Сем 6.

ур-ие Эйлера

$$x^2 \cdot y'' - xy' - 6y = 21x^2$$

Шаг 1. подберём решение

$RHS = 21x^2$

$$y(x) = ax^2$$

$$x^2 \cdot 2a - x \cdot 2ax - 6ax^2 = 21x^2$$

$$a = -\frac{21}{6} = -\frac{7}{2}$$

$$y(x) = -\frac{7}{2}x^2$$

Шаг 2

$$y(x) = \underbrace{-\frac{7}{2}x^2}_{y_1(x)} + \Delta(x) \to$$ подставляем в исходное

$$x^2\left(y_1'' + \Delta''\right) - x\left(y_1' + \Delta'\right) - 6\left(y_1 + \Delta\right) = 21x^2$$

учитываем в диф. урах

$$x^2 y_1'' - xy_1' - 6y_1 + x^2\Delta'' - x\Delta' - 6\Delta = 21x^2$$

$$x^2\Delta'' - x\Delta' - 6\Delta = 0.$$

$$\Delta(x) = x^\lambda \qquad \Delta' = \lambda \cdot x^{\lambda-1} \qquad \Delta'' = \lambda(\lambda-1) \cdot x^{\lambda-2}$$

$$\lambda \cdot (\lambda-1) \cdot x^\lambda - \lambda \cdot x^\lambda - 6x^\lambda = 0$$

$$\lambda^2 - \lambda - \lambda - 6 = 0 \qquad \lambda_1 = -1+\sqrt{7}$$

$$\lambda^2 - 2\lambda - 6 = 0 \qquad \lambda_2 = -1-\sqrt{7}$$

Ответ: $y(x) = -\frac{7}{2}x^2 + \underline{C_1 \cdot x^{-1+\sqrt{7}} + C_2 \cdot x^{-1-\sqrt{7}}}$ где $C_1, C_2 \in \mathbb{R}$

Случай компл. корней.

$$\lambda^2 + 4\lambda + 5 = 0.$$
$$(\lambda+2)^2 = -1$$
$$\lambda_1 = -2 - i$$
$$\lambda_2 = -2 + i$$

$$\exp(\varphi \cdot i) = \cos\varphi + i\sin\varphi$$
$$\exp(-\varphi i) = \cos(-\varphi) + i\sin(-\varphi)$$

$$y(x) = C_1 \cdot x^{-2-i} + C_2 \cdot x^{-2+i} =$$
$$= x^{-2} \cdot \left(C_1 \cdot \exp((\ln x) \cdot (-i)) + C_2 \cdot \exp((\ln x) \cdot i)\right) =$$
$$= x^{-2} \left[C_1 \left(\cos(\ln x) - i\sin(\ln x)\right) + C_2 \left(\cos(\ln x) + i\sin(\ln x)\right) \right]$$
$$= x^{-2} \left(d_1 \cos\ln x + d_2 \sin \ln x \right), \text{ где } d_1, d_2 \in \mathbb{R}$$

Случай

$$C_1 e^{\lambda x} + C_2 \cdot x \cdot e^{\lambda x} + C_3 \cdot x^2 \cdot e^{\lambda x}$$
$$C_1 + C_2 + C_3 = C_4$$
$$C_1 + C_2 x + C_3 x^2 \neq \ldots$$

$$x^2 \cdot y'' - x y' + y = 0$$

$$y = x^\lambda \qquad x^\lambda \cdot \lambda \cdot (\lambda - 1) - \lambda \cdot x^\lambda + x^\lambda = 0$$
$$\lambda^2 - \lambda - \lambda + 1 = 0$$
$$\lambda^2 - 2\lambda + 1 = 0$$
$$\lambda_1 = 1 \quad \lambda_2 = 1$$
$$y_1(x) = x^1 \qquad y_2(x) = x^1 \cdot c(x)$$

$$x^2\underbrace{\left(xc''+2\cdot c'+0\right)}_{y''}-x\underbrace{\left(1\cdot c(x)+x\cdot c'\right)}_{y'}+\underbrace{\left(x\cdot c(x)\right)}_{y}=0$$

$y(x)=x\cdot c(x)$

$$x^2\left(xc''+2c'\right)-x^2\cdot c'=0$$

$$xc''+c'=0$$

$$c'=d(x)$$

$$x\cdot d'+d=0$$

$$x\cdot\frac{dd}{dx}+d=0$$

$$\frac{x}{dx}=-\frac{d}{dd}$$

$$\frac{dx}{x}=-\frac{dd}{d}$$

$$\measuredangle-\ln|x|=\ln|d|\cdot$$

$$d(x)=$$

$$c(x)=$$

$$y(x)=x^!\cdot c_1+x^!\cdot c_2\cdot$$

$$\dot{\begin{pmatrix} y(t) \\ x(t) \end{pmatrix}} = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} y(t) \\ x(t) \end{pmatrix} + \begin{pmatrix} 7 \\ t \end{pmatrix}$$

$$\begin{cases} \dot{y} = 2y + 3x + 7 \\ \dot{x} = y + 5x(t) + t \end{cases}$$

$$\begin{cases} y = \dot{x} - 5x + t \quad \Big| \ \frac{d}{dt} \\ \dot{y} = \ddot{x} - 5\dot{x} + 1 \end{cases}$$

подставим в ур-ние

← это брат ёжика

$$\underbrace{(\ddot{x} - 5\dot{x} + 1)}_{\dot{y}} = 2\underbrace{(\dot{x} - 5x + t)}_{y} + 3 \cdot x + 7$$

$$\underbrace{\ddot{x} - 7\dot{x} + 7x = 6 + 2t}$$

$\downarrow$ ... по старинке.

$$A_{n \times n} = P \cdot D \cdot P^{-1}$$

$$D = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & \ddots \ J_k \end{pmatrix}$$

$$J_i = \begin{pmatrix} \lambda & 1 & & 0 \\ & \lambda & 1 & \\ & & \lambda & 1 \\ & & & \lambda & 1 \\ 0 & & & & \lambda \end{pmatrix}$$

Упр

a) $$\exp\left[\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}\cdot t\right]=\begin{bmatrix}\exp t&0&0\\0&\exp(2t)&0\\0&0&\exp(3t)\end{bmatrix}$$

б) $$\exp\left[\begin{pmatrix}7&1&0\\0&7&1\\0&0&7\end{pmatrix}\cdot t\right]=*$$

$$\begin{pmatrix}7&1&0\\0&7&1\\0&0&7\end{pmatrix}\cdot t=7I\cdot t+N\cdot t$$

$$N^2=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\qquad N^3=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad N=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$$

$$\exp(\square)=1+\square+\frac{\square^2}{2!}+\frac{\square^3}{3!}+\dots$$

$$*=I+(7I\cdot t+Nt)+\frac{(7It+Nt)^2}{2!}+\frac{(7It+Nt)^3}{3!}+\dots$$

без N

$$=\overbrace{I+7It+\frac{I\cdot(7t)^2}{2!}+\frac{I\cdot(7t)^3}{3!}+\dots}^{\text{без } N}\rightarrow\begin{bmatrix}\exp(7t)&0&0\\0&\exp(7t)&0\\0&0&\exp(7t)\end{bmatrix}$$

$$+N\left[t+\frac{2\cdot 7t^2}{2!}+\frac{3\cdot 7^2t^3}{3!}+\frac{4\cdot 7^3t^4}{4!}+\dots\right]=N\cdot t\cdot\exp(7t)=\begin{bmatrix}0&te^{7t}&0\\0&0&te^{7t}\\0&0&0\end{bmatrix}$$

$$+N^2\left[\frac{t^2}{2}+\frac{C_3^2\cdot 7t^3}{3!}+\frac{C_4^2\cdot 7^2\cdot t^4}{4!}+\dots\right]=N^2\cdot\frac{t^2}{2}\cdot\left[1+7t+\frac{(7t)^2}{2!}+\dots\right]$$

$$=N^2\cdot\frac{t^2}{2}\cdot\exp(7t)$$

$$\exp\left[\begin{pmatrix}7&1&0\\0&7&1\\0&0&7\end{pmatrix}\cdot t\right]=\begin{bmatrix}1&t&t^2/2!\\0&1&t\\0&0&1\end{bmatrix}\cdot e^{7t}$$

b) $$\exp\left[\begin{pmatrix} 3 & 0 & 1 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix} \cdot t\right] = \begin{pmatrix} e^{3t} & 0 & t \cdot e^{3t} \\ 0 & e^{6t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix}$$

b^{-1}) $$\exp\left(\left[\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 3 & 0 \\ \hline 0 & 0 & 6 \end{array}\right] \cdot t\right) = \left[\begin{array}{cc|c} e^{3t} & t \cdot e^{3t} & 0 \\ 0 & e^{3t} & 0 \\ \hline 0 & 0 & \boxed{e^{6t}} \end{array}\right]$$