M. Muehlebach, B. Schölkopf, A. Krause, M. Mordig, T. Gebhard:
Beyond i.i.d. learning: Causality, dynamics and interactions (263-5156-00L)

ETH Zurich
Autumn 2021

Lecture 3

This document provides practice problems that are similar to those that will be asked during the final exam. Please note that the document reflects the style and <u>not the number</u> of the questions that will be on the exam.
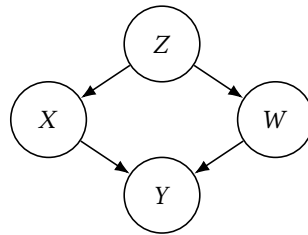
Problem 1

Let X, N be two independent real-valued random variables and let $Y = f(X, N)$, where $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is an arbitrary function. Which of the following statements are correct?

(a) $X \to Y$.

(b) X and Y are correlated.

(c) X and Y are dependent.

Problem 2 (question and answers updated on December 6, 2021)

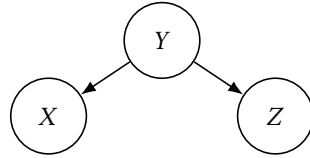Consider the following structural causal model

where X is a discrete random variable that takes values in $\{0, 1, 2, 3\}$ with equal probability and W is a discrete random variable that takes values in $\{0, 1, 2\}$ with equal probability. We further know that $Y = W + X$. Which of the following statements are correct?

(a) $\Pr(Y = 3 \mid \mathrm{do}(X = 1)) = \Pr(Y = 3 \mid X = 1)$.

(b) $\Pr(Y = 4 \mid \mathrm{do}(X = 1)) = \Pr(Y = 4 \mid X = 1)$.

(c) None of the above.

M. Muehlebach, B. Schölkopf, A. Krause, M. Mordig, T. Gebhard:
Beyond i.i.d. learning: Causality, dynamics and interactions (263-5156-00L)

ETH Zurich
Autumn 2021

Problem 3

Consider the following structural causal model:

Which of the following statements are correct (in general)?

(a) $\Pr(X \mid Y = y) = \Pr(X \mid \mathrm{do}(Y = y))$.

(b) $\Pr(Z \mid \mathrm{do}(X = x)) = \Pr(Z \mid X = x)$.

(c) $\Pr(Z \mid \mathrm{do}(X = x)) = \Pr(Z)$.

Problem 4

Consider the following causal model:
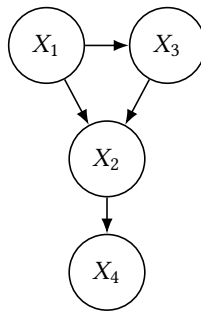
Which of the following statements are correct (in general)?

(a) $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_1 \mid X_2, X_3, X_4) \cdot \Pr(X_2 \mid X_3, X_4) \cdot \Pr(X_3, X_4)$.

(b) $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_1 \mid X_2, X_3, X_4) \cdot \Pr(X_2 \mid X_3) \cdot \Pr(X_3)$.

(c) $\Pr(X_1, X_2, X_3, X_4) = \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3) \cdot \Pr(X_3 \mid X_1) \cdot \Pr(X_1)$.

(d) $\Pr(X_4 \mid \mathrm{do}(X_3 = x)) = \sum_{X_1} \sum_{X_2} \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3 = x) \cdot \Pr(X_1)$.

(e) $\Pr(X_4 \mid \mathrm{do}(X_3 = x)) = \Pr(X_4 \mid X_2) \cdot \Pr(X_2 \mid X_1, X_3 = x) \cdot \Pr(X_1)$.

Problem 5

Imagine you are currently single and looking for a partner. After having gone on a number of dates, you seem to notice a rather frustrating pattern: *All the good-looking people are jerks!* How can this be true? After thinking about your observation for a while and drawing a causal graph, you notice that you are looking at an instance of *Berkson's paradox*, which is ultimately caused by:

    (a)   Conditioning on a collider.

    (b)   Unobserved confounding.

    (c)   Regression to the mean.

Problem 6

Consider the following structural causal model:
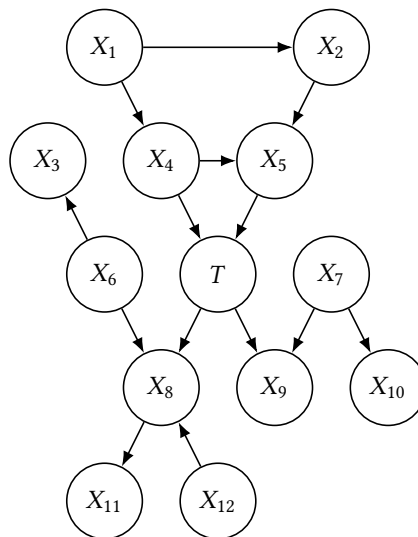
Which of the following statements are correct (in general)?

    (a) T conditioned on $\{X_4, X_5\}$ is independent of X_1.

    (b) T conditioned on $\{X_4, X_5\}$ is independent of X_8.

    (c) X_8 conditioned on T is independent of X_9 conditioned on T.

    (d) X_8 conditioned on $\{X_4, X_5\}$ is independent of X_9 conditioned on $\{X_4, X_5\}$.

    (e)   None of the above.