Q2) For step 1, we have $F_{1}^{ext} = 2\times10^4$ N. We got $d_1 = 6.6667\times10^{-3}$ cm.

For step 2, $[F_2^{ext} = 4\times10^4]$

$$R^{(0)} = F_2^{ext} - [F_{a1}^{int} + F_{b1}^{int}] = 2\times10^4 \text{ N}.$$

K_a, K_b are calculated on basis of Finternal in a and b at end of step 1. Since, these segments remain elastic at end of step 1, we use $K_a = \frac{AE_e}{L_a} = 10^6$ N/cm and $K_b = \frac{AE_e}{L_b} = 2\times10^6$ N/cm². Since we are using modified newton, we keep $K = K_a + K_b = 3\times10^6$ N/cm² constant throughout the step.

$$(K_a + K_b)\,\Delta d^{(0)} = R^{(0)} \longrightarrow \Delta d^{(0)} = 6.667\times10^{-3}$$

Iteration 1:- $\longrightarrow d_2^{(1)} = d_2^{(0)} + \Delta d_2^{(0)} = \boxed{1.333\times10^{-2} \text{ cm}}$.

$\|R^{(1)}\| = 6600$ N

$\Delta d_2^{(1)} = 2.2\times10^{-3}$ cm.

Iteration 2:-

$d_2^{(2)} = d_2^{(1)} + \Delta d_2^{(1)} = \boxed{1.5533\times10^{-2} \text{ cm.}}$

$\|R^{(2)}\| = 4356$ N

Residual check.

$\|R^{(2)}\| = 4356$ N, $\quad \varepsilon\,\|R^{(1)}\| = 0.1\times6600 \text{ N} = 660$ N.

Since $\|R^{(2)}\| \not< \varepsilon\,\|R^{(1)}\| \rightarrow$ convergence not satisfied ∴ calculate $\Delta d_2^{(2)}$

[Note:- in previous notes, prof. had used $\|R^{(i+1)}\| < \varepsilon\,\|R^{(0)}\|$, to define the convergence check. however in this example he used $\varepsilon\,\|R^{(1)}\|$ to define convergence condition and the same has been used for this example]

$\Delta d_2^{(2)} = 1.4521\times10^{-3}$ cm

The above iterations were completed in class, ~~and~~ detailed working is now provided from iteration 3.

Iteration 3.

$$d_2^{(3)} = d_2^{(2)} + \Delta d_2^{(2)} = 1.5533 \times 10^{-2} + 1.4521 \times 10^{-3} = \boxed{1.6985 \times 10^{-2}\ cm}$$

Note: $\varepsilon_{b_2}^{(3)}$ is magnitude of compressive strain

strain: $\varepsilon_{a_2}^{(3)} = \dfrac{d_2^{(3)}}{L_a} = \dfrac{1.6985 \times 10^{-2}}{10} = 1.6985 \times 10^{-3} < \varepsilon_y$ [elastic] [Tensile]

$\varepsilon_{b_2}^{(3)} = \dfrac{d_2^{(3)}}{L_b} = \dfrac{1.6985 \times 10^{-2}}{5} = 3.3971 \times 10^{-3} > \varepsilon_y$ [plastic] [Compressive]

Note: $F_{b_2}^{int(3)}$ is magnitude of compressive internal force.

Fint: $F_{a_2}^{int(3)} = A\,E_e\,\varepsilon_{a_2}^{(3)} = 1 \times 10^7 \times 1.6985 \times 10^{-3} = 1.6985 \times 10^4\ N$ [Tensile]

$F_{b_2}^{int(3)} = [E_p(\varepsilon_{b_2}^{(3)} - \varepsilon_y) + \sigma_y]\,A$ [since it is plastic]

$= [10^5(3.3971 \times 10^{-3} - 2 \times 10^{-3}) + 2 \times 10^4]\,1 = 2.01397 \times 10^4\ N$ [Compressive]

Residual check.

$\|R^{(3)}\| = F_2^{ext} - [F_{a_2}^{int(3)} + F_{b_2}^{int(3)}]$ [In reality, it should be $-F_{b_2}^{int(3)}$, but since we are considering only algebraic value we wrote $+F_{b}^{int(3)}$]

$= 4 \times 10^4 - [1.6985 \times 10^4 + 2.01397 \times 10^4]$

$= \underline{2874.96\ N}$.

$\varepsilon \|R^{(1)}\| = 660\ N$

$\|R^{(3)}\| \not< \varepsilon \|R^{(1)}\|$ [Residual check not satisfied]

$\therefore$ calculate $\Delta d_2^{(3)}$;

$(K_a + K_b)_2\ \Delta d_2^{(3)} = \|R^{(3)}\|$

$\therefore 3 \times 10^6\ \Delta d_2^{(3)} = 2874.96$

$\therefore \Delta d_2^{(3)} = 9.5832 \times 10^{-4}\ cm$.

Iteration 4.

$$d_2^{(4)} = d_2^{(3)} + \Delta d_2^{(3)} = 1.6985 \times 10^{-2} + 9.5832 \times 10^{-4} = \boxed{1.7944 \times 10^{-2} \text{ cm}}$$

strain: $\varepsilon_{a_2}^{(4)} = \dfrac{d_2^{(4)}}{L_a} = \dfrac{1.7944 \times 10^{-2}}{10} = 1.7944 \times 10^{-3} < \varepsilon_y$ [elastic]

$\varepsilon_{b_2}^{(4)} = \dfrac{d_2^{(4)}}{L_b} = \dfrac{1.7944 \times 10^{-2}}{5} = 3.5887 \times 10^{-3} > \varepsilon_y$ [plastic]

Fint :- $F_{a_2}^{int(4)} = A E_e \varepsilon_{a_2}^{(4)} = 1 \times 10^7 \times 1.7944 \times 10^{-3} = 1.7944 \times 10^4 N$

$F_{b_2}^{int(4)} = [E_p(\varepsilon_{b_2}^{(4)} - \varepsilon_y) + \sigma_y] A$ [since it is plastic]

$= [10^5 (3.5887 \times 10^{-3} - 2 \times 10^{-3}) + 2 \times 10^4] \times 1$

$= 2.015887 \times 10^4 N$

Residual check.

$\| R^{(4)} \| = F_2^{ext} - [F_{a_2}^{int(4)} + F_{b_2}^{int(4)}]$

$= 4 \times 10^4 - [1.7944 \times 10^4 + 2.015887 \times 10^4]$

$= \underline{1897.474 N}$.

$\varepsilon \| R^{(1)} \| = 660 N$.

$\| R^{(4)} \| \not< \varepsilon \| R^{(1)} \|$ [Residual check not satisfied].

$\therefore$ calculate $\Delta d_2^{(4)}$.

$(k_a + k_b)_2 \, \Delta d_2^{(4)} = \| R^{(4)} \|$.

$3 \times 10^6 \quad \Delta d_2^{(4)} = 1897.474$

$\Delta d_2^{(4)} = 6.3249 \times 10^{-4}$ cm.

Iteration 5

$$d_2^{(5)} = d_2^{(4)} + \Delta d_2^{(4)} = 1.7944 \times 10^{-2} + 6.3249 \times 10^{-4} = \boxed{1.8576 \times 10^{-2}\ cm}$$

strain:- $\varepsilon_{a_2}^{(5)} = \dfrac{d_2^{(5)}}{L_a} = \dfrac{1.8576 \times 10^{-2}}{10} = 1.8576 \times 10^{-3} < \varepsilon_y$ [elastic]

$\varepsilon_{b_2}^{(5)} = \dfrac{d_2^{(5)}}{L_b} = \dfrac{1.8576 \times 10^{-2}}{5} = 3.7152 \times 10^{-3} > \varepsilon_y$ [plastic]

F^{int} :- $F_{a_2}^{int(5)} = A\, E_e\, \varepsilon_{a_2}^{(5)} = 1 \times 10^7 \times 1.8576 \times 10^{-3} = 1.8576 \times 10^4\ N$

$F_{b_2}^{int(5)} = \left[E_p\left(\varepsilon_{b_2}^{(5)} - \varepsilon_y\right) + \sigma_y\right]A$ [since it is plastic]

$= \left[10^5\left(3.7152 \times 10^{-3} - 2 \times 10^{-3}\right) + 2 \times 10^4\right] \times 1$

$= 2.017152 \times 10^4\ N.$

Residual check.

$\| R^{(5)} \| = F_2^{ext} - \left[F_{a_2}^{int(5)} + F_{b_2}^{int(5)}\right]$

$= 4 \times 10^4 - \left[1.8576 \times 10^4 + 2.017152 \times 10^4\right]$

$= \underline{1252.333\ N.}$

$\varepsilon \| R^{(1)} \| = 660\ N.$

$\| R^{(5)} \| \not< \varepsilon \| R^{(1)} \|$ [Residual check not satisfied].

calculate $\Delta d_2^{(5)}$

$(k_a + k_b)_2\ \Delta d_2^{(5)} = \| R^{(5)} \|$

$3 \times 10^6\ \Delta d_2^{(5)} = 1252.333$

$\Delta d_2^{(5)} = 4.17444 \times 10^{-4}\ cm.$

Iteration 6

$d_2^{(6)} = \boxed{1.8994 \times 10^{-2}\, cm}$

strain: $\varepsilon_{a_2}^{(6)} = 1.8994 \times 10^{-3} < \varepsilon_y$ [elastic] $\varepsilon_{b_2}^{(6)} = 3.4987 \times 10^{-3} > \varepsilon_y$ [plastic]

F^{int}:- $F_{a_2}^{int(6)} = 1.8994 \times 10^4 N$, $F_{b_2}^{int(6)} = 2.017987 \times 10^4 N$

$\|R^{(6)}\| = 826.5395\, N$.

$\|R^{(6)}\| \not< \varepsilon \|R^{(1)}\|$ [Residual check not satisfied].

~~ε~~ $\Delta d_2^{(6)} = 2.75513 \times 10^{-4}\, cm$.

Iteration 7:

$d_2^{(7)} = \boxed{1.9269 \times 10^{-2}\, cm}$ --- ANS.

strain: $\varepsilon_{a_2}^{(7)} = 1.9269 \times 10^{-3} < \varepsilon_y$ [elastic] $\varepsilon_{b_2}^{(7)} = 3.8538 \times 10^{-3} > \varepsilon_y$ [plastic]

F^{int}:- $F_{a_2}^{int(7)} =$ ~~3.853~~ $1.9269 \times 10^4 N$, $F_{b_2}^{int(7)} = 2.018538 \times 10^4 N$

$\|R^{(7)}\| = 545.5161\, N$.

$\|R^{(7)}\| < \underbrace{\varepsilon \|R^{(1)}\|}_{660 N}$ [Residual check satisfied !!]

Step 2.

Iteration no.	$d_2^{(i)}$	$R^{(i)}$
0	$6.6667 \times 10^{-3}\, cm$	$2 \times 10^4\, N$
1	$1.333 \times 10^{-2}\, cm$	6600 N
2	$1.5533 \times 10^{-2}\, cm$	4356 N
3	$1.6985 \times 10^{-2}\, cm$	2874.96 N
4	$1.7944 \times 10^{-2}\, cm$	1897.474 N
5	$1.8576 \times 10^{-2}\, cm$	1252.333 N
6	$1.8944 \times 10^{-2}\, cm$	826.5395 N
7	1.9269×10^{-2}	545.5161 N $< 6600 \times 0.1$ ✓

ANS:- For $\varepsilon = 0.1$, we reach convergence after 7 iterations, with $d_2 = 1.9269 \times 10^{-2}\, cm$.