# Part II

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# Introduction

In Part-I we have learnt about Computation, Universal Turing Machines and Complexity Classes. We have seen how powerful the idea of Reduction can be in classifying "problems" in "certain classes" and have demonstrated the same in the shrewd proof stating that it is NP-Complete to win a generalized level in the Game Celeste.

Now in this part, we shall talk further about Games - how to model them, classify them, provided generalized theorems and discuss how and why having knowledge about such aspects of different games is important not only to our understanding of Complexity Theory but also to the world of Computation, in general.

# Why Games?

Games involve the most vibrant (and in many cases even the oldest) set of computational problems. Chess, for example, is an ancient game believed to have originated in India around 1500 years ago and has been proven to be EXPTIME-Complete (in exclusion of the fifty-move rule) in 1981.

Games serve as models of computation which have been used quite prevalently used to mathematically model real-life scenarios. For example, classical game theory deals with several games involving rational decision making strategies and has found applications in computer science, biology and social sciences.

Games not only serve as a means of understanding computation but also decision making and behavioral relations. This is why they have been often found associated with numerous breakthroughs in artificial intelligence. It is quite astonishing as how games often are found to be embedded in deep computational problems and yet require incredibly less to none formal understanding to be played.

Moreover, puzzles and simulations can often be helpful to encapsulate real life problems especially in fields like bioinformatics.

Thus, it would indeed be surprising if understanding of games and augmented reality would provide us with no further **help in the understanding of the nature** as well as in **trying to answer some of the major philosophical questions** encompassing life and reality.

# Game Theory

Game Theory serves as a good formalism to study a certain category of games (mostly non-cooperative multi-player games). It is very well studied field and we shall start of with it and then move forward to other formalisms and study more categories of games like simulations, puzzles, their video game counterparts as well as multi-player video games.

## Introduction

> An equilibrium is not always an optimum; it might not even be good.

One very important common motivation of both Game Theory and Computer Science is to be able to **model rationality** and solve cognitive tasks such as **negotiation**.

In this module, we shall introduce Game Theory with a Complexity Theory minded approach. As it turns out, Game Theory has given rise to several interesting challenges with respect to computational . complexity especially in the realm of the complexity classes PPAD and PLS.

## Prisoner's Dilemma and Nash Equilibrium

In the movie 'A Beautiful Mind', there is a memorable scene where we find Russell Crowe playing Dr. John Nash say that Adam Smith's statement - "In competition, individual ambition serves the common good" - was incomplete. Instead, Nash postulates that the best result is for everyone in the group doing what’s best for himself and the group. This conversation serves as a very informal introduction to the idea behind Nash Equilibrium. Here, we shall now formalise it starting with an example.

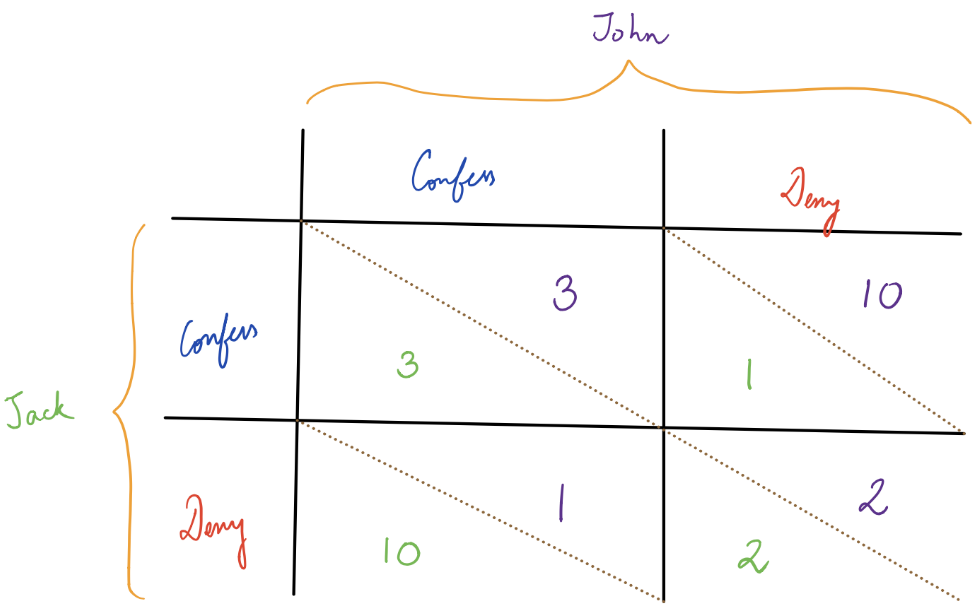
### The Prisoner's Dilemma

Suppose that Jack and John are two two individuals charged and convicted with a minor case of money laundering. They both have to serve 2 years of prison each. However, the judge has a suspicion that both of them (they are acquaintances) are involved in some armed burglary (serious felony).

Now, the judge has no evidence present with him at hand. So, he proposes the following to both of them:

- if both of you deny involvement in the burglary, then both of you receive only 2 years of prison each
- if one confesses while the other denies, then the one who confessed gets 1 year while the other gets 10 years of prison
- if both of you confess, then both of you receive 3 years of prison each

Assuming that the Jack and John have no layalty amongst themselves, we should observe that they will pick a non-optimal scenario.




Here, there exists a global optimum in the case where both the prisoners lie. However, given our predisposed knowledge of the situation it might not be the best rational choice for the prisoners.

The best rational choice for the prisoners would be a sub-optimal choice presented in the case where both of them confess. The case serves as a **sub-optimal stable equilibrium** and is referred to as the Nash Equilibrium.

## Rock-Paper-Scissors

Let us try the above payoff matrix method shown above for the popular game "Rock-Paper-Scissors".

If you have played the game for a lot of times, you may already have a good intuition as to which scenario will give rise to a Nash Equilibrium.

| | | 1/3 | 1/3 | 1/3 |
|---|---|---|---|---|
| | | Rock | Paper | Scissor |
| $\frac{1}{3}$ | R | 0, 0 | -1, 1 | 1, -1 |
| $\frac{1}{3}$ | P | 1, -1 | 0, 0 | -1, 1 |
| $\frac{1}{3}$ | S | -1, 1 | 1, -1 | 0, 0 |

mixed strategy game

Given the payoff matrix, it is evident that only when both players randomize with equal probability, we obtain a Nash Equilibrium.

## Games: Definitions and Notations

**Games** can be defined with a set of **players**, where each player has his own set of allowed **actions** (known as **pure strategies**). Any combination of actions will result in a numerical **payoff** (specified by the game) for each player.

Let the number of players be $1, 2, 3, ..., k$ and $S_i$ denote player $i$'s set of actions. $n$ denotes the size of the largest $S_i$. If $k$ is a constant we look for algorithms that are $O(n^\kappa)$ where $\kappa \in \mathbb{N}$.

Let us look at the above statements in the light of the game of Rock-Paper-Scissors which was discussed above.

- players, $p = \{1, 2\}$
- $\forall i \in p,\ S_i = \{\text{rock, paper, scissor}\}$
- Here, of course $k = 2$, which actually is one of the most studied special case in game theory.

- $n = 3$

Let $S = S_1 \times S_2 \times S_3 \times ... \times S_k$. Then, $S$ is the set of **pure strategy profiles**. Then, each $s \in S$ gives rise to payoff to each player and $u_s^i$ denotes the payoff to player $i$ when all players choose $s$.

- The list of all possible $u_s^i$'s yields a **normal-form game**, which for a $k$-player game should be a list of $kn^k$ numbers.
- We have have other models too. For example, a **Bayesian game** is one where $u_s^i$ can be a probability distribution over $i$'s payoff.

### Nash Equilibrium

A Nash Equilibrium is a set of strategies - one strategy per player - such that no player has an incentive to unilaterally change his strategy (stable sub-optima). In case of two player zero-sum games, Nash Equilibrium is also optimal.

- **Pure** NE: where each player chooses a pure strategy
- **Mixed** NE: where for each player a probability distribution over his pure strategies is applied (in case of MNE we have expected payoff's associated with each player)

### Existence Theorem

Any game with a finite set of players and finite set of strategies has a Nash Equilibrium of Mixed Strategies or MNE.

## Some Computational Problems

### First Desicion Problem

**Input:** A game in normal form.

**Question:** Is there a Pure Nash Equilibrium?

**Solutions**

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### Second Problem