CS 336/436 – Algorithms for Sensor-based Robotics

Lecture II – Bug Path-Planning Algorithms (Part 1)

Erion Plaku

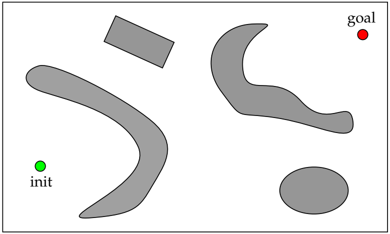
Department of Computer Science
Laboratory for Computational Sensing and Robotics
Johns Hopkins University

January 28, 2010

Outline

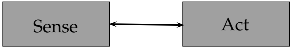
1. General Properties of Bug Path-Planning Algorithms
2. Bug Algorithms with Tactile (Contact) Sensors
   1. `Bug0`
   2. `Bug1`
   3. `Bug2`
3. Bug Algorithms with Range Sensors (next lecture)
   - `TangentBug`
   - `VisBug`
   - `DistBug`
4. Bug Algorithms with Intensity Sensors (next lecture)
   - `I-Bug`

Problem: Compute a collision-free path from an initial to a goal position



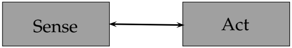
Bug Path-Planning Algorithms

Reactive Paradigm



- No global model of the world, i.e., obstacles are unknown
- Only local information acquired through sensing
- Inspired by insects

Reactive Paradigm

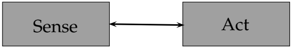


- No global model of the world, i.e., obstacles are unknown
- Only local information acquired through sensing
- Inspired by insects

Properties

- Complete algorithms, i.e., find solution if it exists, report no when there is no solution
- Theoretical lower and upper bounds on path length; optimal paths in certain cases

Reactive Paradigm



- No global model of the world, i.e., obstacles are unknown
- Only local information acquired through sensing
- Inspired by insects

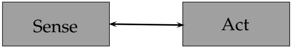
Properties

- Complete algorithms, i.e., find solution if it exists, report no when there is no solution
- Theoretical lower and upper bounds on path length; optimal paths in certain cases

Environment

- Two-dimensional scene filled with unknown obstacles
- Each obstacle is a simple closed curve of finite length and non-zero thickness
- A straight line crosses an obstacle finitely many times
- Obstacles do not touch each other
- Locally finite number of obstacles, i.e., any disc of finite radius intersects a finite set of obstacles
- Initial and goal positions are known

Reactive Paradigm



- No global model of the world, i.e., obstacles are unknown
- Only local information acquired through sensing
- Inspired by insects

Properties

- Complete algorithms, i.e., find solution if it exists, report no when there is no solution
- Theoretical lower and upper bounds on path length; optimal paths in certain cases

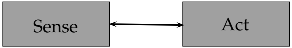
Environment

- Two-dimensional scene filled with unknown obstacles
- Each obstacle is a simple closed curve of finite length and non-zero thickness
- A straight line crosses an obstacle finitely many times
- Obstacles do not touch each other
- Locally finite number of obstacles, i.e., any disc of finite radius intersects a finite set of obstacles
- Initial and goal positions are known

Point Robot, Simple Motions

- Move straight toward `goal`
- Move along obstacle boundary
- Stop

Reactive Paradigm



- No global model of the world, i.e., obstacles are unknown
- Only local information acquired through sensing
- Inspired by insects

Properties

- Complete algorithms, i.e., find solution if it exists, report no when there is no solution
- Theoretical lower and upper bounds on path length; optimal paths in certain cases

Environment

- Two-dimensional scene filled with unknown obstacles
- Each obstacle is a simple closed curve of finite length and non-zero thickness
- A straight line crosses an obstacle finitely many times
- Obstacles do not touch each other
- Locally finite number of obstacles, i.e., any disc of finite radius intersects a finite set of obstacles
- Initial and goal positions are known

Point Robot, Simple Motions

- Move straight toward goal
- Move along obstacle boundary
- Stop

Simple Sensing

- `Bug1`, `Bug2` assume essentially tactile (contact) sensing
- `TangentBug`, `VisBug`, `DistBug` deal with finite distance sensing
- `I-Bug` uses only signal strength emanating from goal

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

`Bug0`, `Bug1`, `Bug2` Algorithms – General Idea

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
  - at some point, leave the obstacle and head again toward `goal`

Bug with Tactile (Contact) Sensor

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

`Bug0`, `Bug1`, `Bug2` Algorithms – General Idea

repeat until `goal` is reached

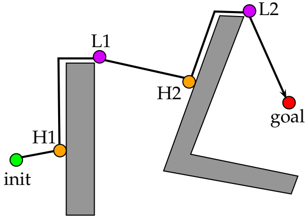
- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
  - at some point, leave the obstacle and head again toward `goal`

Path consists of a sequence of hit (H_i) and leave (L_i) points
Algorithms differ on how leave points are computed

Bug0 Algorithm

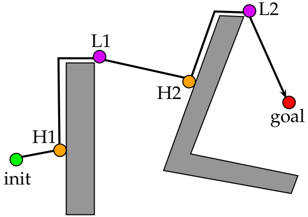
repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward `goal` again



repeat until `goal` is reached

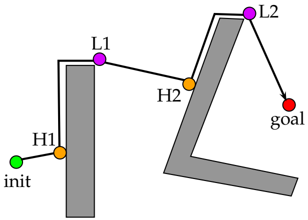
- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward `goal` again



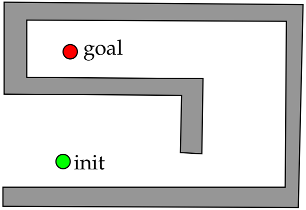
Is Bug0 a complete algorithm?

repeat until goal is reached

- head toward goal
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward goal again

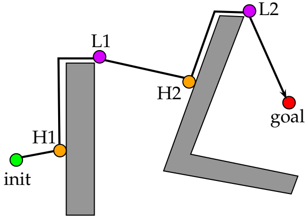


Is Bug0 a complete algorithm?

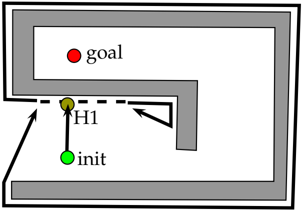


repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward `goal` again



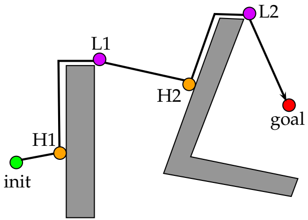
Is Bug0 a complete algorithm?



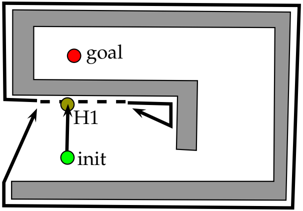
Bug0 fails to find a solution even though a solution exists

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward `goal` again



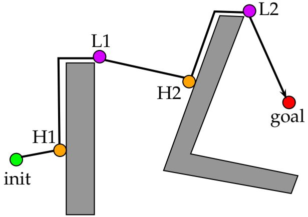
Is Bug0 a complete algorithm?



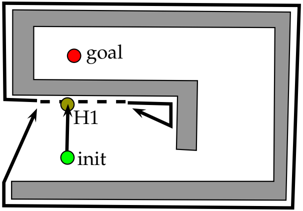
Bug0 fails to find a solution even though a solution exists
Bug0 has no memory

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward `goal` again



Is Bug0 a complete algorithm?



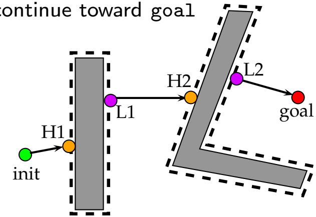
Bug0 fails to find a solution even though a solution exists
Bug0 has no memory

can we obtain a complete algorithm if Bug has some memory?

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

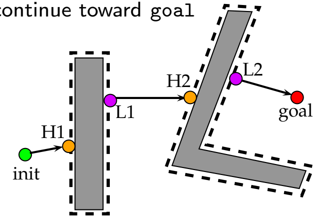
- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



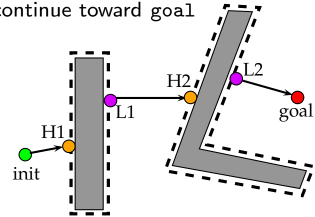
Bug1 Pseudocode

1: $L_0 \leftarrow$ `init`; $i \leftarrow 1$
2: **loop**

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



Bug1 Pseudocode

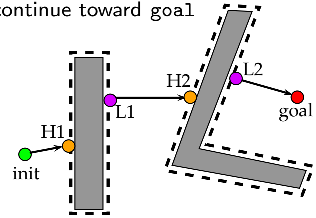
1: $L_0 \leftarrow$ `init`; $i \leftarrow 1$
2: **loop**
3: **repeat** move on a straight line from L_{i-1} to `goal`
4: **until** `goal` is reached or obstacle is encountered at H_i

Bug1 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



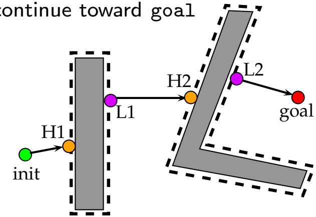
Bug1 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



Bug1 Pseudocode

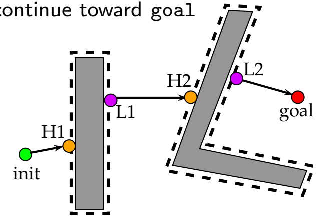
1: $L_0 \leftarrow$ `init`; $i \leftarrow 1$
2: **loop**
3: **repeat** move on a straight line from L_{i-1} to `goal`
4: **until** `goal` is reached or obstacle is encountered at H_i
5: **if** `goal` is reached **then** exit with success
6: **repeat** follow boundary recording point L_i with shortest distance to `goal`
7: **until** `goal` is reached or H_i is re-encountered

Bug1 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



Bug1 Pseudocode

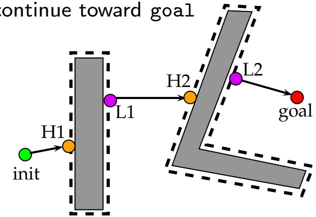
$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary recording point L_i with shortest distance to `goal`
  until `goal` is reached or H_i is re-encountered
  if `goal` is reached **then** exit with success

Bug1 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



Bug1 Pseudocode

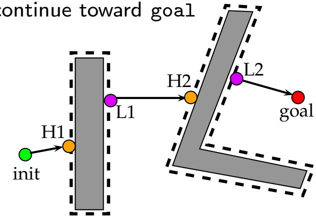
$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
    repeat move on a straight line from L_{i-1} to `goal`
    until `goal` is reached or obstacle is encountered at H_i
    if `goal` is reached **then** exit with success
    repeat follow boundary recording point L_i with shortest distance to `goal`
    until `goal` is reached or H_i is re-encountered
    if `goal` is reached **then** exit with success
    follow boundary from H_i to L_i along shortest route

Bug1 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



Bug1 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary recording point L_i with shortest distance to `goal`
  until `goal` is reached or H_i is re-encountered
  if `goal` is reached **then** exit with success
  follow boundary from H_i to L_i along shortest route
  if move on straight line from L_i toward `goal` moves into obstacle **then** exit with failure
  else $i \leftarrow i + 1$

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch:

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to $\texttt{goal}$

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward $\texttt{goal}$, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to $\texttt{goal}$
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\mathtt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \mathtt{goal}) \geq d(L_i, \mathtt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \mathtt{goal}) > d(L_i, \mathtt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \mathtt{goal}) > d(H_{i+1}, \mathtt{goal})$ since different obstacles do not touch

Therefore,

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$

Thus,

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward $\mathtt{goal}$, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\mathtt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \mathtt{goal}) \geq d(L_i, \mathtt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \mathtt{goal}) > d(L_i, \mathtt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \mathtt{goal}) > d(H_{i+1}, \mathtt{goal})$ since different obstacles do not touch

Therefore, $d(\mathtt{init}, \mathtt{goal}) \geq d(H_1, \mathtt{goal}) > d(L_1, \mathtt{goal}) > d(H_2, \mathtt{goal}) > d(L_2, \mathtt{goal}) > \ldots$

Thus, since $d(L_i, \mathtt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$
Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch:

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward $\texttt{goal}$, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$

Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch: Straight-line segments from L_i to H_{i+1} $(i = 0, 1, \ldots)$ are within the same circle of radius $d(\texttt{init}, \texttt{goal})$ centered at goal since

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$

Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch: Straight-line segments from L_i to H_{i+1} $(i = 0, 1, \ldots)$ are within the same circle of radius $d(\texttt{init}, \texttt{goal})$ centered at goal since

- each hit point is closer than the last leave point

Bug1 Properties

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$

Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch: Straight-line segments from L_i to H_{i+1} $(i = 0, 1, \ldots)$ are within the same circle of radius $d(\texttt{init}, \texttt{goal})$ centered at goal since

- each hit point is closer than the last leave point
- assumption that any finite disc can intersect only a finite number of obstacles

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$

Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch: Straight-line segments from L_i to H_{i+1} $(i = 0, 1, \ldots)$ are within the same circle of radius $d(\texttt{init}, \texttt{goal})$ centered at goal since

- each hit point is closer than the last leave point
- assumption that any finite disc can intersect only a finite number of obstacles

Corollary: `Bug1` *algorithm always terminates in finite time*

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

- $d(H_i, \texttt{goal}) \geq d(L_i, \texttt{goal})$ since L_i closest point on obstacle boundary to goal
- $d(H_i, \texttt{goal}) > d(L_i, \texttt{goal})$ since $H_i \neq L_i$. Why?
  - if straight line is tangent to obstacle, then no circumnavigation
  - otherwise, straight line crosses obstacle at two distinct points (since obstacle has finite thickness)
- $d(L_i, \texttt{goal}) > d(H_{i+1}, \texttt{goal})$ since different obstacles do not touch

Therefore, $d(\texttt{init}, \texttt{goal}) \geq d(H_1, \texttt{goal}) > d(L_1, \texttt{goal}) > d(H_2, \texttt{goal}) > d(L_2, \texttt{goal}) > \ldots$

Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch: Straight-line segments from L_i to H_{i+1} $(i = 0, 1, \ldots)$ are within the same circle of radius $d(\texttt{init}, \texttt{goal})$ centered at goal since

- each hit point is closer than the last leave point
- assumption that any finite disc can intersect only a finite number of obstacles

Corollary: `Bug1` algorithm always terminates in finite time

Proof Sketch: Follows immediately from Lemma 1 and Lemma 2

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch:

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1`

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But. . .

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But...

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But. . .

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time
2. `Bug1` never goes through an obstacle, so it only computes valid paths. So, if `Bug1` reports finding a path to `goal`, then there is a path to `goal`

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But...

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time
2. `Bug1` never goes through an obstacle, so it only computes valid paths. So, if `Bug1` reports finding a path to `goal`, then there is a path to `goal`
3. Then, move from last leave point toward `goal` crosses into obstacle

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But. . .

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time
2. `Bug1` never goes through an obstacle, so it only computes valid paths. So, if `Bug1` reports finding a path to `goal`, then there is a path to `goal`
3. Then, move from last leave point toward `goal` crosses into obstacle
   - But, line must cross obstacle even number of times (Jordan curve theorem)

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But...

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time
2. `Bug1` never goes through an obstacle, so it only computes valid paths. So, if `Bug1` reports finding a path to `goal`, then there is a path to `goal`
3. Then, move from last leave point toward `goal` crosses into obstacle
    - But, line must cross obstacle even number of times (Jordan curve theorem)
    - Then, there is another intersection point on boundary closer to `goal`

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But...

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time
2. `Bug1` never goes through an obstacle, so it only computes valid paths. So, if `Bug1` reports finding a path to `goal`, then there is a path to `goal`
3. Then, move from last leave point toward `goal` crosses into obstacle
    - But, line must cross obstacle even number of times (Jordan curve theorem)
    - Then, there is another intersection point on boundary closer to `goal`
    - Since, we assumed there is a path to `goal`, then `goal` cannot be encircled by obstacle

Bug1 Completeness Analysis

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
- *terminates with failure when there is no path to `goal`*

Proof Sketch: Assume to the contrary that `Bug1` is incomplete. Then

1. `Bug1` does not terminate in finite time, or
2. There is no path to `goal`, but `Bug1` incorrectly reports finding a path, or
3. There is at least a path to `goal`, but `Bug1` incorrectly reports finding no path

But...

1. Lemma 1 and 2 imply that `Bug1` always terminates in finite time
2. `Bug1` never goes through an obstacle, so it only computes valid paths. So, if `Bug1` reports finding a path to `goal`, then there is a path to `goal`
3. Then, move from last leave point toward `goal` crosses into obstacle
    - But, line must cross obstacle even number of times (Jordan curve theorem)
    - Then, there is another intersection point on boundary closer to `goal`
    - Since, we assumed there is a path to `goal`, then `goal` cannot be encircled by obstacle
    - Thus, bug must have encountered this other intersection point (which is supposedly closer to the goal) when circumnavigating obstacle boundary, which contradicts definition of leave point

Lower Bound: What is the shortest distance that Bug1 might travel?

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq$

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why?

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why? (leave point is closest to obstacle)

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why? (leave point is closest to obstacle)
- when going from H_i to L_i, Bug1 first circumnavigates the i-th obstacle and then, after coming back to H_i, selects the shorter route to go to L_i. Thus, $1.5 p_i$, where p_i is the perimeter of the i-th obstacle

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why? (leave point is closest to obstacle)
- when going from H_i to L_i, Bug1 first circumnavigates the i-th obstacle and then, after coming back to H_i, selects the shorter route to go to L_i. Thus, $1.5p_i$, where p_i is the perimeter of the i-th obstacle

Therefore, upper bound

$$d(\texttt{init}, \texttt{goal}) + 1.5 \sum_{i=1}^{n} p_i$$

What is n?

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why? (leave point is closest to obstacle)
- when going from H_i to L_i, Bug1 first circumnavigates the i-th obstacle and then, after coming back to H_i, selects the shorter route to go to L_i. Thus, $1.5p_i$, where p_i is the perimeter of the i-th obstacle

Therefore, upper bound

$$d(\texttt{init}, \texttt{goal}) + 1.5 \sum_{i=1}^{n} p_i$$

What is n?

- number of obstacles intersecting the disc of radius $d(\texttt{init}, \texttt{goal})$ centered at goal

Bug1 Lower and Upper Bounds on Path Length

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why? (leave point is closest to obstacle)
- when going from H_i to L_i, Bug1 first circumnavigates the i-th obstacle and then, after coming back to H_i, selects the shorter route to go to L_i. Thus, $1.5 p_i$, where p_i is the perimeter of the i-th obstacle

Therefore, upper bound

$$d(\texttt{init}, \texttt{goal}) + 1.5 \sum_{i=1}^{n} p_i$$

What is n?

- number of obstacles intersecting the disc of radius $d(\texttt{init}, \texttt{goal})$ centered at goal

Remind me again why it is not necessary to consider obstacles outside this disk?

Lower Bound: What is the shortest distance that Bug1 might travel?

- $d(\texttt{init}, \texttt{goal})$ (straight-line to goal, no obstacles encountered)

Upper Bound: What is an upper bound on the longest distance that Bug1 might travel?

- any path can be looked as consisting of straight-line segments (from L_{i_1} to H_i) and walking around the obstacles
- sum of straight-line segments $\leq d(\texttt{init}, \texttt{goal})$. Why? (leave point is closest to obstacle)
- when going from H_i to L_i, Bug1 first circumnavigates the i-th obstacle and then, after coming back to H_i, selects the shorter route to go to L_i. Thus, $1.5 p_i$, where p_i is the perimeter of the i-th obstacle

Therefore, upper bound

$$d(\texttt{init}, \texttt{goal}) + 1.5 \sum_{i=1}^{n} p_i$$

What is n?

- number of obstacles intersecting the disc of radius $d(\texttt{init}, \texttt{goal})$ centered at goal

Remind me again why it is not necessary to consider obstacles outside this disk?

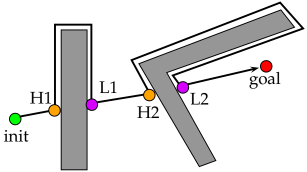
- see proof of Lemma 2, distances from $H_1, L_1, H_2, L_2, \ldots$ to goal become smaller and smaller and are never more than $d(\texttt{init}, \texttt{goal})$. So, bug never encounters obstacles outside this disk

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`



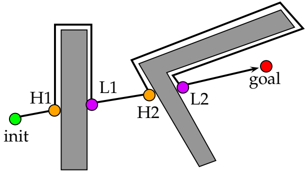
Bug2 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`



Bug2 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success

Bug2 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

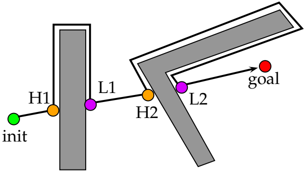
call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`

Bug2 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
    repeat move on a straight line from L_{i-1} to `goal`
    until `goal` is reached or obstacle is encountered at H_i
    if `goal` is reached **then** exit with success
    repeat follow boundary
    until
    (a) `goal` is reached or



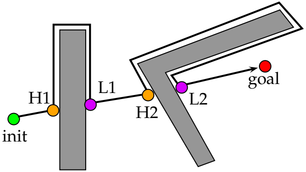
Bug2 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`



Bug2 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary
  until
    (a) `goal` is reached or
    (b) *m*-line is re-encountered at Q such that $Q \neq H_i$, $d(Q, \texttt{goal}) < d(H_i, \texttt{goal})$, and line $(Q, \texttt{goal})$ does not cross the current obstacle at Q or

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

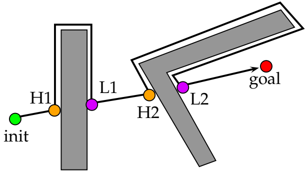
call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`

Bug2 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary
  until
  (a) `goal` is reached or
  (b) *m*-line is re-encountered at Q such that $Q \neq H_i$, $d(Q, \texttt{goal}) < d(H_i, \texttt{goal})$, and line $(Q, \texttt{goal})$ does not cross the current obstacle at Q or
  (c) H_i is re-encountered



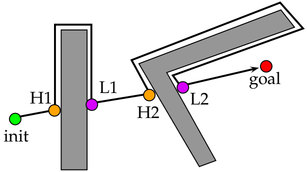
Bug2 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`



Bug2 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
    repeat move on a straight line from L_{i-1} to `goal`
    until `goal` is reached or obstacle is encountered at H_i
    if `goal` is reached **then** exit with success
    repeat follow boundary
    until
        (a) `goal` is reached or
        (b) *m*-line is re-encountered at Q such that $Q \neq H_i$, $d(Q, \texttt{goal}) < d(H_i, \texttt{goal})$, and line $(Q, \texttt{goal})$ does not cross the current obstacle at Q or
        (c) H_i is re-encountered
    if `goal` is reached **then** exit with success

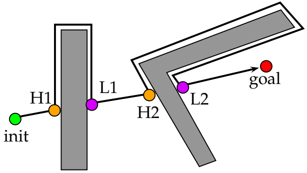
Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`

Bug2 Pseudocode



$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary
  until
    (a) `goal` is reached or
    (b) *m*-line is re-encountered at Q such that $Q \neq H_i$, $d(Q, \texttt{goal}) < d(H_i, \texttt{goal})$, and line $(Q, \texttt{goal})$ does not cross the current obstacle at Q or
    (c) H_i is re-encountered
  if `goal` is reached **then** exit with success
  else if H_i is re-encountered **then** exit with failure

Bug2 Algorithm

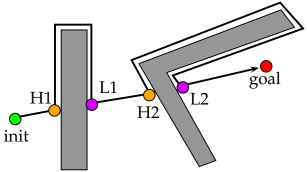
Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`

Bug2 Pseudocode



$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary
  until
   (a) `goal` is reached or
   (b) *m*-line is re-encountered at Q such that $Q \neq H_i$, $d(Q, \texttt{goal}) < d(H_i, \texttt{goal})$, and line $(Q, \texttt{goal})$ does not cross the current obstacle at Q or
   (c) H_i is re-encountered
  if `goal` is reached **then** exit with success
  else if H_i is re-encountered **then** exit with failure
  else $L_i \leftarrow Q$; $i \leftarrow i + 1$

Bug2 Analysis

Lemma 3: $\mathtt{Bug2}$ meets only a finite number of obstacles. Moreover, the only obstacles that can be met are those that intersect the straight-line segment ($init$, $goal$)

Bug2 Analysis

Lemma 3: Bug2 meets only a finite number of obstacles. Moreover, the only obstacles that can be met are those that intersect the straight-line segment $(init, goal)$

Lemma 4: Bug2 will pass any point of the i-th obstacle boundary at most $n_i/2$ *times, where* n_i *is the number of intersections between the straight line* $(init, goal)$ *and the i-th obstacle*

Lemma 3: Bug2 meets only a finite number of obstacles. Moreover, the only obstacles that can be met are those that intersect the straight-line segment $(init, goal)$

Lemma 4: Bug2 will pass any point of the i-th obstacle boundary at most $n_i/2$ times, where n_i is the number of intersections between the straight line $(init, goal)$ and the i-th obstacle

Theorem: Bug2 is a complete path-planning algorithm. Moreover, the length of a path generated by Bug2 never exceeds the limit

$$d(init, goal) + \sum_i \frac{n_i p_i}{2},$$

where p_i's refer to the perimeters of the obstacles intersecting the straight-line segment $(init, goal)$

Bug2 Analysis

Lemma 3: Bug2 meets only a finite number of obstacles. Moreover, the only obstacles that can be met are those that intersect the straight-line segment $(init, goal)$

Lemma 4: Bug2 will pass any point of the i-th obstacle boundary at most $n_i/2$ times, where n_i is the number of intersections between the straight line $(init, goal)$ and the i-th obstacle

Theorem: Bug2 is a complete path-planning algorithm. Moreover, the length of a path generated by Bug2 never exceeds the limit

$$d(init, goal) + \sum_i \frac{n_i p_i}{2},$$

where p_i's refer to the perimeters of the obstacles intersecting the straight-line segment $(init, goal)$

Proof Sketch: Similar to proofs for `Bug1`. Proof of Lemma 4 is slightly different. Maybe an upcoming homework exercise?

Bug1 vs Bug2

Bug1 vs Bug2

Bug1 is an exhaustive search algorithm
– looks at all choices before commiting

Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better

Bug2 often outperforms Bug1, but not always

Bug1 is an exhaustive search algorithm
– looks at all choices before commiting

Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better

Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa

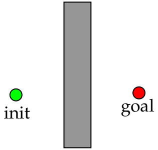
Bug1 is an exhaustive search algorithm
– looks at all choices before commiting

Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better

Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa

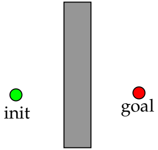


Bug2 beats Bug1

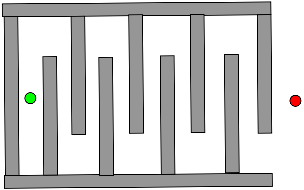
Bug1 is an exhaustive search algorithm
– looks at all choices before commiting
Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better
Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa



Bug2 beats Bug1



Bug1 beats Bug2 in this scene

Bug1 vs Bug2

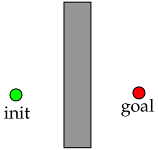
Bug1 is an exhaustive search algorithm
– looks at all choices before commiting

Bug1 has a more stable performance

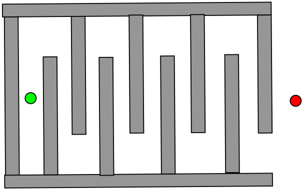
Bug2 is a greedy search algorithm
– takes first choice that looks better

Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa



Bug2 beats Bug1



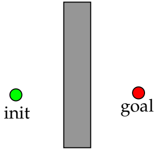
Bug1 beats Bug2 in this scene . . . but only if Bug2 always turns counterclock-wise or always turns clockwise when following boundary

Bug1 vs Bug2

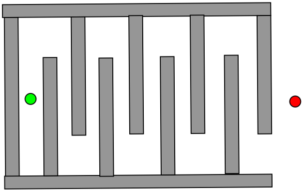
Bug1 is an exhaustive search algorithm
– looks at all choices before commiting
Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better
Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa



Bug2 beats Bug1



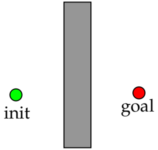
Bug1 beats Bug2 in this scene . . . but only if Bug2 always turns counterclock-wise or always turns clockwise when following boundary

what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

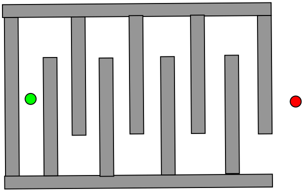
Bug1 is an exhaustive search algorithm
– looks at all choices before commiting
Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better
Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa



Bug2 beats Bug1



Bug1 beats Bug2 in this scene . . . but only if Bug2 always turns counterclock-wise or always turns clockwise when following boundary

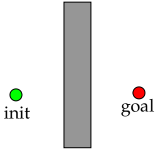
what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

can you draw a scene then where Bug1 beats Bug2 no matter how Bug2 decides to turn each time it has follow an obstacle boundary?

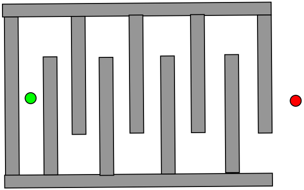
Bug1 is an exhaustive search algorithm
– looks at all choices before commiting
Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better
Bug2 often outperforms Bug1, but not always

Draw scenes in which Bug2 beats Bug1 and vice-versa



Bug2 beats Bug1



Bug1 beats Bug2 in this scene . . . but only if Bug2 always turns counterclock-wise or always turns clockwise when following boundary

what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

can you draw a scene then where Bug1 beats Bug2 no matter how Bug2 decides to turn each time it has follow an obstacle boundary?

next lecture. . . other and better bugs with range or intensity sensors