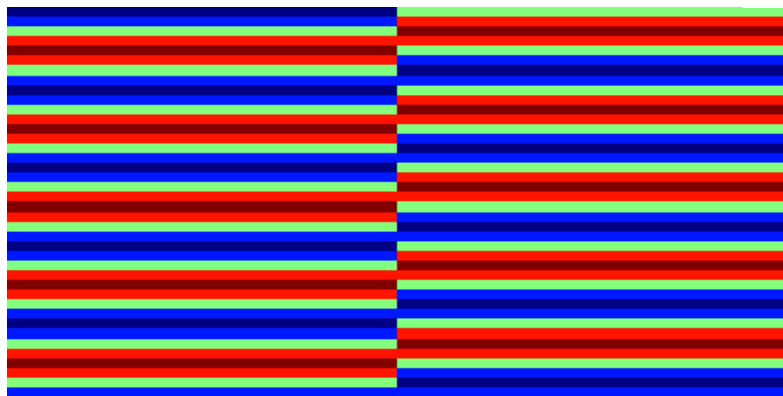
$$f_k = e^{2\pi i k n_0 / N}$$



$$F_n = N \delta_{nn_0}$$



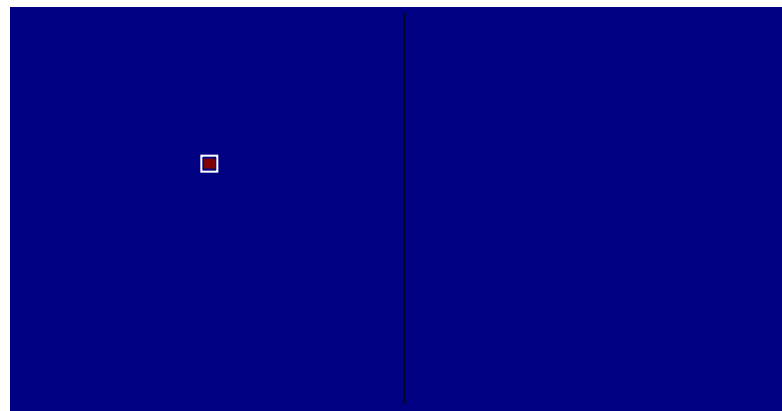
$$f_{kl} = e^{2\pi i k n_0 / N}$$



Re

Im

$$F_{nm} = NM \delta_{nn_0} \delta_{m0}$$

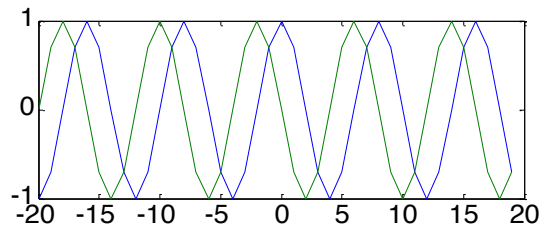


Re

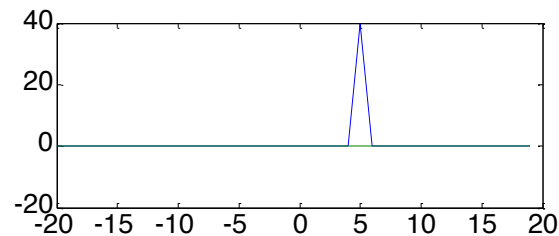
Im

# FFT

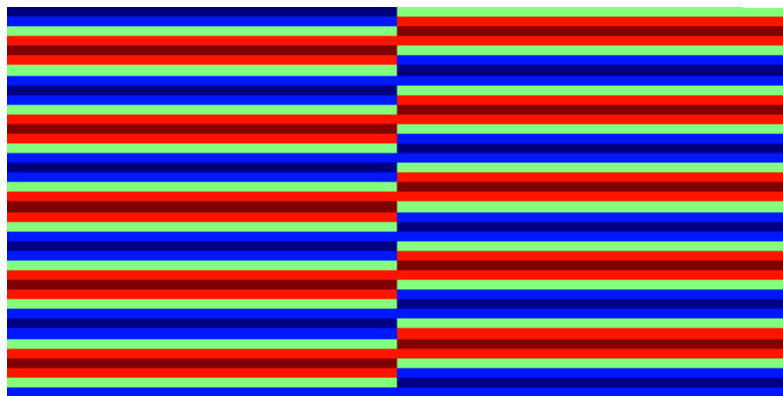
$$f_k = e^{2\pi i k n_0 / N}$$



$$F_n = N \delta_{nn_0}$$



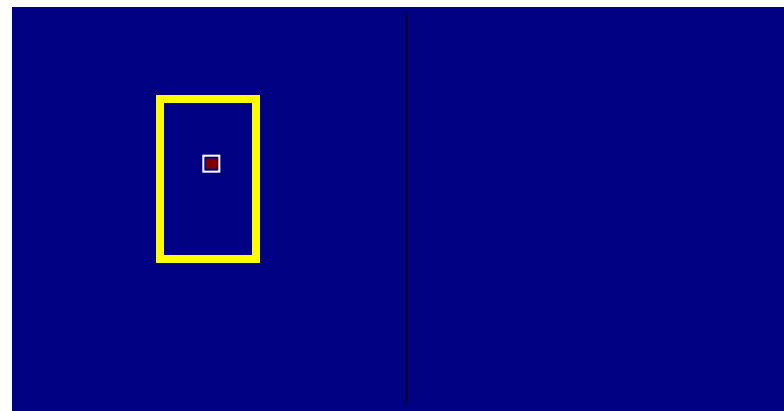
$$f_{kl} = e^{2\pi i k n_0 / N}$$



Re

Im

$$F_{nm} = NM \delta_{nn_0} \delta_{m0}$$



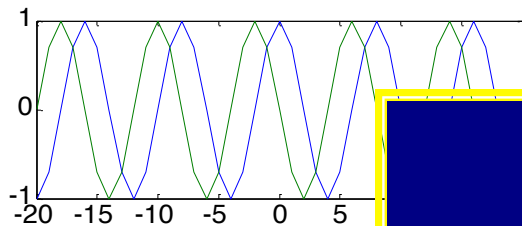
Re

Im

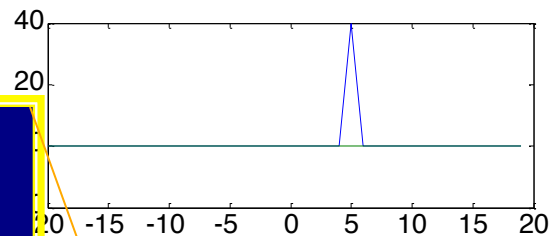


# FFT

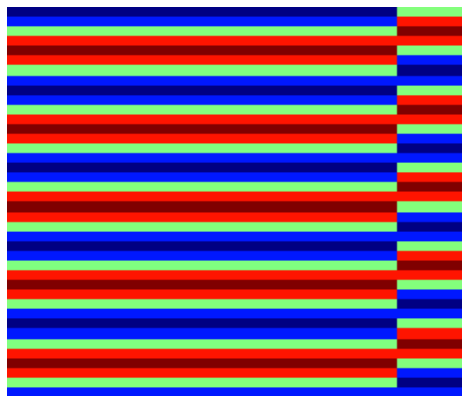
$$f_k = e^{2\pi i k n_0 / N}$$



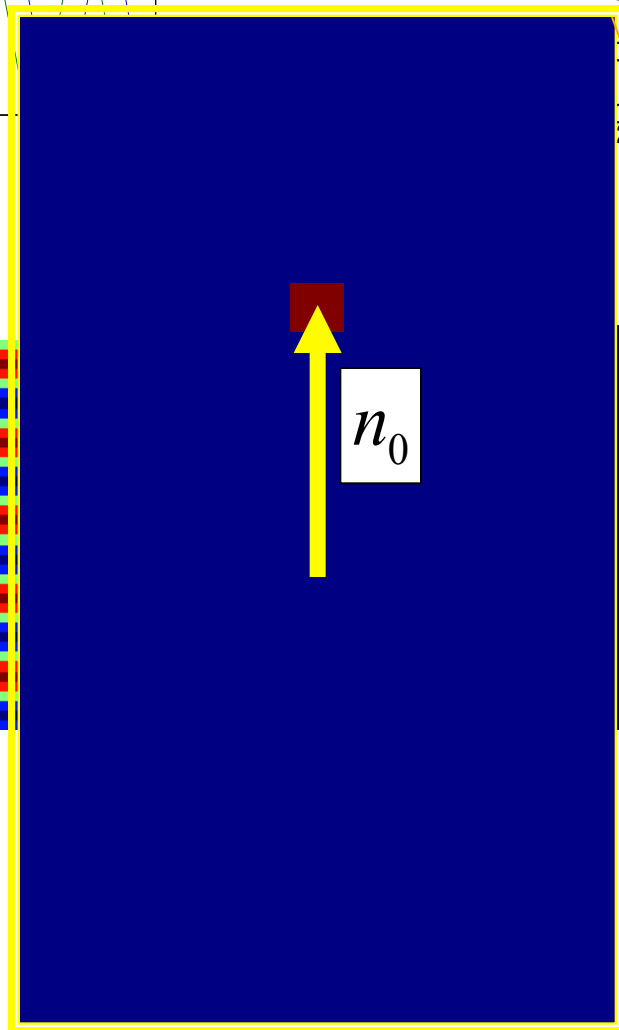
$$F_n = N \delta_{nn_0}$$



$$f_{kl} = e^{2\pi i k n_0 / N}$$

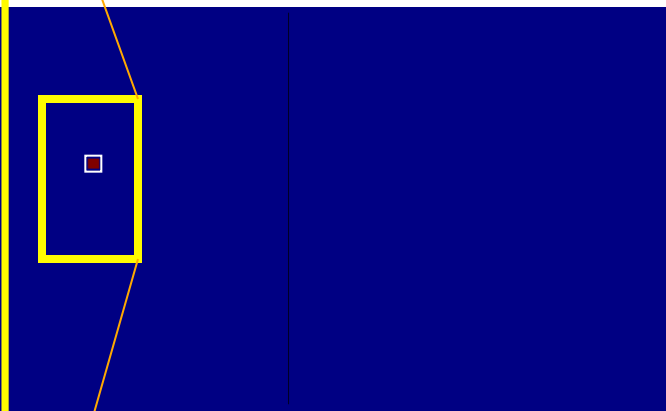


Re



$n_0$

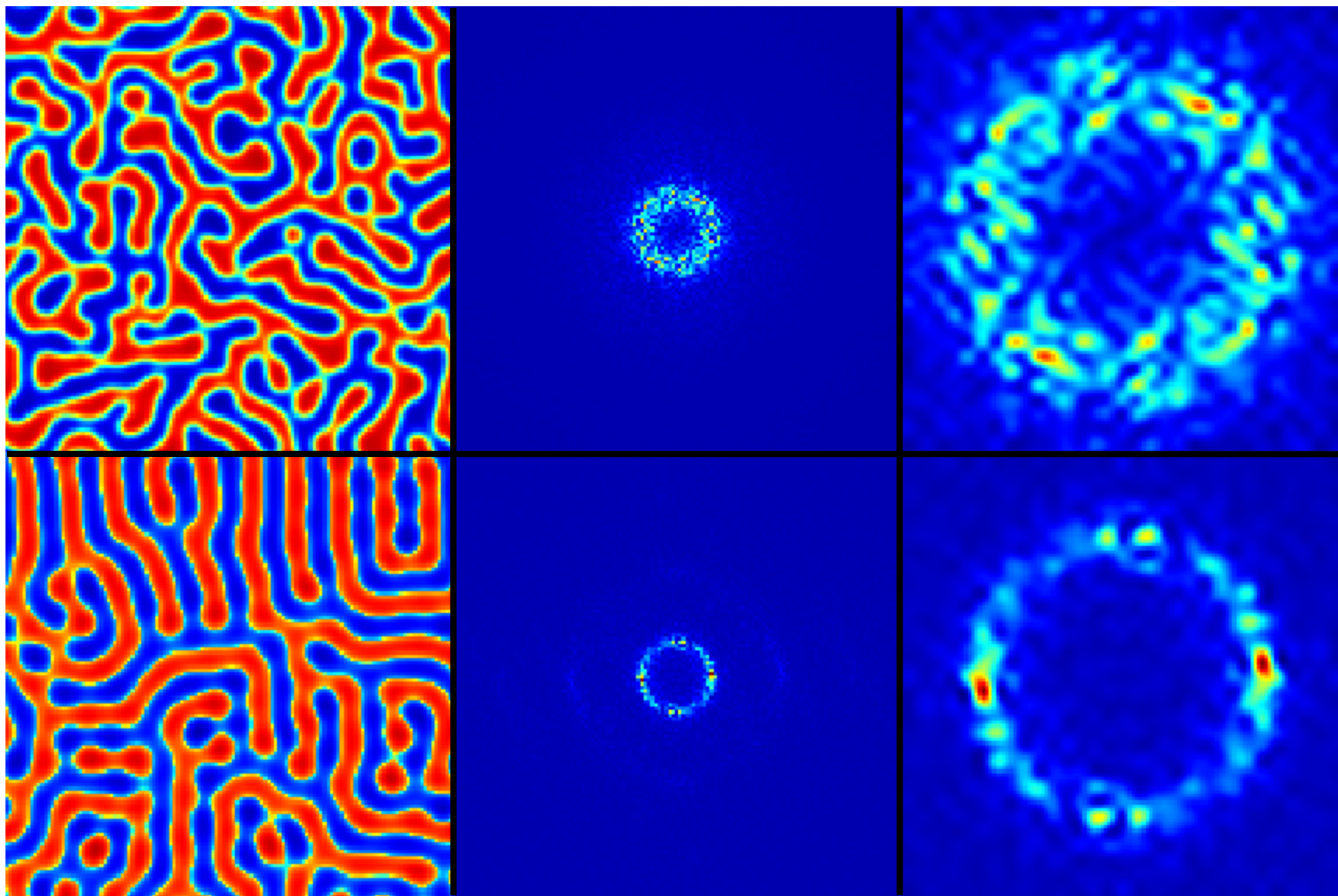
$$F_{nm} = NM \delta_{nn_0} \delta_{m0}$$



Re

Im

# FFT



# Convolution

- Convolution and Cross Correlation

$$C(f, g)_n = \sum_{k=0}^{K-1} f_k g_{(n-k)}$$

$$X(f, g)_n = \sum_{k=0}^{K-1} f_k^* g_{(n+k)}$$

# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1

# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3

# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3 4

# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3 4 7

# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3 4 7 7



# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3 4 7 7 2

# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1

1 3 4 7 7 2 6

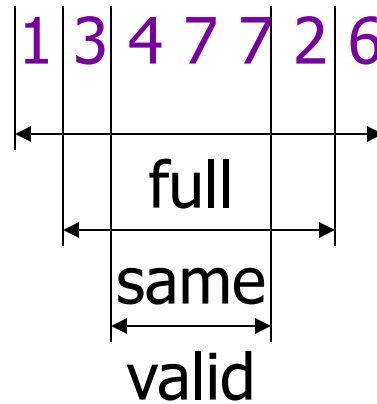
# Convolution

1 3 2 1 3

1 0 2

1 3 2 1 3

2 0 1



# Convolution

- Convolution and Cross Correlation

$$C(f, g)_n = \sum_{k=0}^{K-1} f_k g_{(n-k)}$$

$$X(f, g)_n = \sum_{k=0}^{K-1} f_k^* g_{(n+k)}$$

# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2

# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2 6

# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2 6 5

# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2 6 5 5



# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2 6 5 5 8

# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2 6 5 5 8 1

# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2

2 6 5 5 8 1 3

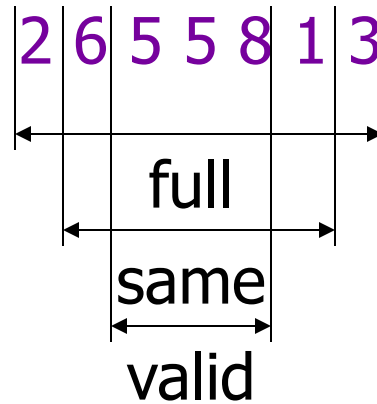
# Correlation

1 3 2 1 3

1 0 2

1 3 2 1 3

1 0 2



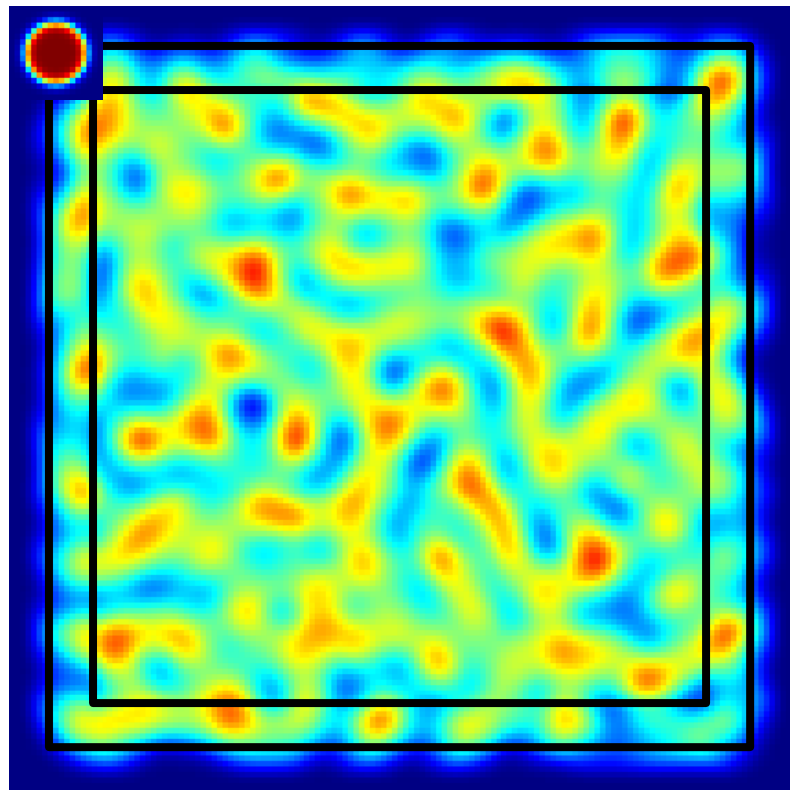
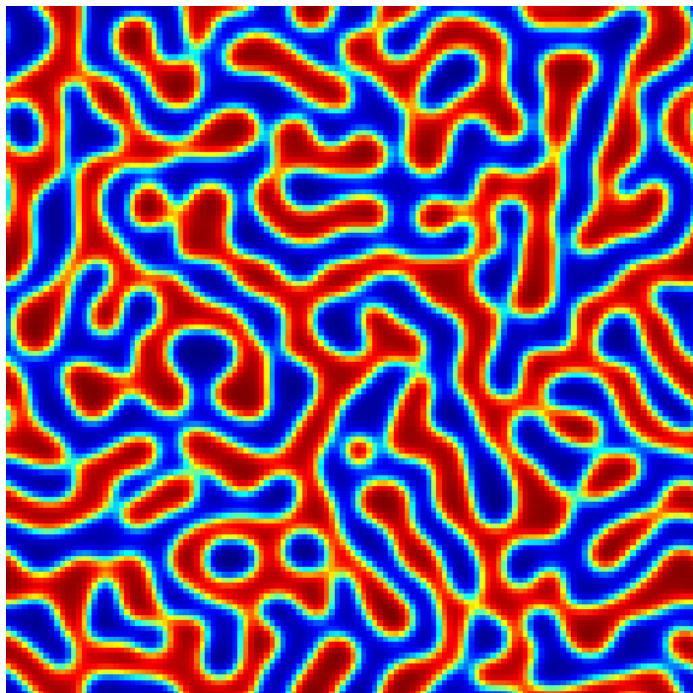
# Convolution

- Convolution and Cross Correlation

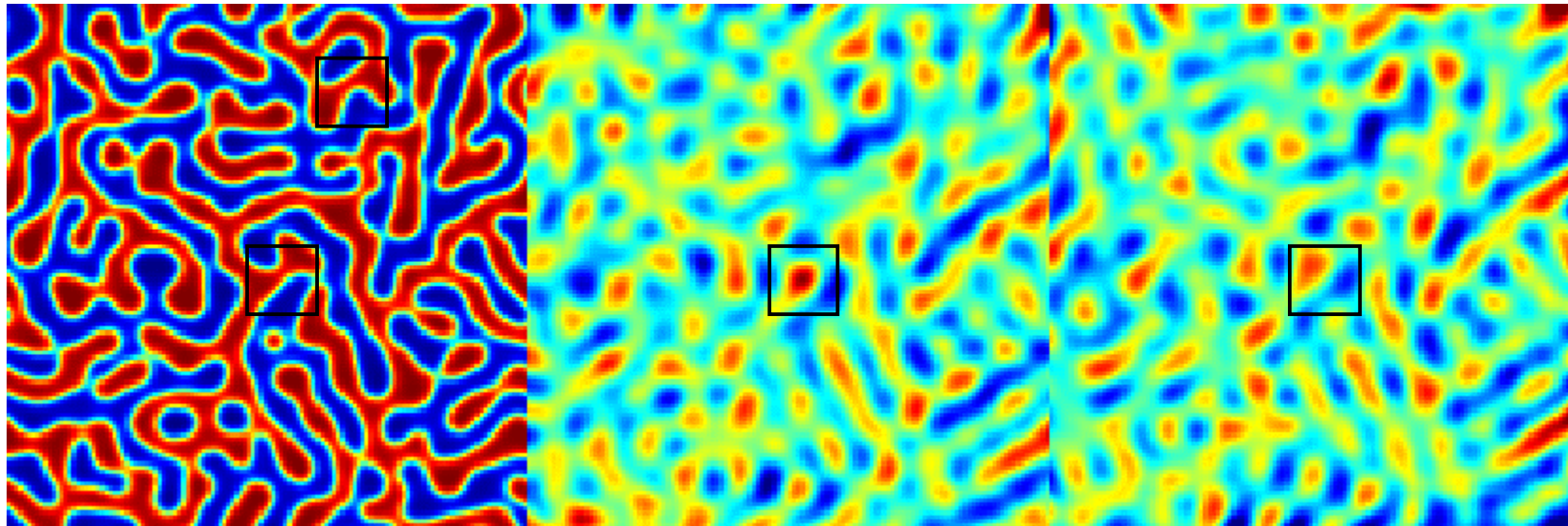
$$C(f, g)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f_{kl} g_{(n-k)(m-l)}$$

$$X(f, g)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f_{kl}^* g_{(n+k)(m+l)}$$

# Convolution



# Correlation vs. Convolution



Correlation

Convolution

# Least-Square Fit

- Minimum Squared Difference

$$\chi^2(I, p)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} [I_{kl} - p_{(n+k)(m+l)}]^2$$

$$\chi^2(I, p)_{nm} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} [I_{kl}]^2 - 2I_{kl} p_{(k+n)(l+m)} + [p_{(k+n)(l+m)}]^2$$

Correlation

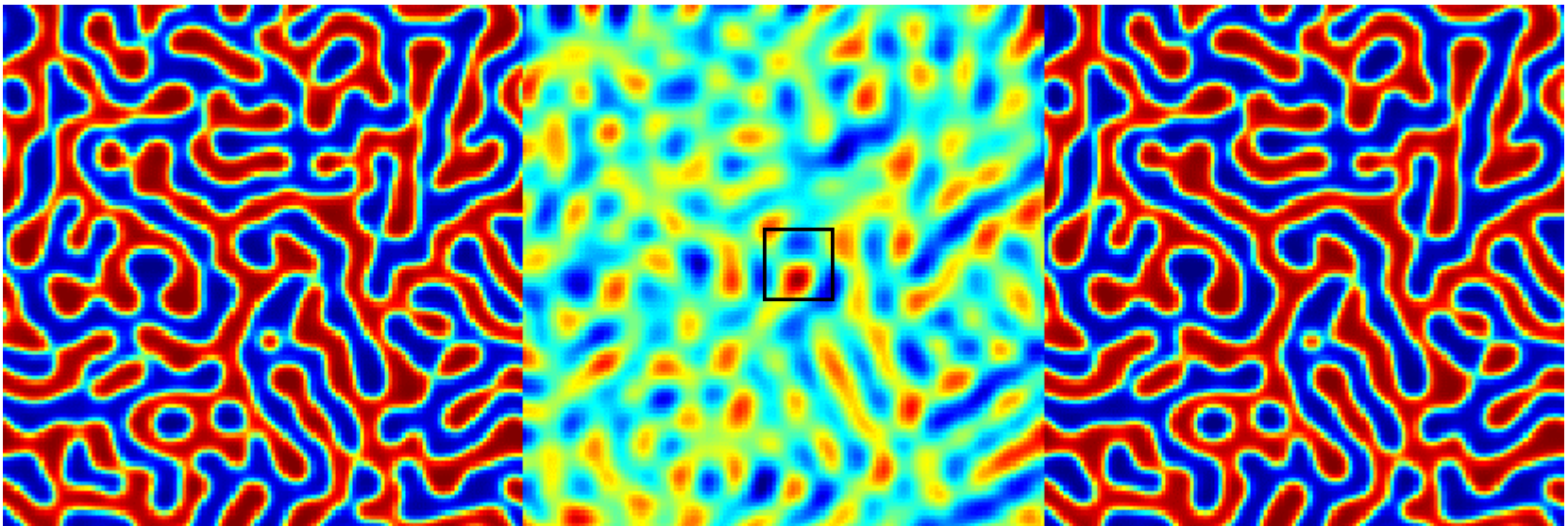
$$\min_{nm} \{ \chi^2(I, p)_{nm} \}$$



# Particle Imaging Velocimetry

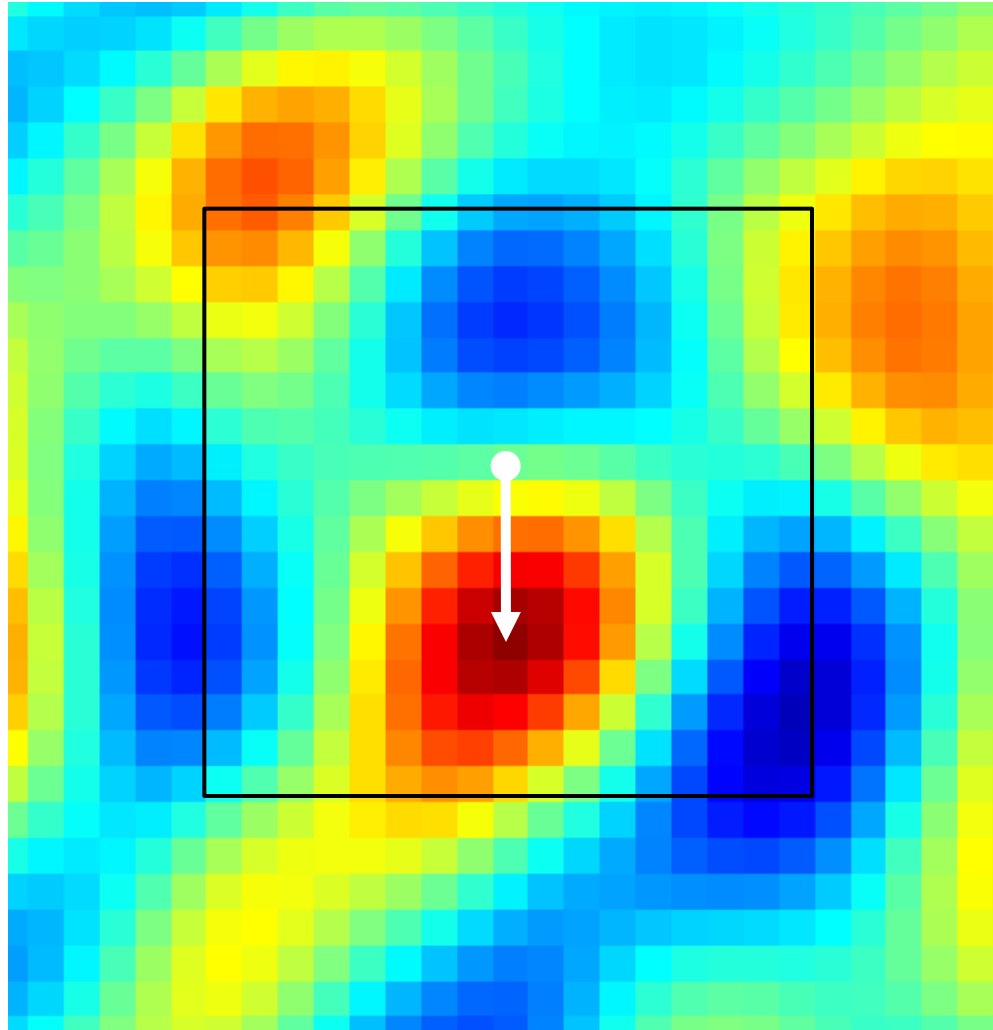
- Take a small patch,  $p$  from image  $n$  and cross-correlate it with image  $n+k$ .
- Find position of maximum.
- Distance from origin of  $p$  to maximum is the best fit for the displacement in time  $k$ .
- Pick new patch and repeat until all patches from frame  $n$  are found.
- Go to frame  $n+1$  and repeat until all frames are done.

# PIV

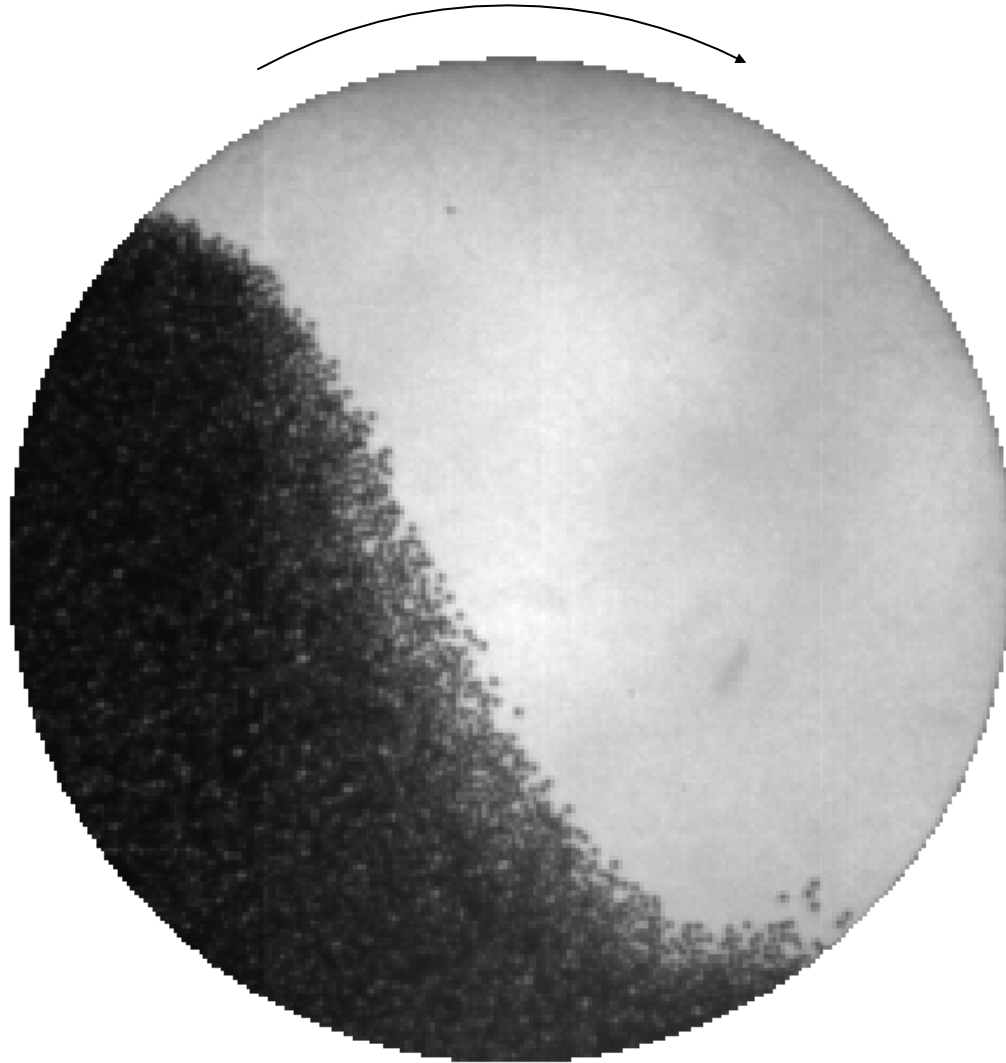


Correlation

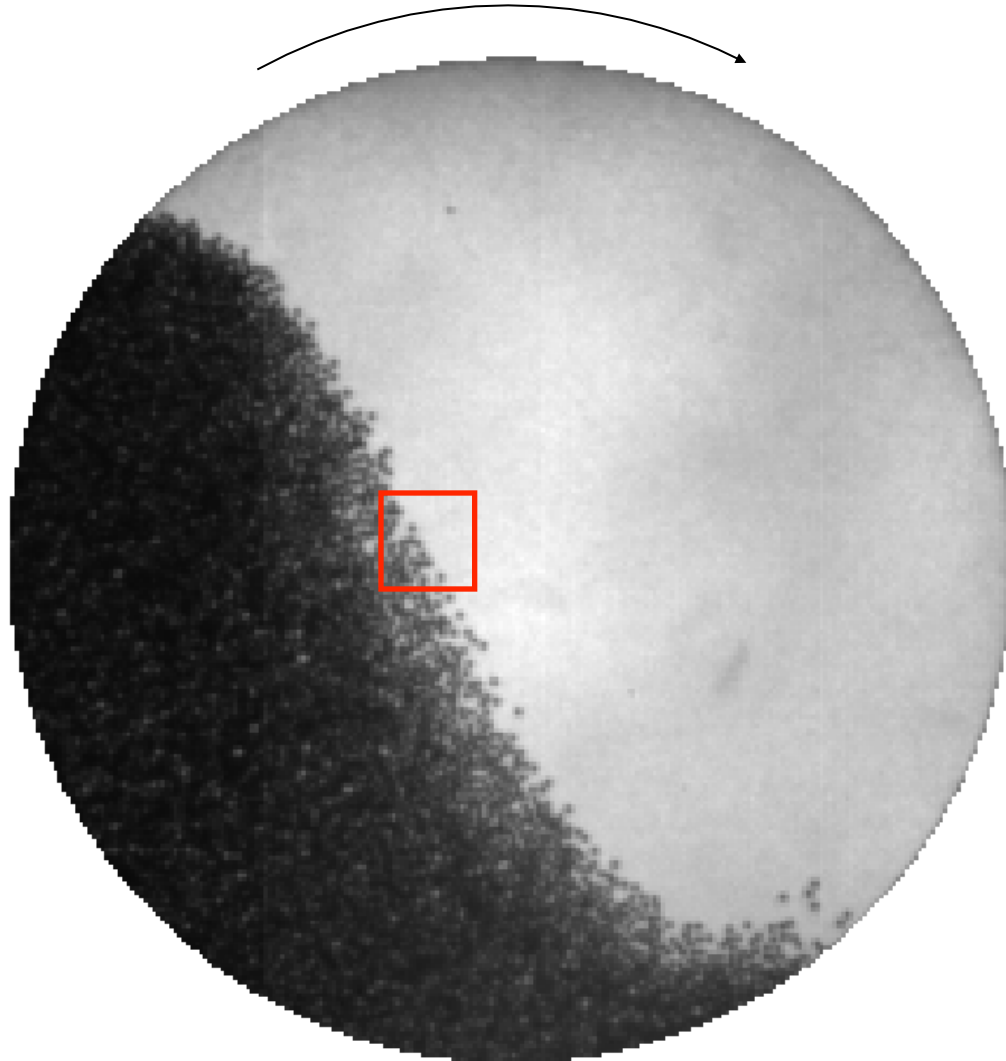
# PIV



# 2D Rotating Drum

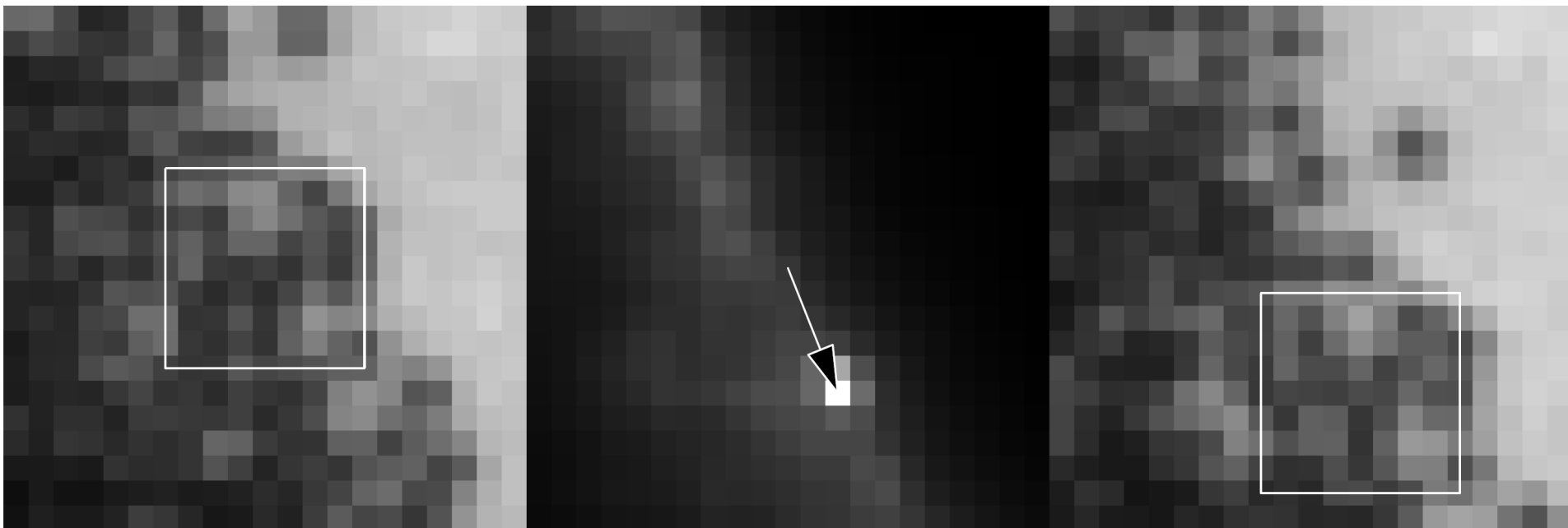


# 2D Rotating Drum



# PIV

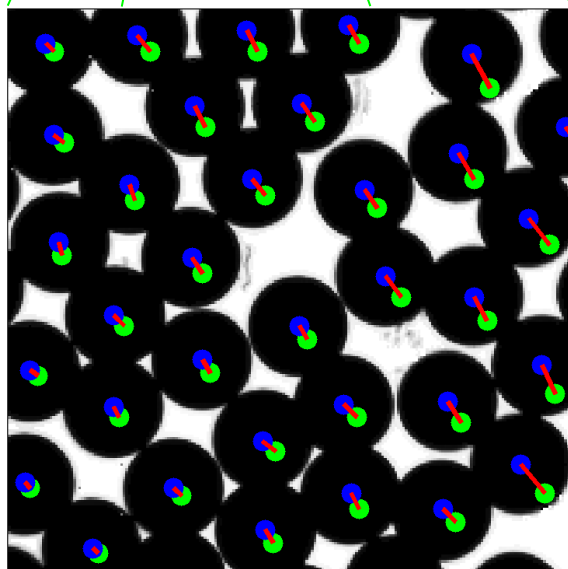
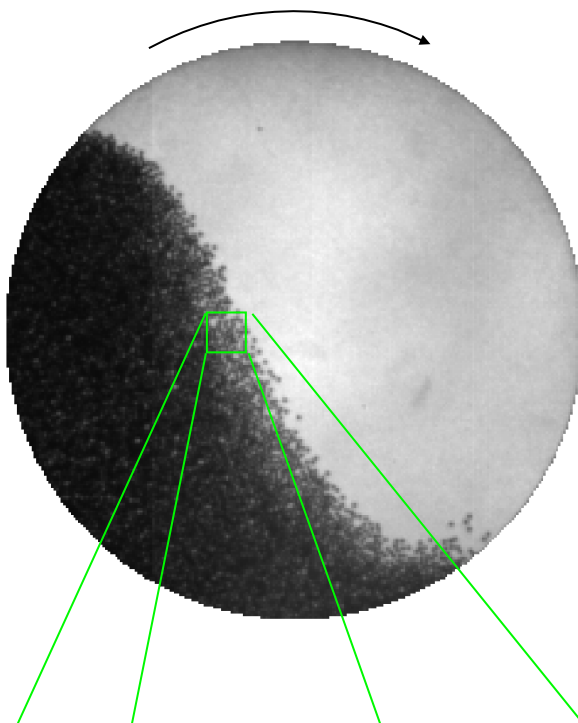
X-Corr



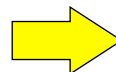
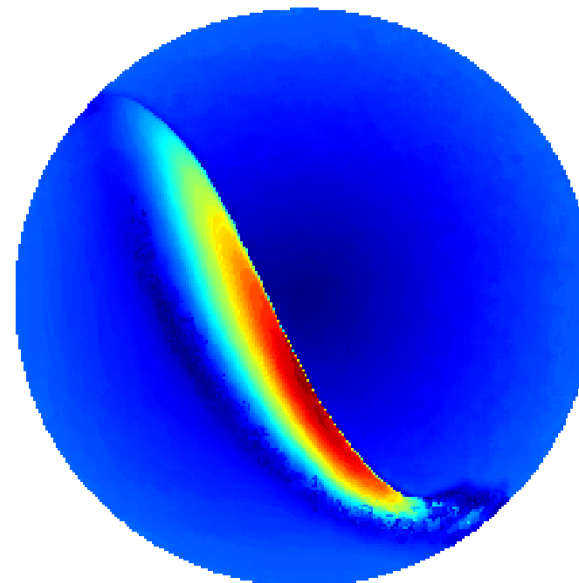
Frame n

Frame n+1

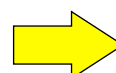
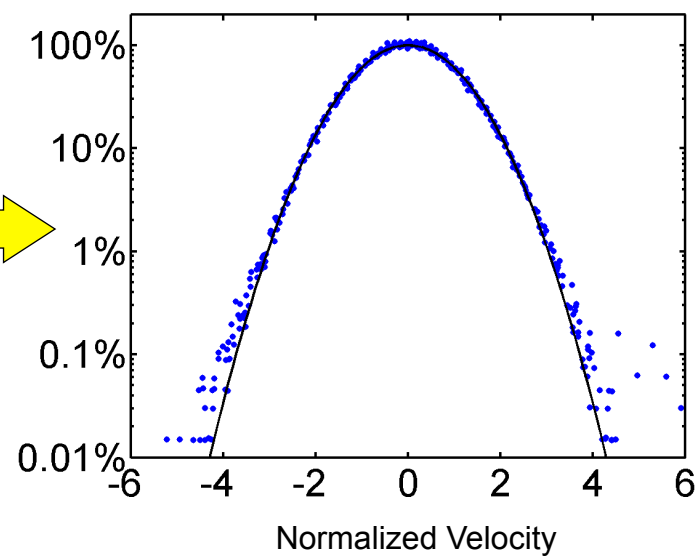
Flow snapshot



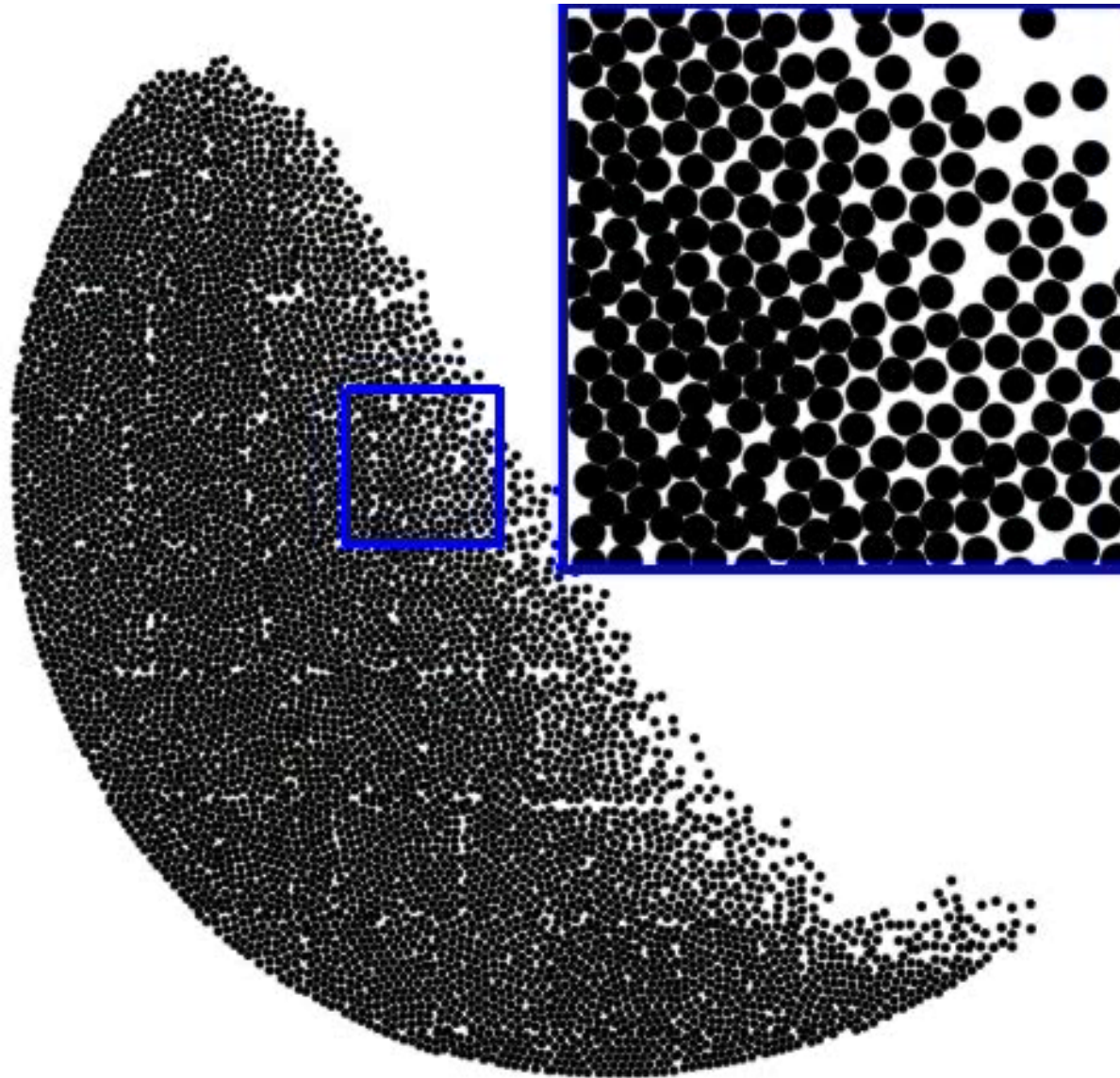
Speed



Velocity Histogram



# 2D Rotating Drum



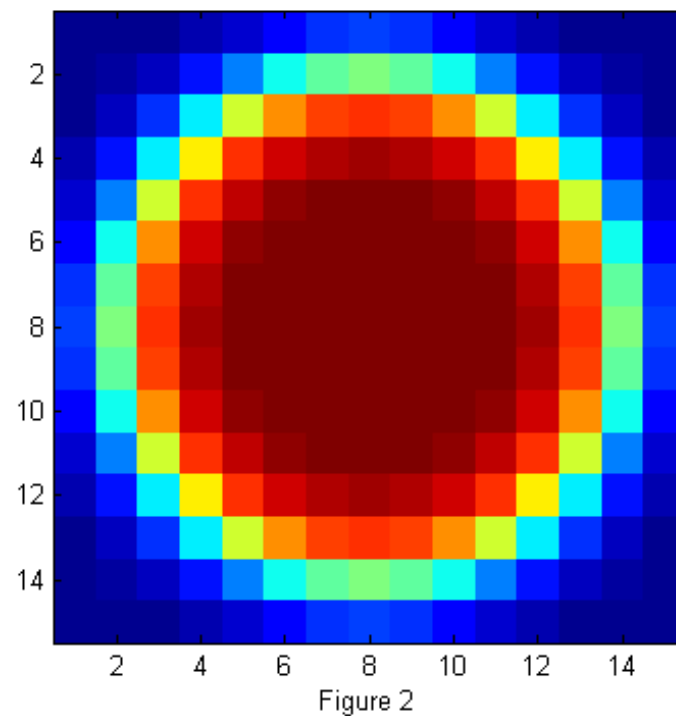
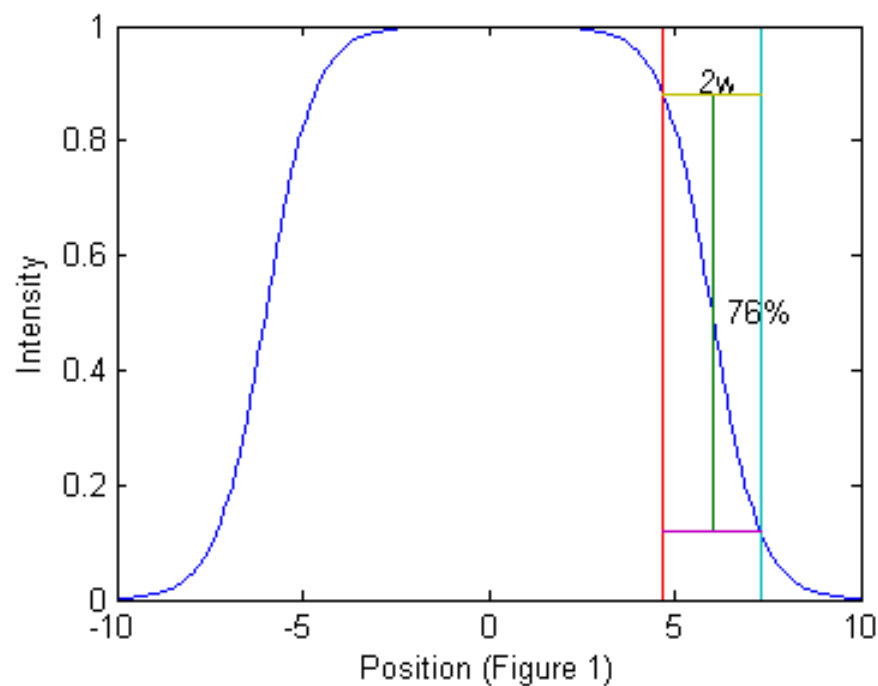


# Particle Tracking

## Pixel Accurate

$$I_c(\vec{x}) = \sum_{n=1}^N I_p(\vec{x} - \vec{x}_n(t); D, \dots),$$

$$I_p(\vec{x}; D, w) = [1 - \tanh(\frac{|\vec{x}| - D/2}{w})]/2.$$



# Particle Tracking

## Pixel Accurate

$$\chi^2(\vec{x}_0; D, w) = \int W(\vec{x} - \vec{x}_0) [I(\vec{x}) - I_p(\vec{x} - \vec{x}_0; D, w)]^2 d\vec{x},$$

$$\chi^2(\vec{x}_0; D, w) = \int W(\vec{x} - \vec{x}_0) [I(\vec{x})^2 - 2I(\vec{x})I_p(\vec{x} - \vec{x}_0; D, w) + I_p(\vec{x} - \vec{x}_0; D, w)^2] d\vec{x},$$

$$\chi^2(\vec{x}_0; D, w) = I^2 \otimes W - 2I \otimes (WI_p) + \langle WI_p^2 \rangle,$$

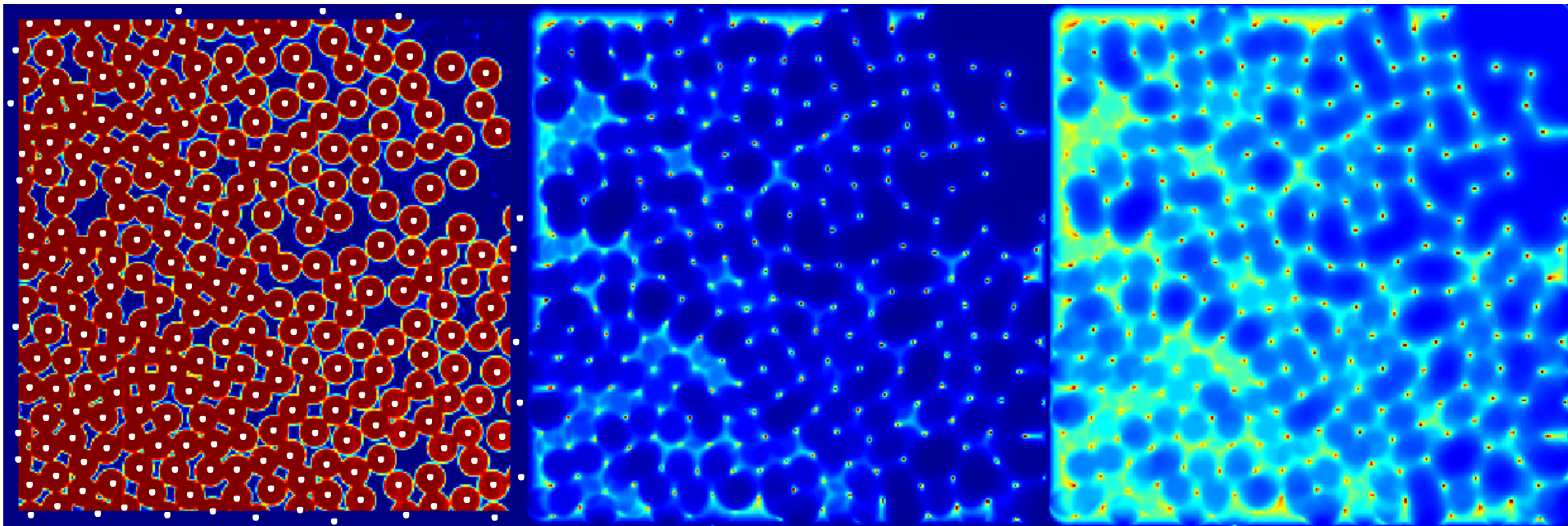
$$W = 1; \quad \chi^2(\vec{x}_0; D, w) = \int I^2 d\vec{x} - 2I \otimes I_p + \langle I_p^2 \rangle,$$

$$W = I_p; \quad \chi^2(\vec{x}_0; D, w) = I^2 \otimes I_p - 2I \otimes I_p^2 + \langle I_p^3 \rangle.$$

# Particle Tracking

## Pixel Accurate

$$W = I_p; \quad \chi^2(\vec{x}_0; D, w) = I^2 \otimes I_p - 2I \otimes I_p^2 + \langle I_p^3 \rangle .$$



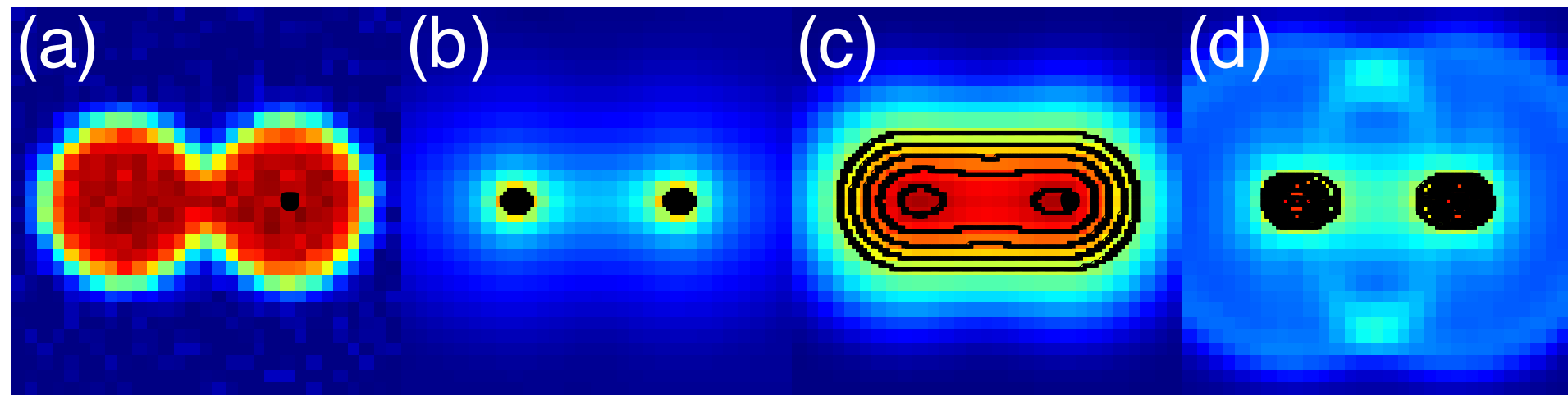
Corrected Image

Chi-Squared Image

# Particle Tracking

## Pixel Accurate

Image    Least-Squares    X-Corr    Hough

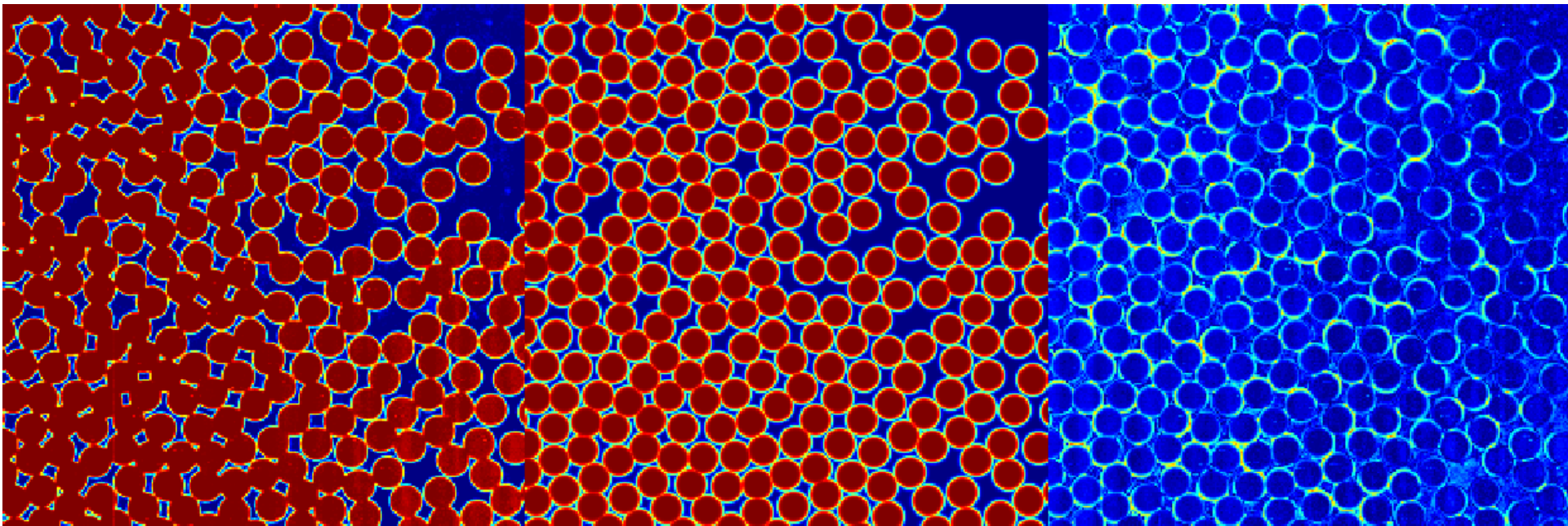


# Particle Tracking

## *Sub-Pixel Accurate*

$$\chi^2(\vec{x}_n; D, w) = \int [I(\vec{x}) - I_c(\vec{x}, \vec{x}_n)]^2 d\vec{x},$$

$$I_c(\vec{x}, \vec{x}_n) = \sum_n W_n(\vec{x}) I_p(\vec{x} - \vec{x}_n; D, w),$$





# Particle Tracking

## *Sub-Pixel Accurate*

$$\frac{\partial \chi^2(\vec{x}_n^*; D, w)}{\partial \vec{x}_n^*} = 0,$$

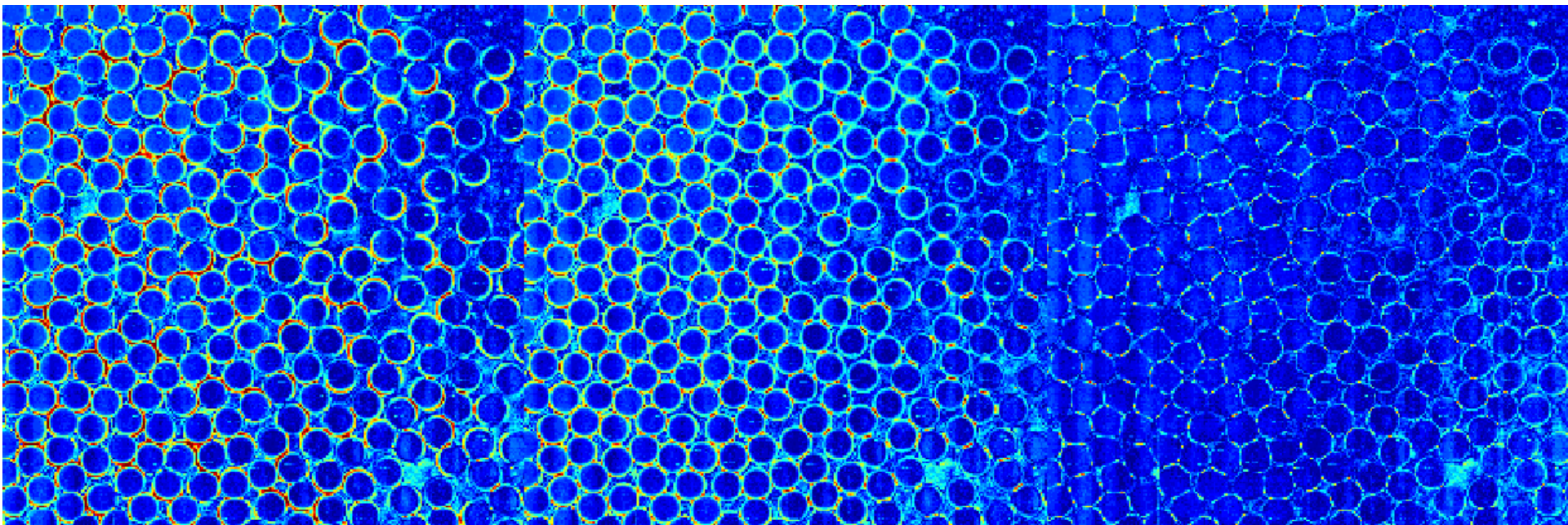
$$\frac{\partial \chi^2(\vec{x}_0; D^*, w^*)}{\partial D^*} = 0$$

$$\frac{\partial \chi^2(\vec{x}_0; D^*, w^*)}{\partial w^*} = 0$$

$$\chi^2 = 937.4$$

$$\chi^2 = 615.2$$

$$\chi^2 = 180.0$$

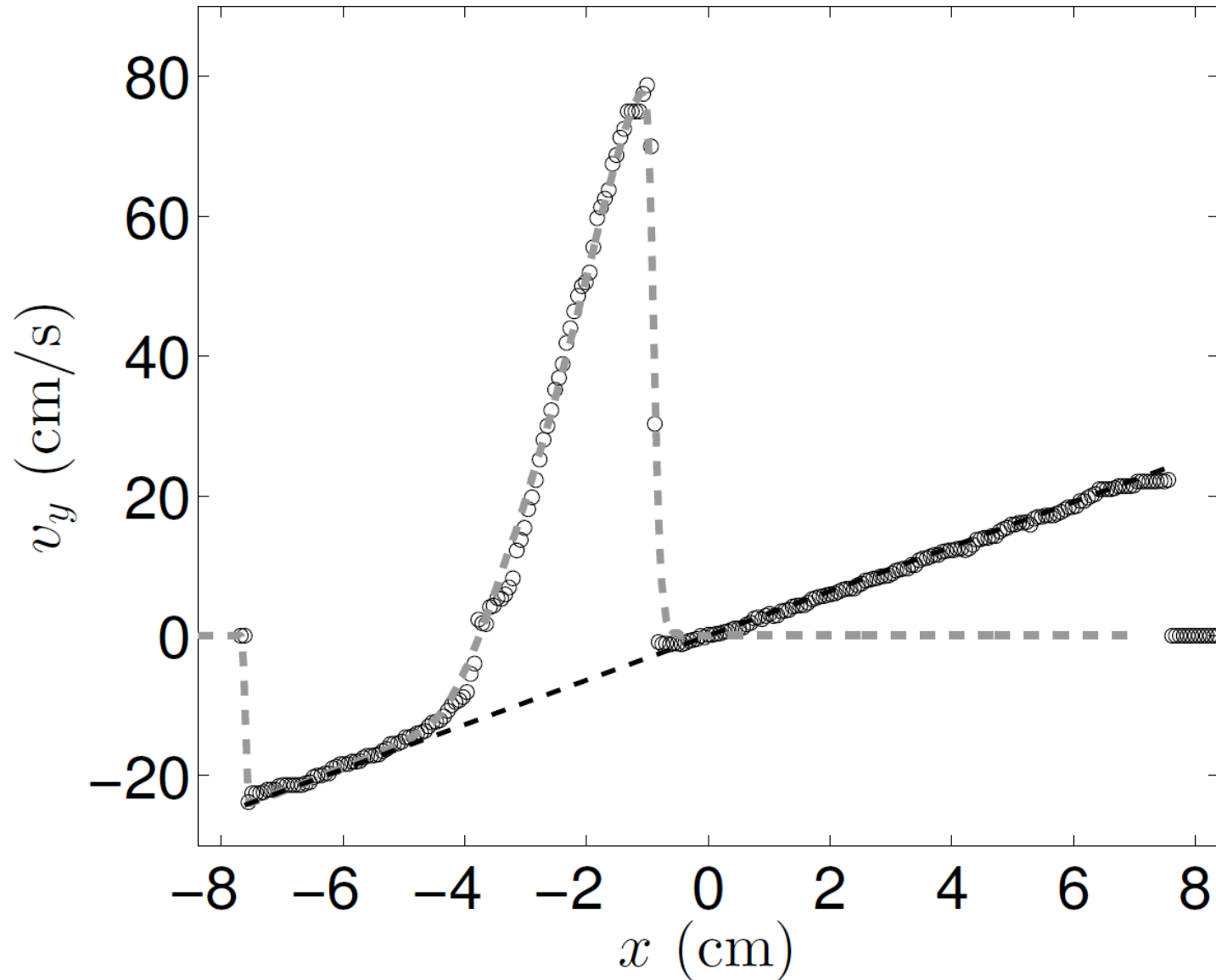


Original

Positions

Diameter-width

# Particle Tracking vs. PIV



# Particle Tracking Extensions (shape)

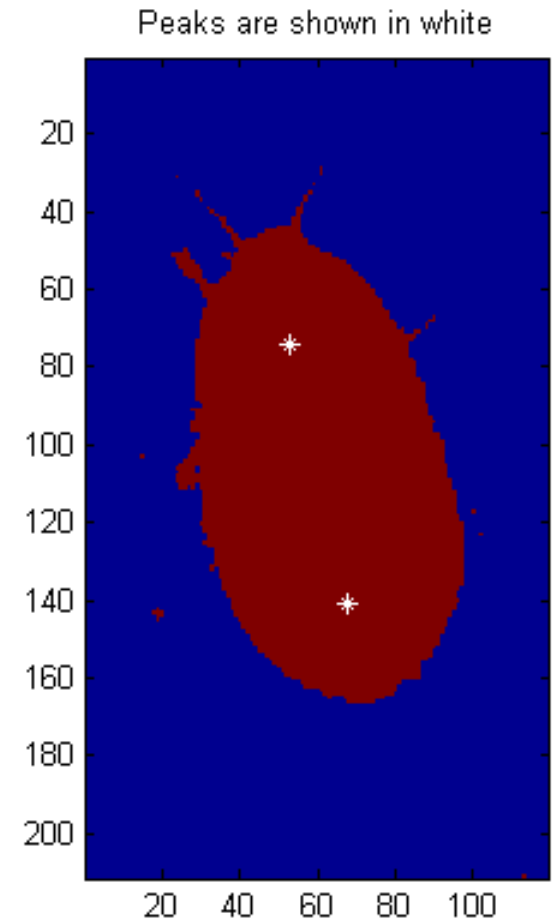
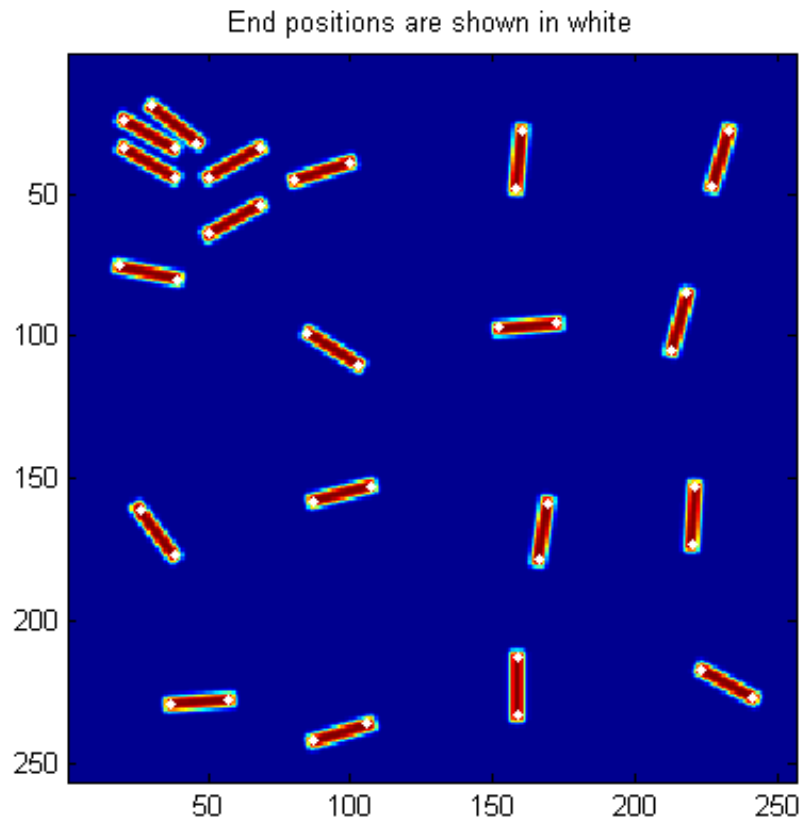
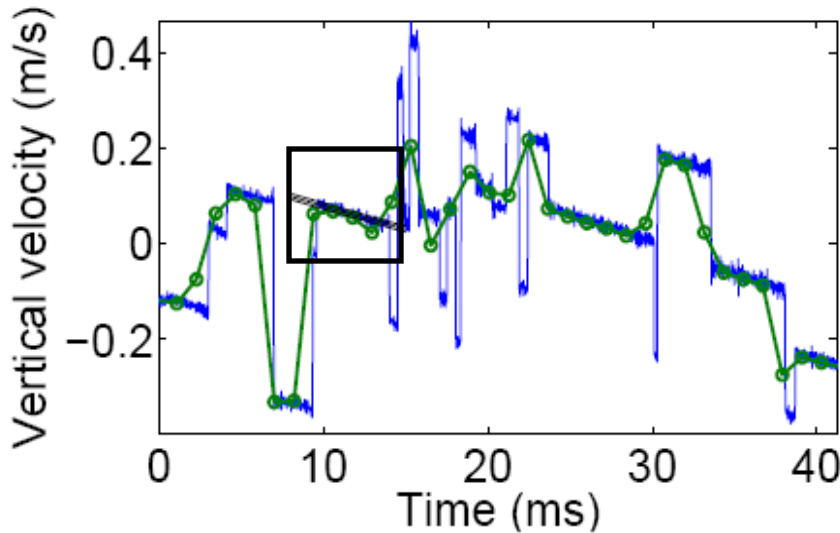


Figure 13.

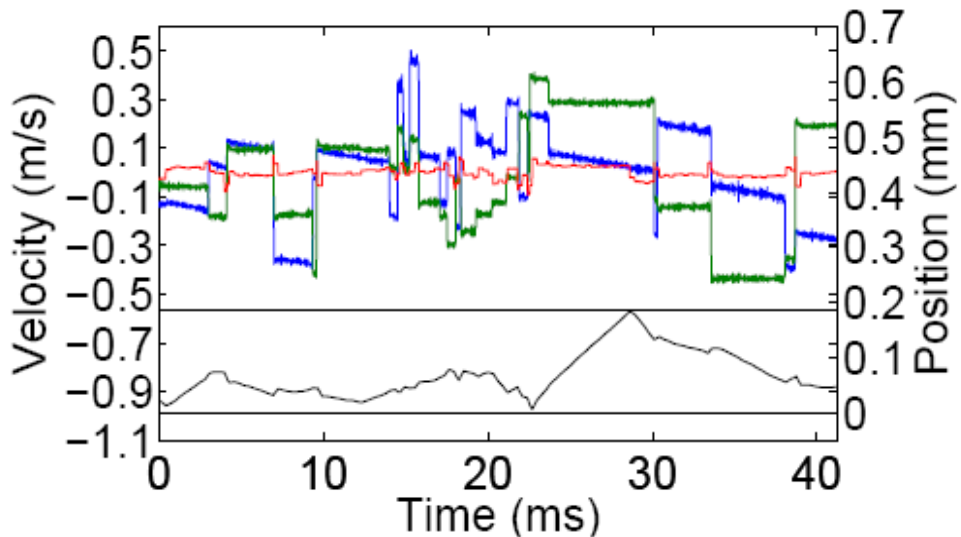
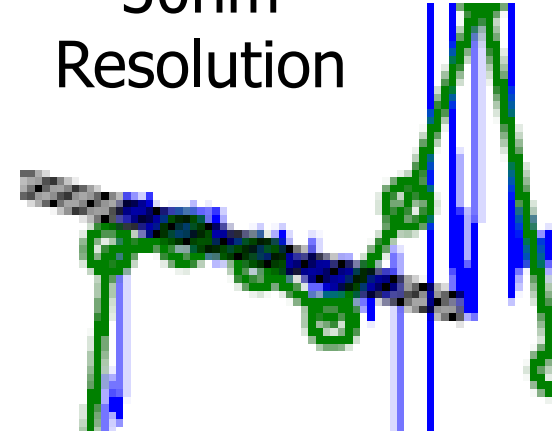


# Particle Tracking

## Extensions (High Accuracy, 3D)

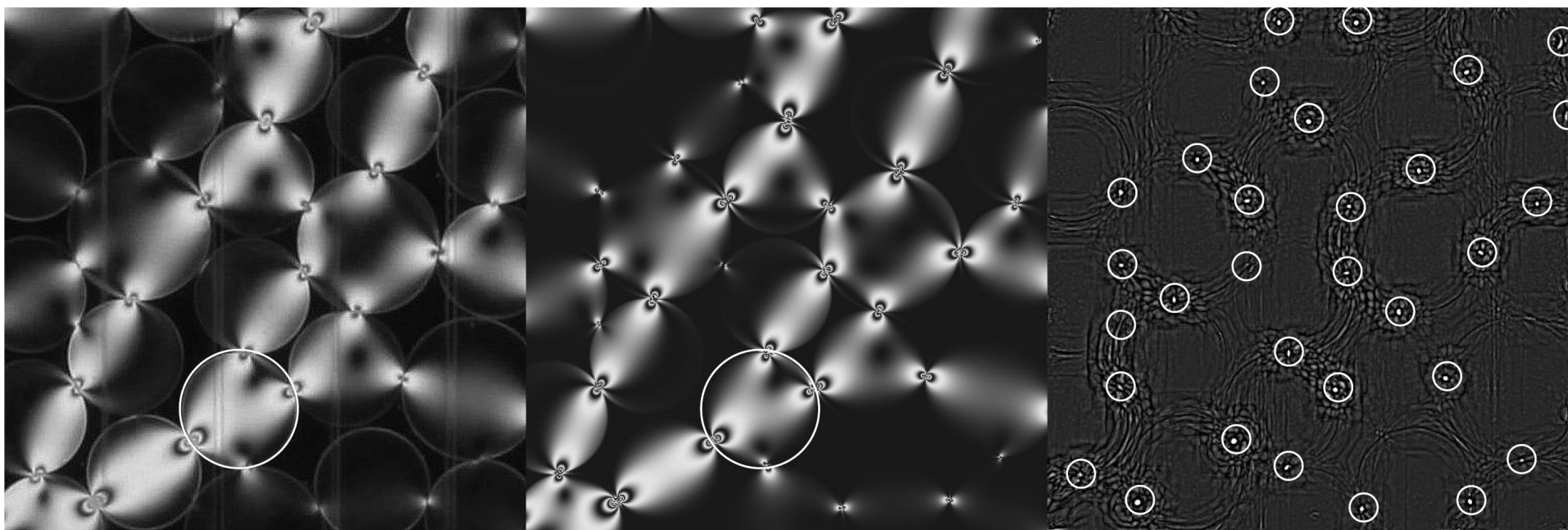


30nm  
Resolution

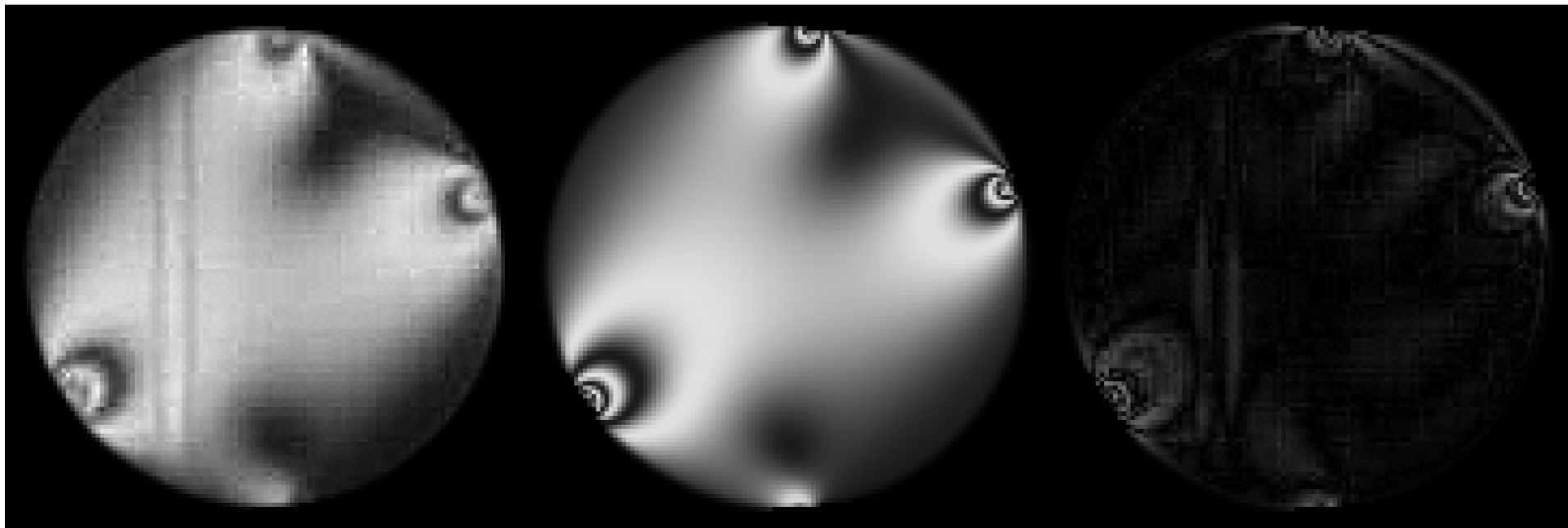


$$30\text{nm}/3\text{mm} = 1/10^5$$

# Particle Tracking Extensions (Force)

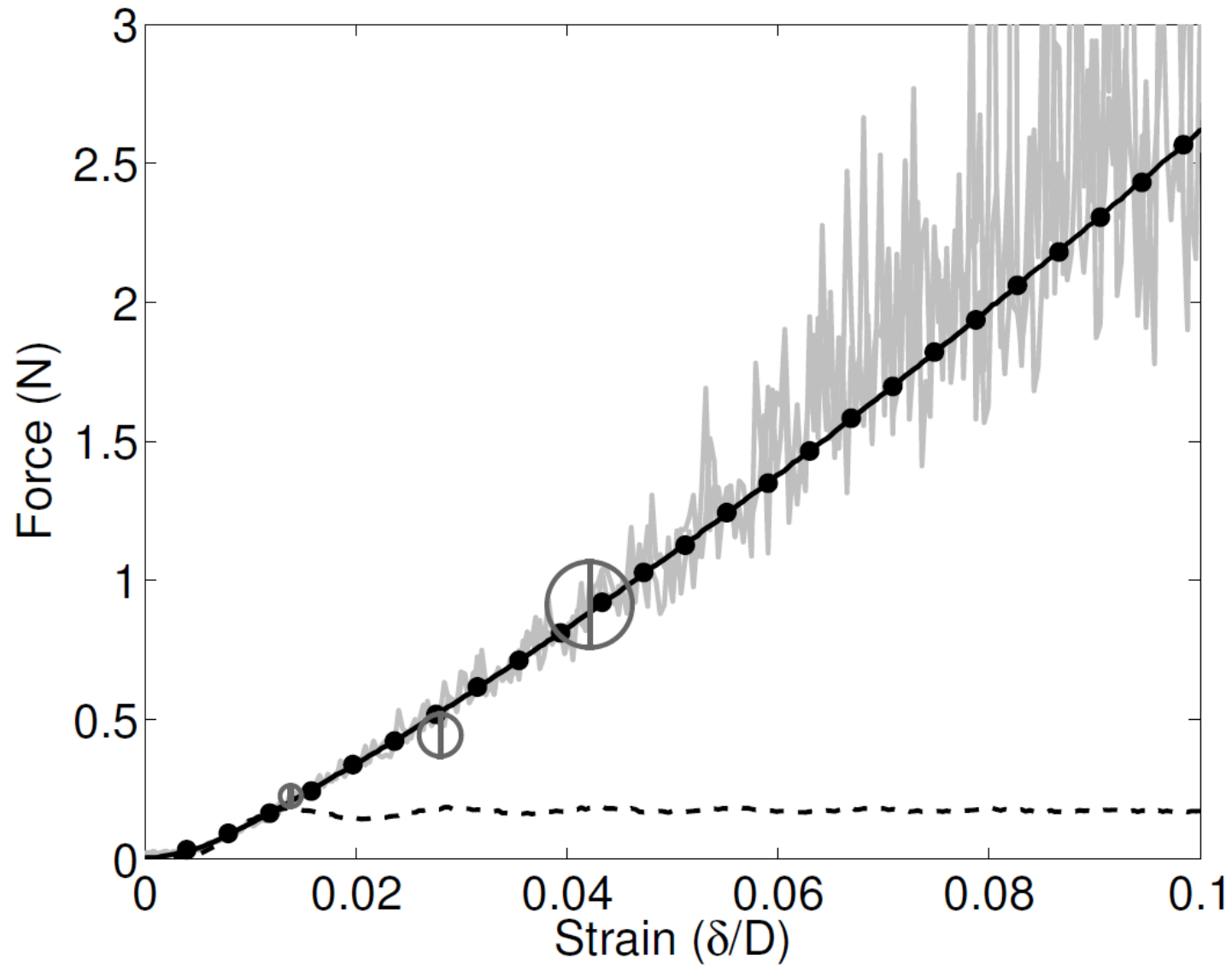


# Particle Tracking Extensions (Force)



# Particle Tracking

## Extensions (Force)



# The End

