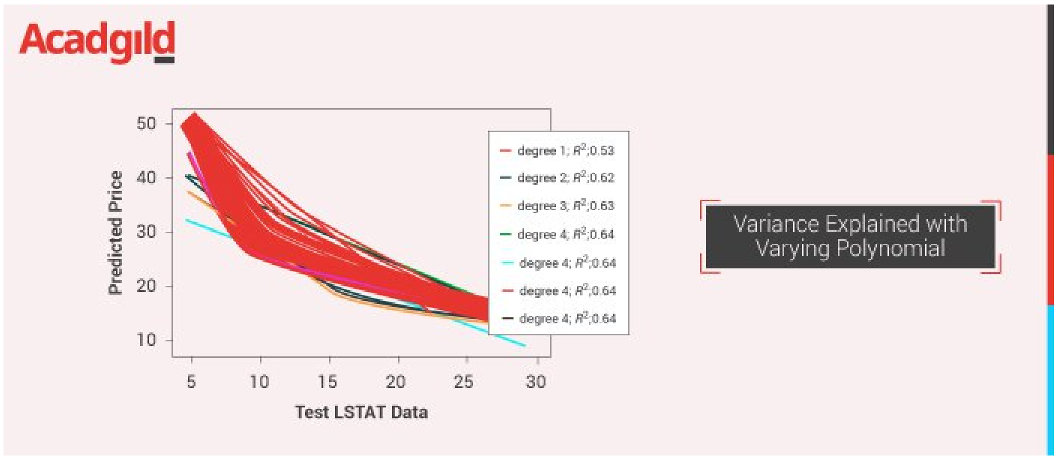
Data Science and Artificial Intelligence

Understand Power of Polynomials with Polynomial Regression

Abhay Kumar • September 13, 2018 1 3,527



This entry is part 4 of 17 in the series Machine Learning Algorithms

Introduction to Polynomial Regression

Polynomial regression is a special case of linear regression. It's based on the idea of how to your select your features. Looking at the multivariate regression with 2 variables: x1 and x2. Linear regression will look like this:

y = a1 * x1 + a2 * x2.

Now you want to have a polynomial regression (let's make 2-degree polynomial). We will create a few additional features: x1*x2, x1^2 and x2^2. So we will get your 'linear regression':

y = a1 * x1 + a2 * x2 + a3 * x1*x2 + a4 * x1^2 + a5 * x2^2

A polynomial term: a quadratic (squared) or cubic (cubed) term turns a linear regression model into a curve. But because it is the data X that is squared or cubed, not the Beta coefficient, it still qualifies as a linear model.

This makes it a nice and straightforward way to model curves without having to model complicated nonlinear models.

One common pattern within machine learning is to use linear models trained on nonlinear functions of the data. This approach maintains the generally fast performance of linear methods while allowing them to fit a much wider range of data.

For example, a simple linear regression can be extended by constructing polynomial features from the coefficients. In the standard linear regression case, you might have a model that looks like this for two-dimensional data:

$$\hat{y}(w, x) = w_0 + w_1 x_1 + w_2 x_2$$

If we want to fit a paraboloid to the data instead of a plane, we can combine the features in second-order polynomials, so that the model looks like this:

$$\hat{y}(w,x) = w_0 + w_1x_1 + w_2x_2 + w_3x_1x_2 + w_4x_1^2 + w_5x_2^2$$

The (sometimes surprising) observation is that this is still a linear model: to see this, imagine creating a new variable

$$z = [x_1, x_2, x_1x_2, x_1^2, x_2^2]$$

With this re-labeling of the data, our problem can be written

$$\hat{y}(w,x) = w_0 + w_1z_1 + w_2z_2 + w_3z_3 + w_4z_4 + w_5z_5$$

We see that the resulting polynomial regression is in the same class of linear models we'd considered above (i.e. the model is linear in w) and can be solved by the same techniques.

By considering, linear fits within a higher-dimensional space built with these basis functions, the model has the flexibility to fit a much broader range of data.

Here is an example of applying this idea to one-dimensional data, using polynomial features of varying degrees:

```
from sklearn.preprocessing import PolynomialFeatures
import numpy as np
X = np.arange(6).reshape(3, 2)
X

poly = PolynomialFeatures(degree=2)
poly.fit_transform(X)
```

In some cases, it's not necessary to include higher powers of any single feature, but only the so-called interaction features that multiply together at most d distinct features. These can be derived from PolynomialFeatures with the setting interaction_only=True.

For example, when dealing with boolean features, x_i^n = x_i for all n, and is therefore useless; but x_i x_j represents the conjunction of two booleans. This way, we can solve the XOR problem with a linear classifier:

```
from sklearn.linear_model import Perceptron
from sklearn.preprocessing import PolynomialFeatures
import numpy as np
X = np.array([[0, 0], [0, 1], [1, 0], [1, 1]])
y = X[:, 0] ^ X[:, 1]
y

X = PolynomialFeatures(interaction_only=True).fit_transform(X).astype(
X
```

Problem Statement:

To understand the relationship between the degree of the independent variable and the dependent variable.

Data details

```
Boston House Prices dataset
===========================

Notes
------
Data Set Characteristics:

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive

    :Median Value (attribute 14) is usually the target

    :Attribute Information (in order):
        - CRIM    per capita crime rate by town
        - ZN   proportion of residential land zoned for lots over 25,0
        - INDUS    proportion of non-retail business acres per town
        - CHAS    Charles River dummy variable (= 1 if tract bounds ri
        - NOX    nitric oxides concentration (parts per 10 million)
        - RM    average number of rooms per dwelling
        - AGE    proportion of owner-occupied units built prior to 1940
        - DIS    weighted distances to five Boston employment centres
        - RAD   index of accessibility to radial highways
        - TAX   full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B   1000(Bk - 0.63)^2 where Bk is the proportion of blacks b
        - LSTAT          - MEDV Median value of owner-occupied homes

    :Missing Attribute Values: None
% lower status of the population

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
http://archive.ics.uci.edu/ml/datasets/Housing

This dataset was taken from the StatLib library which is maintained at

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedon
prices and the demand for clean air', J. Environ. Economics & Manageme
vol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diag
...', Wiley, 1980.   N.B. Various transformations are used in the tabl
pages 244-261 of the latter.
```

The Boston house-price data has been used in many machine learning papers that address regression problems.

Tools used :

- Pandas
- Numpy
- Matplotlib
- scikit-learn

Import necessary libraries

Import the necessary modules from specific libraries.

```
import numpy as np
import pandas as pd
%matplotlib inline
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn import datasets
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
```

Load the data set

Use the pandas module to read the taxi data from the file system. Check few records of the dataset.

```
#
#########################################################################
# Load data
boston = datasets.load_boston()
print(boston.data.shape, boston.target.shape)
print(boston.feature_names)

(506, 13) (506,)
['CRIM' 'ZN' 'INDUS' 'CHAS' 'NOX' 'RM' 'AGE' 'DIS' 'RAD' 'TAX' 'PTRATI
```

```
data = pd.DataFrame(boston.data,columns=boston.feature_names)
data = pd.concat([data,pd.Series(boston.target,name='MEDV')],axis=1)
data.head()

  CRIM    ZN   INDUS CHAS NOX   RM    AGE  DIS    RAD TAX   PTRATIO B
0 0.00632 18.0 2.31  0.0  0.538 6.575 65.2 4.0900 1.0 296.0 15.3    39
1 0.02731 0.0  7.07  0.0  0.469 6.421 78.9 4.9671 2.0 242.0 17.8    39
2 0.02729 0.0  7.07  0.0  0.469 7.185 61.1 4.9671 2.0 242.0 17.8    39
3 0.03237 0.0  2.18  0.0  0.458 6.998 45.8 6.0622 3.0 222.0 18.7    39
4 0.06905 0.0  2.18  0.0  0.458 7.147 54.2 6.0622 3.0 222.0 18.7    39
```

Feature Selection:

Let’s select one of the numerical fields for model fitting here “LSTAT” i.e. % of the population in the neighborhood which comes under lower economic status.

```
X = data[['LSTAT']]
y = data['MEDV']
```

Train test split :

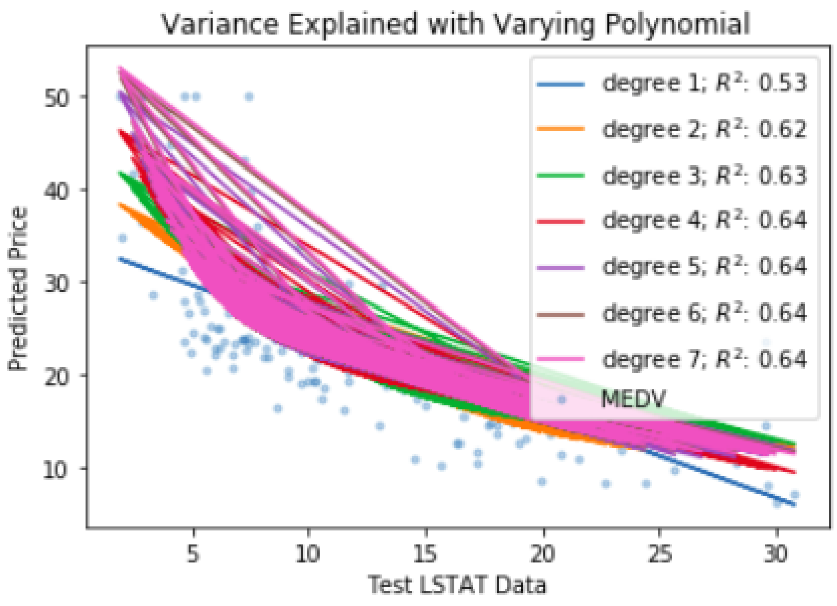
```
x_training_set, x_test_set, y_training_set, y_test_set = train_test_sp
```

Training/model fitting with various degrees :

Fit the model to selected supervised data. We will use the API called Polynomial Features which takes the parameter as the degree of the polynomial. With the given polynomial degree we will fit the data with the linear regression model.

The motive of this fitting is to see if there is a better explanation of the variance with an increase in the degree of the polynomial of the selected feature.

```
# Polynomial Regression-nth order
plt.scatter(x_test_set, y_test_set, s=10, alpha=0.3)

for degree in [1,2,3,4,5,6,7]:

    model = make_pipeline(PolynomialFeatures(degree), LinearRegressior

    model.fit(x_training_set,y_training_set)

    y_plot = model.predict(x_test_set)

    plt.plot(x_test_set, y_plot, label="degree %d" % degree

             +'; $R^2$: %.2f' % model.score(x_test_set, y_test_set))

plt.legend(loc='upper right')

plt.xlabel("Test LSTAT Data")

plt.ylabel("Predicted Price")

plt.title("Variance Explained with Varying Polynomial")

plt.show()
```



We can notice that R^2 has increased from 0.53 – > 0.62 -> 0.63 -> 0.64 with rising in the degree of the polynomial initially and then it has saturated at 0.64.

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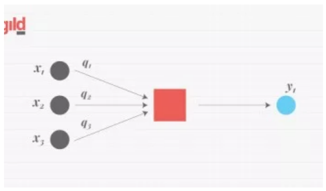

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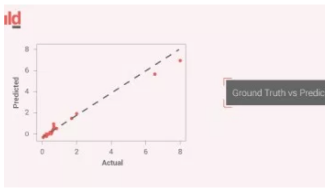

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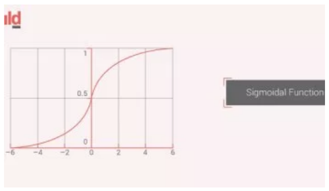

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