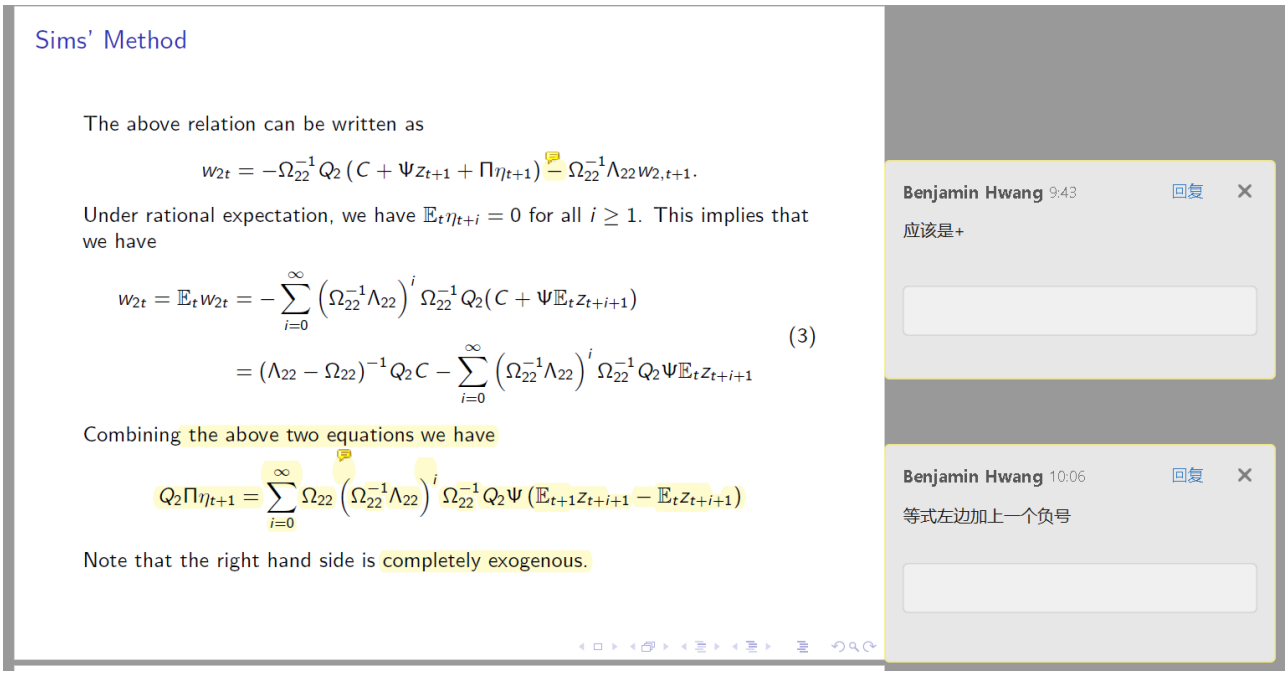
Report and Questions about the Slides, Linear RE

Firstly, I try my best and find two typos.



But I am afraid that the rest part may be not rigorous.

1. There emphasizes $Q_1 \Pi \eta_t = \phi Q_2 \Pi \eta_t$, why there must exist linear relation between the two terms?
2. next, the necessary and sufficient condition can only cause the existence of ϕ, not including the uniqueness.

Sims' Method

In order for η_{t+1} to exists, a necessary and sufficient condition is that

$$\mathcal{R}\left(\sum_{i=0}^{\infty}\Omega_{22}\left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i}\Omega_{22}^{-1}Q_2\Psi\left(\mathbb{E}_{t+1}z_{t+i+1}-\mathbb{E}_t z_{t+i+1}\right)\right)\subset\mathcal{R}(Q_2\Pi),$$

or, equivalently

$$\mathcal{R}\left(\left\{\Omega_{22}\left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i}\Omega_{22}^{-1}Q_2\Psi\right\}_{i=0}^{n-k-1}\right)\subset R(Q_2\Pi).$$

Given that η_t has a solution, in order to completely pin down the whole system, we need to be able to know $Q_1\Pi\eta_t$ from the value of $Q_2\Pi\eta_t$. That is, there exists a unique Φ such that

$$Q_1\Pi\eta_t=\Phi Q_2\Pi\eta_t$$

for all t. A necessary and sufficient is that

$$\mathcal{R}(\Pi' Q_1')\subset\mathcal{R}(\Pi' Q_2').$$

Benjamin Hwang 12:09 回复

不懂为何两者一定要有线性关系

添加回复...

3. the dimension of ϕ is not clear, as I demonstrate below.
4. Is the "Premultiplying" operation called 左乘? I am afraid that I have no idea about how this operation applied to (2), and obtain such formula.

Sims' Method

Premultiplying $[I-\Phi]$ to the system (2) and combining the solution part (3) yields

$$\begin{bmatrix}\Lambda_{11} & \Lambda_{12}-\Phi\Lambda_{22}\\ 0 & I\end{bmatrix}\begin{bmatrix}w_{1t}\\ w_{2t}\end{bmatrix}=\begin{bmatrix}\Omega_{11} & \Omega_{12}-\Phi\Omega_{22}\\ 0 & 0\end{bmatrix}\begin{bmatrix}w_{1,t-1}\\ w_{2,t-1}\end{bmatrix}+\begin{bmatrix}Q_1-\Phi Q_2\\ (\Lambda_{22}-\Omega_{22}^{-1})^{-1}Q_2\end{bmatrix}C+\begin{bmatrix}Q_1-\Phi Q_2\\ 0\end{bmatrix}\Psi z_t+\begin{bmatrix}0\\ -\sum_{i=0}^{\infty}\left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i}\Omega_{22}^{-1}Q_2\Psi\mathbb{E}_t z_{t+i+1}\end{bmatrix}$$

Denote the above equation by

$$B_0 w_t=Bw_{t-1}+B_c C+B_z\Psi z_t+s_t$$

The solution is then given by

$$y_t=Zw_t=ZB_0^{-1}BZ^{*}y_{t-1}+ZB_0^{-1}B_c C+ZB_0^{-1}B_z\Psi z_t+ZB_0^{-1}s_t$$

Benjamin Hwang 11:56 回复

没人说phi一定是方阵呀，如果是方阵这就是说，w_1, w_2有相同的维数，就是说\lambda_i<1的特征根数量和\lambda_i>1的数量相等，没这个假设。。