

## Setting

Choices of variance (without modification to prevent overflow)

$$v_k = \frac{v}{(1-v)^\alpha}, \quad \alpha = 0, \frac{1}{3}, \frac{1}{2}, 1.$$

Additional variances are

$$\beta_k \text{diag}(v_k)$$

Rationale for investigation:

- » When  $\alpha = 1$ ,  $\beta_k$ 's shrink to 0.
- » Choosing  $\alpha \in (0, 1)$  slows the increase of variance with missing data.
- » The variance, except when  $\alpha = 0$ , still achieves infinity as  $v \rightarrow 1$ .
- »  $\alpha = 0$  recovers the original  $v$ .

Observations:

- » optimal  $\beta_k$  decreases as  $\alpha$  increases, as they compensate each other.

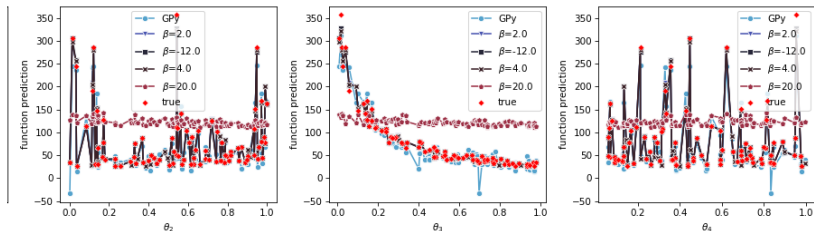
## Summary of emulation comparisons (variance = $v_k$ )

- » Functions: borehole, OTL circuit, Wing weight, and piston
- » Number of locations: 25
- » Number of training parameters: 50
- »  $x$  are sampled uniformly in  $[0, 1]^{d_x}$
- »  $\theta$  are sampled from latin hypercube sampling in  $[0, 1]^{d_\theta}$
- » Test parameters are sampled uniformly in  $[0, 1]^{d_\theta}$
- » Failures are **random**

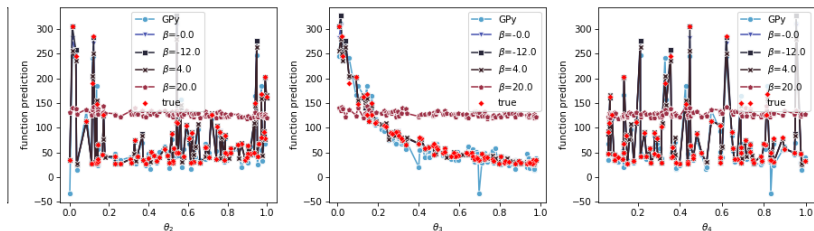
Notice the optimized  $\beta$  value in the legend.

# Example: borehole function

Borehole,  $\alpha = 0.000$



Borehole,  $\alpha = 0.333$

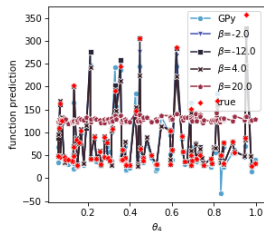
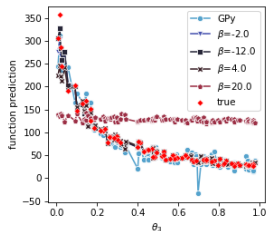
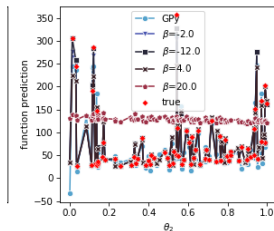


\*legend (top to bottom):

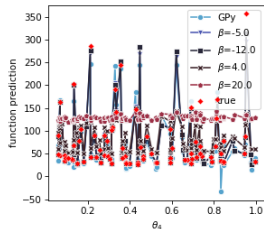
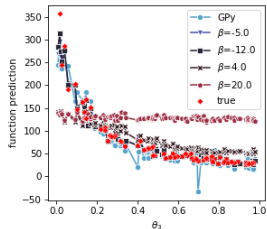
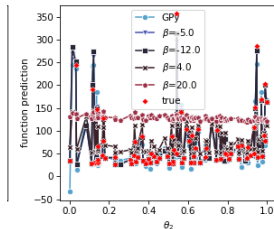
GPy, PCGPwM (optimized,  $\log(\beta_k) = -12, 4, 20$ )

# Example: borehole function

Borehole,  $\alpha = 0.500$



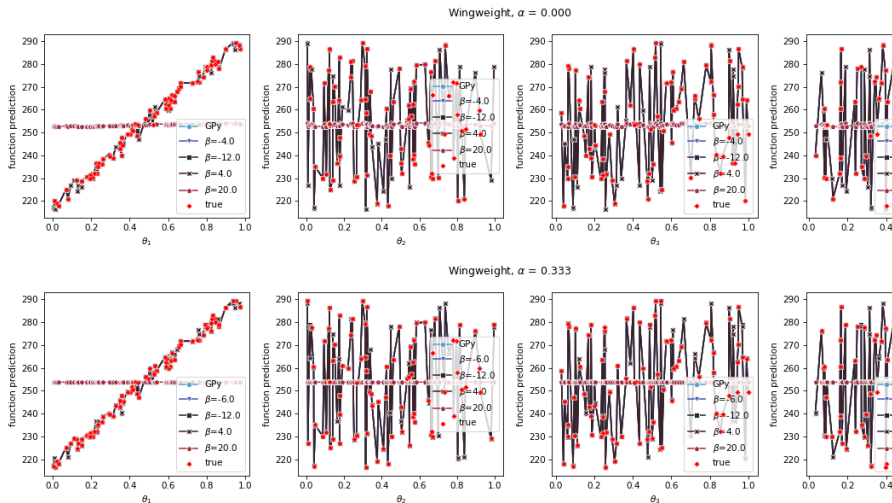
Borehole,  $\alpha = 1.000$



\*legend (top to bottom):

GPy, PCGPwM (optimized,  $\log(\beta_k) = -12, 4, 20$ )

# Example: Wingweight function

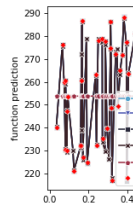
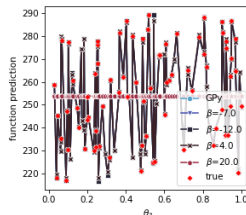
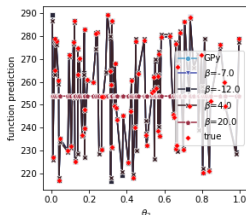
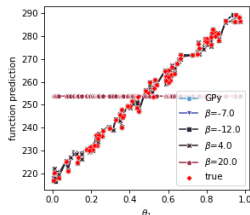


\*legend (top to bottom):

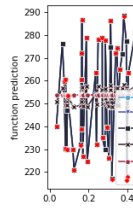
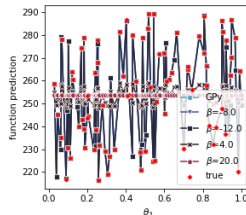
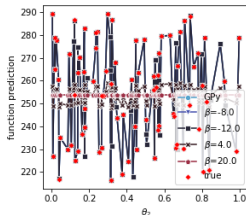
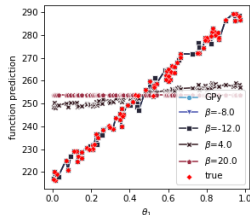
GPy, PCGPwM (optimized,  $\log(\beta_k) = -12, 4, 20$ )

# Example: Wingweight function

Wingweight,  $\alpha = 0.500$



Wingweight,  $\alpha = 1.000$



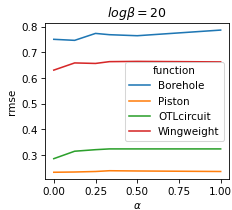
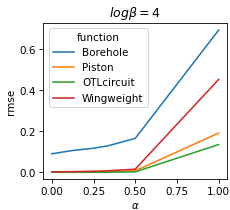
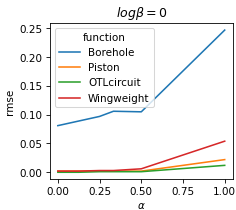
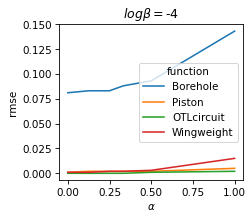
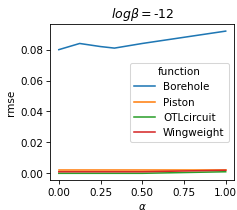
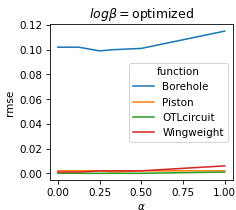
\*legend (top to bottom):

GPy, PCGPwM (optimized,  $\log(\beta_k) = -12, 4, 20$ )

# RMSE and $\alpha$

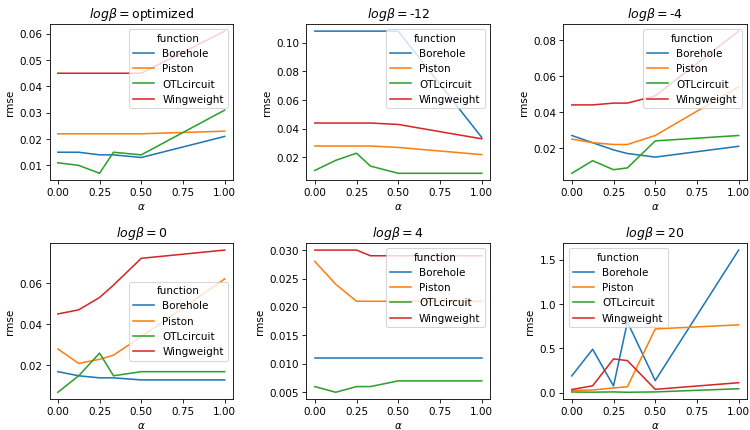
## Random failures

failure random True



## Structured failures

failure random False





# Conclusion

Select  $\alpha$  between  $1/8$  and  $1/2$ .