Econometrics III
Assignment Part 3, 4, 5
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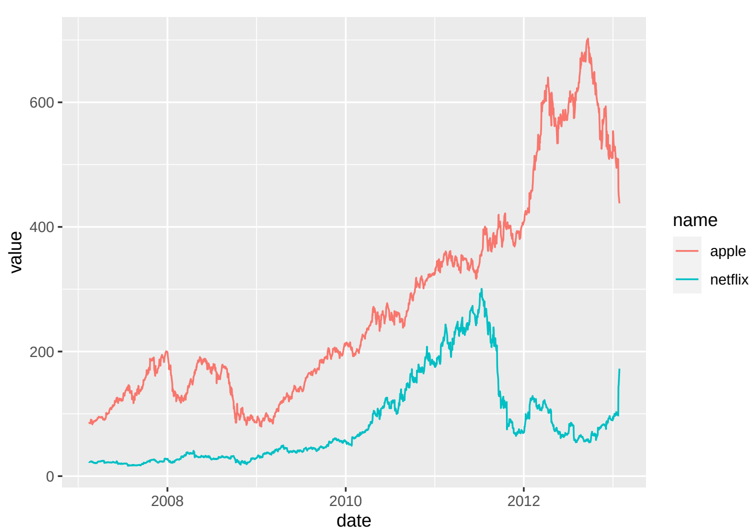
Question 1

Part 1:

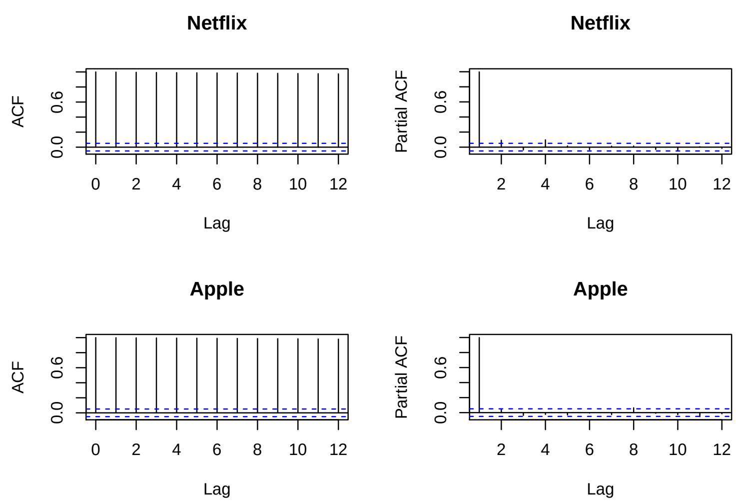
First, we plot the two time series:

```
df3 <- readr::read_csv("./data/data_assign_p3.csv")

twostocks <- c("apple", "netflix")

df3_twostocks <- df3 %>%
    janitor::clean_names() %>%
    pivot_longer(-date) %>%
    filter(is.element(name, twostocks)) %>%
    mutate(date = lubridate::dmy(date))

df3_twostocks %>%
    ggplot(aes(x = date, y = value,
               group = name, color = name)) + geom_line()
```



Then, we show the acf and pacf-functions:

```
n1 <- df3$NETFLIX %>%
    acf(lag.max = 12, plot = F)

n2 <- df3$NETFLIX %>%
    pacf(lag.max = 12, plot = F)

a1 <- df3$APPLE %>%
    acf(lag.max = 12, plot = F)

a2 <- df3$APPLE %>%
    pacf(lag.max = 12, plot = F)

par(mfrow=c(2,2))

plot(n1, main = "Netflix");plot(n2, main = "Netflix")
plot(a1, main = "Apple"); plot(a2, main = "Apple")
```



The ACF's tell us that the stock price is highly dependent on the past stock price, and this dependence decays only very slowly: even the 50 or 100-period lag still shows significant autocorrelation.

Part 2

We now implement a general to specific unit root test function:

```
unit_root_test <- function(column, order){

  # Make the dataset
  series <- ts(column)
  first_differences <- diff(series, differences = 1)
  laggedvar <- stats::lag(series, -1)

  #other lagged first differences, delta x_{t-1}, .., delta x_{t-p+1}
  lagged_fds <- list()

  for(i in 1:(order-1)){

    lagged_fds[[i]] <- stats::lag(first_differences, k = -i)

  }

  df <- cbind(first_differences, laggedvar, purrr::reduce(lagged_fds, cbind)) %>%
    as_tibble()

  colnames(df) <- c("dxt", "xtm1", paste("dxtm", 1:(order-1), sep = ""))

  df <- df %>%
    na.omit()

  # run the stepwise regression - find the best model
  null = lm(data = df, formula = "dxt ~ xtm1")
  full = lm(data = df, formula = paste("dxt ~ xtm1 +",
                                       paste(paste("dxtm", 1:(order-1), sep = ""),
                                             collapse = ' + '),
                                       collapse = " ")
  )

  bestmodel <- step(full,
       scope = list(lower = null, upper = full),
       direction = "backward",
       criterion = "BIC",
       k = log(nrow(df)))

  # perform the unit root test (MacKinnon, 2010)
  b_critical <- -1.6156

  t_value <- bestmodel %>%
    summary() %>%
    .$coefficients %>%
    .[,3] %>%
    .["xtm1"]

  significant = abs(t_value) > abs(b_critical)

  data.frame(stock = deparse(substitute(column)),
             best_model = as.character(bestmodel$call[2]),
             t_value = t_value,
             sig = significant)
}
```

```r
summary_tests <- list()

for(i in 1:length(colnames(df3[,-1]))){
  df <- unit_root_test(df3[, i+1], 12)
  summary_tests[[i]] <- df %>%
    mutate(name = colnames(df3[,-1][i]))
}

summary_tests <- summary_tests %>%
  purrr::reduce(rbind) %>%
  select(-stock)

rownames(summary_tests) <- NULL
```

```r
knitr::kable(summary_tests)
```

best_model	t_value	sig	name
dxt ~ xtm1 + dxtm1 + dxtm3 + dxtm7	-0.6984454	FALSE	APPLE
dxt ~ xtm1 + dxtm1 + dxtm2	-2.0475011	TRUE	EXXON_MOBIL
dxt ~ xtm1	-1.1619768	FALSE	FORD
dxt ~ xtm1 + dxtm1 + dxtm3	-1.5455395	FALSE	GEN_ELECTRIC
dxt ~ xtm1	-2.2598273	TRUE	INTEL
dxt ~ xtm1	-2.3999971	TRUE	MICROSOFT
dxt ~ xtm1 + dxtm2	-1.0642387	FALSE	NETFLIX
dxt ~ xtm1 + dxtm2 + dxtm3	-0.6373731	FALSE	NOKIA
dxt ~ xtm1 + dxtm1	-1.3045653	FALSE	SP500
dxt ~ xtm1 + dxtm1 + dxtm6 + dxtm11	-2.6283893	TRUE	YAHOO

The test seems to be significant for a number of stocks, indicating that for these stocks, the null hypothesis of a unit root is rejected in favor of stationarity. With a 10% α-level, we expect to see a type I-error (rejecting the null while it is true) about one tenth of the time for every test. This means that the probability of having at least 1 type-I error is very large: $(1 - 0.9^{10}) = 0.6513216$. It would be better to use some kind of Bonferroni correction to correct for these compounding type I-errors, but alternatively, we could also lower the α-level.

Part 3

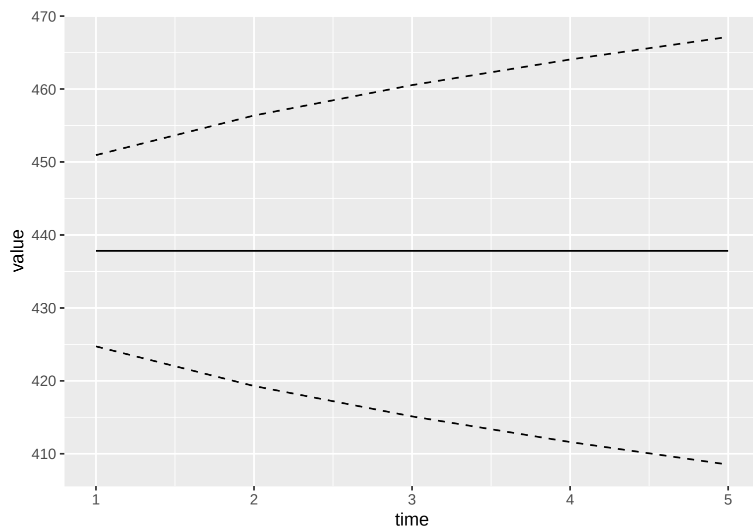
The forecast $\mathbb{E}[P_{t+1}|D_t] = \mathbb{E}[P_{t+1}|P_t] = \mathbb{E}[P_t + \epsilon_t] = P_t$. Similarly, the forecast $\mathbb{E}[P_{t+2}] = \mathbb{E}[P_{t+1} + \epsilon_{t+1}] = \mathbb{E}[P_{t+1}] = P_t$. Generalizing this pattern, the forecast for $P_{t+h} = P_t$. The variance of the forecast is derived using the distribution:

$$
\begin{aligned}
P_{t+1} &= P_t + \epsilon_t \Rightarrow P_{t+1}|P_t \sim N(P_t, \sigma^2) \\
P_{t+2} &= P_{t+1} + \epsilon_{t+1} = \\
&= P_t + \epsilon_t + \epsilon_{t+1} \Rightarrow P_{t+2}|P_t \sim N(P_t, 2\sigma^2)
\end{aligned}
$$

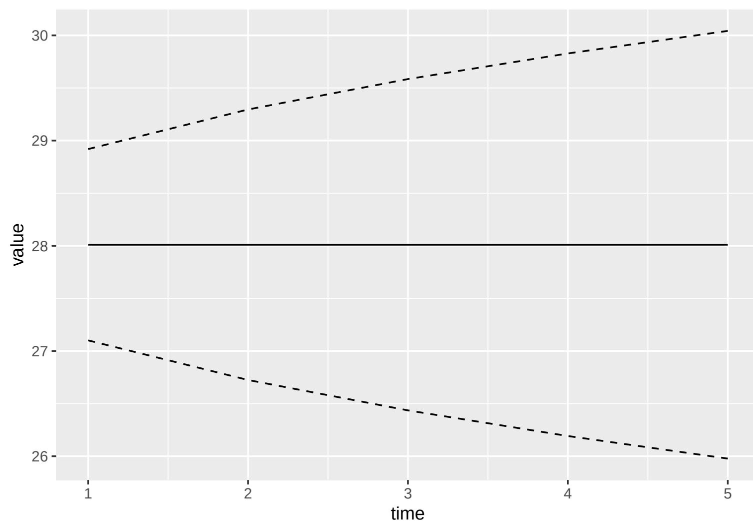
Generalizing this pattern, we can see that $\mathrm{Var}(P_{t+h}) = h \cdot \sigma^2$. Hence, we can implement our forecasts in the following way:

```r
#forecast code
p_tplush <- df3 %>%
  slice_tail(n=1) %>%
```

```r
  select(c("APPLE", "MICROSOFT"))

var_apple <- lm(data = df3 %>%
     select("APPLE"),
   formula = APPLE ~ lag(APPLE, 1)) %>%
  .$residuals %>%
  var()

var_microsoft <- lm(data = df3 %>%
     select("MICROSOFT"),
   formula = MICROSOFT ~ lag(MICROSOFT, 1)) %>%
  .$residuals %>%
  var()

forecasts_apple <- data.frame(time = 1:5,
                              value = rep(p_tplush %>%
                                             pull(1), 5),
                              var = var_apple * 1:5)

forecasts_microsoft <- data.frame(time = 1:5,
                                  value = rep(p_tplush %>%
                                                 pull(2), 5),
                                  var = var_microsoft * 1:5)

forecasts_apple %>%
  ggplot(aes(x = time)) +
  geom_line(aes(y = value)) +
  geom_line(aes(y = value + 1.96*sqrt(var)), lty = "dashed") +
  geom_line(aes(y = value - 1.96*sqrt(var)), lty = "dashed")
```




```r
forecasts_microsoft %>%
  ggplot(aes(x = time)) +
  geom_line(aes(y = value)) +
  geom_line(aes(y = value + 1.96*sqrt(var)), lty = "dashed") +
  geom_line(aes(y = value - 1.96*sqrt(var)), lty = "dashed")
```



Hence, there is no investment advice that we can give: the value is predicted to remain constant, and for all predictions, the probability of an increase in stock price equals the probability of a decrease in stock price. The expected value of any investment strategy is 0, and the value is neither expected to increase, nor to decrease.

Part 4

Do you find a statistically significant contemporaneous relation between Microsoft and Exxon Mobile stock prices?

```
lm(df3, formula = MICROSOFT ~ EXXON_MOBIL) %>%
  stargazer(header = F, omit.stat = c("adj.rsq", "ser", "f"))
```

Table 2:

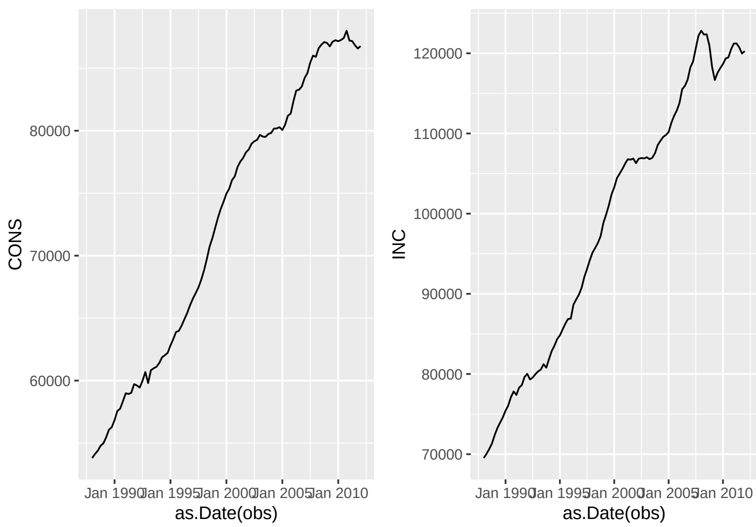
	Dependent variable:
	MICROSOFT
EXXON_MOBIL	0.203***
	(0.009)
Constant	11.245***
	(0.713)
Observations	1,499
R^2	0.251
Note:	*p<0.1; **p<0.05; ***p<0.01

Do you agree that changes in Microsoft stock prices are largely explained by fluctuations in the stock price of Exxon Mobile?

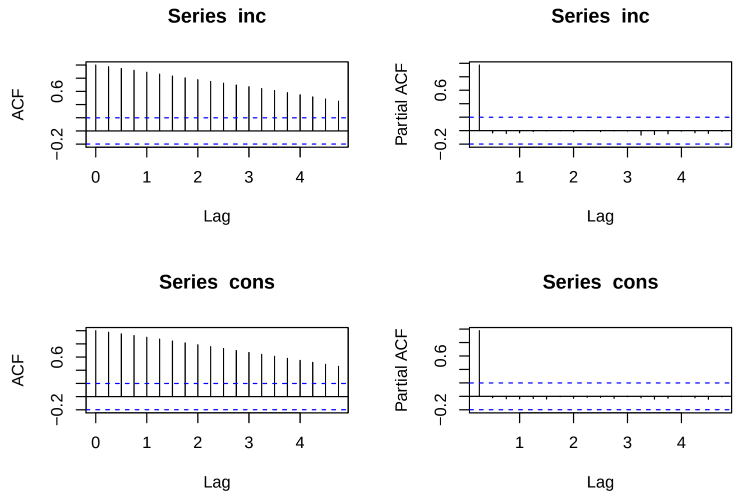
We do not agree that changes in Microsoft stock prices are largely explained by fluctuations in the stock price of Exxon Mobile. It is likely that these results are driven by a shared stochastic trend in both of variables, in other words, the variables might be cointegrated. It can be shown that if two variables share a stochastic trend, the t-value of the estimated coefficient tends to infinity, and the probability of obtaining statistical significance to 1, even under the assumption of completely unrelated trends.

Question 4

```
df4 <- read_csv("data/data_assign_p4.csv")

df4$obs <- ts(df4$obs,
              frequency = 4,
              start = c(1988, 1))

p_cons <- ggplot(data = df4, aes(x = as.Date(obs), y = CONS)) +
  geom_line() +
  scale_x_date(date_labels = "%b %Y")

p_inc <- ggplot(data = df4, aes(x = as.Date(obs), y = INC)) +
  geom_line() +
  scale_x_date(date_labels = "%b %Y")

cowplot::plot_grid(p_cons, p_inc)
```




```
inc <- ts(df4$INC,
   frequency = 4,
   start = c(1988, 1))

cons <- ts(df4$CONS,
   frequency = 4,
   start = c(1988, 1))

inc_acf <- acf(inc, plot = F); inc_pcf <- pacf(inc, plot = F)
cons_acf <- acf(cons, plot = F); cons_pcf <- pacf(cons, plot = F)

par(mfrow=c(2,2))
plot(inc_acf);plot(inc_pcf);plot(cons_acf);plot(cons_pcf)
```



Series inc
ACF
0.6
−0.2
0
1
2
3
4
Lag
Series inc
Partial ACF
0.6
−0.2
1
2
3
4
Lag
Series cons
ACF
0.6
−0.2
0
1
2
3
4
Lag
Series cons
Partial ACF
0.6
−0.2
1
2
3
4
Lag