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CEE 570: Finite Element Method

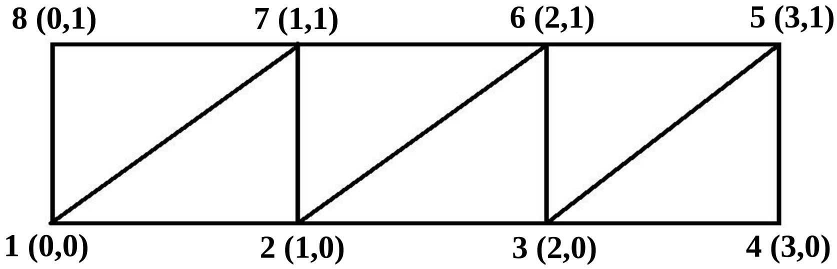
HW #4 **Date: 03/06/2016**

Problem 2:

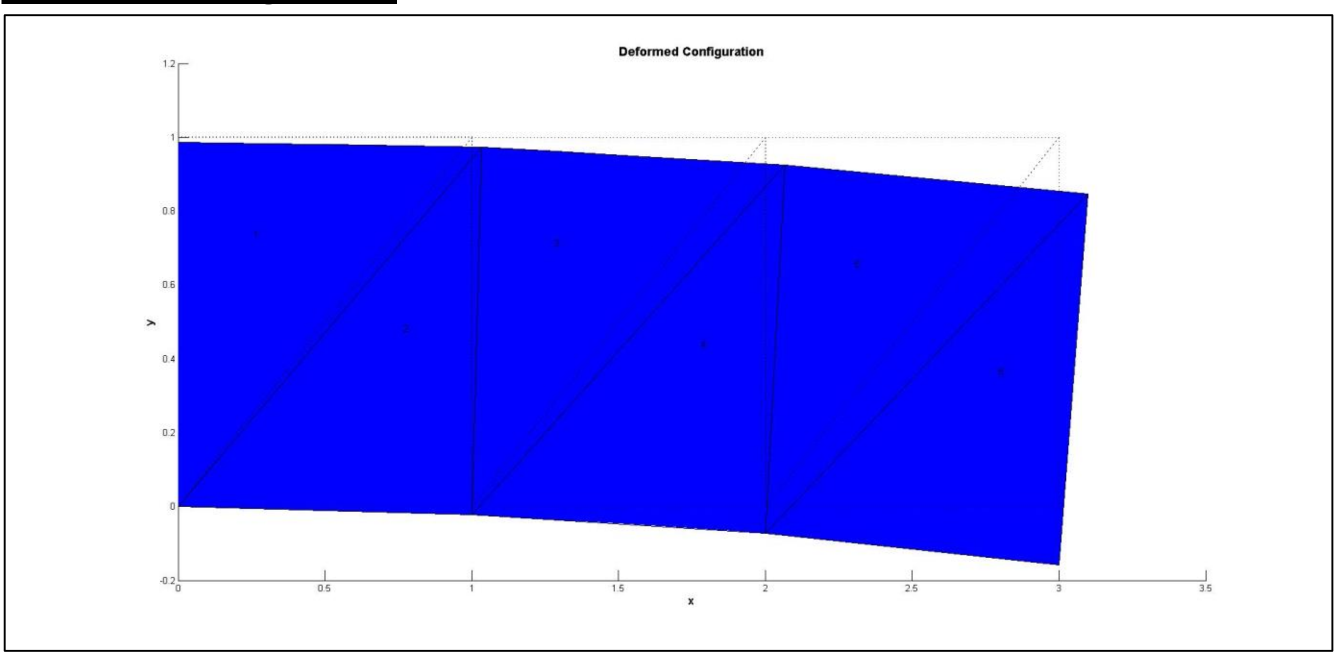
(a)

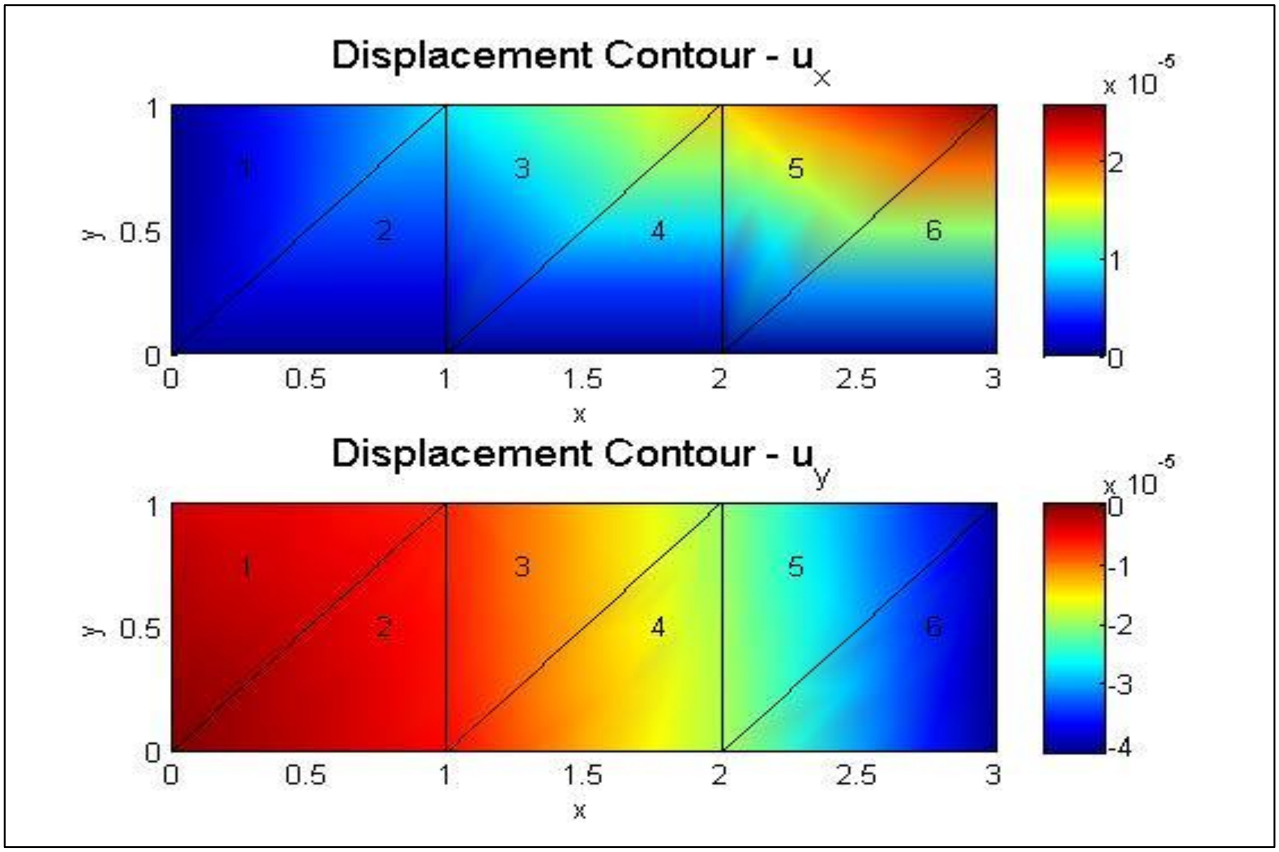
Results:

Element and Nodal Nomenclature considered for the problem



Deformed Configuration





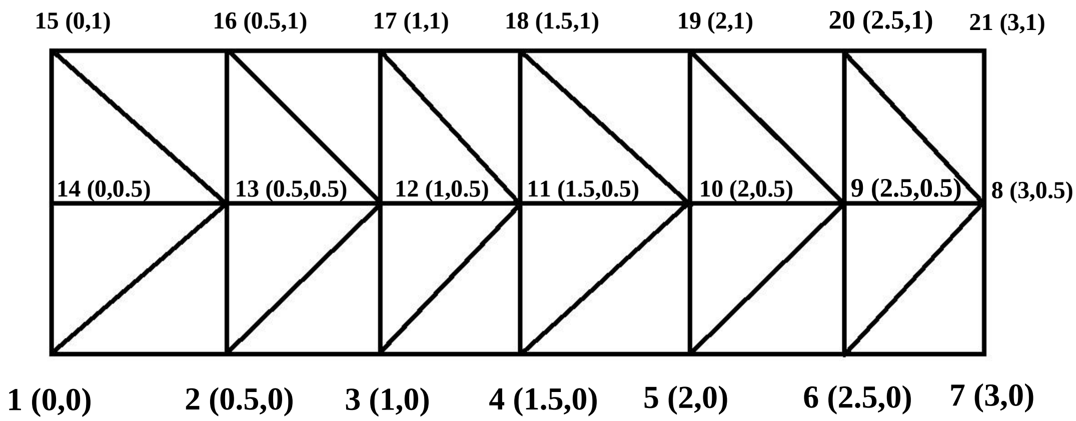
Displacement Values:

Node #	Displacement in x (* 10^{-4})	Displacement in y (* 10^{-4})
1	**0**	**0**
2	**0**	**-0.0568**
3	**0**	**-0.1889**
4	**0**	**-0.4132**
5*	**0.2559**	**-0.4013**
6	**0.1718**	**-0.1979**
7	**0.09**	**-0.0682**
8	**0**	**-0.035**

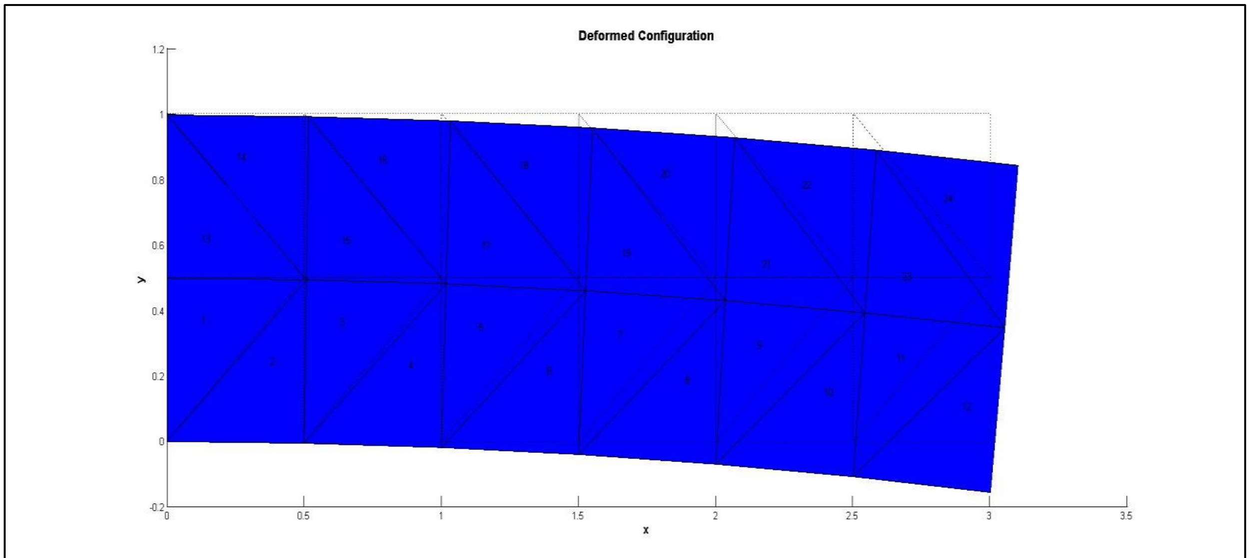
***The displacement of the top right node is highlighted.**

2 (b):

Element and Nodal Nomenclature considered for the problem



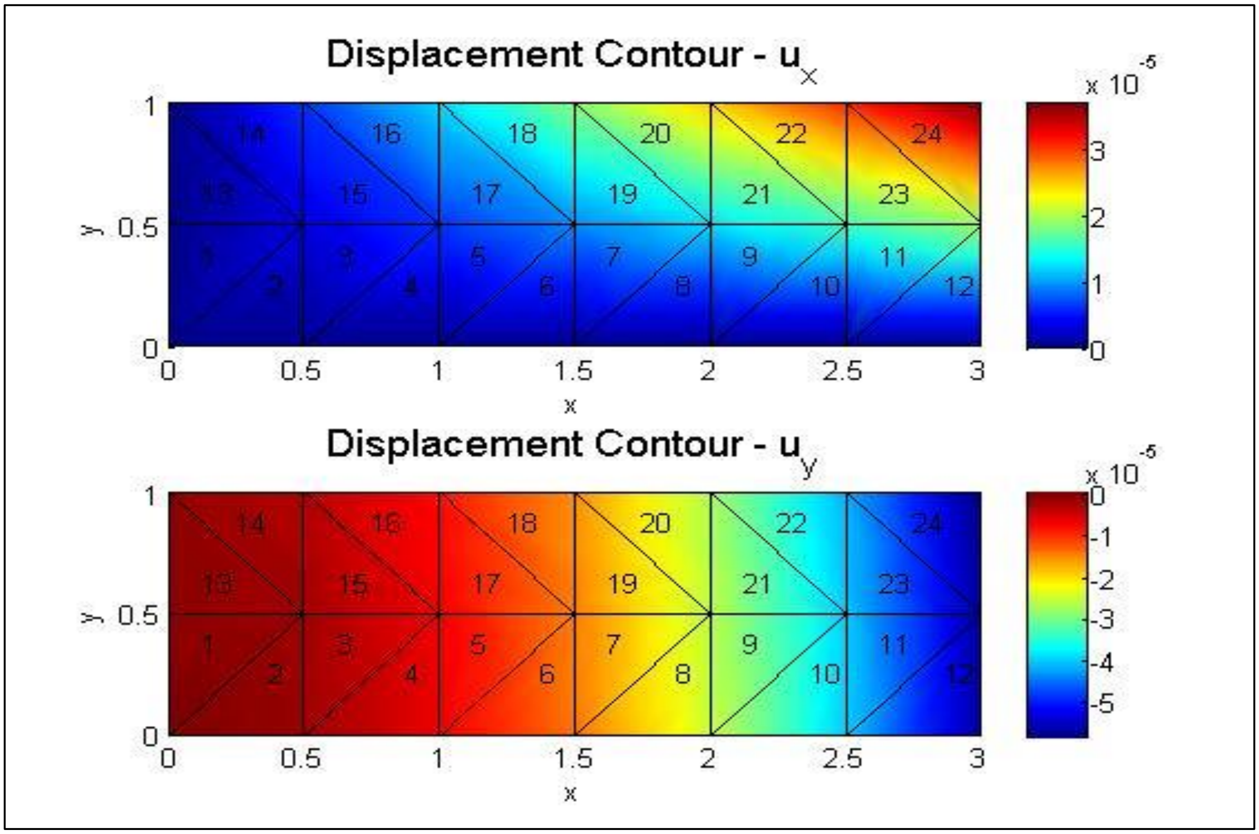
Results:



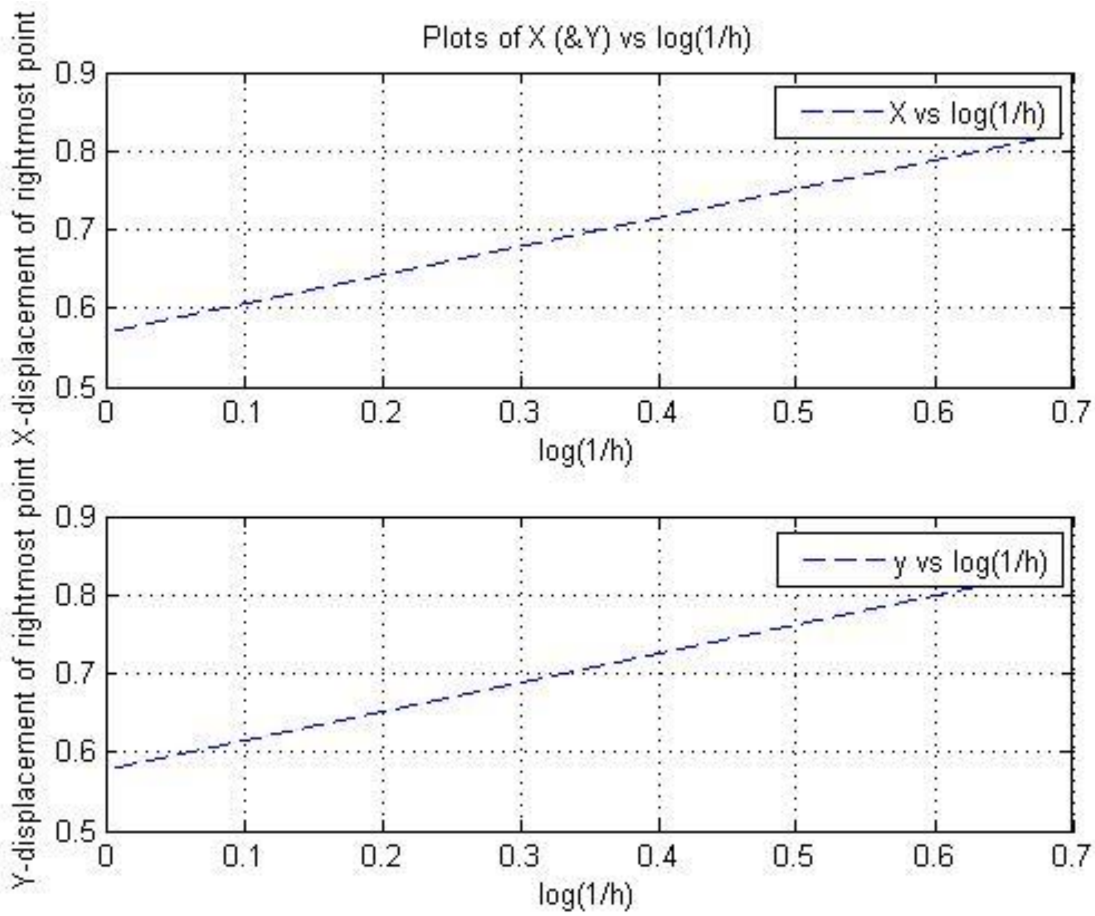
Node #	Displacement in x ($* 10^{-4}$)	Displacement in y ($* 10^{-4}$)
1	0	0
2	0	-0.0195
3	0	-0.0666
4	0	-0.1442
5	0	-0.2525
6	0	-0.3916
7	0	-0.5678
8	0.193	-0.5615

9	0.1577	-0.3957
10	0.1251	-0.2569
11	0.0935	-0.1483
12	0.0624	-0.0706
13	0.0312	-0.024
14	0	-0.0122
15	0	-0.0155
16	0.0628	-0.0352
17	0.1246	-0.0823
18	0.1863	-0.1602
19	0.2478	-0.2691
20	0.3087	-0.4087
21*	0.3694	-0.579

***The displacement of the top right node is highlighted.**



Plot of u vs log(1/h) for x and y displacement values {normalized with exact values}:



Matlab Code for Plotting:

```
%Plot of u (&v) vs log(1/h) for different values of h:
x_plot=[log(1) log(2)];
u_plot=[0.2559 0.3694]/0.45;
v_plot=[-0.4013 -0.579]/(-0.69375);


subplot(2,1,1)
plot(x_plot,u_plot,'--');
grid on
xlabel('log(1/h)');
ylabel('X-displacement of rightmost point');
legend('X vs log(1/h)');
```

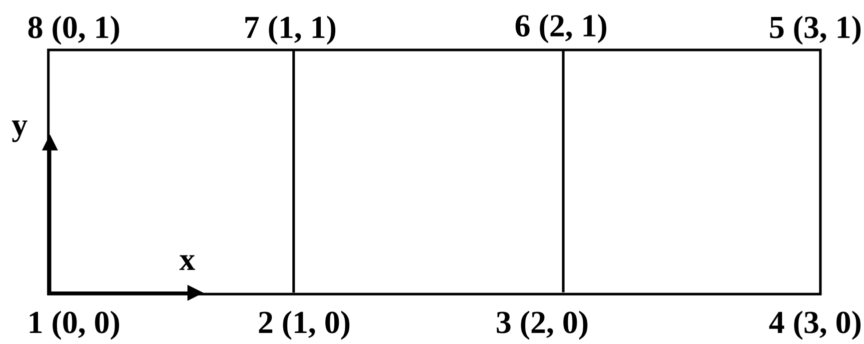
```
title('Plots of X (&Y) vs log(1/h)');

subplot(2,1,2)
plot(x_plot,v_plot,'--');
grid on
xlabel('log(1/h)');
ylabel('Y-displacement of rightmost point');
legend('y vs log(1/h)');
```

Problem 3:

<u>Solution to 2 (a) using quads:</u>

Element and Nodal Nomenclature considered for the problem

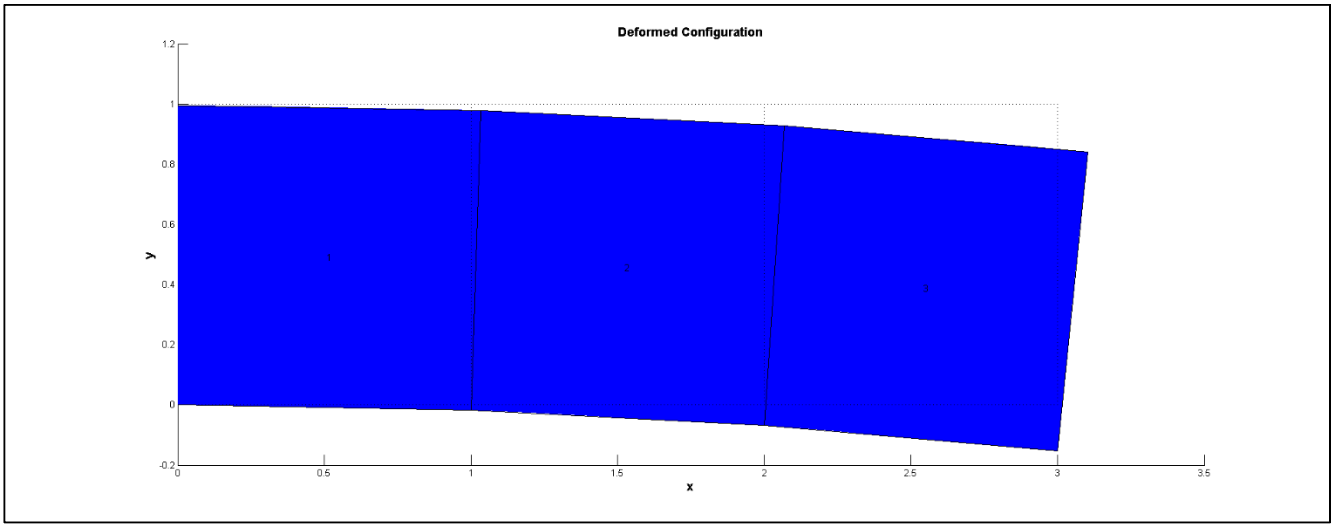


<u>Nodal Displacements:</u>

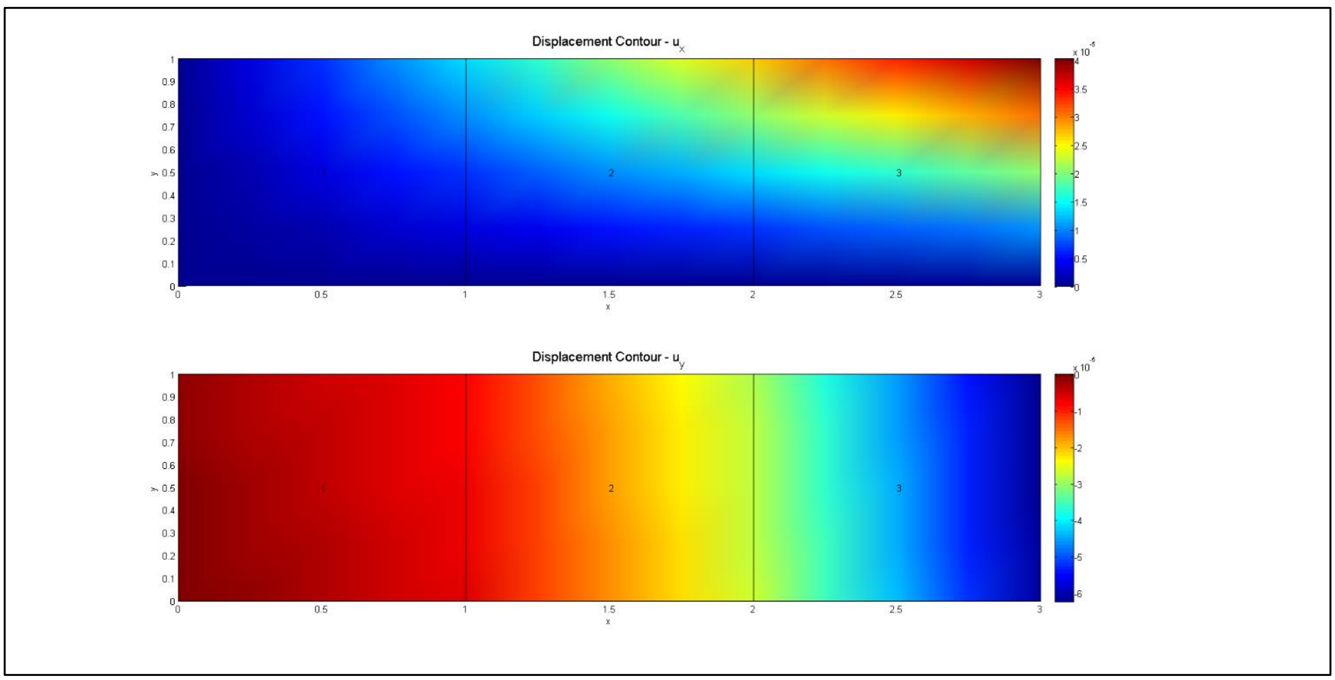
Node #	Displacement in x (* 10^{-4})	Displacement in y (* 10^{-4})
1	0	0
2	0	-0.0672
3	0	-0.2687
4	0	-0.6045
5*	0.403	-0.6213
6	0.2687	-0.2854
7	0.1343	-0.084
8	0	-0.0168

***The displacement of top right node is highlighted.**

Deformed Configuration

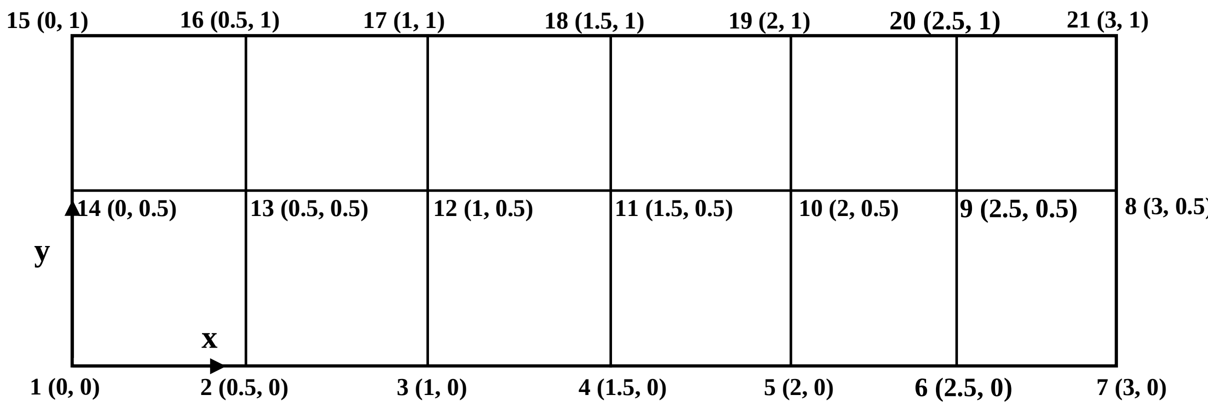


Contour Plots:



Solution to 2(b) using quads:

Element and Nodal Nomenclature considered for the problem

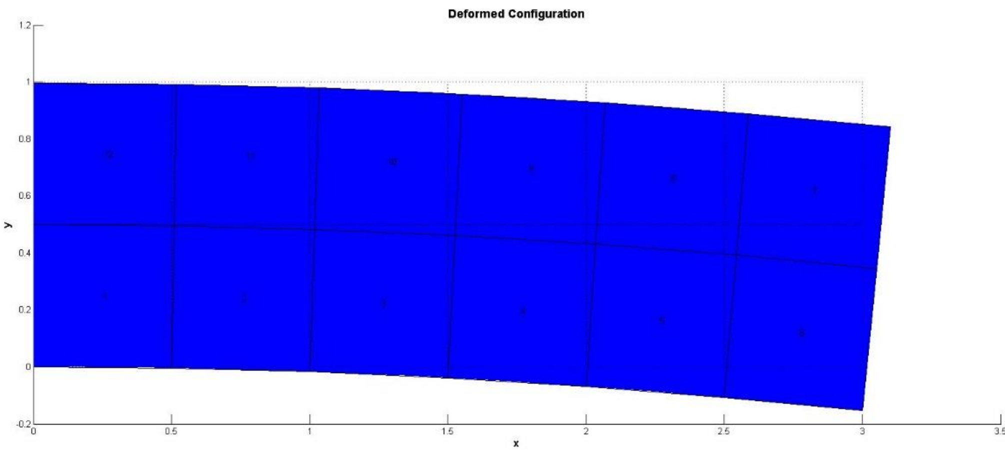


Results:

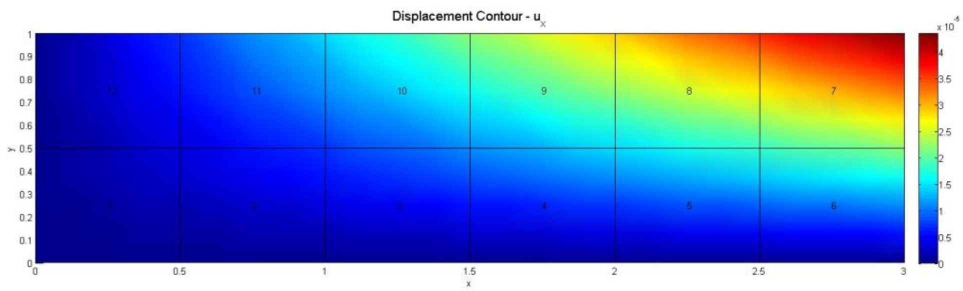
Node #	Displacement in x (* 10^{-4})	Displacement in y (* 10^{-4})
1	0	0
2	0	-0.0182
3	0	-0.0729
4	0	-0.164
5	0	-0.2915
6	0	-0.4551
7	0	-0.6558
8	0.22	-0.6599
9	0.1825	-0.46
10	0.1458	-0.2961
11	0.1093	-0.1685
12	0.0729	-0.0774
13	0.0364	-0.0228
14	0	-0.0046
15	0	-0.0182
16	0.0729	-0.0364
17	0.1457	-0.0911
18	0.2186	-0.1822
19	0.2915	-0.3098
20	0.3641	-0.4739
21*	0.4356	-0.673

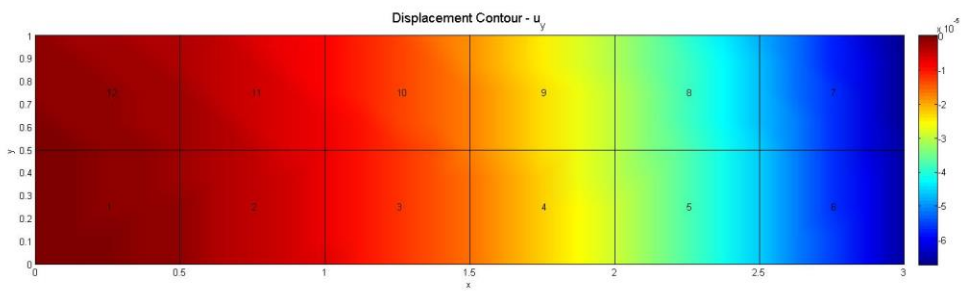
*The displacement of the top right node is highlighted.

Deformed Configuration:

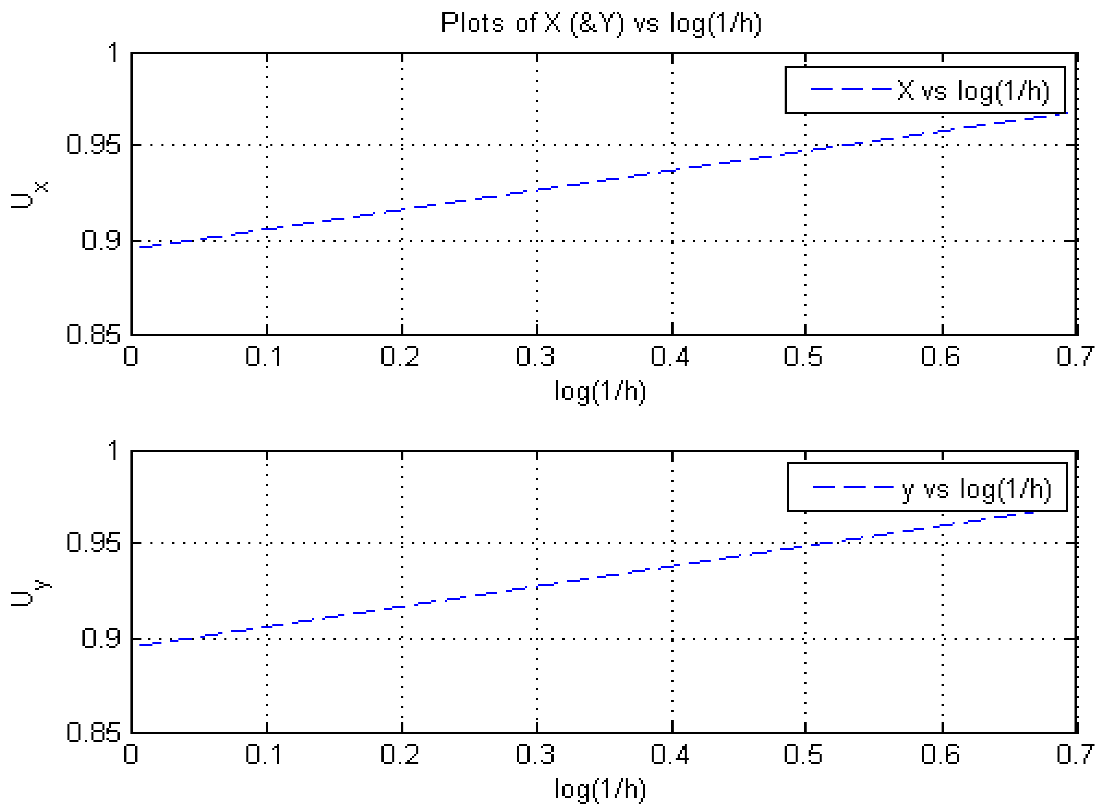


Contour Plots:





Plot of u vs log (1/h) for x and y displacement values {normalized with exact values}:



Problem 3(d): Plane Strain

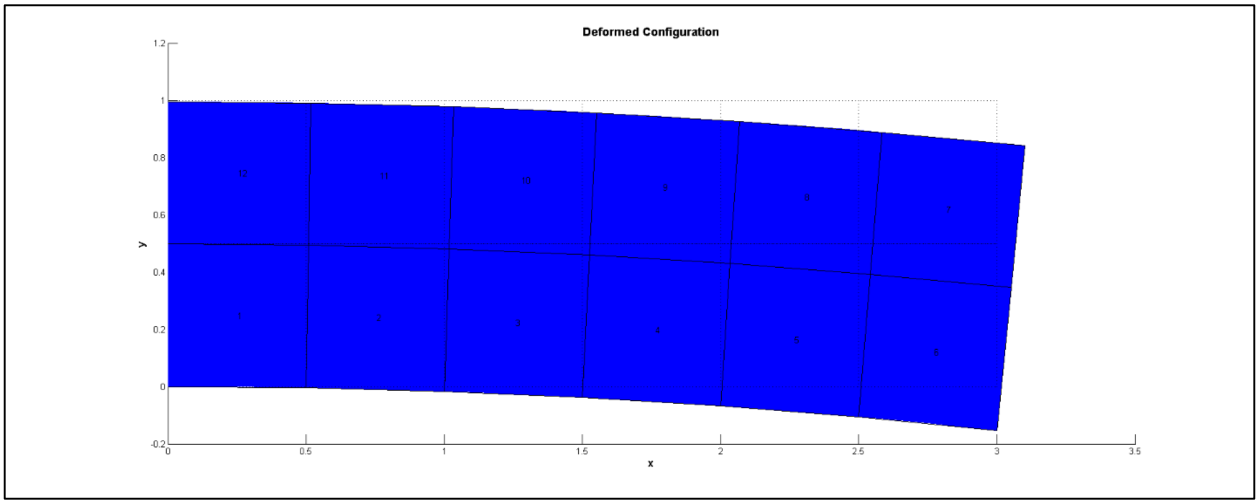
For the plane strain condition, the displacement values obtained for the case of 12 elements is as follows:

Node #	Displacement in x (* 10^{-4})	Displacement in y (* 10^{-4})
1	0	0
2	0	-0.017
3	0	-0.0682
4	0	-0.1534
5	0	-0.2727
6	0	-0.4257
7	0	-0.6135
8	0.206	-0.6188
9	0.1708	-0.4318
10	0.1364	-0.2784
11	0.1023	-0.1591

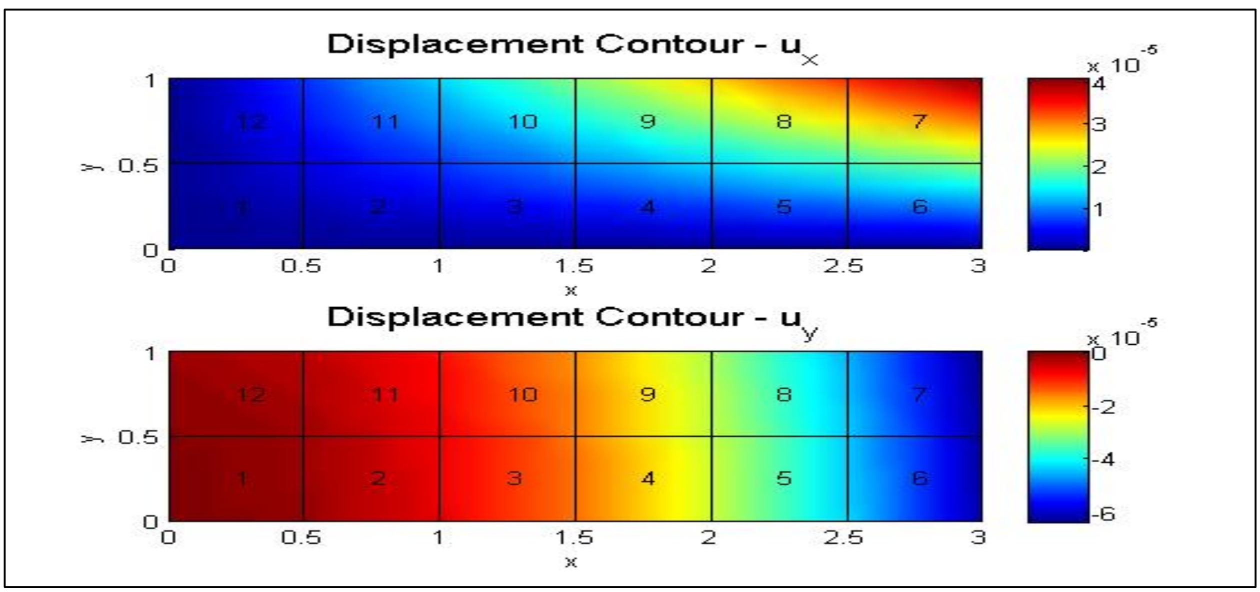
12	0.0682	-0.0739
13	0.0341	-0.0227
14	0	-0.0057
15	0	-0.0227
16	0.0682	-0.0398
17	0.1364	-0.0909
18	0.2045	-0.1761
19	0.2727	-0.2955
20	0.3406	-0.449
21*	**0.4075**	**-0.6353**

***The displacement of the top right node is highlighted:**

Deformed Configuration:



Contour Plots:

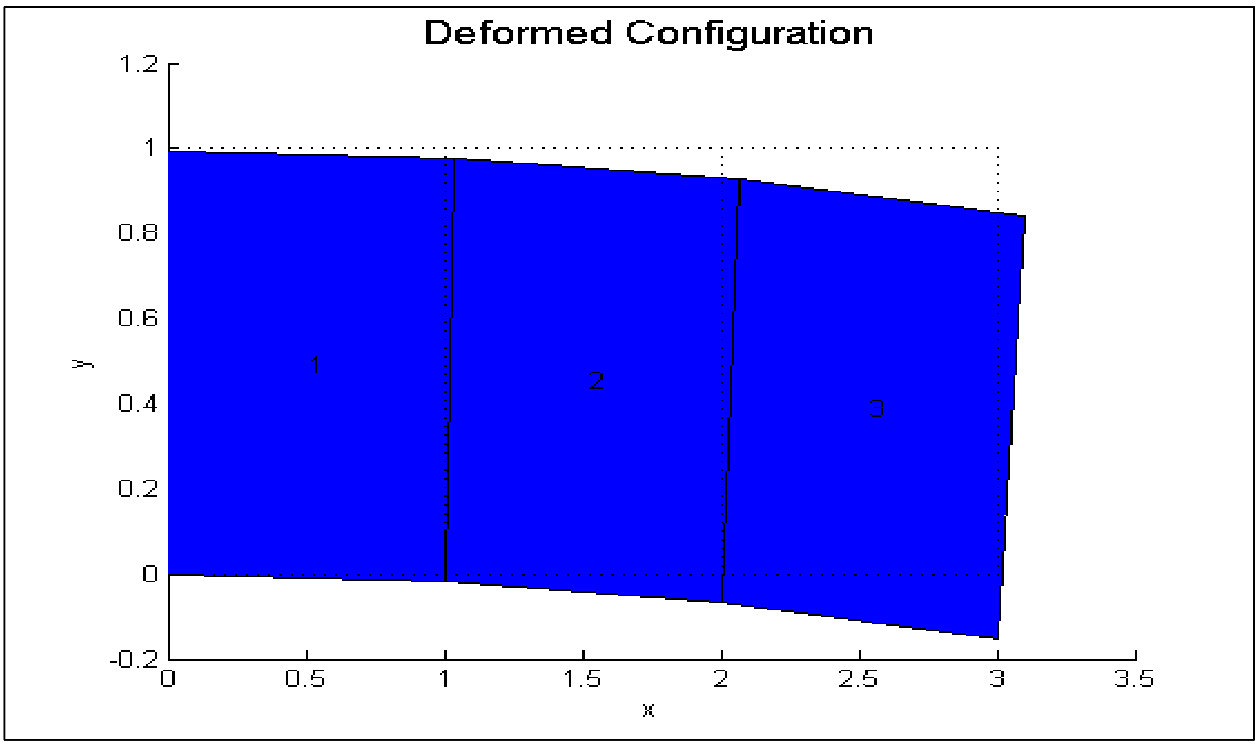


Part (ii)

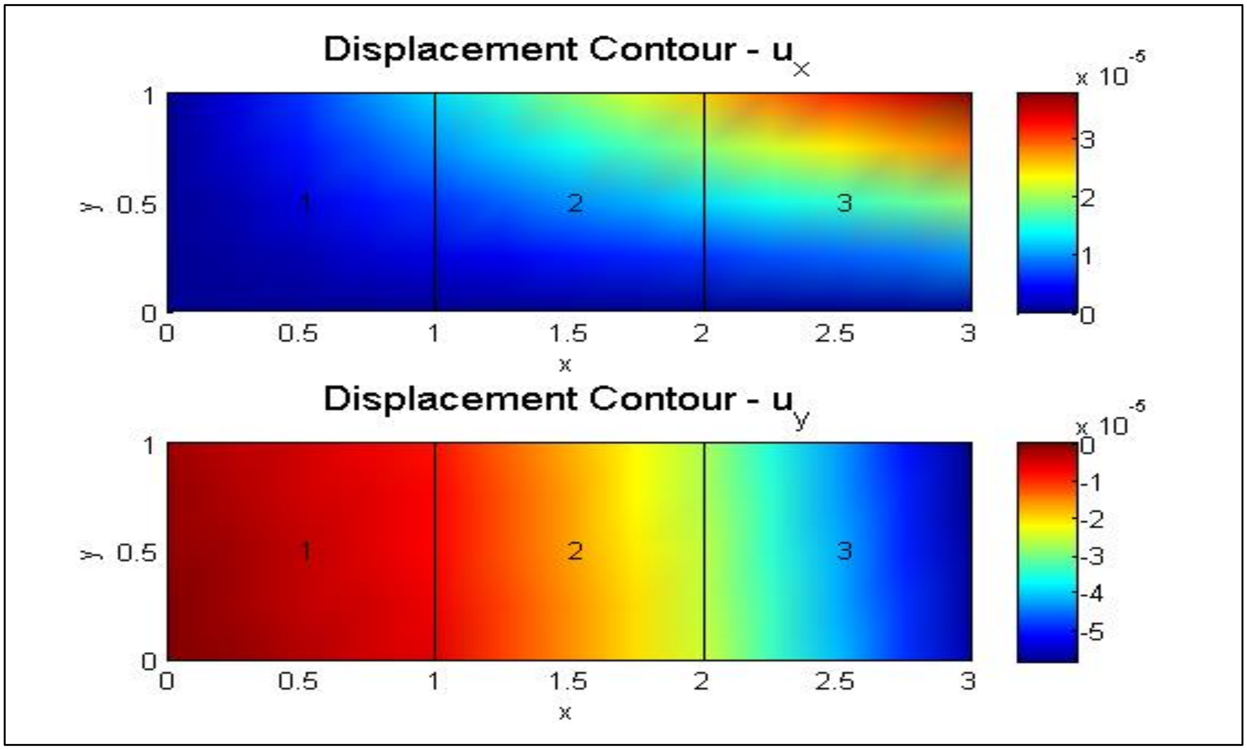
For the plane strain condition, the displacement values obtained for the case of 3 elements is as follows:

Node #	**Displacement in x (* 10^{-4})**	**Displacement in y (* 10^{-4})**
1	**0**	**0**
2	**0**	**-0.0625**
3	**0**	**-0.25**
4	**0**	**-0.5625**
5*	**0.375**	**-0.5833**
6	**0.25**	**-0.2708**
7	**0.125**	**-0.0833**
8	**0**	**-0.0208**

***The displacement of the top right node is highlighted.**



Contour Plot:



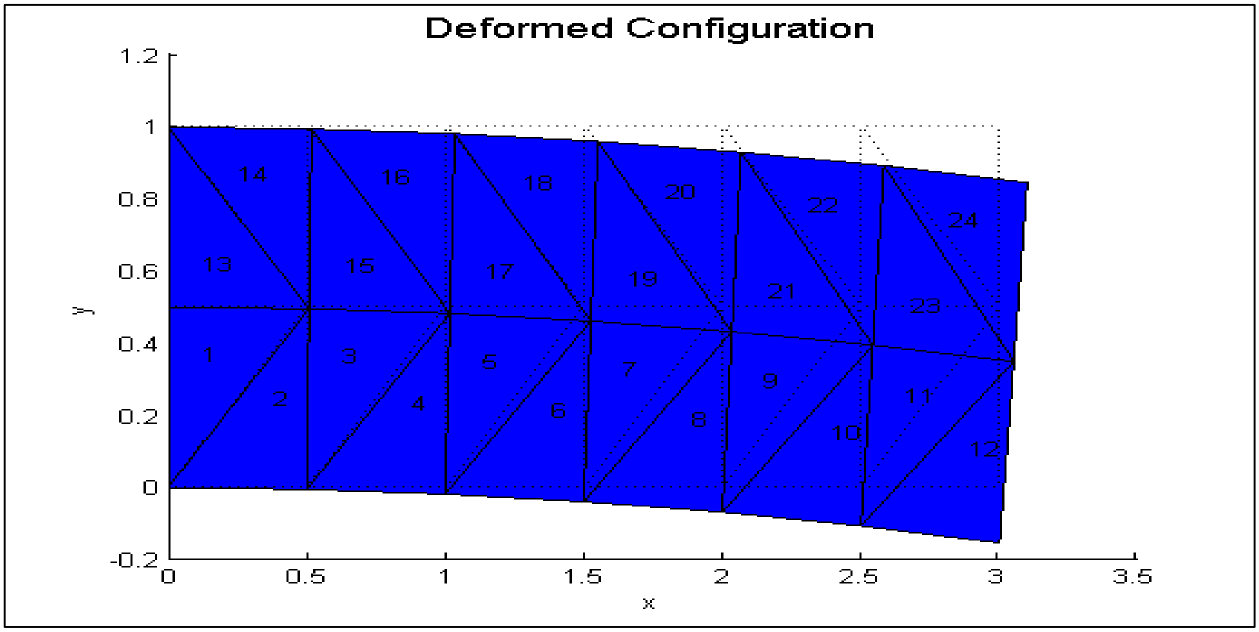
Using Triangular Elements:

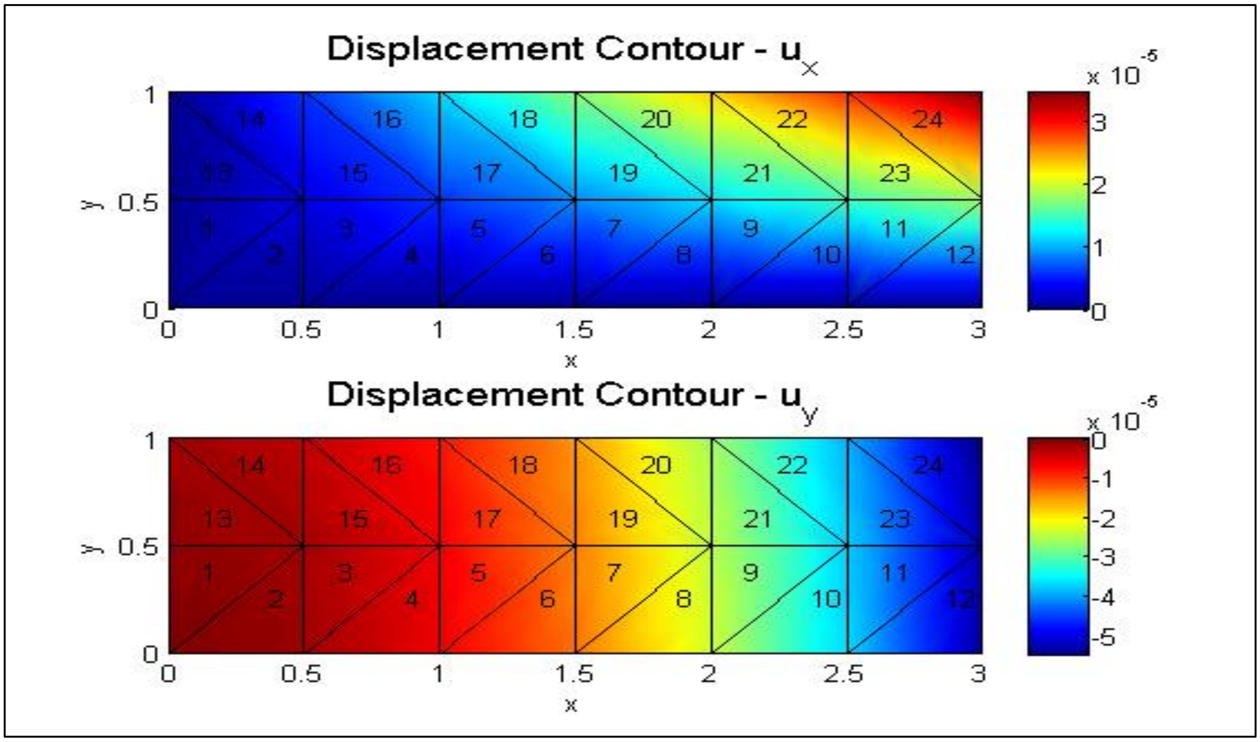
Using triangular elements, and keeping number of elements equal to 24, the following results were obtained for the plane strain condition:

Node #	Displacement in x (* 10^{-4})	Displacement in y (* 10^{-4})
1	0	0
2	0	-0.0185
3	0	-0.0624
4	0	-0.1348
5	0	-0.2357
6	0	-0.3654
7	0	-0.5302
8	0.1804	-0.525
9	0.1474	-0.3706
10	0.1168	-0.2411
11	0.0873	-0.1399
12	0.0582	-0.0674
13	0.0291	-0.024
14	0	-0.0132
15	0	-0.0194
16	0.0586	-0.038
17	0.1162	-0.0819

18	**0.1737**	**-0.1546**
19	**0.2311**	**-0.2561**
20	**0.2879**	**-0.3864**
21*	**0.3452**	**-0.5459**

***The displacement of the top right node is highlighted.**

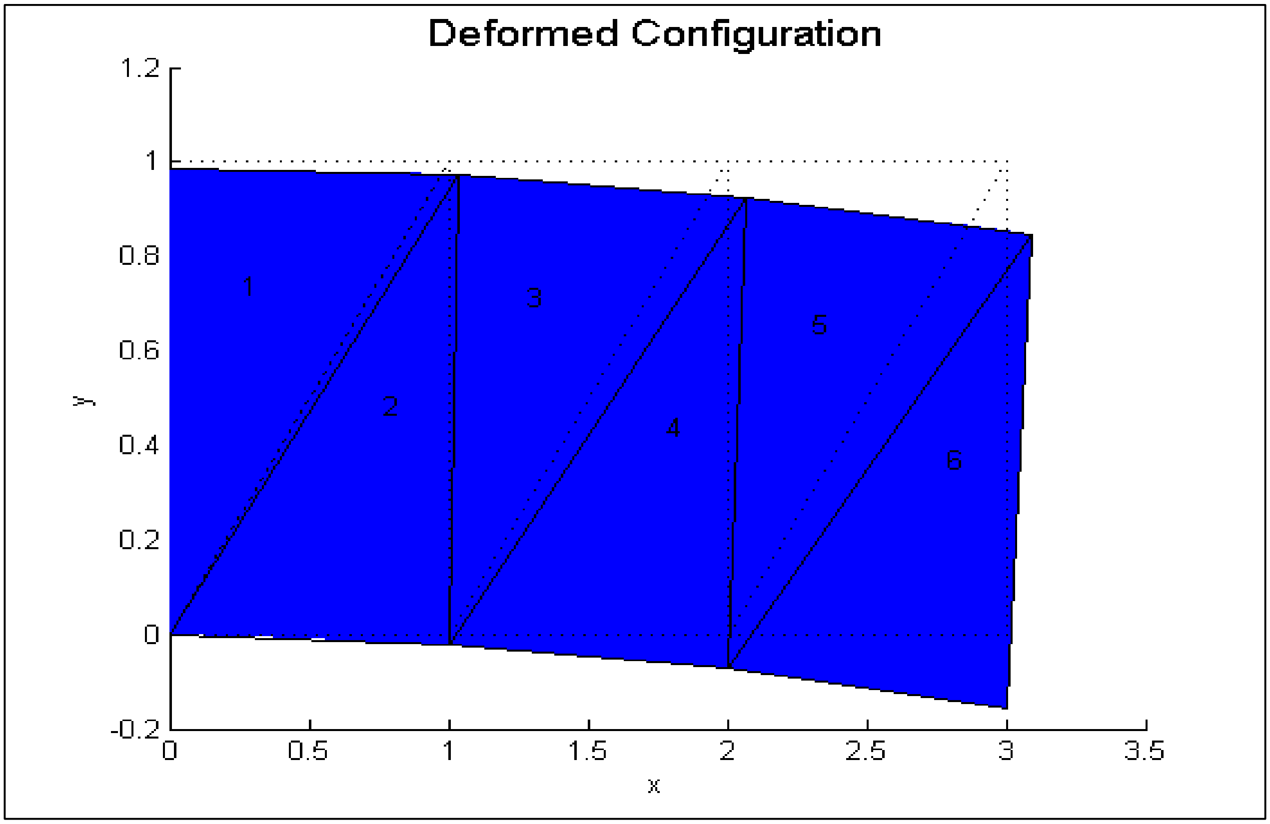




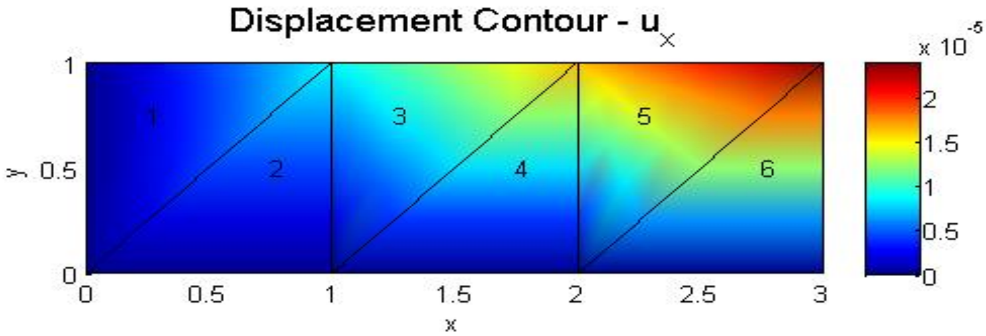
- **For the case of triangular elements, and when the number of elements is equal to 3, the following results were obtained.**

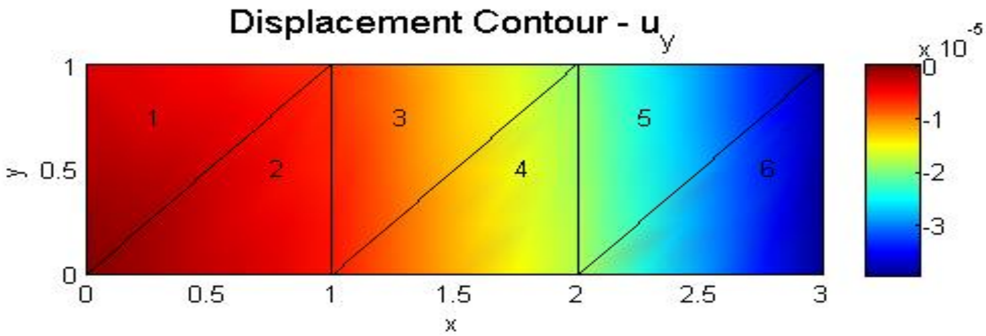
Node #	**Displacement in x (* 10^{-4})**	**Displacement in y (* 10^{-4})**
1	**0**	**0**
2	**0**	**-0.0557**
3	**0**	**-0.1807**
4	**0**	**-0.3918**
5*	**0.2395**	**-0.3824**
6	**0.1618**	**-0.1922**
7	**0.0861**	**-0.0693**
8	**0**	**-0.0389**

***The displacement of the top right node is highlighted.**



Contour Plot:





Summary of the tabulated values of displacement of the top right node for Plane Stress and Plane Strain cases:

Case	Element Type	# of Elements	U_x (*10^{-4})	U_y (10^{-4})
Plane Stress	Triangular	6	0.2559	-0.4013
		24	0.3694	-0.579
	Quadrilateral	3	0.403	-0.6213
		12	0.4356	-0.673
Plane Strain	Triangular	6	0.2395	-0.3824
		24	0.3452	-0.5459
	Quadrilateral	3	0.375	-0.5833
		12	0.4075	-0.6353

The input files used for the assignment are appended below:

For Plane Strain:

(a) Triangular Elements (# of Elements = 6)

```
% Mesh Nodal Coordinates
NodeTable = [0 0
             1 0
             2 0
             3 0
             3 1
             2 1
             1 1
             0 1];
numnp = length(NodeTable);

% Mesh Element Connectivities
ix = [1 7 8 0 1
      1 2 7 0 1
      2 6 7 0 1
      2 3 6 0 1
      3 5 6 0 1
      3 4 5 0 1
      ];

nen = 4;
numel = 6;

% Mesh Boundary Conditions and Loads
BCLIndex = [6 1]';
NodeBC = [1 1 0
          1 2 0
          2 1 0
          3 1 0
          4 1 0
          8 1 0
          ];
NodeLoad = [5 1 5];
% Mesh Material Properties
young = 10e5;
pois = .25;
thick = 1;
PSPS = 'n';
MateT = [young pois thick];
FEA_Program
```

(b) Triangular Elements (# of Elements = 24)

```
% Mesh Nodal Coordinates
NodeTable = [0 0
             0.5 0
             1 0
             1.5 0
             2 0
             2.5 0
             3 0
             3 0.5
             2.5 0.5
             2 0.5
             1.5 0.5
             1 0.5
             0.5 0.5
             0 0.5
             0 1
             0.5 1
             1 1
             1.5 1
             2 1
             2.5 1
             3 1];
numnp = length(NodeTable);

% Mesh Element Connectivities
ix = [1 13 14 0 1
      1 2 13 0 1
      2 12 13 0 1
      2 3 12 0 1
      3 11 12 0 1
      3 4 11 0 1
      4 10 11 0 1
      4 5 10 0 1
      5 9 10 0 1
      5 6 9 0 1
      6 8 9 0 1
      6 7 8 0 1
      13 15 14 0 1
      13 16 15 0 1
      12 16 13 0 1
      12 17 16 0 1
      11 17 12 0 1
      11 18 17 0 1
      10 18 11 0 1
      10 19 18 0 1
      9 19 10 0 1
```

```
      9 20 19 0 1
      8 20 9 0 1
      8 21 20 0 1
      ];

nen = 4;
numel = 24;

% Mesh Boundary Conditions and Loads
BCLIndex = [10 2]';
NodeBC = [1 1 0
          1 2 0
          2 1 0
          3 1 0
          4 1 0
          5 1 0
          6 1 0
          7 1 0
          14 1 0
          15 1 0];
NodeLoad = [8 1 3.75
            21 1 3.125];

% Mesh Material Properties
young = 10e5;
pois = .25;
thick = 1;
PSPS = 'n';
MateT = [young pois thick];
FEA_Program
```

(c) Quadrilateral Elements (# of elements = 3)

```
% Mesh Nodal Coordinates
NodeTable = [0 0
             1 0
             2 0
             3 0
             3 1
             2 1
             1 1
             0 1];
numnp = length(NodeTable);
% Mesh Element Connectivities
ix = [1 2 7 8 1
      2 3 6 7 1
      3 4 5 6 1];
```

```
nen = 4;
numel = 3;
% Mesh Boundary Conditions and Loads
BCLIndex = [6 1]';
NodeBC = [1 1 0
          1 2 0
          2 1 0
          3 1 0
          4 1 0
          8 1 0];

NodeLoad = [5 1 5];

% Mesh Material Properties
young = 10e5;
pois = .25;
thick = 1;
PSPS = 'n';
MateT = [young pois thick];
FEA_Program
```

(i) Modification in the Elas_2D file:

```
if PSPS == 's' %Plane Stress
    Dmat = ElemE/(1-Elemv^2)*[1 Elemv 0; Elemv 1 0; 0 0 (1-
Elemv)/2];

else %Plane Strain
    Dmat = ElemE/((1+Elemv)*(1-2*Elemv))*[1-Elemv Elemv 0;Elemv
1-Elemv 0;0 0 (1-2*Elemv)/2];
end


if nel == 4
        Bmat = [Qxy(1,1) 0        Qxy(1,2) 0        Qxy(1,3) 0        Qxy(1,4) 0
            0        Qxy(2,1) 0        Qxy(2,2) 0        Qxy(2,3) 0        Qxy(2,4)
            Qxy(2,1) Qxy(1,1) Qxy(2,2) Qxy(1,2) Qxy(2,3) Qxy(1,3) Qxy(2,4) Qxy(1,4)
                                                                                ];
        else
        Bmat= [Qxy(1,1) 0        Qxy(1,2) 0        Qxy(1,3) 0
            0        Qxy(2,1) 0        Qxy(2,2) 0        Qxy(2,3)
            Qxy(2,1) Qxy(1,1) Qxy(2,2) Qxy(1,2) Qxy(2,3) Qxy(1,3)];


        end
```

(ii) Modification in the shp_2l.m file:

```
elseif nel == 4

    % shp(i,j). j = denotes the node number,
    % i = 1, derivative w.r.t r
    % i = 2, derivative w.r.t s
    % i = 3, shape funtion
    clear shp
    syms xc eta real
    shp(3,1)=0.25*(1-xc)*(1-eta);
    shp(1,1) = diff(shp(3,1),xc);
    shp(2,1) = diff(shp(3,1),eta);

    shp(3,2) = 0.25*(1+xc)*(1-eta);
    shp(1,2) = diff(shp(3,2),xc);
    shp(2,2) = diff(shp(3,2),eta);

    shp(3,3) = 0.25*(1+xc)*(1+eta);
    shp(1,3) = diff(shp(3,3),xc);
    shp(2,3) = diff(shp(3,3),eta);

    shp(3,4) = 0.25*(1-xc)*(1+eta);
    shp(1,4) = diff(shp(3,4),xc);
    shp(2,4) = diff(shp(3,4),eta);

    xc=r;
    eta=s;
    shp=double(subs(shp));
```

(d) Quadrilateral Elements (# of elements = 12):

```
% Mesh Nodal Coordinates
NodeTable = [0 0
             0.5 0
             1 0
             1.5 0
             2 0
             2.5 0
             3 0
             3 0.5
             2.5 0.5
             2 0.5
             1.5 0.5
             1 0.5
             0.5 0.5
             0 0.5
```

```
             0 1
             0.5 1
             1 1
             1.5 1
             2 1
             2.5 1
             3 1
             ];
numnp = length(NodeTable);
% Mesh Element Connectivities
ix = [1 2 13 14 1
      2 3 12 13 1
      3 4 11 12 1
      4 5 10 11 1
      5 6 9 10 1
      6 7 8 9 1
      8 21 20 9 1
      9 20 19 10 1
      10 19 18 11 1
      11 18 17 12 1
      12 17 16 13 1
      13 16 15 14 1
      ];

nen = 4;
numel = 12;
% Mesh Boundary Conditions and Loads
BCLIndex = [10 2]';
NodeBC = [1 1 0
          1 2 0
          2 1 0
          3 1 0
          4 1 0
          5 1 0
          6 1 0
          7 1 0
          14 1 0
          15 1 0
          ];
NodeLoad = [8 1 3.75
            21 1 3.125];
% Mesh Material Properties
young = 10e5;
pois = .25;
thick = 1;
PSPS = 'n';
MateT = [young pois thick];
```

(e) Calculation of Nodal Forces for Problem #2:

```
clc;
%Calculation of the nodal forces for Problem#2

%Since we know that the shape function varies linearly between
two
%nodes we can directly carry out the integrals.

syms y
f=15*y;

% 1) Using Mesh 1
N=[1-y y];
f_nodal=int(N'*f,y,0,1);

% 2) Using Mesh 2
N12=y*2;N11=2*(0.5-y);
N21=2*(1-y);N22=2*(y-0.5);

N1=[N11 N12];
N2=[N21 N22];

f1_nodal=int(N1'*f,y,0,0.5);
f2_nodal=int(N2'*f,y,0.5,1);
```