Econometrics III
Assignment Part 3, 4, 5
Tinbergen Insitute

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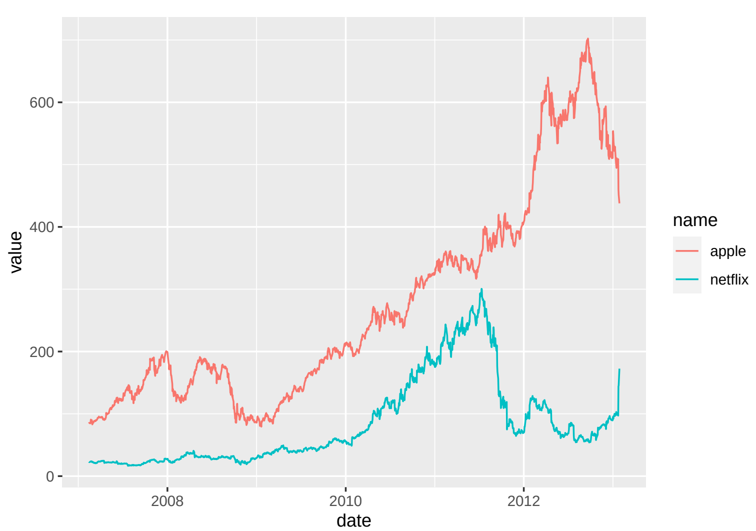
Question 3

Part 1:

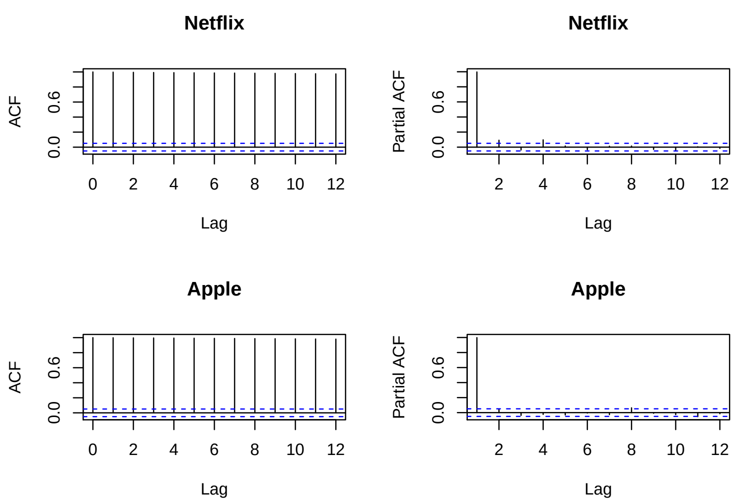
First, we plot the two time series:

```
df3 <- readr::read_csv("./data/data_assign_p3.csv")

twostocks <- c("apple", "netflix")

df3_twostocks <- df3 %>%
    janitor::clean_names() %>%
    pivot_longer(-date) %>%
    filter(is.element(name, twostocks)) %>%
    mutate(date = lubridate::dmy(date))

df3_twostocks %>%
    ggplot(aes(x = date, y = value,
               group = name, color = name)) + geom_line()
```



Then, we show the acf and pacf-functions:

```
n1 <- df3$NETFLIX %>%
    acf(lag.max = 12, plot = F)

n2 <- df3$NETFLIX %>%
    pacf(lag.max = 12, plot = F)

a1 <- df3$APPLE %>%
    acf(lag.max = 12, plot = F)

a2 <- df3$APPLE %>%
    pacf(lag.max = 12, plot = F)

par(mfrow=c(2,2))

plot(n1, main = "Netflix");plot(n2, main = "Netflix")
plot(a1, main = "Apple"); plot(a2, main = "Apple")
```



The ACF's tell us that the stock price is highly dependent on the past stock price, and this dependence decays only very slowly: even the 50 or 100-period lag still shows significant autocorrelation.

Part 2

We now implement a general to specific unit root test function:

```
unit_root_test <- function(column, order, critical_value){

  # Make the dataset
  series <- ts(column)
  first_differences <- diff(series, differences = 1)
  laggedvar <- stats::lag(series, -1)

  #other lagged first differences, delta x_{t-1}, .., delta x_{t-p+1}
  lagged_fds <- list()

  for(i in 1:(order-1)){

    lagged_fds[[i]] <- stats::lag(first_differences, k = -i)

  }

  df <- cbind(first_differences, laggedvar, purrr::reduce(lagged_fds, cbind)) %>%
    as_tibble()

  colnames(df) <- c("dxt", "xtm1", paste("dxtm", 1:(order-1), sep = ""))

  df <- df %>%
    na.omit()

  # run the stepwise regression - find the best model
  null = lm(data = df, formula = "dxt ~ xtm1")
  full = lm(data = df, formula = paste("dxt ~ xtm1 +",
                                     paste(paste("dxtm", 1:(order-1), sep = ""),
                                           collapse = ' + '),
                                     collapse = " ")
  )

  bestmodel <- step(full,
       scope = list(lower = null, upper = full),
       direction = "backward",
       criterion = "BIC",
       k = log(nrow(df)))

  # perform the unit root test (MacKinnon, 2010)
  b_critical <- critical_value

  t_value <- bestmodel %>%
    summary() %>%
    .$coefficients %>%
    .[,3] %>%
    .["xtm1"]

  significant = abs(t_value) > abs(b_critical)

  data.frame(stock = deparse(substitute(column)),
             best_model = as.character(bestmodel$call[2]),
             t_value = t_value,
             sig = significant)
}
```

```r
summary_tests <- list()

for(i in 1:length(colnames(df3[,-1]))){
  df <- unit_root_test(df3[, i+1], 12, -1.6156)
  summary_tests[[i]] <- df %>%
    mutate(name = colnames(df3[,-1][i]))
}

summary_tests <- summary_tests %>%
  purrr::reduce(rbind) %>%
  select(-stock)

rownames(summary_tests) <- NULL
```

```r
knitr::kable(summary_tests)
```

best_model	t_value	sig	name
dxt ~ xtm1 + dxtm1 + dxtm3 + dxtm7	-0.6984454	FALSE	APPLE
dxt ~ xtm1 + dxtm1 + dxtm2	-2.0475011	TRUE	EXXON_MOBIL
dxt ~ xtm1	-1.1619768	FALSE	FORD
dxt ~ xtm1 + dxtm1 + dxtm3	-1.5455395	FALSE	GEN_ELECTRIC
dxt ~ xtm1	-2.2598273	TRUE	INTEL
dxt ~ xtm1	-2.3999971	TRUE	MICROSOFT
dxt ~ xtm1 + dxtm2	-1.0642387	FALSE	NETFLIX
dxt ~ xtm1 + dxtm2 + dxtm3	-0.6373731	FALSE	NOKIA
dxt ~ xtm1 + dxtm1	-1.3045653	FALSE	SP500
dxt ~ xtm1 + dxtm1 + dxtm6 + dxtm11	-2.6283893	TRUE	YAHOO

The test seems to be significant for a number of stocks, indicating that for these stocks, the null hypothesis of a unit root is rejected in favor of stationarity. With a 10% α-level, we expect to see a type I-error (rejecting the null while it is true) about one tenth of the time for every test. This means that the probability of having at least 1 type-I error is very large: $(1 - 0.9^{10}) = 0.6513216$. It would be better to use some kind of Bonferroni correction to correct for these compounding type I-errors, but alternatively, we could also lower the α-level.

Part 3

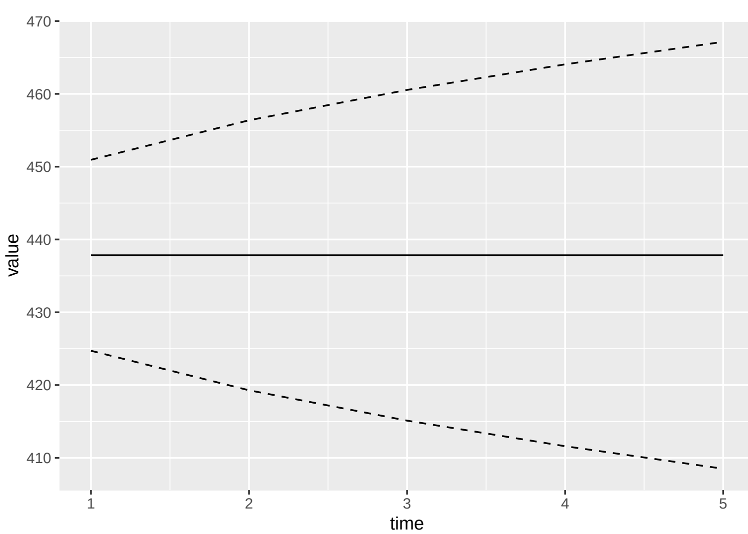
The forecast $\mathbb{E}[P_{t+1}|D_t] = \mathbb{E}[P_{t+1}|P_t] = \mathbb{E}[P_t + \epsilon_t] = P_t$. Similarly, the forecast $\mathbb{E}[P_{t+2}] = \mathbb{E}[P_{t+1} + \epsilon_{t+1}] = \mathbb{E}[P_{t+1}] = P_t$. Generalizing this pattern, the forecast for $P_{t+h} = P_t$. The variance of the forecast is derived using the distribution:

$$
\begin{aligned}
P_{t+1} &= P_t + \epsilon_t \Rightarrow P_{t+1}|P_t \sim N(P_t, \sigma^2) \\
P_{t+2} &= P_{t+1} + \epsilon_{t+1} = \\
&= P_t + \epsilon_t + \epsilon_{t+1} \Rightarrow P_{t+2}|P_t \sim N(P_t, 2\sigma^2)
\end{aligned}
$$

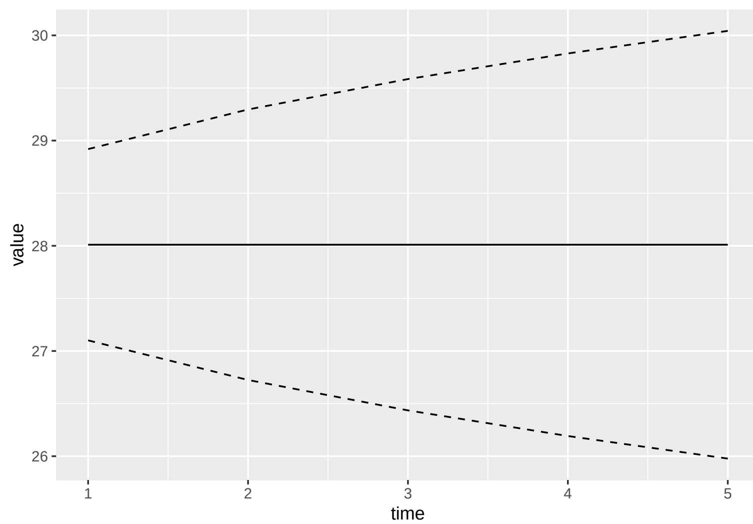
Generalizing this pattern, we can see that $\mathrm{Var}(P_{t+h}) = h \cdot \sigma^2$. Hence, we can implement our forecasts in the following way:

```r
#forecast code
p_tplush <- df3 %>%
  slice_tail(n=1) %>%
```

```
  select(c("APPLE", "MICROSOFT"))

var_apple <- lm(data = df3 %>%
     select("APPLE"),
   formula = APPLE ~ lag(APPLE, 1)) %>%
  .$residuals %>%
  var()

var_microsoft <- lm(data = df3 %>%
     select("MICROSOFT"),
   formula = MICROSOFT ~ lag(MICROSOFT, 1)) %>%
  .$residuals %>%
  var()

forecasts_apple <- data.frame(time = 1:5,
                              value = rep(p_tplush %>%
                                            pull(1), 5),
                              var = var_apple * 1:5)

forecasts_microsoft <- data.frame(time = 1:5,
                                  value = rep(p_tplush %>%
                                                pull(2), 5),
                                  var = var_microsoft * 1:5)

forecasts_apple %>%
  ggplot(aes(x = time)) +
  geom_line(aes(y = value)) +
  geom_line(aes(y = value + 1.96*sqrt(var)), lty = "dashed") +
  geom_line(aes(y = value - 1.96*sqrt(var)), lty = "dashed")
```




```
forecasts_microsoft %>%
  ggplot(aes(x = time)) +
  geom_line(aes(y = value)) +
  geom_line(aes(y = value + 1.96*sqrt(var)), lty = "dashed") +
  geom_line(aes(y = value - 1.96*sqrt(var)), lty = "dashed")
```



Hence, there is no investment advice that we can give: the value is predicted to remain constant, and for all predictions, the probability of an increase in stock price equals the probability of a decrease in stock price. The expected value of any investment strategy is 0, and the value is neither expected to increase, nor to decrease.

Part 4

Do you find a statistically significant contemporaneous relation between Microsoft and Exxon Mobile stock prices?

```
lm(df3, formula = MICROSOFT ~ EXXON_MOBIL) %>%
  stargazer(header = F, omit.stat = c("adj.rsq", "ser", "f"))
```

Table 2:

	Dependent variable:
	MICROSOFT
EXXON_MOBIL	0.203***
	(0.009)
Constant	11.245***
	(0.713)
Observations	1,499
R^2	0.251
Note:	*p<0.1; **p<0.05; ***p<0.01

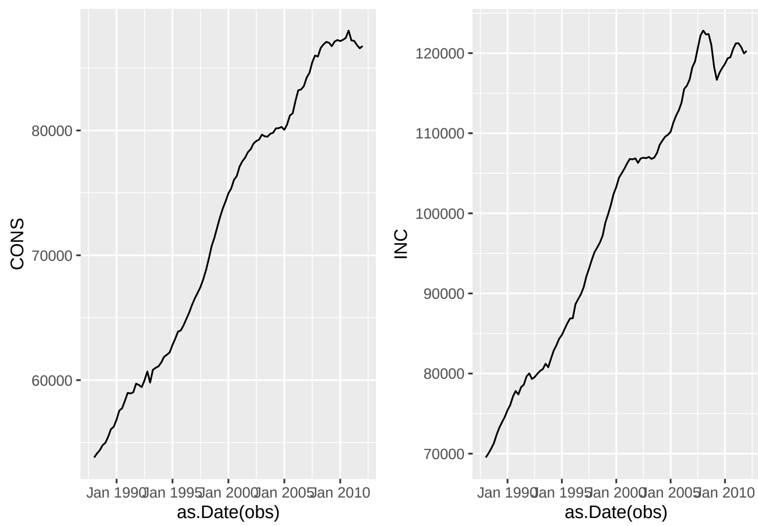
Do you agree that changes in Microsoft stock prices are largely explained by fluctuations in the stock price of Exxon Mobile?

We do not agree that changes in Microsoft stock prices are largely explained by fluctuations in the stock price of Exxon Mobile. It is likely that these results are driven by a shared stochastic trend in both of variables, in other words, the variables might be cointegrated. It can be shown that if two variables share a stochastic trend, the t-value of the estimated coefficient tends to infinity, and the probability of obtaining statistical significance to 1, even under the assumption of completely unrelated trends.

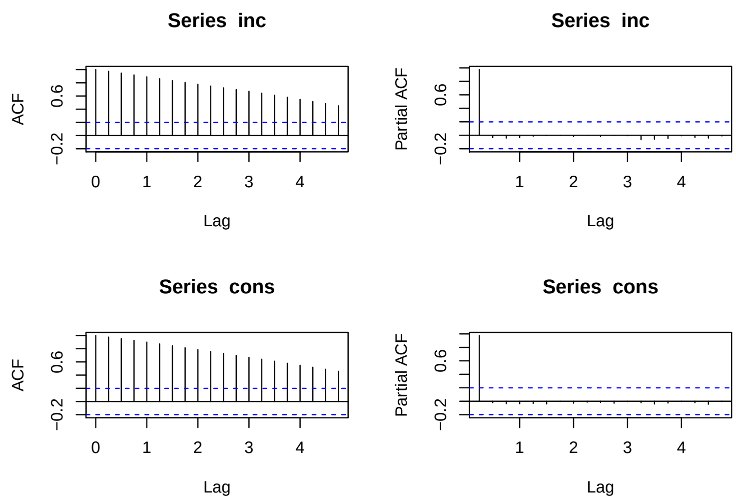
Question 4

Part 1

```r
df4 <- read_csv("data/data_assign_p4.csv")

df4$obs <- ts(df4$obs,
              frequency = 4,
              start = c(1988, 1))

p_cons <- ggplot(data = df4, aes(x = as.Date(obs), y = CONS)) +
  geom_line() +
  scale_x_date(date_labels = "%b %Y")

p_inc <- ggplot(data = df4, aes(x = as.Date(obs), y = INC)) +
  geom_line() +
  scale_x_date(date_labels = "%b %Y")

cowplot::plot_grid(p_cons, p_inc)
```




```r
inc <- ts(df4$INC,
   frequency = 4,
   start = c(1988, 1))

cons <- ts(df4$CONS,
   frequency = 4,
   start = c(1988, 1))

inc_acf <- acf(inc, plot = F); inc_pcf <- pacf(inc, plot = F)
cons_acf <- acf(cons, plot = F); cons_pcf <- pacf(cons, plot = F)

par(mfrow=c(2,2))
plot(inc_acf);plot(inc_pcf);plot(cons_acf);plot(cons_pcf)
```



We observe that in both cases, the autocorrelation function shows extremely high estimates for each subsequent lag: even for a large number of periods, there is still a large degree of autocorrelation in the data. The partial autocorrelations, however, show a completely different view: there is no partial autocorrelation when controlled for other influences. This means that there is a lot of dependence in the series unconditionally, but conditionally on previous values, there seems to be no correlation. This behavior seems to be consistent with random walk behavior.

Part 2

We use the same function as in the previous question to find the best model (according to BIC), and compute the t-value and conduct a Dickey-Fuller test:

```
urcons <- unit_root_test(df4$CONS, order = 12, -1.9393)
urinc <- unit_root_test(df4$INC, order = 12, -1.9393)

unitroots <- rbind(urcons, urinc) %>%
  mutate(stock = str_replace(stock, "df4\\$", ""))

rownames(unitroots) <- NULL
```

```
knitr::kable(unitroots)
```

stock	best_model	t_value	sig
CONS	dxt ~ xtm1 + dxtm3	-1.619306	FALSE
INC	dxt ~ xtm1 + dxtm1	-1.169998	FALSE

As we can see in the table, the unit root hypothesis is not rejected at a 5% level in both cases.

Part 3

Now, we perform a unit root test on the first differences of both series:

```
urcons2 <- unit_root_test(diff(df4$CONS), order = 12, -1.9393)
urinc2 <- unit_root_test(diff(df4$INC), order = 12, -1.9393)

unitroots2 <- rbind(urcons2, urinc2) %>%
  mutate(stock = str_replace(stock, "df4\\$", ""))
```

```r
rownames(unitroots2) <- NULL
```

```r
knitr::kable(unitroots2)
```

stock	best_model	t_value	sig
diff(CONS)	dxt ~ xtm1 + dxtm1 + dxtm2	-2.382383	TRUE
diff(INC)	dxt ~ xtm1 + dxtm8	-5.231291	TRUE

Now, we see that both hypotheses are rejected! We conclude that both of these series are integrated at order $I(1)$, because the hypothesis of a unit root in the first differences is rejected.

Part 4

Assuming both series are I(1), test for cointegration betweenconsumptionandincomebyregressingconsumptiononincomeand performing a unit-root test on the residuals. Reportthe estimated regression coefficients.

Plot the regression residuals. Use the SchwartzInformation Criterion (SIC) to determine the number of ADF lags in your unit-root residualtest. Report the cointegration test statistic. Do you reject cointegration?

```r
consincreg <- lm(data = df4,
   formula = CONS ~ INC)

consincreg %>%
  stargazer(header = F,
            omit.stat = c("adj.rsq", "ser", "f"))
```

Table 5:

	Dependent variable:
	CONS
INC	0.665***
	(0.006)
Constant	6,783.373***
	(554.991)
Observations	97
R^2	0.993
Note:	*p<0.1; **p<0.05; ***p<0.01

```r
urt2 <- unit_root_test(consincreg$residuals,
               order = 12,
               -1.6156)

rownames(urt2) <- NULL
```

```r
knitr::kable(urt2)
```

stock	best_model	t_value	sig
consincreg$residuals	dxt ~ xtm1 + dxtm2 + dxtm8	-2.127911	TRUE

We observe that the hypothesis of a unit root in the residuals is rejected, indicating that the two series are cointegrated: $Cons, Inc \sim CI(1,1)$.

Part 5

As the series are cointegrated, we can estimate an error correction model, incorporate long-run equilibrium information as well as short-run dynamics. The error-correction model is of the following general form:

$$\begin{aligned}\Delta Y_t = \alpha - \gamma \bar{Z}_{t-1} + \phi_1 \Delta Y_{t-1} + \cdots + \phi_p \Delta Y_{t-p} + \\ \beta_0 \Delta X_t + \cdots + \beta_q \Delta X_{t-q} + u_t\end{aligned}$$

We first construct the necessary data matrix in Python:

```python
import pandas as pd

# p for dep var, q for indep var
def generate_datamatrix(p, q, y, x):
  # dependent variable
  fdy = r.df4[str(y)] - r.df4[str(y)].shift(1)
  #ytm1
  ytm1 = r.df4[str(y)].shift(1)
  #xtm1
  xtm1 = r.df4[str(x)].shift(1)

  d = {}

  d['xt'] = r.df4[str(x)]
  d['xtm1'] = xtm1
  # x first differences
  d['dxtm0'] = r.df4[str(x)] - r.df4[str(x)].shift(1)
  # lagged first differences
  for i in range(1, q+1):
    d['dxtm{0}'.format(i)] = d['dxtm0'].shift(i)

  # y first differences
  e = {}

  e['yt'] = r.df4[str(y)]
  e['fdy'] = fdy
  e['ytm1'] = ytm1

  for j in range(1, p+1):
    e['dytm{0}'.format(j)] = e['fdy'].shift(j)

  # put everything in a dataframe
  dfx = pd.DataFrame.from_dict(d)
  dfy = pd.DataFrame.from_dict(e)

  df_out = pd.concat([dfx.reset_index(drop=True), dfy], axis=1)

  return df_out

data_ecm = generate_datamatrix(4, 4, 'CONS', 'INC')
```

Now, we implement the general-to-specific procedure in R. First, we estimate the residuals $\bar{Z}_t$, and compute $\bar{Z}_{t-1}$ which we then lag to add as a predictor in the general-to-specific procedure.

```r
zt <- lm(data = py$data_ecm,
   formula = yt ~ xt)

residuals <- zt$residuals

ztm1 <- lag(residuals, 1)
```

Now, we add $\bar{Z}_{t-1}$ to the dataset, and formulate the g2s procedure (again according to the BIC):

```r
data_ecm <- py$data_ecm %>%
  mutate(ztm1 = ztm1) %>%
  na.omit()

indep_vars <- py$data_ecm %>%
  colnames()

indep_vars <- indep_vars[grepl("dxt|dyt", indep_vars)]

longformula <- paste("fdy ~ ztm1 + 0 +", paste(indep_vars, collapse = " + "))
# run the stepwise regression - find the best model
null = lm(data = data_ecm, formula = "fdy ~ ztm1 + 0")
full = lm(data = data_ecm, formula = longformula)

bestmodel <- step(full,
       scope = list(lower = null, upper = full),
       direction = "backward",
       criterion = "BIC",
       k = log(nrow(data_ecm)))
```

Starting with $p = q = 4$, we get the following model:

```r
stargazer(bestmodel, header = F,
          omit.stat = c("adj.rsq", "ser", "f"))
```

Report and interpret the short-run and long-run multipliers. Report and interpret the error correction coefficient.

The error correction coefficient $\gamma =$ -0.0875952 means that if consumption and income are away from their long-run relationship, there is a correction towards the equilibrium value. For example, if there is a positive residual, meaning consumption is too high relative to the income at a given point in time, in the next period, we tend to observe a *decrease* in consumption to correct for that.

The short-run multipliers can be derived from the coefficients belonging to ΔX_t and ΔY_{t-3}. An (positive) exogenous shock in income causes an increase in consumption (0.19), and the effects of that shock also appear three periods later ΔY_{t-3}.

The long-run multipliers can be derived from the equilibrium relationship in the "first-stage" regression:

$\bar{Y} = 6783.3732069 + 0.6651404 \cdot \bar{X}$.

This indicates that people tend to consume a proportion of 66% of their long-run equilibrium income, plus a fixed amount of 6783.3732069, which can (kind of) be interpreted as the cost of living (even if the long-run equilibrium income is 0, they theoretically consume this amount).

6. How strong is the correction to equilibrium? Is there over-shooting?

Table 7:

	Dependent variable:
	fdy
ztm1	-0.088^{**}
	(0.034)
dxtm0	0.199^{***}
	(0.044)
dytm3	0.378^{***}
	(0.082)
dytm4	0.192^{**}
	(0.076)
Observations	92
R^2	0.678
Note:	*p<0.1; **p<0.05; ***p<0.01

The error correction coefficient γ = -0.0875952 is very close to zero. Hence, there is no overshooting $\gamma < -1$, and only a partial error correction. The error correction is also quite slow, indicated by the low magnitude of the coefficient.

Do you find evidence of Granger causality? Justify your answer.

Lecture 6 lecture slides

Question 5