1 Newton-Raphson Method

- Begin a general step (n). At this stage we have $||\mathbf{R_{step}}^i||$
- Set the iteration counter to an initial value of zero (i=0)
- Using the previously calculated residual, we ascertain Δbfd^i using the expression:

$$\mathbf{K}(\mathbf{\Delta d}^i) = \mathbf{R^i}$$

  . It is to be noted that for the first iteration of a particular step, the residual value at the end of the previous step is used to calculate the consistent tangent.
- Update $\mathbf{d^i}$ as:

$$\mathbf{d^{i+1}} = \mathbf{d^i} + \mathbf{\Delta d^i}$$

- Calculation of Strains, in both the portions of the bar, naturally follows the computation of $\mathbf{d^i}$
- A check is performed, to ascertain whether either of the portions are in the elastic regime or plastic regime, and accordingly determine updated $\mathbf{K_a}$ and/or $\mathbf{K_b}$ and $\mathbf{K} = \mathbf{K_a} + \mathbf{K_b}$
- Calculate $||\mathbf{R_{step}}^i||$ and the check for the convergence criteria. If satisfied move to the next step. If not, the iterate further in the same step.

2 Modified Newton Method

The first line of difference between the Newton-Raphson method and the Modified-Newton Method is that the $\mathbf{K}$ remains constant throughout the step for the former.
The procedure for the latter is summarized as follows:

- Entering a particular step (n). At this stage we have $||\mathbf{R_{step}}^i||$
- Set the iteration counter to an initial value of zero (i=0)
- Using the previously calculated residual, we ascertain $\Delta \mathbf{d}^i$ using the expression:

$$\mathbf{K}(\mathbf{\Delta d}^i) = \mathbf{R^i}$$

  . It is to be noted that for the first iteration of a particular step, the residual value at the end of the previous step is used to calculate the consistent tangent.
- This $\mathbf{K}$ value remains constant throughout the step.
- This is followed by strain, and consequently force, calculations. It is ascertained first whether each of the portions of the bar are in elastic/plastic regions.
- Calculation of the Residual: $\mathbf{R^{i+1}} = \mathbf{F_{ext}} - \mathbf{\Sigma}\ \mathbf{F_{int}}^{\mathbf{i+1}}$
- Check for the Residual: $||\mathbf{R^{i+1}}|| \leqslant \varepsilon ||\mathbf{R^0}||$
- If the residual step is not satisfied the update: $\mathbf{d^i}$ as:

$$\mathbf{d^{i+1}} = \mathbf{d^i} + \mathbf{\Delta d^i}$$

  else move to the next step.

3 Comparison between the Newton-Raphson and Modified Newton

The following section reiterates the comparison between Newton-Raphson and Modified Newton-Method, as observed through the Q-3 in the Assignment.

- The problem begins in the Step 2. This is a nonlinear step and hence, unlike Step 1, which is linear, the value of residual obtained at the end of the Step 2, and further, is nonzero for Modified Newton with $\varepsilon = 10^{-12}$.
- Whereas in case of the Newton-Raphson Method the solution converges exactly after 2 steps and the value of Residual obtained at the end of the second step is exactly equal to zero. The value of displacement obtained in this case is $d = 1.9803922x10^{-2}$
- This can be attributed to the fact that the force-displacement curve is bi-linear and hence the value of updated tangent is exactly equal to the tangent-2 in the force-displacement curve.
- Contrary to what is observed above, the Modified-Newton Method takes 68 iterations to converge to a value less than the tolerance desired, and still is not exactly equal to zero.
- This can be attributed to the fact that the value of initial tangent can never be equal to the tangent-2, effectively saying $\mathbf{E} \neq \mathbf{E_T}$
- It can be concluded that the Newton-Raphson Method, gives much faster convergence. For more complicated and strongly nonlinear problems, the Modified-Newton Method would eventually prove to be computationally efficient, because it would save time for updating the stiffness matrix.