A Note on the Construction of Near-Orthogonal Arrays With Mixed Levels and Economic Run Size

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Due to run-size constraints, some experimental situations cannot be approached using orthogonal-array design. Near-orthogonal arrays can, however, often be used. This article describes a quick and simple method of constructing this type of array. The arrays constructed here are typically better than those of Wang and Wu with respect to D efficiency as well as other design-goodness criteria.

KEY WORDS: Computer-aided designs; Orthogonal arrays; Screening designs.

Consider a screening experiment for factors that may influence the properties of pulps prepared from woodchips by a variant of the kraft pulping process. The 10 factors in this experiment are (1) impregnation temperature (30°, 80°, or 120°C), (2) chip steaming time (0 or 10 minutes), (3) initial concentration of alkali (6% or 12%) and (4) sulfide (2% or 10%) in the impregnation step, (5) impregnation pressure (30 or 120 psi) and (6) time (10 or 40 minutes), (7) ratio of pulping liquor to wood (3.5:1 or 6:1), (8) addition of anthraquinone (none or .05%), (9) temperature of the main cook stage (160°or 170°C), and (10) final quenching of the cook with water (yes/no). The smallest orthogonal array (OA) found for 9 two-level factors and 1 three-level factor requires 24 runs (Wang and Wu 1991). The scientist wants to reduce the cost of this screening experiment and wishes to know which array to use when many fewer than 24 runs can be performed.

The concept of near-orthogonal arrays (near-OA's) (see Taguchi 1959; Tukey 1959; Wang and Wu 1992, hereafter called WW) provides a genuine answer to the preceding question. An OA (of strength 2) of size n with k_i s_i-level columns ($i = 1, \ldots, r$), denoted by $L_n(s_1^{k_1}, \ldots, s_r^{k_r})$, is an $n \times k$ matrix ($k = \sum k_i$) in which all possible combinations of levels in any two columns appear the same number of times (Rao 1947). In a near-OA $L'_n(s_1^{k_1}, \ldots, s_r^{k_r})$, to reduce the run size the orthogonality of some pairs of columns is necessarily sacrificed. A near-OA in 12 runs suitable for the previously mentioned experiment was given in figure 3 of WW. The usefulness of arrays of this type is flexibility in the choice of factor levels and small run size. Perhaps, the best-known use of a near-OA is the one for the integrated circuit (IC) fabrication experiment performed by Phadke and his coworkers in collaboration with Taguchi at AT&T Bell Laboratories (Phadke, Kackar, Speeney, and Greico 1983). This article details a method for constructing near-OA's in which the number of runs is divisible by the number of levels of each factor.

1. A SIMPLE METHOD OF CONSTRUCTING NEAR-OA'S

Nguyen (1996) described an algorithm called NOA for constructing an $L_n^{(1)}(2^m)$, an OA or near-OA with m two-level columns in n runs. NOA can also construct an array with mixed levels by adding additional two-level columns to an existing array with mixed levels. This exercise is carried out by first selecting a *base* array $L_n^{(1)}(s_1^{k_1}, \ldots, s_r^{k_r})$ and building up the $n \times m_0$ ($m_0 = \sum k_i(s_i - 1)$) design matrix X_0 from this array by tables of orthogonal polynomials (see Dey 1985, pp. 18–19). For example, the orthogonal polynomials corresponding to levels 0 and 1 of a two-level factor are -1 and $+1$ and the orthogonal polynomials corresponding to levels $0, 1$, and 2 of a three-level factor are $(-1\,1), (0\,\,-2)$, and $(1\,1)$. Thus, the first two columns of X_0, which correspond to column $(0\,0\,0\,0\,1\,1\,1\,1\,2\,2\,2\,2)'$ of an $L_{12}(3 \cdot 2^4)$, are $(-1\,\,-1\,\,-1\,\,-1\,0\,0\,0\,0\,1\,1\,1\,1)'$ and $(1\,1\,1\,1\,-2\,\,-2\,\,-2\,\,-2\,1\,1\,1\,1)'$. The following steps are then followed:

1. Add $m - m_0$ columns of ± 1's to X_0 to form a *starting* design matrix X. For each column, $\frac{1}{2}n$ entries equal $+1$ and

Table 1. Comparison of Some Near-OA's in Terms of E

Near-OA	*Wang & Wu*	*Nguyen*	$\lvert r\rvert$[b]
$L'_6(3 \cdot 2^3)$	.901 (3[a])	.901 (3)	.333
$L'_{10}(5 \cdot 2^5)$	.883 (10)	.967 (10)	.200
$L'_{12}(6 \cdot 2^5)$	.911 (6)	.959 (4)	.333
$L'_{12}(6 \cdot 2^6)$	—	.947 (6)	.333
$L'_{12}(3 \cdot 2^9)$	.867 (9)	.933 (8)	.333
$L'_{18}(9 \cdot 2^8)$	.981 (28)	.981 (28)	.111
$L'_{18}(3^4 \cdot 2^8)$	.985 (28)	.985 (28)	.111
$L'_{18}(3^7 \cdot 2^3)$	.970 (3)	.970 (3)	.333
$L'_{20}(5 \cdot 2^{15})$	.838 (30)	.922 (25)	.200
$L'_{24}(3 \cdot 2^{21})$	.853 (21)	.968 (8)	.333
$L'_{24}(6 \cdot 2^{15})$	.870 (18)	.994 (1)	.333
$L'_{24}(6 \cdot 2^{16})$	—	.989 (2)	.333
$L'_{24}(6 \cdot 2^{17})$	—	.981 (4)	.333
$L'_{24}(6 \cdot 2^{18})$	—	.974 (6)	.333
$L'_{36}(3^{13} \cdot 2^5)$	—	.996 (1)	.333
$L'_{36}(3^{13} \cdot 2^6)$	—	.993 (2)	.333
$L'_{36}(3^{13} \cdot 2^7)$	—	.989 (3)	.333
$L'_{36}(3^{13} \cdot 2^8)$	—	.986 (4)	.333
$L'_{36}(3^{13} \cdot 2^9)$	—	.956 (8)	.333
$L'_{50}(5^{11} \cdot 2^2)$	—	.999 (1)	.200
$L'_{50}(5^{11} \cdot 2^3)$	—	.998 (3)	.200
$L'_{50}(5^{11} \cdot 2^4)$	—	.996 (6)	.200
$L'_{50}(5^{11} \cdot 2^5)$	—	.994 (10)	.200
$L'_{54}(3^{25} \cdot 2^2)$	—	.998 (1)	.333
$L'_{54}(3^{25} \cdot 2^3)$	—	.990 (3)	.333

[a] Number of nonorthogonal pairs of two-level columns.
[b] $\lvert r\rvert$ of Nguyen's array.

$\frac{1}{2}n$ entries equal -1. Randomize the order of these entries. Form $X'X$ and calculate $f = \sum_{i<j} s_{ij}^2$, where s_{ij} is the element in the ith row and jth column of $X'X$.

2. For column j of X ($j = m_0+1, m_0+2, \ldots, m$), swap the signs of a pair of elements so as to achieve the largest possible reduction in f. Update f, $X'X$, and X accordingly. Repeat this operation for the current column until no further reduction is achieved in f; then proceed to the next column.

Repeat step 2 until $f = 0$ or f cannot be reduced further.

An obvious advantage of the $\sum_{i<j} s_{ij}^2$ criterion over the more familiar D- and A-optimality criteria is that it is computationally cheaper because it works with $X'X$ instead of $(X'X)^{-1}$. As noted by Lin (1993) and Nguyen (1996), however, this criterion is only an approximation of the D- and A-optimality criteria. As such, among designs generated by the preceding algorithm with the same $\sum_{i<j} s_{ij}^2$, the one with the highest $|X'X|$ will be selected. Nguyen (1994) discussed a parallel strategy for constructing incomplete block designs.

As an illustration of this method, consider the following array, columns **A**–**J** of which form the $L'_{12}(3 \cdot 2^9)$ in figure 3 of WW. I construct a new $L'_{12}(3 \cdot 2^9)$ using columns **A**–**H** [an $L'_{12}(3 \cdot 2^7)$] as the base array and columns **I** and **J** as *starting* columns:

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**F′**	**G′**	**H′**	**I′**	**J′**
0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
0	0	1	0	1	0	1	0	1	0	0	1	1	0	0
0	1	0	1	1	1	1	1	0	1	1	1	1	1	1
0	1	1	1	0	1	0	1	1	1	1	0	0	0	0
1	0	0	1	1	1	0	0	0*	1	1	1	0	0	0
1	0	1	1	0	1	1	1	1*	0	0	1	0	1	1
1	1	0	0	1	0	0	1	1	0	0	0	0	1	0
1	1	1	0	0	0	1	0	0	1	1	1	1	1	0
2	0	0	1	0	0	1	1	1	1	0	0	1	1	0
2	0	1	0	1	1	0	1	0	1	1	0	0	1	1
2	1	0	0	0	1	1	0	1	0	0	1	0	0	1
2	1	1	1	1	0	0	0	0	0	0	0	1	0	1.

NOA swaps the signs of two elements of a column of the X matrix that correspond to the fifth and sixth elements of column **I** (those marked with an asterisk). This operation decreases f from 176 to 128 and increases the D-efficiency E, relative to an orthogonal design assuming this design existed, from .886 to .933. The variances of the estimates of the effects of **B**–**J** of WW array are 1, 1, 1.25, 1.25, 1.25, 1.25, 1.75, 2, and 1.75, respectively. This array has 11 nonorthogonal pairs—namely, **DF**, **DH**, **DJ**, **EG**, **EI**, **FH**, **FJ**, **GI**, **HI**, **HJ**, and **IJ**. The variances of the estimates of the effects of **B**–**J** of the improved array are 1, 1.25, 1.25, 1.25, 1.25, 1.25, 1.25, 1.25, and 1.25, respectively. This array has only eight nonorthogonal pairs—namely, **CI**, **DF**, **DH**, **DJ**, **EG**, **FH**, **FJ**, and **HJ**. No further operation can be made in columns **I** and **J** to improve f.

Remarks

1. The preceding display also shows columns **F′**–**J′**. These are columns added to columns **A**–**E** [an $L_{12}(3 \cdot 2^4)$] by NOA when levels 0, 1, and 2 of the three-level columns are coded as (−.5 .5), (0 −1), and (.5 .5). **A**–**E** and **F′**–**J′** form a Type II array, in which all two-level columns are *pairwise* orthogonal and the three-level column is nonorthogonal to some two-level columns (here **F′**–**J′**) (see sec. 3 of WW). This is understandable because, with this choice of coding scheme, the f associated with the array formed by columns **A**–**E** and **F′**–**J′** is smaller than the one associated with the array formed by columns **A**–**H**, new **I** and **J**.

2. To nullify the effect of different coding schemes, the D-efficiency E of arrays in this article is calculated as $|R|^{1/m}$, where R is the correlation matrix of m columns of X [see (4.4) of WW)]. Obviously, for an OA, R becomes I_m and E becomes 1. For a near-OA, the fact that $E < 1$ does not necessarily imply that the array is nonoptimal because the *upper bound* of 1 for E might be unrealistic. The variances of the estimates of the effects of columns of X are diagonal elements of R^{-1}.

3. Adding 3 two-level columns to $L_{32}(4^9 \cdot 2)$ (Taguchi 1987, p. 60) results in a *saturated* $L_{32}(4^9 \cdot 2^4)$. Adding 2 two-level columns to $L_{36}(3^{13} \cdot 2^2)$ [obtained by deleting the last two-level column of $L_{36}(3^{13} \cdot 2^3)$ (Taguchi 1987, p. 67)] results in $L_{36}(3^{13} \cdot 2^4)$. Adding 7 two-level

columns to $L_{36}(3^{12} \cdot 2^4)$ [obtained by deleting the last three-level column of the newly formed $L_{36}(3^{13} \cdot 2^4)$] results in $L_{36}(3^{12} \cdot 2^{11})$. Wang and Wu (1991) discussed a combinatorial approach to the construction of these arrays.

4. A column of an array can also be used for a blocking factor [or a qualitative factor (see Dey 1985, p. 40)]. For the analysis of data, this s-level column corresponds to a set of $s-1$ *dummy* variables in the design matrix X.

5. The problem of adding an s-level column ($s > 2$) to a base array such that the new column is (near-) orthogonal to other columns of this array is the same as that of dividing this array into s (near-) orthogonal blocks because the blocking factor can be assigned to the new factor. Computationally, the orthogonal blocking condition is achieved when $s_{1j} = s_{2j} = \cdots = s_{sj}$ $(j = 1, \ldots, m)$, where s_{wj} is the sum of values in column j of X in block w [see (1.6) of Draper et al. (1993)]. Because the sum of these s terms is constant, the simplest way to make them (near-) equal to one another is to minimize $\sum\sum s_{wj}^2$ $(w = 1, \ldots, s, j = 1, \ldots, m)$.

2. DISCUSSION

Table 1 lists 25 new near-OA's (constructed by adding two-level columns to existing OA's) and corresponding WW arrays (constructed by the method in sec. 6 of WW). The base array $L_{36}(3^{13} \cdot 2^4)$ used for constructing $L'_{36}(3^{13} \cdot 2^q), q = 5, \ldots, 9$, is the one in Remark 3 of Section 1. The base array $L_{50}(5^{11} \cdot 2)$ used to construct $L'_{50}(5^{11} \cdot 2^q), q = 2, \ldots, 5$, is from Taguchi (1987, p. 65). The base array $L_{54}(3^{25} \cdot 2)$ used to construct $L'_{54}(3^{25} \cdot 2^q), q = 2, 3$, is from Taguchi (1987, p. 41).

Each s-level column ($s > 2$) of arrays in Table 1 is orthogonal to other columns. Some two-level columns are not orthogonal to each other, however. If levels 0 and 1 of these columns are coded as -1 and $+1$, the correlation r of any two coded columns is the same in magnitude. I use $|r|$ to measure the degree of nonorthogonality among two-level columns in each near-OA. Only four WW arrays match the new arrays in terms of (a) E, (b) the maximum number of orthogonal two-level columns, (c) the minimum number of nonorthogonal pairs among two-level columns in the array, and (d) the degree of nonorthogonality among two-level columns. The new $L'_{10}(5 \cdot 2^5)$ and $L'_{20}(5 \cdot 2^{15})$ have $|r| = .2$, but the corresponding WW arrays have $|r| = .6$. Most new arrays, because of the way they were constructed, contain more orthogonal two-level columns than do the WW arrays. For example, the new $L'_{12}(3 \cdot 2^9)$ has four orthogonal two-level columns [because its first five columns form an $L_{12}(3 \cdot 2^4)$], but the corresponding WW array in Table 1 has only three orthogonal two-level columns (this array, formed by a near-difference matrix $D'_{3,3;2}$ is different from the one in fig. 3 of WW). Similarly, the new $L'_{20}(5 \cdot 2^{15})$ following has eight orthogonal two-level columns [because its first nine columns form an $L_{20}(5 \cdot 2^8)$], but the corresponding WW array in Table 1 has only three orthogonal two-level columns. The new array is particularly more appealing when approximately half of the number of two-level factors are of primary interest:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
0	0	0	1	1	0	0	1	0	1	1	0	1	0	0	1
0	1	1	0	0	1	1	0	1	1	1	1	1	1	0	0
0	0	1	0	1	0	1	0	0	0	0	1	0	0	1	1
0	1	0	1	0	1	0	1	1	0	0	0	0	1	1	0
1	0	1	1	0	0	1	0	1	1	0	0	1	1	1	1
1	1	1	0	1	0	0	1	1	0	1	1	1	0	1	0
1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
1	1	0	1	1	1	1	1	0	0	1	1	0	1	0	1
2	0	1	1	0	0	0	1	1	0	1	0	0	0	0	1
2	1	0	1	0	0	1	0	0	0	0	1	1	0	0	0
2	0	0	0	1	1	0	0	1	1	1	1	0	1	1	1
2	1	1	0	1	1	1	1	0	1	0	0	1	1	1	0
3	1	0	0	0	0	1	1	1	1	0	1	0	0	1	1
3	0	0	1	1	1	1	0	1	0	1	0	1	0	1	0
3	0	1	0	0	1	0	1	0	0	0	1	1	1	0	1
3	1	1	1	1	0	0	0	0	1	1	0	0	1	0	0
4	0	1	1	1	1	1	1	1	1	0	1	0	0	0	0
4	1	1	1	0	1	0	0	0	1	1	1	1	0	1	1
4	1	0	0	1	0	0	0	1	0	0	0	1	1	0	1
4	0	0	0	0	0	1	1	0	0	1	0	0	1	1	0.

The new $L'_{12}(6 \cdot 2^6)$ in Table 1 can be obtained by adding four columns $(0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0)'$, $(1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\,0)'$, $(0\,1\,0\,1\,1\,0\,1\,0\,1\,0\,0\,1)'$, and $(0\,0\,1\,0\,1\,0\,1\,1\,0\,1\,0\,1)'$ to the $L'_{12}(6 \cdot 2^2)$ on page 453 of Wang and Wu (1991). This array has the same number of nonorthogonal pairs as the $L'_{12}(6 \cdot 2^5)$ of WW, yet it accommodates one additional two-level column and is saturated. Deleting the last column of this array results in an $L'_{12}(6 \cdot 2^5)$ that has only four nonorthogonal pairs instead of six as the one of WW.

As another example, consider the IC fabrication experiment mentioned in Section 1 (Phadke et al. 1983). The nine factors included in this experiment are (1) mask dimension, (2) photoresist viscosity, (3) spin speed, (4) bake temperature, (5) bake time, (6) aperture, (7) exposure time, (8) developing time, and (9) plasma etch time. Factors (1), (2), and (4) are at two levels and the rest are at three levels. The design for this experiment was an $L'_{18}(3^6 \cdot 2^3)$, obtained by *collapsing* the first three-level column **BD** of the $L_{18}(3^7 \cdot 2)$ of Taguchi (1987, p. 36) (columns **A–I** following) to 2 two-level columns (**B** and **D**) using the scheme 0 to 00, 1 to 10, and 2 to 01 and retaining the other factors:

A	BD	C	E	F	G	H	I	B′	D′
0	0	0	0	0	0	0	0	1	0
0	0	1	1	1	1	1	1	1	0
0	0	2	2	2	2	2	2	1	0
0	1	0	0	1	1	2	2	0	1
0	1	1	1	2	2	0	0	0	1
0	1	2	2	0	0	1	1	0	1
0	2	0	1	0	2	1	2	1	1
0	2	1	2	1	0	2	0	1	1
0	2	2	0	2	1	0	1	1	1
1	0	0	2	2	1	1	0	0	0
1	0	1	0	0	2	2	1	0	0
1	0	2	1	1	0	0	2	0	0
1	1	0	1	2	0	2	1	1	1
1	1	1	2	0	1	0	2	1	1
1	1	2	0	1	2	1	0	1	1
1	2	0	2	1	2	0	1	0	0
1	2	1	0	2	0	1	2	0	0
1	2	2	1	0	1	2	0	0	0.

This design has $E = .992$ and one nonorthogonal pair **BD**. An alternative design, obtained by augmenting columns **A** and **C–I** with two-level columns **B′** and **D′**, has $E = .980$ and three nonorthogonal pairs **AB**, **AD**, and **BD**. Despite this, it has three advantages over the former: (1) **B′** and **D′** have an equal number of 0s and 1s, but **B** and **D** each have 12 0s and six 1s; (2) the four level combinations 00, 01, 10, and 11 of **B′** and **D′** appear with frequencies 6:3:3:6 and of **B** and **D** with frequencies 6:6:6:0; and (3) the six level combinations 00, 01, 10, 11, 20, and 21 of each three-level column and **B′** (or **D′**) appear with equal frequencies 3:3:3:3:3: and **B** (or **D**) with unequal frequencies 4:2:4:2:4:2. The advantage is apparent when the high level of **B** (or **D**) happens to be better than the low level of **B** (or **D**) or when it is desirable to study the interaction between **B** and **D**. Note that if there is a 10th two- or three-level factor, the collapsing method fails.

Now, let us suppose that it is essential that factors (2) and (4) in the preceding experiment are at three levels and as such a saturated $L'_{18}(3^8 \cdot 2)$ is required. This array, not found in WW, can be obtained by dividing the $L_{18}(3^7 \cdot 2)$ (columns **A–I**) into three blocks and assigning the blocking factor to the new factor (see Remark 5 of Sec. 1). The newly added three-level column $(000111222111222000)'$ is orthogonal to **A** and **C–I** but nonorthogonal to the three-level column **BD**. The situation is much less favorable with WW arrays, which are not listed in Table 1. These WW arrays contain subarrays of the type 3^k in 12, 15, or 24 runs or 4^k in 24 or 36 runs ($k \geq 3$). The k three- or four-level columns of these arrays are nonorthogonal to one another. This undesirable property heavily complicates analysis and interpretation and therefore might confine their popularity. I will not pursue them further but only give an example to show how a WW near-OA of this type, an $L'_{12}(3^4 \cdot 2^2)$ on page 420 of WW (columns **A–F** following), can be improved:

A	B	C	D	E	F	C′	D′
0	0	0	0	0	0	2	0
0	1	2	0	1	1	0	0
0	2	1	1	0	1	1	1
0	0	2	1	2	1	2	2
0	2	0	2	1	0	1	2
0	1	1	2	2	0	0	1
1	1	1	1	0	0	2	2
1	2	0	1	1	1	2	1
1	0	2	2	0	1	0	2
1	1	0	2	2	1	1	0
1	0	1	0	1	0	1	1
1	2	2	0	2	0	0	0.

The two-level column **A** is orthogonal to columns **B–F**. The two-level column **F** is orthogonal to **A**, **B**, and **E** but not to **C** and **D**. Delete **C** and **D** to form an $L'_{12}(3^2 \cdot 2^2)$. Divide this array into three blocks and add the blocking factor **C′** to this array to form an $L'_{12}(3^3 \cdot 2^2)$. Next, divide this array into three blocks and add the blocking factor **D′** to this array to form an $L'_{12}(3^4 \cdot 2^2)$. The newly formed array, like the original one, has a property that each level combination of any 2 three-level columns (which are nonorthogonal to one another) appears at least once. In addition, **F** is orthogonal to both **C′** and **D′**. This operation has also improved E of the original array from .856 to .941.

3. CONCLUDING REMARKS

This article has taken up the concept of near-OA's and puts forth a quick and simple method of constructing these arrays. Basically, it takes an OA or a near-OA and augments it with additional columns such that the resultant array is very good with respect to E and other design-goodness criteria. The merits of this method are (a) ease of comprehension and use for scientists and engineers, (b) extent of improvement it makes over the combinatorial method of WW, and (c) higher level of flexibility than the combinatorial method of WW. Note that some near-OA's not reported in this article can be constructed by adding columns to nonsaturated OA's $L_{24}(4 \cdot 3 \cdot 2^{13})$, $L_{24}(6 \cdot 4 \cdot 2^{11})$, $L_{36}(4 \cdot 3^{13})$, and so forth.

The computer time is minimal for most near-OA's in Table 1 and for small OA's such as $L_{12}(3 \cdot 2^4)$ or $L_{12}(6 \cdot 2^2)$ [obtained by adding 4 and 2 two-level columns to $L_{12}(3)$ and $L_{12}(6)$, respectively]. $L_{12}(3 \cdot 2^4)$, for example, was obtained 52 times out of 100 tries. The time per try is .01 second on a 66 MHz 486DX2 PC. Other OA's such as the $L_{20}(5 \cdot 2^8)$ require more computer time.

It is essential to note that small arrays are appropriate when the response is measured with sufficient precision to warrant the small number of runs per factor level. In addition, there is a price to pay for use of near-OA's. Section 4 of WW discussed the effects of near-orthogonality as the result of using near-OA's on estimation efficiency and analysis of data.

Further details on the arrays discussed in this article and NOA program and CUT, the blocking program used to construct arrays, can be obtained from the author (E-mail address:namky@forprod.csiro.au).

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