Mixed Strategy Equilibrium: Analysis of Highly Ranked Professional Tennis Players

Match Charting Project

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Outline

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- Defining a Mixed Strategy Equilibrium
- Literature Review
- Methodology
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Do professional men's tennis players behave optimally when serving and returning?

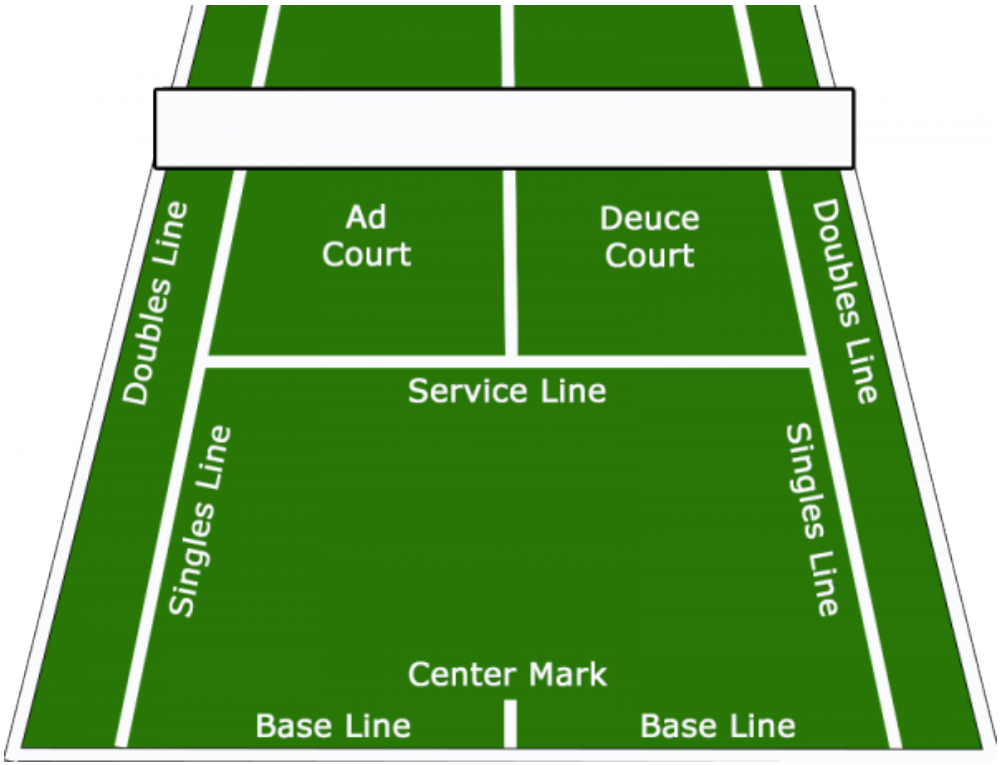
Motivation

- 70% of men's tennis points are between 0-4 shots
- Since 1996 the #1 Player in the World wins just **55% of total points in the entire season**[1]
- Margins are extremely narrow $\rightarrow$ every point matters
- Empirically test for optimization using game theory

[1] Source: Craig O'Shannessey–*braingametennis.com*

Method

- Game Theory Analysis
  - Mixed Strategy Equilibrium Theory
- Serving: Four 'Point-Games' (Deuce/Ad & $1^{st}/2^{nd}$)
- Returning: Four 'Point-Games' (Forehand/Backhand & $1^{st}/2^{nd}$)



What Is A Mixed Strategy Equilibrium?

- Pure Strategy–choosing a strategy with certainty in a deterministic manner
- Mixed Strategy–randomly choosing across strategies
- Ex: Rock, Paper, Scissors
  - Optimal Mix is $\frac{1}{3}$ for each strategy
- Tennis a Zero-Sum Game
  - Minimize opponents' ability to exploit systematic choices in your behavior
- Two Criteria:
  1. $EV(Forehand) = EV(Backhand)$
  2. Randomly mixing between strategies

Results

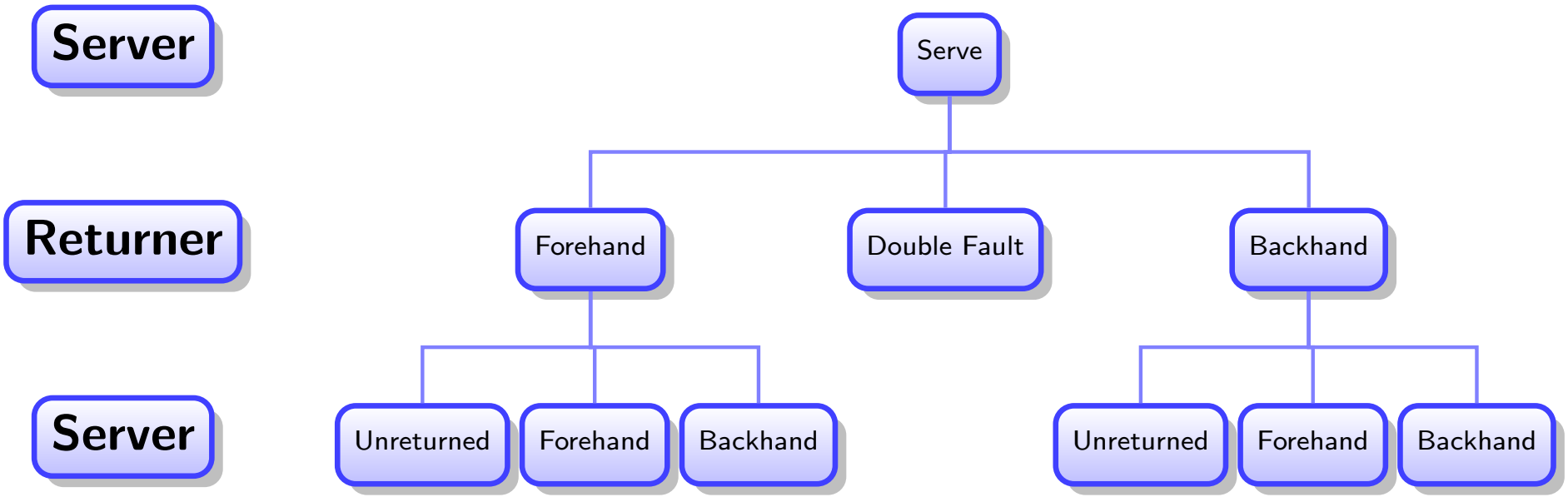
- Players largely optimizing on 1^{st} & 2^{nd} serves when *serving*
- Less optimal when *returning*
  - 1^{st} serves $\rightarrow$ not optimizing
  - 2^{nd} serves $\rightarrow$ optimizing
- Serial correlation in player's strategic decision-making
  - Mixing strategies too often to be considered random

Literature Review

- "Minimax at Wimbledon": Walker & Wooders (2001) *The American Economic Review*
- "David vs. Goliath: An Analysis of Asymmetric Mixed-Strategy Games and Experimental Evidence": Amaldoss (2002) *Management Science*
- "(Mixed) Strategy in Oligopoly Pricing: Evidence from Gasoline Price Cycles Before and Under a Timing Regulation": Wang (2009) *Journal of Political Economy*
- "Mixed Equilibrium in a Downsian Model with a Favored Candidate": Aragones & Palfrey (2002) *Journal of Economic Theory*
- "Games of Strategy: Fourth International Student Edition" Dixit, Avinash K., and Susan Skeath (2015). *WW Norton & Company*

Framework

Figure: Tennis Game Structure



Mixed Strategy Example

Table: Theoretical Point Payoffs

		Returner Leaning To	
		Forehand	Backhand
Server Serving To	Forehand	50,50	80,20
	Backhand	90,10	20,80

- Suppose Server mixed strategy is 75% Forehand, 25% Backhand
- Server Payoffs are then:

$$\textit{Forehand} = 0.75 \cdot 50 + 0.25 \cdot 90 = \mathbf{60} \quad (1)$$

$$\textit{Backhand} = 0.75 \cdot 80 + 0.25 \cdot 20 = 65 \quad (2)$$

- Returner can exploit it by covering the Forehand every time-win 40%

Mixed Strategy Example

Table: Theoretical Point Payoffs

		Returner	
		Forehand	Backhand
Server	Forehand	50,50	80,20
	Backhand	90,10	20,80

- Returner payoffs must be equal to optimize
- Let p = probability Server hits to Forehand $\rightarrow 1-p$ = Probability Server hits to Backhand:

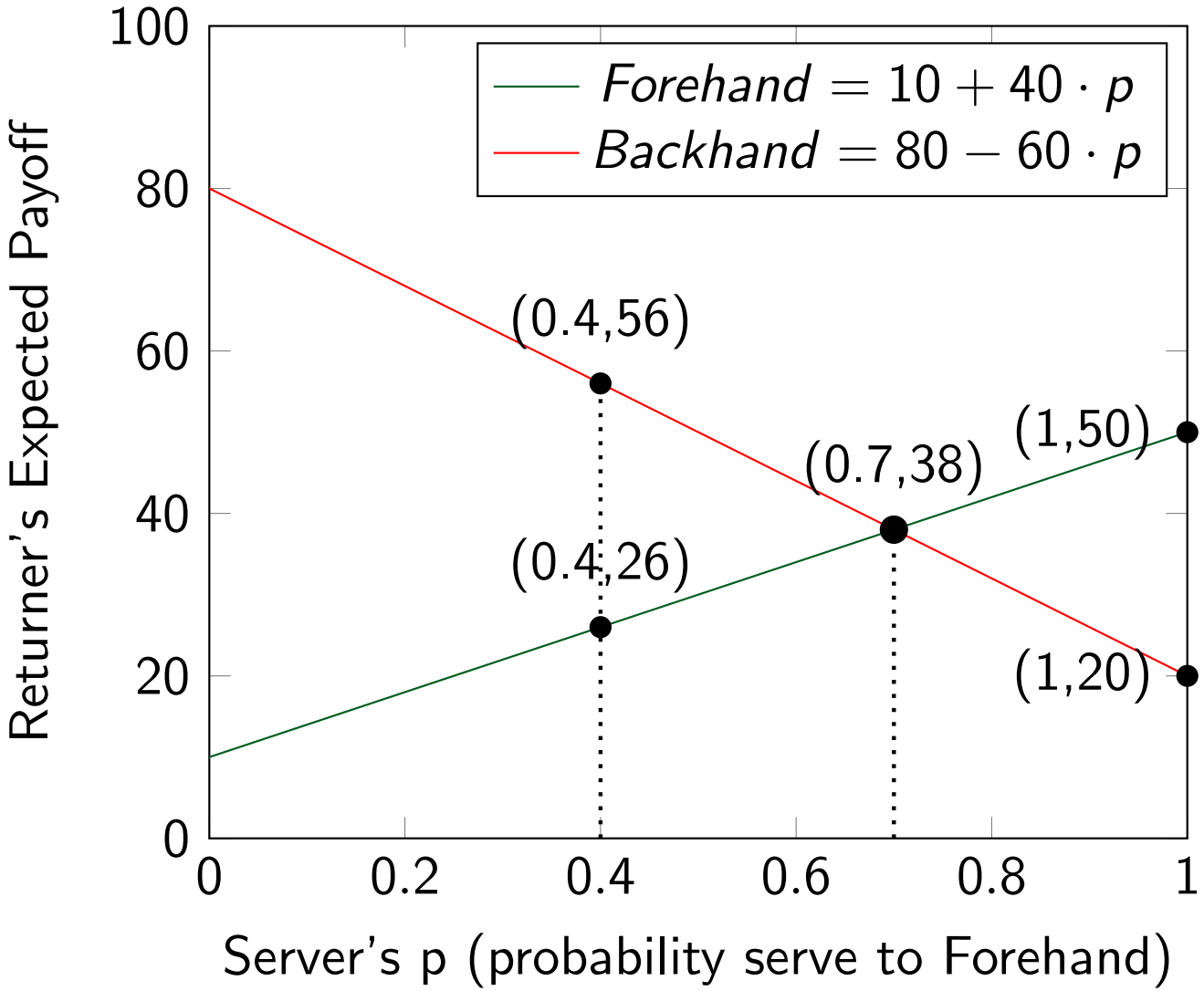
$$Payoff(Forehand) = Payoff(Backhand) \tag{3}$$

$$50p + 10(1-p) = 20p + 80(1-p) \tag{4}$$

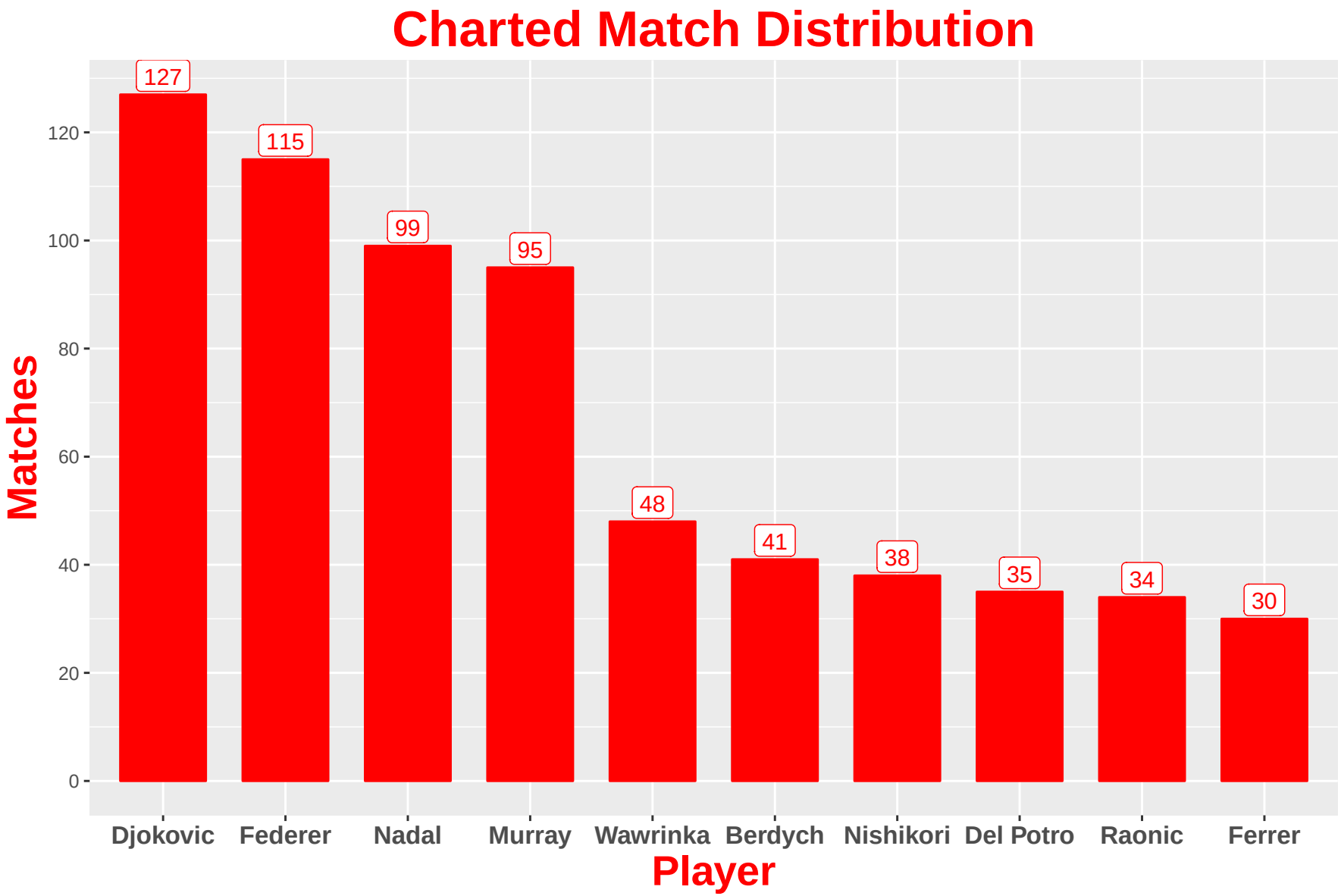
$$10 + 40p = 80 - 60p \tag{5}$$

$$\mathbf{p=0.7} \tag{6}$$

Mixed Strategy Equilibrium Diagram



Data[2]



[2] Jeff Sackmann: "Match Charting Project" from *tennisabstract.com*

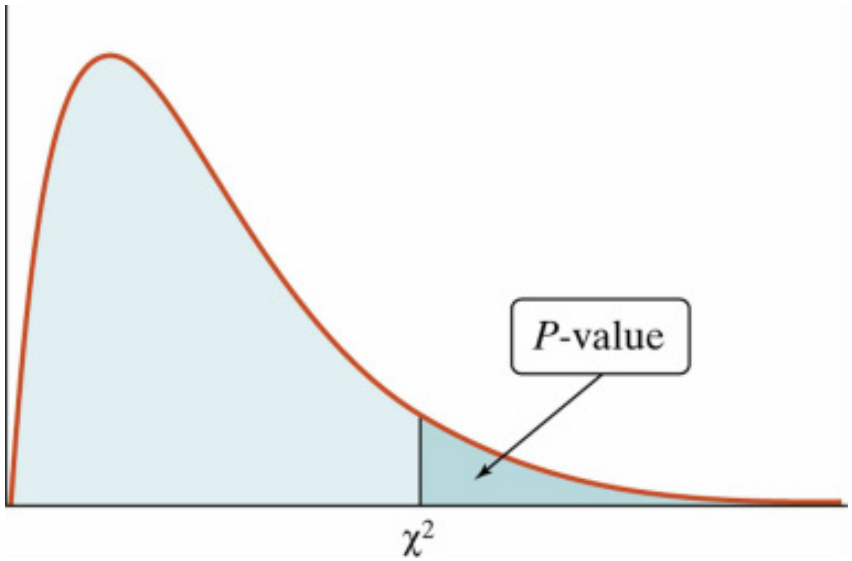
Procedure

- Pearson's χ^2 Equality of Proportions test
  - Tests how the observed data fits theoretical distribution
- Hypothesis:

$$H_0 : P_f^w = P_b^w$$

$$H_1 : P_f^w \neq P_b^w$$

Figure: Chi-Squared Distribution



Example

Table: Roger Federer Service Points

Deuce 1^{st} *Serve*	Points Won	Points Lost	Total
Forehand	1174	376	1550 (51.20% of 3027)
Backhand	1192	285	1477 (48.80% of 3027)
Total	2366	661	3027

- Expected frequencies:

$$0.5120 \cdot 2366 \approx 1212 \tag{7}$$

$$0.5120 \cdot 661 \approx 338 \tag{8}$$

$$0.4880 \cdot 2366 \approx 1154 \tag{9}$$

$$0.4880 \cdot 661 \approx 323 \tag{10}$$

- Observed-Expected difference of **38**

Serving Outcomes

Player	Court	Direction	1st Serve W%	p-value	2nd Serve W%	p-value
Federer	Deuce	Forehand	0.757	0.018*	0.544	0.523
		Backhand	0.807		0.564	
	Ad	Forehand	0.764	0.286	0.569	0.670
		Backhand	0.787		0.583	
Djokovic	Deuce	Forehand	0.710	0.526	0.549	0.166
		Backhand	0.723		0.593	
	Ad	Forehand	0.689	0.259	0.556	0.710
		Backhand	0.714		0.568	
Nadal	Deuce	Forehand	0.703	0.950	0.555	0.948
		Backhand	0.702		0.557	
	Ad	Forehand	0.717	0.870	0.541	0.365
		Backhand	0.720		0.588	
Murray	Deuce	Forehand	0.707	0.006*	0.538	0.280
		Backhand	0.778		0.502	
	Ad	Forehand	0.698	0.080	0.536	0.248
		Backhand	0.751		0.497	
Wawrinka	Deuce	Forehand	0.739	0.901	0.578	0.510
		Backhand	0.743		0.550	
	Ad	Forehand	0.688	0.397	0.457	0.062
		Backhand	0.719		0.545	
Nishikori	Deuce	Forehand	0.725	0.521	0.527	0.701
		Backhand	0.697		0.507	
	Ad	Forehand	0.677	0.879	0.503	0.646
		Backhand	0.683		0.527	
Del Potro	Deuce	Forehand	0.694	0.069	0.534	0.668
		Backhand	0.762		0.509	
	Ad	Forehand	0.737	0.718	0.634	0.267
		Backhand	0.722		0.563	
Ferrer	Deuce	Forehand	0.674	0.521	0.579	0.206
		Backhand	0.642		0.518	
	Ad	Forehand	0.633	0.864	0.548	0.595
		Backhand	0.642		0.510	
Raonic	Deuce	Forehand	0.807	0.901	0.548	0.569
		Backhand	0.802		0.515	
	Ad	Forehand	0.752	0.274	0.533	0.770
		Backhand	0.796		0.551	
Berdych	Deuce	Forehand	0.776	0.246	0.535	0.418
		Backhand	0.727		0.491	
	Ad	Forehand	0.795	0.305	0.426	0.105
		Backhand	0.745		0.521	

Significant Serve

Player	Court	Direction	1st Serve Win Rate	p-value	2nd Serve Win Rate	p-value
Federer	Deuce	Forehand	0.757	0.018*	0.544	0.523
		Backhand	0.807		0.564	
Murray	Deuce	Forehand	0.707	0.006*	0.538	0.280
		Backhand	0.778		0.502	

Return

Player	Serve	Return	1st Serve Win Rate	p-value	2nd Serve Win Rate	p-value
Federer	Forehand	Forehand	0.472	0.005*	0.566	0.456
		Backhand	0.589		0.606	
	Backhand	Forehand	0.429	0.069	0.491	0.056
		Backhand	0.497		0.548	
Djokovic	Forehand	Forehand	0.504	0.723	0.573	0.513
		Backhand	0.516		0.602	
	Backhand	Forehand	0.455	0.001*	0.567	0.366
		Backhand	0.542		0.593	
Nadal	Forehand	Forehand	0.476	0.205	0.568	0.516
		Backhand	0.553		0.598	
	Backhand	Forehand	0.451	0.861	0.555	0.294
		Backhand	0.462		0.617	
Murray	Forehand	Forehand	0.457	0.550	0.474	0.128
		Backhand	0.483		0.551	
	Backhand	Forehand	0.382	0.007*	0.503	0.035*
		Backhand	0.497		0.574	
Wawrinka	Forehand	Forehand	0.429	0.276	0.533	0.893
		Backhand	0.518		0.514	
	Backhand	Forehand	0.358	0.072	0.502	0.879
		Backhand	0.467		0.509	
Nishikori	Forehand	Forehand	0.496	0.643	0.532	0.239
		Backhand	0.457		0.626	
	Backhand	Forehand	0.339	0.004*	0.551	0.602
		Backhand	0.532		0.581	
Del Potro	Forehand	Forehand	0.461	0.617	0.511	0.594
		Backhand	0.503		0.446	
	Backhand	Forehand	0.374	0.353	0.482	0.793
		Backhand	0.449		0.495	
Ferrer	Forehand	Forehand	0.423	0.297	0.600	1.000
		Backhand	0.510		0.600	
	Backhand	Forehand	0.320	0.334	0.503	0.910
		Backhand	0.410		0.510	
Raonic	Forehand	Forehand	0.435	0.920	0.524	0.859
		Backhand	0.445		0.505	
	Backhand	Forehand	0.385	0.978	0.422	0.135
		Backhand	0.382		0.522	
Berdych	Forehand	Forehand	0.431	0.989	0.532	0.483
		Backhand	0.432		0.623	
	Backhand	Forehand	0.329	0.050*	0.538	0.783
		Backhand	0.478		0.555	

Significant Return

- When returns are made, players win **much greater share of points when returned to opponent's backhand**
  - Several are not optimal when returning 1^{st} serves

Table: Return

Player	Serve	Return	1st Serve Win Rate	p-value	2nd Serve Win Rate	p-value
Federer	Forehand	Forehand	0.472	0.005*	0.566	0.456
		Backhand	0.589		0.606	
Djokovic	Backhand	Forehand	0.455	0.001*	0.567	0.366
		Backhand	0.542		0.593	
Murray	Backhand	Forehand	0.382	0.007*	0.503	0.035*
		Backhand	0.497		0.574	
Nishikori	Backhand	Forehand	0.339	0.004*	0.551	0.602
		Backhand	0.532		0.581	
Berdych	Backhand	Forehand	0.329	0.050*	0.538	0.783
		Backhand	0.478		0.555	

Serial Correlation Analysis

- Wald-Wolfowitz runs test
- Ex: Sequence (F, B, B, B, F) represents three runs
- Hypothesis:

  H_0: Shot direction to Forehands and Backhands are randomly mixed in the sequence

  H_1: Shot direction to Forehands and Backhands are not randomly mixed in the sequence
- Standardize into normal distribution: $Z = \frac{R - \bar{R}}{s_R}$
- Too few of runs–Positive Correlation; Too many–Negative Correlation

Runs Testing: Serving

Player	Serve	Court	Forehand	Backhand	Runs	Z-score	p-value	Run Correlation
Federer	1st	Deuce	1550	1477	1407	-3.91	0.000*	(+)
Federer	1st	Ad	1175	1453	1252	-1.91	0.028	
Federer	2nd	Deuce	722	959	886	3.05	0.001*	(−)
Federer	2nd	Ad	642	976	827	2.67	0.004*	(−)
Djokovic	1st	Deuce	1693	1707	1432	-9.23	0.000*	(+)
Djokovic	1st	Ad	1433	1625	1436	-3.19	0.001*	(+)
Djokovic	2nd	Deuce	867	981	975	2.50	0.006*	(−)
Djokovic	2nd	Ad	715	954	864	2.28	0.011*	(−)
Murray	1st	Deuce	1330	1131	1209	-0.587	0.279	
Murray	1st	Ad	1168	919	1044	0.638	0.261	
Murray	2nd	Deuce	580	937	738	1.115	0.132	
Murray	2nd	Ad	497	1007	653	-0.789	0.215	
Del Potro	1st	Deuce	304	609	420	1.002	0.158	
Del Potro	1st	Ad	415	431	444	1.387	0.083	
Del Potro	2nd	Deuce	141	365	208	0.397	0.346	
Del Potro	2nd	Ad	119	315	197	2.810	0.002*	(−)
Nadal	1st	Deuce	821	1792	1212	3.856	0.000*	(−)
Nadal	1st	Ad	869	1603	1138	0.440	0.330	
Nadal	2nd	Deuce	662	548	584	-0.965	0.167	
Nadal	2nd	Ad	379	596	497	2.201	0.014*	(−)
Nishikori	1st	Deuce	490	415	483	2.184	0.014*	(−)
Nishikori	1st	Ad	358	429	411	1.417	0.078	
Nishikori	2nd	Deuce	265	315	302	1.102	0.135	
Nishikori	2nd	Ad	224	349	301	2.383	0.009*	(−)
Raonic	1st	Deuce	452	450	465	0.866	0.193	
Raonic	1st	Ad	342	440	411	1.828	0.034	
Raonic	2nd	Deuce	217	308	262	0.575	0.283	
Raonic	2nd	Ad	139	342	220	2.371	0.009*	(−)
Ferrer	1st	Deuce	344	349	411	4.829	0.000*	(−)
Ferrer	1st	Ad	309	299	350	3.661	0.000*	(−)
Ferrer	2nd	Deuce	185	235	206	-0.201	0.420	
Ferrer	2nd	Ad	112	288	177	1.829	0.034	
Wawrinka	1st	Deuce	618	603	580	-1.799	0.036	
Wawrinka	1st	Ad	482	622	513	-1.905	0.028	
Wawrinka	2nd	Deuce	332	586	451	1.870	0.031	
Wawrinka	2nd	Ad	231	602	355	1.741	0.041	
Berdych	1st	Deuce	443	407	496	4.866	0.000*	(−)
Berdych	1st	Ad	389	300	405	5.060	0.000*	(−)
Berdych	2nd	Deuce	211	365	286	1.580	0.057	
Berdych	2nd	Ad	159	423	253	2.182	0.015*	(−)

Runs Testing: Returning

Player	Court	Forehand	Backhand	Runs, r_i	Z-score	P-value	Run Correlation
Federer	Deuce	662	803	760	1.756	0.040	
Federer	Ad	638	678	678	1.082	0.140	
Djokovic	Deuce	830	760	857	3.144	0.001*	(–)
Djokovic	Ad	641	920	785	1.487	0.069	
Murray	Deuce	455	643	589	3.427	0.000*	(–)
Murray	Ad	350	678	500	2.594	0.005*	(–)
Del Potro	Deuce	263	197	221	-0.502	0.308	
Del Potro	Ad	166	288	185	-2.695	0.004*	(+)
Nadal	Deuce	809	287	436	0.884	0.188	
Nadal	Ad	571	534	555	0.128	0.449	
Nishikori	Deuce	144	260	200	1.483	0.069	
Nishikori	Ad	156	214	196	1.553	0.060	
Raonic	Deuce	217	137	179	1.126	0.130	
Raonic	Ad	149	182	161	-0.429	0.334	
Ferrer	Deuce	85	188	118	-0.010	0.496	
Ferrer	Ad	108	213	140	-0.542	0.294	
Wawrinka	Deuce	335	249	293	0.536	0.296	
Wawrinka	Ad	226	278	255	0.422	0.337	
Berdych	Deuce	169	175	182	1.924	0.027	
Berdych	Ad	241	127	184	0.978	0.164	

Heterogeneity

Conclusions

- Players mixing their serving strategies optimally in line with economic theory
  - Winning equal share of points when employing alternative strategies
- Lack of optimal 1^{st} serve return strategies
  - Returning serve to opponents backhand significantly $\uparrow$ winning %
- 2^{nd} serve return optimization
  - Have more time to deploy strategy $\rightarrow$ more optimal
- Serial correlation present–players are NOT mixing randomly
  - Mixed strategy equilibrium theory fails to completely hold

Further Research

- Include all matches against left handers and return errors/winners$\rightarrow$ are players still behaving according to theory?
- Lower ranked players $\rightarrow$ do they behave as optimally?
- Specific rivalry analysis $\rightarrow$ trends & deviances in strategies over time

Questions?