2022-02-07 Game theory

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Game on matrix $A \in \mathbb{R}^{n \times n}$

Player 1 choose $i \in [n]$ max

2 $j \in [n]$ min

$$\begin{pmatrix} 3 & 2 & -1 \\ \boxed{4 \quad 1 \quad \boxed{0}} \\ 0 & 4 & -3 \end{pmatrix}$$

Outcome : A_{ij}

$$0 = \max_{i \in [n]} \min_{j \in [n]} A_{ij} \overset{?}{\le} \min_{j \in [n]} \max_{i \in [n]} A_{ij} = 1$$

1 goes first

2 goes 2nd

min

$$\begin{array}{cc} & j \\ i & \begin{pmatrix} 0 & 1 \\ \boxed{1} & \boxed{0} \end{pmatrix} \end{array}$$

max

$$\begin{array}{cc} & j \\ & \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \end{array} \qquad \begin{array}{cc} i & \begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

$$v = \max_{i \in [n]} \underbrace{\min_{j \in [n]} A_{ij}}$$

min $\le v$

$$\Rightarrow \left(\forall i \in [n], \ \min_{j \in [n]} A_{ij} \le v \right)$$

$$\exists i^* \in [n] \quad \min_{j \in [n]} A_{i^*j} = v$$

i^*

$$\min_{j \in [n]} \left(\underbrace{\max_{i \in [n]} A_{ij}} \right) \ge \min_{j \in [n]} A_{i^*j} = v$$

1^{st} player choose i w/p p_i

choose 1 w/p p_1 $\quad p_1 + p_2 = 0$

2 w/p p_2

2^{nd}

q_i

choose 1 q_1 $\quad q_1 + q_2 = 0$

2 q_2

$$p^t A q$$

$$\begin{pmatrix} 1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} .9 \\ .1 \end{pmatrix}$$

$$\max_{\cancel{i \in [n]}} \min_{\cancel{j \in [n]}} p^t A q \;\underbrace{\leq}_{=}\; \min_{\cancel{j \in [n]}} \max_{\cancel{i \in [n]}} p^t A q$$

$p \in (\mathbb{R}_{\geq 0})^n$ $\quad q \in (\mathbb{R}_{\geq 0})^n$

$\Sigma p_i = 1$ $\quad \Sigma q_j = 1$

Von Neumann's minimax thm