Optimization module

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Abstract

The objective of this document is to provide a structure for all future documentation for all products.

In this paper, we illustrate some of the optimization techniques which has been implemented namely:

i Travelling salesman problem

ii Transportation problem

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1 Travelling Salesman Problem

The travelling salesman problem (TSP) is about given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

Data Used:

$i \in I$

$j \in I$

D_{ij}

Decision variables:

$$X_{ij} = \begin{cases} 1, & \text{if salesman travels from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

$U_i \in Integer$

$U_j \in Integer$

Objective function:

$$\min \sum_{i \in I} \sum_{j \in I} X_{ij} \times D_{ij} \tag{1}$$

s.t.

Each node should be entered and exited exactly once

$$\sum_i X_{ij} = 1 \qquad \forall j \tag{2}$$

$$\sum_j X_{ij} = 1 \qquad \forall i \tag{3}$$

Eliminate subtours:

$$U_i - U_j + N \times X_{ij} = N - 1 \qquad \forall i \in 1, 2..N-1 \quad j \in 2, 3..N \tag{4}$$

2 Transportation Problem

Transportation problem is about goods being transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hitchcock problem.

Data Used:

$s \in S$

$d \in D$

C_{sd}, Q_s, Q_d

Decision variables:

$X_{sd} \in Integer$

Objective function:

$$\min \sum_{s \in S} \sum_{d \in D} X_{sd} \times C_{sd} \tag{5}$$

s.t.

For a supply node, units shipped must be less than or equal to the supply quantity

$$\sum_{d \in D} X_{sd} <= Q_s \qquad \forall s \tag{6}$$

For a demand node, units shipped must be greater than or equal to the demand quantity

$$\sum_{s \in S} X_{sd} >= Q_d \qquad \forall d \tag{7}$$

Nomenclature

i	Source city
j	Destination city
I	Set of cities
N	Total number of cities (I)
D_{ij}	Distance between source city i and destination city j
X_{ij}	Binary flag, sales man travels from source city i to desination city j
U_i	Integer, artificial variable for source city i
U_j	Integer, artificial variable for destination city j
s	Supply node
S	List of supply nodes s
d	Demand node
D	List of demand nodes d
C_{sd}	Cost to transport one unit from supply node s to demand node d
X_{sd}	Integer, quantity transported from supply node s to demand node d
Q_s	Supply quantity for node s
Q_d	Demand quantity for node d