INCREMENTAL UTILITY ELICITATION FOR ADAPTIVE PERSONALIZATION

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Abstract

Medical devices often contain many tunable parameters. The optimal setting of these parameters depends on the patient's utility function, which is often unknown. This raises two questions. First, how should we optimize the parameters given partial information about the patient's utility? And secondly, what questions do we ask to efficiently elicit this utility information? In this paper, we present a coherent probabilistic decision-theoretic framework to answer these questions. We illustrate the potential of this framework on a toy problem and discuss directions for future research.

1 Introduction

In many cases, a decision maker is uncertain about the user's preferences, but still wishes to recommend (or take) decisions on the user's behalf. Furthermore, by asking additional questions, the decision maker wants to gain further knowledge about these preferences. This problem is often referred to as incremental/adaptive utility/preference elicitation. It appears in many different settings. Examples can be found in, among others, decision support for prenatal diagnosis [5], travel planning [1], and optimal design for heart defibrillators [8]. In this paper we consider the problem of adaptive personalization. More specifically, we will work out the setting in which parameters of a (medical) device have to be tuned such as to adapt them optimally to a user's preferences.

Consider for example the task of an optician, trying to fit eyeglasses or lenses to a client. Not (quite) knowing the client's actual visual condition, he tries to find the optimal parameter setting (e.g., strength, cylinder, axis) within a limited set of experiments. This is a relatively straightforward problem, but for more complex devices, with many different parameters that have to be tuned to different conditions, the problem becomes a lot more challenging. Examples are the tuning of hearing aid devices, pacemakers, functional electrical stimulation (e.g., for cycling), and so on.

In this paper, we will take a decision-theoretic approach towards incremental utility elicitation, following [5, 3] and others. In particular, we will derive incremental utility elicitation as a special case of Bayesian experimental design with a specific goal function. We will spell out what it amounts to in the setting of parameter tuning and illustrate its potential on a toy problem.

In section 2, we introduce the notation and give some definitions, necessary for the exposition that follows. In section 3, we then describe the mathematical framework that joins the two issues above. Section 4 presents the results of a toy experiment and section 5 concludes with directions for further research.

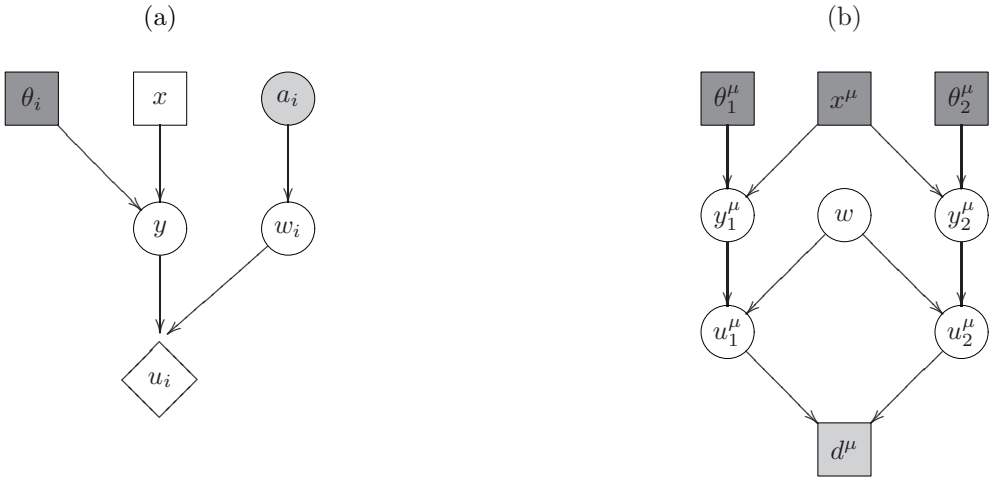

Figure 1: (a) Bayesian decision network specifying the utility function of patient i. (b) Bayesian network visualizing the probability distribution underlying experiment μ in the paired-comparison experimental set-up. The patient index i and profile a have been omitted.

2 Notation and definitions

2.1 The utility model

We will consider the following abstract model of a medical device. A medical device has many tuning parameters, summarized in the vector $\theta \in \Theta$. The value of θ determines how an input $x \in X$ is transformed to an output represented by a vector $y \in Y$: $y = F(x; \theta)$. The goal will be to find the set of parameters θ_i^* that is "optimal" (to be defined below) for patient i. This will depend on the environmental conditions, i.e., the typical inputs that the patient has to deal with. We represent this environment with a probability density $P(x)$. We assume that the domains Θ and X are possibly large but finite sets. We can think of having available a *library* of (relevant) inputs and parameter settings.

Each patient has different preferences. The preference of patient i is represented by the utility function $u_i(y)$, which depends on the output of the device. We assume that utility functions by themselves are models that can be represented by a finite set of parameters $w \in W$. That is, the utility function for patient i reads $u_i(y) = U(y; w_i)$, where $U(y; w)$ is some given functional form, for example, $U(y; w) = \exp(-|w - v(y)|^2)$ with $v(y)$ a fixed set of basis functions or features. The utility of patient i, represented by the *utility state* w_i, is unknown and hence we will treat w_i as a probabilistic variable.

Although we do not know the patient's utility function, following [5] we assume that we have some vague notion of it, that can be represented by a prior probability distribution. This prior probability may depend on other information that we have about the patient (e.g., age, results of other measurements), collected in the *patient profile* $a \in A$. We will write $P(w|a_i)$ for the prior probability over the utility states w given patient profile a_i. Such prior could perhaps be learned from the results on a population of patients.

The model described above can be summarized in the Bayesian decision network sketched in Figure 1(a). Squares indicate discrete variables, ovals continuous variables, and diamonds utilities. We use lighter gray to indicate that the variable is typically observed or given and darker gray to indicate that this variable is typically to be optimized.

2.2 The experimental setup

Without loss of generalization, we consider in the following a single user. For ease of notation we omit the index i and the (conditioning on the) patient profile a from now on.

We consider the following so-called paired-comparison experimental setup. (there are other options, in particular the standard gamble approach as used in [5, 2], see e.g., [12]). An experiment e consists of picking an input x from X in combination with two parameter settings θ_1 and θ_2 from

Θ, i.e., $e = \{x, \theta_1, \theta_2\}$. The patient is then asked to indicate whether he prefers $y_1 = F(x;\theta_1)$ over $y_2 = F(x;\theta_2)$, which we will write as $y_1 \succ y_2$, or the other way around, $y_1 \prec y_2$. A standard modeling assumption [16, 13] is that the patient's decision in such a paired-comparison forced-choice experiment follows a logistic regression model:

$$P(y_1 \succ y_2) = \frac{1}{1 + \exp\{-[u(y_1) - u(y_2)]\}} .$$

In psychophysics and econometrics, this model is often referred to as the Bradley-Terry model [4]. More specifically indicating the dependence on the utility state w, we write

$$P(d|e, w) = \frac{1}{1 + \exp\{-d \times [U(x;\theta_1, w) - U(x;\theta_2, w)]\}} , \tag{1}$$

where we used shorthand $U(x;\theta, w) \equiv U(F(x;\theta); w)$ and where $d \in \{-1, 1\}$ with $d = 1$ if $y_1 \succ y_2$ and $d = -1$ if $y_1 \prec y_2$. The data set D_n consists of n tuples $\{e^\mu, d^\mu\}$, $\mu = 1 \dots n$.

Experiment μ is visualized in Figure 1(b). To display several experiments, one should copy all the variables with index μ. The nodes for the utilities u_1^μ and u_2^μ (here treated as "standard" probabilistic variables) for different μ are then linked to the same w.

3 Framework

3.1 Bayesian updating

Let $P(w|D_n)$ denote the probability density over utility states w after having seen the result of n experiments. The probability density after $n+1$ experiments then follows from Bayes' rule:

$$P(w|D_{n+1}) = P(w|d^{n+1}, e^{n+1}, D_n) = \frac{P(d^{n+1}|e^{n+1}, w)P(w|D_n)}{P(d^{n+1}|e^{n+1}, D_n)} , \tag{2}$$

with $P(d^{n+1}|e^{n+1}, w)$ from (1) and where the likelihood term in the denominator follows from normalization of the product in the numerator:

$$P(d|e, D_n) = \int_W dw\, P(d|e, w)P(w|D_n) . \tag{3}$$

In other words, Bayes' rule allows us to keep track of the probability over utility states and hence utilities when new data becomes available.

Incremental utility elicitation now has to solve the following questions.

1. Suppose that for a particular patient we are given the probability density $P(w|D_n)$ as well as the environmental conditions $P(x)$. How should we set the parameters θ for this patient?

2. Given a probability density, e.g., $P(w|D_n)$, how do we "optimally" choose the next experiment $e^{n+1} = \{x^{n+1}, \theta_1^{n+1}, \theta_2^{n+1}\}$?

3.2 Bayesian experimental design

Incremental utility elicitation can be interpreted as a special case of Bayesian experimental design, as e.g. explained in [7, 6] and first presented by Lindley in [18].

Lindley's framework consists of two decision problems. First an experiment e (in our case the tuple $\{x, \theta_1, \theta_2\}$) is selected from the possible collection of experiments E (in our case $X \times \Theta \times \Theta$). After choosing an experiment e, the outcome d is observed. Based on this new observation d and the experiment e, a terminal decision θ is selected from a set of possible decision rules Θ. A goal function $G(\theta, w, e, d)$ encodes the costs and consequences of using experiment e and decision θ with result d and parameter w. The Bayesian solution to experimental design is now to find the best design and best decision rule that in expectation achieve the highest goal.

The terminal decision problem amounts to finding the best decision θ^* given the observed data d under experiment e that maximizes the *posterior* expected goal

$$G(e, d) \equiv \max_\theta \int_W dw\, G(\theta, w, e, d)P(w|d, e, D_n) , \tag{4}$$

where $P(w|d, e, D_n)$ is the posterior density that follows from Bayesian updating of the density $P(w|D_n)$ as in (2). The second stage optimization problem involves finding the best experiment e that maximizes the so-called *pre-posterior* expected goal that follows by integrating (4) over the possible outcomes of d:

$$G(e) \equiv \sum_{d=\pm 1} P(d|e, D_n) G(e, d) \,, \tag{5}$$

with $P(d|e, D_n)$ from (3). The Bayesian solution to experimental design is provided by the experiment e^* that maximizes $G(e)$:

$$e^* = \arg\max_e G(e) = \arg\max_e \sum_{d=\pm 1} P(d|e, D_n) \max_\theta \int_W dw\, P(w|d, e, D_n) G(\theta, w, e, d) \,. \tag{6}$$

This general formulation can be used to find optimal designs for a single experiment and can (at least in theory) be easily extended to optimal selection of a sequence of experiments and sequential decision making [18].

In the general setting, when the goal is to infer w or functions of w without specification of particular hypotheses, a typical choice for the goal function $G(\theta, w, e, d)$ reads

$$G(\theta, w, e, d) = \log P(w|d, e, D_n, a) - \log P(w|D_n, a) \,,$$

which makes $G(e)$ the expected change in Shannon information or, equivalently, the Kullback-Leibler divergence between the posterior and prior density of w.

3.3 Optimal utility elicitation

The question remains what optimality criterion, i.e., goal function $G(\theta, w, e, d)$ is most suited to our purposes. An obvious choice in the context of utility elicitation is the so-called expected utility, defined as [19, 11]

$$EU(\theta, w) = \sum_{x \in X} P(x) U(x; \theta, w) \,. \tag{7}$$

Namely, if the utility state equals w, the best we can do is to optimize the expected utility with respect to θ. The posterior expected goal then boils down to what has been phrased the *expected* expected utility (EEU) in [3]: it contains two expectations, taking into account both the environment X and the uncertainty over the utility states $w \in W$. The optimal decision with maximum expected expected utility (MEEU) after having observed the data D_n is then

$$\theta_n^* = \arg\max_{\theta \in \Theta} EEU_n(\theta) \,,$$

with

$$EEU_n(\theta) \equiv \sum_{x \in X} P(x) \int_W dw\, P(w|D_n) U(x; \theta, w) \,.$$

EEU seems to be a fairly natural concept given probabilistically quantified uncertainty over utilities and is therefore widely used (e.g., [5, 11, 9]). An alternative is the so-called minimax regret decision criterion as explained in [21].

With this choice, the pre-posterior utility function $G(e)$ in (5) boils down to the *expected value given perfect information*

$$EV|PI_n(e) = \sum_{d=\pm 1} \max_\theta \int_W dw\, P(d|e, w) P(w|D_n) EU(\theta, w) \,, \tag{8}$$

where we substituted the posterior from (2) to note that the likelihood term $P(d|e, D_n)$ drops out. Subtracting the maximum expected expected utility given no further information, we obtain the so-called *expected value of perfect information*:

$$EVPI_n(e) \equiv EV|PI_n(e) - EEU_n(\theta_n^*) \,.$$

Note that the ordering of the sum over d and the max w.r.t. θ is crucial in (8): if we were allowed to interchange them we would obtain EEU_n.

Algorithm 1 Incremental utility elicitation

1: $n = 0$
2: **repeat**
3: **for all** $e \in E$ **do**
4: $EV|PI(e) = 0$
5: **for all** $d \in \{-1, 1\}$ **do**
6: **for all** $\theta \in \Theta$ **do**
7: $S(e, d, \theta) = \sum_{x \in X} P(x) \int_W dw\, P(d|e, w) P(w|D_n) U(x; \theta, w)$
8: **end for**
9: $EV|PI(e) = EV|PI(e) + \max_\theta S(e, d, \theta)$
10: **end for**
11: **end for**
12: $e^{n+1} = \arg\max_e EV|PI(e)$
13: Present e^{n+1} to the patient and observe d^{n+1}
14: $P(w|D_{n+1}) = \dfrac{P(d^{n+1}|e^{n+1}, w) P(w|D_n)}{\int_W dw\, P(d^{n+1}|e^{n+1}, w) P(w|D_n)}$
15: $n = n + 1$
16: **until** some criterion is met

Since $EEU_n(\theta_n^*)$ is independent of e, we conclude that *Bayesian optimal design with the expected utility as goal function boils down to maximizing the expected value of perfect information.* The comparison between the (maximum) expected value of perfect information and the cost of doing an additional experiment provides for a natural stopping criterion. We could also choose other selection criteria, e.g., adapting $G(\theta, w, e, d)$ to incorporate a penalty for unpleasant experiments, which would make $G(\theta, w, e, d)$ indeed depend on e.

3.4 Algorithm

To get a feeling for the algorithmic complexity, we describe the main algorithmic steps in pseudo-code in Algorithm 1. In all this, we assume that the distribution $P(x)$ of inputs, the patient model $P(d|e, w)$, the utility function $U(x; \theta, w)$, and the prior $P(w)$ are all known and given.

The computational complexity of the exact brute-force algorithm is huge. It implies that for each combination of experiments $e \in E$, responses $d \in \{-1, 1\}$, parameter settings $\theta \in \Theta$, and inputs $x \in X$, we have to solve the integral over $w \in W$ in line 7 of Algorithm 1. In many cases, this will be completely unfeasible and we will have to find good approximations to prevent this (see the directions for further research below).

Another important issue is the Bayesian updating of the probability over utility states, line 14 of Algorithm 1. In general this will be intractable, and we will have to use techniques for approximate inference such as variational methods or Monte Carlo sampling.

4 A toy example

To show the potential of this framework, we consider the following toy example. The input-output relationship of the device is modeled by a feedforward neural network, i.e., $y = F(x; \theta) = \theta \tanh(Ax+b)$ with A and b randomly drawn. The utility model is of the form $U(y; w) = \exp(-|Cy - w|^2)$, with C and w randomly drawn. A, b, and C are considered known to the decision maker, w is unknown to him (but is used to simulate the preferences of the patient in line 13 of Algorithm 1), and θ has to be optimized. The environment consists of n_x randomly drawn inputs. All random variables specified above are drawn independently from normal distributions with mean zero and unit variance.

As discussed above, a crucial point is how to keep track of the probability $P(w|D_n)$ over the

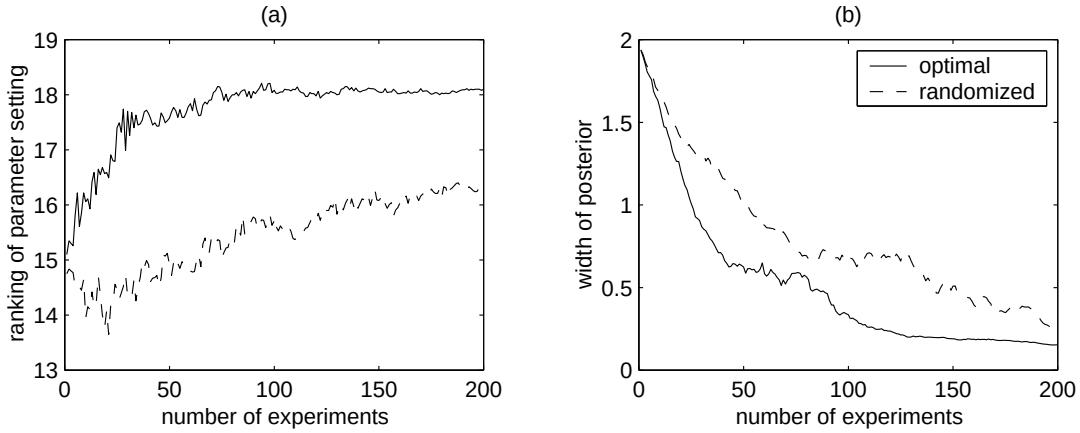

Figure 2: Results on a toy example. (a) Ranking of the parameter setting θ_n^* (20 is best, 1 is worst) as a function of the number of experiments n for the "optimal" design as described in Algorithm 1 compared with a randomized design. (b) Width of the (estimated) probability over utility states $P(w|D_n)$ for the two designs. Averages over 100 trials. See text for further explanation.

utility states w and how to compute the corresponding integrals. Here we consider a sampling approach known as the Gaussian particle filter (see e.g., [17]). For the prior $P(w)$ we take a normal distribution with variance 4. The library Θ consists of n_θ parameter settings θ, each of them optimized for a particular utility state w randomly drawn from the prior. This ensures that the library contains parameter settings that make sense. Note further that this setup is such that the patient model is rather noisy, i.e., for typical w and many experiments $P(d|e,w)$ is closer to 0.5 than to 0 or 1.

Results are displayed for the case in which the inputs x and outputs y are two-dimensional, the device model contains two hidden units, i.e., θ is a two-by-two matrix, w is one-dimensional, $n_x = 10$, $n_\theta = 20$, and we use 100 particles in the Gaussian particle filter. The set of experiments E then consists of $10 \times 20 \times 20 = 4000$ possible combinations of inputs and parameters.

In Figure 2 we compare the optimal design, i.e., where at each iteration we choose the experiment e that maximizes the expected value of perfect information over E, with a randomized design, i.e., where at each iteration we draw an experiment e at random from E. Figure 2(a) displays the rank of the parameter setting θ_n^* chosen by the decision maker, which is the one that maximizes the expected expected utility given the decision maker's estimate of the patient's utility, summarized in $P(w|D_n)$. Note that, knowing the true utility state, we can rank the $n_\theta = 20$ different parameter settings beforehand. A ranking of 20 means that the decision maker chose the best possible parameter setting (in the given library of 20 options), a ranking of 1 is the worst. Averages are over 100 independent trials. It can be seen that the optimal design clearly outperforms the randomized design, much more quickly zooming in on the best parameter settings. It also appears to be saturating at a higher level. Figure 2(b) shows the evolution of the width of the (approximated) posterior $P(w|D_n)$ for the two designs. As expected, the posterior in case of the optimal design tightens more quickly than the one for the randomized design.

5 Directions for further research

In this paper we have sketched a mathematical framework for adaptive personalization. It should be clear that this research has just started: there are many challenges still to tackle before it can be applied to nontrivial practical problems.

- *Bayesian updating.* We have to keep track of the probability density $P(w|D_n)$ and compute integrals over W involving this density (lines 14 and 7 of Algorithm 1). Here we used the Gaussian particle filter, but other approximate inference techniques should also be considered.

- *Efficient optimization.* To compute the next optimal experiment, we have to optimize over the sample spaces Θ and E (lines 9 and 12 of Algorithm 1). Especially the cardinality of E can become huge and we will need optimization methods that can provide good suboptimal solutions within reasonable time, such as simulating annealing or genetic algorithms. For example, in [22] simulated annealing is applied to optimize the design of microarray studies and in [20] genetic algorithms are used to optimize the design of fMRI experiments. Apart from that, mathematical properties can be exploited to restrict the search space dramatically.

- *Replacing expensive on-line calculations by functional mappings.* Even with the above approximations, on-line calculation of the optimal experimental design may still be computationally too demanding in practice. However, following the probabilistic framework laid out in section 2 and 3, we can simulate many experiments and based on those learn relevant functions. Similar ideas are applied in the context of Markov decision processes, in particular for computing the so-called value functions [14, 2].

- *Nonmyopic experimental design.* In the above, we have mainly discussed the case of choosing one experiment at a time, sometimes referred to as the myopic approach [10]. It would be even better if we could solve the more general so-called nonmyopic case of optimally choosing the next k experiments. In this case, however, the computations become much more involved and in fact scale exponentially with k. An approximation, linear in the number of experiments, is described in [15]. In [2] the problem of nonmyopic preference elicitation is formulated and approximately solved in terms of a partially observable Markov decision process (POMDP).

- *Design choices.* Depending on the particular application, many design choices have to be made. Examples are appropriate representations of the device model, utility function, and prior distribution, some of which may be learned from a collection of experiments on different patients. Another issue is the construction of useful libraries X, Θ, and E. For example, for clarity of exposition we have chosen $E = X \times \Theta \times \Theta$, but we might as well base our library of experiments on different sets of inputs and parameter settings (e.g., those that are known to be discriminative). Last but not least, we focused on expected expected utility as the terminal decision criterion. Other criteria, such as the minimax regret decision criterion introduced by [21], deserve to be studied as well.

- *Theoretical analysis.* Even in our toy example, it takes at least about 50 questions to converge. It would be interesting to study how this scales as a function of the number of dimensions, amount of noise, and so on. In simple cases, using information-theoretical arguments, it might be possible to compute the best possible performance of any algorithm. This would give an indication of the practical feasibility. Furthermore, it would tell us when it makes no sense to further improve an existing algorithm.

- *System comparison.* In this paper the target of the analysis was to estimate the optimal parameter value(s) θ^* for algorithm $y = F(x; \theta)$. In the process of searching for the optimal value θ^*, we also obtain the maximum expected expected utility $MEEU = EEU(\theta^*)$, which can be used as a (perceptual) performance metric for algorithm F. The $MEEU$ can be used to compare different algorithms (and select the best). The metric can also be extended to predict performance for a patient population by marginalization of the $MEEU$ over a prior *patient profile* distribution $P(a)$. Performance measures such as these are within our scope of future research.

References

[1] J. Blythe. Visual exploration and incremental utility elicitation. In *Eighteenth national conference on Artificial intelligence*, pages 526–532, Menlo Park, CA, USA, 2002. American Association for Artificial Intelligence.

[2] C. Boutilier. A POMDP formulation of preference elicitation problems. In *Eighteenth national conference on Artificial intelligence*, pages 239–246, Menlo Park, CA, USA, 2002. American Association for Artificial Intelligence.

[3] C. Boutilier. On the foundations of expected expected utility. In *IJCAI-2003*, 2003.

[4] R. Bradley and M. Terry. The rank analysis of incomplete block designs. 1. The method of paired comparisons. *Biometrika*, 39:324–345, 1952.

[5] U. Chajewska, D. Koller, and R. Parr. Making rational decisions using adaptive utility elicitation. In *Proceedings of the Seventeenth National Conference on Artificial Intelligence and Twelfth Conference on Innovative Applications of Artificial Intelligence*, pages 363–369. AAAI Press / The MIT Press, 2000.

[6] K. Chaloner and I. Verdinelli. Bayesian experimental design: a review. *Statistical Science*, 10:273–304, 1995.

[7] M. Clyde. Experimental design: Bayesian designs. In *International Encyclopedia of Social and Behavioral Sciences*, pages 5075–5081. Elsevier, 2004.

[8] M. Clyde, P. Müller, and G. Parmigiani. Optimal design for heart defibrillators. In C. Gatsonis, J. Hodges, R. Kass, and N. Singpurwalla, editors, *Bayesian Statistics in Science and Engineering: Case Studies II*, pages 278–292, New York, NJ, 1995. Springer-Verlag.

[9] R. Cyert and M. de Groot. Adaptive utility. In M. Allais and O. Hagen, editors, *Expected Utility Hypothesis and the Allais Paradox*, pages 223–241. D. Reidel, 1979.

[10] S. Dittmer and F. Jensen. Myopic value of information for influence diagrams. In D. Geiger and P. Shenoy, editors, *Proceedings of the Thirteenth Conference on Uncertainty in Artificial Intelligence*, pages 142–149, San Francisco, 1997. Morgan Kaufmann Publishers Inc.

[11] P. Fishburn. *The foundations of expected utility.* D. Reidel, Dordrecht, 1982.

[12] S. French. *Decision Theory.* Halsted Press, New York, 1986.

[13] M. Glickman and S. Jensen. Adaptive paired comparison design. *Journal of Statistical Planning and Inference*, 127:279–293, 2005.

[14] M. Hauskrecht. Value-function approximations for partially observable Markov decision processes. *Journal of Artificial Intelligence Research*, 13:33–94, 2000.

[15] D. Heckerman, E. Horvitz, and B. Middleton. An approximate nonmyopic computation for value of information. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15(3):292–298, 1993.

[16] B. Kanninen. Optimal design for multinomial choice experiments. *Journal of Marketing Research*, 39:307–317, 2002.

[17] J. Kotecha and P. Djurić. Gaussian particle filtering. *IEEE Transactions on Signal Processing*, 51:2592–2601, 2003.

[18] D. Lindley. *Bayesian Statistics - A Review.* SIAM, Philadelphia, 1972.

[19] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behaviour.* Princeton University Press, Princeton, NJ, 1944.

[20] T. Wager and E. Nichols. Optimization of experimental design in fMRI: a general framework using a genetic algorithm. *NeuroImage*, 18:293–309, 2003.

[21] T. Wang and C. Boutilier. Incremental utility elicitation with the minimax regret decision criterion. In G. Gottlob and T. Walsh, editors, *IJCAI-03, Proceedings of the Eighteenth International Joint Conference on Artificial Intelligence*, pages 309–318, San Mateo, CA, 2003. Morgan Kaufmann.

[22] E. Wit, A. Nobile, and R. Khanin. Near-optimal design for dual-channel microarray studies. *Applied Statistics*, in press, 2005.