

Report and Questions about the Slides, Linear RE

Firstly, I try my best and find two typos.

Sims' Method

The above relation can be written as

$$w_{2t} = -\Omega_{22}^{-1} Q_2 (C + \Psi z_{t+1} + \Pi \eta_{t+1}) - \Omega_{22}^{-1} \Lambda_{22} w_{2,t+1}.$$

Under rational expectation, we have $\mathbb{E}_t \eta_{t+i} = 0$ for all $i \geq 1$. This implies that we have

$$\begin{aligned} w_{2t} &= \mathbb{E}_t w_{2t} = -\sum_{i=0}^{\infty} \left(\Omega_{22}^{-1} \Lambda_{22} \right)^i \Omega_{22}^{-1} Q_2 (C + \Psi \mathbb{E}_t z_{t+i+1}) \\ &= (\Lambda_{22} - \Omega_{22})^{-1} Q_2 C - \sum_{i=0}^{\infty} \left(\Omega_{22}^{-1} \Lambda_{22} \right)^i \Omega_{22}^{-1} Q_2 \Psi \mathbb{E}_t z_{t+i+1} \end{aligned} \quad (3)$$

Combining the above two equations we have

$$Q_2 \Pi \eta_{t+1} = \sum_{i=0}^{\infty} \Omega_{22} \left(\Omega_{22}^{-1} \Lambda_{22} \right)^i \Omega_{22}^{-1} Q_2 \Psi (\mathbb{E}_{t+1} z_{t+i+1} - \mathbb{E}_t z_{t+i+1})$$

Note that the right hand side is completely exogenous.

Benjamin Hwang 9:43 回复 X

应该是+

Benjamin Hwang 10:06 回复 X

等式左边加上一个负号

But I am afraid that the rest part may be not rigorous.

1. There emphasizes $Q_1 \Pi \eta_t = \phi Q_2 \Pi \eta_t$, why there must exist linear relation between the two terms?
2. next, the necessary and sufficient condition can only cause the existence of ϕ , not including the uniqueness.

Sims' Method

In order for η_{t+1} to exist, a necessary and sufficient condition is that

$$\mathcal{R} \left(\sum_{i=0}^{\infty} \Omega_{22}^{-1} \Lambda_{22}^i \Omega_{22}^{-1} Q_2 \Psi (\mathbb{E}_{t+1} z_{t+i+1} - \mathbb{E}_t z_{t+i+1}) \right) \subset \mathcal{R}(Q_2 \Pi),$$

or, equivalently

$$\mathcal{R} \left(\left\{ \Omega_{22}^{-1} \Lambda_{22}^i \Omega_{22}^{-1} Q_2 \Psi \right\}_{i=0}^{n-k-1} \right) \subset \mathcal{R}(Q_2 \Pi).$$

Given that η_t has a solution, in order to completely pin down the whole system, we need to be able to know $Q_1 \Pi \eta_t$ from the value of $Q_2 \Pi \eta_t$. That is, there exists a unique Φ such that

$$Q_1 \Pi \eta_t = \Phi Q_2 \Pi \eta_t$$

for all t . A necessary and sufficient is that

$$\mathcal{R}(\Pi' Q_1') \subset \mathcal{R}(\Pi' Q_2').$$

Benjamin Hwang 12:09

回复 X

不懂为何两者一定要有线性关系

添加回复...

3. the dimension of ϕ is not clear, as I demonstrate below.

4. Is the "Premultiplying" operation called 左乘? I am afraid that I have no idea about how this operation applied to (2), and obtain such formula.

Sims' Method

Premultiplying $[I - \Phi]$ to the system (2) and combining the solution part (3) yields

$$\begin{aligned} \begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi \Lambda_{22} \\ 0 & I \end{bmatrix} \begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} &= \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi \Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} \\ &+ \begin{bmatrix} Q_1 - \Phi Q_2 \\ (\Lambda_{22} - \Omega_{22}^{-1})^{-1} Q_2 \end{bmatrix} C + \begin{bmatrix} Q_1 - \Phi Q_2 \\ 0 \end{bmatrix} \Psi z_t \\ &+ \begin{bmatrix} 0 \\ -\sum_{i=0}^{\infty} (\Omega_{22}^{-1} \Lambda_{22})^i \Omega_{22}^{-1} Q_2 \Psi \mathbb{E}_t z_{t+i+1} \end{bmatrix} \end{aligned}$$

Denote the above equation by

$$B_0 w_t = B w_{t-1} + B_c C + B_z \Psi z_t + s_t$$

The solution is then given by

$$y_t = Z w_t = Z B_0^{-1} B Z^* y_{t-1} + Z B_0^{-1} B_c C + Z B_0^{-1} B_z \Psi z_t + Z B_0^{-1} s_t$$

Benjamin Hwang 11:56

回复 X

没人说phi一定是方阵呀，如果是方阵这就是说，w_1, w_2有相同的维数，就是说\lambda_i < 1的特征根数量和\lambda_i > 1的数量相等，没这个假设。。