Bayesian Machine Learning for Personalization of Hearing Aid Algorithms

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Problem Statement

- I have a set of HA's from Phonak, Oticon, ReSound, Widex, etc.
- Next to me I have a patient.
- Goal: Pick the best hearing aid and optimize the parameter settings for this patient.
- I am allowed to measure the patient's auditory profile and conduct listening experiments.

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Outline

We will provide a solution for this problem (algorithm evaluation and fitting). Our solution features the following properties:-

- A completely general (algorithm-independent) computable solution, based on elementary mathematics.
- Complete fitting and algo evaluation results including any statistic (e.g. confidence limits etc.) in seconds.
- There is no loss of information: all patient data will be optimally processed. (more measurements $\Rightarrow$ better results).
- Lots of room for algorithm designers, auditory modelers and audiologists to be creative and affect the outcome.

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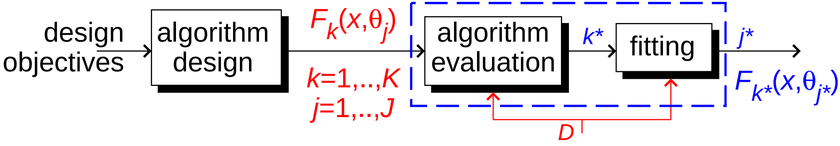

- We have K HA algorithms $\mathcal{F} = \{F_1(x,\theta),\ldots,F_K(x,\theta)\}$.
- For each algorithm F_k, there are J candidate parameter values $\Theta = \{\theta_1,\ldots,\theta_J\}$.
- We will do a set of experiments $E = \{e_1,\ldots,e_N\}$ and collect the measurements (auditory profile and listening test responses) in variable $D = \{d_1,\ldots,d_N\}$.
- Goal: Select the best algo index k^* and the best parameter value index j^*.

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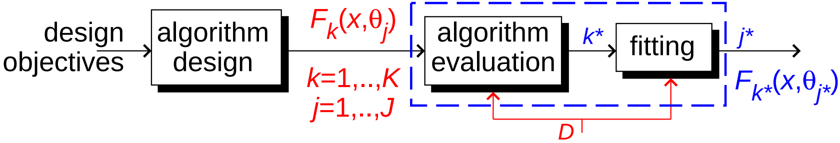

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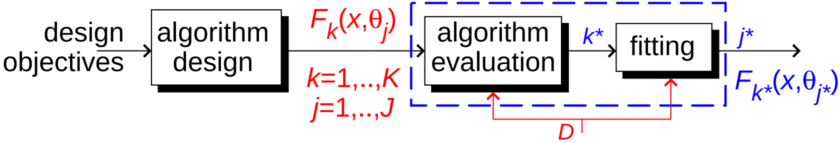

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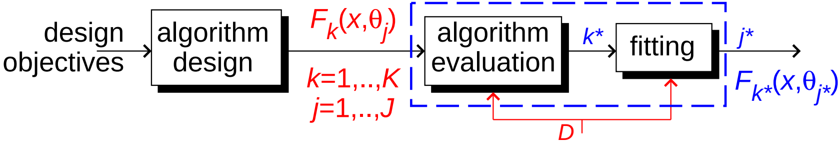

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Rating the Candidate Parameter Values

- We cannot hope to pick the best unless we can evaluate all candidates.
- Assign a rating $\in \mathbb{R}^+$ to each candidate θ_j that expresses our belief that θ_j is the best value.
  - Think of this as an ELO rating for each candidate.
- Normalize such that the sum of the ratings equals 1.
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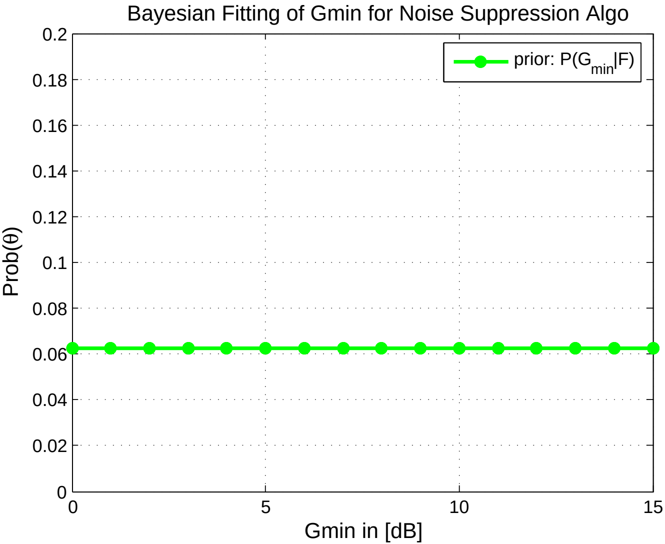

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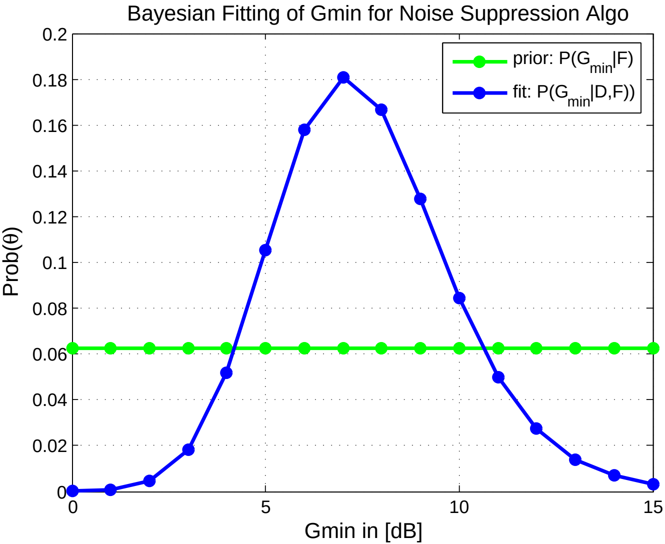

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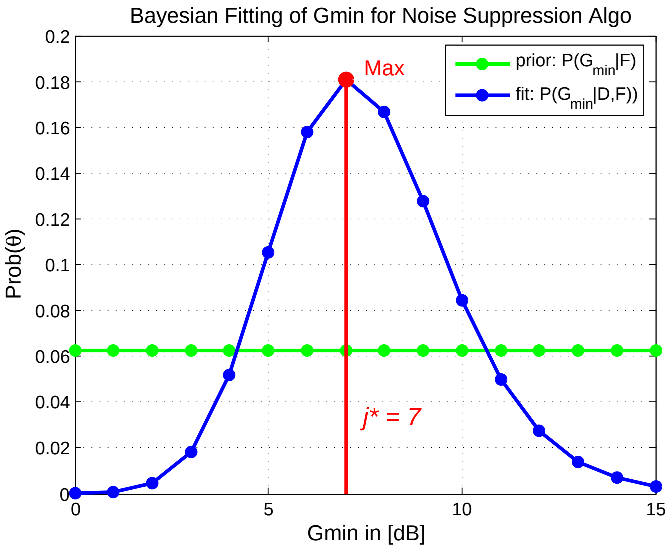

Probability Theory

- The conditional probability $P(CR = 3|HL = 50\text{dB})$ expresses our belief that $CR = 3$, given a hearing loss of 50 (dB).
- The joint probability $P(CR = 3, HL = 50\text{dB})$ expresses our belief that $CR = 3$ and $HL = 50$ (dB).
- Sum Rule. For $\{A_1, \ldots, A_K\}$ mutually exclusive and exhaustive,

$$\boxed{\sum_{k=1}^{K} P(A_k|I) = 1}$$

- Product Rule.

$$\boxed{P(A,B|I) = P(A|B,I) \cdot P(B|I) = P(B|A,I) \cdot P(A|I)}$$

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Bayes for Fitting and Algo Evaluation

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- Rewrite as

$$\mathrm{FIT} \times \mathrm{EVAL} = \mathrm{PCM} \times \mathrm{PRIOR}$$

- We can show that

$$\begin{aligned} \mathrm{EVAL} &= P(D|\{E,F_k\}) = \sum_j \mathrm{PCM} \times \mathrm{PRIOR} \\ \mathrm{FIT} &= P(\theta_j|D,\{E,F_k\}) = \frac{\mathrm{PCM} \times \mathrm{PRIOR}}{\mathrm{EVAL}} \end{aligned}$$

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Proof for Bayesian Fitting (OPTIONAL)

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The Algorithm Evaluation factor $\mathrm{EVAL} \equiv P(D|\{E, F_k\})$ (OPTIONAL)

- Technically, $P(F_k|D)$ is more accurate than $\mathrm{EVAL} \equiv P(D|F_k)$.
- Consider 2 algorithm candidates, F_k and F_l; we can evaluate F_k versus F_l by the ratio

$$\frac{P(F_k|D)}{P(F_l|D)} = \frac{\frac{P(D|F_k)P(F_k)}{P(D)}}{\frac{P(D|F_l)P(F_l)}{P(D)}} = \frac{P(D|F_k)}{P(D|F_l)} \times \frac{P(F_k)}{P(F_l)}$$
$$= \frac{\mathrm{EVAL}_k}{\mathrm{EVAL}_l}$$

  if we have no prior preference for either F_k or F_l (i.e. $P(F_k) = P(F_l) = 0.5$).
- Hence, EVAL is a proper measure for algorithm evaluation.

More about $\mathrm{PRIOR} \equiv P(\theta_j|\{E, F_k\})$

- $P(\theta_j|\{E, F_k\})$ describes what we know (and don't know) about the parameter value candidates after design of algorithm F_k, but still before any patient measurements.
- Example: 'CR lies somewhere between 1 and 4'
- This is the place for algo engineers to shine.
- Moral: algo engineer must properly define the candidate parameter value space.
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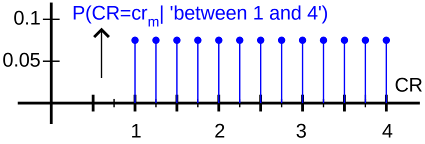

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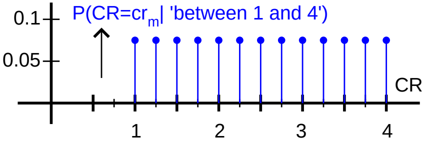

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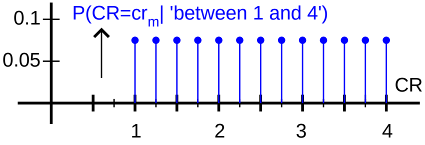

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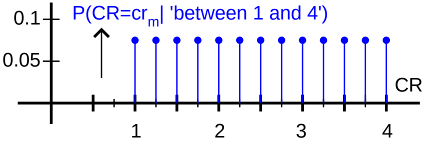

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Probabilistic Choice Model $\mathrm{PCM} \equiv P(D|\theta_j, \{E, F_k\})$

- $P(D|\theta_j, \{E, F_k\})$ describes what we know about patient decisions (D), given experimental design E and a given algorithm $F_k(\cdot, \theta_j)$.
- Example: Pairwise listening protocol for noise suppression.

- Bradley-Terry model

$$P(d|\theta, e, F) = \frac{e^{U(x,\theta^1)}}{e^{U(x,\theta^1)} + e^{U(x,\theta^2)}}$$

where the utility function $U(\cdot)$ is $\mathrm{PESQ}(\cdot)$, $\mathrm{PEMO}(\cdot)$, $U_{\mathrm{camfit}(\cdot)}$, $\mathrm{CSII}_3(\cdot)$ or any other utility model that you desire.

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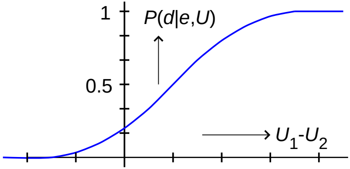

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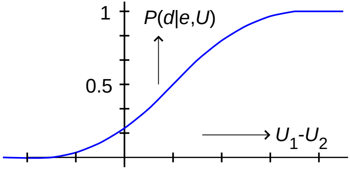
Probabilistic Choice Model $\mathrm{PCM} \equiv P(D|\theta_j, \{E, F_k\})$

- $P(D|\theta_j, \{E, F_k\})$ describes what we know about patient decisions (D), given experimental design E and a given algorithm $F_k(\cdot, \theta_j)$.
- Example: Pairwise listening protocol for noise suppression.

- Bradley-Terry model

$$P(d|\theta, e, F) = \frac{e^{U(x,\theta^1)}}{e^{U(x,\theta^1)} + e^{U(x,\theta^2)}}$$



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Bayesian Fitting and Evaluation: How to do it

1. Engineer designs HA algo $y = F_k(x, \theta)$ and specifies what he doesn't know in $\mathrm{PRIOR} \equiv P(\theta_j | F_k)$.
2. The audiologist designs experiments E and collects patient measurements D. These results are encoded in $\mathrm{PCM} \equiv P(D | E, \theta_j, F_k)$.
3. PT produces fitting and algo evaluation instantly, BOOM:

$$P(D | \{E, F_k\}) = \sum_j \mathrm{PCM} \times \mathrm{PRIOR} \qquad \text{(EVAL)}$$

$$P(\theta_j | D, \{E, F_k\}) = \frac{\mathrm{PCM} \times \mathrm{PRIOR}}{\mathrm{EVAL}} \qquad \text{(FIT)}$$

- End of story, in principle

4. In a practical (HA) setting, choose

$$k^* = \arg\max_k P(D | \{E, F_k\}) \quad \text{and} \quad j^* = \arg\max_j P(\theta_j | F_{k^*}, D)$$

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Conservation of Information

- Recall Bayes for $\mathrm{ALL} = P(\theta_j, D | \{E, F_k\}) =$

$$P(\theta_j|D, \{E, F_k\}) \cdot P(D|\{E, F_k\}) = P(D|\theta_j, \{E, F_k\}) \cdot P(\theta_j|\{E, F_k\})$$
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$\Longrightarrow$ Everything we know is contained in $\mathrm{PCM} \times \mathrm{PRIOR}$ and everything we know is also exactly contained in $\mathrm{FIT} \times \mathrm{EVAL}$;

- No new information was forged and nothing was lost.
- This is not optimization; this is inference.
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- The algo engineer contributes F, the audiologist E, the auditory modeler U, (and the patient D).
- Pool all contributions into the same currency (probability theory).
- All pooled information gets transferred into fitting and algo evaluation results.

$\Longrightarrow$ It's not about algorithms, it's about FEUD

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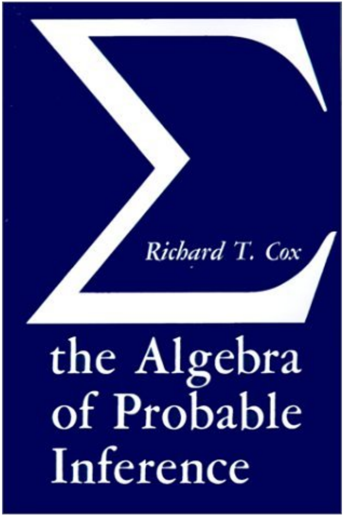
- (corollary). Any alternative ‘intelligent reasoning’ system is either equivalent to PT (in which case it is redundant), or inconsistent.



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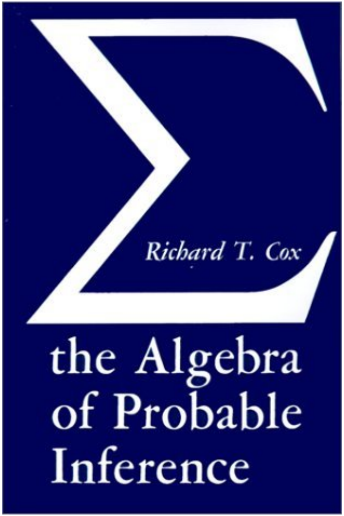

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Conclusions

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