

# ML Assignment 3

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3.1 BC 3.2 D 3.3 B

3.4. (a). Sol.  $\because x(t)$  is misclassified, so  $w^T(t)x(t) \neq y(t)$

When  $y(t) = \text{sgn}(w^T(t)x(t)) = 1, \Rightarrow w^T(t)x(t) = -1 < 0$

When  $y(t) = \text{sgn}(w^T(t)x(t)) = -1, \Rightarrow w^T(t)x(t) = 1 > 0$

$\therefore y(t)w^T(t)x(t) < 0$  holds.

(b). Sol.  $\because w(t+1) = w(t) + y(t)x(t)$

$$\therefore w^T(t+1) = w^T(t) + y(t)x^T(t)$$

$$\begin{aligned} \therefore y(t)w^T(t+1)x(t) &= y(t)w^T(t)x(t) + y(t)y(t)x^T(t)x(t) \\ &= y(t)w^T(t)x(t) + y^2(t)|x(t)|^2 \end{aligned}$$

$\therefore y^2(t)|x(t)|^2 > 0$  holds

$$\therefore y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$$

(c). Sol. Since for every move from  $t$  to  $t+1$ ,  $y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$  holds,

And initially  $y(t)w^T(t)x(t) < 0$  (misclassified).

This move will keep increase  $y(t)w^T(t)x(t)$  until it passes 0, in which case it's no longer misclassified.

3.5. 解:  $\arg \min_{w_1, w_2, b} \frac{1}{N} \sum_{i=1}^N |f(x_i) - y_i|^2$

$$= \frac{1}{N} \sum_{i=1}^N (w_1 x_{1i} + w_2 x_{2i} + b - y_i)^2$$

$$\therefore E(w_1, w_2, b) = \frac{1}{N} \sum_{i=1}^N (w_1 x_{1i} + w_2 x_{2i} + b - y_i)^2$$

$E(w_1, w_2, b)$  为凸函数,  $\therefore$  在取得全局最优解时, 由 KKT 条件:

$$\nabla E(w_1, w_2, b) = 0$$

$$\text{求} \cdot \left\{ \begin{array}{l} \frac{\partial E}{\partial w_1} = \frac{1}{N} \sum_{i=1}^N 2x_{1i}(w_1x_{1i} + w_2x_{2i} + b - y_i) = \frac{2}{N} \left[ \sum_{i=1}^N (w_1x_{1i} + b - y_i)x_{1i} + w_1 \sum_{i=1}^N x_{1i}^2 \right] \quad (1) \\ \frac{\partial E}{\partial w_2} = \frac{2}{N} \left[ \sum_{i=1}^N (w_1x_{1i} + b - y_i)x_{2i} + w_2 \sum_{i=1}^N x_{2i}^2 \right] \quad (2) \\ \frac{\partial E}{\partial b} = \frac{2}{N} \sum_{i=1}^N (w_1x_{1i} + w_2x_{2i} + b - y_i) = \frac{2}{N} \left[ Nb + \sum_{i=1}^N (w_1x_{1i} + w_2x_{2i} - y_i) \right] \quad (3) \end{array} \right.$$

令①、②、③式为零解得：

$$w_1 = \frac{\sum_{i=1}^N y_i (x_{1i} - \bar{x}_1)}{\sum_{i=1}^N x_{1i}^2 - \frac{1}{N} (\sum_{i=1}^N x_{1i})^2 + \sum_{i=1}^N x_{2i}^2 - \frac{1}{N} (\sum_{i=1}^N x_{2i})^2}$$

$$b = \frac{1}{N} \sum_{i=1}^N (y_i - w_1 x_{1i} - w_2 x_{2i}), \text{ 其中 } \bar{x}_1 = \frac{1}{N} \sum_{i=1}^N x_{1i}.$$

3.6 预测： $f(x) = \text{sig}\left(\sum_{i=1}^d w_i x_i + b\right)$ , 这里为二分类模型

$$\therefore f(x) = \text{sig}(w_1 x_1 + w_2 x_2 + b)$$

$$t=0: \text{假设 } w_1 = w_2 = 0.5, b = 0.5, \text{ 则 } w(0) = (0.5, 0.5, 0.5)$$

$$f(x) = \text{sig}(0.5x_1 + 0.5x_2 + 0.5) \text{ 检查:}$$

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2, 4)	3.5	1	1	✓
(3, 3)	3.5	1	1	✓
(0, 1)	1	1	-1	✗

$$\therefore w(1) = w(0) + \eta \sum_{i \in M} y_i x_i = w(0) + 0.5 (0, -1, -1) = (0.5, 0, 0)$$

$$t=1: f(x) = \text{sig}(0.5x_1 + 0.5x_2 + 0), \text{ 检查:}$$

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2, 4)	1	1	1	✓
(3, 3)	1.5	1	1	✓
(0, 1)	0	1	-1	✗

$$\therefore w(1) = w(0) + \eta \sum_{i \in M} y_i x_i = w(0) + 0.5 (0, -1, -1) = (0.5, -0.5, -0.5)$$

$$t=2 : f_1(x) = \text{sig}(0.5x_1 - 0.5x_2 - 0.5), \text{ XORf};$$

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2,4)	-1.5	-1	1	X
(3,3)	-0.5	-1	1	X
(0,1)	-1	-1	-1	✓

$$\therefore w(1) = w(0) + \eta \sum_{i \in M} y_i x_i = w(0) + 0.5((2,4,1) + (3,3,1)) = (3, 3, 0.5)$$

$$t=3 : f_1(x) = \text{sig}(3x_1 + 3x_2 + 0.5), \text{ XORf};$$

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2,4)	18.5	1	1	✓
(3,3)	18.5	1	1	✓
(0,1)	3.5	1	-1	X

$$\therefore w(1) = w(0) + \eta \sum_{i \in M} y_i x_i = w(0) + 0.5(0, -1, -1) = (3, 2.5, 0)$$

$$t=4 : f_1(x) = \text{sig}(3x_1 + 2.5x_2 + 0), \text{ XORf};$$

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2,4)	16	1	1	✓
(3,3)	16.5	1	1	✓
(0,1)	2.5	1	-1	X

$$\therefore w(4) = w(3) + \eta \sum_{i \in M} y_i x_i = w(3) + 0.5(0, -1, -1) = (3, 2, -0.5)$$

$$t=5 : f_1(x) = \text{sig}(3x_1 + 2x_2 - 0.5), \text{ XORf};$$

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2,4)	13.5	1	1	✓
(3,3)	14.5	1	1	✓
(0,1)	1.5	1	-1	X

$$w(5) = w(4) + \eta \sum_{i \in M} y_i x_i = w(4) + 0.5(0, -1, -1) = (3, 1.5, -1)$$

$t = b$ :  $f_1(x) = \text{sgn}(3x_1 + 1.5x_2 - 1)$ . 正确;

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2, 4)	11	1	1	✓
(3, 3)	12.5	1	1	✓
(0, 1)	0.5	1	-1	✗

$$w(b) = w(s) + \eta \sum_{i \in M} y_i x_i = w(s) + 0.5(0, -1, -1) = (3, 1, -1.5)$$

$t = 7$ :  $f_1(x) = \text{sgn}(3x_1 + x_2 - 1.5)$ . 正确;

$x$	$\hat{y}$	$\text{sgn}(\hat{y})$	$y$	Correct
(2, 4)	8.5	1	1	✓
(3, 3)	10.5	1	1	✓
(0, 1)	-0.5	-1	-1	✓

发现  $t = 7$  后，所有数据分类正确，此时  $w = (3, 1)$ ,  $b = -1.5$

∴ 感知机模型为  $f(x) = \text{sgn}(3x_1 + x_2 - 1.5)$ .