

Data Science for Economists

Fixed Effects and Difference-in-differences

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Today's lecture explores

Software requirements

R packages

It's important to note that "base" R already provides all of the tools to implement a fixed effects regression, **but** you'll quickly hit walls due to memory caps. Instead, I want to introduce **fixest**, short for Fixed-Effects Estimation, which provides lightning fast fixed effects estimation and make your life much easier.

- New: **fixest**, **wooldridge**
- Already used: **modelsummary**, **broom**, **tidyverse**

A convenient way to install (if necessary) and load everything is by running the below code chunk.

```
## Load and install the packages that we'll be using today
if (!require("pacman")) install.packages("pacman")
pacman::p_load(modelsummary, broom, fixest, wooldridge, tidyverse)

## My preferred ggplot2 plotting theme (optional)
theme_set(theme_minimal())
```

Note on fixest and feols I'll be using **fixest** and **feols** throughout these notes. The **fixest** package is a new package that is very fast and has a lot of functionality. It has several bits of functionality like **feols()** and **etable()**, which are powerful functions for making regressions and putting the output into tables that work well together. **feols()** works very much like **lm()** in base R, but with a few added bonuses.

Panel models

A panel dataset is one in which we view a single unit over multiple periods of time, so a balanced panel has the same number of observations for each unit. For example, we might have data on 100 countries over 10 years, or 50 US states over 20 years. We can then take unit fixed effects, which lets us compare between years within a single unit. Similarly, we can take time fixed effects to compare between units within a given point in time. If our dataset has other dimensions that vary in a way that is not collinear with unit or time, we can also take a fixed effect for that – though again, you want to be careful about throwing in fixed effects.

Dataset

Let me introduce the dataset we'll be using, `crime4`. It comes from Jeffrey Wooldridge's R package – Dr. Wooldridge is one of the most accomplished professors of econometrics on the planet. I was tipped off about his package by Nick Huntington-Klein's own [lecture notes](#). The dataset shows county probability of arrest and county crime rate by year.

```
data(crime4)
crime4 %>%
  dplyr::select(county, year, crmrte, prbarr) %>%
  rename(County = county,
         Year = year,
         CrimeRate = crmrte,
         ProbofArrest = prbarr) %>%
  slice(1:9)
```

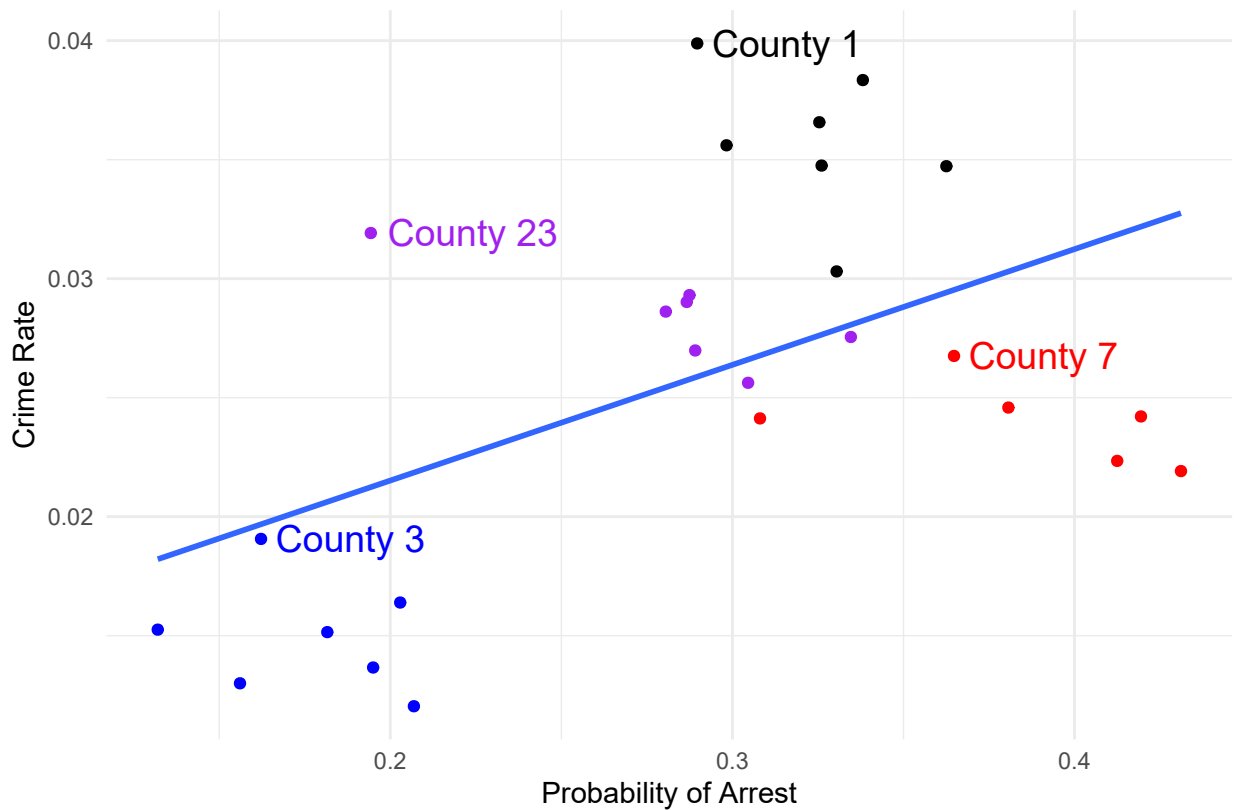
```
## # A tibble: 630 x 4
## # Groups:   County [90]
##   County  Year CrimeRate ProbofArrest
##   <int> <int>     <dbl>      <dbl>
## 1     1     81    0.0399      0.290
## 2     1     82    0.0383      0.338
## 3     1     83    0.0303      0.330
## 4     1     84    0.0347      0.363
## 5     1     85    0.0366      0.325
## 6     1     86    0.0348      0.326
## 7     1     87    0.0356      0.298
## 8     3     81    0.0164      0.203
## 9     3     82    0.0191      0.162
## 10    3     83    0.0151      0.182
## # i 620 more rows
```

Let's visualize it

Below I visualize the data for just a few counties. Note the positive slope when pooling! Is that surprising?

```
crime4 %>%
  filter(county %in% c(1,3,7, 23),
         prbarr < .5) %>%
  group_by(county) %>%
  mutate(label = case_when(
    crmrte == max(crmrte) ~ paste('County',county),
    TRUE ~ NA_character_
  )) %>%
  ggplot(aes(x = prbarr, y = crmrte, color = factor(county), label = label)) +
  geom_point() +
  geom_text(hjust = -.1, size = 14/.pt) +
  labs(x = 'Probability of Arrest',
       y = 'Crime Rate',
       caption = 'One outlier eliminated in County 7.') +
  #scale_x_continuous(limits = c(.15, 2.5)) +
  guides(color = FALSE, label = FALSE) +
  scale_color_manual(values = c('black','blue','red','purple')) +
  geom_smooth(method = 'lm', aes(color = NULL, label = NULL), se = FALSE)
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



One outlier eliminated in County 7.

Let's try the de-meaning approach

We can use `group_by` to get means-within-groups and subtract them out.

```
crime4 <- crime4 %>%
  group_by(county) %>%
  mutate(mean_crime = mean(crmrte),
         mean_prob = mean(prbarr)) %>%
  mutate(demeaned_crime = crmrte - mean_crime,
         demeaned_prbarr = prbarr - mean_prob)
```

And Regress!

```
orig_data <- feols(crmrte ~ prbarr, data = crime4)
de_mean <- feols(demeaned_crime ~ demeaned_prbarr, data = crime4)
msummary(list(orig_data, de_mean))
```

Note the coefficient has flipped!

Interpreting a Within Relationship

How can we interpret that slope of -0.02 ? This is all *within variation* so our interpretation must be *within-county*. So, “comparing a county in year A where its arrest probability is 1 (100 percentage points) higher than it is in year B, we expect the number of crimes per person to drop by .02.” Or if we think we’ve causally identified it (and want to work on a more realistic scale), “raising the arrest probability by 1 percentage point in a county reduces the number of crimes per person in that county by .0002”. We’re basically “controlling for county” (and will do that explicitly in a moment). So your interpretation should think of it in that way - *holding county constant* i.e. *comparing two observations with the same value of*

	(1)	(2)	
(Intercept)	0.043 (0.001)	0.000 (0.000)	
prbarr	-0.038 (0.004)		
demeaned_prbarr		-0.002 (0.002)	
Num.Obs.	630	630	
R2	0.129	0.001	
R2 Adj.	0.127	-0.001	
AIC	-3349.3	-4551.6	
BIC	-3340.4	-4542.8	
RMSE	0.02	0.01	
Std.Errors	IID	IID	

	(1)	(2)	(3)
prbarr	-0.038 (0.004)		-0.002 (0.003)
demeaned_prbarr		-0.002 (0.002)	
Num.Obs.	630	630	630
R2	0.129	0.001	0.871
R2 Adj.	0.127	-0.001	0.849
AIC	-3349.3	-4551.6	-4373.6
BIC	-3340.4	-4542.8	-3969.1
RMSE	0.02	0.01	0.01
Std.Errors	IID	IID	IID

county i.e. comparing a county to itself at a different point in time.

Concept checks

- Do you think the model we've presented is sufficient to have a causal interpretation of the effect of arrest probability on crime?
- What assumptions would we need to make to have a causal interpretation?
- What potential confounders are there?
- Why does subtracting the within-individual mean of each variable “control for individual”?
- In a sentence, interpret the slope coefficient in the estimated model $(Y_{it} - \bar{Y}_i) = 2 + 3(X_{it} - \bar{X}_i)$ where Y is “blood pressure”, X is “stress at work”, and i is an individual person, and \bar{Y}_i means average of Y_i
- Is this relationship causal? If not, what assumptions are required for it to be causal?

Can we do that all at once? Yes, with the Least Squares Dummy Variable Approach

De-meaning takes some steps which could get tedious to write out. Another way is to include a dummy or category variable for each county. This is called the Least Squares Dummy Variable approach.

You end up with the same results as if we de-meaned.

```
lsdv <- feols(crmrte ~ prbarr + factor(county), data = crime4)
msummary(list(orig_data, de_mean, lsdv), keep = c('prbarr', 'demeaned_prob'))
```

Hey look, the coefficient is the same!

Why LSDV?

- A benefit of the LSDV approach is that it calculates the fixed effects α_i for you
- We left those out of the table with the `coefs` argument of `msummary` (we rarely want them) but here they are:

```
lsdv
```

```
##
## Call:
## lm(formula = crmrte ~ prbarr + factor(county), data = crime4)
##
## Coefficients:
##      (Intercept)          prbarr  factor(county)3  factor(county)5
##      0.0363976      -0.0020232      -0.0211038      -0.0227439
##  factor(county)7  factor(county)9  factor(county)11  factor(county)13
##     -0.0125058     -0.0240486     -0.0183143     -0.0032912
##  factor(county)15  factor(county)17  factor(county)19  factor(county)21
##     -0.0179836     -0.0146255     -0.0185499      0.0035485
##  factor(county)23  factor(county)25  factor(county)27  factor(county)33
##     -0.0073943     -0.0034639     -0.0012558     -0.0198379
##  factor(county)35  factor(county)37  factor(county)39  factor(county)41
##      0.0070240     -0.0143802     -0.0212591     -0.0115589
##  factor(county)45  factor(county)47  factor(county)49  factor(county)51
##     -0.0008915     -0.0053747     -0.0015888      0.0318754
##  factor(county)53  factor(county)55  factor(county)57  factor(county)59
##     -0.0186603      0.0221664     -0.0063204     -0.0178825
##  factor(county)61  factor(county)63  factor(county)65  factor(county)67
##     -0.0149666      0.0381621      0.0198140      0.0214212
##  factor(county)69  factor(county)71  factor(county)77  factor(county)79
##     -0.0211463      0.0228639      0.0022599     -0.0215523
##  factor(county)81  factor(county)83  factor(county)85  factor(county)87
##      0.0205261     -0.0064776      0.0051594     -0.0078661
##  factor(county)89  factor(county)91  factor(county)93  factor(county)97
##     -0.0088413     -0.0040777     -0.0018436      0.0021169
##  factor(county)99  factor(county)101  factor(county)105  factor(county)107
##     -0.0192747     -0.0027612      0.0143055      0.0108018
##  factor(county)109  factor(county)111  factor(county)113  factor(county)115
##     -0.0170930     -0.0187163     -0.0239391     -0.0301032
##  factor(county)117  factor(county)119  factor(county)123  factor(county)125
##     -0.0169581      0.0526182     -0.0023063     -0.0091250
##  factor(county)127  factor(county)129  factor(county)131  factor(county)133
##      0.0028419      0.0386488     -0.0179728      0.0098405
##  factor(county)135  factor(county)137  factor(county)139  factor(county)141
##      0.0188796     -0.0220273     -0.0066127      0.0337109
##  factor(county)143  factor(county)145  factor(county)147  factor(county)149
##     -0.0139798     -0.0071850      0.0166929     -0.0200991
##  factor(county)151  factor(county)153  factor(county)155  factor(county)157
##     -0.0114062     -0.0047028     -0.0026681     -0.0058717
##  factor(county)159  factor(county)161  factor(county)163  factor(county)165
##     -0.0043145     -0.0154759     -0.0147833      0.0082355
##  factor(county)167  factor(county)169  factor(county)171  factor(county)173
##     -0.0128534     -0.0232628     -0.0141934     -0.0242636
##  factor(county)175  factor(county)179  factor(county)181  factor(county)183
##     -0.0175234     -0.0077435      0.0232585      0.0175664
##  factor(county)185  factor(county)187  factor(county)189  factor(county)191
```

```
##          -0.0243118          -0.0078490          -0.0071590          0.0015451
## factor(county)193 factor(county)195 factor(county)197
##          -0.0152095          0.0097064          -0.0209701
```

The interpretation is exactly the same as with a categorical variable - we have an omitted county, and these show the difference relative to that omitted county

NOTE: See how I put factor() around county? That is to ensure it reads county, which is the county fips code as a categorical variable instead of as a numerical variable. If you don't do that, it will read it as a numerical variable and you'll get a different result:

```
feols(crmrte ~ prbarr + county, data = crime4)
```

```
## OLS estimation, Dep. Var.: crmrte
## Observations: 630
## Standard-errors: IID
##          Estimate Std. Error   t value Pr(>|t|)
## (Intercept)  0.042131   0.001829  23.028937 < 2.2e-16 ***
## prbarr       -0.037882   0.003944  -9.605290 < 2.2e-16 ***
## county       0.000011   0.000012   0.940199   0.34748
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 0.016891   Adj. R2: 0.127011
```

This is saying that as FIPS code increases by one, the crime rate increases by 0.000011... that's nonsense. There's an urban legend of an economist who took the log of the NAICS industry classification code for quite some time before realizing they meant to use a categorical variable. Correcting that mistake completely changed their results.

Why LSDV?

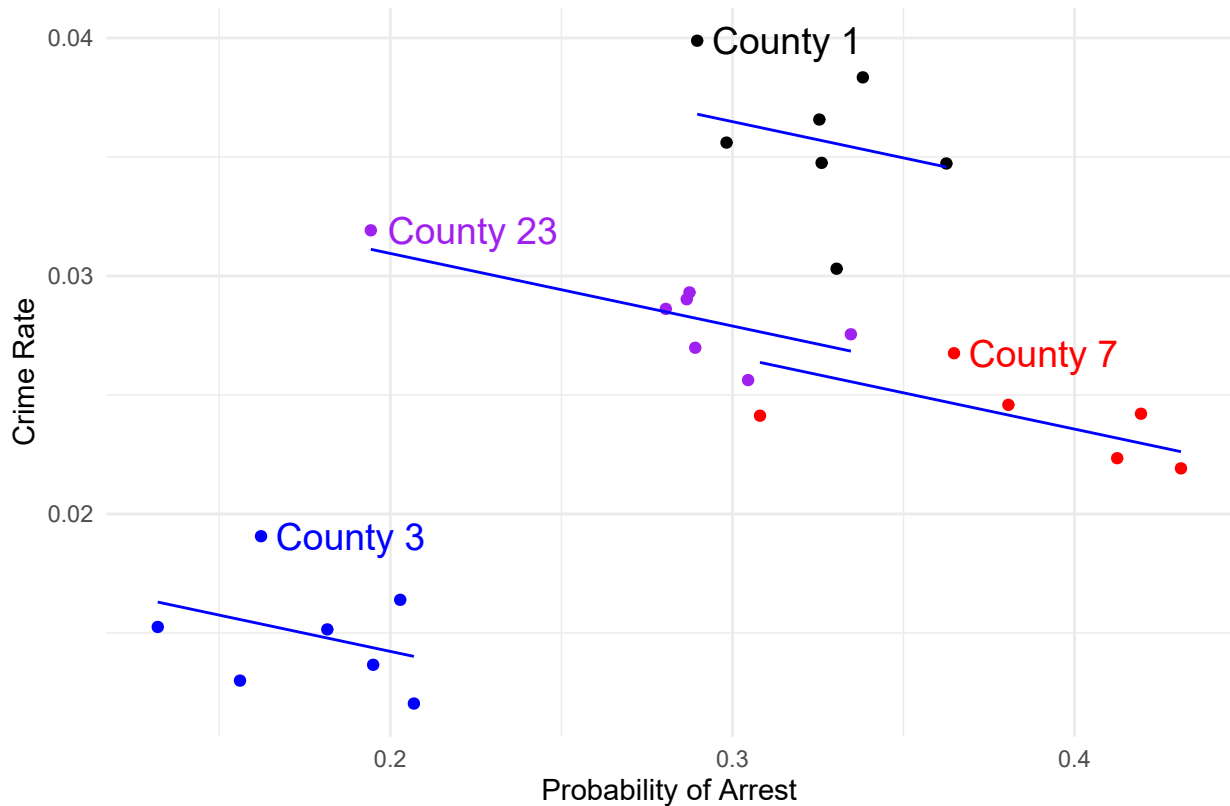
This also makes clear another element of what's happening! Just like with a categorical var, the line is moving *up and down* to meet the counties. Graphically, de-meaning moves all the points together in the middle to draw a line, while LSDV moves the line up and down to meet the points

```
crime4_small <- crime4 %>%
  filter(county %in% c(1,3,7, 23), # filter down data points
    prbarr < .5) %>%
  ungroup()
# Make lsdv for this small dataframe
lsdv_small <- feols(crmrte ~ prbarr + factor(county),
  data = crime4_small)

crime4_small %>%
  mutate(pred = predict(lsdv_small)) %>%
  group_by(county) %>%
  mutate(label = case_when(
    crmrte == max(crmrte) ~ paste('County',county),
    TRUE ~ NA_character_
  )) %>%
  ggplot(aes(x = prbarr, y = crmrte, color = factor(county), label = label)) +
  geom_point() +
  geom_text(hjust = -.1, size = 14/.pt) +
  geom_line(aes(y = pred, group = county), color = 'blue') +
  labs(x = 'Probability of Arrest',
    y = 'Crime Rate',
    caption = 'One outlier eliminated in County 7.') +
  #scale_x_continuous(limits = c(.15, 2.5)) +
```

```
guides(color = FALSE, label = FALSE) +
scale_color_manual(values = c('black', 'blue', 'red', 'purple'))
```

```
## Warning: Removed 23 rows containing missing values (`geom_text()`).
```



One outlier eliminated in County 7.

The “Pros” don’t use LSDV

Most people do not use LSDV – it is computationally expensive. If you get too many fixed effects or too big of data, it just will not work. The professionally-written commands use de-meaning, like **fixest**, which is less computationally expensive. See for yourself! Look, we even used the **etable** function.

```
pro <- feols(crmrte ~ prbarr | county, data = crime4)
de_mean <- feols(demeaned_crime ~ demeaned_prbarr, data = crime4)
etable(de_mean, pro)
```

```
##                               de_mean                pro
## Dependent Var.:      demeaned_crime      crmrte
##
## Constant      -1.01e-20 (0.0003)
## demeaned_prbarr -0.0020 (0.0025)
## prbarr                                -0.0020 (0.0026)
## Fixed-Effects:  -----
## county                                No                Yes
## -----
## S.E. type      IID                by: county
## Observations    630                630
## R2              0.00106            0.87076
## Within R2      --                0.00106
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

To explain the **fixest** package, let's dive a bit deeper into the crime data. It has tons of variables we could use. We could account for variation by year for example.

```
crime_county_fe <- feols(crmrte ~ prbarr | county, data = crime4)
crime_year_fe <- feols(crmrte ~ prbarr | year, data = crime4)
crime_county_year_fe <- feols(crmrte ~ prbarr | county+year, data = crime4)

etable(list('County FE'=crime_county_fe,
            'Year FE'=crime_year_fe,
            'County and Year FE'=crime_county_year_fe))
```

```
##               County FE               Year FE County and Yea..
## Dependent Var.:      crmrte              crmrte              crmrte
##
## prbarr               -0.0020 (0.0026) -0.0378** (0.0090) -0.0011 (0.0026)
## Fixed-Effects: -----
## county                Yes                No                Yes
## year                  No                 Yes                Yes
## -----
## S.E.: Clustered      by: county          by: year          by: county
## Observations         630                 630                 630
## R2                   0.87076              0.13347              0.87735
## Within R2            0.00106              0.12764              0.00034
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Pretty neat right? Just sticking something after the | allows you to residualize its fixed effect!

```
dict = c('prbarr'='Prob. Arrest',
          'avgsen'='Avg. Sentence',
          'county'='County',
          'year'='Year',
          'crmrte'='Crime Rate',
          'prbconv'='Prob. Conviction')

etable(list('County FE'=crime_county_fe,
            'Year FE'=crime_year_fe,
            'County and Year FE'=crime_county_year_fe),
        notes='Note: Estimates from various fixed effects regressions on the Crime Data',
        dict=dict
    )
```

```
##               County FE               Year FE County and Yea..
## Dependent Var.:      Crime Rate          Crime Rate          Crime Rate
##
## Prob. Arrest         -0.0020 (0.0026) -0.0378** (0.0090) -0.0011 (0.0026)
## Fixed-Effects: -----
## County                Yes                No                Yes
## Year                  No                 Yes                Yes
## -----
## S.E.: Clustered      by: County          by: Year          by: County
## Observations         630                 630                 630
## R2                   0.87076              0.13347              0.87735
```

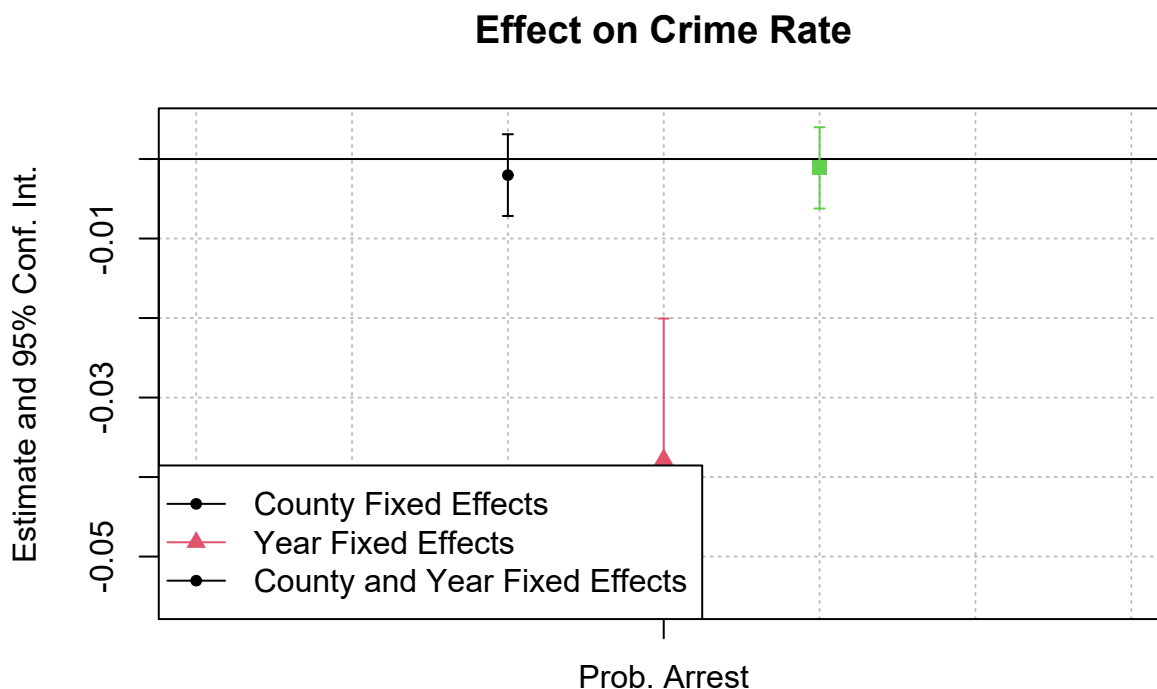


```
## Within R2          0.00106          0.12764          0.00034
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
# I don't want to keep writing in ,dict=dct. So I'll use setFixestDict
# This applies to every etable in the session
setFixest_dict(dict)
```

Visualization Similarly, the `fixest::coefplot()` function for plotting estimation results:

```
coefplot(list(crime_county_fe, crime_year_fe, crime_county_year_fe))

## Add legend (optional)
legend("bottomleft", col = 1:2, lwd = 1, pch = c(20, 17),
       legend = c("County Fixed Effects", "Year Fixed Effects", "County and Year Fixed Effects"))
```



`coefplot()` is especially useful for tracing the evolution of treatment effects over time, as in a difference-in-differences setup (see [Examples](#)). However, I realise some people may find it a bit off-putting that it produces base R plots, rather than a **ggplot2** object. We'll get to an automated **ggplot2** coefficient plot solution further below with `modelsummary::modelplot()`. Nevertheless, let me close this out this section by demonstrating the relative ease with which you can do this “manually”. Consider the below example, which leverages the fact that we have saved (or can save) regression models as data frames with `broom::tidy()`. As I suggested earlier, this makes it simple to construct our own bespoke coefficient plots.

```
# library(ggplot2) ## Already loaded

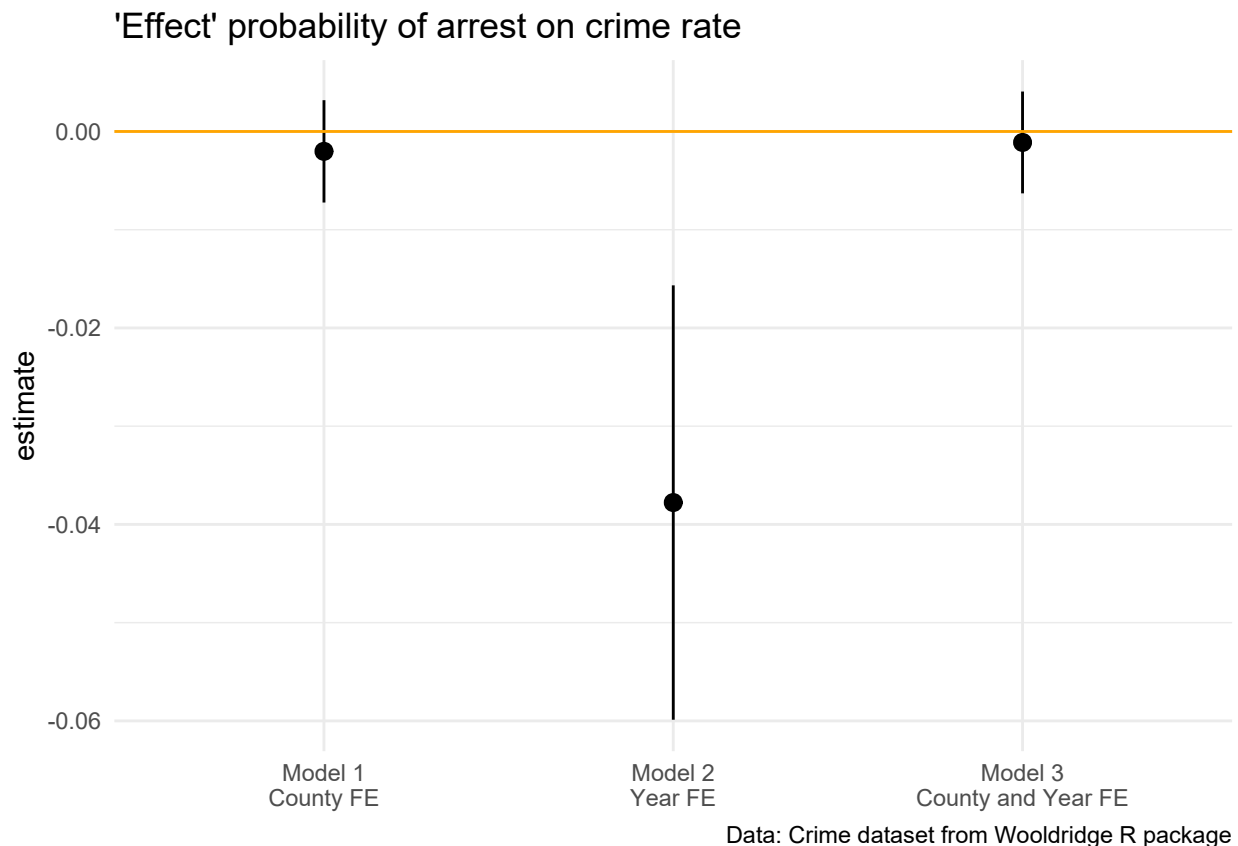
## First get tidied output of the ols_hdfe object
coefs_crime_county_fe = tidy(crime_county_fe, conf.int = TRUE)
coefs_crime_year_fe = tidy(crime_year_fe, conf.int = TRUE)
coefs_crime_county_year_fe = tidy(crime_county_year_fe, conf.int = TRUE)

bind_rows(
  coefs_crime_county_fe %>% mutate(reg = "Model 1\nCounty FE"),
  coefs_crime_year_fe %>% mutate(reg = "Model 2\nYear FE"),
```

```

coefs_crime_county_year_fe %>% mutate(reg="Model 3\nCounty and Year FE")
) %>%
ggplot(aes(x=reg, y=estimate, ymin=conf.low, ymax=conf.high)) +
geom_pointrange() +
labs(title = "Marginal effect of probability of arrest on crime rate") +
geom_hline(yintercept = 0, col = "orange") +
labs(
  title = "'Effect' probability of arrest on crime rate",
  caption = "Data: Crime dataset from Wooldridge R package"
) +
theme(axis.title.x = element_blank())

```



What if we wanted to change the clustering of the standard errors? Did you notice the S.E. type above? It auto-clustered by the fixed effects – specifically the fixed effect with the most levels. **fixest** does that by default, but maybe you disagree!

Sometimes you want to cluster standard errors a new way. Well that is something you can do with **fixest** and its delightfully well-designed `etable()` function. You can specify the cluster variable with `cluster()` or the type of standard errors you want with `se()` and get different types of standard errors. Below I specify standard errors clustered by state and then an assumption of independent and identically distributed errors. (The most vanilla standard errors you can assume and rarely the ones we believe explain real world phenomena.)

```

# IID standard errors
etable(list('County FE'=crime_county_fe,
           'Year FE'=crime_year_fe,
           'County and Year FE'=crime_county_year_fe),
       se='IID')

```

```
##               County FE               Year FE County and Yea..
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##
## Prob. Arrest    -0.0020 (0.0027) -0.0378*** (0.0040) -0.0011 (0.0026)
## Fixed-Effects:  -----
## County                Yes                No                Yes
## Year                  No                Yes                Yes
## -----
## S.E. type          IID                IID                IID
## Observations        630                630                630
## R2                  0.87076            0.13347            0.87735
## Within R2           0.00106            0.12764            0.00034
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
etable(list('County FE'=crime_county_fe,
            'Year FE'=crime_year_fe,
            'County and Year FE'=crime_county_year_fe),
        cluster='county')
```

```
##               County FE               Year FE County and Yea..
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##
## Prob. Arrest    -0.0020 (0.0026) -0.0378*** (0.0103) -0.0011 (0.0026)
## Fixed-Effects:  -----
## County                Yes                No                Yes
## Year                  No                Yes                Yes
## -----
## S.E.: Clustered    by: County          by: County          by: County
## Observations        630                630                630
## R2                  0.87076            0.13347            0.87735
## Within R2           0.00106            0.12764            0.00034
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We'd normally expect our standard errors to blow up with clustering and we see something similar here. Why is that?

Yes, I know this is a lot on stuff you've only barely experienced before. But you're going to come across these terms when you read papers and I want you to know how to play with them when you're trying to learn by doing.

Aside on standard errors We've now seen the various options that **fixest** has for specifying different standard error structures. In short, you invoke either of the `se` or `cluster` arguments. Moreover, you can choose to do so either at estimation time, or by adjusting the standard errors for an existing model post-estimation (e.g. with `summary.fixest(mod, cluster = ...)`). There are two additional points that I want to draw your attention to.

First, if you're coming from another statistical language, adjusting the standard errors post-estimation (rather than always at estimation time) may seem slightly odd. But this behaviour is actually extremely powerful, because it allows us to analyse the effect of different error structures *on-the-fly* without having to rerun the entire model again. **fixest** is already the fastest game in town, but just think about the implied time savings for really large models.¹ I'm a huge fan of the flexibility, safety, and speed that on-the-fly standard error adjustment offers us. I even wrote a whole [blog post](#) about it if you'd like to read more.

Second, reconciling standard errors across different software is a much more complicated process than you may realise. There are a number of unresolved theoretical issues to consider — especially when it comes to multiway clustering — and package maintainers have to make a number of arbitrary decisions about the best way to account for these. See [here](#)

¹To be clear, adjusting the standard errors via, say, `summary.fixest()` completes instantaneously.

for a detailed discussion. Luckily, Laurent (the **fixest** package author) has taken the time to write out a [detailed vignette](#) about how to replicate standard errors from other methods and software packages.²

Multiple estimations But won't it get tedious writing out all these variations of fixed effects over and over with the `feols()` repeated? Sure will. That's where the **fixest** package comes in handy.

fixest allows you to do multiple estimations in one command and it does it fast! Why is it so fast? It leverages the de-meaning trick mentioned above. If a fixed effect is used in multiple estimations, it saves the outcome variable de-meant of that fixed effect to use in all the other estimations. That saves a bunch of time!

This is also a really smart big data technique we'll get into more later in the course. It does a task once instead of multiple times to save time and processing power.

Here's a demo using the stepwise `sw0()` function, which adds fixed effects – starting with none step-by-step:

```
crime_many_fes <- feols(crmrte ~ prbarr |
  sw0(county,year,county+year),
  data=crime4)
etable(crime_many_fes)
```

```
##               crime_many_fes.1 crime_many_fes.2  crime_many_fes.3
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##
## Constant           0.0432*** (0.0014)
## Prob. Arrest       -0.0379*** (0.0039) -0.0020 (0.0026) -0.0378** (0.0090)
## Fixed-Effects: -----
## County              No              Yes              No
## Year                No              No              Yes
## -----
## S.E. type          IID              by: County      by: Year
## Observations        630              630              630
## R2                  0.12856          0.87076          0.13347
## Within R2           --              0.00106          0.12764
##
##               crime_many_fes.4
## Dependent Var.:      Crime Rate
##
## Constant
## Prob. Arrest       -0.0011 (0.0026)
## Fixed-Effects: -----
## County              Yes
## Year                Yes
## -----
## S.E. type          by: County
## Observations        630
## R2                  0.87735
## Within R2           0.00034
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

These results are the same as above. Oh and guess what? You can get a lot more complicated than that!

Wouldn't it be nice to have better names of our variables? We can do that using a dict, which is just a fancy vector with names.

²If you want a deep dive into the theory with even more simulations, then [this paper](#) by the authors of the **sandwich** paper is another excellent resource.

Here's the basics of how it works.³ You can specify:

1. One or more rhs variable using `c(var1, var2, var3)`
2. One or more fixed effects using the stepwise functions `sw()`, `sw0()`, `csw()`, and `csw0()`.
3. One or more independent variable using the stepwise functions `sw()`, `sw0()`, `csw()`, and `csw0()`.
4. Different samples using the `split` or `fsplit` option.

And here's multiple estimations used to their "fuller" potential:

```
crime_many_estimations <- feols(c(crmrte, prbconv) ~ csw(prbarr, avgsen, polpc) |
  sw0(county, year, county+year),
  data=crime4,
  fsplit=~urban)

etable(crime_many_estimations[lhs='crmrte', sample=1], title='Crime Rate', notes='Note: Estimates from var
```

```
##              crime_many_estim..1 crime_many_estim..2 crime_many_estim..3
## Sample (urban)          Full sample          Full sample          Full sample
## Dependent Var.:          Crime Rate          Crime Rate          Crime Rate
##
## Constant              0.0432*** (0.0014)  0.0406*** (0.0026)  0.0397*** (0.0025)
## Prob. Arrest          -0.0379*** (0.0039) -0.0381*** (0.0039) -0.0478*** (0.0039)
## Avg. Sentence                0.0003 (0.0003)      0.0003 (0.0002)
## polpc                      2.089*** (0.2442)
## Fixed-Effects: -----
## County                  No                  No                  No
## Year                    No                  No                  No
## -----
## S.E. type              IID                  IID                  IID
## Observations            630                  630                  630
## R2                      0.12856              0.13055              0.22159
## Within R2              --                  --                  --
##
##              crime_many_es..4 crime_many_es..5 crime_many_est..6
## Sample (urban)          Full sample          Full sample          Full sample
## Dependent Var.:          Crime Rate          Crime Rate          Crime Rate
##
## Constant
## Prob. Arrest          -0.0020 (0.0026) -0.0019 (0.0027) -0.0043 (0.0028)
## Avg. Sentence                7.12e-5 (0.0001)  0.0002* (0.0001)
## polpc                      1.735*** (0.3191)
## Fixed-Effects: -----
## County                  Yes                  Yes                  Yes
## Year                    No                  No                  No
## -----
## S.E. type              by: County          by: County          by: County
## Observations            630                  630                  630
## R2                      0.87076              0.87084              0.89669
## Within R2              0.00106              0.00164              0.20150
##
##              crime_many_esti..7 crime_many_esti..8 crime_many_esti..9
## Sample (urban)          Full sample          Full sample          Full sample
## Dependent Var.:          Crime Rate          Crime Rate          Crime Rate
##
```

³You can find a more in-depth explanation at the [Multiple Estimation vignette](#).

```

## Constant
## Prob. Arrest      -0.0378** (0.0090) -0.0379** (0.0088) -0.0478** (0.0086)
## Avg. Sentence                0.0002 (0.0002)      0.0002 (0.0002)
## polpc                                2.134* (0.7683)
## Fixed-Effects: -----
## County                      No                      No                      No
## Year                        Yes                      Yes                      Yes
## -----
## S.E. type                  by: Year                  by: Year                  by: Year
## Observations                630                      630                      630
## R2                          0.13347                  0.13463                  0.22896
## Within R2                   0.12764                  0.12881                  0.22377
##
## crime_many_e..10 crime_many_e..11 crime_many_es..12
## Sample (urban)          Full sample          Full sample          Full sample
## Dependent Var.:        Crime Rate          Crime Rate          Crime Rate
##
## Constant
## Prob. Arrest      -0.0011 (0.0026) -0.0012 (0.0026) -0.0038 (0.0027)
## Avg. Sentence                -9.4e-5 (0.0001)  5.05e-5 (0.0001)
## polpc                                1.821*** (0.3223)
## Fixed-Effects: -----
## County                      Yes                      Yes                      Yes
## Year                        Yes                      Yes                      Yes
## -----
## S.E. type                  by: County                  by: County                  by: County
## Observations                630                      630                      630
## R2                          0.87735                  0.87746                  0.90563
## Within R2                   0.00034                  0.00125                  0.23086
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```
etable(crime_many_estimations[lhs='prbconv',sample=1],title='Probability of Conviction',notes='Note: Es
```

```

## crime_many_esti..1 crime_many_es..2 crime_many_est..3
## Sample (urban)          Full sample          Full sample          Full sample
## Dependent Var.:        Prob. Conviction Prob. Conviction Prob. Conviction
##
## Constant      0.5807*** (0.1385) 0.5018. (0.2628) 0.3759 (0.2338)
## Prob. Arrest    0.3512 (0.3937) 0.3464 (0.3942) -1.029** (0.3662)
## Avg. Sentence                0.0090 (0.0254) 0.0068 (0.0226)
## polpc                                296.5*** (22.92)
## Fixed-Effects: -----
## County                      No                      No                      No
## Year                        No                      No                      No
## -----
## S.E. type                  IID                      IID                      IID
## Observations                630                      630                      630
## R2                          0.00127                  0.00146                  0.21216
## Within R2                   --                      --                      --
##
## crime_many_es..4 crime_many_es..5 crime_many_es..6
## Sample (urban)          Full sample          Full sample          Full sample
## Dependent Var.: Prob. Conviction Prob. Conviction Prob. Conviction
##

```

```

## Constant
## Prob. Arrest      -2.941 (2.064)    -2.940 (2.074)    -3.394 (2.559)
## Avg. Sentence      0.0008 (0.0342)    0.0301 (0.0299)
## polpc              328.8* (142.5)
## Fixed-Effects: -----
## County              Yes              Yes              Yes
## Year                No              No              No
## -----
## S.E. type          by: County        by: County        by: County
## Observations        630              630              630
## R2                   0.33114          0.33114          0.43784
## Within R2           0.04762          0.04762          0.19955
##
## crime_many_es..7 crime_many_es..8 crime_many_es..9
## Sample (urban)      Full sample      Full sample      Full sample
## Dependent Var.: Prob. Conviction Prob. Conviction Prob. Conviction
##
## Constant
## Prob. Arrest        0.3665 (0.3999)    0.3571 (0.4001)    -1.008 (0.7845)
## Avg. Sentence        0.0138 (0.0358)    0.0074 (0.0274)
## polpc                294.5. (125.8)
## Fixed-Effects: -----
## County              No              No              No
## Year                Yes              Yes              Yes
## -----
## S.E. type          by: Year          by: Year          by: Year
## Observations        630              630              630
## R2                   0.01355          0.01398          0.22043
## Within R2           0.00139          0.00182          0.21082
##
## crime_many_e..10 crime_many_e..11 crime_many_e..12
## Sample (urban)      Full sample      Full sample      Full sample
## Dependent Var.: Prob. Conviction Prob. Conviction Prob. Conviction
##
## Constant
## Prob. Arrest        -2.939 (2.077)    -2.931 (2.079)    -3.388 (2.552)
## Avg. Sentence        0.0104 (0.0277)    0.0361 (0.0269)
## polpc                324.6* (139.9)
## Fixed-Effects: -----
## County              Yes              Yes              Yes
## Year                Yes              Yes              Yes
## -----
## S.E. type          by: County        by: County        by: County
## Observations        630              630              630
## R2                   0.34289          0.34305          0.44589
## Within R2           0.04784          0.04807          0.19709
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

etable(crime_many_estimations[lhs='crmrte',sample=2],title='Crime Rate in Urban Areas',notes='Note: Est.

```

```

## crime_many_estim..1 crime_many_estim..2 crime_many_estim..3
## Sample (urban)      0              0              0
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##

```

```

## Constant      0.0370*** (0.0012)  0.0392*** (0.0023)  0.0385*** (0.0021)
## Prob. Arrest  -0.0270*** (0.0034) -0.0267*** (0.0034) -0.0356*** (0.0033)
## Avg. Sentence                -0.0003 (0.0002)      -0.0003 (0.0002)
## polpc                                1.819*** (0.2036)
## Fixed-Effects: -----
## County                        No                        No                        No
## Year                          No                        No                        No
## -----
## S.E. type                    IID                        IID                        IID
## Observations                  574                      574                      574
## R2                           0.10023                  0.10240                  0.21267
## Within R2                     --                       --                       --
##
## crime_many_es..4 crime_many_es..5 crime_many_est..6
## Sample (urban)                0                        0                        0
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##
## Constant
## Prob. Arrest  -0.0017 (0.0026) -0.0017 (0.0026) -0.0041 (0.0027)
## Avg. Sentence                1.95e-5 (0.0001)  0.0002. (0.0001)
## polpc                                1.722*** (0.3264)
## Fixed-Effects: -----
## County                        Yes                       Yes                       Yes
## Year                          No                        No                        No
## -----
## S.E. type                    by: County          by: County          by: County
## Observations                  574                      574                      574
## R2                           0.80689                  0.80690                  0.84780
## Within R2                     0.00084                  0.00088                  0.21251
##
## crime_many_esti..7 crime_many_esti..8 crime_many_esti..9
## Sample (urban)                0                        0                        0
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##
## Constant
## Prob. Arrest  -0.0268** (0.0058) -0.0264** (0.0059) -0.0355** (0.0064)
## Avg. Sentence                -0.0004* (0.0001)  -0.0004. (0.0002)
## polpc                                1.865* (0.7238)
## Fixed-Effects: -----
## County                        No                        No                        No
## Year                          Yes                       Yes                       Yes
## -----
## S.E. type                    by: Year          by: Year          by: Year
## Observations                  574                      574                      574
## R2                           0.10602                  0.10988                  0.22501
## Within R2                     0.09934                  0.10323                  0.21922
##
## crime_many_e..10 crime_many_e..11 crime_many_es..12
## Sample (urban)                0                        0                        0
## Dependent Var.:      Crime Rate      Crime Rate      Crime Rate
##
## Constant
## Prob. Arrest  -0.0010 (0.0026) -0.0011 (0.0026) -0.0036 (0.0026)
## Avg. Sentence                -0.0001 (0.0001)  3.91e-5 (0.0001)

```



```
## polpc 1.805*** (0.3295)
## Fixed-Effects: -----
## County Yes Yes Yes
## Year Yes Yes Yes
## -----
## S.E. type by: County by: County by: County
## Observations 574 574 574
## R2 0.81420 0.81446 0.85886
## Within R2 0.00030 0.00168 0.24061
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
etable(crime_many_estimations[lhs='crrmte',sample=3],title='Crime Rate in Rural Areas',notes='Note: Est.
```

```
## crime_many_estim..1 crime_many_estim..2 crime_many_estim..3
## Sample (urban) 1 1 1
## Dependent Var.: Crime Rate Crime Rate Crime Rate
##
## Constant 0.1055*** (0.0078) 0.0990*** (0.0100) 0.0775*** (0.0138)
## Prob. Arrest -0.1995*** (0.0368) -0.2033*** (0.0369) -0.2014*** (0.0357)
## Avg. Sentence 0.0007 (0.0007) 0.0004 (0.0007)
## polpc 11.94* (5.472)
## Fixed-Effects: -----
## County No No No
## Year No No No
## -----
## S.E. type IID IID IID
## Observations 56 56 56
## R2 0.35269 0.36588 0.41909
## Within R2 -- -- --
##
## crime_many_est..4 crime_many_est..5 crime_many_est..6
## Sample (urban) 1 1 1
## Dependent Var.: Crime Rate Crime Rate Crime Rate
##
## Constant
## Prob. Arrest -0.1903. (0.0823) -0.1871. (0.0801) -0.1811* (0.0741)
## Avg. Sentence 0.0008 (0.0005) 0.0007 (0.0005)
## polpc 8.647 (6.132)
## Fixed-Effects: -----
## County Yes Yes Yes
## Year No No No
## -----
## S.E. type by: County by: County by: County
## Observations 56 56 56
## R2 0.88722 0.89616 0.90231
## Within R2 0.20282 0.26602 0.30946
##
## crime_many_estim..7 crime_many_estim..8 crime_many_estim..9
## Sample (urban) 1 1 1
## Dependent Var.: Crime Rate Crime Rate Crime Rate
##
## Constant
## Prob. Arrest -0.1979*** (0.0149) -0.2011*** (0.0175) -0.1982*** (0.0137)
## Avg. Sentence 0.0005 (0.0007) -4.93e-5 (0.0007)
```

```

## polpc 13.42** (3.493)
## Fixed-Effects: -----
## County No No No
## Year Yes Yes Yes
## -----
## S.E. type by: Year by: Year by: Year
## Observations 56 56 56
## R2 0.39994 0.40426 0.46292
## Within R2 0.36403 0.36861 0.43078
##
## crime_many_es..10 crime_many_es..11 crime_many_es..12
## Sample (urban) 1 1 1
## Dependent Var.: Crime Rate Crime Rate Crime Rate
##
## Constant
## Prob. Arrest -0.1694. (0.0771) -0.1723. (0.0733) -0.1709* (0.0703)
## Avg. Sentence 0.0002 (0.0005) 3.84e-5 (0.0006)
## polpc 11.27 (6.369)
## Fixed-Effects: -----
## County Yes Yes Yes
## Year Yes Yes Yes
## -----
## S.E. type by: County by: County by: County
## Observations 56 56 56
## R2 0.93501 0.93554 0.94120
## Within R2 0.23564 0.24188 0.30840
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Concept check In our second table, the probability of conviction regressed on probability of arrest is almost certainly not causal. It is a pretty bogus regression since both that are heavily affected by government decisions.

Can we say any of the above are causal? What would we need to assume?

Difference-in-differences

One of the most popular uses of fixed effects is to implement difference-in-difference designs we've discussed. Here's a quick visualization. Let's walk through an example that uses the Earned Income Tax Credit.

Earned Income Tax Credit

The Earned Income Tax Credit (EITC) was increased for parents in 1993. The EITC is a tax credit for low-income workers. It is a refundable tax credit, meaning that if the credit exceeds the amount of taxes owed, the excess is returned to the taxpayer. The EITC is designed to supplement wages for low-to-moderate income workers. The amount of the credit depends on income and number of children.

The EITC is also designed to incentivize work. It initially increases as earnings increase, before leveling off and falling once earnings reach a threshold level and the worker transitions out of "low-income."

Effectively at low-income levels, the EITC increases the dollars earned from working – either on the intensive margin (one more hour) or extensive margin (working vs. not working). But does it effect labor supply?

Let's focus on how this affects labor supply of single mothers who are the primary beneficiaries of . This example is borrowed from Nick Huntington-Klein and pulled from work by [Bruce Meyer \(2002\)](#).

We walked through this example in the lecture, but let's do it again.

Questions to keep in mind

1. How do you interpret each coefficient?
2. What assumptions provide a causal interpretation?
3. Are we able to test any of these assumptions?
4. If not, what can we test?

Diff-in-diff with data

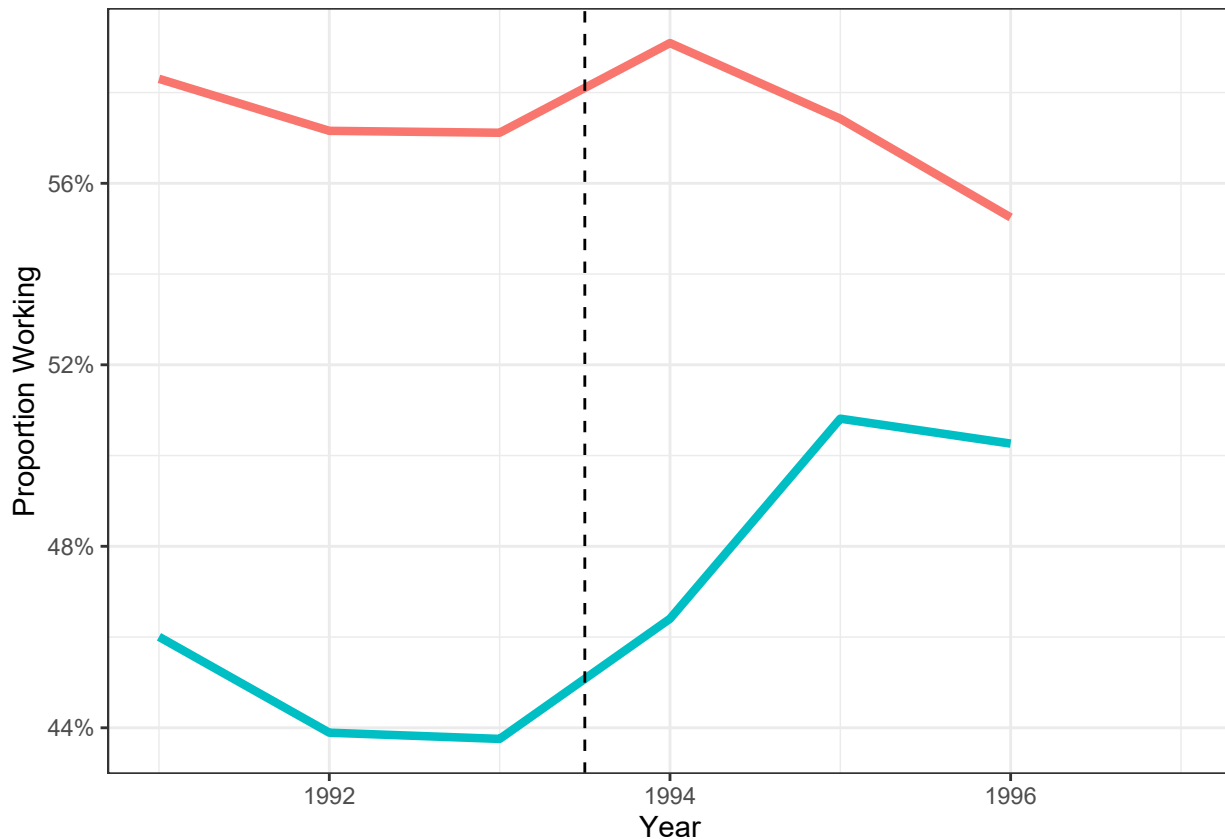
Let's load in the data.

```
df <- read.csv('http://nickchk.com/eitc.csv') %>%
  mutate(after = year >= 1994,
         treated = children > 0) %>%
  as_tibble()
```

We do not have an individual identifier in these data, so we can't add an individual fixed effect. We can add other fixed effects if we believe there is endogenous variation in the treatment between the groups of the fixed effect.

Still, let's work through how to visualize the data to check for no pre-trends and treatment effects change over time. We checked averages for our two groups before – not bad!

```
df %>%
  mutate(tlabel = factor(treated, labels = c('Untreated', 'Treated'))) %>%
  group_by(year, treated, tlabel) %>%
  summarize(proportion_working = mean(work)) %>%
  ggplot(aes(x = year, y = proportion_working, color = treated)) +
  geom_line(size = 1.5) +
  geom_vline(aes(xintercept = 1993.5), linetype = 'dashed') +
  theme_bw() +
  scale_x_continuous(limits = c(1991, 1997)) +
  scale_y_continuous(labels = function(x) scales::percent(x, accuracy = 1)) +
  guides(color = FALSE) +
  labs(x = "Year", y = "Proportion Working")
```



But the lines are a little far apart, so it makes it tricky to visualize the difference. And we don't know the confident interval on the difference between these. Let's try to get that!

Introducing the `i()` function. This handy little guy is a function that creates an interaction term. It's a little tricky to use, but it's worth it. Basically, what you do is you feed it a factor variable, an interacted variable, then a reference value of the factor variable – all coefficients will relative to the level when the factor variable equals the reference value.

```
eitc_did <- feols(work ~ i(year,treated,1993) | year + treated,
  data = df,
  se='hc1' # Heterogeneity robust standard errors
)
eitc_did
```

```
## OLS estimation, Dep. Var.: work
## Observations: 13,746
## Fixed-effects: year: 6, treated: 2
## Standard-errors: Heteroskedasticity-robust
##
##               Estimate Std. Error t value Pr(>|t|)
## year::1991:treated 0.010618    0.028518  0.372334 0.7096501
## year::1992:treated 0.000952    0.028960  0.032880 0.9737710
## year::1994:treated 0.006720    0.029484  0.227919 0.8197125
## year::1995:treated 0.067488    0.030154  2.238086 0.0252314 *
## year::1996:treated 0.083754    0.030552  2.741312 0.0061274 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 0.496477      Adj. R2: 0.012584
##                      Within R2: 0.001075
```

So what does this output mean? Well it tells us the difference between the treated and untreated groups over time! But

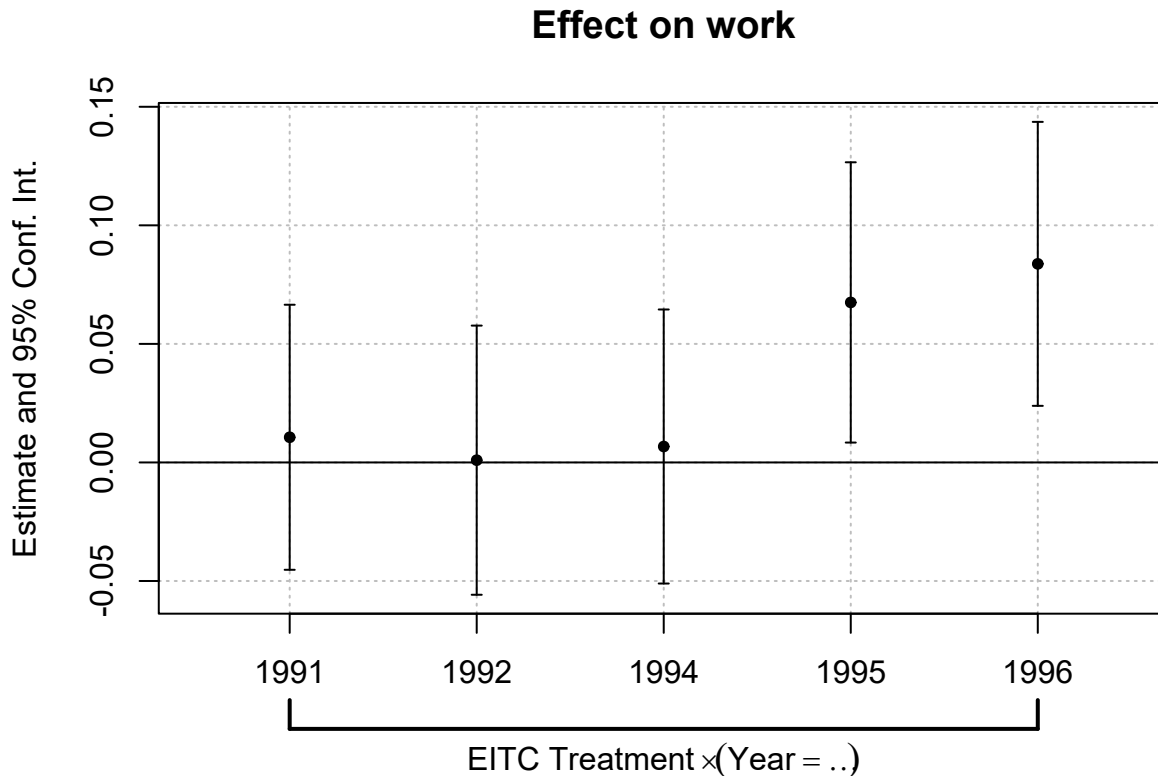
relative to when? It is all relative to the reference value, when year=1993. That is often called the “omitted” year. I chose the period just before the EITC expansion.

Challenge: What regression did we just run? Write it out. We have a year fixed effect and a treated fixed effect. Note the treated fixed effect is defined across individuals because we do not have an individual identifier!

But how do we visualize this? We have a few options. They both work the same way as the examples with `coefplot()` and `ggplot()` above though. Note, I introduce a dict to improve the labels.

The plots show that prior to 1994, the labor supply decisions of women with and without children were on a similar trend (though it is a fairly short trend).

```
coefplot(eitc_did, dict=c('treated'='EITC Treatment', 'year'='Year'))
```

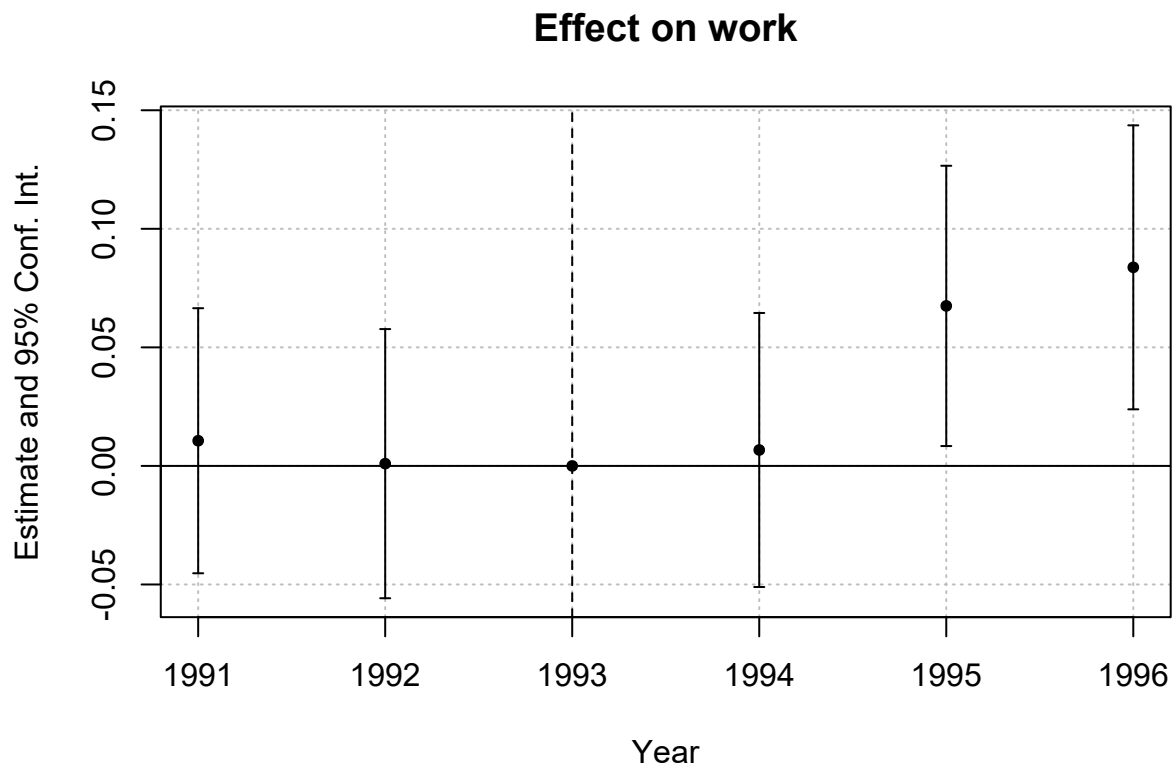


```
coef_eitc_did <- tidy(eitc_did, conf.int = TRUE) %>%  
  mutate(year=str_extract(term, '\\d{4}')) # Regular expressions to extract year  
  
ggplot(coef_eitc_did, aes(x=year, y=estimate, ymin=conf.low, ymax=conf.high)) +  
  geom_pointrange() +  
  geom_hline(yintercept = 0, col = "black") +  
  labs(  
    title = "Effect of EITC on labor supply",  
    caption = "Data: EITC dataset from Nick Huntington-Klein"  
  ) +  
  theme(axis.title.x = element_blank())
```



And then we also have `iplot()`, which works directly with `i()`. It works well for quick visualization, but it can be a little clunky to make as beautiful plots as you can with `ggplot`.

```
iplot(eitc_did, dict=c('treated'='EITC Treatment', 'year'='Year'))
```



Further resources

- [Ed Rubin](#) has outstanding [teaching notes](#) for econometrics with R on his website. This includes both [undergrad-](#) and [graduate-](#)level courses. Seriously, check them out.
- Several introductory texts are freely available, including [Introduction to Econometrics with R](#) (Christoph Hanck *et al.*), [Using R for Introductory Econometrics](#) (Florian Heiss), and [Modern Dive](#) (Chester Ismay and Albert Kim).
- [Tyler Ransom](#) has a nice [cheat sheet](#) for common regression tasks and specifications.
- [Itamar Caspi](#) has written a neat unofficial appendix to this lecture, [recipes for Dummies](#). The title might be a little inscrutable if you haven't heard of the `recipes` package before, but basically it handles “tidy” data preprocessing, which is an especially important topic for machine learning methods. We'll get to that later in course, but check out Itamar's post for a good introduction.
- I promised to provide some links to time series analysis. The good news is that R's support for time series is very, very good. The [Time Series Analysis](#) task view on CRAN offers an excellent overview of available packages and their functionality.
- Lastly, for more on visualizing regression output, I highly encourage you to look over Chapter 6 of Kieran Healy's [Data Visualization: A Practical Guide](#). Not only will learn how to produce beautiful and effective model visualizations, but you'll also pick up a variety of technical tips.