Prof. A. Masud

CEE-576: Nonlinear Finite Elements

Handout for Assignment # 6

As derived in the class, the element level matrix systems for the quasi-static-cases using 2-node linear elements with 1-point numerical integration are as follows.

The Quasi-Static case for the Isothermal split

Element level matrix system

$$\begin{bmatrix} \frac{E}{h} & 0 & -\frac{E}{h} & 0 \\ -\frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t}+\frac{\bar{k}}{2h} & \frac{m}{2\Delta t} & -\frac{\bar{k}}{2h} \\ -\frac{E}{h} & 0 & \frac{E}{h} & 0 \\ -\frac{m}{2\Delta t} & -\frac{\bar{k}}{2h} & \frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t}+\frac{\bar{k}}{2h} \end{bmatrix} \begin{Bmatrix} d_1^e \\ d_2^e \\ d_3^e \\ d_4^e \end{Bmatrix}_{n+1} = \begin{bmatrix} 0 & -\frac{m}{2} & 0 & -\frac{m}{2} \\ -\frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t}-\frac{\bar{k}}{2h} & \frac{m}{2\Delta t} & \frac{\bar{k}}{2h} \\ 0 & \frac{m}{2} & 0 & \frac{m}{2} \\ -\frac{m}{2\Delta t} & \frac{\bar{k}}{2h} & \frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t}-\frac{\bar{k}}{2h} \end{bmatrix} \begin{Bmatrix} d_1^e \\ d_2^e \\ d_3^e \\ d_4^e \end{Bmatrix}_n$$

The Quasi-Static case for the Adiabatic split

Element level matrix system

$$\begin{bmatrix} \frac{E}{h} & \frac{m}{4} & -\frac{E}{h} & \frac{m}{4} \\ -\frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t}+\frac{\bar{k}}{h} & \frac{m}{2\Delta t} & -\frac{\bar{k}}{h} \\ -\frac{E}{h} & -\frac{m}{4} & \frac{E}{h} & -\frac{m}{4} \\ -\frac{m}{2\Delta t} & -\frac{\bar{k}}{h} & \frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t}+\frac{\bar{k}}{h} \end{bmatrix} \begin{Bmatrix} d_1^e \\ d_2^e \\ d_3^e \\ d_4^e \end{Bmatrix}_{n+1} = \begin{bmatrix} 0 & -\frac{m}{4} & 0 & -\frac{m}{4} \\ -\frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t} & \frac{m}{2\Delta t} & 0 \\ 0 & \frac{m}{4} & 0 & \frac{m}{4} \\ -\frac{m}{2\Delta t} & 0 & \frac{m}{2\Delta t} & \frac{\bar{c}h}{2\Delta t} \end{bmatrix} \begin{Bmatrix} d_1^e \\ d_2^e \\ d_3^e \\ d_4^e \end{Bmatrix}_n$$