CS 336/436 – Algorithms for Sensor-based Robotics

Lecture II, III – Bug Path-Planning Algorithms

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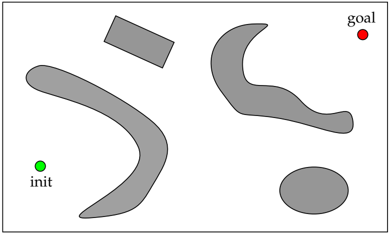
Lecture II: January 28, 2010 Lecture III: February 2, 2010

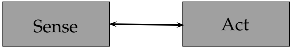
Outline

1. General Properties of Bug Path-Planning Algorithms
2. Bug Algorithms with Tactile (Contact) Sensors
   - `Bug0`
   - `Bug1`
   - `Bug2`
3. Bug Algorithms with Range Sensors
   - `TangentBug`
4. Summary

Problem: Compute a collision-free path from an initial to a goal position



Reactive Paradigm



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- Only local information acquired through sensing
- Inspired by insects

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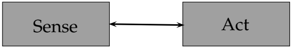

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- Theoretical lower and upper bounds on path length; optimal paths in certain cases

Bug Path-Planning Algorithms

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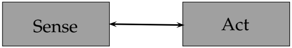

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Environment

- Two-dimensional scene filled with unknown obstacles
- Each obstacle is a simple closed curve of finite length and non-zero thickness
- A straight line crosses an obstacle finitely many times
- Obstacles do not touch each other
- Locally finite number of obstacles, i.e., any disc of finite radius intersects a finite set of obstacles
- Initial and goal positions are known

Bug Path-Planning Algorithms

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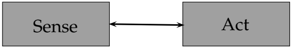

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Point Robot, Simple Motions

- Move straight toward `goal`
- Move along obstacle boundary
- Stop

Bug Path-Planning Algorithms

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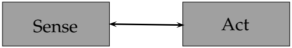

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Simple Sensing

- `Bug1`, `Bug2` assume essentially tactile (contact) sensing
- `TangentBug`, `VisBug`, `DistBug` deal with finite distance sensing
- `I-Bug` uses only signal strength emanating from goal

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

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`Bug0`, `Bug1`, `Bug2` Algorithms – General Idea

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
  - at some point, leave the obstacle and head again toward `goal`

Bug with Tactile (Contact) Sensor

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

`Bug0`, `Bug1`, `Bug2` Algorithms – General Idea

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Path consists of a sequence of hit (H_i) and leave (L_i) points

Algorithms differ on how leave points are computed

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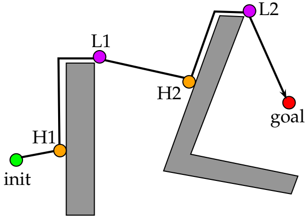
  - follow obstacle boundary
    until can head toward `goal` again



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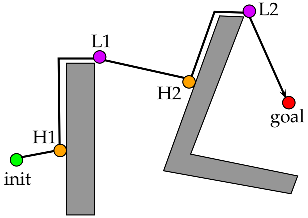

Is `Bug0` a complete algorithm?

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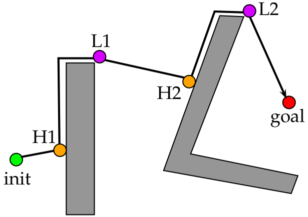

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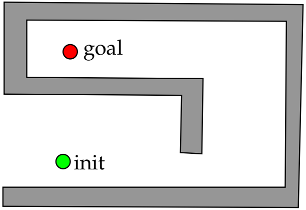

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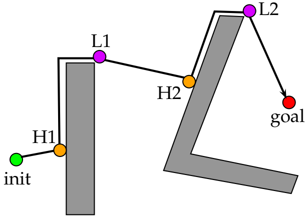

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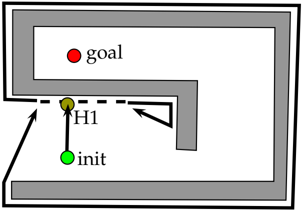

Bug0 fails to find a solution even though a solution exists

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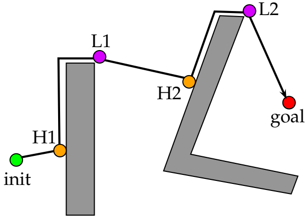

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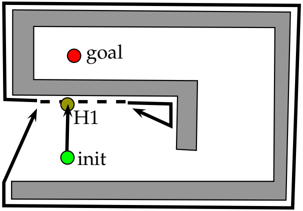

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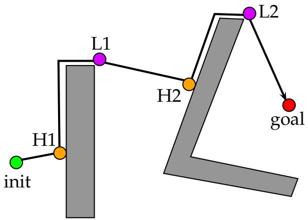

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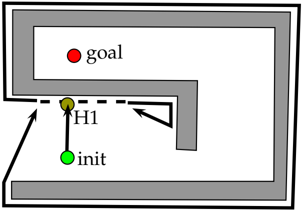

Bug0 fails to find a solution even though a solution exists
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can we obtain a complete algorithm if Bug has some memory?

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
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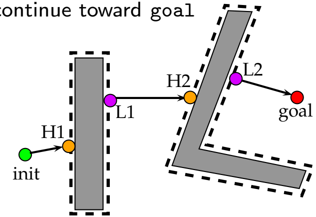
  - return to that closest point (by wall following) and continue toward `goal`



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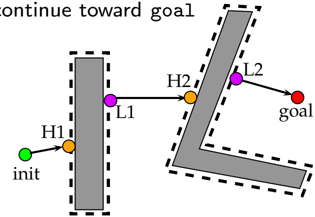

Bug1 Pseudocode

1: $L_0 \leftarrow$ `init`; $i \leftarrow 1$
2: **loop**

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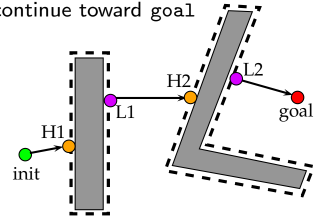

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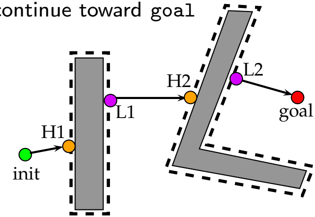

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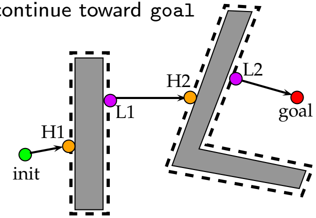

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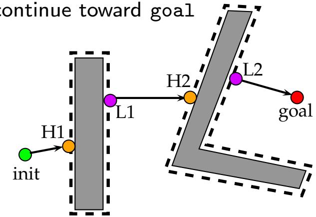

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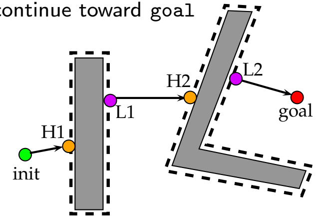

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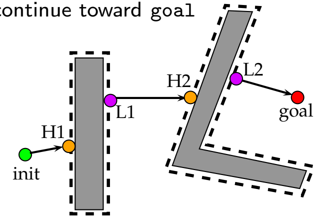

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  follow boundary from H_i to L_i along shortest route
  if move on straight line from L_i toward `goal` moves into obstacle **then** exit with failure
  else $i \leftarrow i + 1$

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch:

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`*, the bug never returns to this obstacle again*

Proof Sketch: Consider the sequence of points visited by bug: $\texttt{init}, H_1, L_1, H_2, L_2, \ldots$

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Therefore,

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Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to `goal` and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

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Lemma 2: Bug meets only a finite number of obstacles

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Lemma 2: Bug meets only a finite number of obstacles

Proof Sketch: Straight-line segments from L_i to H_{i+1} $(i = 0, 1, \ldots)$ are within the same circle of radius $d(\texttt{init}, \texttt{goal})$ centered at goal since

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Proof Sketch: Follows immediately from Lemma 1 and Lemma 2

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

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    - Since, we assumed there is a path to `goal`, then `goal` cannot be encircled by obstacle
    - Thus, bug must have encountered this other intersection point (which is supposedly closer to the goal) when circumnavigating obstacle boundary, which contradicts definition of leave point

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$$d(\texttt{init}, \texttt{goal}) + 1.5 \sum_{i=1}^{n} p_i$$

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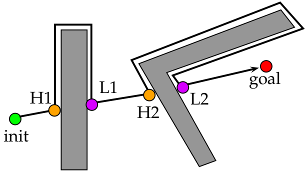
- see proof of Lemma 2, distances from $H_1, L_1, H_2, L_2, \ldots$ to goal become smaller and smaller and are never more than $d(\texttt{init}, \texttt{goal})$. So, bug never encounters obstacles outside this disk

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow the obstacle until it encounters the *m*-line again
  - leave the obstacle and continue straight toward `goal`



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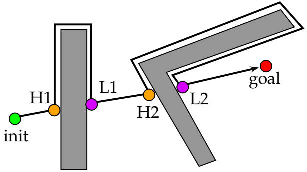

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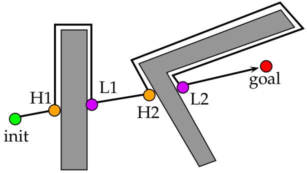

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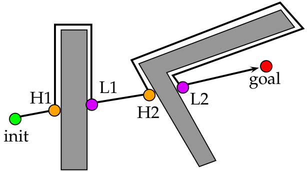

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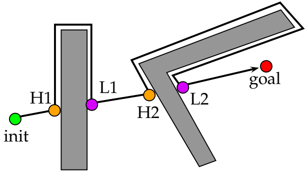

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Bug2 Algorithm

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
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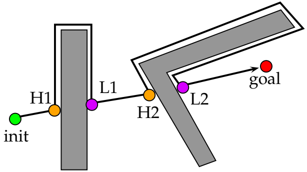

Bug2 Pseudocode

$L_0 \leftarrow$ `init`; $i \leftarrow 1$
loop
  repeat move on a straight line from L_{i-1} to `goal`
  until `goal` is reached or obstacle is encountered at H_i
  if `goal` is reached **then** exit with success
  repeat follow boundary
  until
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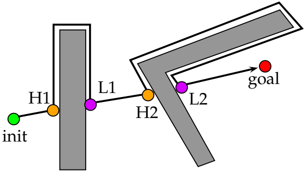

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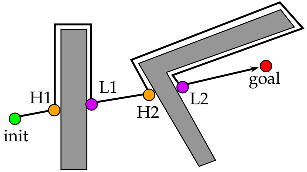

Bug2 Pseudocode

```
L_0 ← init; i ← 1
loop
    repeat move on a straight line from L_{i−1} to goal
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    if goal is reached then exit with success
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    until
      (a) goal is reached or
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          line (Q, goal) does not cross the current obstacle at Q or
      (c) H_i is re-encountered
    if goal is reached then exit with success
    else if H_i is re-encountered then exit with failure
    else L_i ← Q; i ← i + 1
```

Bug2 Analysis

Lemma 3: Bug2 meets only a finite number of obstacles. Moreover, the only obstacles that can be met are those that intersect the straight-line segment $(init, goal)$

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Theorem: $\mathcal{Bug2}$ is a complete path-planning algorithm. Moreover, the length of a path generated by $\mathcal{Bug2}$ never exceeds the limit

$$d(\mathit{init}, \mathit{goal}) + \sum_i \frac{n_i p_i}{2},$$

where p_i's refer to the perimeters of the obstacles intersecting the straight-line segment $(\mathit{init}, \mathit{goal})$

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$$d(init, goal) + \sum_i \frac{n_i p_i}{2},$$

where p_i*'s refer to the perimeters of the obstacles intersecting the straight-line segment* $(init, goal)$

Proof Sketch: Similar to proofs for Bug1. Proof of Lemma 4 is slightly different. Maybe an upcoming homework exercise?

Bug1 vs Bug2

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Bug1 is an exhaustive search algorithm
– looks at all choices before commiting

Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better

Bug2 often outperforms Bug1, but not always

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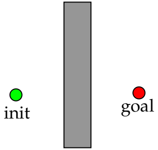

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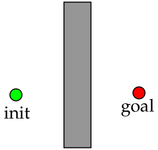

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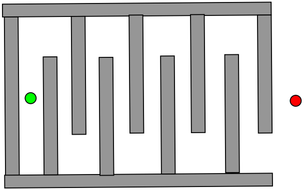

Bug1 beats Bug2 in this scene

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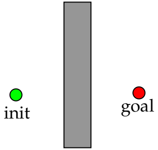

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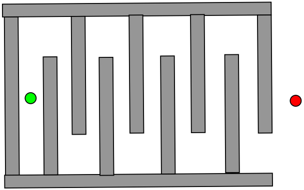

Bug1 beats Bug2 in this scene . . . but only if Bug2 always turns counterclock-wise or always turns clockwise when following boundary

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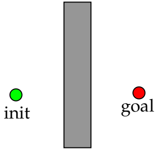

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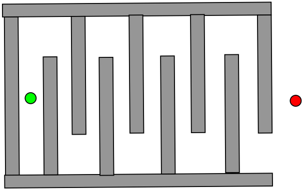

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what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

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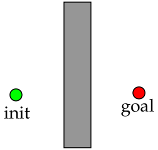

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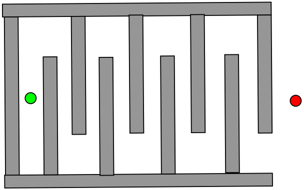

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what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

can you draw a scene then where Bug1 beats Bug2 no matter how Bug2 decides to turn each time it has follow an obstacle boundary?

Bug with Range Sensor

Raw Distance Function $\rho : \mathbb{R}^2 \times [0, 2\pi) \rightarrow \mathbb{R}$

$$\rho(x, \theta) = \min_{\alpha \in [0, \infty)} \text{ such that the point } x + \alpha \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \in \bigcup_i \texttt{Boundary}(O_i)$$

- $\rho(x, \theta)$ is the distance to the closest obstacle along the ray emanating from point $x \in \mathbb{R}^2$ at an angle $\theta \in [0, 2\pi)$

Saturated Raw Distance Function $\rho_R : \mathbb{R}^2 \times [0, 2\pi) \rightarrow \mathbb{R}$ with Sensing Range $R \in \mathbb{R}^{\geq 0}$

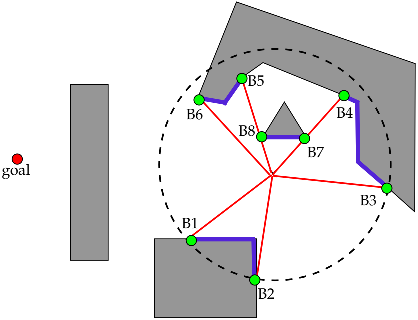
$$\rho_R(x, \theta) = \begin{cases} \rho(x, \theta), & \text{if } \rho(x, \theta) < R \\ \infty, & \text{otherwise} \end{cases}$$

- ρ_R has same value as ρ when obstacle is within sensing range R
- ρ_R has ∞ value when obstacles are outside the sensing range R

Ishay Kamon, Elon Romon, and Ehud Rivlin: IJRR (1998) 17:934–953

`TangentBug` relies on range sensor ρ_R to compute endpoints of finite continuous segments on obstacle boundaries

These segments constitute its local model of the world



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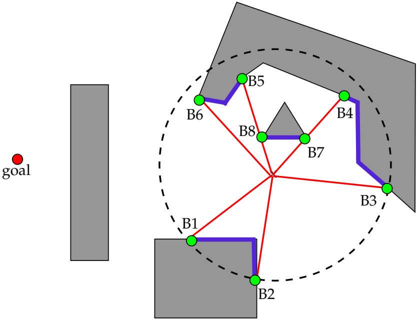

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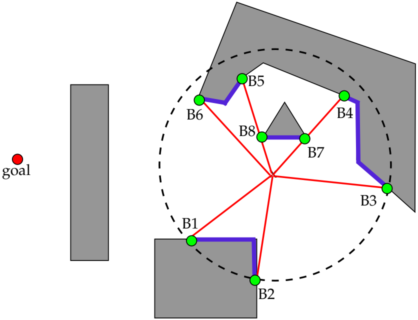

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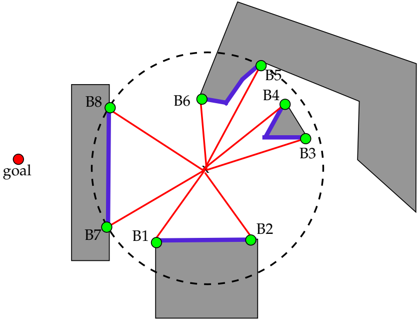

- TangentBug now sees that it can't go straight to the goal. What can it do?

TangentBug Algorithm – Idea

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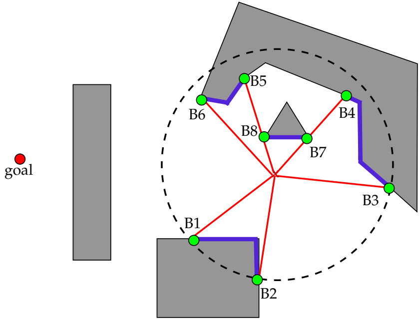

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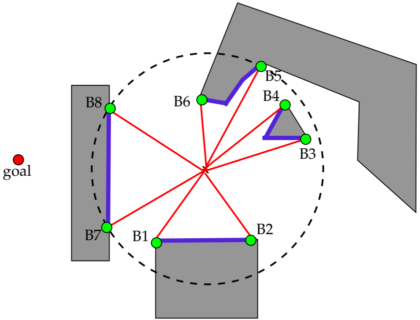

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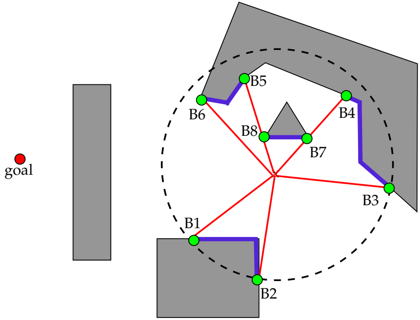

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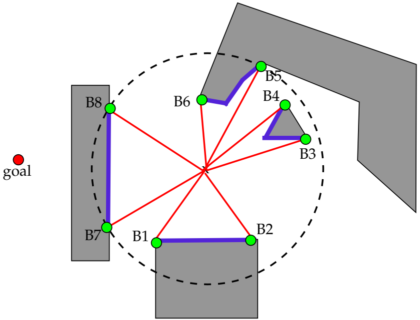

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- What if this distance starts increasing?

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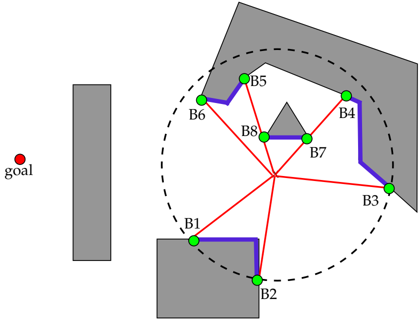

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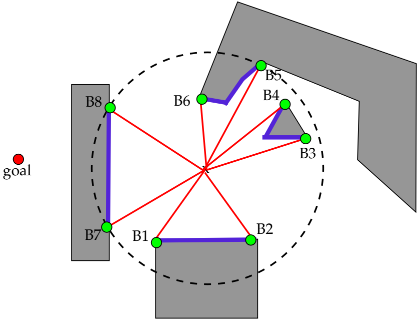

- TangentBug now sees that it can't go straight to the goal. What can it do?
- Choose the point B_i that minimizes heuristic distance $d(x, B_i) + d(B_i, \mathtt{goal})$
- What if this distance starts increasing? Then, start following some boundary

- A motion-to-goal behavior as long as way is clear or there is a visible obstacle boundary point that decreases heuristic distance
- A boundary following behavior invoked when heuristic distance increases
- A value $d_{followed}$ which is the shortest distance between the sensed boundary and goal
- A value d_{reach} which is the shortest distance between blocking obstacle and goal (or distance to goal if no blocking obstacle visible)
- Terminate boundary following behavior when $d_{reach} < d_{followed}$

repeat until goal is reached

1. repeat
   - take sensor-range reading and compute continuous range segments
   - move toward point $n \in \{\texttt{goal}, B_1, B_2, \ldots\}$ that minimizes $h(x, n) = d(x, n) + d(n, \texttt{goal})$

   until
   - goal is reached, or
   - value of $h(x, n)$ begins to increase
2. follow boundary continuing in same direction as before repeating
   - update discontinuity points $\{B_1, B_2, \ldots\}$, d_{reach}, $d_{followed}$

   until
   - goal is reached, or
   - a complete cycle is performed (goal is unreachable)
   - $d_{reach} < d_{followed}$

Completeness proof similar to other bug-algorithm proofs, although the definition of hit and leave points is trickier

Basic problem: compute tangent to curve forming boundary of obstacle at any point, and drive the robot in that direction

- Let $D(x) = \min_c d(x, c), c \in \bigcup \texttt{Boundary}(O_i)$
- Let $G(x) = D(x) - W$, where W is some safe following distance
- Note that $\nabla G(x)$ points radially away from the object
- Define $T(x) = (\nabla G(x))$ the tangent direction
  - in a real sensor, this is just the tangent to the array element with lowest reading
- We could just move in the direction $T(x)$
  - open-loop control

Summary

- `Bug0` is incomplete
- `Bug1` is complete, safe, and reliable
- `Bug2` is complete, better in some cases than `Bug1`, but worse in others
- `TangentBug` is complete, supports range sensors

Reactive paradigm with minimal global information

Point Robot, Simple Motions

- Move straight toward `goal`
- Move along obstacle boundary
- Stop