Lecture 3

This document provides practice problems that are similar to those that will be asked during the final exam. Please note that the document reflects the style and not the number of the questions that will be on the exam.

Problem 1

Let X, N be two independent real-valued random variables and let $Y = f(X, N)$, where $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is an arbitrary function. Which of the following statements are correct?

(a) $X \to Y$.

(b) X and Y are correlated.

(c) X and Y are dependent.

Problem 2

Consider the causal model $W \to Y \leftarrow X$ with $Y := X + W$, where X is a discrete random variable that takes values in $\{0, 1, 2, 3\}$ with equal probability and the noise W is a discrete random variable that takes values in $\{0, 1, 2\}$ with equal probability. Which of the following statements are correct?

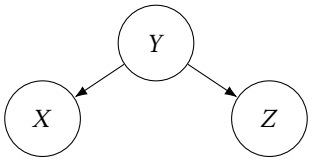
(a) $\Pr(Y = 3 \mid \mathrm{do}(X = 1)) = \Pr(Y = 3 \mid X = 1)$.

(b) $\Pr(Y = 3 \mid \mathrm{do}(X = 1)) = \Pr(Y = 3 \mid X = 1)$ holds in case X and W are independent.

(c) $\Pr(Y = 4 \mid \mathrm{do}(X = 1)) = \Pr(Y = 4 \mid X = 1)$.

Problem 3

Consider the following structural causal model:



Which of the following statements are correct (in general)?

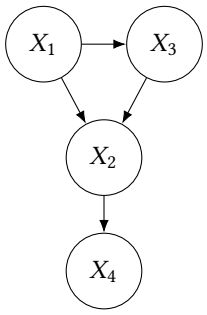
(a) $\Pr(X \mid Y = y) = \Pr(X \mid \mathrm{do}(Y = y))$.

(b) $\Pr(Z \mid \mathrm{do}(X = x)) = \Pr(Z \mid X = x)$.

(c) $\Pr(Z \mid \mathrm{do}(X = x)) = \Pr(Z)$.

Problem 4

Consider the following causal model:



Which of the following statements are correct (in general)?

(a) $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_1 \mid X_2, X_3, X_4) \cdot \Pr(X_2 \mid X_3, X_4) \cdot \Pr(X_3, X_4)$.

(b) $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_1 \mid X_2, X_3, X_4) \cdot \Pr(X_2 \mid X_3) \cdot \Pr(X_3)$.

(c) $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3) \cdot \Pr(X_3 \mid X_1) \cdot \Pr(X_1)$.

(d) $\Pr(X_4 \mid \mathrm{do}(X_3 = x)) = \sum_{X_1} \sum_{X_2} \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3 = x) \cdot \Pr(X_1)$.

(e) $\Pr(X_4 \mid \mathrm{do}(X_3 = x)) = \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3 = x) \cdot \Pr(X_1)$.

Problem 5

Imagine you are currently single and looking for a partner. After having gone on a number of dates, you seem to notice a rather frustrating pattern: *All the good-looking people are jerks!* How can this be true? After thinking about your observation for a while and drawing a causal graph, you notice that you are looking at an instance of *Berkson's paradox*, which is ultimately caused by:

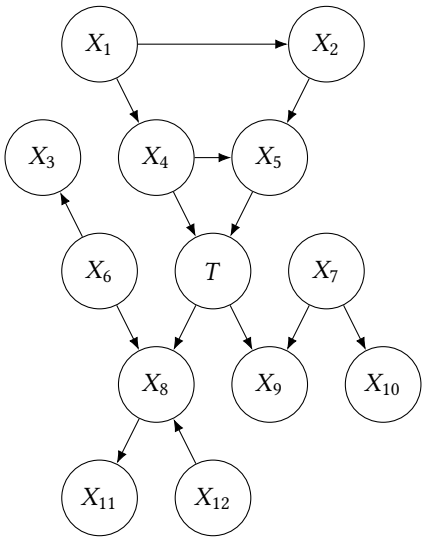
(a) Conditioning on a collider.

(b) Unobserved confounding.

(c) Regression to the mean.

Problem 6

Consider the following structural causal model:



Which of the following statements are correct (in general)?

(a) T conditioned on $\{X_4, X_5\}$ is independent of X_1.

(b) T conditioned on $\{X_4, X_5\}$ is independent of X_8.

(c) X_8 conditioned on T is independent of X_9 conditioned on T.

(d) X_8 conditioned on $\{X_4, X_5\}$ is independent of X_9 conditioned on $\{X_4, X_5\}$.

(e) None of the above.