# Lecture 3

This document provides practice problems that are similar to those that will be asked during the final exam. Please note that the document reflects the style and not the number of the questions that will be on the exam.

## Problem 1

Let $X, N$ be two independent real-valued random variables and let $Y = f(X, N)$, where $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is an arbitrary function. Which of the following statements are correct?

**(a)** $X \to Y$. **[False]**

**Explanation:** Equations such as $Y = f(X, N)$ are undirected, whereas structural causal models are directed.

**(b)** $X$ and $Y$ are correlated. **[False]**

**Explanation:** Dependence does not imply correlation; correlation only looks at the *linear* dependence between variables.

**(c)** $X$ and $Y$ are dependent. **[True]**

## Problem 2

Consider the causal model $W \to Y \leftarrow X$ with $Y := X + W$, where $X$ is a discrete random variable that takes values in $\{0, 1, 2, 3\}$ with equal probability and the noise $W$ is a discrete random variable that takes values in $\{0, 1, 2\}$ with equal probability. Which of the following statements are correct?

**(a)** $\Pr(Y = 3 \mid \mathrm{do}(X = 1)) = \Pr(Y = 3 \mid X = 1)$. **[False]**

**(b)** $\Pr(Y = 3 \mid \mathrm{do}(X = 1)) = \Pr(Y = 3 \mid X = 1)$ holds in case $X$ and $W$ are independent. **[True]**

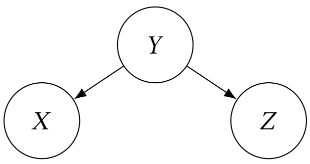
**Explanation:** $X$ has no parents.

**(c)** $\Pr(Y = 4 \mid \mathrm{do}(X = 1)) = \Pr(Y = 4 \mid X = 1)$. **[True]**

**Explanation:** Both probabilities are 0, as $Y$ cannot take on a value of 4 if X is 1.

## Problem 3

Consider the following structural causal model:




Which of the following statements are correct (in general)?

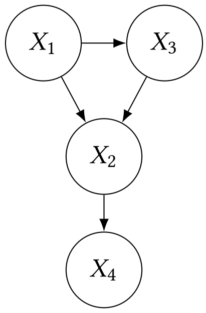
**(a)** $\Pr(X \mid Y = y) = \Pr(X \mid \mathrm{do}(Y = y))$. **[True]**

**(b)** $\Pr(Z \mid \mathrm{do}(X = x)) = \Pr(Z \mid X = x)$. **[False]**

**(c)** $\Pr(Z \mid \mathrm{do}(X = x)) = \Pr(Z)$. **[True]**

## Problem 4

Consider the following causal model:




Which of the following statements are correct (in general)?

**(a)** $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_1 \mid X_2, X_3, X_4) \cdot \Pr(X_2 \mid X_3, X_4) \cdot \Pr(X_3, X_4)$. **[True]**

**(b)** $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_1 \mid X_2, X_3, X_4) \cdot \Pr(X_2 \mid X_3) \cdot \Pr(X_3)$. **[False]**

**(c)** $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3) \cdot \Pr(X_3 \mid X_1) \cdot \Pr(X_1)$. **[True]**

**(d)** $\Pr(X_4 \mid \mathrm{do}(X_3 = x)) = \sum_{X_1} \sum_{X_2} \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3 = x) \cdot \Pr(X_1)$. **[True]**

**(e)** $\Pr(X_4 \mid \mathrm{do}(X_3 = x)) = \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3 = x) \cdot \Pr(X_1)$. **[False]**

## Problem 5

Imagine you are currently single and looking for a partner. After having gone on a number of dates, you seem to notice a rather frustrating pattern: *All the good-looking people are jerks!* How can this be true? After thinking about your observation for a while and drawing a causal graph, you notice that you are looking at an instance of *Berkson's paradox*, which is ultimately caused by:

**(a)** Conditioning on a collider. **[True]**

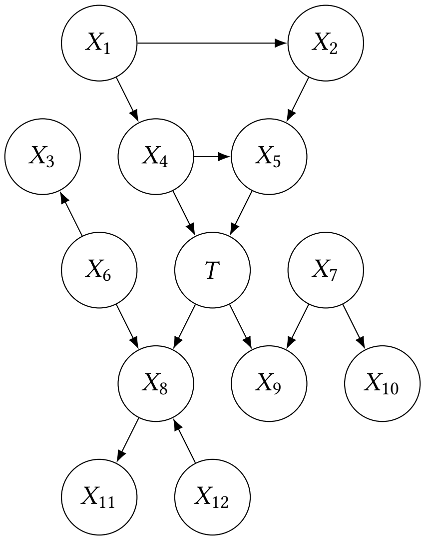
**Explanation:** The idea is that "being in a relationship" is a function of two factors, "looks" and "character", and that you need a high score on at least one of the two to find a partner. Basically, "nobody wants to date an ugly jerk". Conversely, people who are *both* good-looking and have a nice character are likely to be in a relationship already. When you go on dates, you (usually) condition on "not in a relationship", which then creates a negative correlation between looks and character: The good-looking people you meet are more likely to be jerks, because the good-looking non-jerks are mostly not "on the market" anymore.

**(b)** Unobserved confounding. **[False]**

**(c)** Regression to the mean. **[False]**

## Problem 6

Consider the following structural causal model:




Which of the following statements are correct (in general)?

**(a)** $T$ conditioned on $\{X_4, X_5\}$ is independent of $X_1$. **[True]**

**(b)** $T$ conditioned on $\{X_4, X_5\}$ is independent of $X_8$. **[False]**

**(c)** $X_8$ conditioned on $T$ is independent of $X_9$ conditioned on $T$. **[True]**

**(d)** $X_8$ conditioned on $\{X_4, X_5\}$ is independent of $X_9$ conditioned on $\{X_4, X_5\}$. **[False]**

**(e)** None of the above. **[False]**