Big Data and Economics

Causal Inference

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Bates College | ECON/DCS 368

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Prologue

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- We see in the Opportunity Atlas that neighborhood income mobility is correlated with many outcomes
- But are any of these correlations causal?
- If so, we should be able to change neighborhood characteristics to change outcomes
- Problem set 3 has been posted to the course website, get started early!

Hack-a-thon update

- Lewiston's Department of Economic Development has been in touch about the hack-a-thon
- We'll be helping the city with a project to understand where to put street lights to help reduce crime
- Two options for scheduling hack-a-thon
    - Last week of class - kick off Tuesday (4/9), team presentations Thursday (4/11), participation optional
    - Last two weeks of class - kick off Tuesday (4/2), team presentations Thursday (4/11) and converted to your last problem set, so no longer optional
- I'll poll Thursday to see what works best for everyone

Goals today

1. Separate causality and correlation
2. Discuss common challenges to establishing causality
3. Discuss approaches and assumptions to establish causality
    - Control for all unobserved variables correlated with treatment
    - Use treatment that is truly random
    - Something between these two

Warning

- This causality stuff is **really** tricky
- A causal paper may be intuitive -- that means it is a great paper, but finding your own intuitive causal relationship in the wild is hard
- Beyond intuition, the math and statistics are also hard
  - There are many interrelated frameworks to put some structure on the problem
  - Connections between frameworks can be hard to see and sometimes not particularly illuminating at first
- Be patient and comfortable with the fact that you won't understand everything at first, second, third, or even when you're trying to teach the material†

† cough, cough --- me

Attribution

- These slides are adapted from work by Ed Rubin and Nick Huntington-Klein

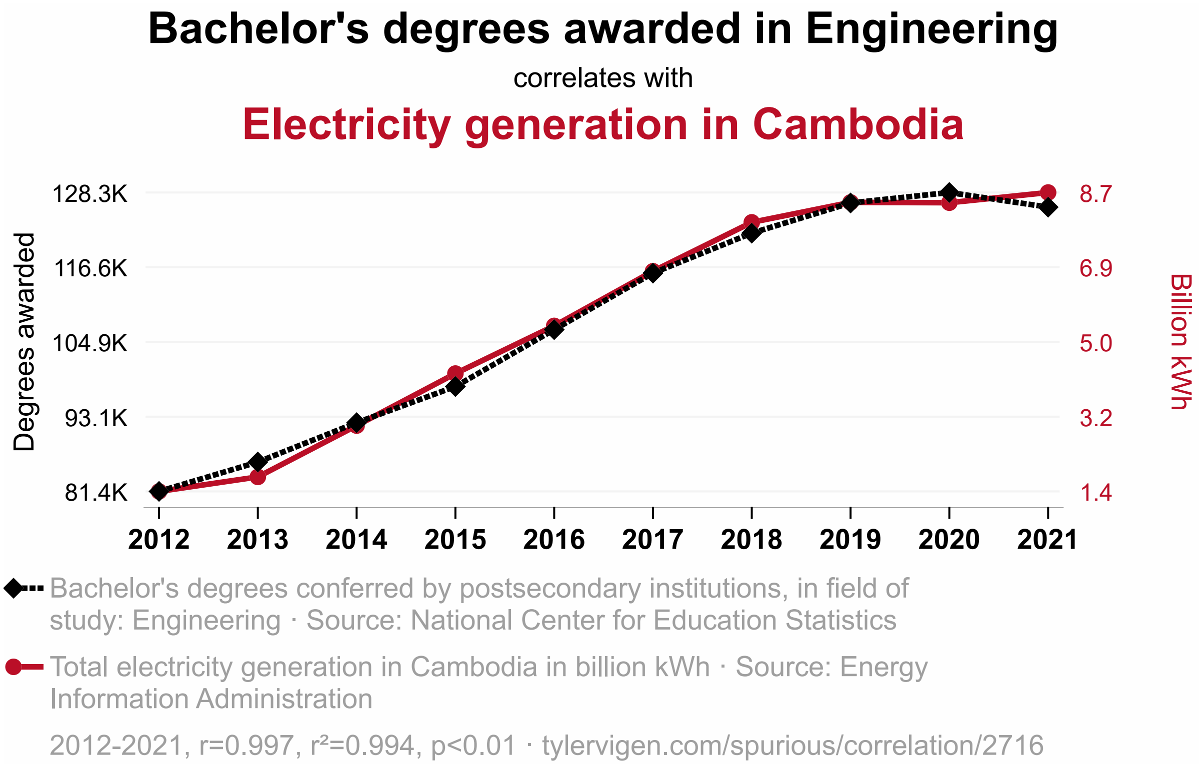
- They're both superb econometric instructors and I highly recommend their work

Correlation vs. Causation

Spurious Correlations

- Everyone submit a correlation that you know about in the world
- If you're not sure, please check out this delightful https://www.tylervigen.com/spurious-correlations

Spurious correlation and bad policy



Someone with this graph argues Cambodia should disincentivize engineering to fight climate change. Does that make sense?

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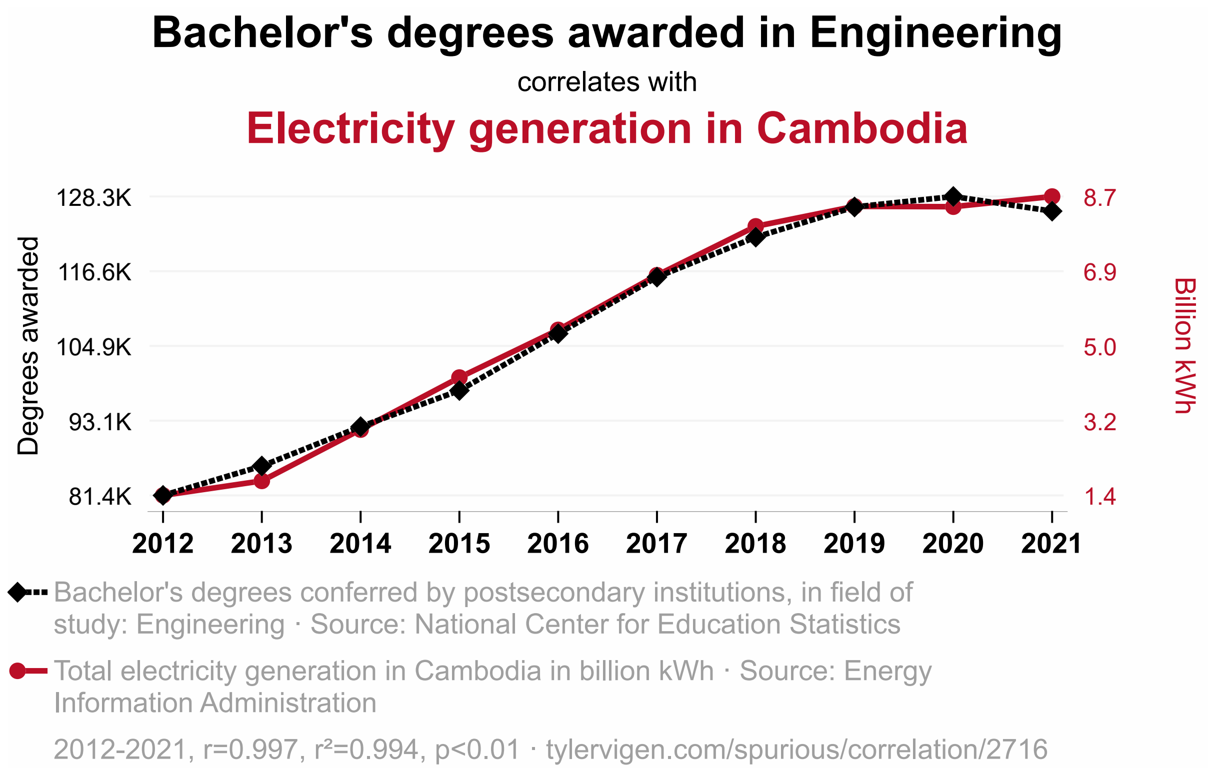

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No! But this is why this matters. One nice-looking correlation plus a bad actor = very bad policy.

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> Correlation is not causation.

- What does it mean?

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New saying:

> Correlation plus **exogeneity** is causation.

- Today we're going to unpack this a bit to kick off a unit on causal inference methods

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Generally, ***causal*** relationships are complex and challenging to answer, *e.g.*,

- Does job growth cause higher economic mobility?
- What caused the capital riot?
- How does the number of police officers affect crime?
- What is the effect of better air quality on test scores?
- Do tariffs reduce the amount of trade?
- How did cannabis legalization affect mental health/opioid addiction?

Non-causal correlations

Examples of non-zero *correlations* that are not *causal* (or may be causal in the other direction!)

Some obvious:

- People tend to wear shorts on days when ice cream trucks are out
- Rooster crowing sounds are followed closely by sunrise*

Some less obvious:

- Colds tend to clear up a few days after you take Emergen-C
- The performance of the economy tends to be lower or higher depending on the president's political party

Find more at https://www.tylervigen.com/spurious-correlations

*This case of mistaken causality is the basis of the film Rock-a-Doodle which I understand is extremely entertaining.

So what is causality?

- We say that `X` *causes* `Y` if...
- Were we to intervene and *change* the value of `X` without changing anything else...
- then `Y` would also change as a result

Important Note

- "X causes Y" *doesn't* mean that X is necessarily the *only* thing that causes Y
- And it *doesn't* mean that all Y must be X
- For example, using a light switch causes the light to go on
- But not if the bulb is burned out (no Y, despite X), or if the light was already on (Y without X), and it ALSO needs electricity (something else causes Y)
- But still we'd say that using the switch causes the light! The important thing is that X *changes the distribution* of Y, not that it necessarily makes it happen for certain

Prediction vs. causation

Most tasks in econometrics boil down to one of two goals:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + u$$

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1. **Prediction:** Accurately and dependably predict/forecast y using on some set of explanatory variables—doesn't need to be x_1 through x_k. Focuses on $\hat{y}$. β_j doesn't really matter.

2. **Causal inference:**[†] Estimate the true, population model that explains how y changes when we change x_j—focuses on β_j. Accuracy of $\hat{y}$ is not important. (So R^2 concerns can often take a hike.)

† Often called *causal identification*.

Why Causality?

- Many interesting questions to answer with data are causal
- Some are non-causal - for example, "how can we predict whether this photo is of a dog or a cat" is vital to how Google Images works, but it doesn't care what *caused* the photo to be of a dog or a cat
- Nearly every *why* question is causal and what we want to know!
- Also, this is economists' comparative advantage!
    - Plenty fields do statistics. But few make causal inference standard training for students
- This understanding of causality makes economists useful! *This* is one big reason why tech companies have whole economics departments

Fundamental Problem of Causal Inference

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Causal inference can be pretty difficult—both practically and econometrically.

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Many of these challenges relate to **exogeneity**, *i.e.*, $\boldsymbol{E}[u_i|X] = 0$.
Causality requires us to **hold all else constant** (*ceterus paribus*) on average, i.e.

- The amount our model misses the mark (u) is equally likely to be positive as negative, or unbiased

Fund. Problem of Causal Inference

- The econometric problems largely fall under the umbrella problem that is fundamental to causal inference

- In short, it is impossible to observe a treated unit in the counterfactual world where they were not untreated

- Unless your name is Evelyn Quan, Marty McFly, Loki, or Miles Morales, this sort of multiversal experimentation is not possible

- You're stuck with the rest of in 2024, using an extremely clever, but limited causal infernce toolbox that relies on **exogeneity**

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$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + u$$

- Let's break this equation down into its component parts
  - y is the outcome/dependent variable
  - x_k are the independent/explanatory variables
  - β_k are the coefficients on the explanatory variables
  - u is the error term: anything else that affects y that we didn't/couldn't include
- Formally, exogeneity means $\mathbb{E}[u_i|X] = 0$: in expectation ("on average") the error term is zero after controlling for all the x variables
- Intuition: anything we left out that explains y is uncorrelated with our x variables

Causal inference approaches

- So how can we get $\mathbb{E}[u_i|X] = 0$ to make a causal claim?

1. **Random assignment**: Randomly assign units to treatment/control
    - The treatment is completely exogenous by design
2. **Conditional independence assumption (CIA)**: Control for everything that could possibly affect y that is related x
    - The treatment is then "as good as random," but you can't prove it
    - Sometimes called "selection on observables" and is often a tough sell
3. **Natural/quasi experiments**: A treatment is not randomly assigned, but due to something that "as good as random" with respect to treatment
    - This is the bread and butter of applied microeconomics

Assumptions

- All causal inference tools require an assumption about the world
- Your goal is to pick the least objectionable assumption possible
- You **cannot** prove these assumptions, that's why they're assumptions
- You can potentially see whether other patterns in the data are consistent with your assumption
    - e.g. Check placebo outcomes like parent's income for those who do/do not win a school lottery
    - These tests will change depending on your assumption/question/topic

Selection on observables

Prince Charles and Ozzy Osbourne are very similar. Source: Andrew Heiss' Mastodon

Causation

Some examples

- Let's explore the three causal inference approaches with two simple examples

1. What is the effect of fertilizer on crop yield?

2. What is the effect of education on income mobility of those born at the 25th percentile of the income distribution?

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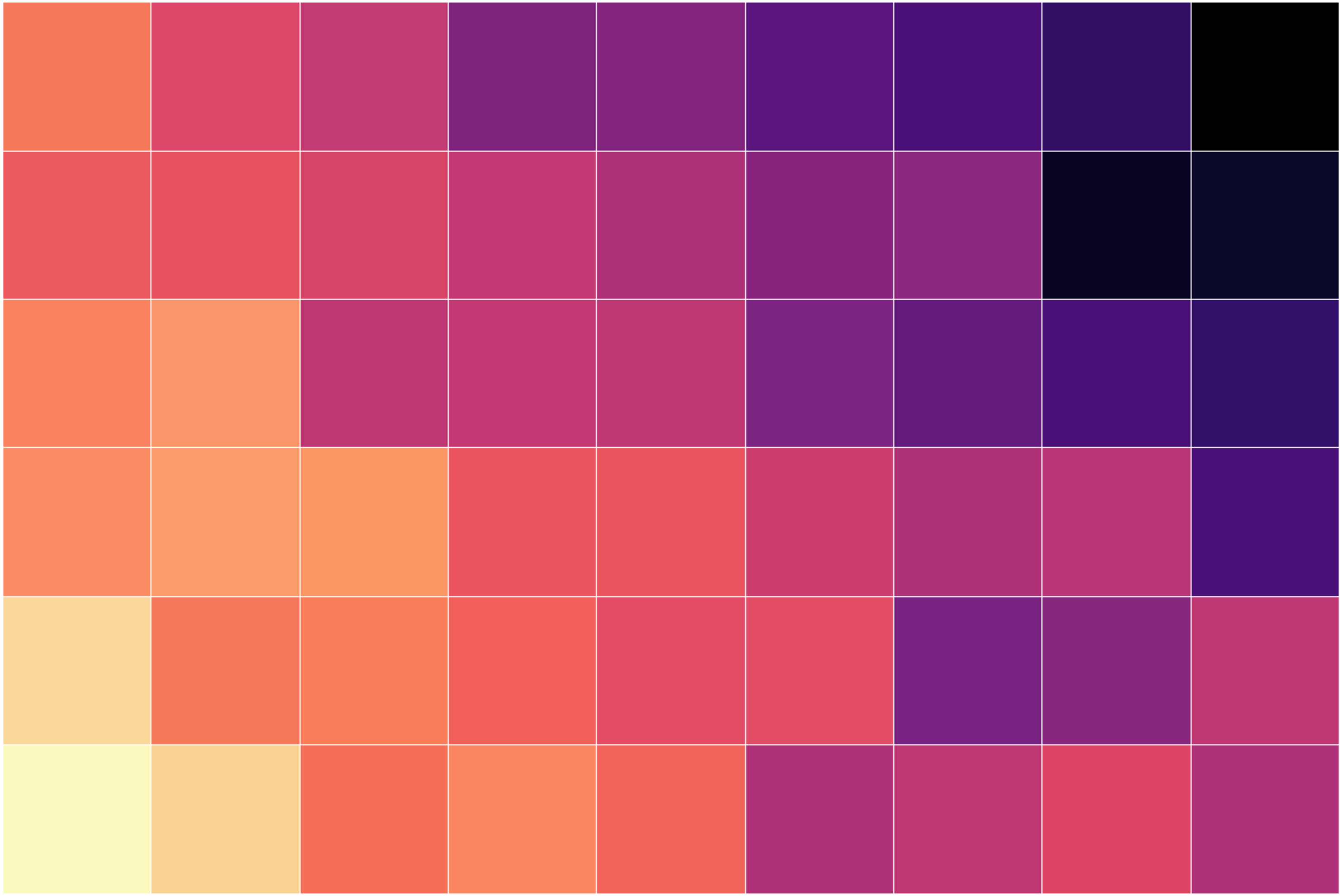
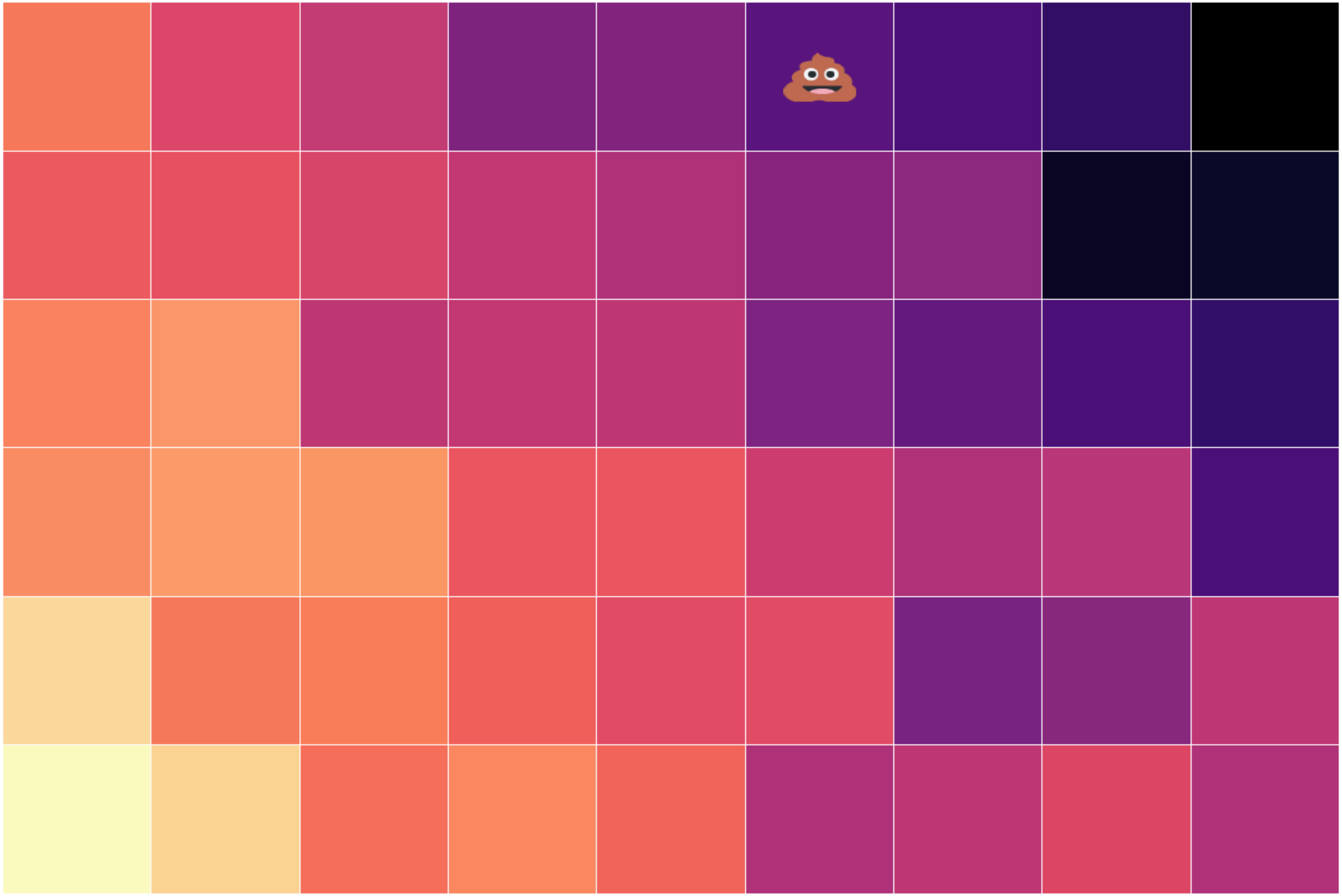
54 equal-sized plots

01	02	03	04	05	06	07	08	09
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54

54 equal-sized plots of varying quality



54 equal-sized plots of varying quality plus randomly assigned treatment



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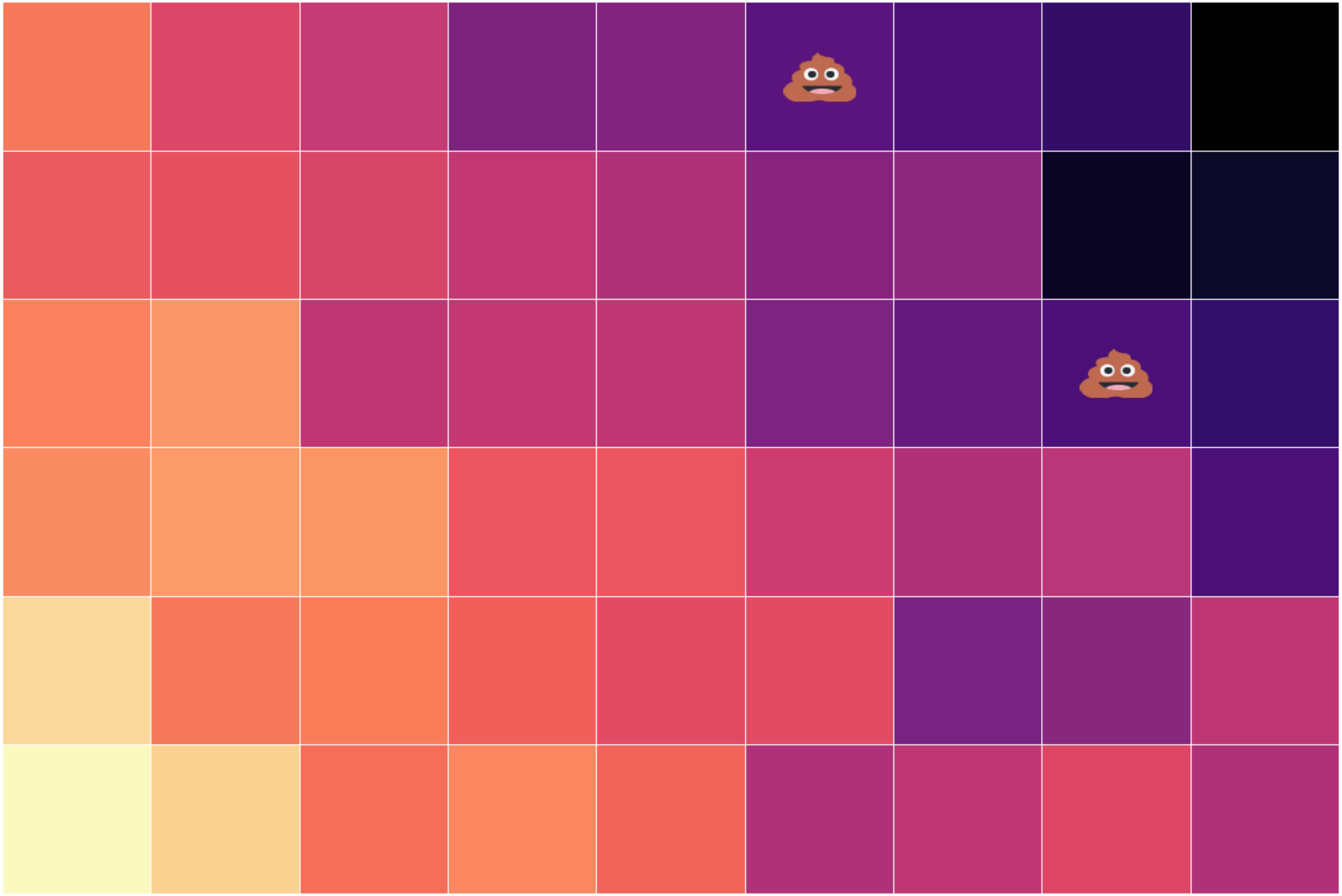

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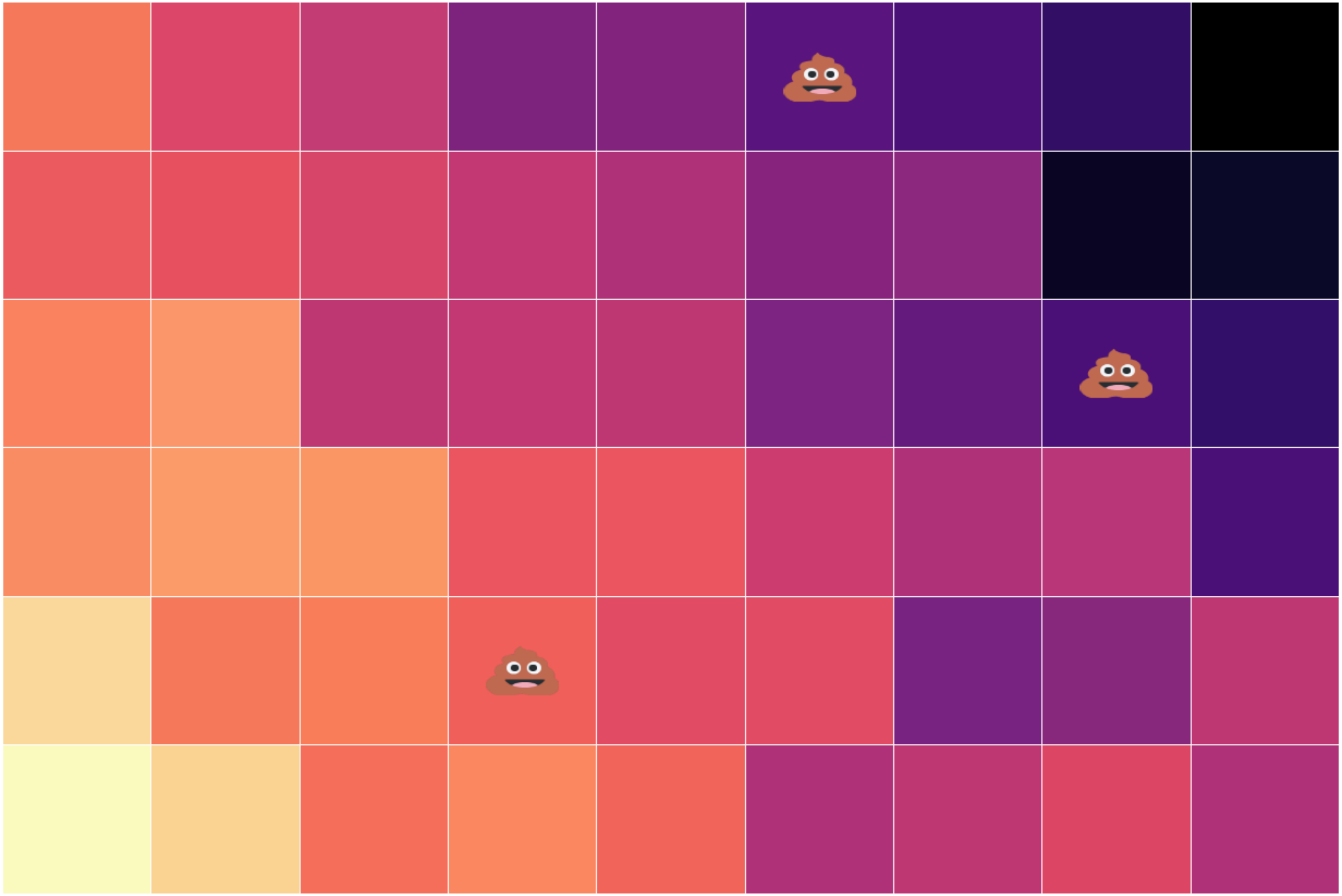

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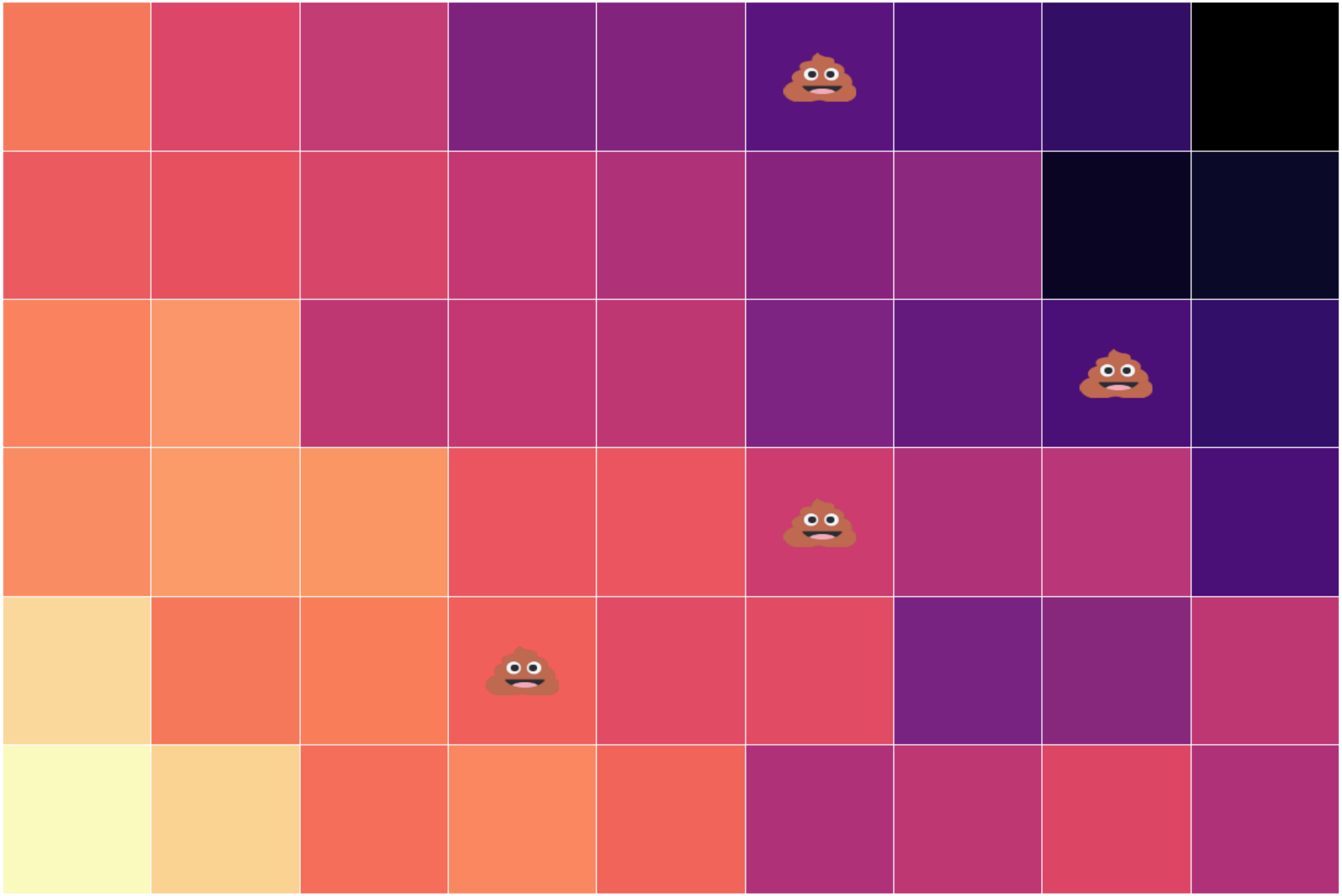

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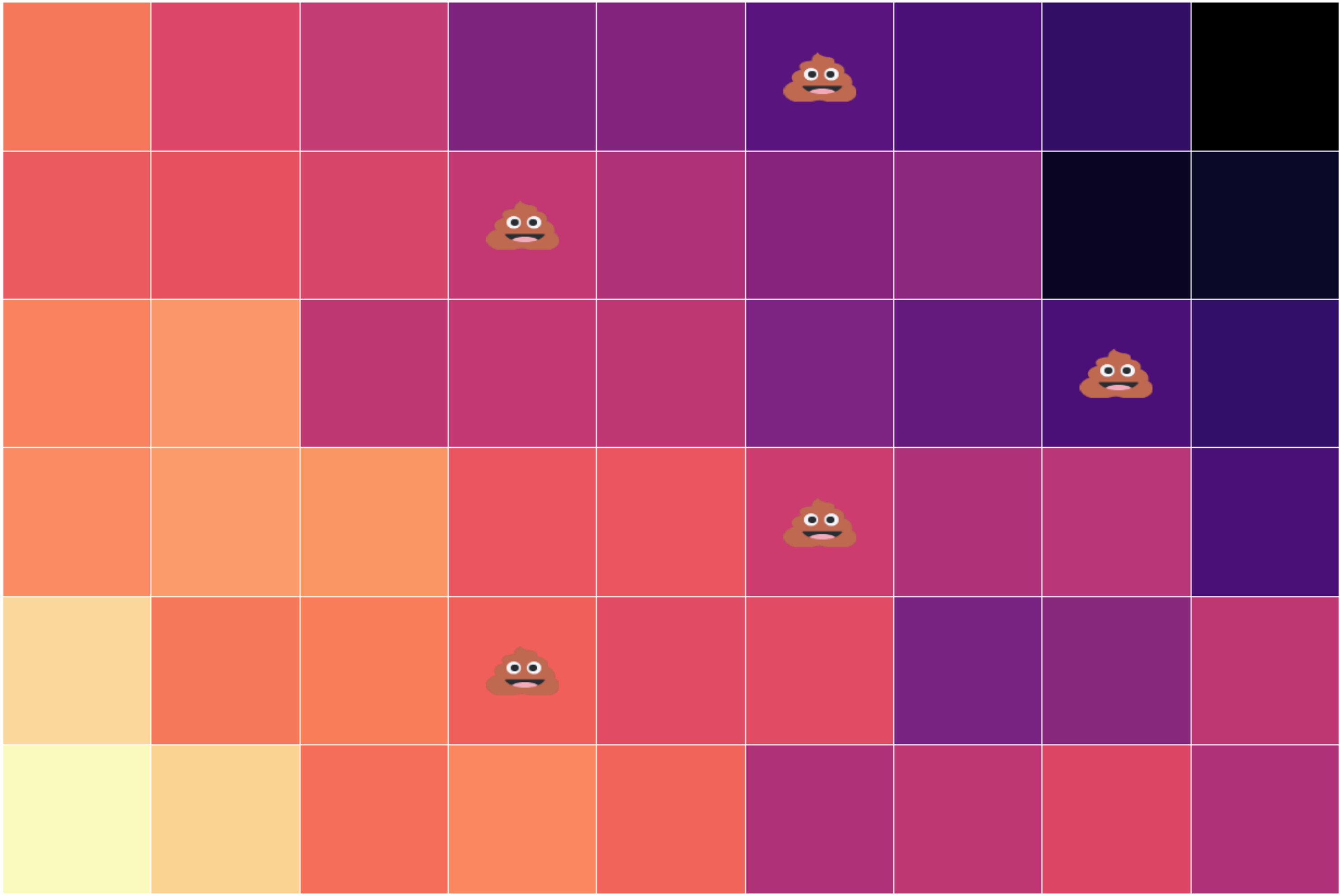

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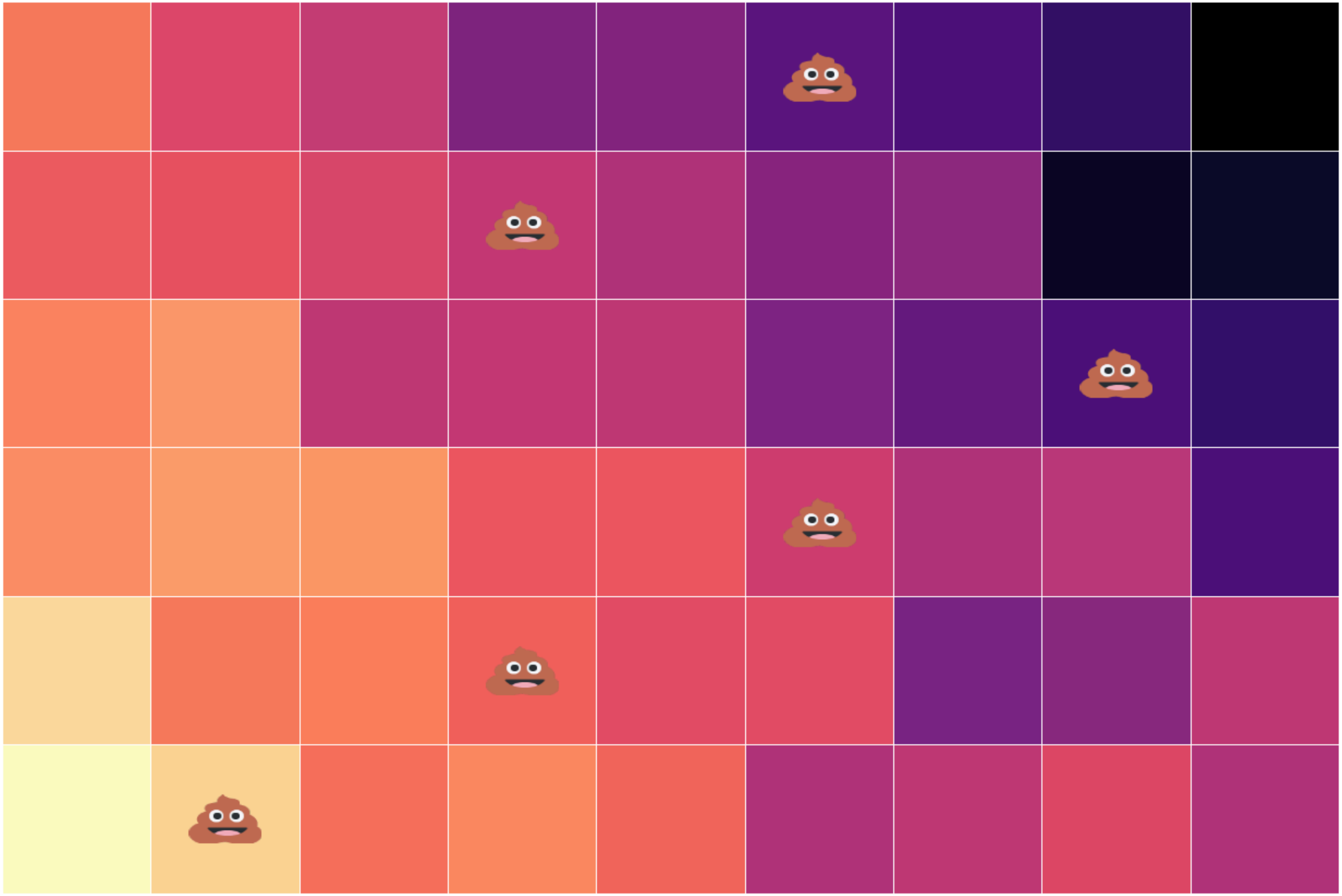

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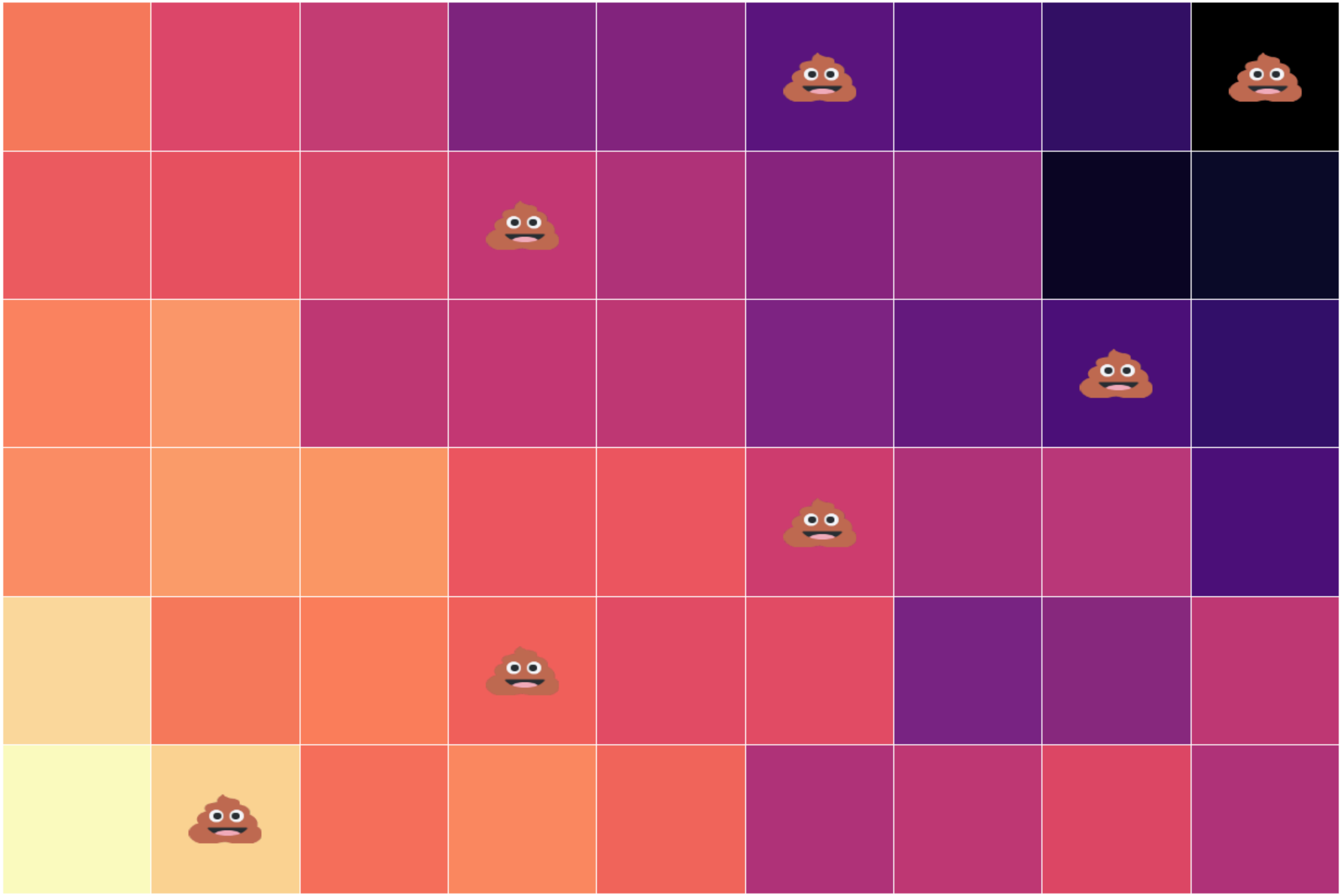

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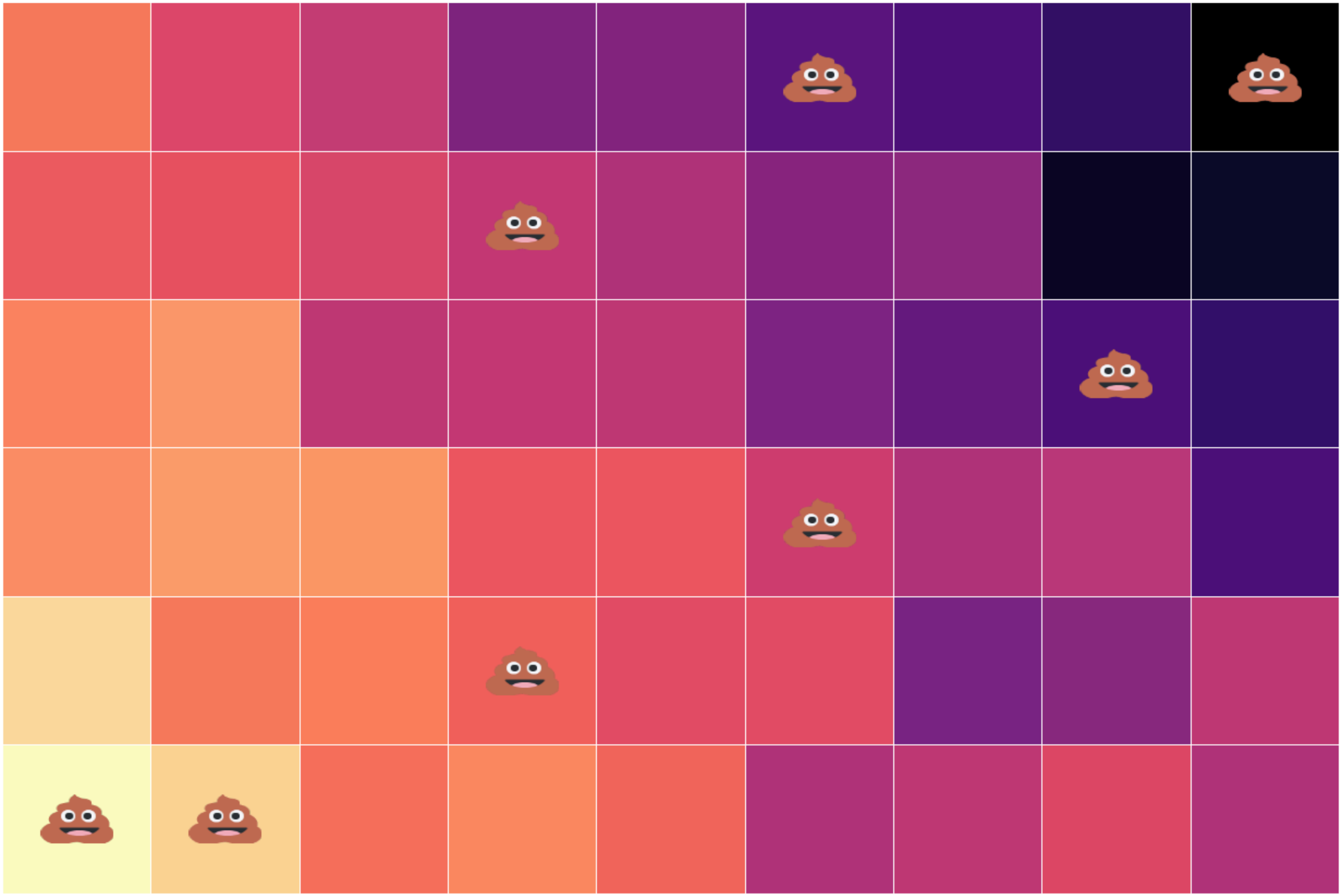

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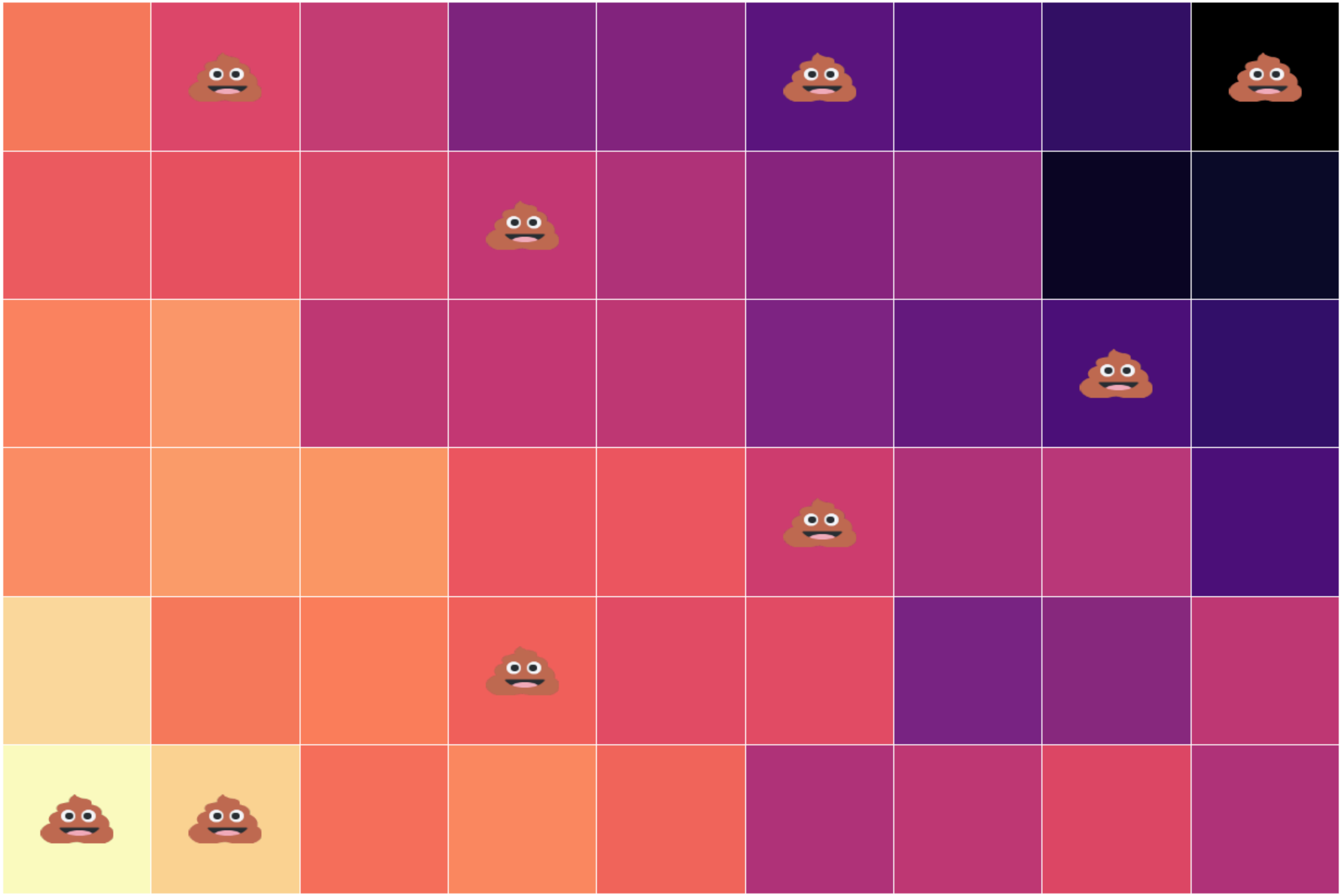

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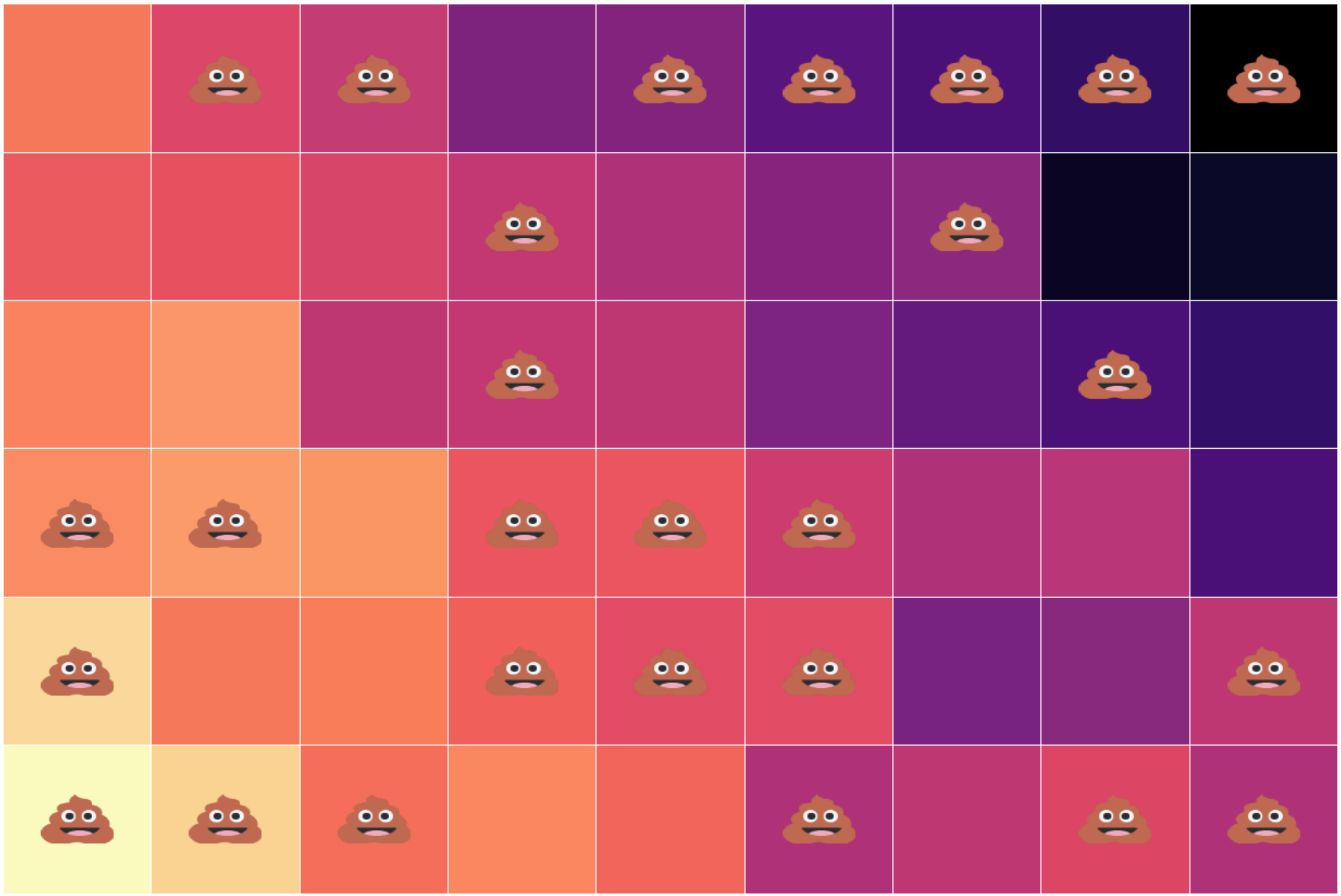

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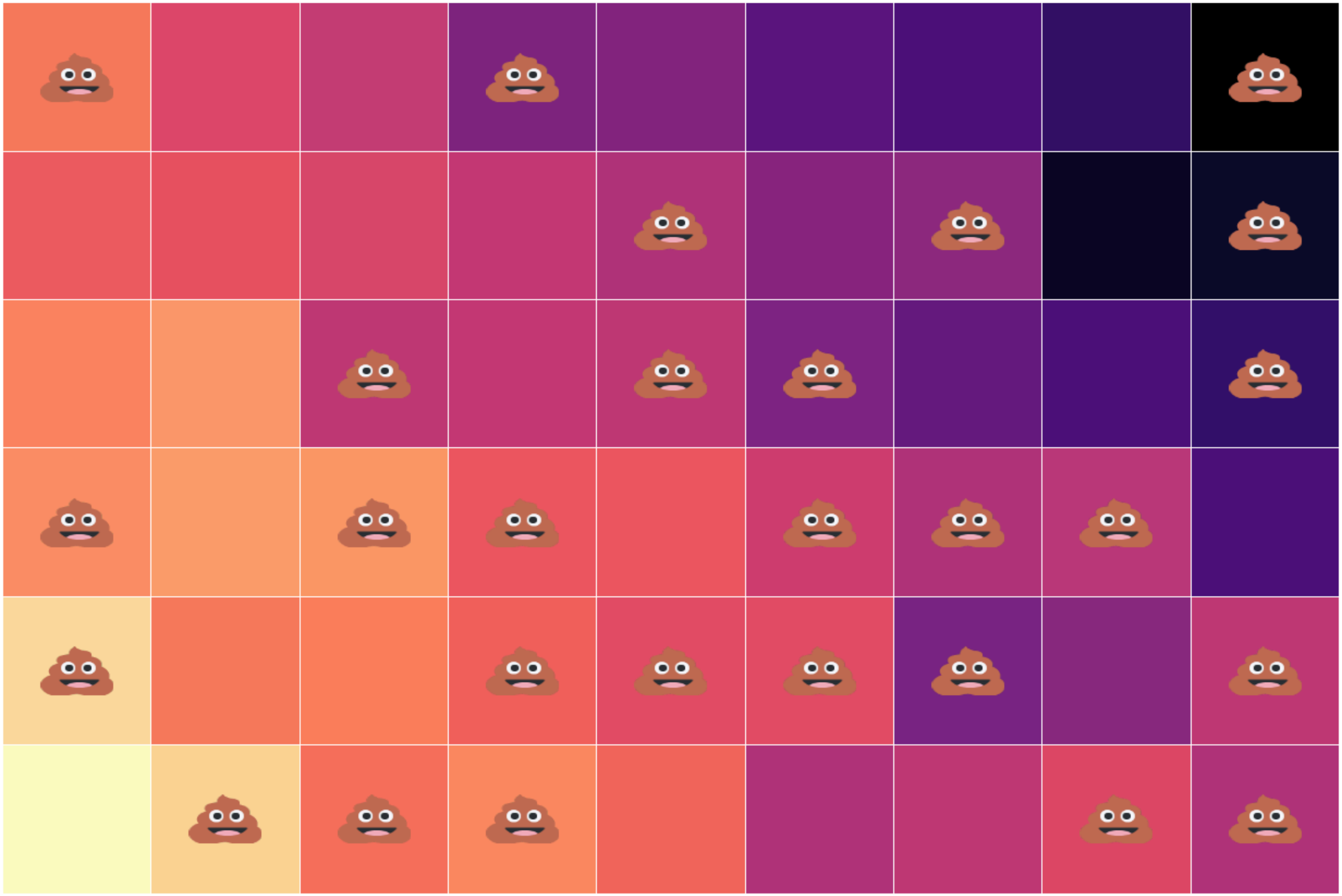

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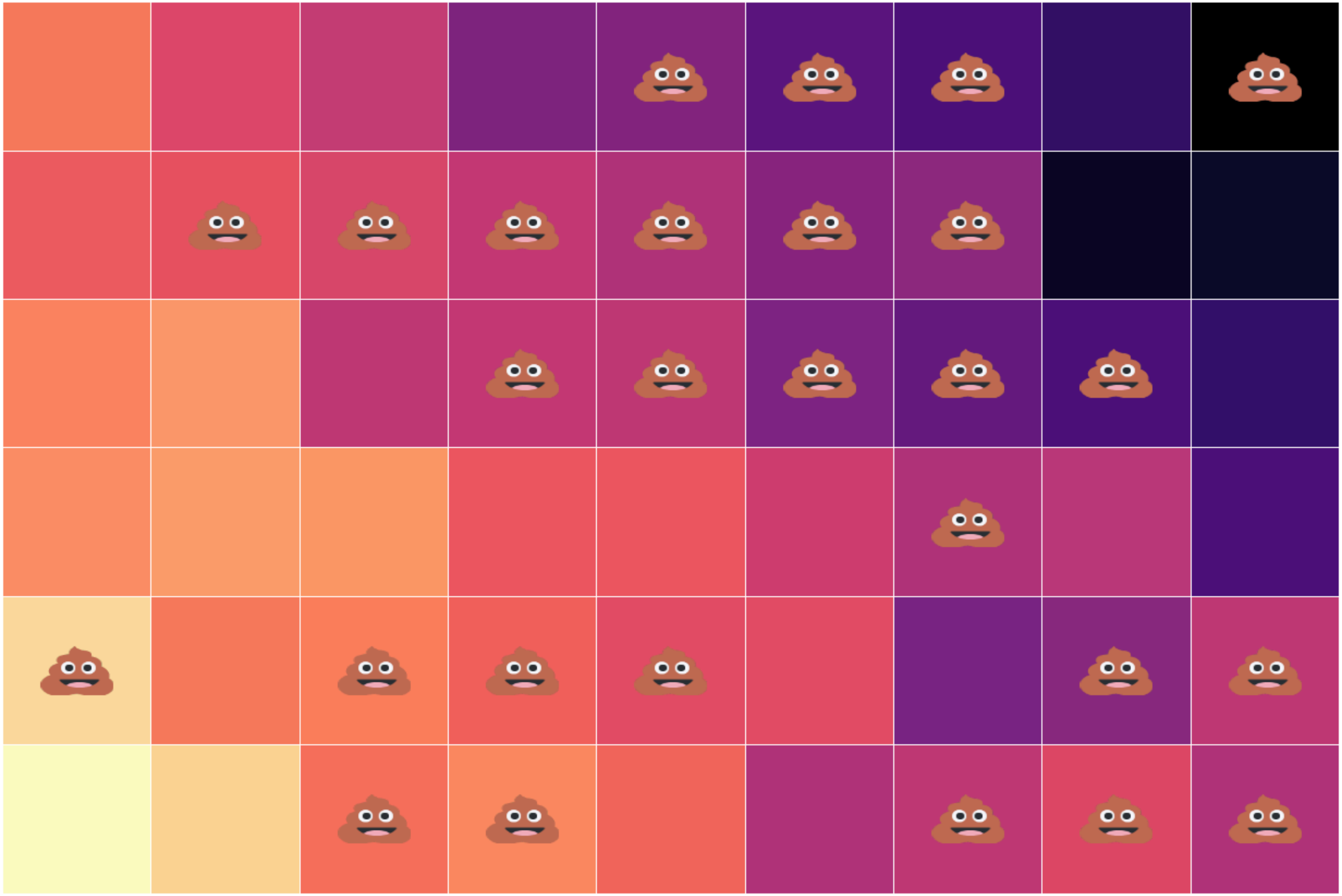

Causality

We can estimate the **causal effect** of fertilizer on crop yield by comparing the average yield in the treatment group (💩) with the control group (no 💩).

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A: On average, **randomly assigning treatment should balance** trt. and control across the other dimensions that affect yield (soil, slope, water).

Causality

Example: Returns to education

Labor economists, policy makers, parents, and students are all interested in the (monetary) *return to education*.

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Thought experiment:

- Randomly select an individual.
- Give her an additional year of education.
- How much do her earnings increase?

This change in earnings gives the **causal effect** of education on earnings.

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2. Education likely reduces experience (time out of the workforce).
3. Education is ***endogenous*** (violates *exogeneity*).

The point (2) above also illustrates the difficulty in learning about educations while *holding all else constant*.

Many important variables have the same challenge—gender, race, income.

Causality

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- Randomly encourage education (might work).
- Randomly assign programs that affect education (*e.g.*, mentoring).

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Option 3: Look for a ***natural experiment***—a policy or accident in society that arbitrarily increased education for one subset of people.

Causality

- Let's try controlling for every variable that affects both education and earnings, under CIA it should work!

$$\begin{gathered} Earnings = \beta_0 + \beta_1 Edu + \beta_2 Ability + \cdots + \\ \beta_{k-1} Race + \beta_k Gender + u \end{gathered}$$

- Anyone see any problems?

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$$\boldsymbol{Earnings} = \beta_0 + \beta_1 \boldsymbol{Edu} + \beta_2 \boldsymbol{Ability} + \cdots + \beta_{k-1} \boldsymbol{Race} + \beta_k \boldsymbol{Gender} + \boldsymbol{u}$$

- Anyone see any problems?
- Should race and gender be interacted? Race or gender and education?
- How do we measure ability? Specialized tests? Do those tests capture everything?
- Should we control for experience in a job?
- Uh oh, this is getting complicated and I'm not even sure we learn much

Causality

- Natural experiment approach: what policies arbitrarily increase education for a subset of people?

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- Natural experiment approach: what policies arbitrarily increase education for a subset of people?
- Admissions cutoffs: people around the cutoff are similar, but above gets more education
  - Regression discontinuity
- Lottery enrollment and/or capacity constraints: people who get in get more education
  - Instrumental variables
- New school built: people near school get more education
  - Difference-in-differences

Real-world experiments

RCTs and certain policy changes yield **real experiments** to isolate causal effects.

Characteristics

- Feasible—we can actually (potentially) run the experiment.
- Compare individuals randomized into treatment against individuals randomized into control.
- Require "good" randomization to get *all else equal* (exogeneity).

Real-world experiments

RCTs and certain policy changes yield **real experiments** to isolate causal effects.

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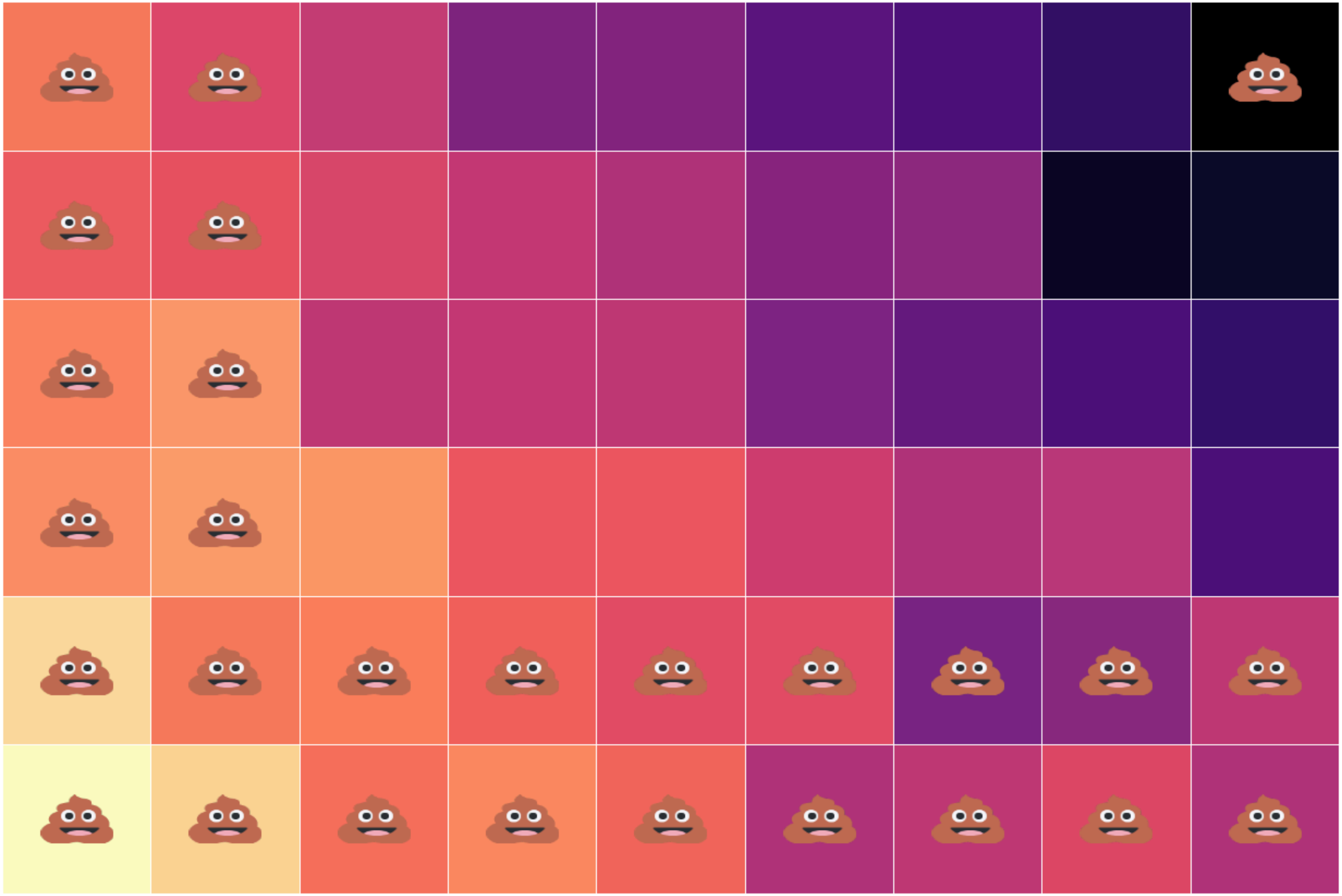
Note: Your experiment's results are only as good as your randomization.

Unfortunate randomization



The ideal experiment

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This *ideal experiment* is clearly infeasible[†], but it creates nice notation for causality (the Rubin causal model/Neyman potential outcomes framework).

† Without (1) God-like abilities and multiple universes or (2) a time machine.

Causality

The *ideal* data for 10 people

```
##     i trt  y1i  y0i
## 1   1   1 5.01 2.56
## 2   2   1 8.85 2.53
## 3   3   1 6.31 2.67
## 4   4   1 5.97 2.79
## 5   5   1 7.61 4.34
## 6   6   0 7.63 4.15
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Calculate the causal effect of trt.

$$\tau_i = y_{1,i} - y_{0,i}$$

for each individual i.

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##     i trt  y1i  y0i effect_i
## 1   1   1 5.01 2.56     2.45
## 2   2   1 8.85 2.53     6.32
## 3   3   1 6.31 2.67     3.64
## 4   4   1 5.97 2.79     3.18
## 5   5   1 7.61 4.34     3.27
## 6   6   0 7.63 4.15     3.48
## 7   7   0 4.75 0.56     4.19
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for each individual i.

The mean of τ_i is the **average treatment effect** (ATE).

Thus, $\bar{\tau} = 3.82$

Causality

This model highlights the fundamental problem of causal inference.

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The challenge:

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##     i trt  y1i  y0i
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But, we do observe

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- $y_{0,j}$ for j in 6, 7, 8, 9, 10

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- $y_{0,j}$ for j in 6, 7, 8, 9, 10

Q: How do we "fill in" the `NA`s and estimate $\overline{\tau}$?

Causally estimating the treatment effect

Notation: Let D_i be a binary indicator variable such that

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Time for math! 🎉

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Assumption: Let $\tau_i = \tau$ for all i.

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Note: We defined

$$\tau_i = \tau = y_{1,i} - y_{0,i}$$

which implies

$$y_{1,i} = y_{0,i} + \tau$$

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So our proposed group-difference estimator give us the sum of

1. τ, the **causal, average treatment effect** that we want
2. **Selection bias:** How much trt. and control groups differ (on average).

Inference: Did we just get lucky?

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- Most of today's lecture covered causal identification
- That's how you know whether the average treatment effect is causal
- But the other key part is inference: how do you know whether the average treatment effect is *statistically* different from zero?
- That's where "inference" comes in
- Inference is the practice of determining how special your results are.
- Generally you get a confidence interval and p-value (except Bayesian inference)

Types of inference

1. **Asymptotic** inference: what you saw in econometrics
   - Under a few assumptions, you can make inferences
2. **Randomization**: maybe you saw it?
   - Assign placebo treatments to see if results are unique
   - Are your results are driven by something about the treated group?
3. **Bootstrapping**: maybe you saw it?
   - Resample data to see if your results are sensitive to the sample
   - Are your results are driven by something about the sample?
4. **Bayesian**: I doubt you've seen this
   - Assume a prior distribution for β and update it
   - Generates a "credibility" interval

Next lecture: Regression analysis