货币经济学 第二次大作业

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货币经济学 HW2

1. (4.4) 对数线性化：

$$\hat{P}_t^* = \frac{\theta}{\theta-1}\frac{E_t\sum_{i=0}^{\infty}w^i\beta^i C_{t+i}^{1-\sigma}\varphi_{t+i}P_{t+i}^{\theta}}{E_t\sum_{i=0}^{\infty}w^i\beta^i C_{t+i}^{1-\sigma}P_{t+i}^{\theta-1}} \qquad \varphi_t=\frac{w_t}{Z_t},\ \forall t.$$

$$= \frac{E_t\sum_{i=0}^{\infty}w^i\beta^i(C^{ss})^{1-\sigma}\varphi^{ss}(P^{ss})^{\theta}\cdot((1-\sigma)\hat{C}_{t+i}+\hat{\varphi}_{t+i}+\theta\hat{P}_{t+i})}{\frac{1}{1-w\beta}(C^{ss})^{1-\sigma}(P^{ss})^{\theta}\varphi^{ss}} - \frac{E_t\sum_{i=0}^{\infty}w^i\beta^i(C^{ss})^{1-\sigma}(P^{ss})^{\theta-1}\cdot((1-\sigma)\hat{C}_{t+i}+(\theta-1)\hat{P}_{t+i})}{\frac{1}{1-w\beta}(C^{ss})^{1-\sigma}(P^{ss})^{\theta-1}}$$

$$=(1-w\beta)E_t\sum_{i=0}^{\infty}w^i\beta^i((1-\sigma)\hat{C}_{t+i}+\hat{\varphi}_{t+i}+\theta\hat{P}_{t+i})-(1-w\beta)E_t\sum_{i=0}^{\infty}w^i\beta^i((1-\sigma)\hat{C}_{t+i}+(\theta-1)\hat{P}_{t+i})$$

$$=(1-w\beta)E_t\sum_{i=0}^{\infty}w^i\beta^i(\hat{\varphi}_{t+i}+\hat{P}_{t+i})$$

$$\Rightarrow \hat{P}_t^* = (1-w\beta)\left(w^0\beta^0(\hat{\varphi}_t+\hat{P}_t)+\frac{1}{1-w\beta}\overset{w\beta}{\cancel{(1-w\beta)}}E_t\sum_{i=0}^{\infty}w^i\beta^i(\hat{\varphi}_{t+1+i}+\hat{P}_{t+1+i})\right)$$

$$=(1-w\beta)\left(w^0\beta^0(\hat{\varphi}_t+\hat{P}_t)+\frac{w\beta}{1-w\beta}\hat{P}_{t+1}^*\right)$$

$$=(1-w\beta)w^0\beta^0(\hat{\varphi}_t+\hat{P}_t)+w\beta\hat{P}_{t+1}$$

$$\Rightarrow \underline{\hat{P}_t^*-\hat{P}_{t-1}}=(1-w\beta)(\hat{\varphi}_t+\hat{P}_t-\hat{P}_{t-1})+w\beta(\underline{\hat{P}_{t+1}^*-\hat{P}_t}+\hat{P}_t-\hat{P}_{t-1}) \qquad ①$$

(4.5)：首先看稳态：

$$P_t^{1-\theta}=(1-w)P_t^{*(1-\theta)}+wP_{t-1}^{1-\theta} \Rightarrow P^{ss}=P^{*ss}$$

log linear：

$$\hat{P}_t=\frac{(1-w)(P^{*ss})^{1-\theta}(\hat{P}_t^*)+w(P^{ss})^{1-\theta}\cdot\hat{P}_{t-1}}{(1-w)(P^{*ss})^{1-\theta}+w(P^{ss})^{1-\theta}}\overset{P^{ss}=P^{*ss}}{=\!=\!=}(1-w)\hat{P}_t^*+w\hat{P}_{t-1} \qquad ②$$

~~将②代入①~~，同时注意到 $\pi_t=\pi_t-0=\tilde{\pi}_t \Rightarrow \widetilde{\frac{P_t-P_{t-1}}{P_{t-1}}}=\frac{\tilde{P}_t-\tilde{P}_{t-1}}{P^{ss}}=\hat{P}_t-\hat{P}_{t-1}$

故而有②式变为：$\pi_t=(1-w)(\hat{P}_t^*-\hat{P}_{t-1})$

代①式变为：$\frac{\pi_t}{1-w}=(1-w\beta)(\hat{\varphi}_t+\pi_t)+w\beta(\frac{\pi_{t+1}}{1-w}+\pi_t)$

$$\Rightarrow \pi_t=\beta E_t\pi_{t+1}+\frac{(1-w)(1-w\beta)}{w}\hat{\varphi}_t \qquad ③$$

此即为 New Keynesian Phillips Curve.

2. $\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\psi_t \frac{C_t^{1-\sigma}}{1-\sigma} + \gamma \frac{m_t^{1-b}}{1-b} - \chi_t \frac{n_t^{1+\eta}}{1+\eta} \right]$ s.t. $C_t + m_t + b_t = w_t n_t + \frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1}) b_{t-1}}{1+\pi_t} + \pi_t$

F.O.C. $\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\psi_t \frac{C_t^{1-\sigma}}{1-\sigma} + \gamma \frac{m_t^{1-b}}{1-b} - \chi_t \frac{n_t^{1+\eta}}{1+\eta} \right] + \sum_{t=1}^{\infty} \lambda_t \left(w_t n_t + \frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1}) b_{t-1}}{1+\pi_t} + \pi_t - (C_t + m_t + b_t) \right)$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t \psi_t C_t^{-\sigma} - \lambda_t = 0. \Rightarrow \lambda_t = \beta^t \psi_t C_t^{-\sigma}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = \beta^t \gamma m_t^{-b} + \frac{\lambda_{t+1}}{1+\pi_{t+1}} - \lambda_t = 0 \Rightarrow \mathbb{E}_t \frac{\beta \psi_{t+1} C_{t+1}^{-\sigma}}{1+\pi_{t+1}} + \gamma m_t^{-b} = \psi_t C_t^{-\sigma} \quad ④$$

$$\frac{\partial \mathcal{L}}{\partial b_t} = \mathbb{E}_t \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} - \lambda_t = 0 \Rightarrow \mathbb{E}_t \beta \frac{\psi_{t+1} C_{t+1}^{-\sigma} (1+i_t)}{(1+\pi_{t+1})} = \psi_t C_t^{-\sigma} \quad ⑤$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = -\beta^t \chi_t n_t^{\eta} + \lambda_t w_t = 0 \Rightarrow \chi_t n_t^{\eta} = \psi_t C_t^{-\sigma} w_t \quad ⑥$$

$$n_t^{\eta} = \frac{\psi_t C_t^{-\sigma} w_t}{\chi_t} \Rightarrow \text{劳动供给: } n_t = \left(\frac{\psi_t C_t^{-\sigma} w_t}{\chi_t} \right)^{\frac{1}{\eta}}$$

$\eta > 0$, 那么, 在正向消费偏好冲击下, 将增加劳动供给.

在正向劳动厌恶冲击(χ_t上升), 将减少劳动供给.

3. 由于中间产品需求: $c_{tj} = \left(\frac{P_{tj}}{P_t} \right)^{-\theta} c_t$

市场出清有: $y_t = C_t$. ⑦

$$n_t = \int_0^1 n_{tj} dj = \int_0^1 \frac{c_{tj}}{z_t} dj = \frac{c_t}{z_t} \int_0^1 \left(\frac{P_{tj}}{P_t} \right)^{-\theta} dj \quad ⑧$$

⑧ ⇒ log linear: $\hat{n}_t = \hat{c}_t - \hat{z}_t + \widehat{\int_0^1 \left(\frac{P_{tj}}{P_t} \right)^{-\theta} dj} = \hat{c}_t - \hat{z}_t + \int_0^1 \frac{\left(\frac{P_j^{ss}}{P^{ss}} \right)^{-\theta} \times (-\theta) \times (\hat{P}_{tj} - \hat{P}_t)}{\int_0^1 \left(\frac{P_j^{ss}}{P^{ss}} \right)^{-\theta} dj} dj$

$$= \hat{c}_t - \hat{z}_t + (-\theta) \cdot \int_0^1 (\hat{P}_{tj} - \hat{P}_t) dj$$

又由于定义: $\int_0^1 (\hat{P}_{tj} - \hat{P}_t) dj = 0$.

$$\Rightarrow \hat{n}_t = \hat{c}_t - \hat{z}_t \quad ⑨.$$

这时 $\hat{\varphi}_t = \widehat{\left(\frac{w_t}{z_t} \right)} = \hat{w}_t - \hat{z}_t$.

由⑥ log linear: $\hat{\chi}_t + \eta \hat{n}_t = \hat{\psi}_t - \sigma \hat{c}_t + \hat{w}_t$

$$\Rightarrow \hat{\varphi}_t = \hat{\chi}_t + \eta \hat{n}_t - \hat{\psi}_t + \sigma \hat{c}_t - \hat{z}_t \cancel{= \hat{\chi}_t - \hat{\psi}_t + (\eta + \sigma)(\hat{y}_t - \hat{y}_t^f)}$$

由于价格完全弹性时: $\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{w_t}{z_t} \Rightarrow \hat{\varphi}_t = 0$.

此时 $0 = \hat{\chi}_t + \eta(\hat{y}_t^f - \hat{z}_t) - \hat{\psi}_t + \sigma \hat{y}_t^f - \hat{z}_t \Rightarrow \hat{y}_t^f = \frac{-\hat{\chi}_t + \hat{\psi}_t + (\eta + 1)\hat{z}_t}{\eta + \sigma}$

可以看到 如有正向消费偏好冲击(ψ_t上升). $\hat{y}_t^f$ 提高

如有正向劳动厌恶冲击(χ_t上升) $\hat{y}_t^f$ 下降.

4. $\hat{\varphi}_t = (\eta + \sigma)(\hat{y}_t - \hat{y}_t^f)$

而 Phillips Curve: $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\omega)(1-\omega\beta)}{\omega} (\eta + \sigma)(\hat{y}_t - \hat{y}_t^f)$

记 $K = \frac{(1-\omega)(1-\omega\beta)}{\omega} (\eta + \sigma)$. i.e.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + K \cdot (\hat{y}_t - \hat{y}_t^f)$$

这时冲击将影响 Phillips Curve.

5. 我将ψ_t设置为$AR(1)$的形式，回归系数为0.9。

5.

1 π
2 x
3 $\psi \rightarrow$ 自己设
$\chi=1$ 常数.
4 z
5 i
6 v
7 $\hat{c}$
8 $\tilde{r}$
9 $\hat{n}$
10 $\hat{m}$

Phillips Curve: 定义 $x_t = \hat{y}_t - \hat{y}_t^f$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$$

$$\Rightarrow -\beta \pi_{t+1} = \beta(\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) - \pi_t - \kappa x_t \qquad (1)$$

IS: Curve:

Euler Equation: ⑤: $\mathbb{E}_t \beta \frac{\psi_{t+1} C_{t+1}^{-\sigma}(1+i_t)}{1+\pi_{t+1}} = \psi_t C_t^{-\sigma}$

$\Rightarrow$ 对数线性化:

$$\hat{\psi}_{t+1} - \pi_{t+1} - \sigma \hat{C}_{t+1} = \hat{\psi}_t - \sigma \hat{C}_t - \hat{i}_t + (\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) + \sigma(\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_{t+1})$$

$$\Rightarrow \frac{\eta}{\eta+\sigma}\hat{\psi}_{t+1} + \cancel{\frac{\sigma}{\eta+\sigma}\hat{\chi}_{t+1}} - \frac{\sigma(\eta+1)}{\eta+\sigma}\hat{z}_{t+1} - \sigma x_{t+1} - \pi_{t+1} = -\hat{i}_t + \frac{\eta}{\eta+\sigma}\hat{\psi}_t + \cancel{\frac{\sigma}{\eta+\sigma}\hat{\chi}_t} - \frac{\sigma(\eta+1)}{\eta+\sigma}\hat{z}_t - \sigma x_t$$
$$+ \frac{\sigma}{\eta+\sigma}(\mathbb{E}_t \hat{\psi}_{t+1} - \hat{\psi}_{t+1}) - \cancel{\frac{\sigma}{\eta+\sigma}(\mathbb{E}_t \hat{\chi}_{t+1} - \hat{\chi}_{t+1})} + \frac{\sigma(\eta+1)}{\eta+\sigma}(\mathbb{E}_t \hat{z}_{t+1} - \hat{z}_{t+1})$$
$$+ \sigma(\mathbb{E}_t x_{t+1} - x_{t+1}) + (\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) \qquad (2)$$

$\hat{i}_t$ 动态: (Monetary Policy)

$$\hat{i}_t = \alpha \pi_t + \delta x_t + v_t \quad \Rightarrow t+1 \qquad (3)$$

v_t 为AR(1) 过程:

$$\hat{v}_{t+1} = \rho_v \hat{v}_t + e_{t+1}^v \qquad (\rho_v = 0.5) \qquad (4)$$

z_t 动态:

$$\hat{z}_{t+1} = \rho_z \hat{z}_t + e_{t+1}^z \qquad (\rho_z = 0.5) \qquad (5)$$

ψ_t 动态: AR(1)

$$\hat{\psi}_{t+1} = \rho_\psi \hat{\psi}_t + e_{t+1}^\psi \qquad (\rho_\psi = 1) \qquad (6)$$

为了研究以下各量: c_t, r_t, n_t, m_t.

我们引入:

Output: $\hat{C}_t = x_t + \frac{1+\eta}{\sigma+\eta}\hat{z}_t \quad \Rightarrow t+1 \qquad (7)$

Real, inter: $\tilde{r}_t = \hat{i}_t(1+i^{ss}) - \mathbb{E}_t \pi_{t+1} \qquad i^{ss} = \frac{1}{\beta} - 1 \qquad (8)$ 8

$$= \hat{i}_t\left(\frac{1}{\beta}\right) - \mathbb{E}_t \pi_{t+1} \Rightarrow \pi_{t+1} = -\tilde{r}_t + \left(\frac{1}{\beta}\right)\hat{i}_t - (\mathbb{E}_t \pi_{t+1} - \pi_{t+1})$$

Employment $\hat{n}_t = \hat{y}_t - \hat{z}_t \quad \Rightarrow t+1 \qquad (9)$

Money Stock: 由④: $\gamma m_t^{-b} = \psi_t C_t^{-\sigma} \frac{i_t}{1+i_t}$

$$\Rightarrow -b\hat{m}_t = \hat{\psi}_t - \sigma \hat{C}_t + \hat{i}_t \frac{1}{i^{ss}}$$

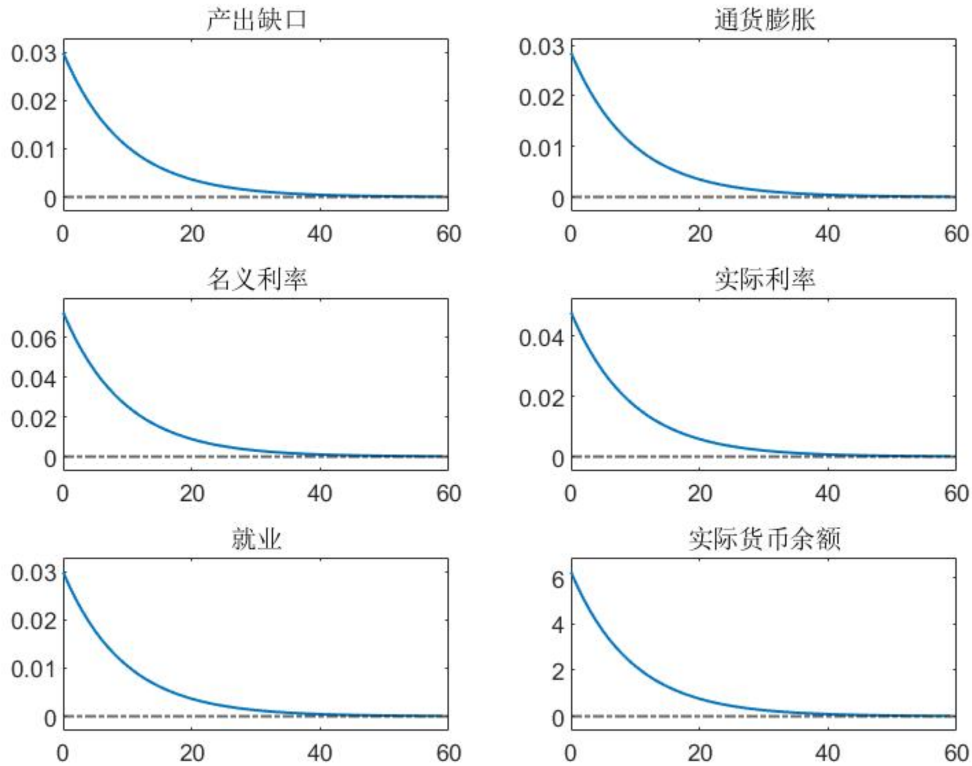
$$\Rightarrow -b\hat{m}_t = \frac{\hat{\psi}_t}{-b} + \frac{\sigma}{b}\hat{C}_t + \hat{i}_t \frac{\beta}{1-\beta} \cdot \frac{1}{b} \qquad (10)$$

$$\Rightarrow 0 = b\hat{m}_t + \hat{\psi}_t - \sigma \hat{C}_t + \frac{\beta}{1-\beta}\hat{i}_t$$

$\Downarrow$

$t+1$

在加入了大小为1的preference冲击后，这里的产出增长，通货膨胀上升，名义利率上升，实际利率上升，就业上升，实际货币存量也上升。——总体来说经济更加景气。



我将自回归系数改为0.5后，发现收敛速度更快了。