# On the Discrepancy between Density Estimation and Sequence Generation

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### Is Log-likelihood correlated to BLEU?

Task: sequence-to-sequence generation

Metrics: test log-likelihood, BLEU

$$L(F) = \frac{1}{N} \sum_{n=1}^{N} \log p_F(y_n | x_n).$$

Models: autoregressive and latent variable models

Log-likelihood is highly correlated with BLEU when considering models within the same family. Log-likelihood is not correlated with BLEU when comparing models from different families.

#### Data

#### Datasets:

- IWSLT'16 De→En (197K training, 2K development, 2K test)
- WMT'16 En→Ro (612K training, 2K development, 2K test)
- WMT'14 En→De 4 (4.5M training, 3K development, 3K test)

Wordpiece tokenization

Knowledge distillation (Transformer-base, Transformer-small)

## **Autoregressive Models**

Learning: 
$$\log p_{\mathrm{AR}}(\mathbf{y}|\mathbf{x}) = \sum_{t=1}^{T} \log p_{\theta}(y_t|y_{< t},\mathbf{x}).$$
 
$$L_{\mathrm{AR}}(\theta) = \frac{1}{N} \sum_{n=1}^{N} \log p_{\mathrm{AR}}(\mathbf{y}_n|\mathbf{x}_n).$$

$$L_{AR}(\theta) = \frac{1}{N} \sum_{n=1}^{N} \log p_{AR}(\mathbf{y}_n | \mathbf{x}_n).$$

Inference: 
$$\operatorname{argmax}_{\mathbf{y}} \log p_{\theta}(\mathbf{y}|\mathbf{x}) = \operatorname{argmax}_{y_{1:T}} \sum_{t=1} \log p_{\theta}(y_t|y_{< t}, \mathbf{x}).$$

## **Autoregressive Models**

- Transformer-big (Tr-L)
- Transformer-base (Tr-B)
- Transformer-small (Tr-S)

Beam search is used

#### Latent Variable Models

Learning: 
$$\log p_{\text{LVM}}(\mathbf{y}|\mathbf{x}) = \log \int_{-}^{} p_{\theta}(\mathbf{y}|\mathbf{z},\mathbf{x}) \ p_{\theta}(\mathbf{z}|\mathbf{x}) d\mathbf{z}.$$

$$\log p_{\text{LVM}}(\mathbf{y}|\mathbf{x}) \ge \text{ELBO}(\mathbf{y}, \mathbf{x}; \theta, \phi) = \mathbb{E}_{\mathbf{z} \sim q_{\phi}} \left[ \log p_{\theta}(\mathbf{y}|\mathbf{z}, \mathbf{x}) \right] - \text{KL} \left[ q_{\phi}(\mathbf{z}|\mathbf{y}, \mathbf{x}) \mid p_{\theta}(\mathbf{z}|\mathbf{x}) \right]$$

Inference:

$$\delta(\mathbf{z}|\boldsymbol{\mu}) = \begin{cases} 1, & \text{if } \mathbf{z} = \boldsymbol{\mu} \\ 0, & \text{otherwise} \end{cases}$$

Then, the ELBO reduces to:  $\log p_{\theta}(\mathbf{y}|\boldsymbol{\mu}, \mathbf{x}) + \log p_{\theta}(\boldsymbol{\mu}|\mathbf{x})$ .

Factorization:  $p_{\theta}(\mathbf{y}|\mathbf{z}, \mathbf{x}) = \prod_{t=1}^{T} p_{\theta}(y_t|\mathbf{z}, \mathbf{x}).$ 

# Diagonal Gaussian vs Normalizing Flow

$$\log p_{\theta}(z_{1:T}|\mathbf{x}) = \sum_{t=1}^{T} \log \mathcal{N}\Big(z_t \Big| \mu_{\theta,t}(\mathbf{x}), \sigma_{\theta,t}(\mathbf{x})\Big)$$

a base distribution  $p_b(\epsilon)$ 

$$f(\mathbf{z}) = \epsilon, \ f^{-1}(\epsilon) = \mathbf{z}$$
  
 $f(\mathbf{z}; \mathbf{x}) = \epsilon, \ f^{-1}(\epsilon; \mathbf{x}) = \mathbf{z}.$ 

$$\log p_{\theta}(\mathbf{z}|\mathbf{x}) = \log p_{b}(f(\mathbf{z};\mathbf{x})) + \log \left| \det \frac{\partial f(\mathbf{z};\mathbf{x})}{\partial \mathbf{z}} \right|$$

$$\mathbf{z}_{\mathrm{id}}, \mathbf{z}_{\mathrm{tr}} = \mathrm{split}(\mathbf{z})$$
 $\mathbf{s}, \mathbf{b} = g_{\mathrm{param}}(\mathbf{z}_{\mathrm{id}})$ 
 $f(\mathbf{z}) = \mathrm{concat}(\mathbf{z}_{\mathrm{id}}; \ \mathbf{s} \cdot \mathbf{z}_{\mathrm{tr}} + \mathbf{b})$ 

#### Latent Variable Models

encoder — Transformer encoder

length predictor — a 2-layer MLP

prior:

- Gauss (Transformer, a sequence of positional encodings of length T as input, outputs the mean and standard deviation)
- Normalizing Flow (Transformer decoder + Linear with weight-normalization, )

decoder — Transformer decoder

posterior — final Linear layer with weight normalization

### Test BLEU score and log-likelihood of each model

		BLEU (†)		LL (†)	
		RAW	DIST.	RAW	DIST.
	TR-S	24.54	24.94	-1.77	-2.36
П	TR-B	28.18	27.86	-1.44	-2.19
Q	TR-L	29.39	28.29	-1.35	-2.23
WMT'14 EN→DE	GA-B	15.74	24.54	-1.51	-2.44
4 F	GA-L	17.33	25.53	-1.47	-2.24
7	FL-S	18.17	21.98	-1.41	-2.13
M	FL-B	18.57	21.82	-1.23	-2.05
≥	FL-B(*)	18.55	21.45		
	$FL-L^{(*)}$	20.85	23.72		
3. <del>.</del>	TR-S	29.15	28.40	-1.66	-2.24
Z	TR-B	32.21	32.24	-1.42	-2.12
一	TR-L	33.16	32.24	-1.35	-2.05
)E-	GA-B	21.64	29.29	-1.41	-2.17
4 I	GA-L	23.03	30.30	-1.31	-2.04
,	FL-S	23.17	27.14	-1.28	-1.73
WMT'14 DE→EN	FL-B	23.12	26.72	-1.20	-1.71
≥	FL-B <sup>(*)</sup>	23.36	26.16		
	FL-L(*)	25.40	28.39		

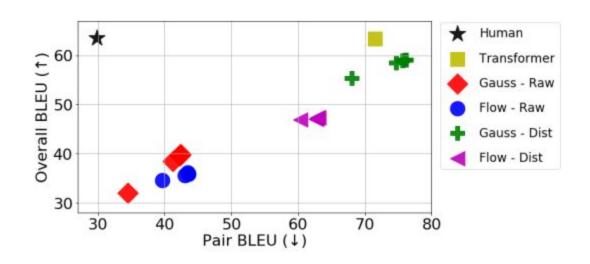
# Pearson's correlation between log-likelihood and BLEU

	TR-B	GA-B	FL-B
RAW	0.926	0.831	0.678
DIST.	-0.758	-0.897	-0.873

BLEU scores and log-likelihoods on out-of-distribution test sets.

		BLEU (↑)		LL (†)	
		RAW	DIST.	RAW	DIST.
.14 SLT	TR-S TR-B TR-L	29.15 32.29 33.16	28.40 31.75 32.24	-1.65 -1.42 -1.35	-2.25 -2.12 -2.06
WMT'14 →IWSL7	GA-B GA-L FL-S FL-B	24.26 25.46 24.35 24.25	28.77 <b>29.60</b> 26.79 27.12	-1.37 -1.28 -1.26 <b>-1.19</b>	-2.10 -2.01 -1.76 -1.73
<b>†</b> 4	TR-S	18.50	18.94	-2.79	-3.41
IWSLT WMT' 1	GA-B FL-S FL-B	12.12 11.78 12.56	13.78 <b>14.35</b> 14.30	-3.10 -2.81 <b>-2.62</b>	-3.83 -3.22 -3.43

#### Results



#### Results

BLEU			SPEED				Size				
k =	0	1	2	4	8	0	1	2	4	8	-
TR-S	24.54					2.69					17M
TR-B	28.18					2.58					60M
TR-L	29.39					1.93					208M
GA-B	23.15	24.54	24.87	24.94	24.92	28.77	20.52	16.51	12.00	8.11	75M
GA-L	24.31	25.53	25.69	25.68	25.68	19.83	14.72	10.25	7.88	4.91	95M
FL-B	21.57	21.82	21.79	21.81	21.80	5.82	5.60	4.84	3.60	3.37	75M
$FL-L^{(*)}$	23.72										258M

#### Вопросы

1) Перед вами таблица с результатами эксперимента для трёх моделей на одних и тех же данных. Предположите, из каких семейств (из одного или из разных) эти модели, и объясните, почему вы так думаете.

BLE	U (†)	LL (†)		
RAW	DIST.	RAW	DIST.	
24.54	24.94	-1.77	-2.36	
28.18	27.86	-1.44	-2.19	
29.39	28.29	-1.35	-2.23	

- 2) Расскажите про корреляцию между BLEU и LL для моделей внутри одного семейства и из разных семейств.
- 3) Опишите модели LVM, которые использовались для эксперимента.