Final Exam

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STAT 4025: Design and Analysis of Experiments

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Final Exam

The questions from the assignment are also written out in this document for convenience. Questions will always be labeled and written on level 1 and level 2 headings. Answers will always be clearly labeled and written below level 3 headings; code that is highly relevant to the answer will be included in the given answer. The last section of this paper contains a transcript of all code used.

Question 1

"Members of the same species living at different locations are called ecotypes. A study investigated the effect of ecotype and hydrocarbon pollution on the metabolism of a common frog species. Frog eggs were randomly collected from 6 ecotypes. Several hatchery tanks were used in the study. Tanks were randomly assigned to one of 4 hydrocarbon chemicals - control, A, B and a mix of 50% A and 50% B. The response variable was oxygen consumption in milliliters per gram per hour. Assume that 2 eggs from each ecotype were placed in 16 tanks – all of the eggs hatched and developed into mature frogs. Assume there are 96 (6x4x4) tanks used for this experiment. The mean oxygen consumption of the 2 frogs in each tank is used as the response."

Part A

What are the factors, treatment combinations, replicates, experimental units and sampling units for this study? [4 pts]

Answer:

For this study, the factors are the 6 ecotypes and 4 hydrocarbon chemicals mixtures. The replicates are the 4 repeated tanks for each combination of ecotype and chemical. Those twenty-four combinations of 6 ecotypes and 4 chemical mixtures are the treatment combinations. The experimental units are the individual tanks, this is because the treatment is applied on a tank-by-tank basis. The sampling units are the individual frogs/eggs placed in each tank.

Part B

What is the null hypothesis for a test of whether the effect of chemical treatment differs among the ecotypes and what is the test statistic and its associated degrees of freedom for conducting this test? [5 pts – 1 hyp; 2 test stat; 2 df]

Answer:

Treating i, j, k as ecotype, chemical, and n^{th} response respectively, the null hypothesis is:

$H_0 = Y_{1..} = Y_{2..} = Y_{3..} = Y_{4..} = Y_{5..} = Y_{6..}$

Error df = (a * b) (n – 1) = 72

qtukey(p=.95, 10, 72) = 4.617241

Part C

How many degrees of freedom are associated with the error term for this design? [3 pts]

Answer:

Error df = (a * b) (n – 1) = 72

Part D

Briefly explain how you would do a permutation test to test for the interaction effect in a two-factor experiment. [3 pts]

Answer:

The permutation test we would be applying is for one-factor studies we first create a linear model T such that T = lm(respons~effect1:effect2), a one factor model respective only to the interaction of the effects. This gives us the f-value needed for our permutation test. Next, we would need to create reference data for the test, so we add an additional column onto our reference data that is the difference between effect 1 and effect 2 for each row. Then we can apply our permutation test function with our treatment variable as the added difference column, our response variable as the response column and our fperm value equal to the f-stat found earlier.

Part E

Suppose that the effect of chemical treatment differs among the ecotypes and that you want to investigate all pairwise comparisons involving ecotype 1 and the chemical treatment. What value of alpha will you need to use to control the experiment wise error rate at 0.10? [3 pts]

Answer:

N = total comparisons = 4 * 1 = 4

- ⇨ $0.10 > 1 - (1 - a)^4$
- ⇨ …
- ⇨ a > 1.97400
- ⇨ Therefore, alpha must be greater than 1.974 for our experiment wise error rate to remain below 0.10

Part F

Suppose it was important to keep the tanks at a constant temperature and that temperature control chambers are used for this purpose. Each chamber can contain multiple tanks. If 96 tanks are still available, how many chambers will you need and how many tanks per chamber are required for conducting a randomized complete block design? (Chambers are blocks) [4 pts]

Answer:

Experimental Units = (treatments) * (blocks)

- ⇨ 96 = (ecotype) * (chemical) * (blocks)

⇨ 96 = 6 * 4 * B

⇨ 96 = 24 * B

⇨ B = 4

We will need four chambers and each chamber will need to hold 24 tanks, this is because there are 24 combinations of our treatments, so each block must have 24 tanks.

Question 2

"A machine is used to fill 5-gallon metal containers with soft drink syrup. The response variable of interest is the amount of frothing (higher frothing implies greater syrup loss). Ideally, frothing will be close to zero which means no loss of syrup in the filling process. Suppose two factors are thought to influence frothing: A- nozzle type and B- filling speed and that 18 5-gallon tanks are available for the experiment. Assume that there are three levels of A: 1, 2, and 3 and three filling speeds: 100, 120 and 140 litres/minute. The table below provides the data from this experiment."

Nozzle	Speed		
	100	120	140
1	4 6	7 13	13 17
2	4 6	13 17	8 12
3	23 27	14 16	8 12

Part A

Suppose you are told that the design used to collect the data was a two-factor completely randomized design. Explain the randomization process. [3 pts]

Answer:

There are 18 tanks available for the experiment, our factors being applied are the three filling speeds, and three nozzle types. This means we have exactly nine total combinations and therefore 9 total treatments, we now construct a list of these 9 treatments. Then we randomly select a treatment and randomly select a tank and then pair them together, removing the treatment from our list as we go. When our list of treatments is empty, we 'reset' the list to its original state and repeat the process until we are out of tanks to pair. In this case it will take exactly two cycles and there will be exactly two tanks for every treatment combination. This can be easily done in R by constructing the list of all possible treatments as described and applying the 'sample' function without replacement onto a list of tanks.

Part B

The output below is the result of using R to analyze the data. Answer the following regarding the output:

Response: Frothing

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Nozzle	2	177.78	88.889	12.5000	0.0025260 **
Speed	2	11.11	5.556	0.7812	0.4865652
Nozzle:Speed	4	422.22	105.556	14.8437	0.0005345 ***
Residuals	9	64.00	7.111		

Part I: What set of hypotheses is being tested with the p-value corresponding to Nozzle:Speed? Offer a practical interpretation of the test result assuming $\alpha = 0.05$. Given this result, what would be your next step so that you can further explain this result? [7 pts – 2 hypotheses, 3 for interpretation, 2 for next steps]

Answer: The hypothesis being tested is if whether the interaction between the factors is equal to zero. Our p-value for Nozzle:Speed is well below 0.05, this means that there is a significant interaction between Nozzle type and Speed. This invalidates our individual p-values for Nozzle and Speed. To investigate this further we should create a main effect plot and interaction plot, we can interpret these plots to then inform a contrast approach, and/or apply a Tukey's cell mean comparison. This allows us to narrow down and explain the specific interactions and/or identify if there is a true difference in means.

Part II: What feature of the design provides 9 df for the Residuals? [3 pts]

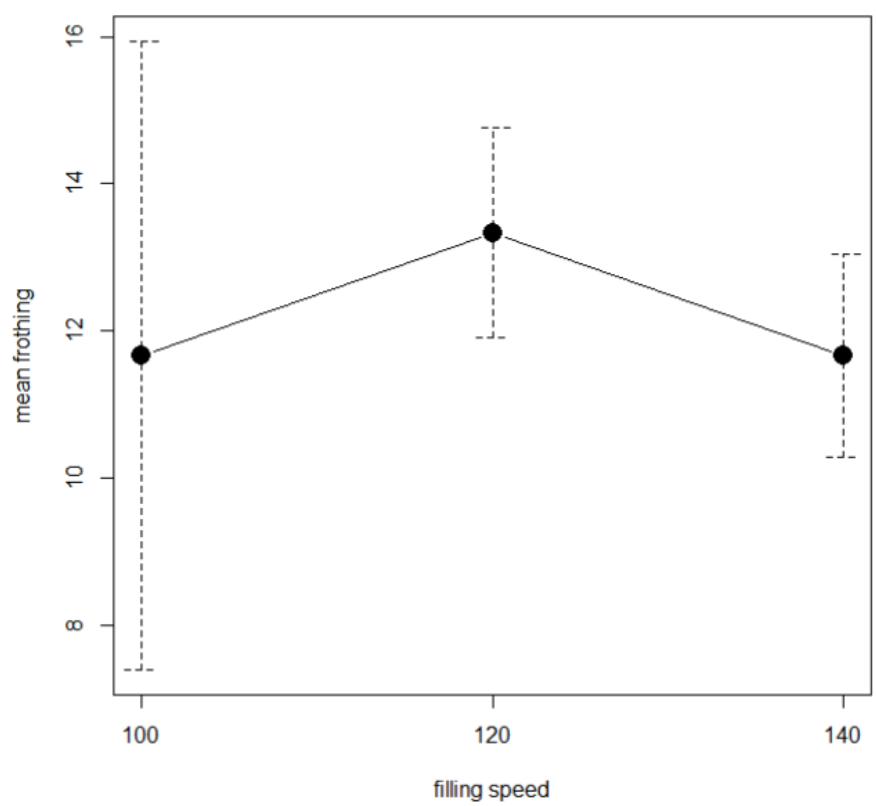
Answer: This is because it is a two-factor design, this multiplies the levels of each factor together to create a total 9 degrees of freedom. The function call in R contained both Nozzle and Speed as well as their interaction.

Part III: The mean frothing score when the Speed is 140 and the Nozzle type is 2 is 10. Give a practical interpretation of the square root of the MSE Residuals as it relates to this combination of Speed and Nozzle type. [4 pts]

Answer: This MSE residual has a square root of 2.66, this means that the average response recorded was ± 2.66 from the regression line. Without knowing the specific regression line, and quickly looking at the given data, we can presume a froth value close to 12.66 was predicted by the model for speed 140 nozzle type 2.

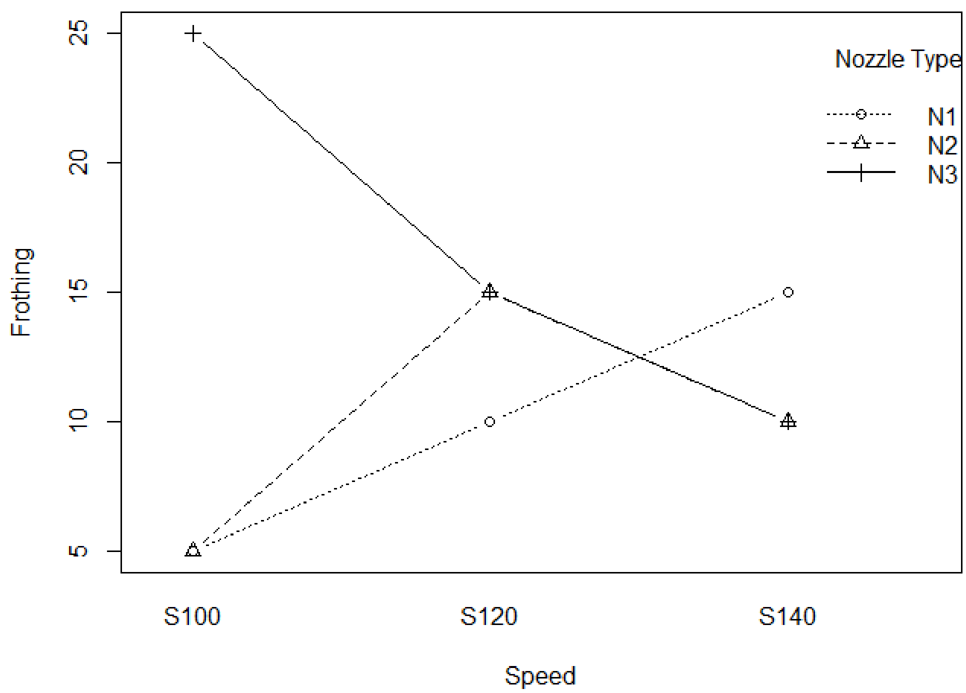
Part C

Someone who doesn't know as much about statistics as you do, produced a main effects plot for Filling Speed in order to show others that it really doesn't matter what Filling Speed one operates at if the goal is to have minimal frothing. This person, who has not taken STAT 4025/5025, concludes that "filling speed doesn't impact frothing and so don't worry about the speed at which the containers are filled". Explain why this statement is not exactly correct. In your explanation, use both a hypothesis test as well as an appropriate plot. [6 pts – 3 for appropriate plot and 3 for the appropriate test]



Answer:

Although they have the wrong plot, the wide interval for speed 100 already indicates that there could be other effects in play, as we know from the ANOVA table it is the interaction. So, we will produce an interaction plot for the data:



Here the interaction plot shows a distinct interaction between nozzle type and speed. The wide interval shown on the original plot is clearly explained as speed 100 is being influenced by nozzle type, and with nozzle 3 being substantially worse than nozzles 1 and 2. We also see that at higher speeds nozzle 3 begins to increase in its effectiveness and overtakes nozzle 1. Each speed has a different best nozzle(s) showing that all speeds have some interaction with nozzle type. To test this hypothesis, we can perform a TukeyHSD test, and identify if all nozzles and speeds have a true difference. Doing so we get the following output:

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = m1)

$Speed
               diff       lwr      upr     p adj
S120-S100  1.666667 -2.631907 5.965241 0.5475786
S140-S100  0.000000 -4.298574 4.298574 1.0000000
S140-S120 -1.666667 -5.965241 2.631907 0.5475786

$Nozzle
              diff       lwr       upr    p adj
N2-N1 1.776357e-15 -4.298574  4.298574 1.000000
N3-N1 6.666667e+00  2.368093 10.965241 0.004859
N3-N2 6.666667e+00  2.368093 10.965241 0.004859
```

We see that overall speed does in fact not have a significant impact on the frothing, but nozzle selection does. However, we know that speed 100 seemed to have the widest variation and most interaction. Looking more closely at the specific p-values we do affirm that nozzle choice is important, but we also see that for nozzle 3, speed can make a significant difference.

Comparison	P-Val
S120:N3-S100:N3	0.0661415
S140:N3-S100:N3	0.0057597

Overall, the student's conclusions were not incorrect- but they were incomplete. ***On average*** speed does not make a significant difference. However, there are important cases where speed does make a significant difference depending on the nozzle chosen, excluding this information leaves the conclusion incomplete.

Question 3

"An experiment was designed to investigate the effects of three factors, each at two levels, on a quality response related to the manufacturing of truck leaf springs. The factors were furnace temperature (A), furnace heating time (B) and transfer time between the furnace and the forming machine (C). Higher quality measures are better than lower values. A 2^3 factorial experiment was run. Raw levels of the factors are A – 1840-1880; B – 23-25 seconds; C- 10-12 seconds. The data is given below."

A	B	C	y1
-1	-1	-1	32
1	-1	-1	35
-1	1	-1	28
1	1	-1	31
-1	-1	1	48
1	-1	1	39
-1	1	1	28
1	1	1	29

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	33.75	1.25	27.0	0.0000112 ***
Heat_Time	-4.75	1.25	-3.8	0.0191 *
Trans_Time	2.25	1.25	1.8	0.1462
Heat_Time:Trans_Time	-2.75	1.25	-2.2	0.0927 .

Part A

Show how to obtain the coded value of -1 from the natural units associated with the Heat_Time variable. [5 pts]

Answer:

With B representing the coded value for Heat_Time we can produce the coded value following the algorithm given for all responses:

⇨ Coded B = (response – median_response) / (0.5 * response_range)

⇨ Coded B = (R_n – 24) / (0.5 * 2)

⇨ Coded B = (R_n – 24) / 1

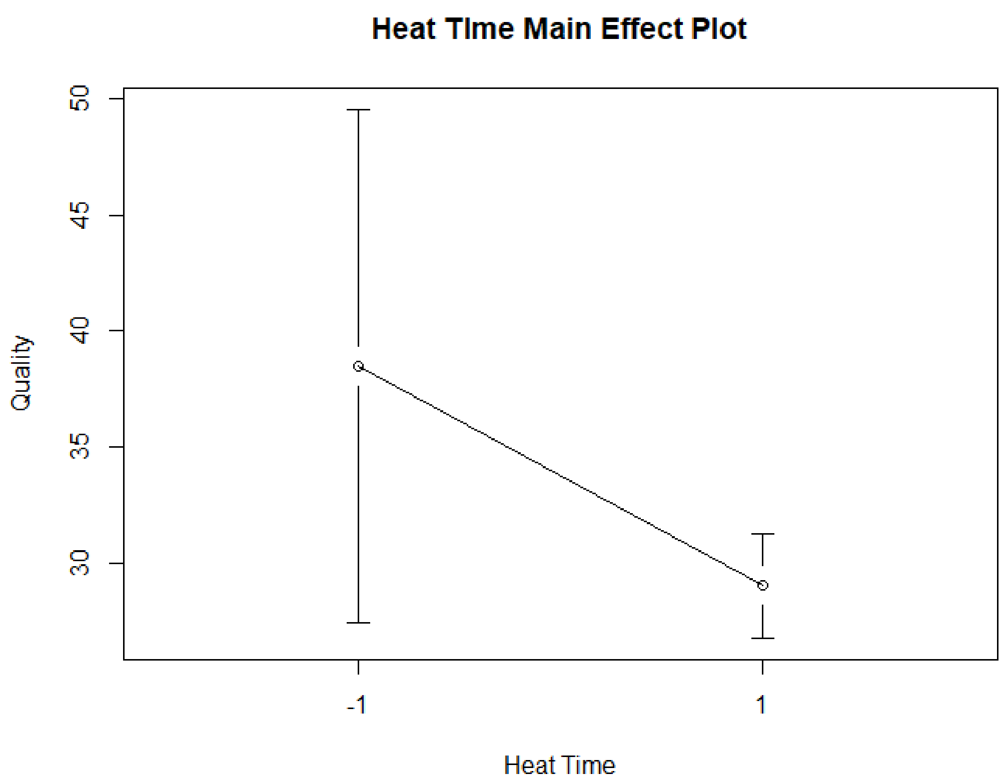
⇨ Coded B = 1 or -1

If the resultant value is not already 1 or -1 then Coded(R) ≤ 0 it is **represented** in the table as "--" or -1, otherwise it is **represented** in the table as "++" or 1. These non 1/-1 values can be used raw in other calculations. The treatments are administered, and responses recorded in the order respective to the factorial model. That is alternating code A, followed by twice that ordering for B, followed by twice the previous ordering and so forth. This covers all combinations of high and low factors.

Part B

Provide the main effect plot for Heat_Time and interpret the estimated coefficient of -4.75 provided in the table above – make sure that your interpretation of the slope is in terms of the natural units (ex. "As the heating time increases…."). [8 pts – 5 pts for the plot and 3 pts interpretation]

Answer:



As shown in the above main effect plot, lower heat time appears to produce generally better quality. The wide range hints towards a potential interaction. Looking at the model summary we see there is a significant (< 0.10) interaction. The estimated coefficient of -4.75 for

Heat_Time supports what we find in the main effect plot. As the heat time increases the estimated quality is reduced, the reducing factor has a slope of -4.75. In a regression equation other factors could still result in an increasing response.

Part C

This problem is what is known as a response surface problem. What is the fundamental difference in response surface problems than all of the other problems we have looked at this semester? [5 pts]

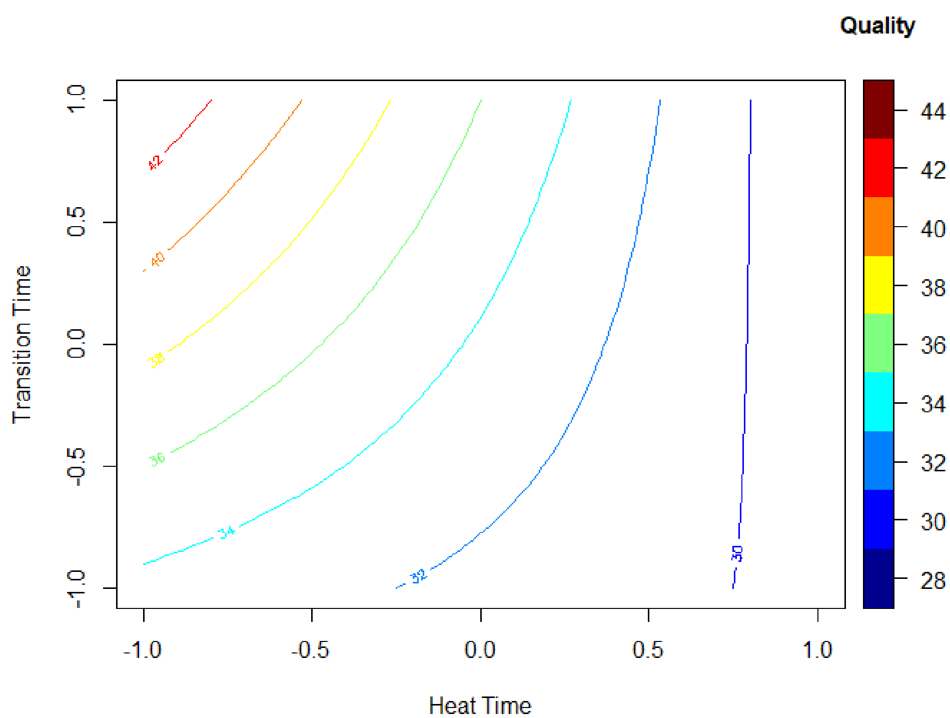
Answer:

A response surface problem is when two effects are compared against a response AND those two effects have some complex interaction. Response surface problems are represented in some 3-dimensional model (contour plot, three axis, etc.). Using the response surface method, we can identify the ‘sweet spot’ for combination factors, we can also find areas with the highest relative average (high robustness).

Part D

Interpret the interaction effect (i.e. assume $\alpha = 0.10$) using a plot as well as discussion. [7 pts - 4 for plot; 3 for interpretation]

Answer:



According to the p-value returned in the model summary, the interaction p-value is below 0.10 which means that it is a significant interaction. Referring to our contour plot we see that the interaction pertains primarily to low heat time. At heat times greater than 0.5 we see almost zero interaction; the quality is consistently low. While at heat times less than -0.5 the quality becomes highly dependent on transition time. It appears that at high heat times the leaf springs get too hot and permanently damage their quality, while at lower and more appropriate heat times it becomes important for transitions to be close to 12 seconds. If I were designing a second experiment, I would study transition times past 12 seconds to see if the benefits drop off/reverse.

The contour plot does not appear to be flattening in either direction at the lowest transition and heat times, so there may be room to improve. Potentially a spot with less variance and a higher average quality can be found.

Question 4

"An experiment was conducted to study the effects of three types of fertilizers (1,2, and 3) and two tomato varieties (A and B) on tomato yield in an agricultural research station. Three square fields were randomly selected. Each field was divided into three strips of equal size that were randomly treated by the fertilizers. Each strip was further divided into two areas, where the two varieties were planted, respectively. The response was the yield of tomato over the whole season. Sums of squares from the data generated from this experiment are found using the lm() function and the results are provided in the table below."

Source	df	SSQ
Field | r = 3	(r-1)= 2	514
Fertilizer | a = 3	(a-1)= 2	522
Variety | b = 2	(b-1)= 1	76
Fertilizer*Variety | a x b	(a-1)(b-1)= 2	17
Field*Fertilizer | error(a)	(r-1)(b-1)= 4	47
Residual | error(b)	a(r-1)(b-1)= 4	29

Part I

The experiment is a split-plot design. Which factor is the whole plot factor? Which is the subplot factor? What are the whole plots? What are the subplots? [4 pts]

Answer:

For this study, the whole plots are the three strips divided from the three fields, there are 9 whole plots. The whole plot factor is fertilizer. The subplots are the two areas chosen from each whole plot, there are 18 subplots. The subplot factor is tomato variety.

Part II

Explain the ways that blocking was used in this design [4 pts]

Answer:

Firstly, three square fields were randomly selected for the first design. Each one of these fields is the original blocking. Then each field (block) was split into three strips that would have fertilizer applied to them. Each of these strips (the whole plots) were the blocks for the subplot design. The strips were split in half and each variety of tomato was applied. Our main blocks are the 3 square fields, and the inner blocks are the 9 divisions of the square fields.

Part III

Fill in the degrees of freedom column for the design [3 pts]

Answer:

Refer to the table at the beginning of question 4.

Part IV

Test if the main effects for Fertilizers are significant using $\alpha = 0.05$ and use a p-value to make your decision. Show all work for credit. [3 pts]

Answer:

F-value of Fertilizer = MSa / MSerrorA = 261 / 11.75 = 22.213

Df between = k – 1 = 2

Df within = N - k = 6

P-value = 0.001685

With p-values less than 0.05 we conclude that the main effects for fertilizers are significant.

Part V

Test if the main effect of Variety is significant using $\alpha = 0.05$ and use a p-value to make your decision. Show all work for credit. [3 pts]

Answer:

F-value of Variety = MSb / MSerrorB = 76 / 7.25 = 10.483

Df between = k – 1 = 8

Df within = N - k = 9

P-value = 0.000959

With p-values less than 0.05 we conclude that the main effects for variety are significant.

Part VI

Test if the interaction effect of Variety*Fertilizer is significant using $\alpha = 0.05$ and use a p-value to make your decision. Show all work for credit. [3 pts]

Answer:

F-value of Variety = MSab / MSerrorB = 8.5 / 7.25 = 1.172

Df between = k – 1 = 10

Df within = N - k = 17

P-value = 0.371738

With p-values greater than 0.05 we conclude that the interaction between variety and fertilizer are not significant.

Part 'VIII'

Provide the line of code for using the lmer() function in R to fit the model for this data. Which R library do you need to have loaded in order for R to perform the correct tests for the Fertilizer effect? [4 pts]

Answer:

The code I would use would be:

```
M3 <- lmer(Yield ~ Field + Fertilizer + Tomato_Variant + Fertilizer:Tomato_Variant +
        (1 | Filed:Fertilizer), data = q4)
anova(M3)
```

For this code to run properly we need the library 'lmerTest' which itself includes/requires the library 'lme4'.

Code Transcript:

```
library(tidyverse)


## problem 2

p2 <- read.csv("exam2p2.csv", header = T)

m1 <- lm(Froth~Speed+Nozzle+Speed:Nozzle,data = p2)

summary(m1)

anova(m1)


p2$resids <- residuals(m1)


interaction.plot(p2$Speed, p2$Nozzle, p2$Froth,
        fun = mean,
        type = "b",
        pch=c(1:3),
        legend = TRUE,
        trace.label = "Nozzle Type",
        fixed = FALSE,
        xlab = "Speed",
        ylab = "Frothing")


TukeyHSD(aov(m1),conf.level = 0.95)
```

```
# problem 3b

p3 <- read.csv("Part3b.csv", header = T)

m2 <- lm(Qual~Heat_Time+Trans_Time+Heat_Time:Trans_Time,data=p3)


summary(m2)

anova(m2)


p3$resids <- residuals(m2)


library(gplots)


plotmeans(Qual~Heat_Time,xlab="Heat Time",ylab="Quality", p=.95,
      main="Heat TIme Main Effect Plot",barcol="black", n.label=F,data=p3)





B_coded<-seq(-1,1,length.out=50)

A_coded<-seq(-1,1,length.out=50)
```

```
z<- outer(B_coded,A_coded,function(a,b)
  predict(m2,newdata=
          data.frame(Heat_Time=a,Trans_Time=b)))
z


library(plot3D)


contour2D(x = B_coded,y = A_coded, z = z,
        xlab = "Heat Time",
        ylab = "Transition Time",
        clab = "Quality")


# problem 4


pf(22.213,df1=2,df2=6,lower.tail=F)
pf(10.483,df1=8,df2=9,lower.tail=F)
pf(1.172,df1=10,df2=17,lower.tail=F)
```