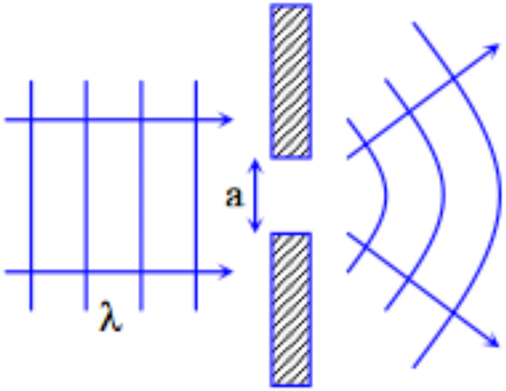
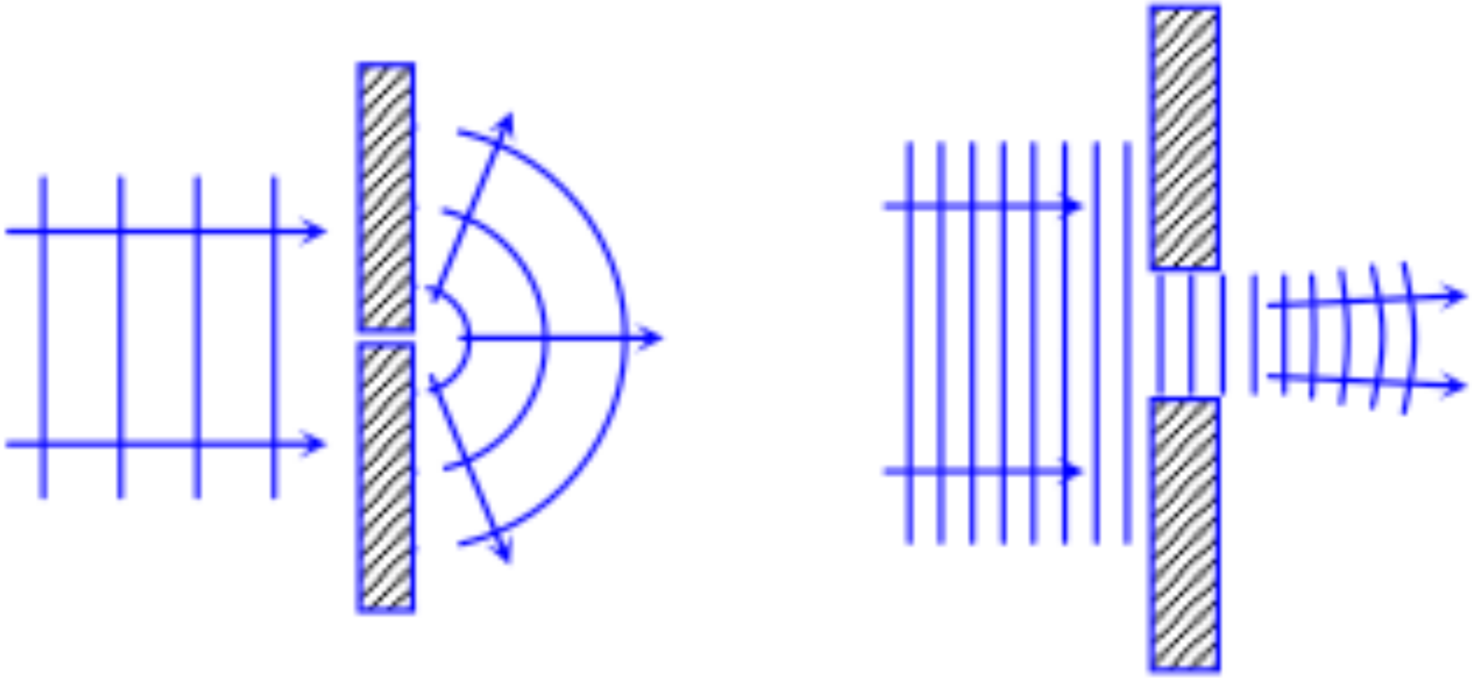
Introduction:

Diffraction is bending of waves around an obstacle (barrier) or spreading of waves passing through a narrow slit.



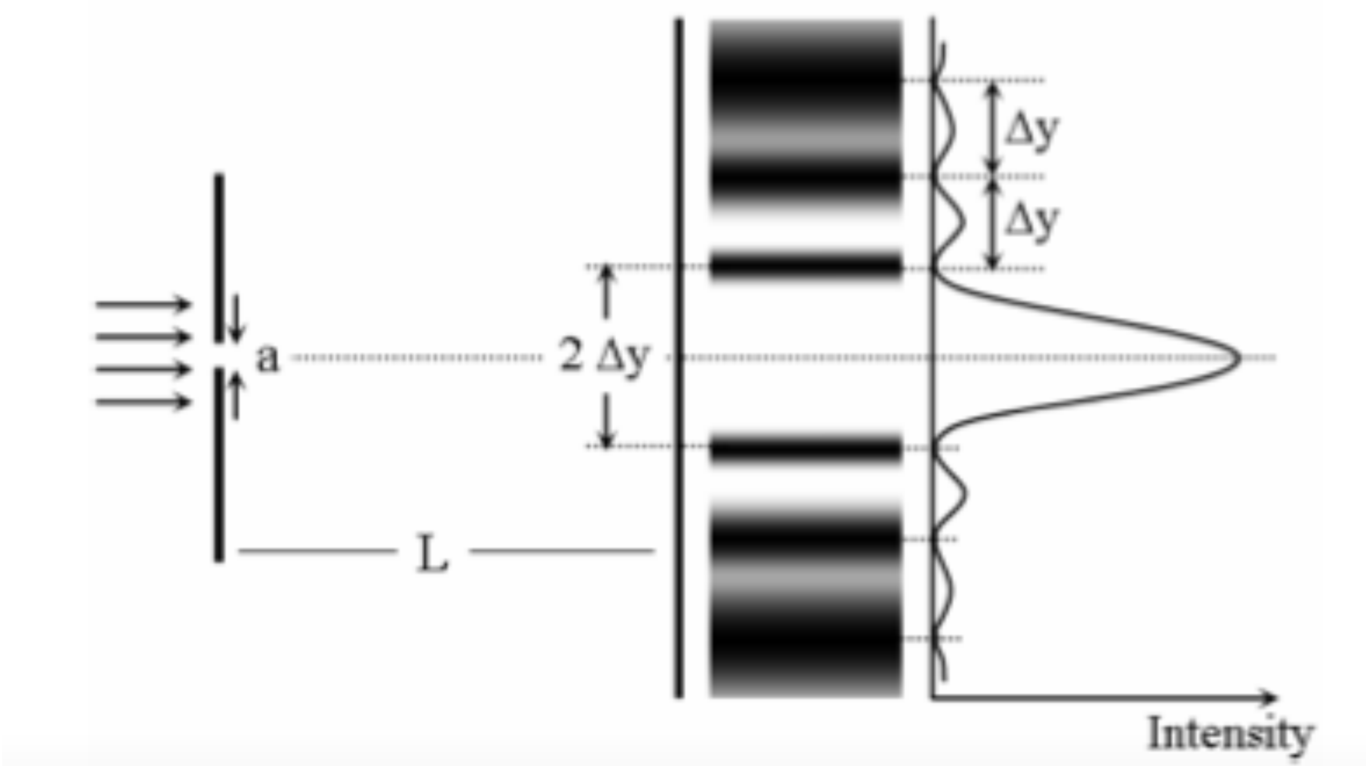
Diffraction amount depends on **λ/a** proportion

If a >> λ diffraction is negligible

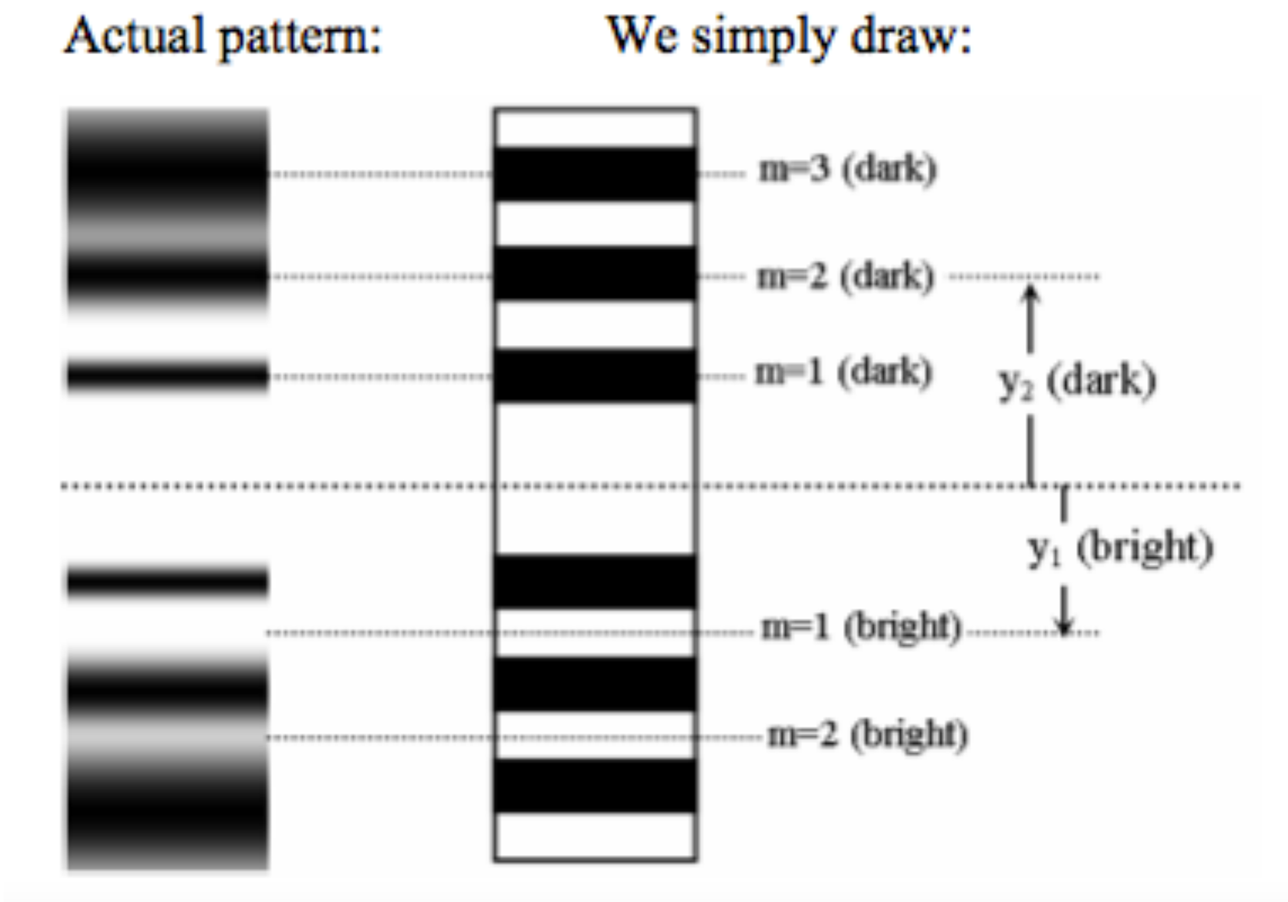


- Same phenomenon is observable with light waves.
- Since **λ** of light is very small, the opening of the slit must also be very small, something like 0.1 mm.

Single slit diffraction:

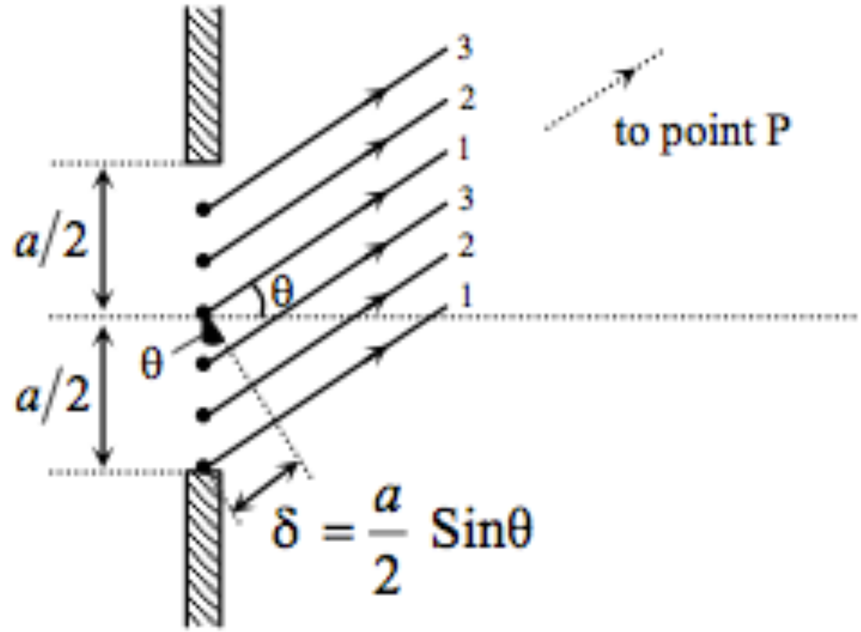


- Most of the light energy is concentrated at the central maximum.
- Actually it is possible to say that all the light passing the slit is spread as wide as the central maximum simply omitting the other bright fringes.



- We still have dark fringes although there is only one slit.
- Therefore light waves coming from different portions of the slit must be canceling.

If we divide the slit into two equal portions:



- Now condition for first dark:

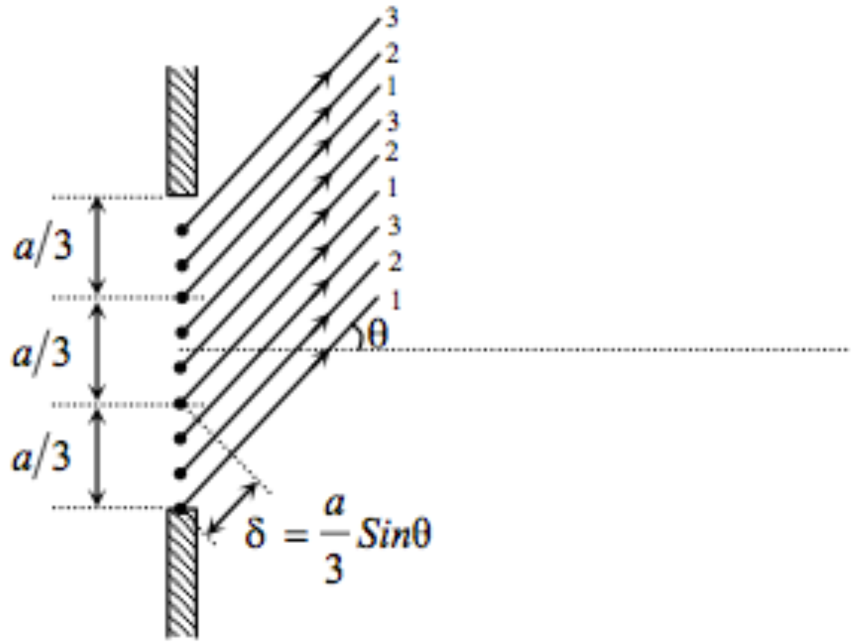
$$\delta = \frac{\lambda}{2} \Rightarrow \frac{a}{2} Sin\theta = \frac{\lambda}{2}$$

$$\boxed{a\ Sin\ \theta = \lambda \text{ (first dark)}}$$

First dark is important, because between two first darks we have the central bright, which receives nearly all the light energy passing through the slit.

For the bright condition (the slit is divided into 3, 5, 7, (odd number)

- For the first bright ($m = 1$), we divide the slit into 3 equal portions



- For the first bright ($m = 1 \rightarrow \delta = a/3 \sin\theta = \lambda/2$, waves from two portions cancel but the remaining third portion illuminates the point on screen.

- So far the first bright: $m = 1 \rightarrow a \sin\theta = 3/2\ \lambda = (m + \frac{1}{2})\lambda$

If we divide the slit into 4, 6, 8, (even number) equal parts [and set $\delta=\lambda/2$ we will have $(a/4)\sin\theta=\lambda/2$, $(a/6)\sin\theta=\lambda/2$, $(a/8)\sin\theta=\lambda/2$, …] we get condition for other darks.

Dark	Bright
$a\,\mathrm{Sin}\theta = m\lambda$	$a\,\mathrm{Sin}\theta=(m+1/2)\lambda$
$a\dfrac{y_m}{L} = m\lambda$	$a\dfrac{y_m}{L} = \left(m+\dfrac{1}{2}\right)\lambda$
$\boxed{y_m = \dfrac{\lambda L}{a}m}$	$\boxed{y_m = \dfrac{\lambda L}{a}\left(m+\dfrac{1}{2}\right)}$
m=1,2,3…	m=1,2,3…

$$\Delta y = \frac{\lambda L}{a}$$

Example: 500 nm monochromatic light passes through a slit having 0.01 mm width. How much does it spread?

Solution:

Given: $\lambda = 500$ nm $= 5\text{x}10^{-7}$ m, a = 0.01 mm $= 1\text{x}10^{-5}$ m

Find: θ = ?

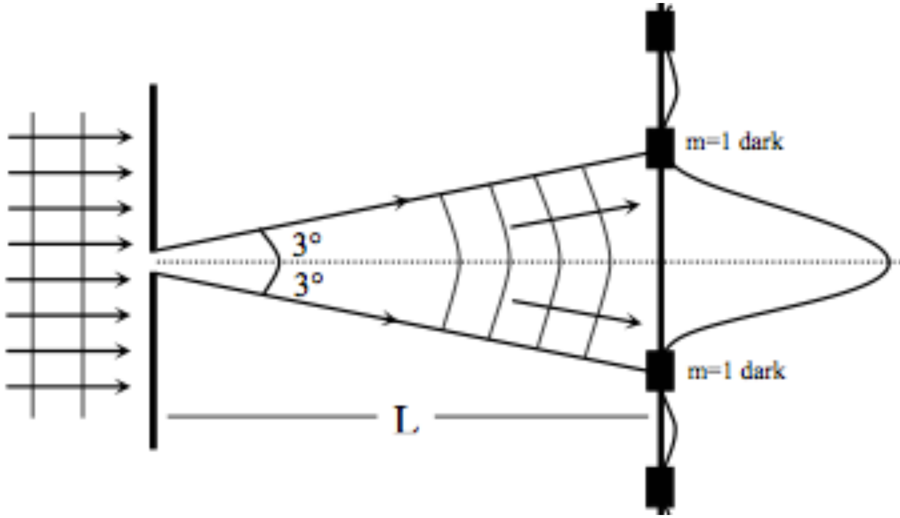
Answer:

For the first dark: $a \sin \theta = m\lambda$

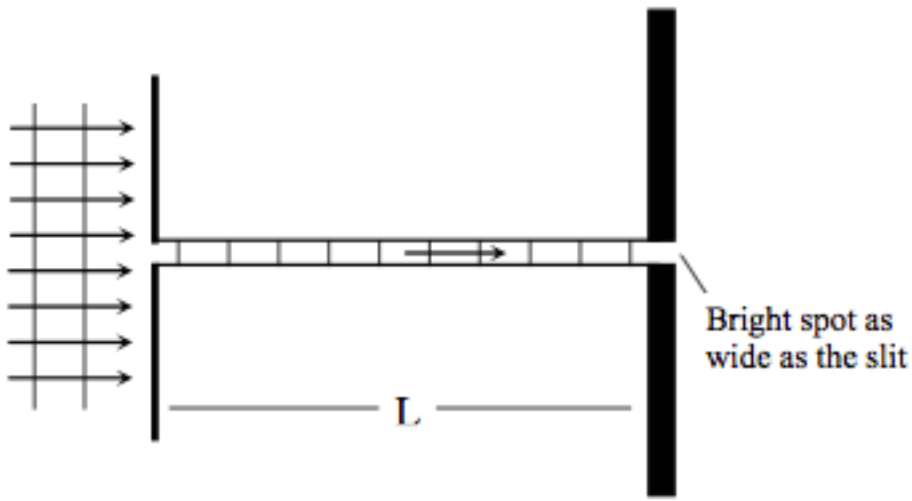
$a \sin \theta_1 = \lambda$

$\sin \theta_1 = \lambda/a = 5\text{x}10^{-7}/ 1\text{x}10^{-5}$

$\theta_1 \approx 3^o$



If there is no diffraction



Example: Monochromatic light, λ = 600 nm, passes through a slit 0.1 mm wide and illuminates a screen 2 m away. Find the width of central bright on screen!

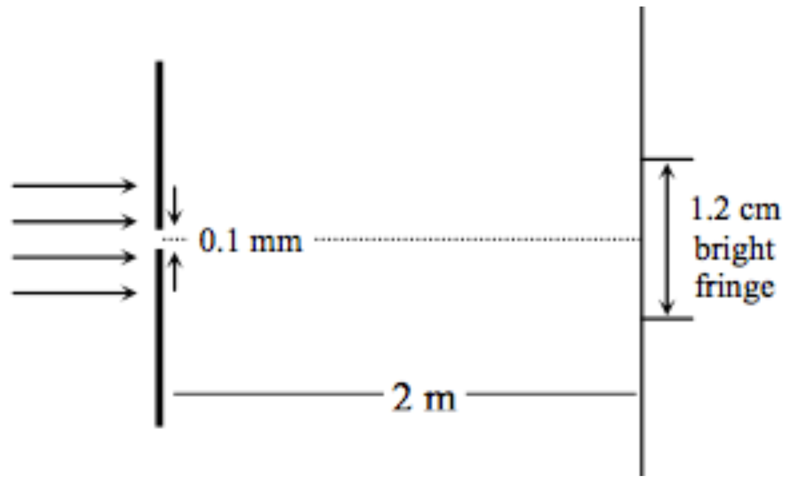
Solution:

Given: λ = 600 nm = $6x10^{-7}$ m, a = 0.1 mm = $1x10^{-4}$ m, L = 2 m

Find: Δy = ?

Answer:

$\Delta y = \lambda L/a = (6x10^{-7})x(2)/(1x10^{-4}) = 0.012$ m = 1.2 cm



If there is no diffraction, there would be a bright spot 0.1 mm wide on the screen.

What are the effect of diffraction?

- We can not send a light ray along a straight path for a long distance. It will spread and lose intensity.
- We do not have sharp shadows of objects even with a point light source.

What is the maximum slit width for diffraction?

- If the light source is coherent, diffraction always occurs for all openings even if the slit is large.
- But, according to formula: $\sin\theta = \lambda/a$ (first dark), if $a >> \lambda$, then θ is very small.
- So diffraction effect becomes negligible over small distance. But, for large distance a small angle causes a large separation.
- If we are trying to send a 5 mm wide laser ray from earth to moon for example, the spreading of the beam will be ~ 0.01^{o}, which is negligible at the beginning, but, when it reaches the moon, the beam will be as wide as~ 80 km!!

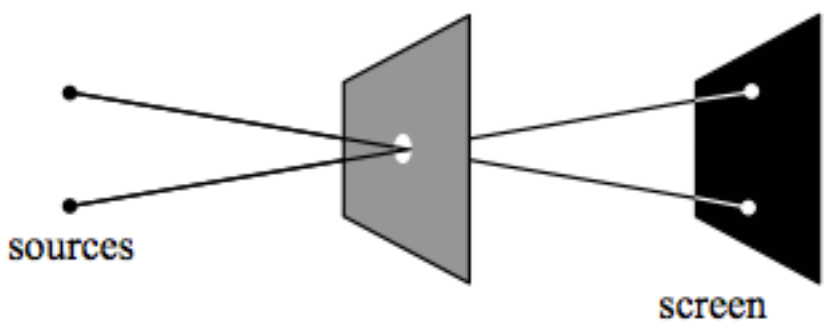
Resolving Power

Two light sources are seen as a single source if they are far away enough.

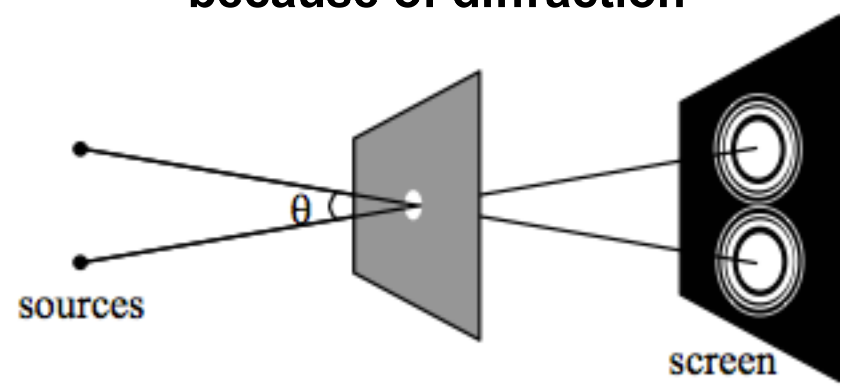
- Many bright dots in the night sky are actually star pairs – not single star.
- Another example can be the two headlights of a car approaching from a distance.We can not send a light ray along a straight path for a long distance. It will spread and lose intensity.

The reason is diffraction. When light from the sources passes through the pupil of the eye, (which is a circular opening of ~ 2-3 mm) diffraction occurs. The retina acts as a screen.

If there is no diffraction

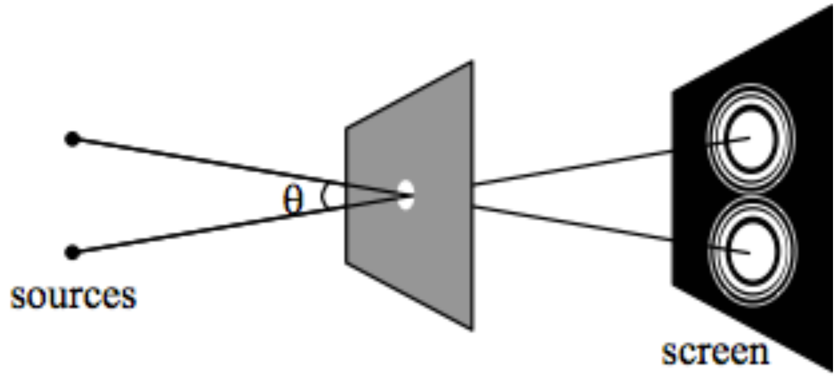


because of diffraction

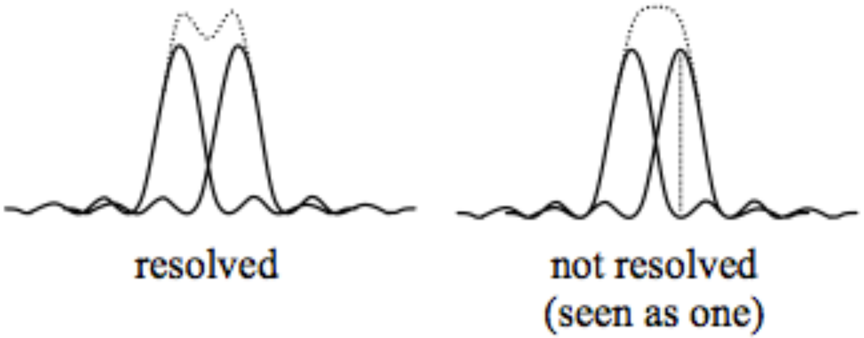


Resolving Power

because of diffraction



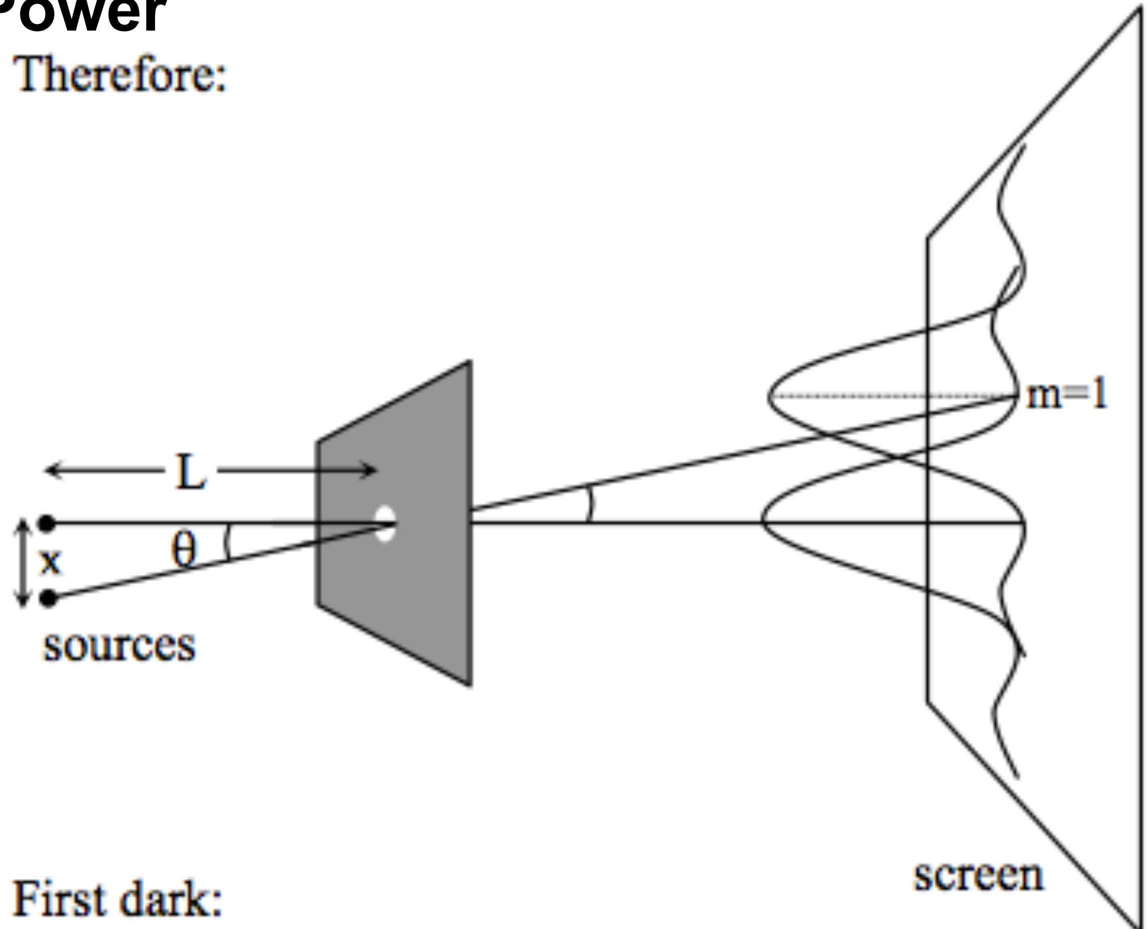
When θ gets smaller patterns overlap and seen as one:



Two sources seen as one when central bright of one pattern is on the first dark of the other.

Resolving Power

Therefore:



First dark:

$d \, Sin\theta = \lambda \quad \Rightarrow$

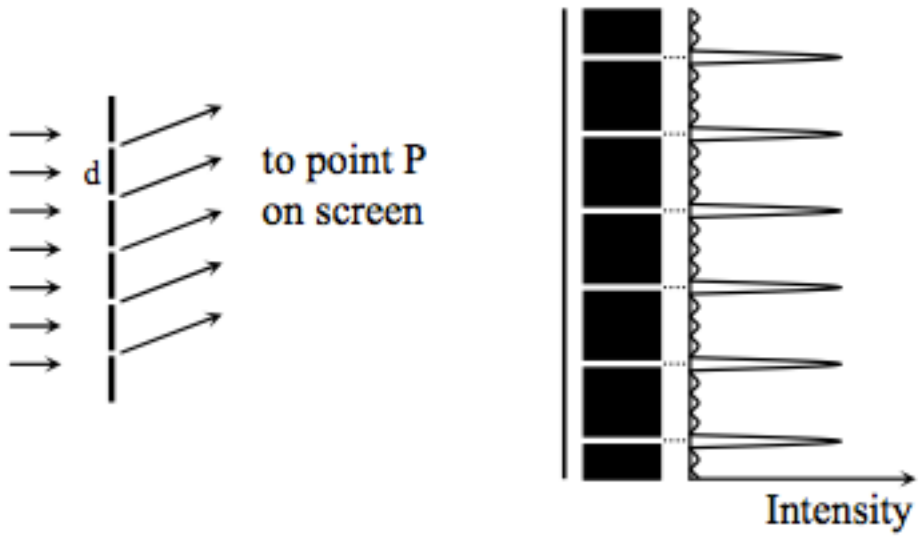
$Sin\theta = \frac{\lambda}{a}$ $\qquad$ $\frac{x}{L} > \frac{\lambda}{a}$ two sources seen

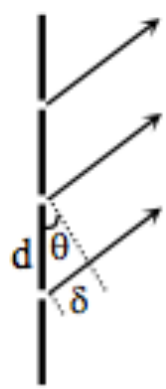
$\frac{x}{L} = \frac{\lambda}{a}$ (just resolved) $\qquad$ $\frac{x}{L} < \frac{\lambda}{a}$ seen as one source

Caution: These formulas are for slits, for circular apertures we have a factor of 1.22 which we negelected here!!

Diffraction grating

The diffraction grating is a more useful device to analyze light sources, because the interference maximums (bright fringes) are thin lines, making the measurements easier.





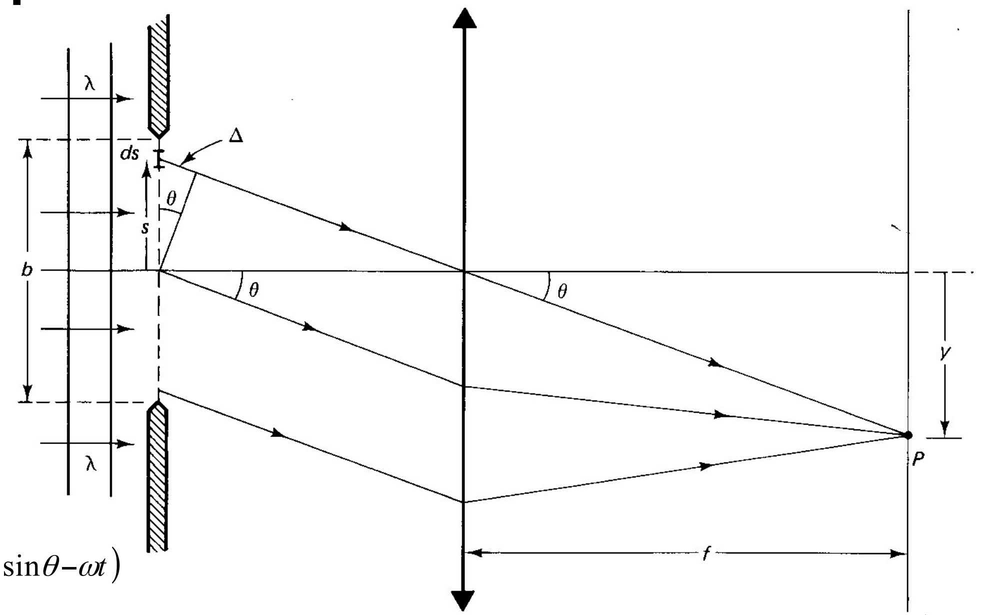
$\delta = d\,\mathrm{Sin}\theta$

Therefore; m'th BRIGHT fringe:

$\boxed{d\,\mathrm{Sin}\theta = m\,\lambda}$ $\quad (m = 0, 1, 2, 3 \ldots)$

If both the source of light and observation screen are effectively far enough from the diffraction aperture so that the wavefronts arriving at the aperture and observation screen may be considered plane, it is called **Fraunhofer**, or **far-field diffraction**. When the curvature of the wavefront must be taken into account, it is called **Fresnel**, or **near-field diffraction**.

Fraunhofer Diffraction at Single Apertures:



Each interval contributes spherical wavelets at P of the form,

$$dE_p = \left(\frac{dE_0}{r}\right)e^{i(kr-\omega t)}$$

$$dE_p = \left(\frac{dE_0}{r}\right)e^{i[k(r_0+\Delta)-\omega t]}$$

$$dE_p = \left(\frac{E_L ds}{r}\right)e^{i[k(r_0+\Delta)-\omega t]} = \left(\frac{E_L ds}{r_0}\right)e^{i(kr_0+ks\sin\theta-\omega t)}$$

$$E_p = \left(\frac{E_L}{r_0}\int_{-b/2}^{b/2} e^{iks\sin\theta} ds\right)e^{i(kr_0-\omega t)}$$

$$I \propto \left|E_p\right|^2 = I_0\left(\frac{\sin^2\beta}{\beta^2}\right)$$

$$\beta = \frac{1}{2}kb\sin\theta$$

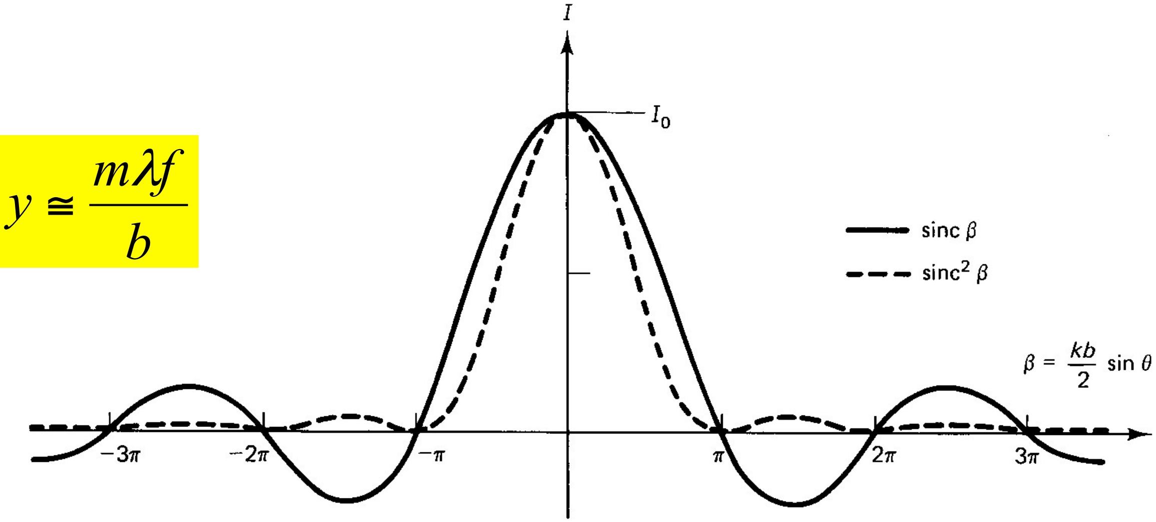
Fringe Pattern:

Dark fringe:

$$\beta = \frac{1}{2} kb \sin\theta = m\pi$$

$$y \cong \frac{m\lambda f}{b}$$

The second, third and fourth maxima of the diffraction pattern occur at β=1.43π, 2.46π and 3.47π, respectively.



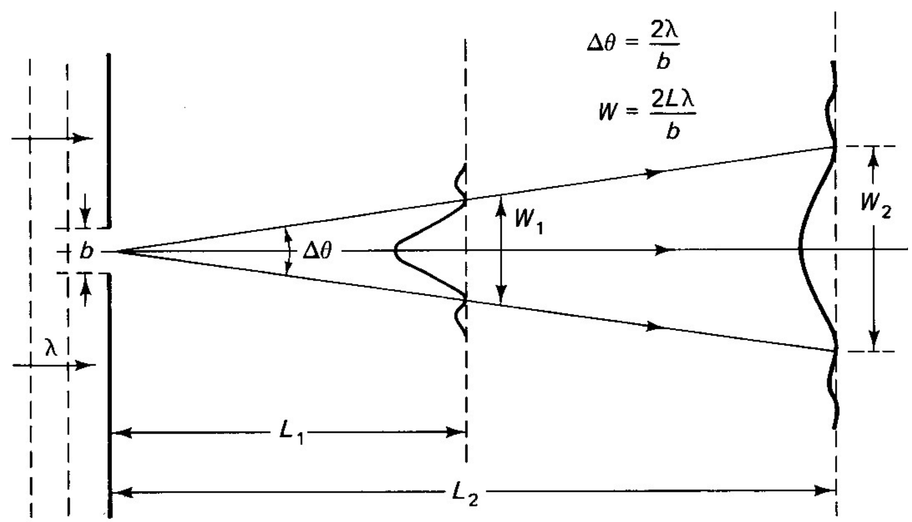
The central maximum represents essentially the image of the slit on a distant screen.

The angular width of the central maximum Is

$$\Delta\theta = \frac{2\lambda}{b}$$

The linear width of the central maximum is

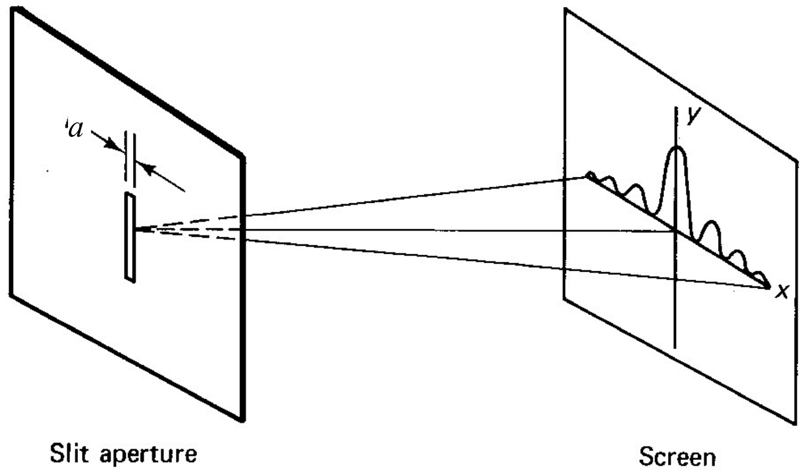
$$W = L\Delta\theta = \frac{2L\lambda}{b}$$

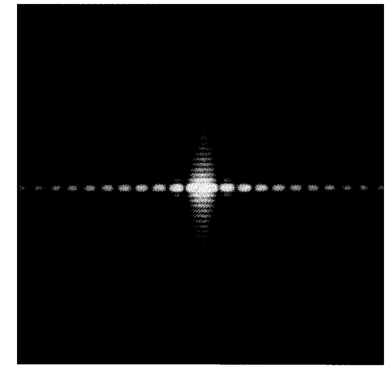


Rectangular Slits:

$$I = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2\beta}{\beta^2}\right)$$

$$y = \frac{m\lambda f}{b} \qquad x = \frac{n\lambda f}{a}$$





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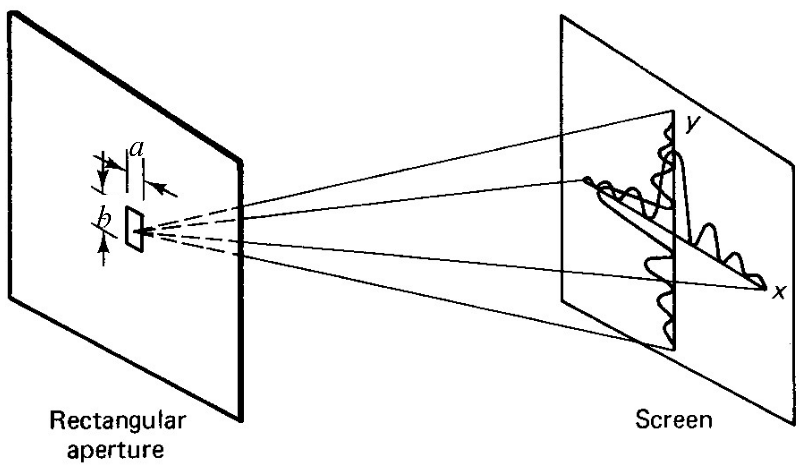
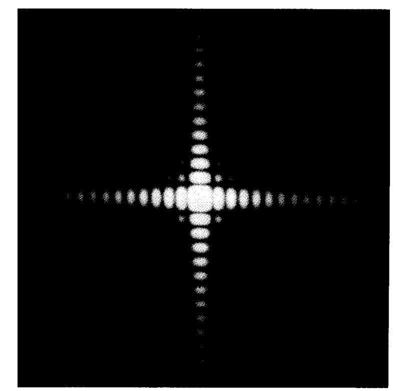




Circular Slits:

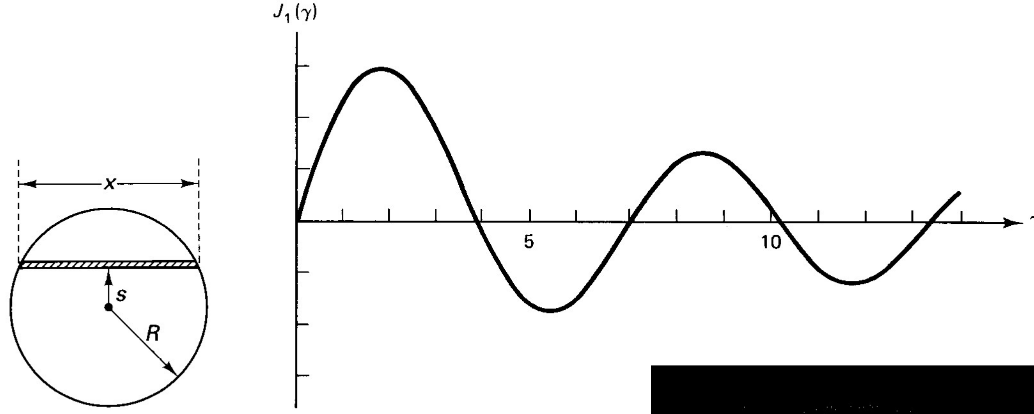
$$I = I_0\left(\frac{2J_1(\gamma)}{\gamma}\right)^2$$

$$\gamma = \frac{1}{2}kD\sin\theta$$

$$D\sin\theta = 1.22\lambda$$

The far-field angular radius Airy disc is,

$$\Delta\theta = \frac{1.22\lambda}{D}$$



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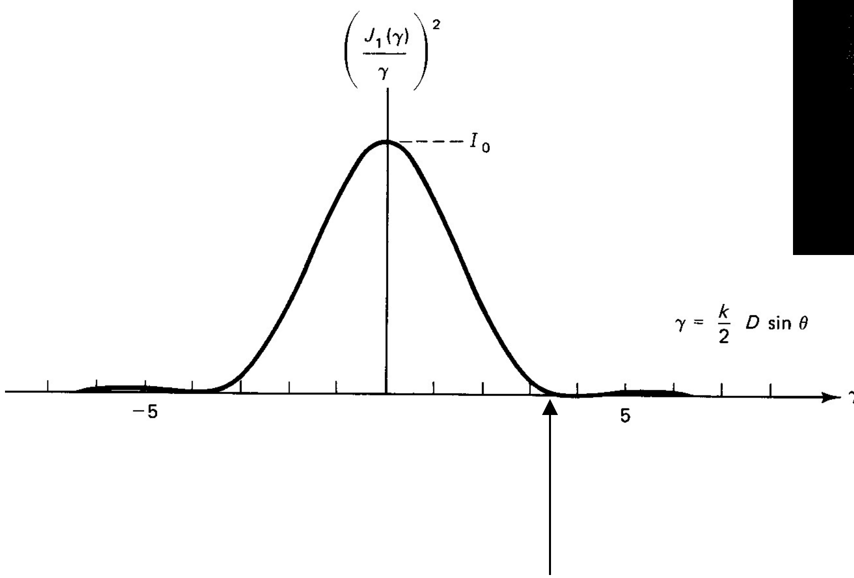

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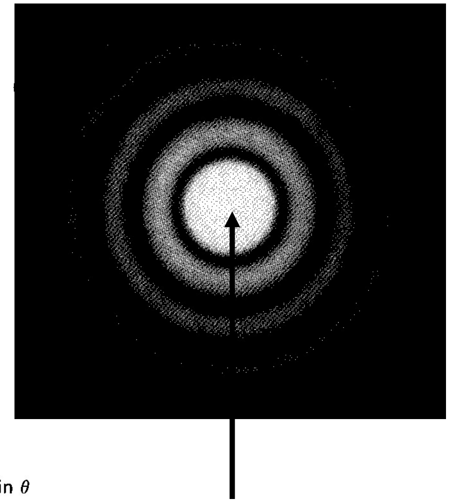
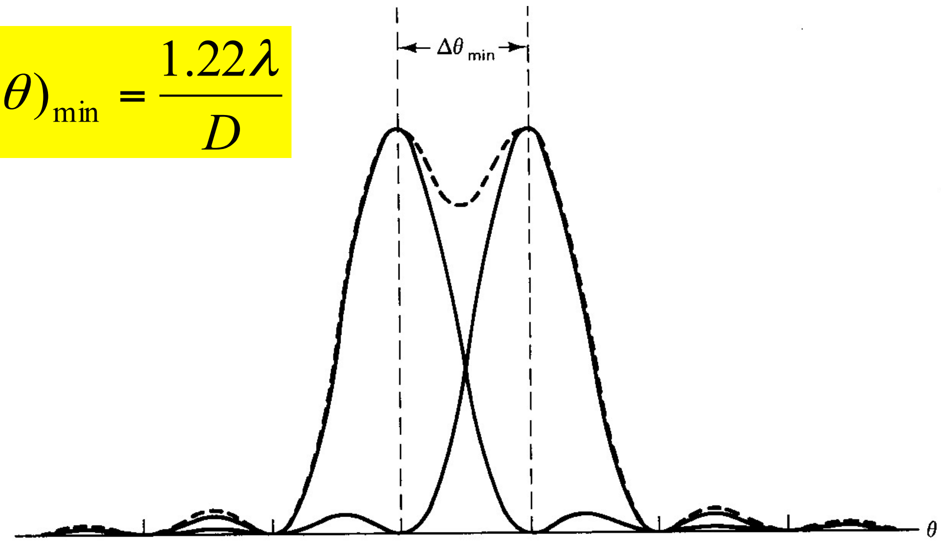
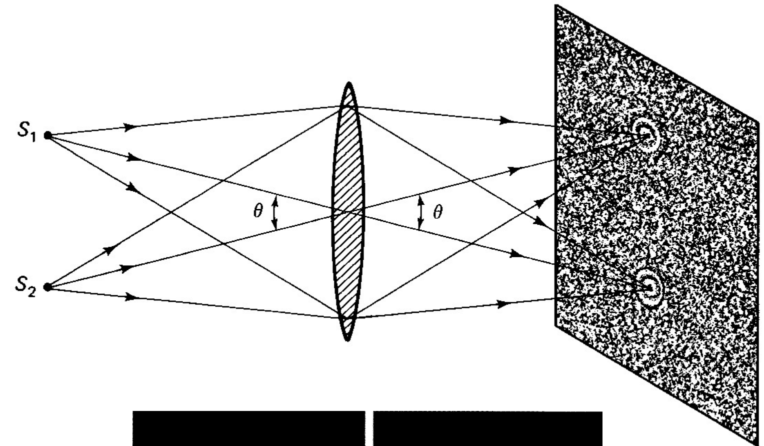


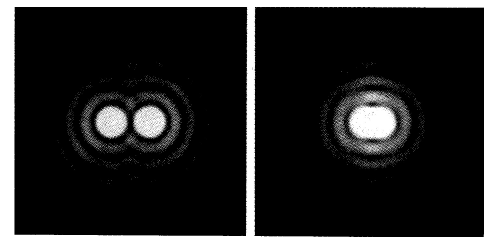
Airy Disc

Rayleigh's Criterion:

$$(\Delta\theta)_{\min} = \frac{1.22\lambda}{D}$$

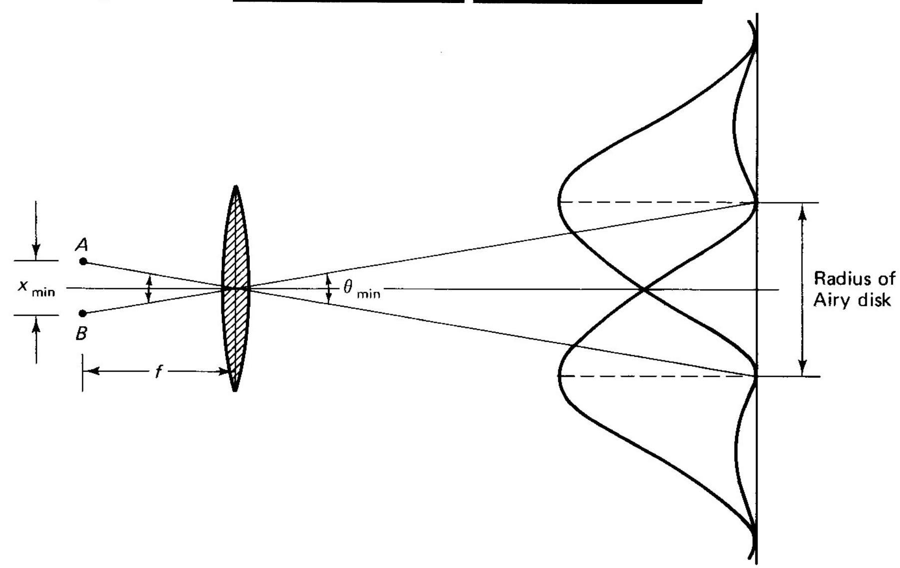






For a microscope,

$$x_{\min} = f\theta_{\min} = f\,\frac{1.22\lambda}{D}$$



The ratio D/f is the **numerical aperture**.

Example:

Two stars have an angular separation of 44.73×10^{-7} radian. Find the minimum diameter of the telescope objective which can just resolve the stars in light of 550 nm wavelength.

$$\theta_{\min} = 1.22\frac{\lambda}{d} \le 44.73\times10^{-7} \qquad d \ge 15 \text{ cm}$$

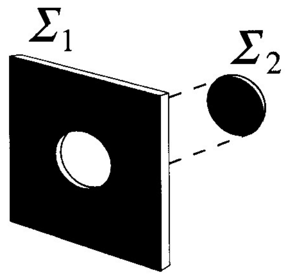
Example:

Calculate the minimum angular subtense of two points which can be just resolved by an eye with a 6 mm diameter pupil in light of 555 nm wavelength.

$$\theta_{\min} = 1.22\frac{\lambda}{d} = 1.129\times10^{-4} \text{ radian}$$

Diffraction by Small Particles:

Babinet's Principle



Σ_1 and Σ_2 are complementary apertures.

Suppose that monochromatic plane wavefronts are incident normally on Σ_1 and the diffracted light is imaged on a screen. In a direction θ to the normal let the magnitude of the electric vector be E_1. Replace Σ_1 with Σ_2 and let the magnitude of the electric vector be E_2. Apparently,

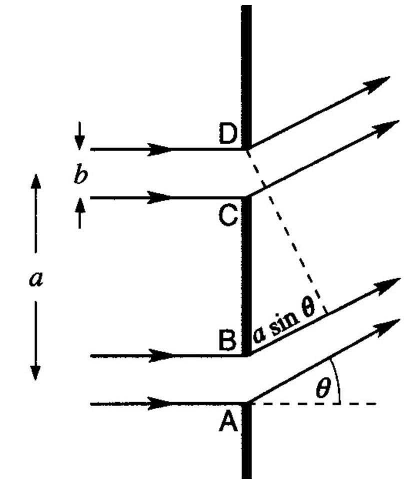
$$E_1+E_2=0 \qquad \text{and} \qquad E_1=-E_2$$

Therefore, $I_1 = I_2$

It means that the diffraction patterns for Σ_1 and Σ_2 are identical.

Fraunhofer Diffraction at Two Slits:

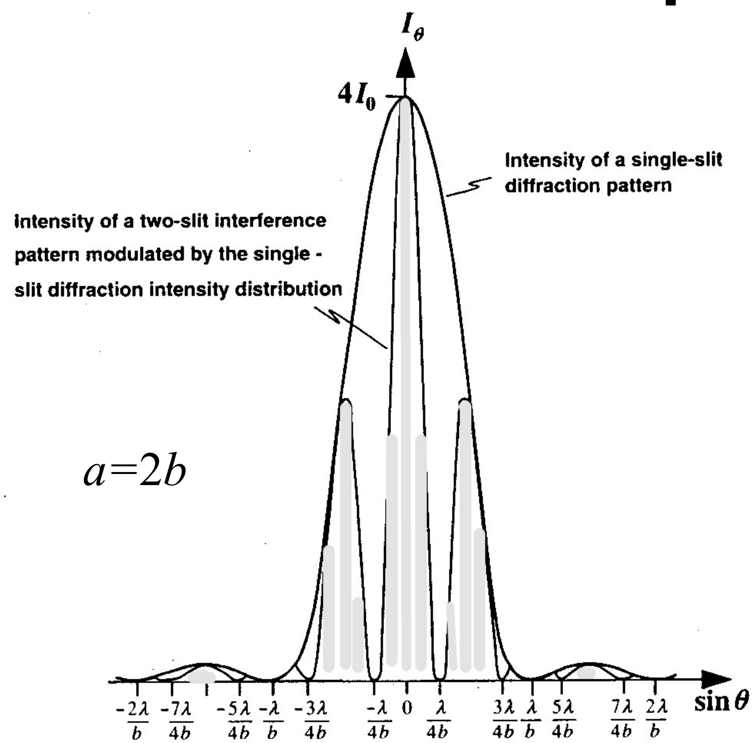
$$E_R = \frac{E_L}{r_0}\int_{-(1/2)(a+b)}^{-(1/2)(a-b)} e^{isk\sin\theta}ds + \int_{(1/2)(a-b)}^{(1/2)(a+b)} e^{isk\sin\theta}ds$$



$$\alpha = \frac{1}{2}ka\sin\theta \qquad \beta = \frac{1}{2}kb\sin\theta$$

$$I = 4I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\alpha$$

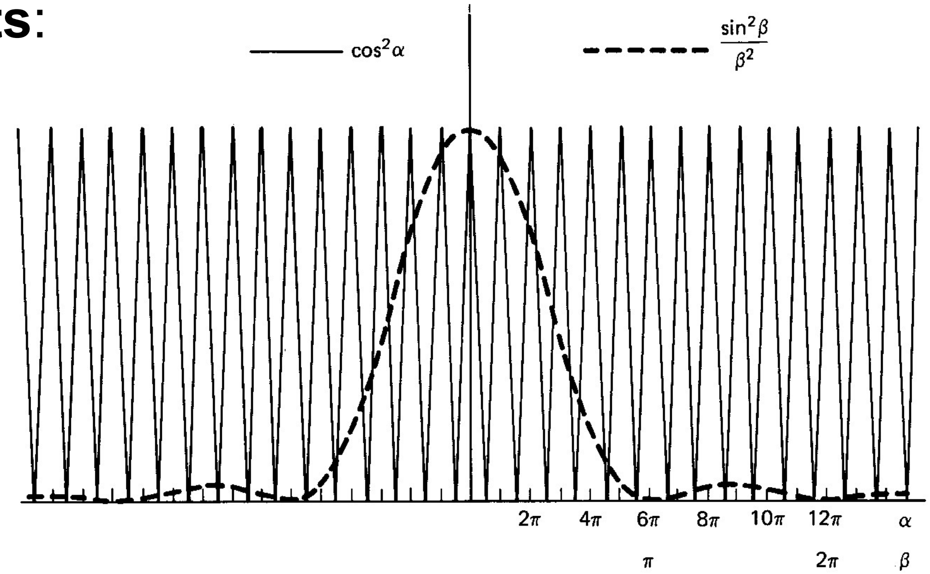
$$\cos^2\alpha = \cos^2\left[\frac{ka\sin\theta}{2}\right] = \cos^2\left[\frac{\pi a\sin\theta}{\lambda}\right]$$

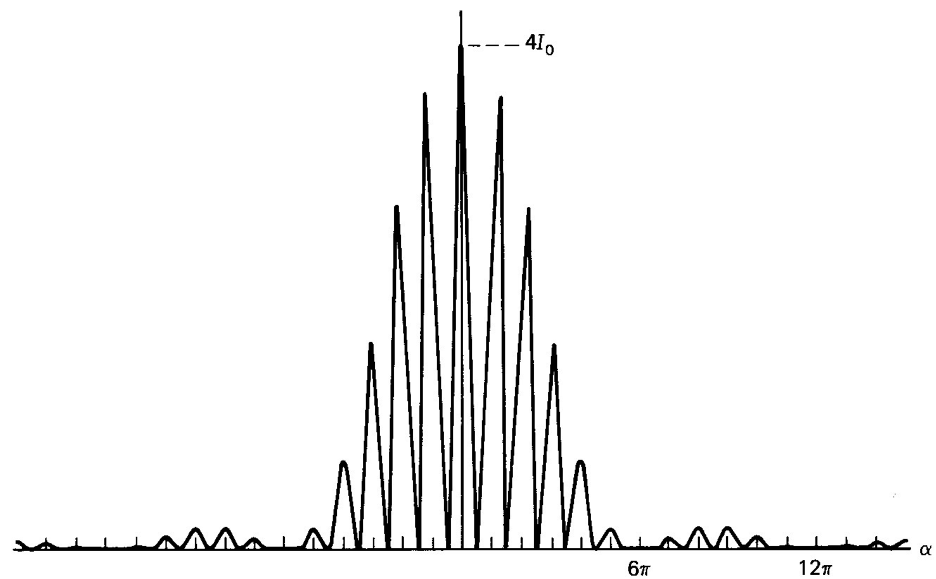


Fraunhofer Diffraction at Two Slits:

a=6*b*

$$I = 4I_0\left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha$$



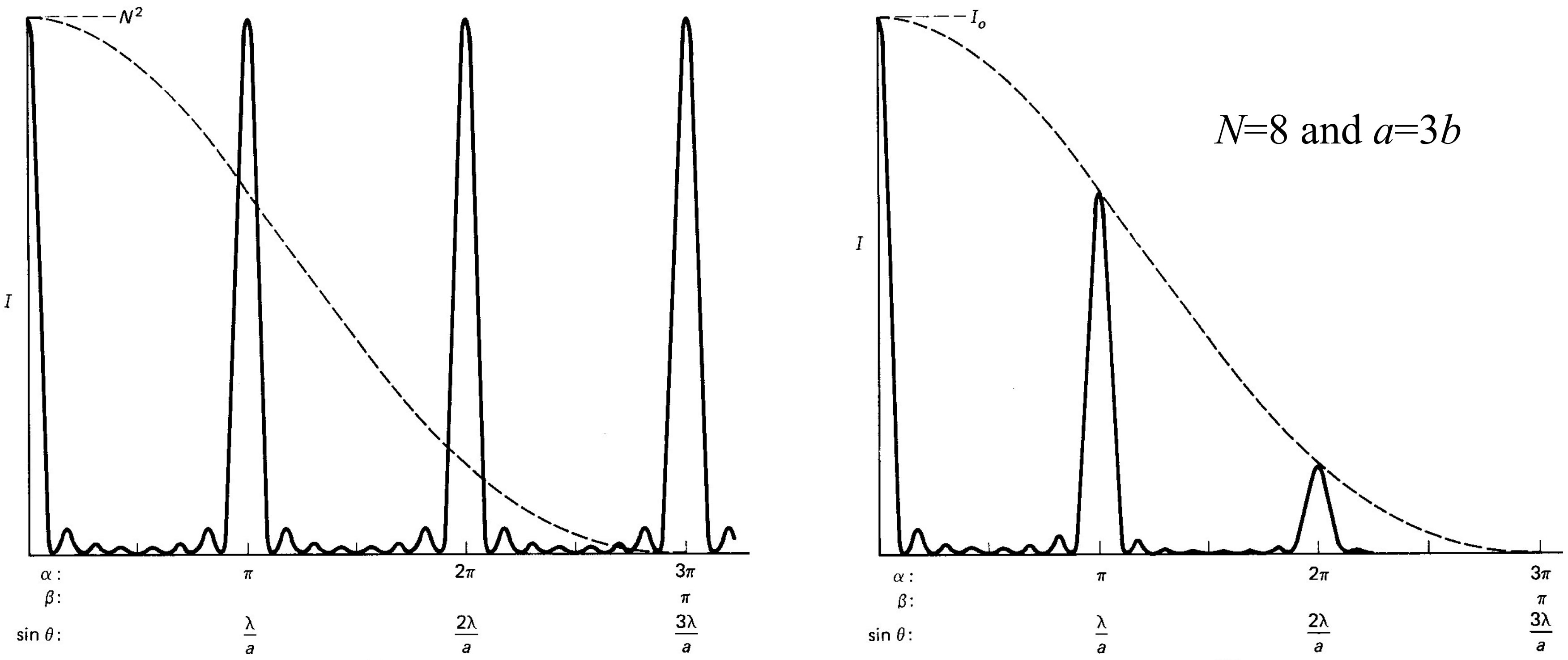


Diffraction at Many Slits:

$$E_R = \frac{E_L}{r_0}\sum_{j=1}^{N/2}\left\{\int_{[-(2j-1)a-b]/2}^{[-(2j-1)a+b]/2} e^{isk\sin\theta}ds + \int_{[(2j-1)a-b]/2}^{[(2j-1)a+b]/2} e^{isk\sin\theta}ds\right\}$$

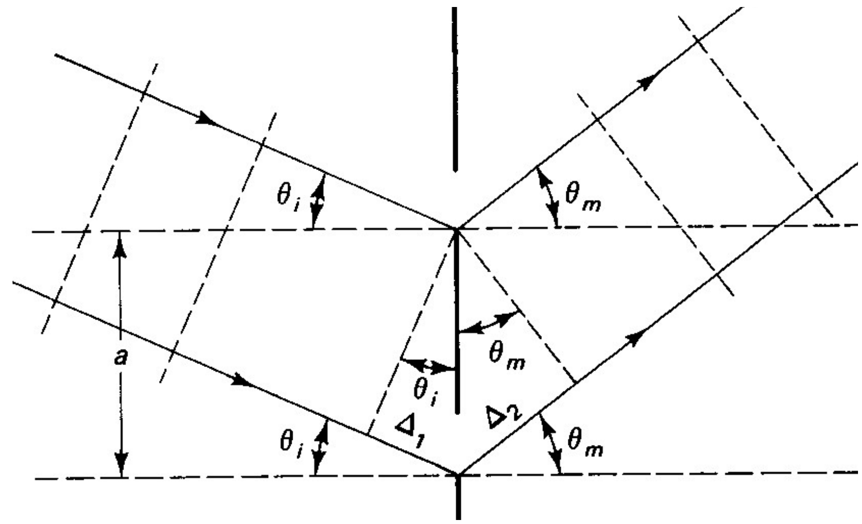
$$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

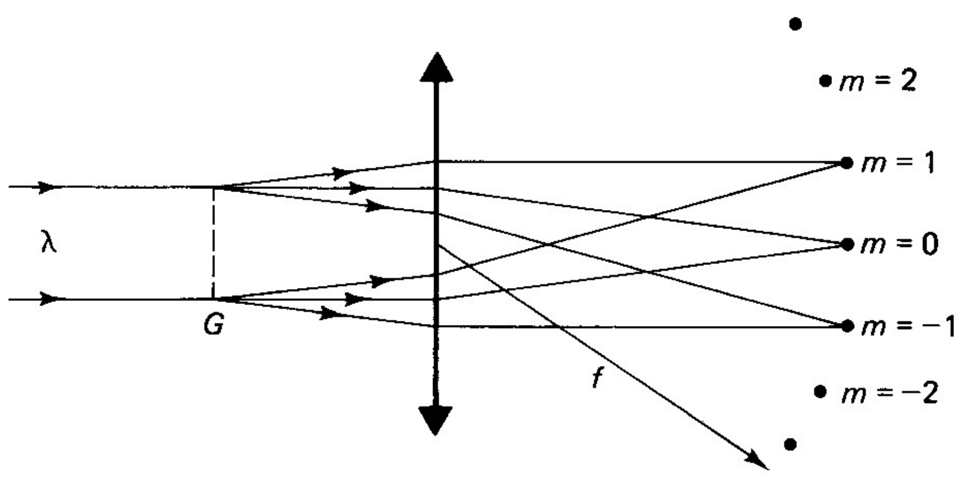
principal maxima at $a\sin\theta = m\lambda$



Diffraction Grating:

$$a\left(\sin\theta_i + \sin\theta_m\right) = m\lambda$$



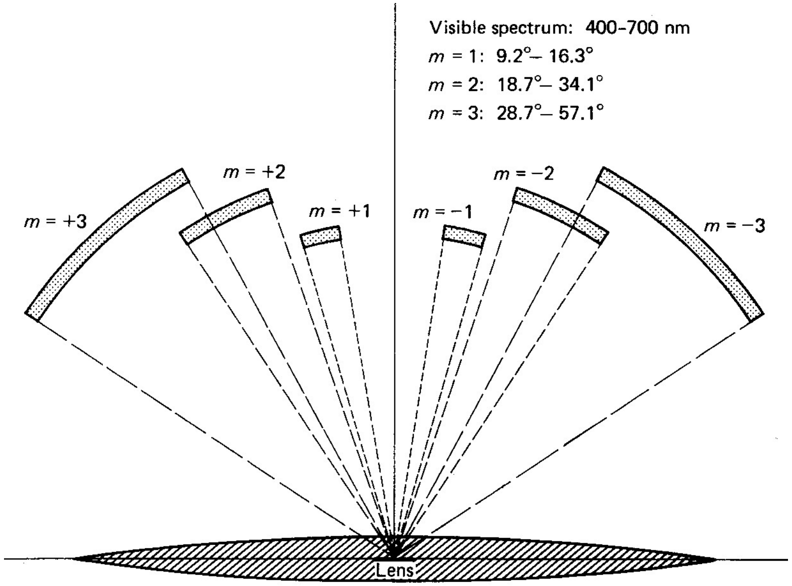


Free Spectral Range:

—the non-overlapping wavelength range in a particular order.

The non-overlapping spectral region is smaller for higher orders.

$$a \sin\theta = m\lambda$$



The wavelength are better separated as their order increases. This property is described by **angular dispersion**, or **dispersive power** of a grating,

$$D = \frac{d\theta_m}{d\lambda} \qquad D = \frac{m}{a\cos\theta_m}$$

Linear dispersion is,

$$\frac{dy}{d\lambda} = f\frac{d\theta_m}{d\lambda} = fD$$

Resolving power of a grating is defined as,

$$R \equiv \frac{\lambda}{(\Delta\lambda)_{\min}}$$

Using the Rayleigh's criterion, suppose the number of grating grooves is *N*, we have

$$R = mN$$

Example:

A grating has 4000 grooves or lines per centimeter. Calculate the dispersive power in the second order spectrum in the visible range.

We take the mean wavelength to be 550 nm.

$$a = \frac{1}{4000} = 2.5\times10^{-6}\ \text{m}; \qquad m = 2 \qquad \because a\sin\theta_m = m\lambda \qquad \therefore \theta_2 = 26.10^\circ$$

$$D = \frac{m}{a\cos\theta_2} = 8.9\times10^5\ \text{rad/m}$$

Example:

Find the number of lines (grooves) required on a grating to just resolve the two sodium lines, λ_1=589.592 nm and λ_2=588.995 nm, in the second order spectrum of a grating.

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} \qquad (\Delta\lambda)_{\min} = \lambda_1 - \lambda_2$$

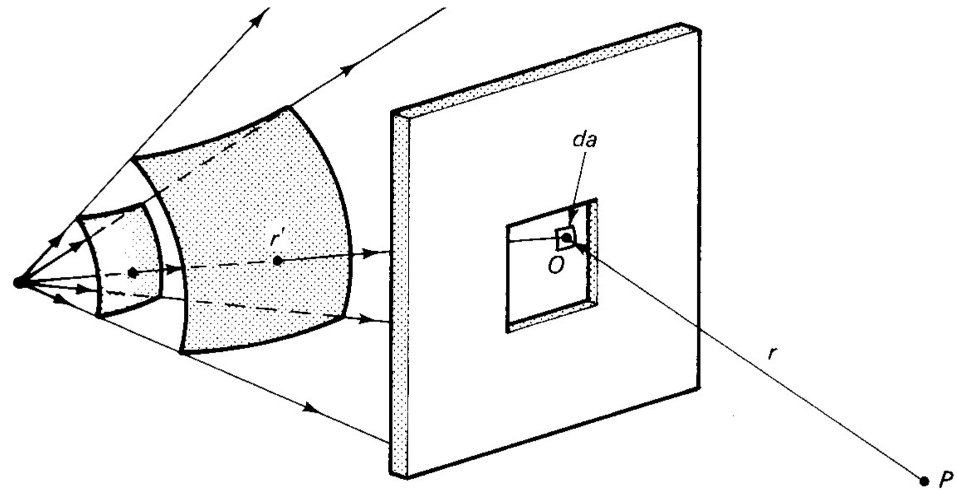
$$R = \frac{\lambda}{(\Delta\lambda)_{\min}} = mN \qquad N = 494$$

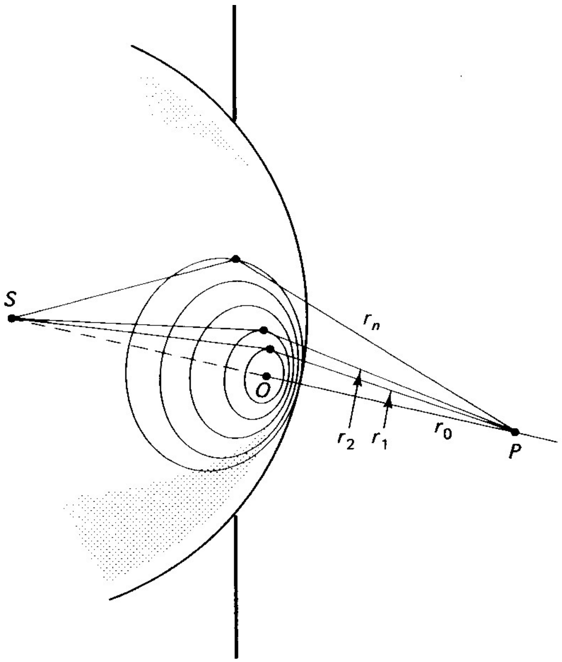
Fresnel Diffraction:

$$dE_p = \left(\frac{dE_0}{r}\right)e^{ikr} \qquad dE_0 \propto E_L da$$

$$E_L = \left(\frac{E_S}{r'}\right)e^{ikr'} \qquad dE_p = \left(\frac{E_S}{rr'}\right)e^{ik(r+r')}da$$

$$E_p = \frac{-ikE_S}{2\pi}\iint F(\theta)\frac{e^{ik(r+r')}}{rr'}da$$





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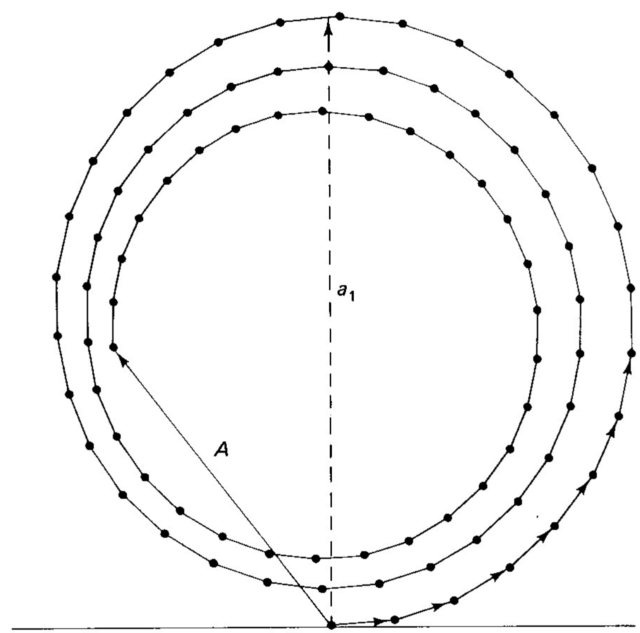


(b)

$$r_1 = r_0 + \frac{\lambda}{2}$$

$$r_2 = r_0 + \lambda$$

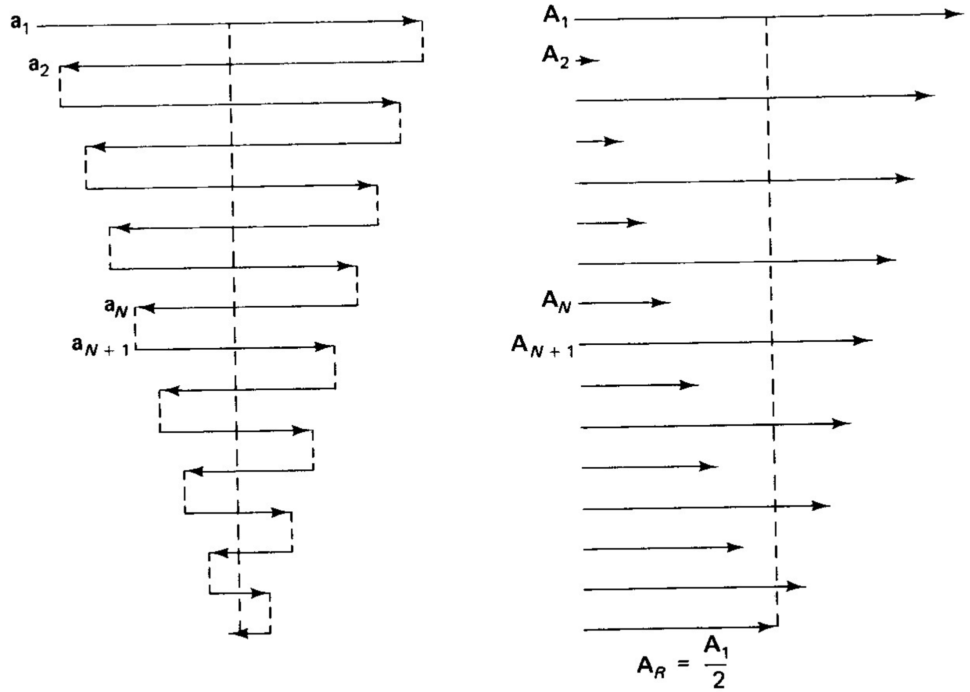
$$r_N = r_0 + \frac{N\lambda}{2}$$

$$A = a_1 + a_2 e^{i\pi} + a_3 e^{i2\pi} + a_4 e^{i3\pi} + \ldots$$
$$= a_1 - a_2 + a_3 - a_4 + \ldots$$

$$A_1 = a_1$$

$$A_2 = a_1 - a_2$$

$$A_3 = a_1 - a_2 + a_3$$



Two conclusions

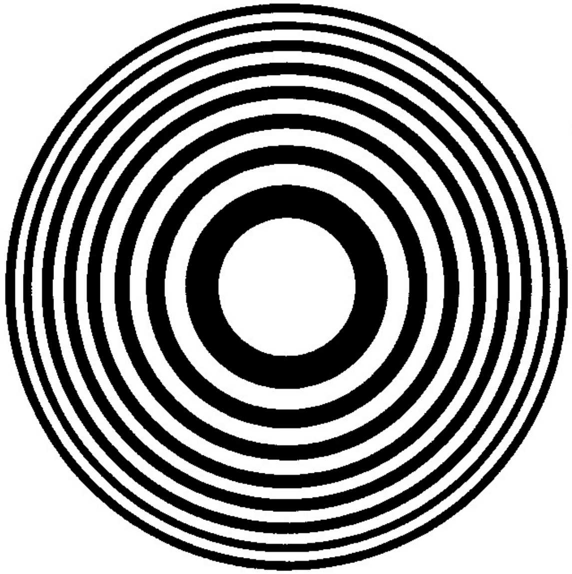
(1) If N is small, there is large changes in the resultant phasor A_N as the contribution from each new zone is added. The resultant amplitude seems to oscillate between magnitudes that are larger and smaller than the limiting value of $a_1/2$. As the aperture gradually increases, one can see oscillations between bright and dark in a fixed position of the screen.

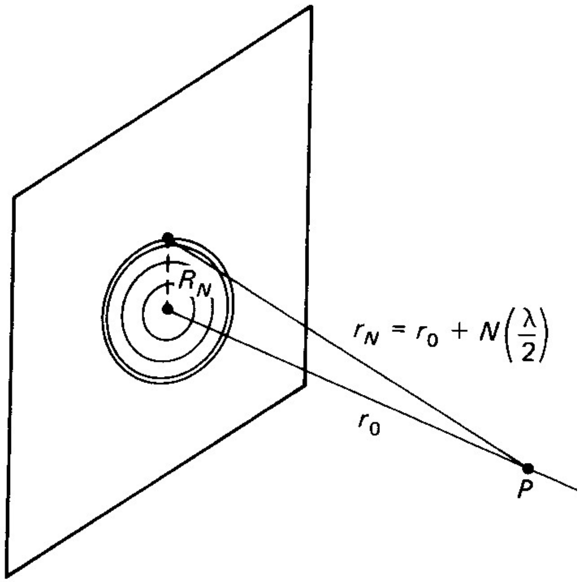
(2) If N is large, as in the case of unlimited aperture, the resultant amplitude is half that of the first contribution zone, $a_1/2$.

Fresnel zone plate:

very other Fresnel zone is blocked

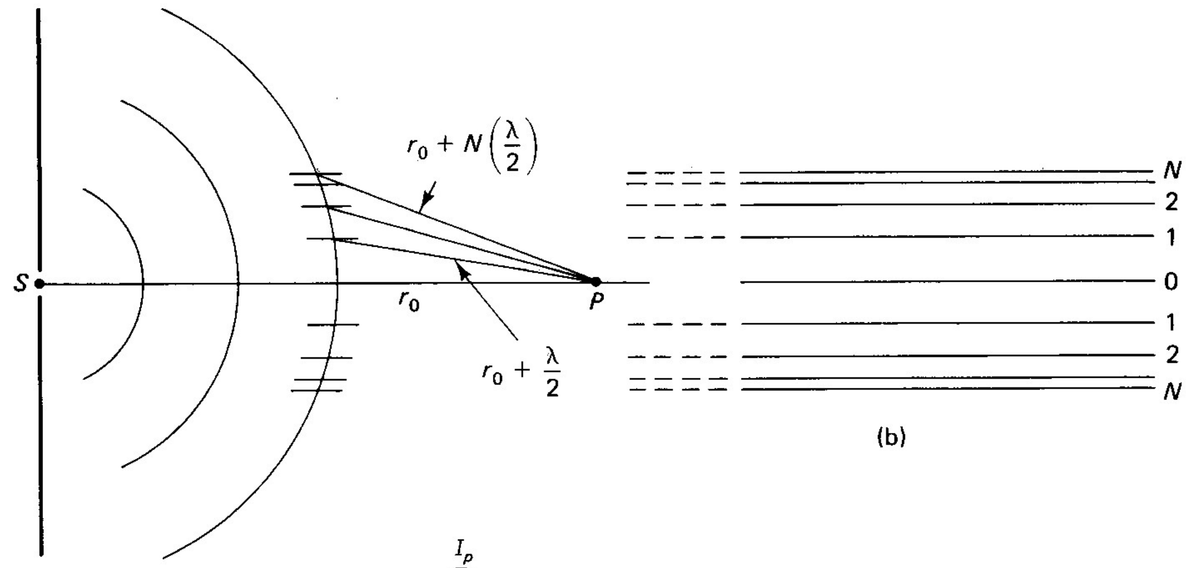
The zone plate radii are approximately given by, $R_N = \sqrt{N r_0 \lambda}$

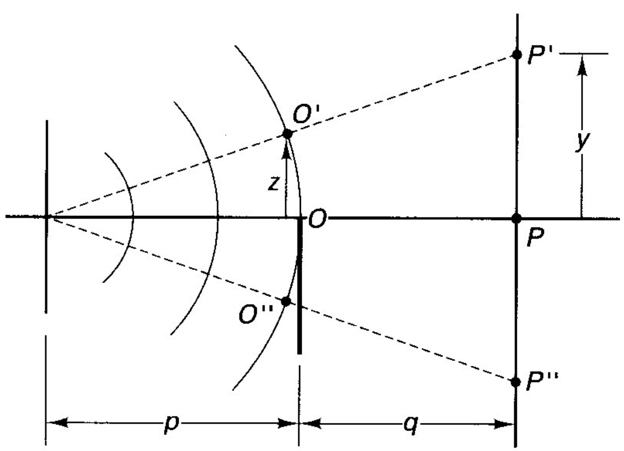


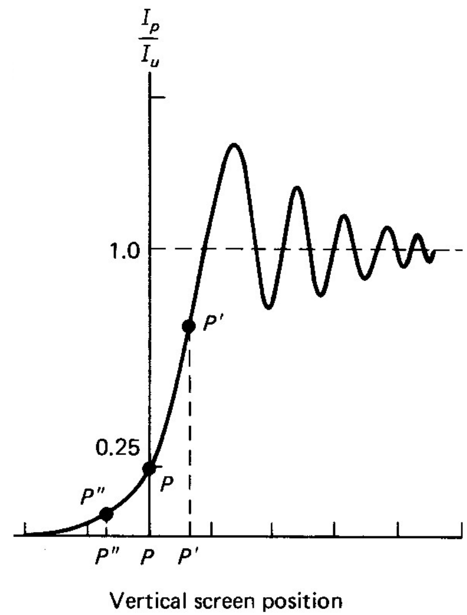


Diffraction by Straight Edges :

Use cylindrical waves.







Example:

Plane wave of λ=550 nm are incident normally on a circular aperture of radius $\sqrt{11}$ mm. Does a bright or a dark spot appear at the point *P* on the axis 4 m from the hole? If the intensity of the incident light is I_0, calculate the intensity at *P*.

$$R_N = \sqrt{N r_0 \lambda} \qquad \therefore N = \frac{R_N^2}{r_0 \lambda} = 5 \qquad \text{odd number} \quad \therefore \text{bright spot}$$

$$E = E_1 - E_2 + E_3 - E_4 + E_5 \approx E_1 \qquad \frac{I}{I_0} = \left(\frac{E_1}{E_1/2}\right)^2 = 4 \qquad I = 4I_0$$

Example:

A 4 mm diameter circular hole in an opaque screen is illuminated by plane waves of wavelength 500 nm. If the angle of incidence is zero, find the positions of the first two intensity maxima and the first intensity minimum along the central axis.

The first two maxima will occur when *N*=1 and 3, respectively. The first minimum occurs when *N*=2.

$$N = 1: \quad r = \frac{R^2}{N\lambda} = 8\ \text{m} \qquad N = 3: \quad r = \frac{R^2}{N\lambda} = 2.67\ \text{m}$$

$$N = 2: \quad r = \frac{R^2}{N\lambda} = 4\ \text{m}$$

Example:

Plane waves of 550 nm wavelength are incident normally on a narrow slit of width 0.25 mm. Calculate the distance between the first minima on either side of the central maximum when the Fraunhofer diffraction pattern is imaged by a lens of focal length 60 cm.

$$W = f\Delta\theta = f\frac{2\lambda}{b} = 2.64 \text{ mm}$$

Example:

Plane waves (λ=550 nm) fall normally on a slit 0.25 mm wide. The separation of the fourth order minima of the Fraunhofer diffraction pattern in the focal plane of the lens is 1.25 mm. Calculate the focal length of the lens.

$$\beta = \frac{1}{2}kb\sin\theta = 8\pi \qquad \therefore \theta \cong \frac{8\lambda}{b} \qquad f = \frac{W}{\theta} = \frac{Wb}{8\lambda} = 7.10 \text{ cm}$$

Example:

Light from a distant point source enters a converging lens of focal length 22.5 cm. How large must the lens be if the Airy disc is to be 10^{-6} m in diameter? λ=450 nm

$$f(2\Delta\theta) = 2f\frac{1.22\lambda}{D} = 10^{-6} \qquad D = \frac{2.44 f\lambda}{10^{-6}} = 24.7 \text{ cm}$$

Example:

A telescope objective is 12 cm in diameter and has a focal length of 150 cm. Light of mean wavelength 550 nm from a star is imaged by the objective. Calculate the size of the Airy disc.

$$f\left(2\Delta\theta\right) = 2f\,\frac{1.22\lambda}{D} = 0.017\ \text{mm}$$

Example:

Assuming Rayleigh's criterion can be applied to the eye, how far apart must two small lights be in order to be just resolved at a distance of 1000 m? Take the pupil diameter as 2.5 mm, the wavelength to be 555 nm, and the eye's refractive index 1.333. Assume a single surface model eye with the pupil at the surface.

$$\Delta\theta_{\min} = \frac{1.22\lambda_m}{D} = \frac{1.22\lambda}{nD} \qquad \sin\frac{\Delta\theta_i}{2} = n\sin\frac{\Delta\theta_{\min}}{2} \qquad \therefore \Delta\theta_i \cong n\Delta\theta_{\min}$$

$$x_{\min} = L\Delta\theta_i = Ln\Delta\theta_{\min} = L\frac{1.22\lambda}{D} = 27.1\ \text{cm}$$