Tokamak grid generator in IDL

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1 Using the grid generator

The function `create_grid` takes a 2D array of ψ values, and produces an orthogonal mesh aligned with the flux-surfaces.

Settings to control the resulting mesh are:

- `psi_inner`, the normalised ψ of the innermost flux surface. This can be either a scalar or an array:
  - **scalar**: This value is used for the core and all PF regions
  - **array[0]**: The inner normalised ψ for the core
  - **array[1..n_xpoint]**: Inner ψ to use for each PF region (see section 4)
- `psi_outer`, normalised ψ of outermost surface. Can also be either a scalar or array:
  - **scalar**: This value is used for the core and all PF regions

- **array[0..(n_xpoint-1)]**: Outer normalised ψ for each SOL region (one per x-point)

- `nrad` Number of radial grid points
  - **scalar**: Total number of radial grid points. Automatically divides this between regions.
  - **array[0]**: Number of radial grid points in the core
  - **array[1..(n_xpoint-1)]**: Radial grid points between separatrices (going outwards from core to edge)
  - **array[n_xpoint]**: Radial grid points outside last separatrix

- `npol` Number of poloidal grid points.
  - **scalar**: Total number of points. Divides between regions based on poloidal arc lengths
  - **array[0..(3*n_xpoint-1)]**: Number of points in each poloidal region. See section 4 for numbering scheme.

- `rad_peaking`

- `pol_peaking`

2 DCT

DCT of 2D NxM $f\left(x, y\right)$

$$F\left(u,v\right) = \sqrt{\frac{2}{N}}\sqrt{\frac{2}{M}}\Lambda\left(u\right)\Lambda\left(v\right)\sum_{i=0}^{N-1}\sum_{j=0}^{M-1} f\left(i,j\right)\cos\left[\frac{\pi u}{2N}\left(2i+1\right)\right]\cos\left[\frac{\pi v}{2M}\left(2j+1\right)\right]$$

where $\Lambda\left(i\right) = 1/\sqrt{2}$ for $i = 0$, and $\Lambda\left(i\right) = 1$ otherwise

3 Finding critical points

To find x- and o-points,

4 Region numbering

5 Separatrices

Having found the x-point locations, the separatrices need to be found. First step is to calculate the lines going through the x-point:

Close to an x-point, approximate the change in ψ by

$$\delta\psi = \frac{1}{2}\psi_{xx}x^2 + \frac{1}{2}\psi_{yy}y^2 + \psi_{xy}xy$$

The two lines through the x-point are then given by where this is zero:

$$\frac{1}{2}\psi_{yy}y^2 + \psi_{xy}xy + \frac{1}{2}\psi_{xx}x^2 = 0$$

Which has the solution

$$y = \frac{-\psi_{xy}x \pm \sqrt{\psi_{xy}^2x^2 - \psi_{yy}\psi_{xx}x^2}}{\psi_{yy}}$$

i.e.

$$y = \frac{1}{\psi_{yy}}\left(-\psi_{xy} \pm \sqrt{\psi_{xy}^2 - \psi_{yy}\psi_{xx}}\right)x$$

Note that if $\psi_{yy} = 0$ then the solutions are $x = 0$ and $y = -\frac{\psi_{xx}}{2\psi_{xy}}x$