Data-Driven Design & Analysis of Structures & Materials (3dasm)

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Homework 1

Deliver **a short PDF report** of this assignment containing the answers to the questions listed here. **UPLOAD to CANVAS in the Assignments section (Homework 2) by the due date.**

Due date: September 20, 2023 (until 11:59pm)

Exercise 1

1. Consider the function $x\sin(x)$ in the domain $x \in [0, 10]$.

   1.1. Plot the function using matplotlib.

   Here's a little reminder of the steps to do this:

   - Define the function $f(x) = x\sin(x)$.
   - Create a vector "x_data" of 50 points that are uniformly spaced between 0 and 10.
   - Calculate the values of $f(x) = x\sin(x)$ for each of the 50 points of vector "x_data" and save them in "y_data".
   - Plot the function from the 50 points that you defined, label the x-axis as "x" and the y axis as "y", and include a title "Exercise 1" in the plot.

   1.2. Create a pandas dataframe called "my_df" with the data needed to create the plot, and save it in a pickle file called "exercise1_data.pkl" that needs to be in the "your_data" folder located at the same level as the "your_Assignments" folder in the 3dasm_course repository. Print this dataframe.

   1.3. Load the pandas dataframe from the "exercise1_data.pkl" file you saved previously into a variable called "(my_loaded_df)". Print this dataframe and check that it is the same one you saved.

Exercise 2

2. Consider the car stopping distance problem discussed in the Lectures for which we know the governing equation: $y = zx + 0.1x^2$

   where $z \sim \mathcal{N}(\mu_z = 1.5, \sigma_z^2 = 0.5^2)$

   2.1. Calculate the expected value for the stopping distance y: $\mathbb{E}[y] =?$

   2.2. Calculate the variance for the stopping distance y: $\mathbb{V}[y] =?$

Exercise 3

3. Use the change of variables formula to demonstrate that $p(y)$ is a Gaussian distribution with the expected value and variance determined previously. In other words, that $p(y) = \mathcal{N}(y|\mu_y = x\mu_z + 0.1x^2, \sigma_y^2 = \sigma_z^2 x^2)$ when $y = zx + 0.1x^2$.

Exercise 4

4. Knowing that $p(y, z) = \delta\left(y - (zx + 0.1x^2)\right)\mathcal{N}(z|\mu_z, \sigma_z^2)$, calculate $p(y)$ and $p(z)$.

Exercise 5

5. Consider two **independent** rv's x_1 and x_2 where each of them is a univariate Gaussian pdf:

$$x_1 = \mathcal{N}(x_1|\mu_{x_1}, \sigma^2_{x_1})$$

$$x_2 = \mathcal{N}(x_2|\mu_{x_2}, \sigma^2_{x_2})$$

where $\mu_{x_1} = 10$, $\sigma^2_{x_1} = 5^2$, $\mu_{x_2} = 0.5$ and $\sigma^2_{x_2} = 2^2$.

   5.1. What is the joint pdf $p(x_1, x_2)$?

   5.2. Calculate the covariance matrix for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Exercise 6

6. Consider again the car stopping distance problem of Exercise 2, but for a constant velocity of $x = 75$ m/s.

   6.1. Calculate the mean vector and covariance matrix for y and z. Remember that y is dependent on z.

   6.2. Calculate the determinant of the covariance matrix and interpret it.

Exercise 7

7. Consider the car stopping distance problem at a constant velocity of $x = 75$ m/s but now with two hidden rv's: $y = 75z_1 + 75^2 z_2$, where $z_1 \sim \mathcal{N}(\mu_{z_1} = 1.5, \sigma^2_{z_1} = 0.5^2)$, and $z_2 \sim \mathcal{N}(\mu_{z_2} = 0.1, \sigma^2_{z_2} = 0.01^2)$.

   7.1. Write the joint pdf $p(y, z_1, z_2)$.

   7.2. Calculate the covariance matrix for $\mathbf{x} = \begin{bmatrix} y \\ z_1 \end{bmatrix}$, i.e. $\text{Cov}\left(\begin{bmatrix} y \\ z_1 \end{bmatrix}\right)$

   7.3. Calculate the covariance matrix for $\mathbf{x} = \begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix}$, i.e. $\text{Cov}\left(\begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix}\right)$