

How to get good at learning

What is this and who is this for?

This is a document to help you get better at *learning* (math, but also hopefully other things). Because I get many requests asking “how to study” or (more honestly!) “how to study for the exam,” I decided to start a collection of the things that actually work, and update it with the experience and feedback from every new class.

There are 2 situations where my advice is completely useless:

1. If you think you are already “good enough” at learning, already know the best way for you to learn, or have a very strong identity of “this is who I am,” this guide is **not** for you. I am not trying to make you feel guilty – I have much better things to do with my life than being passive-aggressive with you; I am just saying that advice is kind of useless if you don’t think you need it. Honestly, I had many students who actively did not want advice, kept doing what they did without listening to my advice, and got A’s. That’s great! (sadly, some of these people were getting F’s, kept doing what they did, and kept getting F’s.)
2. If you simply don’t care about getting better at learning, then my advice is also useless. Again, this is not to make you feel guilty – many people don’t care about becoming better students. They care about other things more, like finances, personal problems, love and relationships, their band, etc. I honestly think it’s your right to choose what you care and what you do not care about. Nobody is trying to offend you. I want to value your time.

For everyone else: i.e. those of you who are open to learning about learning and are interested in getting better, I (along with the spirits of my old students!) will now try my best to help you.

What is learning?

What do I mean by *learning*? I mean how good you are at getting *better* at something. Note this is not about how good you *are*. You can be strong at math but weak at *learning* math. Learning is hard. It is hard because of many Painful Truths and Dangerous Traps, starting with this one:

Painful Truth: Most people are weak at learning. If you are weak at learning, you will likely find it very hard to get a competitive job that requires real skills. This is because if you do not learn skills quickly, you are at a disadvantage compared to people who learn skills quicker than you, even if you have a head start.

Mental experiment: if you “just played” volleyball everyday for an hour or two, then in 2000 hours you are bound to be pretty good compared to normal folk. But I think you’d be better if you spent 1000 hours in a team, with each practice session having a goal (and drills to hit the goal, games to train the goal, and a coach to give you goals). This means you could have saved 1000 hours. 1000 hours is a very very long time.

I’m not pretending to be above you, or to have figured it all out. It took me a long time myself to realize how **weak** I was at learning math (and I am finding inefficiencies in my own learning even now). The hardest part for me was admitting to myself I was bad at learning math, **EVEN THOUGH** I was pretty good at math! I felt really stupid when I realized this, but once I did, I got a lot better.

Proposed Solution to Painful Truth: Be honest to yourself. This allows you to find weaknesses. For each weakness, make a goal to attack it. **Practice things that train you for your goal.** Practice LESS (not “don’t practice”; just less) things that **DON’T** directly train you for your goal.

“What should my goal be?”

You can actually set your goal to be anything, but a good guiding principle is “**what is my weakest skill?**” because that’s probably what’s holding you back the most. Then you ask “What is my weakest part of that skill?” **and keep going.** (for those of you who dabble in business: this is similar to “The Five Whys”)

For example:

1. Most people who are weak at math classes are weak at **taking exams.**
2. This is too broad, so let’s now find the weakest part of taking exams. What’s an exam? It is many things (like dealing with time pressure, psychology, etc.), but I think the main part is usually **being able to solve new problems.**
3. This is still too broad. So you ask “what’s my weakest point when solving new problems?” ...
4. You keep doing until you find something *specific*, and train that. What does it mean to be specific? It means you can make a *drill*. (see next section)
5. (Yes yes, I would actually prefer your goals to more broad, like an appreciation of mathematics, being a good person, etc., but I’m respecting the fact that you may have different goals from mine.)

Deadly trap: The main reason that most people in the world are bad at learning is that they practice what they are *good* at instead of what they are *weak* at. For example, they’re weak at solving problems and good at reading. So when they review, they read. They’ll then feel good (since they’re good at reading). Then on the actual exam they are not asked to read, but to solve problems. Since they were always afraid to actually solve problems by themselves, they never got good at solving problems. Then they fail the exam. 9 times out of 10 this is because they didn’t actually solve their own problems on homework (hey, I warned you).

“I found a specific weakness. Now what?”

Mental experiment: Suppose Stephanie Curry shoots 90% accurate 3-pointers from the top of the key, 95% from the left baseline, and 84% from the right baseline. What do you think her coach does with her during shooting practice?

Hint: it’s not shooting from the left baseline.

After you find a specific weakness, your goal is now to train that weakness. How do you do that? You do drills. By that, I mean **a small activity that isolates your weakness.** (athletes are one of the best examples of human excellence. Their coaches have figured out how to improve them in a very competitive setting. I learn a lot about teaching and learning from reading what coaches have to say; for example, I learned ideas about drilling from coaching and deliberate practice literature)

Example (continuing the previous section’s examples):

1. The most common “big” weakness my old students report is “I can’t solve problems.”
 - So how do you train it? Find a drill.
 - Maybe a drill is “Solve a problem.”
 - Go solve 1 problem. Take a break. You now got better at solving problems.
2. Maybe this doesn’t work. Then we need to get deeper (like the last section). Ask yourself why you can’t solve problems. My old students report one common cause is **not understanding definitions.**
 - So how do you train it? Find a drill.
 - One drill is answering “What is an X?”
 - So take a random word X, ask “What is an X” and try to answer it. If you can’t, look it up, and answer it later.
 - Do this 20 times. Take a break. You now got better at definitions.
 - (I hope you now know why I do this in class! I’m trying to give you good habits on drilling yourself. The students who figure this out early learn better.)

3. Another reported weakness related to above is “I don’t really understand the problem.” Now you can identify your weakness as **reading abstraction / formalism**. If you read a math sentence and don’t remember what you just read, you are weak at this skill.
- So how do you train it? Find a drill. (If you find my repetitiveness offensive, I’m unapologetic. I’m repeating things because the lessons here sound simple but are TRULY difficult, and you need repetition to learn difficult things.)
 - Maybe a drill is “Read and understand 1 sentence completely.”
 - Now you have a plan. Go get a book or problems or whatever, and just read. Your goal is to understand the sentences *completely*. Read word by word and no moving on until you understand everything in a sentence so far. When you finish reading a sentence and think you understand it, you did 1 drill. Read 25 sentences. Take a break. You now got better at reading.

There are no shortcuts.

“How much should I read books/notes?”

Well - this depends on your weaknesses! (that’s why you do that first.) - If your weakness is solving new problems, then if you are just reading your textbook/notes, **you are practicing the skill of reading, not directly understanding how to solve new problems**. Thus, while reading is good, to succeed at your goal of solving new problems, you must be spending more time solving problems, and less time reading. - If you are actually pretty strong on solving problems, but weak at understanding problem statements, maybe your goal should be to **get better at reading**, so instead of doing problems, you should actually read more books!

Here’s an analogy: you can practice swimming by watching swimming videos on Youtube or swimming in the pool. The former will probably help you, but if you have a swimming exam in a pool (as opposed to an exam where you are tested on your knowledge of Youtube swimming videos), you’d have better spent time swimming in the pool. The ideal would be to actually go into the pool **and** watch swimming videos, probably watching videos that address your weaknesses that you learn from actually swimming into the pool.

What this means is you should read your book/notes to understand **specific** things. It is much easier for us to learn when we have specific goals: If you find a definition you don’t know, read your book. If you think the problem looks like something you saw before, read your book to find the similarity. Occasionally, read your book broadly in order to find the holes in your knowledge; this won’t train you to solve problems, but will find you problems to solve.

“How do I learn better in class?”

Do you know why I always pause and ask you guys questions? It is to train you guys the skill of solving new problems. Those periods of silence are probably actually the most useful periods in the class for you for learning, because you get to train the skill of solving new problems. The parts where you listen to me talk are actually LESS important, because you are training the skill of following my logic, which is a good skill but not the skill of solving new problems.

When you choose to, in those moments of silence, do something else (stare blankly, look around the room to see who knows the answer, or whatever) instead of using the time to practice solving new problems, **you are wasting your time**. Those are actually the only times in lectures when you are training your fundamental goal (solving new problems). Don’t waste those times. They are awkward, but they are the most important.

“I understand things in lecture, but not on homework/exams...”

This is probably the biggest **Dangerous Trap** of them all. (I hear this from about 5 students every year).

Painful Truth: If you catch yourself saying this, 10 (not even 9) times out of 10 this simply means **your skill of following my logic is good (and important!), but your skill of solving new problems is weak**. These are **different** skills, in the same way that reading and writing French are different skills. To get better, you **must** honestly admit to yourself that this is a weakness that you have to work on. You need to set your goal to solving new problems (or as said before, another goal if you are really weak at something else) and start there.

“I don’t know what questions to ask (either when studying or in lecture)”

I said before the best thing to train is probably your weakest thing, which you have been avoiding but won’t train itself. Similarly, the best questions to ask are **the most basic/“dumb” thing you don’t understand**, which you have been avoiding but won’t train itself either. The best thing you can do for your own learning is to be honest with yourself and ask these questions to yourself.

For example: if you read a sentence containing the word ‘foo’, which came up a lot in your class, and don’t really know what it means, train yourself to stop and say “Hmm. I see that I don’t understand what ‘foo’ means. I should learn it now.” and look it up immediately. Think about it: suppose you didn’t know what “permutation” means and you feel you’re wasting your time reading about permutations. Why would you keep reading to “cyclic permutation” if you’ll be wasting even more time there?

This also applies to lectures. Suppose I say “permutation” during a lecture several times, and you really have no idea what it means. In the next 15 minutes I’ll probably still be talking about permutations. Do you really not care about your 15 minutes? Value your time! It literally takes you 5 seconds to raise your hand and ask “what is a permutation?” Now you saved 15 minutes of your life. If you do this 50 times during a semester, you have saved 12+ hours.

If you understand psychology, you also understand that all this means that **every time you don’t look up something, you are actually training yourself to NOT look up things**, which directly hurts you on your foundation, which directly hurts your goal of solving new problems (because you need foundation). Similarly, every time you don’t ask a question that would have helped you in class because you’re afraid to look dumb, you are **training yourself to not ask questions**, which means you’ll just ask fewer questions later.

Painful Truth: People don’t really care about how dumb you look. They’re too busy worrying about themselves.

“How do I read theorems / books better?”

When you read a proof of a theorem or a homework, think “**How can I use this to solve new problems?**” Most of the time, you are probably training the skill of “remembering the proof.” This is not a skill you want on the exam, because the exam is **not about you remembering proofs**. It is – yeah, yeah – **about you solving new problems**. This is why I tell you to listen to my thinking process, not just write down my proofs, because the thinking process is the thing that allows you to solve new problems, not memorizing my proof.

“I want some secret sauce”

Well, not exactly that, but a group of questions people **always** ask that are usually much less important than they sound:

- “what other books can I use?”
- “do you have extra problems?”
- “should I do book problems or problems from somewhere else?”
- etc.

I do have a stash of awesome problems that will make you learn faster. In fact, they’re my best collection of problems. Do you know what it is? It’s **the homework and the exams**. I’m trying my best to teach you and not hide things from you! So... I really don’t have many secrets.

You already have almost an infinite source of problems:

- Pick a random problem from the book that seems relevant. Try to solve it.
- Pick a random theorem from the book that seems relevant. Try to prove it.
- Pick an old homework problem. Try to solve it. The BEST problems to solve are those where you got help from somewhere else. Now you can ask yourself the painful but good question “Did I actually understand, or did I just copy someone else?” (Yes, I did this exercise and learned a lot about myself.)
- Look up problems on the internet. There are many other combinatorics courses out there. Many of them have solutions.
- (For advanced students) Wonder about something, like “hmm, are there more surjections than injections?” and solve your own problem. For beginner/intermediate students this is not that helpful, because you won’t really know if your solution is right. For advanced students who have confidence checking their own problems though, this is a great strength builder.

Important: I’m **not** saying it is bad to want to do more problems (especially if you are advanced) and you are **always** welcome to ask. My point is that the “Pokemon collector” mindset of trying to collect more problems and “secret sources” is probably not going to help you **if you aren’t using what you already have**. If you only have one book but practice 30 problems from the book, you’ll be stronger than if you downloaded 20 books but do no problems.

One exception to my advice is that because of learning style differences, sometimes a book is just impossible for your learning style, and you should find a book with a different style / language (this also applies to teachers, when you have the ability to change teachers). However...

Painful Truth: If none of the 3 or 4 books you found fit your learning style, the Painful Truth is that **the main problem here is not the style of the books, but your skill level in reading books is weak**. You have to be honest to yourself, admit that, and change your **goal** to practice reading books, which probably means your real weakness is reading abstract math.

If you suggested 5 different Spanish books to your friend who says they read Spanish, and they said “I couldn’t understand any of these books because they didn’t fit my learning style.” It’s probably not the learning style – they probably doesn’t understand Spanish.

It is really convenient to blame the skill (instead of yourself) in this case. This is because most of us (hey, me too) hate admitting to ourselves that we are simply weak at something, and would love to blame something else instead. The painful truth is that reading math books and abstraction is a totally obtainable skill (it just takes time), in the same way reading an adult-level book in a different language is a totally obtainable skill (it just takes time). Don’t blame the skill if you don’t put in the time.

“What do your best students do?”

First of all, congrats if you are asking this kind of question. It means you want to actively emulate strong people, instead of being satisfied with doing whatever you have been doing. This is a great start.

Painful truth: some of my students just do whatever, and they get really good grades. I’m not going to pretend they are all hard workers. I can’t give you advice to become them. They’re just different, and very unique, people.

However, I **can** give you advice in the form of habits that I see over multiple people **who start with C’s and F’s and end up with A’s and B’s at the end of the course:**

1. They will try to read and solve problems on their own before the lecture, this way they get stuck on specific things. Because they’re stuck on specific things, they then know exactly what they are looking for during the lecture, which makes them learn faster. Also, because they had to wrestle with definitions before the lecture, when I do definitions during lecture they get to see the definitions twice, and thus learn them twice as well.
 2. They will not let me go on, whether in lecture or email or live conversation, if they don’t understand any word I am saying. This way, after I answer all their questions, they have a solid foundation and absorb everything else like a sponge. (the fact that my teaching style revolve around questions is because this is such a prevalent trait, repeatedly observable across many actual strong students)
 3. and 4. and 5... they do the other things in this post. How do you think I found the material for this document? I looked at people who *improved* and summarized their patterns.
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“I understand the intuition, but not the definitions.”

This is the second most common Dangerous Trap after “I understand the lectures, but not the homework/exams”. (2-3 direct reports a year). So again, it gets its own section.

Painful Truth: Saying these words is **almost 100% a sign that your skill in reading abstractions is weak**. This is usually caused by you avoiding abstraction and becoming reliant on examples and visual notation, which are like training wheels. Everyone can (and should) ride bikes with training wheels, but eventually, it holds you back from actually learning to ride the bike. It is a very common problem, because having the examples and visual notation in your head *feels* like intuition... but it can hurt you. Why?

If you understand how to read abstraction, then you understand how to do new problems, because you can just read the words in the new problem and understand what they mean. If you only understand examples and visual notation, then you only understand how to face new problems if they look exactly like your old problems. If I add any twist, then the picture will look different, and you will not be able to understand the problem because you are not able to read abstraction. There’s a reason I risk low teaching review scores to force abstraction onto you – they’re hard, you grumble, but that’s where the real kung fu is and I want you to learn the actual kung fu.

True artists and masters who talk about “intuition” about their art are **all** backed up by **tens of thousands** of hours of foundation (brush strokes, arpeggios, chords, memorizing chess endgame, hours in the weightroom, whatever). If you did 10, 20, or 100 examples and visual notation, sure, you might have some intuition. But if you think you “understand the intuition” without understanding the definitions... **you probably don’t actually have enough intuition to solve any new problems.**

“Everything feels hard...”

First of all, my classes are hard. They’re hard because I have high expectations of you and think you can succeed. You’re not going to hit high expectations unless you work hard. My goal is not to make you **feel** good. Your friends can do that. My goal is to make you **be** good.

A recurring weird thing in education theory is that many students who think they’re struggling have actually learned a lot. Think of the problems you are successfully able to solve now; could you solve those problems a year ago? If not, then you’ve learned something. Be proud of that. Maybe you can’t solve all my problems. But I told you, my problems are hard.

Painful Truth: Learning does not feel easy, except at the very beginning. You’re learning a hard skill, so it shouldn’t feel easy. If everything feels easy, the material is probably too easy for you and you are just getting the illusion of learning. It is really easy to confuse actually learning with feeling like you are learning. Real learning is a mix between mostly feeling awkward/difficult and **occasionally** having things “click” because your awkward and difficult hard work paid off. Like physical exercise (which is just another way to say your body is learning), real learning feels awkward, tiring, stumbling, failing most of the time – and the next day you could magically do a little (and just a little) more.

Of course, you have to – again – be honest to yourself. Don’t use what I said as an excuse to feel good about yourself. **If you think you’re struggling and you can’t seem to find any concept you understand, or think of the last time you solved a problem by yourself instead of having the internet or a friend tell you the answer, then you probably are actually not learning.** The way out of that starts with – you guessed it – admitting to yourself that you’re doing something wrong.

The one weird trick that helps you take exams 200% better:

Well, it’s an exercise. But if you do it, I guarantee you’ll become amazing.

Which means you have to do it.

Ready? Here’s the exercise.

Think of a word X, say “permutation,” that you hear a lot in class. Do you understand X? Can you give me a definition right now, as if you were teaching it to a younger sibling? **Yes, right now.** Stop what you are doing for 15 seconds and do it.

Now, if that was too easy, I want you to pick another word that you hear a lot, and try to define it. I want you to do this until you find something hard. Something that you think will probably on the final, but you can’t define perfectly.

Now that you have the hard word, go look it up **now**.

No, really, now.

If you don’t have a definition now and ignore my advice to look it up now, just remember that **you are training yourself right now that it is okay to be content with mediocre knowledge.**

1. If your knowledge is weak, you just trained it to stay weak.
2. If you train it to stay weak, then on the exam, when I say somewhere “give me an X such that...” your brain is going to freeze.
3. Why did your brain freeze?
4. Because you **chose**, when you read this post, to throw away an opportunity to understand X.

Painful Truth: Your brain looooooves to tell you that it’s fine “the way it is,” and it will say **anything** to stay in your comfort zone and not change. Is it making an excuse right now?

- “who really does things like this anyway” (I won’t pretend it’s all of them, but many strong students do.)
- “ugh.” (Learning is not supposed to be easy or comfortable, except maybe at the very beginning.)
- “he talks so much” (I do. Does that take away from my trying to help you?)
- “I’ll cram later and I’ll be fine.” (Do you agree that if you study **now** AND cram, you’ll then be better than fine? Also, did cramming work last time?)
- “I did cram last time and I was fine.” (If so, good luck! You’ll probably do fine.)
- “I’m going to ignore the potential value of this post, and text my classmate about how lame it is so I can feel better about myself.” (Sigh. Go ahead.)
- “this isn’t worth any points” (On the exam, when I ask you to “give me an X that...” your work **now** suddenly will be worth points)
- “I’m in a car / walking / eating” (You are reading this. You have a phone or a computer and internet. If you can do this, you can look up a word.)
- “why is he trying so hard” (Because I care about your learning.)
- “it’s not cool to try hard” (You have the right to be cool instead of good. But if you want good, you better be ready to try hard.)
- “this may be useful and I may do it later, but not now” (Are you really going to do it later?)

If you’ve actually done the exercise, skip to the line. Everything inbetween is not for you. For everyone else, keep reading.

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If you haven’t done it, ask yourself this: “If I’m not willing to actually do things differently from what I usually do, why do I expect to get a different grade from what I usually get?”

If that changes your mind, go do it now. Otherwise, keep scrolling.

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Do you see yourself scrolling through all of this? Are you on autopilot, or do you have control over yourself? Can you really not fight the autopilot?

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One last chance.

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Did you do it?

If so, congrats. You've found the one weird trick.

No, **I'm not talking about practicing the definition.** That was awesome for you to do, and I'm proud of you, and you should do it more, but that's not the point.

The point is that you found out how to trick yourself to actually **stop your autopilot of "scroll down and read a post like a mindless zombie" and actually do something different.** You found out how to be a person instead of a zombie, a person who can choose to do things, new things, different things. If you can do this for a math definition, you can do it for anything else.

The beautiful thing is I don't know what the actual trick you used was, because you had to come up with it yourself! (Feel free to message/email me and tell me. I'm interested in what works and what doesn't work. My experience is that the trick really changes from person to person, and very few things work for everyone)

Next time you need to get yourself to do something (anything, math or not), use whatever you did that actually got yourself to practice the definition, because that will be the thing you can always rely on to stop being a zombie. That will be the thing you can rely on to stop being mediocre and actually try to be good.

But if you just scrolled here just to see what I had to say and didn't actually do the exercise... well, I think you worked on the skill of **reading a document I wrote** instead of **getting better at learning**.

Summary:

I make no claims that all of these things will help everyone, because people are pretty diverse in learning styles. However, I do claim that they'll **probably** help you, because... honestly, we are all people, and we make very similar mistakes in learning.

If you have anything to add, for sake of future students, contact me. People like you help make this document better.

I repeat: these are HARD lessons to learn and took me many many years, and I'm still myself a student. Don't feel guilty if you feel you are making mistakes or have fallen into these traps – I may have spent much more time in these traps than you have. Feeling guilty isn't productive and I don't expect you, or anybody really, to already know these simple-sounding but VERY-hard-to-actually do things. What **is** productive is if you change the way you learn and improve and then do better (whether it be this class or elsewhere in life). That would actually be rewarding. Both for you and for me.

Good luck.

Yan X Zhang (last update: 5/2018)