Finding Circles by an Array of Accumulators

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We describe an efficient procedure for detecting approximate circles and approximately circular arcs of varying gray levels in an edge-enhanced digitized picture. This procedure is an extension and improvement of the circle-finding concept sketched by Duda and Hart [2] as an extension of the Hough straight-line finder [6].

The application of the Hough transformation to the detection of circular objects in a digitized picture requires a three-dimensional array of accumulators. In the Duda and Hart circle-finding algorithm, the array is indexed by three parameters specifying the location and size of a circle: the circle's center $\mathbf{a}$, where $\mathbf{a} = (a_1, a_2)$, and the circle's radius r. The radius is limited to a set of discrete values: $\{r_k \mid k = 1, \ldots, m\}$. Restrict the range and quantization steps of the circle centers to those of the digitized picture. We can visualize the accumulator array as layered, one layer for each member of $\{r_k\}$.

Let $p(\mathbf{x})$ denote the digitized picture function entering the Duda-Hart circle-finding algorithm, where $\mathbf{x} = (x_1, x_2)$ equals the pair of coordinates of a pixel. It is assumed in that algorithm that $p(\mathbf{x})$ is the result of a digital edge-enhancement of the raw digitized picture data, followed by a thresholding for noise reduction. As a result $p(\mathbf{x})$ consists primarily of digitized lines and curves.

For each $\mathbf{x}$ there is a set $\mathcal{C}_\mathbf{x}$ of circles each of which passes through $\mathbf{x}$. Let $\mathcal{X}_p$ denote the set of points $\{\mathbf{x} \mid p(\mathbf{x}) \neq 0\}$. The algorithm suggested by Duda and Hart finds the center $\mathbf{a}(\mathbf{x})$ and radius $r(\mathbf{x})$ for each member of $\mathcal{C}_\mathbf{x}$ such that $\mathbf{x} \in \mathcal{X}_p$. We denote the resulting set of center-radius pairs as $\{(\mathbf{a}, r) \mid \mathcal{X} \in \mathcal{X}_p\}$. For each member of $\{(\mathbf{a}, r) \mid \mathbf{x} \in \mathcal{X}_p\}$, an accumulator at $(\mathbf{a}(\mathbf{x}), r(\mathbf{x}))$ in $(\mathbf{a}, r)$-space is incremented by unity. After all members of $\mathcal{X}_p$ have been processed in this way, the accumulator at $(\mathbf{a}(\mathbf{x}), r(\mathbf{x}))$ will contain the number of elements of $\mathcal{X}_p$ lying on the circle of radius $r(\mathbf{x})$ centered at $\mathbf{a}(\mathbf{x})$.

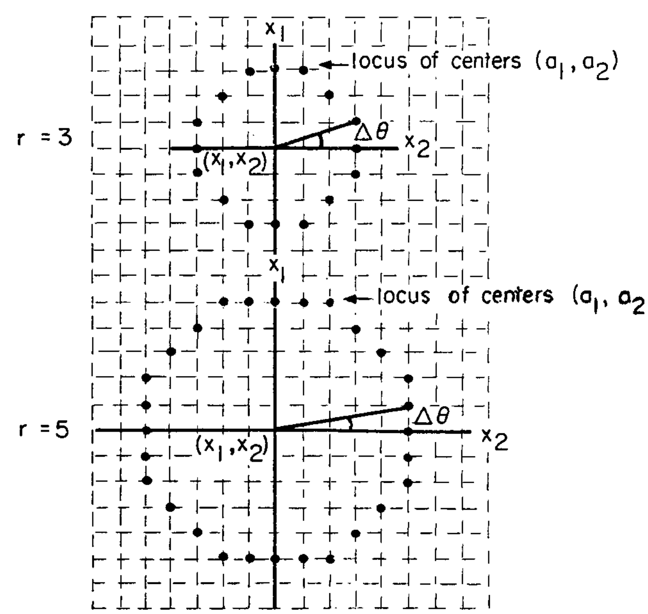
To implement this algorithm in a computationally efficient manner, we used (a) a procedure for generating the digitized members of $\mathcal{C}_\mathbf{x}$, and (b) the direction of the gradient to eliminate portions of each member of $\mathcal{C}_\mathbf{x}$.

To generate each member of $\mathcal{C}_\mathbf{x}$, we compute the digitization of $(r \cos \theta, r \sin \theta)$ for a fixed r, while θ increases in increments of $\Delta\theta$, where $\Delta\theta$ is a stored function of r. This digitization produces a locus of centers of the members of $\mathcal{C}_\mathbf{x}$ for a fixed r. Two such loci, for $r = 3$ and $r = 5$, are shown in Figure 1. This figure illustrates that $r\Delta\theta$ is approximately equal to the spacing between adjacent points of the accumulator array. Thus the size of $\Delta\theta$ varies approximately as $1/r$. Note that each member of $\mathcal{C}_\mathbf{x}$ requires $2\pi/\Delta\theta$ computations.

A threshold T is applied to the contents of the accumulator array to detect those points that are likely to be the centers of circles in $p(\mathbf{x})$. We set the value of T empirically by applying our algorithm to prototype pictures containing known circles. Because the number of points in each member of $\mathcal{C}_\mathbf{x}$ is proportional to r, the size of T is proportional to r (i.e. T/r is approximately constant).

To obtain further computational economy, we use the direction of the gradient [3, 5] to select those portions of $\mathcal{C}_\mathbf{x}$ that are likely to be circle centers.

Fig. 1.



$\Delta\theta$ as a function of r

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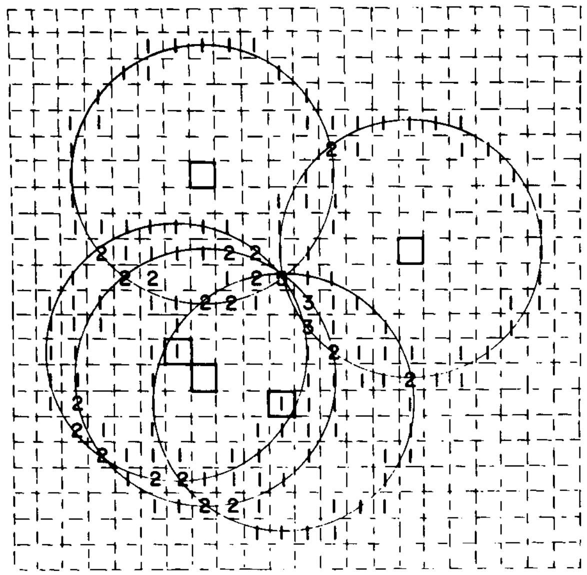
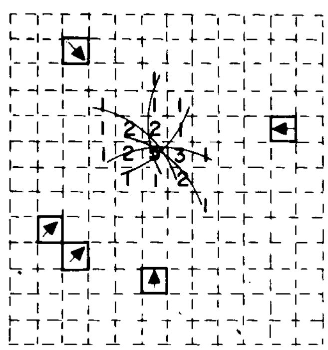
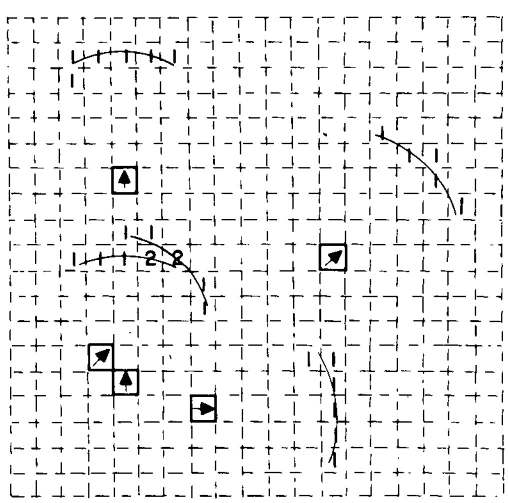
Fig. 2.



a. Contents of accumulator array
No gradient direction information R = 5



b. Contents of accumulator array
Gradient direction information
for circle $\Delta\phi$ = 45



c. Contents of accumulator array
Gradient direction information for
artifact $\Delta\phi$ = 45

☐ Denotes a pixel in $P(\underline{X})$ superimposed on accumulator array
➚ Denotes the gradient direction

(This technique is similar to O'Gorman and Clowes' use of gradient direction to detect straight lines [4].) Let $f(\mathbf{x})$ denote the gray level at $\mathbf{x}$ in the original picture. (Note the distinction between $f(\mathbf{x})$ and the edge-enhanced picture function $p(\mathbf{x})$.) Let $\mathbf{g}(\mathbf{x})$ denote the digitized gradient of $f(\mathbf{x})$, and let the modulus and direction of $\mathbf{g}(\mathbf{x})$ be denoted by $g(\mathbf{x})$ and $\phi(\mathbf{x})$ respectively. Thus in complex number notation, $\mathbf{g}(\mathbf{x}) = g(\mathbf{x}) \exp (j\phi(\mathbf{x}))$. Let $g_1(\mathbf{x})$ and $g_2(\mathbf{x})$ denote the horizontal and vertical components, respectively, of $\mathbf{g}(\mathbf{x})$. Thus $\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}))$. We define $\mathbf{g}(\mathbf{x})$ in terms of $g_i(\mathbf{x})$:

$$g_1(\mathbf{x}) = \frac{1}{2s}\,[f(\mathbf{x} + (s, 0)) - f(\mathbf{x} - (s, 0))]$$

$$g_2(\mathbf{x}) = \frac{1}{2s}\,[f(\mathbf{x} + (0, s)) - f(\mathbf{x} - (0, s))]$$

where s equals a positive integer. The gradient $\mathbf{g}(\mathbf{x})$ is computed from $f(\mathbf{x})$ by

$g(\mathbf{x}) = [g_1^2\,(\mathbf{x}) + g_2^2\,(\mathbf{x})]^{\frac{1}{2}}$,

$\phi(\mathbf{x}) = \tan^{-1}\,(g_2(\mathbf{x})/g_1(\mathbf{x}))$.

If $\mathbf{x}_1$ lies on a circle embedded in $f(\mathbf{x})$, then the gradient of $f(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}_1$ will point to the center of the circle. Our procedure selects only that portion of each member of $\mathcal{C}_{\mathbf{x}}$ such that $\mathbf{g}(\mathbf{x})$ points to $(\mathbf{a}, r \mid \mathbf{x} \in \mathbf{X}_p)$ within a range of angle $\Delta\phi$. This not only reduces the errors in choosing circle centers from the accumulator array, but also reduces the number of computations by a factor of $2\pi/\Delta\phi$.

The effectiveness of use of gradient directions is shown in Figure 2. Part (a) of this figure shows the content of a plane of the accumulator array for the case r = 5, with the directions of the gradient unused. Part (b) shows the effect of using the directions of the gradient when these directions are approximately those appearing along the edge of a circular disc. Part (c) shows the effect of using the directions of the gradient when these directions are those of a noncircular object. Note that without the use of these directions, both parts (b) and (c) would be replaced by part (a).

We applied our algorithm to the detection of the images of tumors in chest radiographs (X-ray photographs of the human chest). We implemented this algorithm on an HP2114B computer with about 4000, 16-bit words. Since tumors in chest radiographs are usually nearly spherical, their images are often approximately circular. The original image (Figure 3) is Fourier filtered to enhance high spatial frequencies (Figure 4). A gradient operator with a noise threshold of 10 is applied to the filtered, digitized radiograph to obtain the edge-enhanced picture (Figure 5). At this point the algorithm is applied to produce Figure 6. The resolution of the digitization is 6 pixels per cm. The algorithm was applied to overlapping windows of 64 by 64 pixels each, and the range of radii was 2, 3, 4, 6, 8, and 10 pixels, corresponding to tumor sizes of .7, 1.0, 1.3, 2, 2.7, and 3.3 cm, respectively, in diameter.

Fig. 3. Grey scale digitized radiograph.

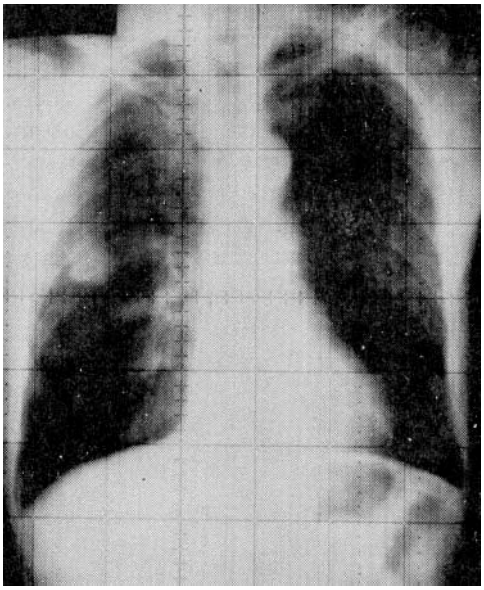
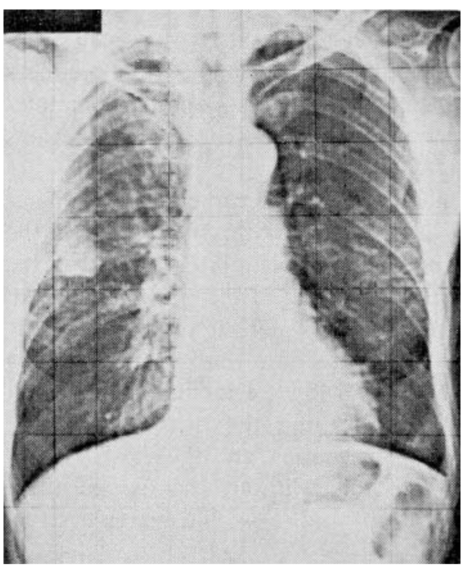


Fig 4. Same radiograph after Fourier filtering.



With a data base of six radiographs, the algorithm located all the large tumors (r = 6, 8, and 10) and no false tumors of this range of sizes.

At smaller radii our algorithm detected some false tumors as well as most of the true tumors. Further research in classification procedures is aimed toward removing these false tumors [1].

The use of the direction of the gradient to eliminate portions of each member of $\mathcal{C}_x$ achieved a six-to-one reduction of computation time on the HP2114B computer in the processing of the radiograph illustrated in Figures 3 to 6. Specifically, the algorithm consumed 75 minutes without the use of the directional information, and 13 minutes with the use of this information. We

Fig. 5. Gradient Modulus of Fourier filtered radiograph (white = large modulus).

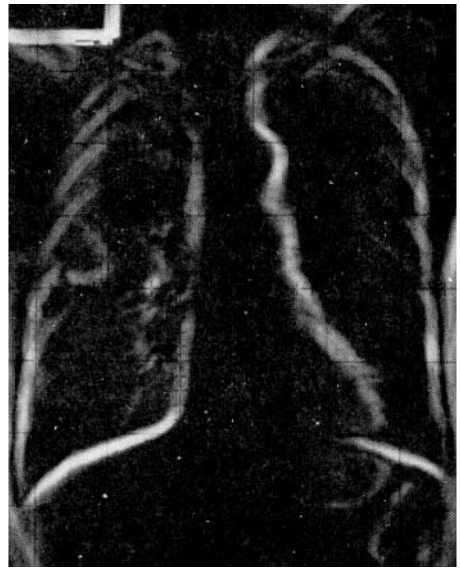
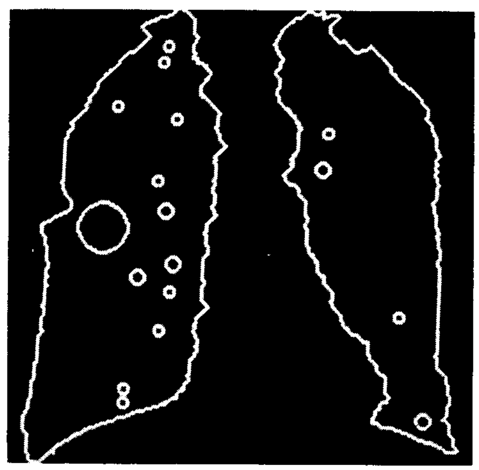


Fig. 6 Threshold results of applying algorithm to Figure 5 data (with gradient direction).



have no timing information for the Duda-Hart algorithm because our early implementations of this algorithm on a PDP-10 computer were too expensive.

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