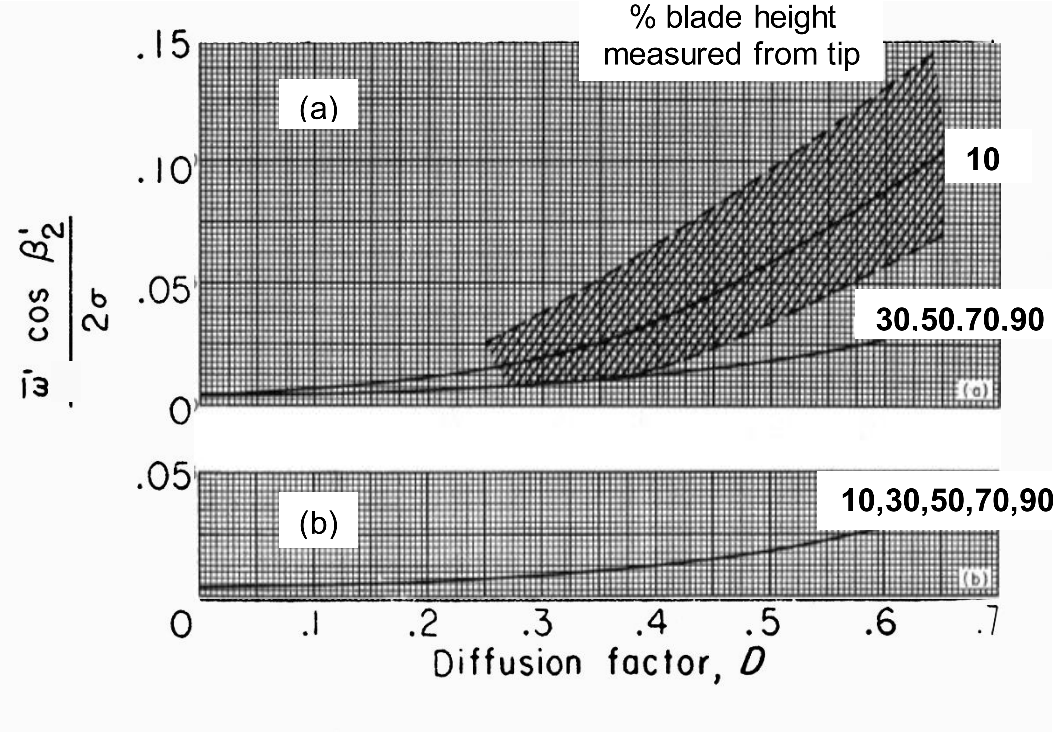
Mini Project

Based on blade mean line analysis it is required to design an axial flow stage at SLS conditions. Two alternatives will be considered:

a) Subsonic stage with maximum inlet relative Mach number of 0.77. NACA 65 blades may be used.
b) Supersonic stage with maximum inlet relative Mach number of 1.25. DCA blades may be used.

Evaluation of rotor pressure loses

The subsonic 3D pressure losses may be evaluated by weighting of the loss coefficient ω radially as given by Fig.(1).



(a) Rotor.
(b) Stator.

Shock stagnation pressure loss may be approximated by that corresponding to a normal shock at the relative inlet Mach number (see normal shock table).

where, $\omega_{2D-subsonic} = (P_{trel1} - P_{trel2-subsoinc})/(P_{trel1} - P_1)$

$\omega_{2D} = (P_{trel1} - \pi_{shock} P_{trel2-subsonic})/(P_{trel1} - P_1)$, this includes supersonic losses.

ω_{3D} is then obtained by the subsonic radial weighting of the loss coefficient. Possibly similar to what is done for subsonic range.

Shock wave stagnation pressure losses may be conservatively estimated by normal shock stagnation pressure ratio $\frac{P_{t2}}{P_{t1}}$ (gas dynamics tables) at the inlet relative Mach number.

The mass flow rate is to be varied (for 8 groups) from 20 kg/s-37.5 kg/s in steps of 2.5 kg/s.
The following constraints must be imposed:

- $C_x \leq 150$ m/s
- $U_2/U_1 \leq 1.05$
- DF (for both rotor and stator) ≤ 0.47
- For transonic stage tip rotational velocity is ≤ 400 m/s.
- For subsonic stage tip rotational velocity is ≤ 300 m/s.
- $AN^2 \leq 3.0 \cdot 10^7$ [m^2 rpm^2]

It is required to find the maximum pressure ratio for each type of stage (subsonic-transonic) with appropriate weight given to stage efficiency.
The polytropic efficiency for both designs may be compared.

Full data of the two designs must be reported:

- Velocity triangles data.
- Blade geometry at the mean line and number of blades.
- Blade incidence and deviation angles at design conditions.
- Annulus shape.
- Compressor efficiency.
- Solidity- hub to tip ratio- height to chord ratio, …..

Subsonic & Supersonic Compressor Design

JET PROPULSION FINAL REPORT

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Givens

We run our code on mass flow rate of 30 kg/s. Also total temperature is 288.16 kelvin, and total pressure is 1.013e5 Pascal.

```
%consts
cp=1004.5;
Rconst=287;
G=1.4;
```

```
%Givens
Tt1=288.16;
Pt1=1.013e5;
mdot=30;
```

Assumptions

1- Constant Cx
2- Repeating stage
3- t/c = 0.1 (we noticed that varying this number along its range causes insignificant changes so we just assumed this number).

Subsonic Design

Inputs

We define ranges for the inputs as follows:

$5 \le \alpha_1 \le 30$ $\quad$ $0.4 \le \sigma_s \le 2$ $\quad$ $0.4 \le \sigma_r \le 2$

$0.4 \le \phi_1 \le 0.6$ $\quad$ $0.6 \le \zeta_r \le 0.8$ $\quad$ $100 \le C_{x_1} \le 150$ $\quad$ $1 \le \frac{h}{c_r} \le 3$

$0.1 \le \psi \le 0.4$ $\quad$ $0.6 \le \zeta_s \le 0.8$ $\quad$ $1 \le \frac{U_2}{U_1} \le 1.05$ $\quad$ $1 \le \frac{h}{c_s} \le 3$

```
xbound =[5 30; %alpha1
    0.4 0.6; %phi1
    0.1 0.4; %psi
    100 150; %Cx1
    1 1.05; %U22U1
    0.6 0.8; %zetar
    0.4 2; %soldr
    1 3; %h2cr
    0.6 0.8; %zetas
    0.4 2; %solds
    1 3]; %h2cs
```

Constraints

$0.6 \leq M_{rel1} \leq 0.77$ $\quad U_{t_r} \leq 300$ $\quad 100 \leq C_{x_2} \leq 150$

$0.3 \leq DF_r \leq 0.47$ $\quad 0.5 \leq R \leq 1$ $\quad 0.4 \leq \phi_2 \leq 0.6$

```
if((Mrel1>=0.6)&&(Mrel1<=0.77)&&(DFr>=0.3)&&(DFr<=0.47)&&(utr<=300)&&...
        (Cx2>=100)&&(Cx2<=150)&&(R<=1)&&(R>=0.5)&&(phi2>=0.4)&&(phi2<=0.6))
 % Satisfied
    criteria=min(-(0.6.*(Pratio)+0.4.*(eff)));
else
    criteria=455567;
end
```

Equations

First, we use trigonometry to calculate the velocity triangle:

```
C1=Cx1./cosd(alpha1);   %Absolute Inlet Velocity
U1=Cx1./phi1;           %Rotor Speed 1
Cth1=C1.*sind(alpha1);  %Absolute Inlet Radial Velocity
Wth1=U1-Cth1;           %Relative Inlet Radial Velocity
W1=sqrt(Cx1.^2+Wth1.^2);%Relative Inlet Velocity
beta1=asind(Wth1./W1);  %Relative Inlet Angle
```

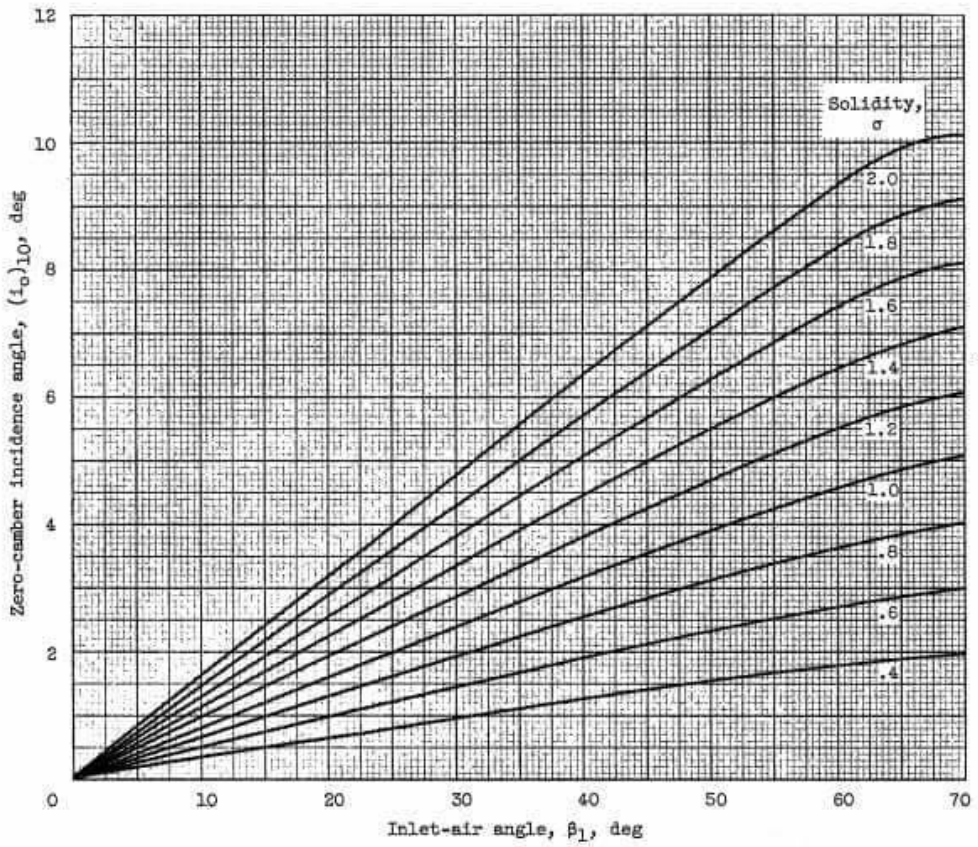
and calculate Flow Coefficient 2: ϕ_2 to work out the second triangle:

```
%Vel Tri 2
U2=U22U1.*U1;           %Rotor Speed 2
phi2=Cx2./U2;           %Flow Coefficient 2
beta2=atand((1-(psi+(1-phi1.*tand(beta1))).*(1./(r22r1.^2)))./phi2);    %Relative Outlet Angle
Wth2=Cx2.*tand(beta2);  %Relative Outlet Radial Velocity
W2=sqrt(Wth2.^2+Cx2.^2);%Relative Outlet Velocity
Cth2=U2-Wth2;           %Absolute Outlet Radial Velocity
C2=sqrt(Cx2.^2+Cth2.^2);%Absolute Outlet Velocity
alpha2=asind(Cth2./C2); %Absolute Outlet Angle
```

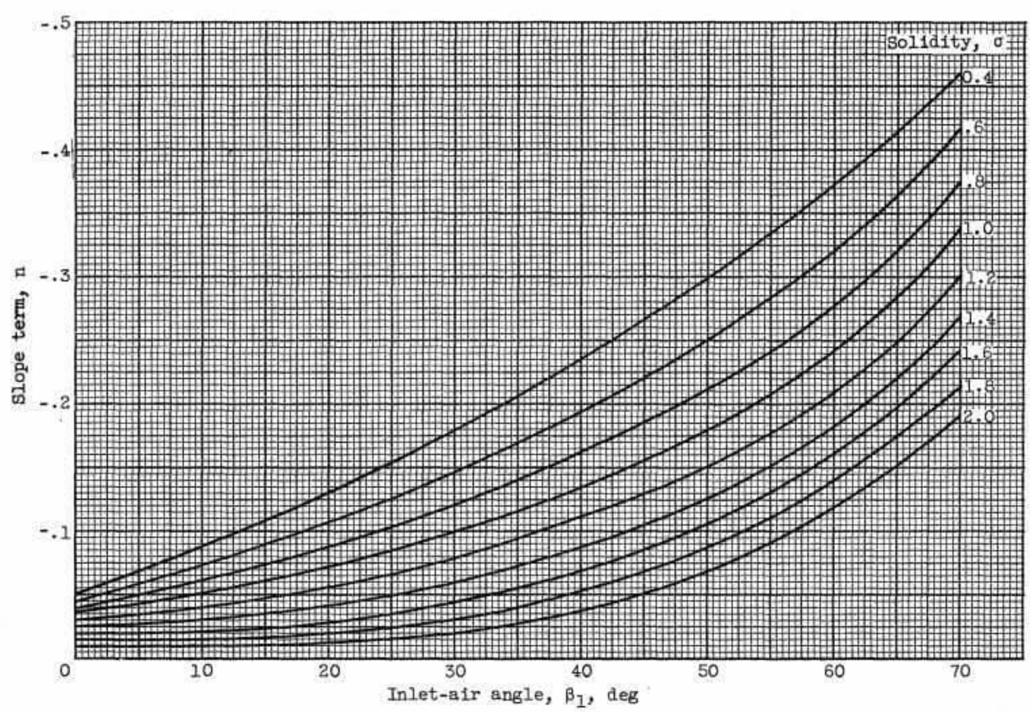
Then we use the isentropic relations to calculate the static inlet temperature and calculate from that the inlet Mach number, first mass flow parameter, and inlet area.

```
T1=Tt1-(C1.^2)/(2.*cp);     %Inlet Static Temperature
M1=C1./sqrt(G.*Rconst.*T1); %Inlet Mach Number
MFP1=sqrt(G./Rconst).*M1.*(1+((G-1)./2).*(M1.^2)).^(-(G+1)./(2.*(G-1))); %Mass Flow Parameter 1
A1=(mdot.*sqrt(Tt1))./(MFP1.*Pt1.*cosd(alpha1));  %Area 1
```

Earlier, we had used grabit to obtain graphs for io_{10} and n for the incidence relation for different solidities (0.4, 0.5, 0.6, 0.8, 1.0, 1.2, 1.4, 1.5, 1.6, 1.8, 2.0) and we load them as functions (using fit function as smoothing spline).



Zero-camber incidence angle, $(i_0)_{10}$, deg
Inlet-air angle, β_1, deg
Solidity, σ
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4



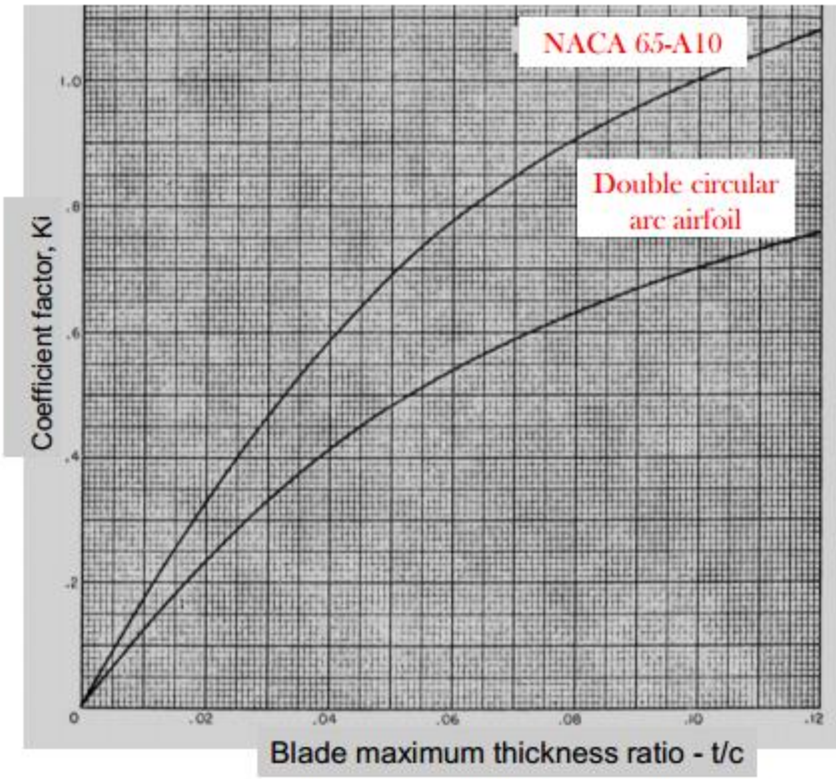
Slope term, n
Inlet-air angle, β_1, deg
Solidity, σ
0.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0

Since the values we have graphs for are discrete, but the input is not, we use a function called “discretize_sold.m” to make the values discrete, which goes like this (the complete function is not shown here)

```
function [ sold ] = discretize_sold( a )
%sold=[0.4,0.5,0.6,0.8,1,1.2,1.4,1.5,1.6,1.8,

if a>=0.4&&a<=0.5
    lower_diff=abs(a-0.4);
    upper_diff=abs(a-0.5);
    if lower_diff<=upper_diff
        sold=0.4;
    else
        sold=0.5;
    end
elseif a>0.5&&a<=0.6
    lower_diff=abs(a-0.5);
    upper_diff=abs(a-0.6);
    if lower_diff<=upper_diff
        sold=0.5;
    else
        sold=0.6;
    end
elseif a>0.6&&a<=0.8
```

We then define Carter’s deviation and use it along with the incidence defined above to calculate the blade angles: β_1\`, $\beta_2$\`, and then the incidence and deviation.

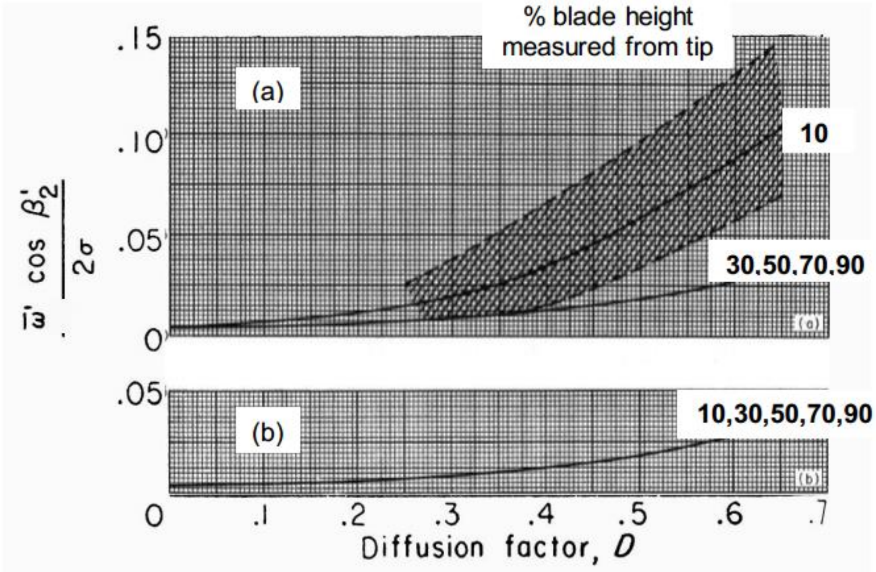


Since this is NACA 65 then its $K_{i_{sh}}$ is 1 and K_{i_t} for thickness ratio of 0.1 is 1.

```
kish=1;
kit=1;
i0=i010*kish*kit;

a_c=0.5; %circular arc
m=0.23.*((2.*a_c).^2)+beta2./500;
kk=m./((soldr).^0.5);
A=[(n+1),-n;kk,(1-kk)];
b=[beta1-i0;beta2];
XX=A\b;
beta1d=XX(1);
beta2d=XX(2);
i_r=beta1-beta1d;
dev_r=beta2-beta2d;
```

We had also used grabit to obtain the graphs of rotor and stator loss against the diffusion factor.



(a) **Rotor.**
(b) **Stator.**

So next we calculate the diffusion factor and use it to obtain the rotor and stator loss.

```
%Diffusion factor & losses
%DFr=1-(W2/W1)+(abs(r2*Wth2-r1*Wth1))/((2*sold*((r1+r2)/2)*W1));
DFr=1-(W2./W1)+(abs(r22r1.*Wth2-Wth1))./((2.*soldr.*((r22r1./2)+1).*W1));
```

```
wrcos=0.7.*f_RL(DFr)+0.3.*f_RL10(DFr);
wr=wrcos.*((2.*soldr)./(cosd(beta2d)));
```

Since we have the losses, we can now use the usual isentropic relations and velocities to get the stage pressure ratio. In the middle, we calculate the relative Mach number 2 to obtain the second mass flow rate and the outlet area.

```
%Transition
P1=Pt1./((Tt1./T1).^(G./(G-1)));
Ttrel1=T1+(W1.^2)./(2.*cp);
Mrel1=W1./sqrt(G.*Rconst.*T1);
Ttrel2=Ttrel1+(U2.^2-U1.^2)./(2.*cp);
Ptrel1=P1.*(Ttrel1./T1).^(G./(G-1));
Ptrel2d=Ptrel1.*(Ttrel2./Ttrel1).^(G./(G-1));
Ptrel2=Ptrel2d-wr.*(Ptrel1-P1);
```

```
T2 = Ttrel2 - (W2.^2)./(2.*cp);
Mrel2 = W2./sqrt(G.*Rconst.*T2);
MFPrel2=sqrt(G./Rconst).*Mrel2.*(1+((G-1)./2).*(Mrel2.^2)).^(-(G+1)./(2.*(G-1)));
A2=(mdot.*sqrt(Ttrel2))./(MFPrel2.*Ptrel2.*cosd(beta2));
```

With this information, we can also calculate the Area Mean Square, Radius Mean Square, and **r_1** & **r_2**.

```
Amr = (A1+A2)./2 ;       %Area Mean
rmr=sqrt((Amr.*(1+zetar))./(4.*pi.*(1-zetar)));%Radius Mean
r2r=2.*rmr./(1+1./r22r1);%Radiua Rotor
r1=r2r./r22r1;            %Radius 1
```

Then we get the values of pressure, mach number, and temperatures at stage 2.

```
P2=Ptrel2./((1+((G-1)./2).*Mrel2.^2)).^(G./(G-1));
M2=C2./sqrt(G.*Rconst.*T2);
Tt2=T2.*(1+((G-1)./2).*M2.^2);
Pt2=P2.*(1+((G-1)./2).*M2.^2).^(G./(G-1));
```

The rotor now has all the calculations needed and the stator will be done next.

```
%%Stator
%Facts
Tt3=Tt2;

%Assumption: repeating stage
alpha3=alpha1;
C3=C1;
Cth3=Cth1;
```

Again, we start with the incidence and deviation relations to get the blade angles.

```
i0s=i010s*kish*kit;

a_c=0.5; %circular arc
m=0.23.*((2.*a_c).^2)+alpha3./500;
kk=m./((solds).^0.5);
A=[(ns+1),-ns;kk,(1-kk)];
b=[alpha2-i0s;alpha3];
XX=A\b;
alpha2d=XX(1);
alpha3d=XX(2);
i_s=alpha2-alpha2d;
dev_s=alpha3-alpha3d;
alpha1d=alpha3d;
```

```
staggerR=(beta1d+beta2d)./2;
staggerS=(alpha2d+alpha3d)./2;
```

In order to get the stator diffusion factor we need to iterate and check using the following loop:

```
%solving for diffusion factor stator
DFsassumed=0.4:0.001:0.47;
tol=0.001;
load ('RotorLoss.mat'); DFs_SL=RotorLoss(:,1); wscos_SL=RotorLoss(:,2);
f_SL=fit(DFs_SL,wscos_SL,'smoothingspline');
for j=1:length(DFsassumed)
    wscos=f_SL(DFsassumed(j));
    ws=wscos.*((2.*solds)./(cosd(alpha3d)));  %Stator Loss
    Pt3=Pt2-ws.*(Pt2-P2);                      %Total Pressure 3
    T3 = Tt3 - (C3.^2 /(2.*cp));               %Static Temperature 3
    M3 = C3./sqrt(G.*Rconst.*T3);              %Mach Number 3
    MFP3=sqrt(G./Rconst).*M3.*(1+((G-1)./2).*(M3.^2)).^(-(G+1)./(2.*(G-1)));%Mass Flow Parameter
    A3=(mdot.*sqrt(Tt3))./(MFP3.*Pt3.*cosd(alpha3));  %Area 3
    Ams =(A2+A3)./2;                                  %Area Mean
    rms=sqrt((Ams.*(1+zetas))./(4.*pi.*(1-zetas)));   %Radius Mean
    r2s=r2r+(rms-r2r)/(2.*cosd(staggerS)+1);          %Radius Stator 2
    ratio=2.*(cosd(staggerS)+0.25)/(cosd(staggerS)+0.5);
    r3s=r2r+ratio.*(rms-r2r);                         %Radius Stator 3
    DFscheck=1-C3./C2+(abs(r3s.*Cth3-r2s.*Cth2))./(2.*((r2s+r3s)./2).*solds.*C2);
    %Check
    if abs(DFsassumed(j)-DFscheck) < tol
        break;
    end
end
```

Get the stage efficiency using the values calculated thus far:

```
%Calculating pressure ratio and efficiency
Pratio=Pt3./Pt1;     %Stage Pressure Ratio
Tratio=Tt3./Tt1;     %Stage Temperature Ratio
eff=((Pratio.^((G-1)./G))-1)./(Tratio-1);
```

Calculate Tip Radius and Hub Radius to calculate number of blades

```
%calculating no. of blades for
%rotor
rtr = 2.*rmr./(1+zetar); %rtip rotor
rhr = 2.*rmr-rtr; %rhub rotor
chr=(rtr-rhr)./h2cr; %chord rotor
Sr=chr./soldr; %spacing rotor
Nr=(2.*pi.*rmr)./Sr; %Number of blades in rotor
%stator
rts = 2.*rms./(1+zetas); %rtip stator
rhs = 2.*rms-rts; %rhub stator
chs=(rts-rhs)./h2cs; %chord stator
Ss=chs./solds; %spacing stator
Ns=(2.*pi.*rms)./Ss; %Number of blades in stator
```

Calculate final check for the constraints:

```
%calculating tip velocities for rotor
omega = U1./r1;     %Radial Velocity
utr = omega.* rtr; %Tip Velocity
```

```
R=(((W1.^2)./2)-((W2.^2)./2)+(((U2.^2)-(U1.^2))./2))./((U2.^2)-(U1.^2)+(U1.*Wth1)-(U2.*Wth2));
```

Optimization

Number of variables = 11

```
%xbound=[alpha1, phi1, psi, Cx1, U22U1, zetar, soldr, h2cr, zetas, solds, h2cs]
```

Initiating the initial value vector

```
X0 = xbound(:,1) + (xbound(:,2) - xbound(:,1)).*rand(Nv,1);
```

Setting the options of the optimizer to converge at e-5, or after time limit of 12000 seconds. Of a total of 800 generations and a population size of 300.

```
X0 = xbound(:,1) + (xbound(:,2) - xbound(:,1)).*rand(Nv,1);
Lb = xbound(:,1);
Ub = xbound(:,2);
options = gaoptimset('TimeLimit',12000,'PopulationSize',300,'Generations',800, 'TolCon', 1e-5,...
'TolFun', 1e-5,'PlotFcns',@gaplotbestf);  %%%%%%%%
options.Display='iter';
[x, fval, exitflag,output] = ga(@optimize_subsonic,11, [], [], [], [], Lb, Ub, [],options);
[Pratio, eff, Mrel1, DFr, utr, Cx2, R, phi2, criteria]=subsonic(x)
```

Solving at limits

Lower Limit

1- Velocity triangles data:
   a) alpha1 = 5
   b) alpha2 = 18.6490
   c) beta1 = 67.4857
   d) beta2 = 65.1830
   e) C1 = 100.3820
   f) C2 = 105.5414
   g) W1 = 261.1553
   h) W2 = 238.2531
2- Blade geometry at mean line
   a) Rotor: A_m = 0.2068, r_m = 0.2566
   b) Stator: A_m = 0.1575, r_m = 0.2239

3- Number of blades
   a) Rotor: 5
   b) Stator: 5
4- Blade incidence and deviation
   a) Rotor: i = 21.2006, σ= -25.0291
   b) Stator: i = -2.6644, σ= 9.9762
5- Annulus shape: A1 = 0.1572, A2 = 0.1578, A3 = 0.2559
6- Solidity
   a) Rotor = 0.4000
   b) Stator = 0.4000
7- Hub to tip ratio
   a) Rotor = 0.6
   b) Stator = 0.6
8- Height to chord ratio
   a) Rotor = 1
   b) Stator = 1

Lower Limit

1- Velocity triangles data:
   a) alpha1 = 30
   b) alpha2 = 49.8343
   c) beta1 = 47.4479
   d) beta2 = 29.4761
   e) C1 = 173.2051
   f) C2 = 232.5580
   g) W1 = 221.8079
   h) W2 = 172.3027
2- Blade geometry at mean line
   a) Rotor: A_m = 0.1775, r_m = 0.3565
   b) Stator: A_m = 0.1619, r_m = 0.3406
3- Number of blades
   a) Rotor: 169
   b) Stator: 169
4- Blade incidence and deviation
   a) Rotor: i = 6.5902, σ= 2.9226
   b) Stator: i = 6.6578, σ= 3.3990
5- Annulus shape: A1 = 0.1865, A2 = 0.1684, A3 = 0.1554
6- Efficiency = 0.8724
7- Solidity
   a) Rotor = 2

b) Stator = 2

8- Hub to tip ratio

a) Rotor = 0.8000

b) Stator = 0.8000

9- Height to chord ratio

a) Rotor = 3

b) Stator = 3

Run 1

Options:

20 populations – 200 generations

Results:

- Pratio = 1.3585
- eff = 0.9548
- Mrel1 = 0.7326
- DFr = 0.4464
- Utr = 300
- Cx2 = 127.8708
- R = 0.6367
- phi2 = 0.4625
- criteria = -1.1971

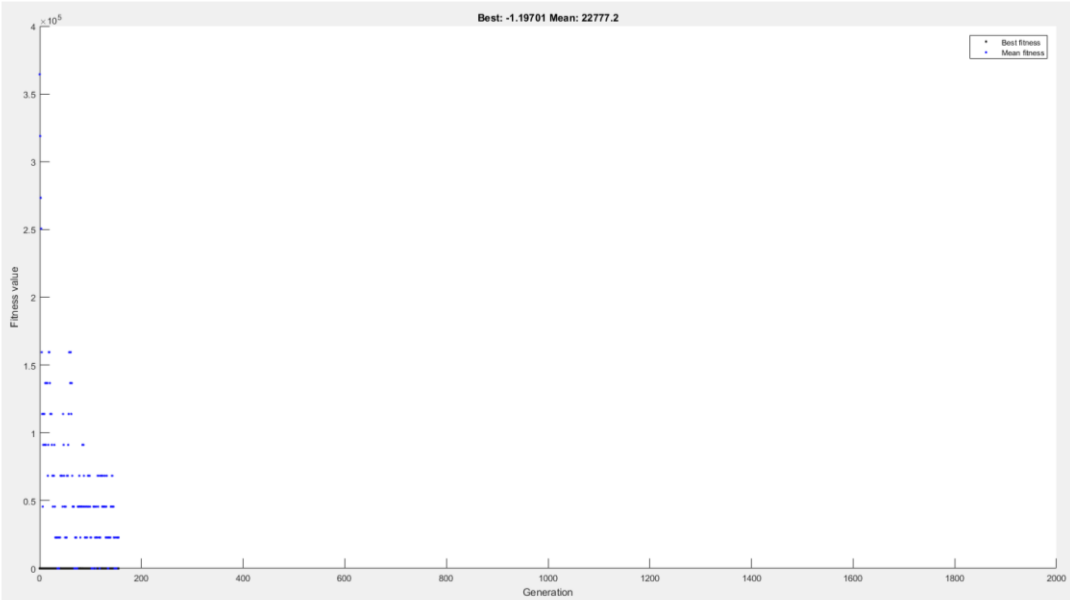
Algorithm chosen inputs:

- alpha1 = 22.9837
- phi1 = 0.4856
- psi = 0.4000
- Cx1 = 127.8708
- U22U1 = 1.0500
- zetar = 0.7993
- soldr = 0.6000
- h2cr = 1.4192
- zetas = 0.7972
- solds = 0.5000
- h2cs = 2.9712

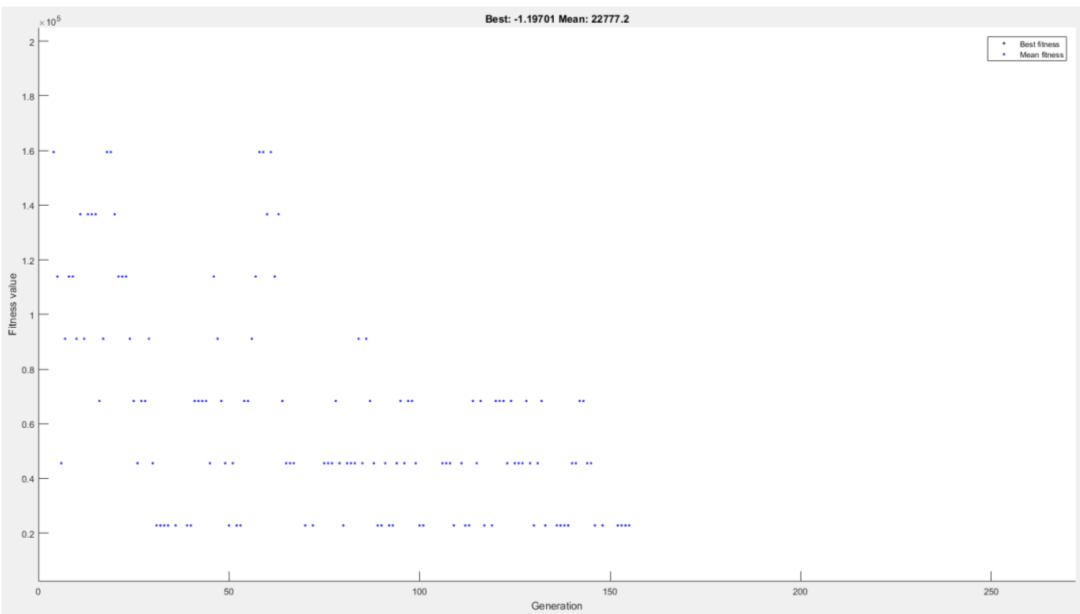
Required results:

1- Velocity triangles data:
   a) alpha1 = 22.9837
   b) alpha2 = 49.9214
   c) beta1 = 58.5518
   d) beta2 = 44.2405
   e) C1 = 138.8970
   f) C2 = 198.6073
   g) W1 = 245.0913
   h) W2 = 178.4863
2- Blade geometry at mean line
   a) Rotor: A_m = 0.1946, r_m = 0.3726
   b) Stator: A_m = 0.1738, r_m = 0.3501
3- Number of blades
   a) Rotor: 23
   b) Stator: 41
4- Blade incidence and deviation
   a) Rotor: i = -10.3459, σ= 17.2168
   b) Stator: i = -18.2006, σ= 28.8925
5- Annulus shape: A1 = 0.2085, A2 = 0.1807, A3 = 0.1669
6- Efficiency = 0.9548
7- Solidity
   a) Rotor = 0.6
   b) Stator = 0.5
8- Hub to tip ratio
   a) Rotor = 0.7993
   b) Stator = 0.7972
9- Height to chord ratio
   a) Rotor = 1.4192
   b) Stator = 2.9712

Graph:



Zoomed in:



Exit flag:

Optimization terminated: average change in the fitness value less than options.FunctionTolerance.

Run 2

Options:

300 populations – 800 generations

Results:

- Pratio = 1.3643
- eff = 0.9578
- Mrel1 = 0.7007
- DFr = 0.4544
- Utr = 300
- Cx2 = 147.1640
- R = 0.5272
- phi2 = 0.5348
- criteria = -1.2017

Algorithm chosen inputs:

- alpha1 = 29.9998
- phi1 = 0.5557
- psi = 0.3999
- Cx1 = 147.1640
- U22U1 = 1.0391
- zetar = 0.8000
- soldr = 0.6000
- h2cr = 2.7467
- zetas = 0.6744
- solds = 0.4000
- h2cs = 2.1458

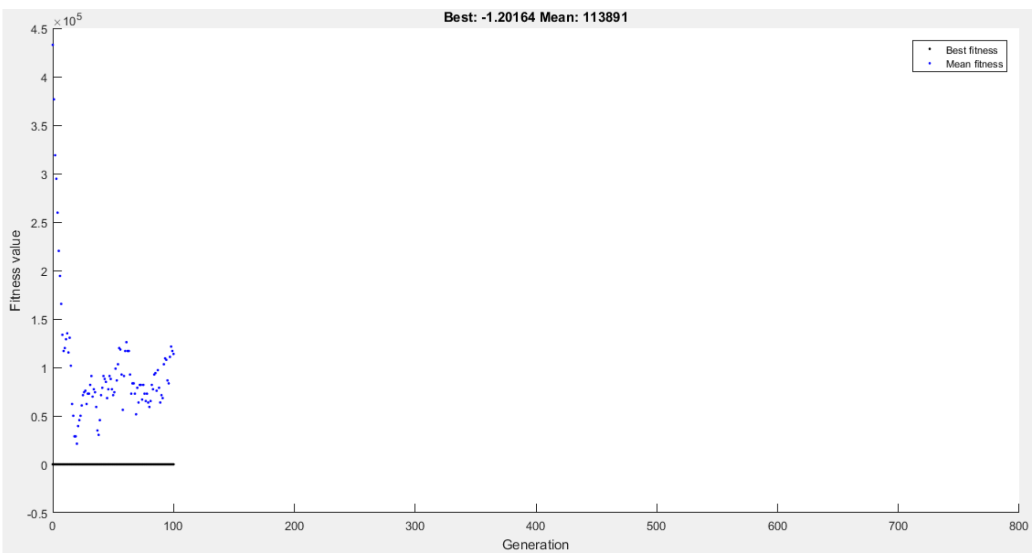
Required results:

1- Velocity triangles data:
   a) alpha1 = 29.9998
   b) alpha2 = 51.2993
   c) beta1 = 50.7098
   d) beta2 = 31.8700
   e) C1 = 169.9300
   f) C2 = 235.3677
   g) W1 = 232.3952
   h) W2 = 173.2874

2- Blade geometry at mean line
   a) Rotor: A_m = 0.1783, r_m = 0.3573
   b) Stator: A_m = 0.1589, r_m = 0.2550

3- Number of blades
   a) Rotor: 46
   b) Stator: 13
4- Blade incidence and deviation
   c) Rotor: i = -9.1902, σ= 17.1226
   d) Stator: i = -24.6342, σ= 38.8978
5- Annulus shape: A1 = 0.1891, A2 = 0.1674, A3 = 0.1503
6- Efficiency = 0.9578
7- Solidity
   a) Rotor = 0.6
   b) Stator = 0.4
8- Hub to tip ratio
   a) Rotor = 0.8000
   b) Stator = 0.6744
9- Height to chord ratio
   a) Rotor = 2.7467
   b) Stator = 2.1458

Graph:



Exit flag:

Optimization terminated: time limit exceeded.

Elapsed time is 12095.296320 seconds.

Supersonic Design

Inputs

We define ranges for the inputs as follows:

$0 \le \alpha_1 \le 15$ $\quad$ $0.4 \le \sigma_s \le 2$ $\quad$ $0.4 \le \sigma_r \le 2$

$0.25 \le \phi_1 \le 0.5$ $\quad$ $0.4 \le \zeta_r \le 0.9$ $\quad$ $100 \le C_{x_1} \le 150$ $\quad$ $3 \le \frac{h}{c_r} \le 5$

$0.1 \le \psi \le 0.4$ $\quad$ $0.6 \le \zeta_s \le 0.8$ $\quad$ $1 \le \frac{U_2}{U_1} \le 1.05$ $\quad$ $3 \le \frac{h}{c_s} \le 5$

```
xbound =[0 15; %alpha1
    0.25 0.5; %phi1
    0.1 0.4; %psi
    100 150; %Cx1
    1 1.05; %U22U1
    0.4 0.9; %zetar
    0.4 2; %soldr
    3 5; %h2cr
    0.6 0.8; %zetas
    0.4 2; %solds
    3 5]; %h2cs
```

Constraints

$1 < M_{rel1} \le 1.25$ $\quad$ $U_{t_r} \le 400$ $\quad$ $100 \le C_{x_2} \le 150$

$0.3 \le DF_r \le 0.47$ $\quad$ $0.5 \le R \le 1$ $\quad$ $0.25 \le \phi_2 \le 0.5$

```
if((Mrel1>1)&&(Mrel1<=1.25)&&(DFr>=0.3)&&(DFr<=0.47)&&(utr<=400)&&...
        (Cx2>=100)&&(Cx2<=150)&&(R<=1)&&(R>=0.5)&&(phi2>=0.25)&&(phi2<=0.5))
 % Satisfied
    criteria=min(-(0.6.*(Pratio)+0.4.*(eff)));
else
    criteria=455567;
end
```

Equations

The equations of the supersonic design are almost the same as that of the subsonic design, so we will just mention the differences here.

First, for supersonic the blade is DCA which has a $K_{i_{sh}}$ is 0.7 and K_{i_t} for thickness ratio of 0.1 is 0.7.

```
kish=0.7;
kit=0.7;
i0=i010*kish*kit;
```

Second, after getting the total relative pressure we have to multiply it by a factor "pi_shock" to account for the normal shock losses.

```
Ptrel2_sub=Ptrel2d-wr2d_sub.*(Ptrel1-P1);
pi_shock=NSW_totalP(Mrel1);
Ptrel2_super=pi_shock.*Ptrel2_sub;
```

This function "NSW_totalP.m" takes the mach number as an unput and calculates the ratio between total pressure after the shock and total pressure before the shock.

```
function [P_02_div_P_01_NSW]=NSW_totalP (M1)
if (M1<=1)
    P_02_div_P_01_NSW=1;
else
    gamma=1.4;
    M2=sqrt((1+(((gamma-1)./2).*M1.^2))./((gamma.*M1.^2)-((gamma-1)./2)));
    a=(1+((gamma-1)./2).*M1.^2).^(gamma./(gamma-1));
    b=(1+((gamma-1)./2).*M2.^2).^(gamma./(gamma-1));
    P_02_div_P_01_NSW=(b./a).*(1+((2.*gamma)./(gamma+1)).*((M1.^2)-1));
end
end
```

Optimization

Number of variables = 11

```
%xbound=[alpha1, phi1, psi, Cx1, U22U1, zetar, soldr, h2cr, zetas, solds, h2cs]
```

Initiating the initial value vector

```
X0 = xbound(:,1) + (xbound(:,2) - xbound(:,1)).*rand(Nv,1);
```

Setting the options of the optimizer to converge at e-5, or after time limit of 12000 seconds. Of a total of 800 generations and a population size of 300.

```
Lb = xbound(:,1);
Ub = xbound(:,2);
options = gaoptimset('TimeLimit',12000,'PopulationSize',300,'Generations',800, 'TolCon', 1e-5,...
'TolFun', 1e-5,'PlotFcns',@gaplotbestf);
options.Display='iter';
[x, fval, exitflag,output] = ga(@optimize_supersonic,11, [], [], [], [], Lb, Ub, [],options);
[Pratio, eff, Mrel1, DFr, utr, Cx2, R, phi2, criteria]=supersonic(x)
```

Solving at limits

PHI1 INPUT, MREL1 CONSTRAINT

Lower Limit

10- Velocity triangles data:
  i) alpha1 = 15
  j) alpha2 = 51.4368
  k) beta1 = 66.9173
  l) beta2 = 55.9831
  m) C1 = 147.0341
  n) C2 = 227.8297
  o) W1 = 362.2512
  p) W2 = 253.8694

11- Blade geometry at mean line
  c) Rotor: A_m = 0.1641, r_m = 0.4981
  d) Stator: A_m = 0.1312, r_m = 0.2153

12- Number of blades
  c) Rotor: 156
  d) Stator: 25

13- Blade incidence and deviation
  e) Rotor: i = -19.5751, σ= 24.1156
  f) Stator: i = -38.9075, σ= 52.5956

14- Annulus shape: A1 = 0.1897, A2 = 0.1385, A3 = 0.1239

15- Efficiency = 0.9554

16- Solidity
  c) Rotor = 0.6000
  d) Stator = 0.4000

17- Hub to tip ratio
  c) Rotor = 0.9000

d) Stator = 0.6323

18- Height to chord ratio

c) Rotor = 4.3563

d) Stator = 4.6054

Upper Limit

1- Velocity triangles data:
   a) alpha1 = 15
   b) alpha2 = 51.4368
   c) beta1 = 66.9173
   d) beta2 = 55.9831
   e) C1 = 147.0341
   f) C2 = 227.8297
   g) W1 = 362.2512
   h) W2 = 253.8694
2- Blade geometry at mean line
   a) Rotor: A_m = 0.1641, r_m = 0.4981
   b) Stator: A_m = 0.1312, r_m = 0.2153
3- Number of blades
   a) Rotor: 156
   b) Stator: 25
4- Blade incidence and deviation
   a) Rotor: i = -19.5751, σ= 24.1156
   b) Stator: i = -38.9075, σ= 52.5956
5- Annulus shape: A1 = 0.1897, A2 = 0.1385, A3 = 0.1239
6- Efficiency = 0.9554
7- Solidity
   a) Rotor = 0.6000
   b) Stator = 0.4000
8- Hub to tip ratio
   a) Rotor = 0.9000
   b) Stator = 0.6323
9- Height to chord ratio
   a) Rotor = 4.3563
   b) Stator = 4.6054

MREL1 INPUT, PHI1 CONSTRAINT

In this run, as a check we let Mrel1 be our input instead of phi1 and had constraints over phi.

Lower Limit

1- Velocity triangles data:
   a) alpha1 = 0
   b) alpha2 = 17.8565
   c) beta1 = 72.7551
   d) beta2 = 70.9706
   e) C1 = 100
   f) C2 = 105.0611
   g) W1 = 337.3169
   h) W2 = 306.6985
2- Blade geometry at mean line
   a) Rotor: A_m = 0.2471, r_m = 0.2142
   b) Stator: A_m = 0.2380, r_m = 0.2752
3- Number of blades
   a) Rotor: 8
   b) Stator: 15
4- Blade incidence and deviation
   a) Rotor: i = 6.9844, σ= -7.4240
   b) Stator: i = -3.8568, σ= 12.4090
5- Annulus shape: A1 = 0.2558, A2 = 0.2384, A3 = 0.2375
6- Efficiency = 0.8726
7- Solidity
   a) Rotor = 0.4000
   b) Stator = 0.4000
8- Hub to tip ratio
   a) Rotor = 0.4000
   b) Stator = 0.6000
9- Height to chord ratio
   a) Rotor = 3
   b) Stator = 3

Upper Limit

1- Velocity triangles data:
   a) alpha1 = 15
   b) alpha2 = 53.3438
   c) beta1 = 68.8847
   d) beta2 = 58.8826
   e) C1 = 155.2914
   f) C2 = 251.2509

g) W_1 = 416.3825
h) W_2 = 290.2515

2- Blade geometry at mean line
   a) Rotor: A_m = 0.1541, r_m = 0.4826
   b) Stator: A_m = 0.1192, r_m = 0.2922
3- Number of blades
   a) Rotor: 596
   b) Stator: 282
4- Blade incidence and deviation
   a) Rotor: i = 3.3297, σ= 2.1759
   b) Stator: i = 0.1921, σ= 8.5941
5- Annulus shape: A_1 = 0.1816, A_2 = 0.1265, A_3 = 0.1119
6- Efficiency = 0.8563
7- Solidity
   a) Rotor = 0.6000
   b) Stator = 0.4000
8- Hub to tip ratio
   a) Rotor = 2
   b) Stator = 2
9- Height to chord ratio
   a) Rotor = 5
   b) Stator = 5

Run 1

Options:

300 populations – 800 generations

Results:

- Pratio = 1.7947
- eff = 0.9554
- Mrel1 = 1.0852
- DFr = 0.4698
- Utr = 399.9916
- Cx2 = 142.0240
- R = 0.7252
- phi2 = 0.3654
- criteria = -1.4590

Algorithm chosen inputs:

- alpha1 = 15
- phi1 = 0.3825
- psi = 0.3996
- Cx1 = 142.0240
- U22U1 = 1.0465
- zetar = 0.9
- soldr = 0.6
- h2cr = 4.3563
- zetas = 0.6323
- solds = 0.4
- h2cs = 4.6054

Required results:

1- Velocity triangles data:
   a) alpha1 = 15
   b) alpha2 = 51.4368
   c) beta1 = 66.9173
   d) beta2 = 55.9831
   e) C1 = 147.0341
   f) C2 = 227.8297
   g) W1 = 362.2512
   h) W2 = 253.8694
2- Blade geometry at mean line
   a) Rotor: A_m = 0.1641, r_m = 0.4981
   b) Stator: A_m = 0.1312, r_m = 0.2153
3- Number of blades
   a) Rotor: 156
   b) Stator: 25
4- Blade incidence and deviation
   a) Rotor: i = -19.5751, σ= 24.1156
   b) Stator: i = -38.9075, σ= 52.5956
5- Annulus shape: A1 = 0.1897, A2 = 0.1385, A3 = 0.1239
6- Efficiency = 0.9554
7- Solidity
   a) Rotor = 0.6000
   b) Stator = 0.4000
8- Hub to tip ratio
   a) Rotor = 0.9000

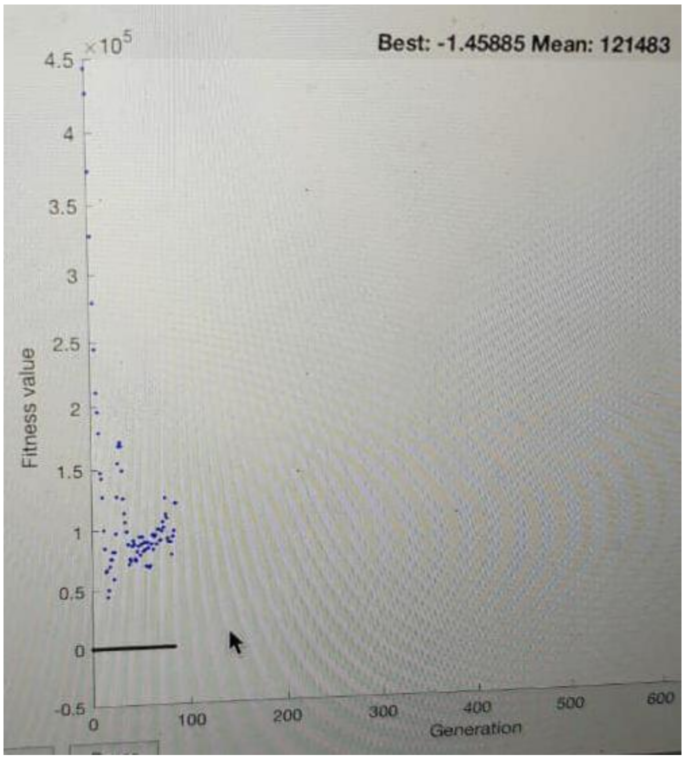
b) Stator = 0.6323

9- Height to chord ratio

a) Rotor = 4.3563

b) Stator = 4.6054

Graph:



Exit flag:

Optimization terminated: average change in the fitness value less than options.FunctionTolerance.

Elapsed time is 5826.067384 seconds.

Run 2

In this run, as a check we let Mrel1 be our input instead of phi1 and had constraints over phi.

Options:

300 populations – 800 generations

Results:

- Pratio = 1.7980
- eff = 0.9578
- phi1 = 0.3116
- DFr = 0.4663
- Utr = 400
- Cx2 = 115.6980
- R = 0.7957
- phi2 = 0.2977
- criteria = -1.4620

Algorithm chosen inputs:

- alpha1 = 4.3796
- Mrel1 = 1.1314
- psi = 0.4000
- Cx1 = 115.6980
- U22U1 = 1.0467
- zetar = 0.9
- soldr = 0.6
- h2cr = 3.3375
- zetas = 0.7824
- solds = 0.5
- h2cs = 3.7454

Required results:

1- Velocity triangles data:
   a) alpha1 = 4.3796
   b) alpha2 =52.4239
   c) beta1 = 72.2967
   d) beta2 = 64.1022
   e) C1 = 116.0368
   f) C2 = 189.7267
   g) W1 = 380.4765
   h) W2 = 264.8960

2- Blade geometry at mean line
   a) Rotor: A_m = 0.1921, r_m = 0.5389
   b) Stator: A_m = 0.1534, r_m = 0.3163

3- Number of blades
   a) Rotor: 119

b) Stator: 48

4- Blade incidence and deviation

a) Rotor: i = -67.0020, σ= 64.6884

b) Stator: i = -35.1260, σ= 42.3994

5- Annulus shape: A1 = 0.2245, A2 = 0.1596, A3 = 0.1473

6- Efficiency = 0.9578

7- Solidity

a) Rotor = 0.6000

b) Stator = 0.5000

8- Hub to tip ratio

a) Rotor = 0.9000

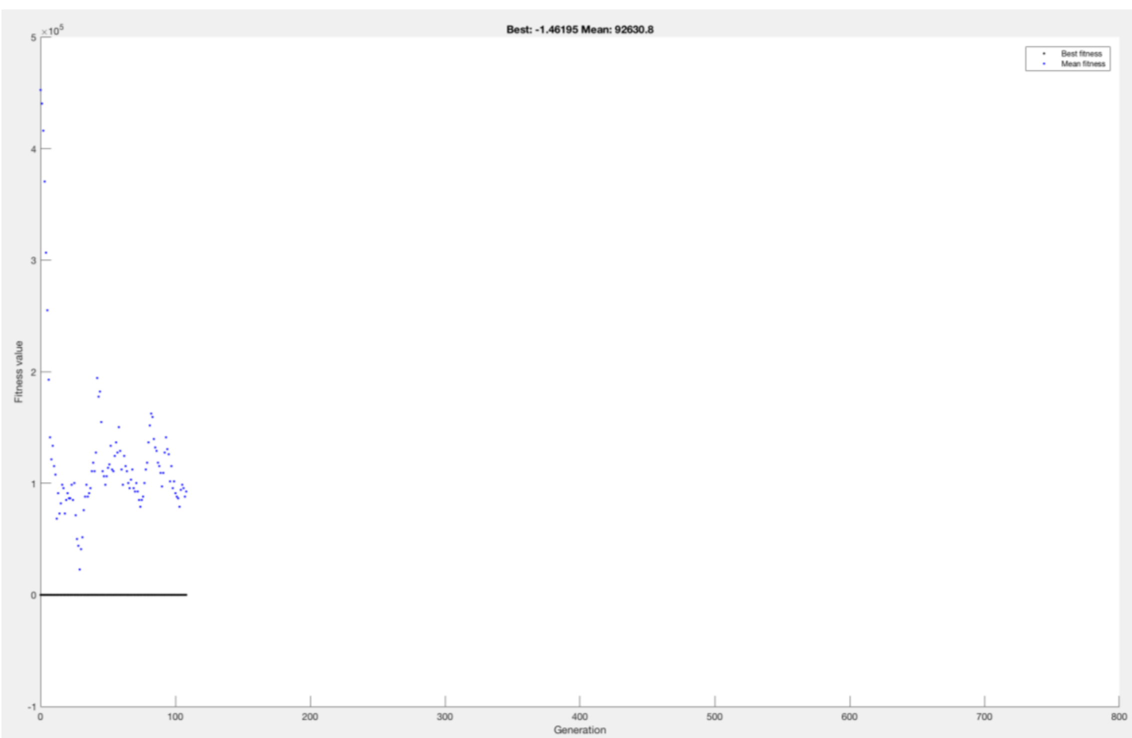
b) Stator = 0.7824

9- Height to chord ratio

a) Rotor = 3.3375

b) Stator = 3.7454

Graph:



Exit flag:

Optimization terminated: average change in the fitness value less than options.FunctionTolerance.

Elapsed time is 8192.706709 seconds.