# Generalisations in Complexity Analysis of 2-D Platform Games

*An analysis of the paper: "Computational Complexity of Two-Dimensional Platform Games" by Michal Forišek*

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The analysis of 2-D platform games [1] are of great mathematical and computational interest in the recent times. Many of the traditional games and puzzles have undergone rigorous mathematical analysis over the last century and are proved to be hard ie: if $P \neq NP$ then there is no efficient algorithm to solve/play them optimally. A similar approach to video games, in particular 2-D platform games, has been fairly recent. The complexity of many video games are yet to be studied and remains largely unexplored.

The article approaches the analysis of 2-D platform games by observing that all such games have certain common features. Any 2-D platform game typically involves a sequential set of puzzles ("levels"). At each level, the player must manipulate his/her avatar to the predesignated end point, performing tasks along the way.The avatar is controlled by the player using a limited simple commands (step, jump, crouch etc.). Further, the avatar is affected by some form of gravity, as a consequence of which maximum jump height is limited and avatar could potentially fall from a height.

[1] Each level in such games are represented by a 2-D map representing a vertical slice of a virtual world which usually consists of horizontal platforms, hence the name "platform game".

In addition, many of these games have certain common features, as enlisted below:

- **long fall:** The height of the longest safe fall (that does not hurt the avatar), which is taken to be larger than the maximum jump height.
- **opening doors:** The game world may contain a variable number of doors and suitable mechanisms to open them.
- **closing doors:** The game world contains a mechanism to close doors, and a way to force the player to trigger such a mechanism.
- **collecting items:** The game world contains items that must be collected.
- **enemies:** The game world contains enemy characters that must be killed or avoided in order to solve the level.
- **time limit:** The given level of the game must be completed within the a given time limit

Video games are essentially understood as decision problems, with fixed states. Generalisation of the features common to many games helps in understanding and approaching the problem with greater abstraction. While certain or combination of features are easy from an algorithmic point, other features, or certain combinations of thereof, are hard to manipulate. Thus having such a feature within the game essentially imply that the game itself is hard to solve, regardless of other details. This abstraction can be used to form "*meta-theorems*" which hold for all games having the features. For any given game, these meta-theorems can be adjusted according to details of the particular game.

The meta-theorems are stated here without proof:

- **Meta-Theorem 1:** *A 2-D platform game where the levels are constant and there is* ***no time limit*** *is in P, even if the collecting items feature is present.*
- **Meta-Theorem 2:** *A 2-D platform game where the* ***collecting items*** *feature is present and a* ***time limit is present*** *as a part of the instance is NP-hard*

- **Meta-Theorem 3:** *Any 2-D platform game that exhibits the features* ***long fall*** *and* ***opening doors*** *is NP-hard.*[2]
- **Meta-Theorem 4:** *Any 2-D platform game that exhibits the features long fall, opening doors and closing doors is PSPACE-hard.*

These results can be use to directly analyse the complexiety of several 2-D platform games. For instance, *Commander Keen, Crystal Caves, Secret Agent* and *Bio-Menace* can be shown to be NP-hard due to presence of switches for activation of moving platforms. Similarly, *Jill of the Jungle* and *Hocus Pocus* are also NP-hard due to the presence of switches that toggle walls, so are *Crash Bandicoot* and *Jazz Jackrabbit 2*, due to crates on breaking which sets of new floor tiles are activated. *Duke Nukem* with the traditionally observed 4 key-cards will be P, while the generalized case with 'n'-keys will be NP-hard.

The game *Prince of Persia* is taken in particular in the article. An instance of the word problem for linear bound automata (WordLBA) is reduced to a instance of the game. The general layout of an instance of the game *Prince of Persia* is as shown below:

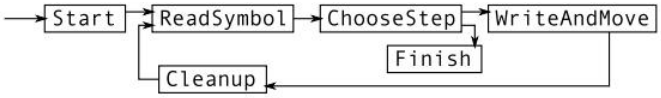


**Fig 1:** Layout of gadgets in the *Prince of Persia* instance

The reduction allows us to conclude that the game *Prince of Persia* is PSPACE-hard. Further, it is known that the game is in NPSPACE. These results, along with Savitch's theorem which states that NPSPACE = PSPACE [2], can be used to show that the game *Prince of Persia* belongs to PSPACE-complete.

[2]This meta-theorem also apply to games that may not include any jumping. For example, this result can be extended to dungeon exploration games that involve one-way trapdoors to lower levels.

The article also considers the case of the generalised version of the game *Lemmings*. A proof was given by Cormode in his 2004 paper in which *Lemmings* is shown to belong to NP.[3] However, the author of the article argues against the proof by noting that the setup used Cormode's proof with an exhaustive trial and error algorithm fails if the number of theoretically possible configurations is not polynomial in the input size. [1] Using the earlier given meta-theorems, the author goes on to show the sequence of instances $\{I_n\}^{\infty}{}_{n=1}$ is a valid for the game *Lemmings* and a counter example to the Cormode's proof. Further, this result is shown to be true for a simple (non generalised version) of the game *Lemming* too. The author goes on to conclude that while Concorde's proof of the game being NP-hard is valid, we cannot comment whether the problem belongs NP or not.

# References

[1] Forišek M. (2010) *Computational Complexity of Two-Dimensional Platform Games* In: Boldi P., Gargano L. (eds) Fun with Algorithms. FUN 2010. Lecture Notes in Computer Science, vol 6099. Springer, Berlin, Heidelberg.

[2] Savitch, Walter J. (1970), *Relationships between nondeterministic and deterministic tape complexities*, Journal of Computer and System Sciences

[3] Cormode, G. (2004) *The Hardness of the Lemmings Game, or Oh no, more NP-Completeness Proofs.* In: Proceedings of Third International Conference on Fun with Algorithms, pp. 65–76