Functional Dependencies and Schema Refinement

CS 186, Fall 2008, R&G Chapter 19

Science is the knowledge of consequences, and dependence of one fact upon another.

Thomas Hobbes (1588-1679)

Review: Database Design

- Requirements Analysis
  - user needs; what must database do?
- Conceptual Design
  - high level descr (often done w/ER model)
- Logical Design
  - translate ER into DBMS data model
- Schema Refinement
  - consistency,normalization
- Physical Design - indexes, disk layout
- Security Design - who accesses what

Review: Database Design

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  - **consistency,normalization**
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The Evils of Redundancy

- ***Redundancy*** **is at the root of several problems associated with relational schemas:**
  - *redundant storage, insert/delete/update anomalies*
- **Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.**
- **Main refinement technique: *decomposition***
  - replacing ABCD with, say, AB and BCD, or ACD and ABD.
- **Decomposition should be used judiciously:**
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

Functional Dependencies (FDs)

- **A functional dependency X → Y holds over relation schema R if, for every allowable instance *r* of R:**

$$t1 \in r,\ t2 \in r,\ \pi_X(t1) = \pi_X(t2) \ \text{implies}\ \pi_Y(t1) = \pi_Y(t2)$$

(where t1 and t2 are tuples;X and Y are sets of attributes)

- **In other words: X → Y means**

  Given any two tuples in *r*, if the X values are the same, then the Y values must also be the same. (but not vice versa)
- **Can read "→" as "determines"**

FD's Continued

- **An FD is a statement about *all* allowable relations.**
  - Must be identified based on semantics of application.
  - Given some instance *r1* of R, we can check if *r1* violates some FD *f*, but we cannot determine if *f* holds over R.
- **Question: How related to keys?**
- **if "K → all attributes of R" then K is a *superkey* for R**

  (does not require K to be *minimal*.)
- **FDs are a generalization of keys.**

Example: Constraints on Entity Set

- **Consider relation obtained from** Hourly_Emps:
  Hourly_Emps (<u>*ssn*</u>, *name, lot, rating, wage_per_hr, hrs_per_wk*)

- We sometimes denote a relation schema by listing the attributes: e.g., SNLRWH
- This is really the *set* of attributes {S,N,L,R,W,H}.
- Sometimes, we refer to the set of *all attributes* of a relation by using the relation name. e.g., "Hourly_Emps" for SNLRWH

What are some FDs on Hourly_Emps?

ssn is the key: S → SNLRWH
rating determines *wage_per_hr*: R → W
lot determines *lot*: L → L ("trivial" dependnency)

Problems Due to R → W

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

Hourly_Emps

- ***Update anomaly*: Can we modify W in only the 1st tuple of SNLRWH?**
- ***Insertion anomaly*: What if we want to insert an employee and don't know the hourly wage for his or her rating? (or we get it wrong?)**
- ***Deletion anomaly*: If we delete all employees with rating 5, we lose the information about the wage for rating 5!**

Detecting Reduncancy

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

Hourly_Emps

Q: Why was R → W problematic, but S→W not?

Decomposing a Relation

- **Redundancy can be removed by "chopping" the relation into pieces.**
- **FD's are used to drive this process.**
  R → W is causing the problems, so decompose SNLRWH into what relations?

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

Hourly_Emps2

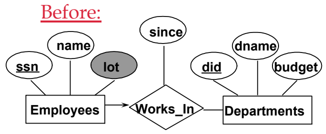
R	W
8	10
5	7

Wages

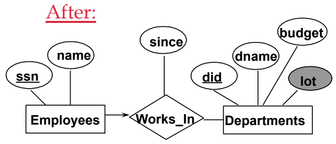
Refining an ER Diagram

- **1st diagram becomes:**
  Workers(S,N,L,D,Si)
  Departments(D,M,B)
  - Lots associated with workers.
- **Suppose all workers in a dept are assigned the same lot: D → L**
- **Redundancy; fixed by:**
  Workers2(S,N,D,Si)
  Dept_Lots(D,L)
  Departments(D,M,B)
- **Can fine-tune this:**
  Workers2(S,N,D,Si)
  Departments(D,M,B,L)

Before:



After:



Reasoning About FDs

- **Given some FDs, we can usually infer additional FDs:**
  title → *studio, star* implies *title* → *studio* and *title* → *star*
  title → *studio* and *title* → *star* implies *title* → *studio, star*
  title → *studio*, *studio* → *star* implies *title* → *star*

  But,
  title, star → *studio* does NOT necessarily imply that *title* → *studio* or that *star* → *studio*
- **An FD *f* is *implied by* a set of FDs *F* if *f* holds whenever all FDs in *F* hold.**
- **F^+ = *closure of F* is the set of all FDs that are implied by *F*. (includes "trivial dependencies")**

Rules of Inference

- **Armstrong's Axioms (X, Y, Z are <u>sets</u> of attributes):**
  - *Reflexivity*: If $X \subseteq Y$, then $X \rightarrow Y$
  - *Augmentation*: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
  - *Transitivity*: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- **These are *sound* and *complete* inference rules for FDs!**
  - i.e., using AA you can compute all the FDs in F+ and only these FDs.
- **Some additional rules (that follow from AA):**
  - *Union*: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Decomposition*: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

Example

- **Contracts(<u>*cid*</u>,*sid,jid,did,pid,qty,value*), and:**
  - C is the key: $C \rightarrow CSJDPQV$
  - Proj purchases each part using single contract: $JP \rightarrow C$
  - Dept purchases at most 1 part from a supplier: $SD \rightarrow P$
- **Problem: Prove that SDJ is a key for Contracts**
- **$JP \rightarrow C$, $C \rightarrow CSJDPQV$ imply $JP \rightarrow CSJDPQV$**
  (by transitivity) (shows that JP is a key)
- **$SD \rightarrow P$ implies $SDJ \rightarrow JP$** (by augmentation)
- **$SDJ \rightarrow JP$, $JP \rightarrow CSJDPQV$ imply $SDJ \rightarrow CSJDPQV$**
  (by transitivity) thus SDJ is a key.

Q: can you now infer that $SD \rightarrow CSDPQV$ (i.e., drop J on both sides)?

No! FD inference is not like arithmetic multiplication.

Attribute Closure

- **Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)**
- **Typically, we just want to check if a given FD $X \rightarrow Y$ is in the closure of a set of FDs *F*. An efficient check:**
  - Compute *attribute closure* of X (denoted X^+) wrt *F*. X^+ = Set of all attributes A such that $X \rightarrow A$ is in F^+
    - $X^+ := X$
    - Repeat until no change: if there is in fd $U \rightarrow V$ in *F* such that U is in X^+, then add V to X^+
  - Check if Y is in X^+
  - Approach can also be used to find the keys of a relation.
    - If all attributes of R are in the closure of X then X is a superkey for R.
    - Q: How to check if X is a "candidate key"?

Attribute Closure (example)

- **R = {A, B, C, D, E}**
- **F = { $B \rightarrow CD$, $D \rightarrow E$, $B \rightarrow A$, $E \rightarrow C$, $AD \rightarrow B$ }**
- **Is $B \rightarrow E$ in F^+ ?**
  - $B^+ = B$
  - $B^+ = BCD$
  - $B^+ = BCDA$
  - $B^+ = BCDAE$ … Yes! and B is a key for R too!
- **Is D a key for R?**
  - $D^+ = D$
  - $D^+ = DE$
  - $D^+ = DEC$
  - … Nope!
- **Is AD a key for R?**
  - $AD^+ = AD$
  - $AD^+ = ABD$ and B is a key, so Yes!
- **Is AD a *candidate* key for R?**
  - $A^+ = A$
  - A not a key, nor is D so Yes!
- **Is ADE a *candidate* key for R?**
  - No! AD is a key, so ADE is a superkey, but not a cand. key

Functional Dependencies (Review)

- **A <u>functional dependency</u> $X \rightarrow Y$ holds over relation schema R if, for every allowable instance *r* of R:**

  $t1 \in r$, $t2 \in r$, $\pi_X(t1) = \pi_X(t2)$
  implies $\pi_Y(t1) = \pi_Y(t2)$

  (where t1 and t2 are tuples;X and Y are sets of attributes)
- **In other words: $X \rightarrow Y$ means**
  Given any two tuples in *r*, if the X values are the same, then the Y values must also be the same. (but not vice versa)
- **Can read "$\rightarrow$" as "determines"**

Keys (review)

- **A key is a set of attributes that uniquely identifies each tuple in a relation.**
  key $\rightarrow$ all attributes in the relation
- **A candidate key is a key that is minimal.**
  If AB is a candidate key, then neither A nor B is a key on its own.
- **A superkey is a key that is not necessarily minimal (although it could be)**
  If AB is a candidate key then ABC, ABD, and even AB are superkeys.

Normal Forms

- **Back to schema refinement…**
- **Q1: is any refinement is needed??!**
- **If a relation is in a *normal form* (BCNF, 3NF etc.):**
  - we know that certain problems are avoided/minimized.
  - helps decide whether decomposing a relation is useful.
- **Role of FDs in detecting redundancy:**
  - Consider a relation R with 3 attributes, ABC.
    - No (non-trivial) FDs hold: There is no redundancy here.
    - Given A → B: If A is not a key, then several tuples could have the same A value, and if so, they'll all have the same B value!
- **1st Normal Form – all attributes are atomic**
- **1st ⊃2nd (of historical interest) ⊃ 3rd ⊃ Boyce-Codd ⊃ …**

Boyce-Codd Normal Form (BCNF)

- **Reln R with FDs *F* is in BCNF if, for all X → A in F+**
  - A ∈ X (called a *trivial* FD), or
  - X is a superkey for R.
- **In other words: "R is in BCNF if the only non-trivial FDs over R are *key constraints.*"**
- **If R in BCNF, then every field of every tuple records information that cannot be inferred using FDs alone.**
  - Say we know FD X → A holds for this example relation:

X	Y	A
x	y1	a
x	y2	?

- Can you guess the value of the missing attribute?
- Yes, so relation is not in BCNF

Boyce-Codd Normal Form - Alternative Formulation

"The key, the whole key, and nothing but the key, so help me Codd."

Decomposition of a Relation Scheme

- If a relation is not in a desired normal form, it can be *decomposed* into multiple relations that each are in that normal form.
- Suppose that relation R contains attributes *A1 … An.* A *decomposition* of R consists of replacing R by two or more relations such that:
  - Each new relation scheme contains a subset of the attributes of R, and
  - Every attribute of R appears as an attribute of at least one of the new relations.

Example (same as before)

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
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612-67-4134	Madayan	35	8	10	40

Hourly_Emps

- SNLRWH has FDs S → SNLRWH and R → W
- Q: Is this relation in BCNF?

No, The second FD causes a violation;
W values repeatedly associated with R values.

Decomposing a Relation

- Easiest fix is to create a relation RW to store these associations, and to remove W from the main schema:

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
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612-67-4134	Madayan	35	8	40

Hourly_Emps2

R	W
8	10
5	7

Wages

- Q: Are both of these relations are now in BCNF?
- **Decompositions should be used only when needed.**
  - Q: potential problems of decomposition?

Problems with Decompositions

- There are three potential problems to consider:
  1) May be impossible to reconstruct the original relation! (Lossiness)
     - Fortunately, not in the SNLRWH example.
  2) Dependency checking may require joins.
     - Fortunately, not in the SNLRWH example.
  3) Some queries become more expensive.
     - e.g., How much does Guldu earn?

***Tradeoff*: Must consider these issues vs. redundancy.**

Lossless Decomposition (example)

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
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131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

$\bowtie$

R	W
8	10
5	7

=

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
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612-67-4134	Madayan	35	8	10	40

Lossy Decomposition (example)

A	B	C
1	2	3
4	5	6
7	2	8

$\Rightarrow$

A	B
1	2
4	5
7	2

B	C
2	3
5	6
2	8

$A \rightarrow B;\ C \rightarrow B$

A	B
1	2
4	5
7	2

$\bowtie$

B	C
2	3
5	6
2	8

=

A	B	C
1	2	3
4	5	6
7	2	8
1	2	8
7	2	3

Lossless Join Decompositions

- **Decomposition of R into X and Y is *lossless-join* w.r.t. a set of FDs F if, for every instance *r* that satisfies F:**

  $\pi_X(r) \bowtie \pi_Y(r) = r$

- **It is always true that $r \subseteq \pi_X(r) \bowtie \pi_Y(r)$**
  - In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- **Definition extended to decomposition into 3 or more relations in a straightforward way.**
- ***It is essential that all decompositions used to deal with redundancy be lossless! (Avoids Problem #1)***

More on Lossless Decomposition

- The decomposition of R into X and Y is **lossless with respect to F** *if and only if* the closure of F contains:

  $X \cap Y \rightarrow X$, **or**

  $X \cap Y \rightarrow Y$

  in example: decomposing ABC into AB and BC is lossy, because intersection (i.e., "B") is not a key of either resulting relation.
- Useful result: If $W \rightarrow Z$ holds over R and $W \cap Z$ is empty, then decomposition of R into R-Z and WZ is loss-less.

Lossless Decomposition (example)

A	B	C
1	2	3
4	5	6
7	2	8

$\Rightarrow$

A	C
1	3
4	6
7	8

B	C
2	3
5	6
2	8

$A \rightarrow B;\ C \rightarrow B$

A	C
1	3
4	6
7	8

$\bowtie$

B	C
2	3
5	6
2	8

=

A	B	C
1	2	3
4	5	6
7	2	8

But, now we can't check $A \rightarrow B$ without doing a join!

Dependency Preserving Decomposition

- **Dependency preserving decomposition (Intuitive):**
  - If R is decomposed into X, Y and Z, and we enforce the FDs that hold individually on X, on Y and on Z, then all FDs that were given to hold on R must also hold. *(Avoids Problem #2 on our list.)*
- ***Projection of set of FDs F*: If R is decomposed into X and Y the projection of F on X (denoted F_X) is the set of FDs U → V in F^+ (*closure of F*, *not just F*) such that all of the attributes U, V are in X. *(same holds for Y of course)***

Dependency Preserving Decompositions (Contd.)

- **Decomposition of R into X and Y is *dependency preserving* if $(F_X \cup F_Y)^+ = F^+$**
  - i.e., if we consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+.
- **Important to consider F^+ in this definition:**
  - ABC, A → B, B → C, C → A, decomposed into AB and BC.
  - Is this dependency preserving? Is C → A preserved?????
    - note: F^+ contains F ∪ {A → C, B → A, C → B}, so…
- F_{AB} contains A → B and B → A; F_{BC} contains B → C and C → B
- So, $(F_{AB} \cup F_{BC})^+$ contains C → A

Decomposition into BCNF

- **Consider relation R with FDs F. If X → Y violates BCNF, decompose R into R - Y and XY (guaranteed to be loss-less).**
  - Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
  - e.g., CSJDPQV, key C, JP → C, SD → P, J → S
  - *{contractid, supplierid, projectid,deptid,partid, qty, value}*
  - To deal with SD → P, decompose into SDP, CSJDQV.
  - To deal with J → S, decompose CSJDQV into JS and CJDQV
  - So we end up with: SDP, JS, and CJDQV
- **Note: several dependencies may cause violation of BCNF. The order in which we ``deal with'' them could lead to very different sets of relations!**

BCNF and Dependency Preservation

- **In general, there may not be a dependency preserving decomposition into BCNF.**
  - e.g., CSZ, CS → Z, Z → C
  - Can't decompose while preserving 1st FD; not in BCNF.
- **Similarly, decomposition of CSJDPQV into SDP, JS and CJDQV is not dependency preserving (w.r.t. the FDs JP → C, SD → P and J → S).**
- ***{contractid, supplierid, projectid,deptid,partid, qty, value}***
  - However, it is a lossless join decomposition.
  - In this case, adding JPC to the collection of relations gives us a dependency preserving decomposition.
    - but JPC tuples are stored only for checking the f.d. (*Redundancy!*)

Third Normal Form (3NF)

- **Reln R with FDs *F* is in 3NF if, for all X → A in F^+**
  - A ∈ X (called a *trivial* FD), or
  - X is a superkey of R, or
  - A is part of some candidate key (not superkey!) for R. (sometimes stated as "A is *prime*")
- ***Minimality* of a key is crucial in third condition above!**
- **If R is in BCNF, obviously in 3NF.**
- **If R is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no ``good'' decomp, or performance considerations).**
  - *Lossless-join, dependency-preserving decomposition of R into a collection of 3NF relations always possible.*

What Does 3NF Achieve?

- **If 3NF violated by X → A, one of the following holds:**
  - X is a subset of some key K ("partial dependency")
    - We store (X, A) pairs redundantly.
    - e.g. Reserves SBDC (C is for credit card) with key SBD and S → C
  - X is not a proper subset of any key. ("transitive dep.")
    - There is a chain of FDs K → X → A, which means that we cannot associate an X value with a K value unless we also associate an A value with an X value (different K's, same X implies same A!) – problem with initial SNLRWH example.
- **But: even if R is in 3NF, these problems could arise.**
  - e.g., Reserves SBDC (note: "C" is for credit card here), S → C, C → S is in 3NF (why?), but for each reservation of sailor S, same (S, C) pair is stored.
- **Thus, 3NF is indeed a compromise relative to BCNF.**

Decomposition into 3NF

- **Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier) but does not ensure dependency preservation.**
- **To ensure dependency preservation, one idea:**
  - If X → Y is not preserved, add relation XY.

  Problem is that XY may violate 3NF! e.g., consider the addition of CJP to `preserve' JP → C. What if we also have J → C ?
- **Refinement: Instead of the given set of FDs F, use a *minimal cover for F*.**

Minimal Cover for a Set of FDs

- ***Minimal cover* G for a set of FDs F:**
  - Closure of F = closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- **Intuitively, every FD in G is needed, and ``*as small as possible*'' in order to get the same closure as F.**
- **e.g., A → B, ABCD → E, EF → GH, ACDF → EG has the following minimal cover:**
  - A → B, ACD → E, EF → G and EF → H
- **M.C. implies Lossless-Join, Dep. Pres. Decomp!!!**
  - (in book)

Summary of Schema Refinement

- **BCNF: each field contains information that cannot be inferred using only FDs.**
  - ensuring BCNF is a good heuristic.
- **Not in BCNF? Try decomposing into BCNF relations.**
  - Must consider whether all FDs are preserved!
- **Lossless-join, dependency preserving decomposition into BCNF impossible? Consider 3NF.**
  - Same if BCNF decomp is unsuitable for typical queries
  - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.
- **Note: even more restrictive Normal Forms exist (we don't cover them in this course, but some are in the book.)**