Assignment # 6

Due: Wednesday, November 30, 2016

Q. 1 Q1 is based on class lectures on the Quasi-Static case for the Isothermal split of the thermoelasticity problem. The element stiffness matrix was developed for 1D linear shape functions using 1-point integration rule. A linear system was developed (in equation (44) of class lectures) that corresponds to the situation of zero Dirichlet BCs at the right node (node-2) of the one-element mesh in Figure 1.

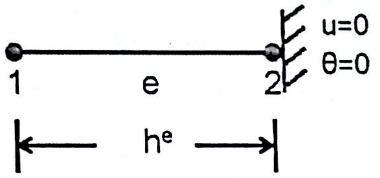

Figure 1. One element mesh. (2-node linear element).

1) Write the element level matrix system (it will help you assemble the system equations).
2) Now consider the 4-element mesh with zero Dirichlet BCs at global node-5 (shown in Fig 2 below). Develop the linear system for this problem. Show all your work.

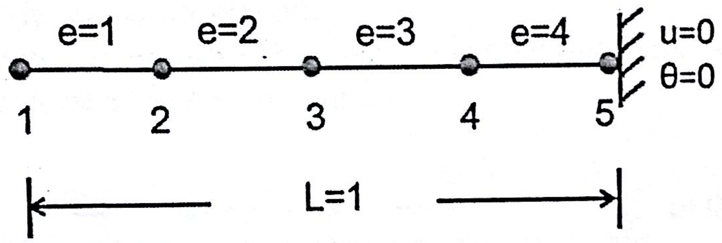

Figure 2. Four element mesh. (2-node linear elements).

3) Now let us do some simple numerical calculations. Consider the following parameters E=100, m=0.3, k=1, c=1. Solution vector at time t_n as $d_n = 0.1$, $\theta_n = 0.25$,
   a) You have explicit form of the element level matrices. Write a matlab code (Note: you can explicitly assemble the element matrices to make the system matrix)
   b) For $\Delta t = 0.05$, find solution at t_{n+1}. Repeat the process for 10 steps till $n\Delta t = 0.5$,
   c) Repeat the process with $\Delta t = 0.1$ (i.e., in 5 steps).
   d) Present in a tabular form, the evolution of the displacement and the temperature fields at the mesh nodal points for parts 3-b and 3-c.

Q. 2 Q2 is based on class lectures on the Quasi-Static case for the Adiabatic split of the thermoelasticity problem. The element stiffness matrix was developed for 1D linear shape functions using 1-point integration rule. A linear system was developed (in equation (51) of class lectures) that corresponds to the situation of zero Dirichlet BCs at the right node (node-2) of the one-element mesh.

1) Write down the element level matrix (as it will help you assemble the system equations).
2) Now consider the 4-element mesh with zero Dirichlet BCs at global node-5 (shown in Fig 3 below). Develop the linear system for this problem. Show all your work.

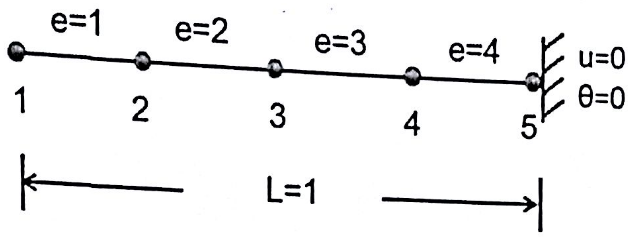

Figure 3. Four element mesh. (2-node linear elements).

3) Let us do some simple numerical calculations. Consider the following parameters E=100, m=0.3, k=1, c=1. Solution vector at time t_n as $d_n = 0.1$, $\theta_n = 0.25$,
   a) You have explicit form of the element level matrices. Write a matlab code (Note: you can explicitly assemble the element matrices to make the system matrix)
   b) For $\Delta t = 0.05$, find solution at t_{n+1}. Repeat the process for 10 steps till $n\Delta t = 0.5$,
   c) Repeat the process with $\Delta t = 0.1$ (i.e., in 5 steps).
   d) Present in a tabular form, the evolution of the displacement and the temperature fields at the mesh nodal points for parts 3-b and 3-c.

Q. 3 Again consider the Quasi-Static case for the Adiabatic split discussed in Q2. Using 3-node quadratic shape functions, and employing 2 point numerical quadrature, evaluate the element stiffness matrix.

1) Write the element level matrix system.
2) For the 2-element mesh with zero Dirichlet BCs at the right end (shown in Fig 4 below), develop the linear system. Show all your work.

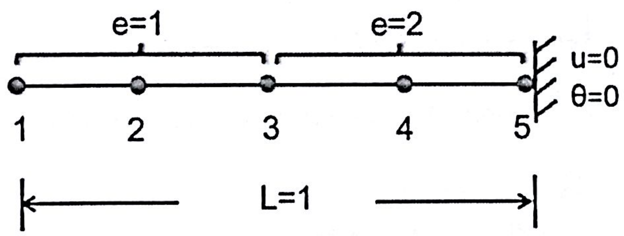

Figure 4. Two element mesh. (3-node quadratic element).

3) Let us do some simple numerical calculations. Consider the following parameters =100, m=0.3, k=1, c=1. Solution vector at time t_n as $d_n = 0.1$, $\theta_n = 0.25$,
   a) You have explicit form of the element level matrices. Write a matlab code (Note: you can explicitly assemble the element matrices to make the system matrix)
   b) For $\Delta t = 0.05$, find solution at t_{n+1}. Repeat the process for 10 steps till $n\Delta t = 0.5$,
   c) Repeat the process with $\Delta t = 0.1$ (i.e., in 5 steps).
   d) Present in a tabular form, the evolution of the displacement and the temperature fields at the mesh nodal points for parts 3-b and 3-c.

4) Provide detailed comments on any difference in the linear and quadratic element systems for the Adiabatic Case (i.e., from Q2 and Q3).

BONUS (10 points): Comment on the differences between the Isothermal and Adiabatic systems as observed in Q1, Q2 and Q3.