

# Data-Driven Design & Analysis of Structures & Materials (3dasm)

**Instructor:** Miguel A. Bessa and **TAs:** Martin van der Schelling & Bruno Ribeiro

Brown University

## Homework 2

Deliver a **short PDF report** of this assignment containing the answers to the questions listed here.  
**UPLOAD to CANVAS in the Assignments section (Homework 2) by the due date.**

**Due date: September 20, 2023 (until 11:59pm)**

### Exercise 1<sup>1</sup>

1. Consider the function  $x \sin(x)$  in the domain  $x \in [0, 10]$ .

- 1.1. Plot the function using matplotlib.

Here's a little reminder of the steps to do this:

- Define the function  $f(x) = x \sin(x)$ .
  - Create a vector "x\_data" of 50 points that are uniformly spaced between 0 and 10.
  - Calculate the values of  $f(x) = x \sin(x)$  for each of the 50 points of vector "x\_data" and save them in "y\_data".
  - Plot the function from the 50 points that you defined, label the x-axis as "x" and the y axis as "y", and include a title "Exercise 1" in the plot.
- 1.2. Create a pandas dataframe called "my\_df" with the data needed to create the plot, and save it in a pickle file called "exercise1\_data.pkl" that needs to be in the "your\_data" folder located at the same level as the "your\_Assignments" folder in the 3dasm\_course repository. Print this dataframe.
    - 1.3. Load the pandas dataframe from the "exercise1\_data.pkl" file you saved previously into a variable called "(my\_loaded\_df)". Print this dataframe and check that it is the same one you saved.

### Exercise 2<sup>2</sup>

2. Consider the car stopping distance problem discussed in the Lectures for which we know the governing equation:  $y = zx + 0.1x^2$

where  $z \sim \mathcal{N}(\mu_z = 1.5, \sigma_z^2 = 0.5^2)$

- 2.1. Calculate the expected value for the stopping distance  $y$ :  $\mathbb{E}[y] = ?$
  - 2.2. Calculate the variance for the stopping distance  $y$ :  $\mathbb{V}[y] = ?$

### Exercise 3

3. Use the change of variables formula to demonstrate that  $p(y)$  is a Gaussian distribution with the expected value and variance determined previously. In other words, that  $p(y) = \mathcal{N}(y | \mu_y = x\mu_z + 0.1x^2, \sigma_y^2 = \sigma_z^2 x^2)$  when  $y = zx + 0.1x^2$ .

### Exercise 4

4. Knowing that  $p(y, z) = \delta(y - (zx + 0.1x^2)) \mathcal{N}(z | \mu_z, \sigma_z^2)$ , calculate  $p(y)$  and  $p(z)$ .

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<sup>1</sup>Exercise 1 is a simple coding exercise.

<sup>2</sup>Exercises 2 to 7 are analytical, i.e. pen and paper.

## Exercise 5

5. Consider two **independent** rv's  $x_1$  and  $x_2$  where each of them is a univariate Gaussian pdf:

$$x_1 = \mathcal{N}(x_1 | \mu_{x_1}, \sigma_{x_1}^2)$$

$$x_2 = \mathcal{N}(x_2 | \mu_{x_2}, \sigma_{x_2}^2)$$

where  $\mu_{x_1} = 10$ ,  $\sigma_{x_1}^2 = 5^2$ ,  $\mu_{x_2} = 0.5$  and  $\sigma_{x_2}^2 = 2^2$ .

- 5.1. What is the joint pdf  $p(x_1, x_2)$ ?

- 5.2. Calculate the covariance matrix for  $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ .

## Exercise 6

6. Consider again the car stopping distance problem of Exercise 2, but for a constant velocity of  $x = 75$  m/s.

- 6.1. Calculate the mean vector and covariance matrix for  $y$  and  $z$ . Remember that  $y$  is dependent on  $z$ .

- 6.2. Calculate the determinant of the covariance matrix and interpret it.

## Exercise 7

7. Consider the car stopping distance problem at a constant velocity of  $x = 75$  m/s but now with two hidden rv's:  $y = 75z_1 + 75^2z_2$ , where  $z_1 \sim \mathcal{N}(\mu_{z_1} = 1.5, \sigma_{z_1}^2 = 0.5^2)$ , and  $z_2 \sim \mathcal{N}(\mu_{z_2} = 0.1, \sigma_{z_2}^2 = 0.01^2)$ .

- 7.1. Write the joint pdf  $p(y, z_1, z_2)$ .

- 7.2. Calculate the covariance matrix for  $\mathbf{x} = \begin{bmatrix} y \\ z_1 \end{bmatrix}$ , i.e.  $\text{Cov} \left( \begin{bmatrix} y \\ z_1 \end{bmatrix} \right)$

- 7.3. Calculate the covariance matrix for  $\mathbf{x} = \begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix}$ , i.e.  $\text{Cov} \left( \begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix} \right)$