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Stat 4015
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1. An experiment is to be conducted to compare 4 cooking temperatures and 3 mixtures of alloys on strength measurements of steel rods. The cooking period is 2 hours, so that only 4 cooking periods can be conducted on a business day (the temperatures are randomly assigned to periods). The experimenter decides she will assign the 3 mixtures at random to the 3 positions in the oven (experience implies there are no position effects), separately for the 4 runs on a given day. She repeats the experiment over 3 days (randomizing separately on each day). Her assistant provides her the following sequences of random numbers for temperature and mixtures – using the random numbers, she decides the highest numbers go first. [14 pts]

A. Give the assignment of treatments to experimental positions (in each cell, enter T_i/M_j where i =Temperature level, j =Mixture level). [10 pts]

		Day 1				Day 2				Day 3			
	Temp	1	2	3	4	1	2	3	4	1	2	3	4
	Ran#	0.96	0.91	0.16	0.22	0.21	0.34	0.28	0.27	0.75	0.23	0.37	0.20
		Period 1			Period 2			Period 3			Period 4		
Day 1	Mix	1	2	3	1	2	3	1	2	3	1	2	3
	Ran#	0.10	0.54	0.58	0.59	0.70	0.79	0.26	0.74	0.72	0.83	0.17	0.56
Day 2	Mix	1	2	3	1	2	3	1	2	3	1	2	3
	Ran#	0.29	0.64	0.43	0.76	0.65	0.56	0.97	0.28	0.06	0.83	0.76	0.64
Day 3	Mix	1	2	3	1	2	3	1	2	3	1	2	3
	Ran#	0.91	0.55	0.83	0.43	0.69	0.19	0.98	0.79	0.36	0.26	0.78	0.11

	Day1	Day1	Day1	Day2	Day2	Day2	Day3	Day3	Day3
	Pos1	Pos2	Pos3	Pos1	Pos2	Pos3	Pos1	Pos2	Pos3
Pe r1	T1 M3	T1 M2	T1 M1	T2 M2	T2 M3	T2 M1	T1 M1	T1 M3	T1 M2
Pe r2	T2 M3	T2 M2	T2 M1	T3 M1	T3 M2	T3 M3	T3 M2	T3 M1	T3 M3
Pe r3	T4 M2	T4 M3	T4 M1	T4 M1	T4 M2	T4 M3	T2 M1	T2 M2	T2 M3
Pe r4	T3 M1	T3 M3	T3 M2	T1 M1	T1 M2	T1 M3	T4 M2	T4 M1	T4 M3

B. What is the name of this design? Explain.

This is design is a 'split plot design' having whole plot treatments and subplot treatments separated by blocks. The blocks are days and the whole plot is the period treated with/by temperature and subplot are treated with/by mixtures.

2. The file laundry.csv contains data from an experiment that was conducted to determine the impact of wool treatment (1- untreated, 2- 15 seconds alcoholic potash, 3- 5 minutes alcoholic potash, and 4- 10 minutes alcoholic potash) and drying cycle speed (200 revolutions/minute – 1400 revolutions/min) on the percent of shrinkage in a wool fabric. There were 4 dryers available for the study. A total of 16 pieces of wool were available for the study – 4 were untreated, 4 were exposed to 15 seconds of alcoholic potash, 4 were exposed to 5 minutes of alcoholic potash and 4 were exposed to 10 minutes of alcoholic potash. Each dryer was randomly assigned one piece of wool from each of the wool treatment groups. After this assignment, each piece of wool was cut into 7 strips – 28 strips available for each dryer.

- A. Determine if there is a difference in mean percent shrinkage among the four wool treatment types using output from the `lm()` function. Write your hypotheses. Provide the appropriate test statistic value and determine your p-value [6 pts – 2 for hypotheses, 2 for test statistic and 2 for the p-value].

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \text{ vs } H_A: \text{At least one } \mu \text{ is different for } i = 1, 2, 3, 4$$

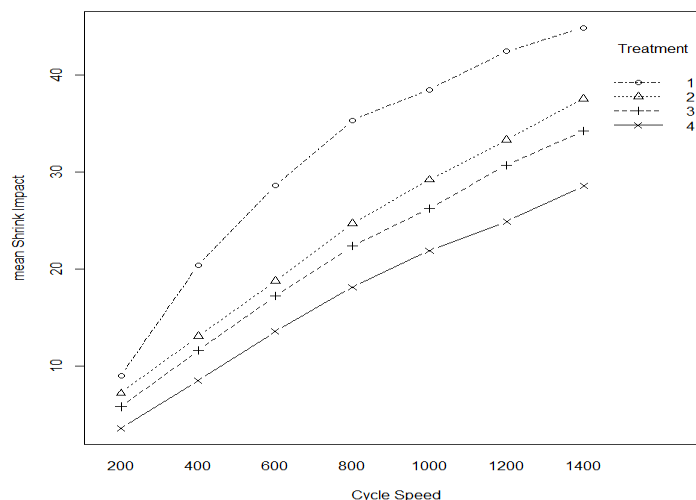
- B. Repeat your test from part A but use the `lmer()` function. Simply show the appropriate output and show that your test statistic value is the same as it is in part A. [2 pts]

Type III Analysis of Variance Table with Satterthwaite's method

	Sum Sq	Mean Sq	NumDF	DenDF	F value	Pr(>F)
Treatment	1086.9	362.29	3	9	78.8399	8.803e-07 ***
Machine	44.8	14.95	3	9	3.2526	0.07388 .
Cycle_Speed	11051.8	1841.96	6	72	400.8395	< 2.2e-16 ***
Machine:Cycle_Speed	37.8	2.10	18	72	0.4571	0.96769

 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
 F = 78.839
 P = 2.2e-16

- C. Produce the treatment*cycle speed interaction plot with cycle speed on the horizontal axis. [3 pts]



- D. Write out the betas associated with the mean shrinkage percentage for treatment 1 at a cycle speed of 1400 RVM. Do the same for mean shrinkage percentage for treatment 2 at a cycle speed of 1400 RVM. [4 pts].

$$T_i = \text{Treatment}, M_j = \text{Machine}, S_k = \text{Speed of Cycle}$$

$$\bar{Y}_{T1,M,S1400} = \beta_0 + \beta_{M2} + \beta_{M3} + \beta_{M4} + \beta_{S1400} + \beta_{M2S1400} + \beta_{M3S1400} + \beta_{M4S1400} + \varepsilon_{ijk}$$

$$\bar{Y}_{T2,M,S1400} = \beta_0 + \beta_{T2} + \beta_{M2} + \beta_{M3} + \beta_{M4} + \beta_{S1400} + \beta_{T2M2S1400} + \beta_{T2M3S1400} + \beta_{T2M4S1400} + \varepsilon_{ijk}$$

- E. Provide a 95% confidence interval for the difference in mean shrinkage percentage between treatment 1/1400 RVM and treatment 2/1400 and interpret your interval. [4 pts- 2 pts for correct R and 2 pts for correct interpretation].

Treat 1 Speed 1400 vs. Treat 2 Speed 1400

Lwr - Est - Upr

4.89 7.30 9.70

With 95% confidence we can say that the true mean shrinkage percentage difference between treatments 1 and 2 at 1400 ROM is between 4.9 and 7.9%.

3. An experiment was conducted to study the efficacy of five different pesticides upon the yield of corn. Ten Nebraska farms were identified for participation and each farm set aside 5 one-acre plots for application of the pesticides (1 pesticide on each plot). Pesticides were randomly assigned to each of the five one acre plotson each farm. The partially completed ANOVA table from the analysis of this data is below: [11 pts for 4025 and 14 pts for 5025]

Source	Df	SS	MS	F
Blocks	9	135	15	
Treatments	4	100	25	12.5
Residual	36	72	2	
Total	49	307	6.27	

- a. What is the name of this experimental design? Justify your answer. [2 pts]

This is a randomized complete block design. This is because there are 10 farms (blocks) of which 5 pesticides are randomly assigned to the 5 plots on each farm.

- b. Complete the ANOVA table above. [4 pts]
c. Write the linear model which corresponds to the appropriate analysis of this data.[2 pts]

$$y_{ij} = \mu + \alpha_{ij} + \beta_j + \varepsilon_{ij}$$

- d. Determine if there is a significance difference among the means of the five treatments. Be sure to state your hypotheses, provide the test-statistic, computethe p-value and state a conclusion. Use a significance level of 0.05. [3 pts]

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \quad vs \quad H_A: \text{At least one } \mu \text{ is different}$$

$$F_{Treatment} = 12.5$$

$$> \text{pf}(12.5, \text{df1}=4, \text{df2}=36, \text{lower.tail}=F)$$

$$[1] 1.785876\text{e-}06$$

Because the p-value is less than 0.05 we must reject the null hypothesis that there is no significant difference among treatments.