Numerical Integration

Performs numerical integration on x-y data. Uses **Simpson's Rule** where applicable, and the **Trapezoidal Rule** in all other cases.

Returns the **result**, as well as a string indicating the **method** used.

```
[method, result] = NumIntgrl(x, y);
```

Methods

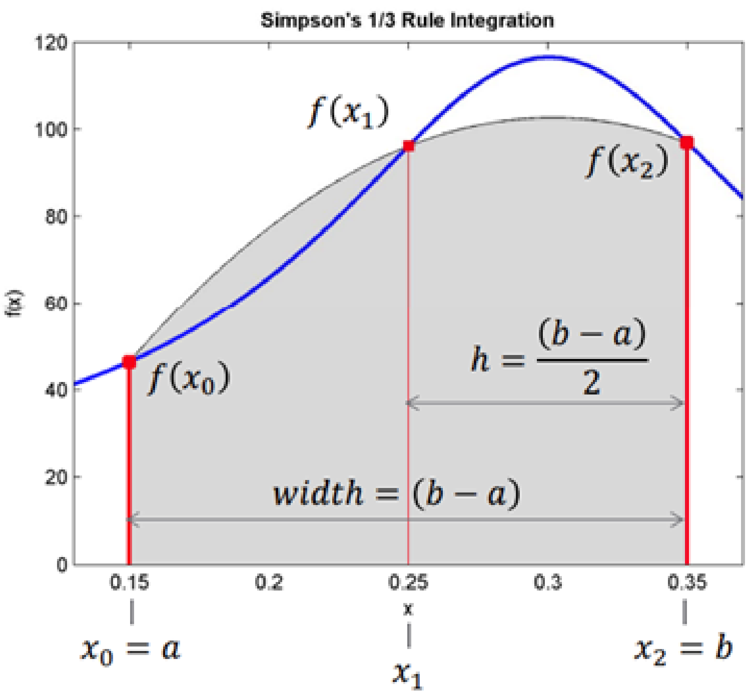
Simpson's 1/3 Rule:

Approximates a function using **2nd order polynomials**.

For ***n*** equal segments (*i.e.*, ***n+1*** evenly spaced data points), the definite integral can be approximated as follows. Note that ***n*** must be **even**.

$$\int_a^b f(x)dx \cong \frac{1}{3}\frac{(b-a)}{n}\left[f(x_0) + 4\sum_{i=1,3,5...}^{n-1} f(x_i) + 2\sum_{j=2,4,6...}^{n-2} f(x_j) + f(x_n)\right]$$

Here `(b-a)/n` is the spacing between data points.



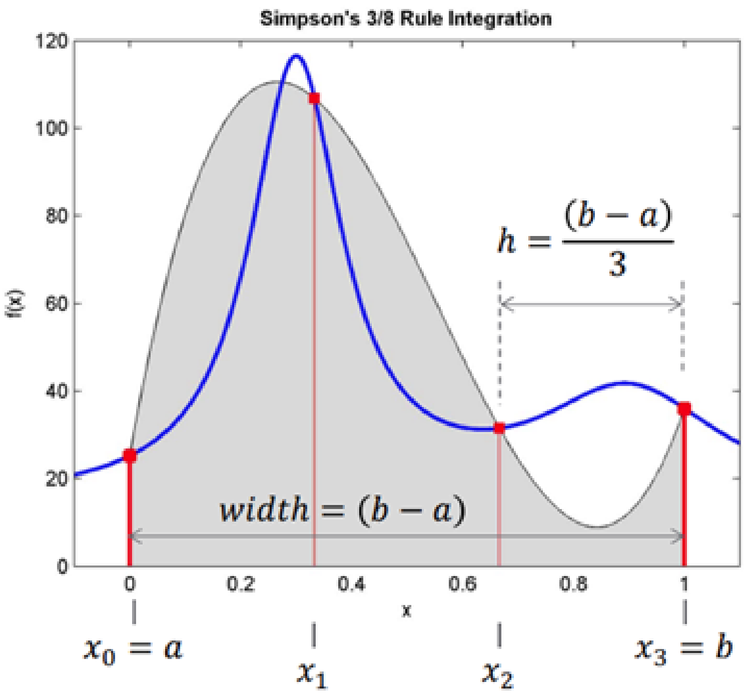
Simpson's 3/8 Rule:

Approximates a function using **3rd order polynomials**.

For ***n*** equal segments (*i.e.*, ***n+1*** evenly spaced data points), the definite integral can be approximated as follows. Note that ***n*** must be **an integer multiple of 3**.

$$\int_a^b f(x)dx \cong \frac{3}{8}\frac{(b-a)}{n}\left[f(x_0) + 3\sum_{i=1,4,7\ldots}^{n-2} f(x_i) + 3\sum_{j=2,5,8\ldots}^{n-1} f(x_j) + 2\sum_{k=3,6,9\ldots}^{n-3} f(x_k) + f(x_n)\right]$$

Here `(b-a)/n` is the spacing between data points.



Trapezoidal Rule:

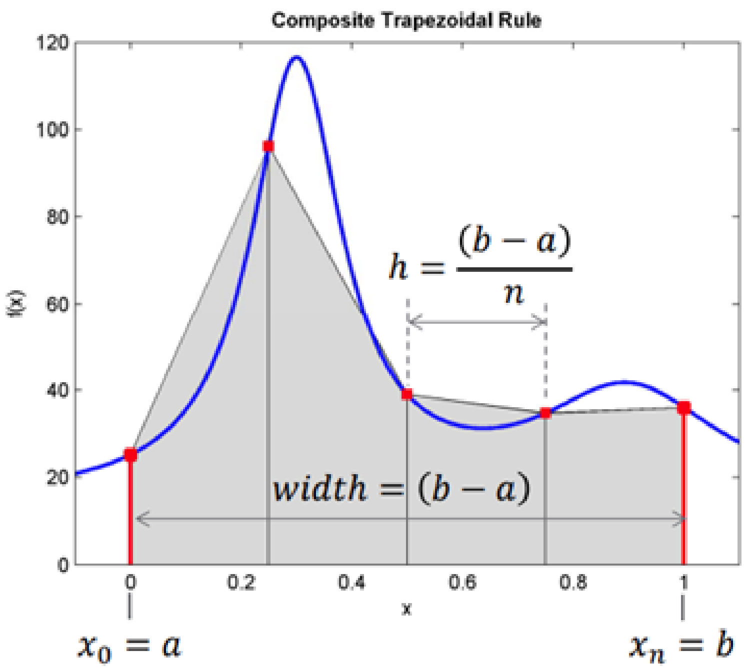
Approximates a function using **trapezoids**.

For ***n*** equal segments (*i.e.*, ***n+1*** evenly spaced data points), the definite integral can be approximated as follows.

$$\int_a^b f(x)dx \cong \frac{b-a}{n} \sum_{i=1}^{n} \frac{1}{2} \left[f(x_i) + f(x_{i+1}) \right]$$

$$= \frac{b-a}{n} \left[\sum_{j=1}^{n+1} f(x_j) - \frac{1}{2} [f(x_1) + f(x_{n+1})] \right]$$

Here `(b-a)/n` is the spacing between data points.



Test code:

$$\int_0^{\pi} \sin x \, dx \; = \; 2$$

```
for Npts = logspace(1, 4, 4) + 1
    x = pi*linspace(0, 1, Npts);
    y = sin(x);
    [method, result] = NumIntgrl(x, y);
    fprintf('n = %-6d \tTrapz = %.6f \t%s = %.6f\n', Npts-1, trapz(x, y), method,
result)
end

n = 10      Trapz = 1.983524    Simp 1/3 = 2.000110
n = 100     Trapz = 1.999836    Simp 1/3 = 2.000000
n = 1000    Trapz = 1.999998    Simp 1/3 = 2.000000
n = 10000   Trapz = 2.000000    Simp 1/3 = 2.000000
```

```
for Npts = logspace(1, 4, 4)
    x = pi*linspace(0, 1, Npts);
    y = sin(x);
    [method, result] = NumIntgrl(x, y);
    fprintf('n = %-6d \tTrapz = %.6f \t%s = %.6f\n', Npts-1, trapz(x, y), method,
result)
end

n = 9       Trapz = 1.979651    Simp 3/8 = 2.000382
n = 99      Trapz = 1.999832    Simp 3/8 = 2.000000
n = 999     Trapz = 1.999998    Simp 3/8 = 2.000000
n = 9999    Trapz = 2.000000    Simp 3/8 = 2.000000
```

```
for Npts = logspace(1, 4, 4) + 2
    x = pi*linspace(0, 1, Npts);
    y = sin(x);
    [method, result] = NumIntgrl(x, y);
    fprintf('n = %-6d \tTrapz = %.6f \t%s = %.6f\n', Npts-1, trapz(x, y), method,
result)
end

n = 11      Trapz = 1.986387    Trap     = 1.986387
n = 101     Trapz = 1.999839    Trap     = 1.999839
n = 1001    Trapz = 1.999998    Trap     = 1.999998
n = 10001   Trapz = 2.000000    Trap     = 2.000000
```

Resources:

https://web.engr.oregonstate.edu/~webbky/MAE4020_5020_files/Section%208%20Integration.pdf