

Data-Driven Design & Analysis of Structures & Materials (3dasm)

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Homework 2

Deliver a **short PDF report** of this assignment containing the answers to the questions listed here.
UPLOAD to CANVAS in the Assignments section (Homework 1) by the due date.

Due date: until 11:59pm of day announced in [course home page](#).

Exercise 1¹

1. Consider the function $x \sin(x)$ in the domain $x \in [0, 10]$.

- 1.1. Plot the function using matplotlib.

Here's a little reminder of the steps to do this:

- Define the function $f(x) = x \sin(x)$.
 - Create a vector "x_data" of 50 points that are uniformly spaced between 0 and 10.
 - Calculate the values of $f(x) = x \sin(x)$ for each of the 50 points of vector "x_data" and save them in "y_data".
 - Plot the function from the 50 points that you defined, label the x-axis as "x" and the y axis as "y", and include a title "Exercise 1" in the plot.
- 1.2. Create a pandas dataframe called "my_df" with the data needed to create the plot, and save it in a pickle file called "exercise1_data.pkl" that needs to be in the "your_data" folder located at the same level as the "your_Assignments" folder in the 3dasm_course repository. Print this dataframe.
 - 1.3. Load the pandas dataframe from the "exercise1_data.pkl" file you saved previously into a variable called "(my_loaded_df)". Print this dataframe and check that it is the same one you saved.

Exercise 2²

2. Consider the car stopping distance problem discussed in the Lectures for which we know the governing equation: $y = zx + 0.1x^2$

where $z \sim \mathcal{N}(\mu_z = 1.5, \sigma_z^2 = 0.5^2)$

- 2.1. Calculate the expected value for the stopping distance y : $\mathbb{E}[y] = ?$
 - 2.2. Calculate the variance for the stopping distance y : $\mathbb{V}[y] = ?$

Exercise 3

3. Use the change of variables formula to demonstrate that $p(y)$ is a Gaussian distribution with the expected value and variance determined previously. In other words, that $p(y) = \mathcal{N}(y | \mu_y = x\mu_z + 0.1x^2, \sigma_y^2 = \sigma_z^2 x^2)$ when $y = zx + 0.1x^2$.

Exercise 4

4. Knowing that $p(y, z) = \delta(y - (zx + 0.1x^2)) \mathcal{N}(z | \mu_z, \sigma_z^2)$, calculate $p(y)$ and $p(z)$.

¹Exercise 1 is a simple coding exercise.

²Exercises 2 to 7 are analytical, i.e. pen and paper.

Exercise 5

5. Consider two **independent** rv's x_1 and x_2 where each of them is a univariate Gaussian pdf:

$$x_1 = \mathcal{N}(x_1 | \mu_{x_1}, \sigma_{x_1}^2)$$

$$x_2 = \mathcal{N}(x_2 | \mu_{x_2}, \sigma_{x_2}^2)$$

where $\mu_{x_1} = 10$, $\sigma_{x_1}^2 = 5^2$, $\mu_{x_2} = 0.5$ and $\sigma_{x_2}^2 = 2^2$.

- 5.1. What is the joint pdf $p(x_1, x_2)$?

- 5.2. Calculate the covariance matrix for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Exercise 6

6. Consider again the car stopping distance problem of Exercise 2, but for a constant velocity of $x = 75$ m/s.

- 6.1. Calculate the mean vector and covariance matrix for y and z . Remember that y is dependent on z .

- 6.2. Calculate the determinant of the covariance matrix and interpret it.

Exercise 7

7. Consider the car stopping distance problem at a constant velocity of $x = 75$ m/s but now with two hidden rv's: $y = 75z_1 + 75^2z_2$, where $z_1 \sim \mathcal{N}(\mu_{z_1} = 1.5, \sigma_{z_1}^2 = 0.5^2)$, and $z_2 \sim \mathcal{N}(\mu_{z_2} = 0.1, \sigma_{z_2}^2 = 0.01^2)$.

- 7.1. Write the joint pdf $p(y, z_1, z_2)$.

- 7.2. Calculate the covariance matrix for $\mathbf{x} = \begin{bmatrix} y \\ z_1 \end{bmatrix}$, i.e. $\text{Cov} \left(\begin{bmatrix} y \\ z_1 \end{bmatrix} \right)$

- 7.3. Calculate the covariance matrix for $\mathbf{x} = \begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix}$, i.e. $\text{Cov} \left(\begin{bmatrix} y \\ z_1 \\ z_2 \end{bmatrix} \right)$