What is Algorithm ?

→ finite set of steps to solve a problem is called algorithm.
→ Analysis is process of comparing two algos w.r.t time, space etc.

Analysis

- priory → Approximate value → machine independent
- posterior → exact value → machine dependent

S1 : Read A
S2 : Read B
S3 : sum = A+B
S4 : print (sum)

```
{
int a=1;     → infinite loop.
while (1)
a=a+1;
}
}
```

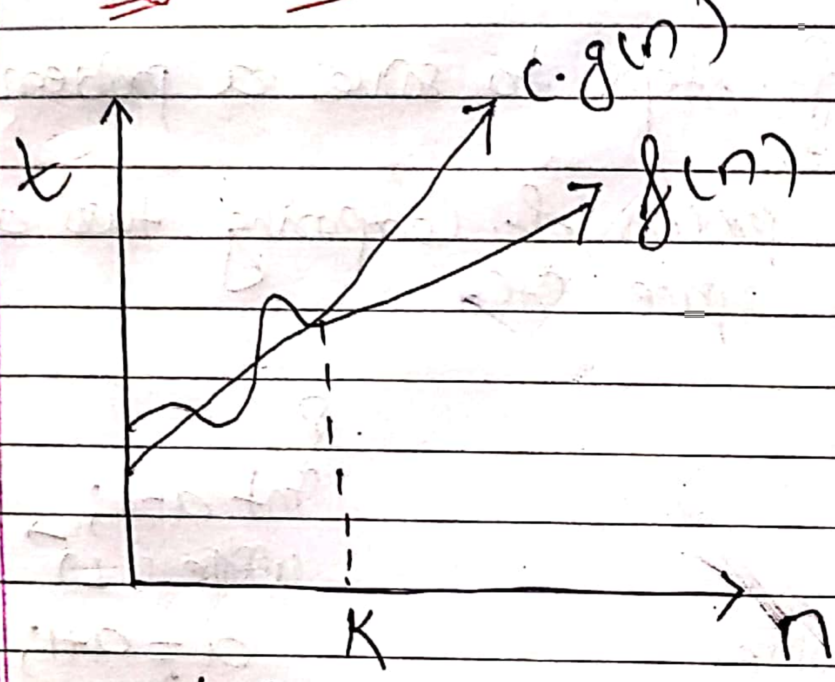
Asymptotic notation

(I) Big-oh (O)
(II) Big-omega (Ω)
(III) Theta (θ)

1) Big - Oh (O)

The function $f(n) = O\,g(n)$ iff $\exists$ +ve constant c & n_0 s.t.:

$$f(n) \le c * g(n) \;\forall\; n \ge n_0$$



$$f(n) = O\,g(n)$$
$$f(n) \le c \cdot g(n)$$
$$c > 0$$
$$n \ge K$$
$$K \ge 0$$

→ worst case

→ upper Bound (At most)

for e.g :- $f(n) = 2n^2 + n$

$$f(n) = O\,g(n)$$

$$2n^2 + n \le c \cdot g(n^2)$$

∴ take $c = 2$

$$2n^2 + n \le 2 \cdot n^2 \quad ✗$$

$$2n^2 + n \le 3 \cdot n^2 \quad \therefore O(n^2)$$

$$n \le n^2$$

$$\boxed{n \ge 1}$$

$$2 \times (5)^2 + 5 \le 3 \cdot 5^2$$
$$55 \le 75$$

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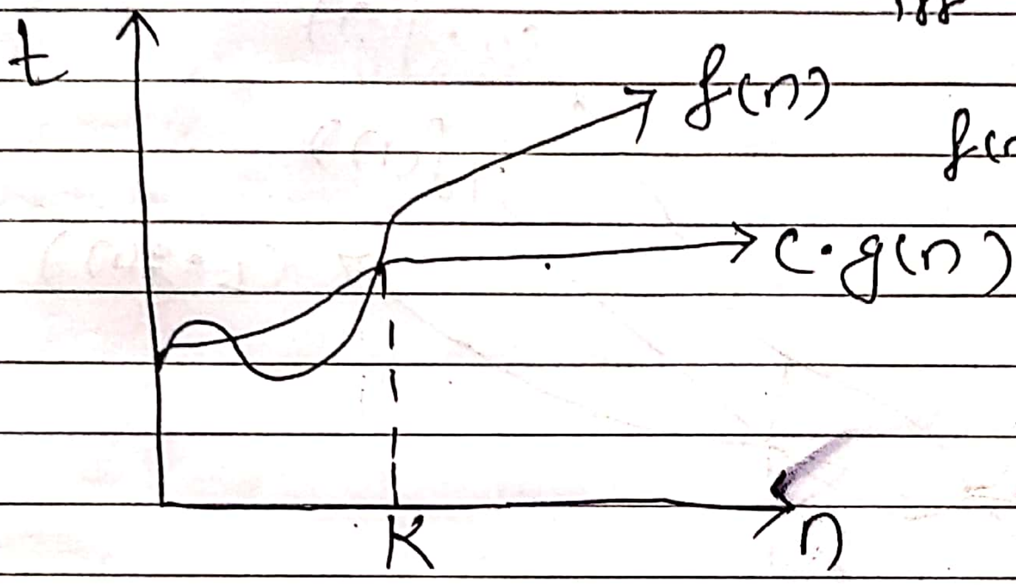
2) Big - omega (Ω)

The function $f(n) = \Omega\, g(n)$ iff $\exists$ +ve constant c & n_0 st: $f(n) \geq c \cdot g(n)\ \forall n \geq n_0$



→ Best Case

→ Lower Bound (At least)

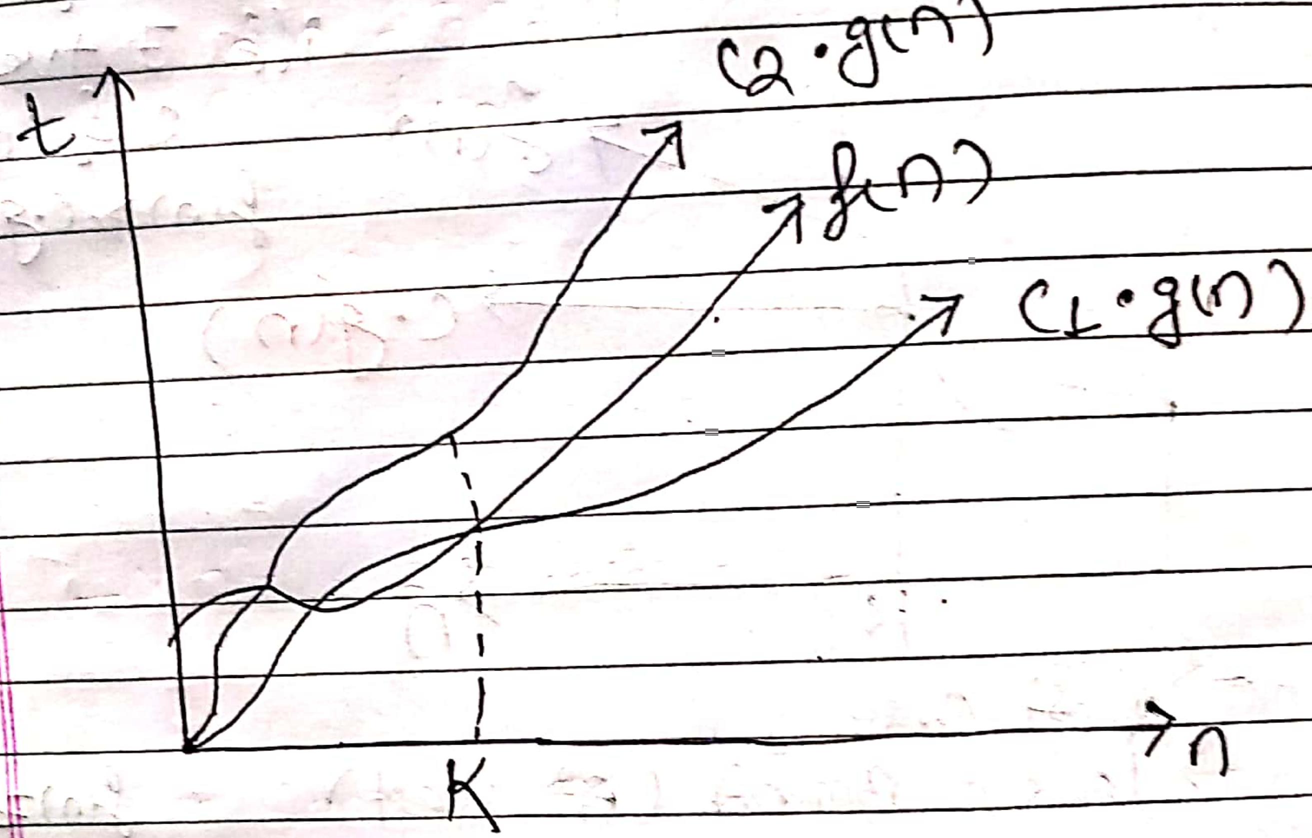
$f(n) = 2n^2 + n$

$$f(n) = \Omega\, g(n)$$
$$f(n) \geq c \cdot g(n)$$

$$2n^2 + n \geq c \cdot n^2 \quad \text{if } c = 2.$$
$$2n^2 + n \geq 2 \cdot n^2$$

$\therefore \Omega(n^2)$ always True.

3) Theta (Θ)



→ Average Case
→ Exact Case

$$c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$$

$$f(n) = 2n^2 + n.$$

$$c_1 \cdot n^2 \le 2n^2 + n \le c_2 \cdot n^2$$

$$2n^2 \le 2n^2 + n \le 3n^2 \qquad \therefore \Theta(n^2)$$

The function $f(n) = \Theta(g(n))$ iff $\exists$ +ve constant c_1, c_2 & n_0. such that,

$$c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$$

Eg $f(n) = 2n+3$

$$1 \times n \le 2n+3 \le 5 \times n$$
$$c_1\, g(n) \quad f(n) \quad c_2\, g(n)$$

✓ $\therefore f(n) = \theta(n)$

X $\therefore f(n) = \theta(n^2)$

$$\boxed{1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \dots < 2^n < 3^n < \dots < n^n}$$

lower Bound — 1; Average Bound — n; upper Bound — 2^n

Eg :-

$f(n) = 2n^2 + 3n + 4$

$$2n^2+3n+4 \le 2n^2+3n^2+4n^2$$
$$\underset{f(n)}{2n^2+3n+4} \le \underset{c}{9}\, \underset{g(n)}{n^2}$$

$f(n) = O(n^2)$

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$f(n) = 2n^2+3n+4$

$2n^2+3n+4 \geq 1 \times n^2$

$f(n)$ $\quad c \quad g(n)$

$f(n) = \Omega(n^2)$

$1 \times n^2 \leq 2n^2+3n+4 \leq 9n^2$

$f(n) = \Theta(n^2)$

E.g

$f(n) = n^2 \log n + n$

$1 \times n^2 \log n \leq n^2 \log n + n \leq 10\, n^2 \log n$

$O(n^2 \log n)$ $\quad \Omega(n^2 \log n)$ $\quad \Theta(n^2 \log n)$

E.g. $f(n) = n! = n \times (n-1) \times \cdots \times 3 \times 2 \times 1.$
$= 1 \times 2 \times 3 \times \cdots \times n$

$1 \times 1 \times 1 \times \cdots \times 1 \leq 1 \times 2 \times 3 \times \cdots \times n \leq n \times n \times n \times \cdots \times n$

$$\underbrace{1 \leq n! \leq n^n}_{\text{diff.}}$$

Here, $\Omega(1)$ $O(n^n)$
L.H.S $\neq$ R.H.S so we can't write average Bound. so, Θ is not possible.

E.g $f(n) = \log n!$

$\log(1 \times 1 \times 1 \times \cdots 1) \leq \log(1 \times 2 \times 3 \times \cdots \times n) \leq \log(n \times n \times \cdots \times n)$

$1 \leq \log n! \leq \log n^n$
$$\underbrace{1 \leq \log n! \leq n \log n}_{\text{not same.}}$$

So, we can't write 'Θ'.

$\Omega(1)$, $O(n \log n)$

Properties of Asymptotic Notations

General properties

- If $f(n)$ is $O(g(n))$ then $a*f(n)$ is $O(g(n))$.

  Eg :- $f(n) = 2n^2+5$ is $O(n^2)$.
  then,
  $$7 \cdot f(n) = 7(2n^2+5)$$
  $$= 14n^2+35 \text{ is } O(n^2).$$

and same for Ω and Θ.

Reflexive

if $f(n)$ is given then $f(n)$ is $O(f(n))$

Eg $f(n) = n^2$ $O(n^2)$

Transitive

if $f(n)$ is $O(g(n))$ & $g(n)$ is $O(h(n))$ then,
$$f(n) = O(h(n))$$

Eg $f(n) = n$ $g(n) = n^2$ $h(n) = n^3$

n is O(n3) and n^2 is O(n3) then n is O(n3).

Symmetric

if f(n) is $\Theta(g(n))$ then g(n) is $\Theta(f(n))$

It is only true for Θ notation.

Eg $f(n) = n^2$ $g(n) = n^2$

$f(n) = \Theta(n^2)$

$g(n) = \Theta(n^2)$

Transpose Symmetric

if $f(n) = O(g(n))$ then g(n) is $\Omega(f(n))$

Eg $f(n) = n$ $g(n) = n^2$

n is $O(n^2)$ and

n^2 is $\Omega(n)$.

Eg

if $f(n) = O(g(n))$

& $f(n) = \Omega(g(n))$

$g(n) \le f(n) \le g(n)$

$\therefore f(n) = \Theta(g(n))$

if $f(n) = O(g(n))$
$d(n) = O(e(n))$
then $f(n) + d(n) = O(\max(g(n), e(n)))$

Eg $f(n) = n = O(n)$
$d(n) = n^2 = O(n^2)$

$f(n) + d(n) = n + n^2 = O(n^2)$

if $f(n) = O(g(n))$
& $d(n) = O(e(n))$
then, $f(n) * d(n) = O(g(n) * e(n))$

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