Detailed Algorithm for Cagetti, De Nardi, 2006

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Abstract

This file documents the detailed algorithm of Cagetti and De Nardi (2006) to solve the stationary equilibrium with entrepreneurs.

1 Algorithm for the Baseline Model

1. Construct a grid for the state variables. The maximum asset level is chosen so that it is not binding for the household's saving decisions.
2. Fix a tax rate τ for labor's wage.
3. Fix the risk-free rate r and a wage w. Taking these as given, solve for the value functions using value function iteration.
4. This process of VFI is nonstandard due to the existence of the borrowing constraint

$$u(c) + \beta\pi_o \mathbb{E}W(a', \theta') + \eta\beta(1-\pi_o)\mathbb{E}V(a', y', \theta') \geq W_r(fk)$$

and

$$u(c) + \beta\pi_y \mathbb{E}V(a', y', \theta') + \beta(1-\pi_y)\mathbb{E}V(a', \theta') \geq V_w(fk, y, \theta).$$

. Thus, initialize the maximum capital of entrepreneur as $\widehat{k}(\mathbf{x}) = k_{max}$.

   (a) Guess 4 value functions below

$$V_e(a, y, \theta), V_w(a, y, \theta), W_e(a, \theta), \text{ and } W_r(a).$$

   (b) Solve the occupation choice by

$$V(a, y, \theta) = \max\left\{V_e(a, y, \theta), V_w(a, y, \theta)\right\}$$

   and

$$W(a, \theta) = \max\left\{W_e(a, \theta), W_r(a)\right\}.$$

   Then, with 6 value functions above, we can solve the intertemporal maximization problems of each agent.

(c) As for the young worker $(s = 1)$, the consumption is given by

$$c = -a' + (1+r)a + (1-\tau)wy.$$

Plug consumption into the Bellman equation, then, we can solve the new V_w.

$$V_w(a, y, \theta) = \max_{a'} \left\{ u(c) + \beta \left(\pi_y \mathbb{E} V(a', y', \theta') + (1-\pi_y) W_r(a') \right) \right\}.$$

(d) As for the young entrepreneur $(s = 2)$, the consumption is given by

$$c = -a' + (1-\delta)k + \theta k^v - (1+r)(k-a).$$

Similarly, substitute it into the corresponding Bellman equation, then, we can update the V_e

$$V_e(a, y, \theta) = \max_{a', k < \widehat{k}} \left\{ u(c) + \beta \left(\pi_y \mathbb{E} V(a', y', \theta') + (1-\pi_y) \mathbb{E} W(a', \theta') \right) \right\}.$$

(e) As for the old entrepreneur $(s = 3)$, the consumption is given by

$$c = -a' + (1-\delta)k + \theta k^v - (1+r)(k-a).$$

Similarly, substitute it into the corresponding Bellman equation, then, we can update the W_e

$$W_e(a, \theta) = \max_{a', k < \widehat{k}} \left\{ u(c) + \beta \left(\pi_o \mathbb{E} W(a', \theta') + \eta(1-\pi_o) \mathbb{E} V(a', y', \theta') \right) \right\}.$$

(f) As for the retirees $(s = 4)$, the consumption is given by

$$c = -a' + (1+r)a + p,$$

where $p = 0.4\mathbb{E}y \cdot w$. Then, we can update W_r by

$$W_r(a) = \max_{a'} \left\{ u(c) + \beta \left(\pi_o W_r(a') + \eta(1-\pi_o) \mathbb{E} V(a', y', \theta') \right) \right\}.$$

5. Check the endogenous borrowing constraint. If it is not satisfied under the restriction of $\widehat{k}(\mathbf{x})$, update it until it is convergent.

6. Summarize the policy functions and compute the transition matrix.

7. Compute the invariant distribution by iteration.

8. Compute total savings and total capital invested by the entrepreneurial sector implied by invariant distribution and hence capital in the non-corporate sector. Same for labor.

9. Compute implied wages and interest rate, update w and r. Iterate these steps until w and r converge.

10. Iterate steps above and update tau, until social security system outlays equal revenues.

References

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, frictions, and wealth," *Journal of political Economy*, 2006, *114* (5), 835–870.